Lecture 4
Hadronic heavy-quark decays

Hsiang-nan Li

Oct. 22, 2012

Outlines

- Naïve factorization
- QCD-improved factorization
- Perturbative QCD approach
- Strong phases and CP asymmetries
- Puzzles in B decays

Introduction

- Why B physics?

  Constrain standard-model parameters

    CKM matrix elements, weak phases

  Explore heavy quark dynamics

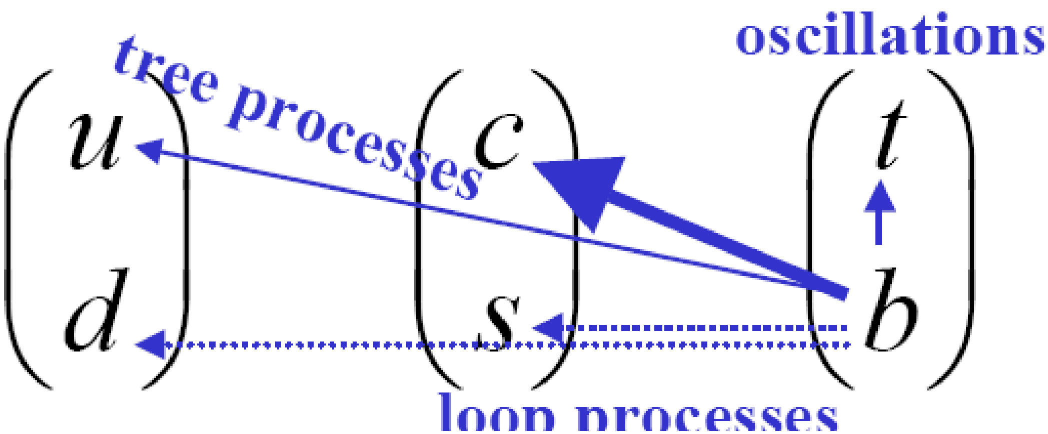
    Form factors, penguins, strong phases

  Search for new physics

    SUSY, 4th generation, Z'…

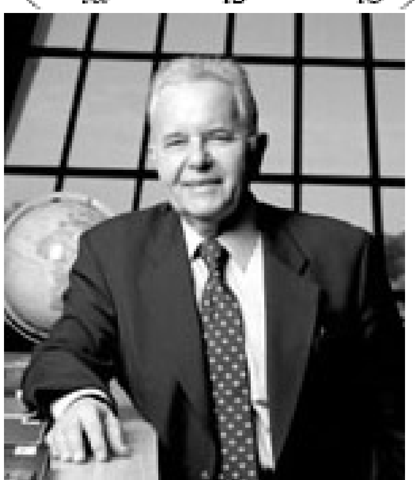
- Need B factories and critical comparison between data and QCD theories.

Cabbibo-Kobayashi-Maskawa Matrix



(Wolfenstein parametrization)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$



$$; \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ -0.23 & 0.97 & 0.04 \\ 0.004 & -0.04 & 1 \end{pmatrix}$$

Weak phase, $\cancel{CP}$

(magnitudes only)

CP violation

- Thumb only on the right---P violation
- Thumbs on the right of right hand, and on the left of left hand---CP conservation
- God is fair: He gives L to particle and R to antiparticle.
- CP conserved here? ⟶
- If she loses one arm, ~~CP~~ at 10^{-3}

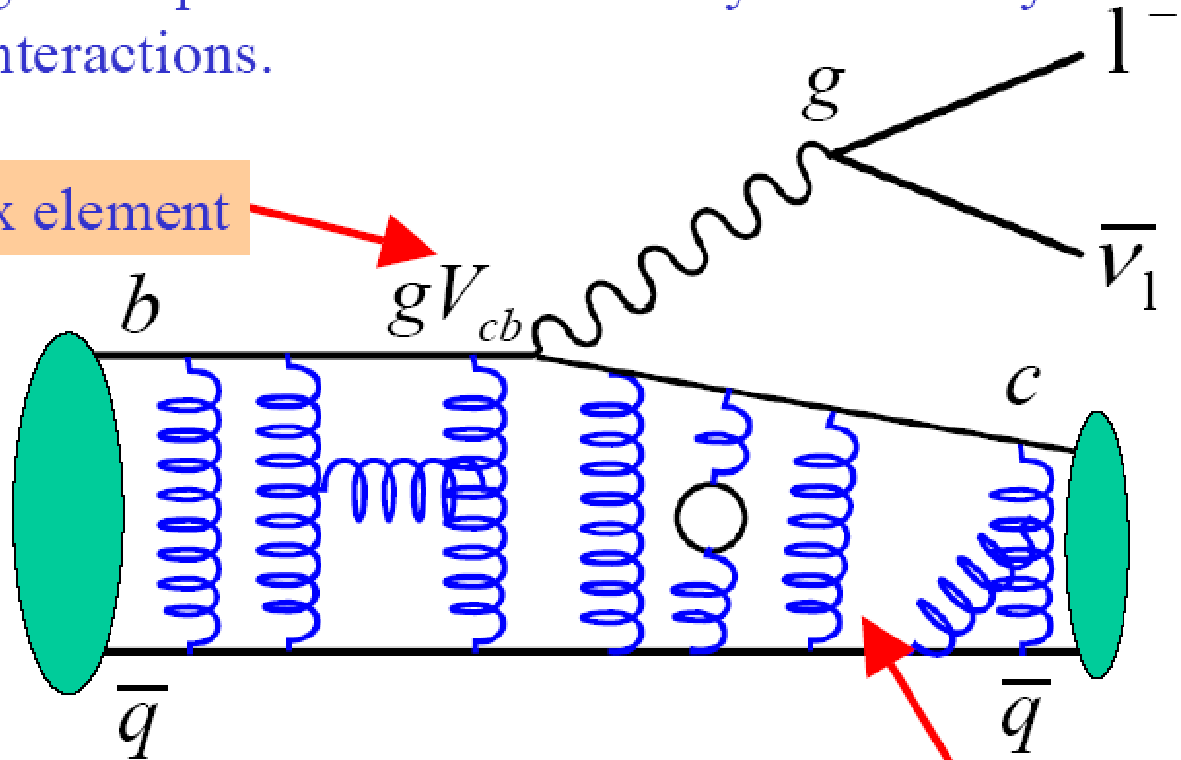
Anti- particle particle

Thousand-hand Guan Yin

B transition form factors

Underlying weak process is substantially affected by an overlay of strong interactions.

CKM matrix element

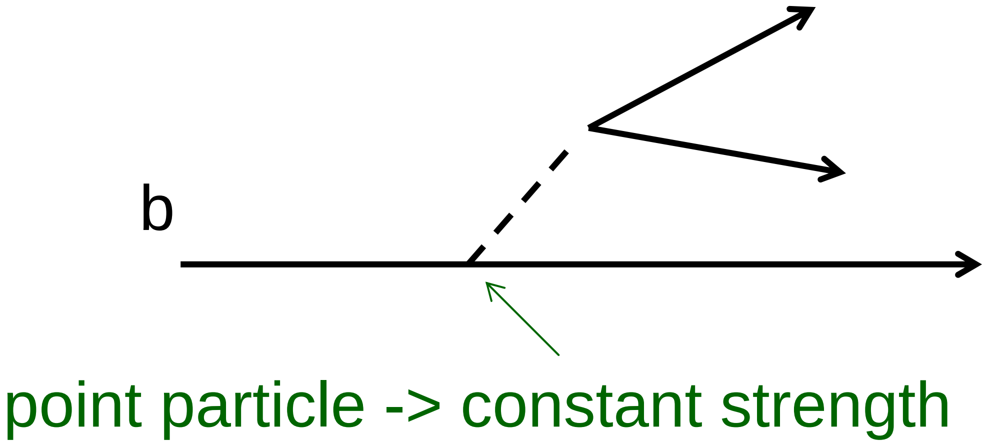


Initial system is bound state: *b*-quark is not at rest in B frame.

Exchange of gluons is between daughter quark and spectator quark to form the final state meson.

Form factors

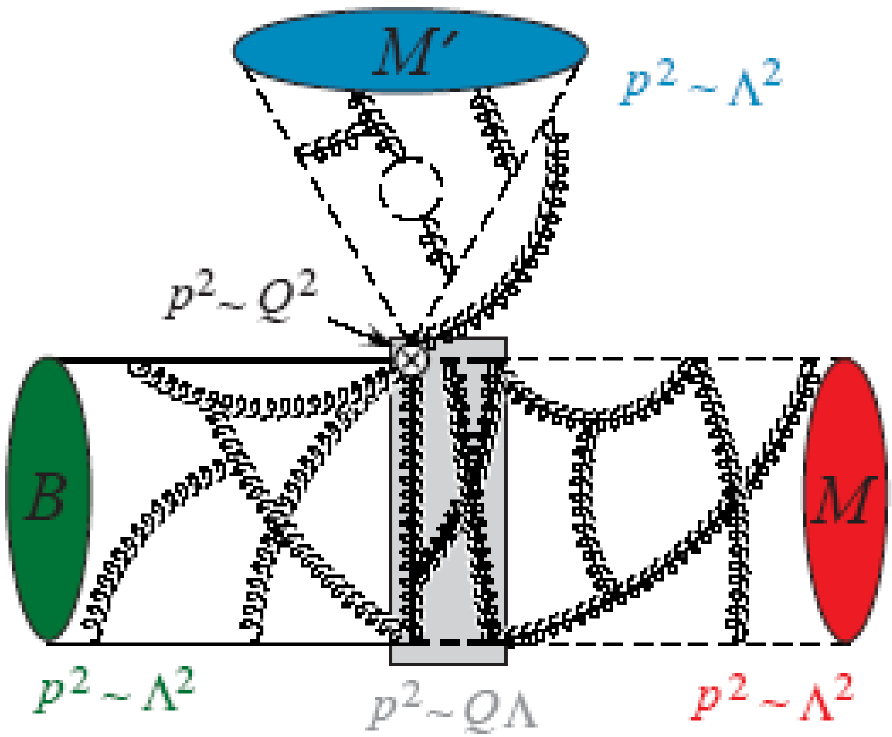
- Measurement of structure of a bound state



- If bound state, effective strength dependent on momentum transfer
- It is like structure functions in DIS
- Are form factors calculable?

Nonleptonic decays

- Much more complicated
- Involve scales m_W, $m_b \sim Q$, and Λ
- Are they calculable?



Naïve factorization

Effective Hamiltonian

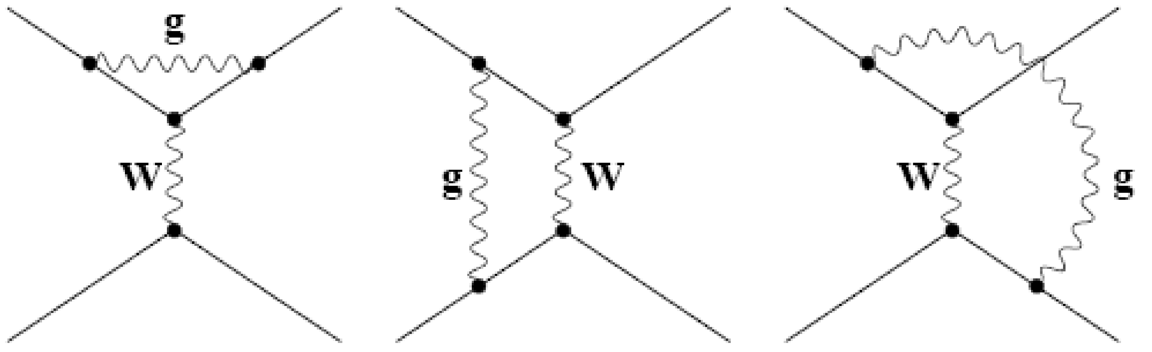
- First step, derive effective weak Hamiltonian by integrating out m_W
- Dynamics with scale m_W is organized into Wilson coefficient C_i
- The rest dynamics lower than m_W goes into 4-fermion operators O_i
- $H=V_{CKM}\Sigma_i C_i(\mu)O_i(\mu)$ Buras,Buchalla, 1995

$$O_1 = (\bar{d}b)_{V-A}(\bar{c}u)_{V-A} \qquad O_2 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$

- Their μ (factorization scale) dependence cancels.

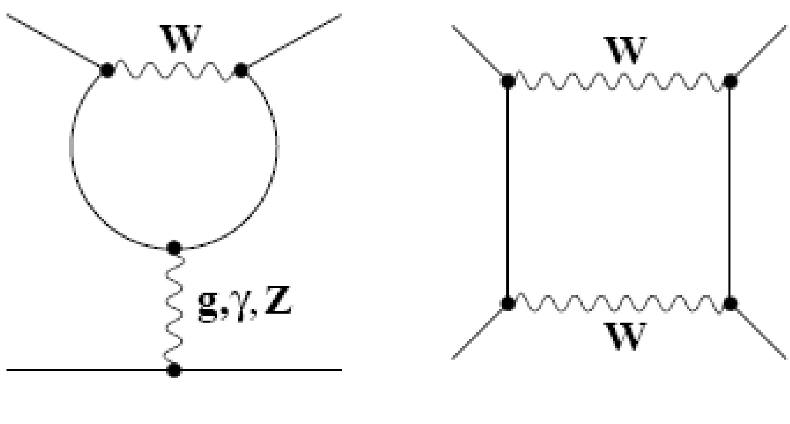
Diagrams at scale m_W

b



(a) (b) (c)

penguin



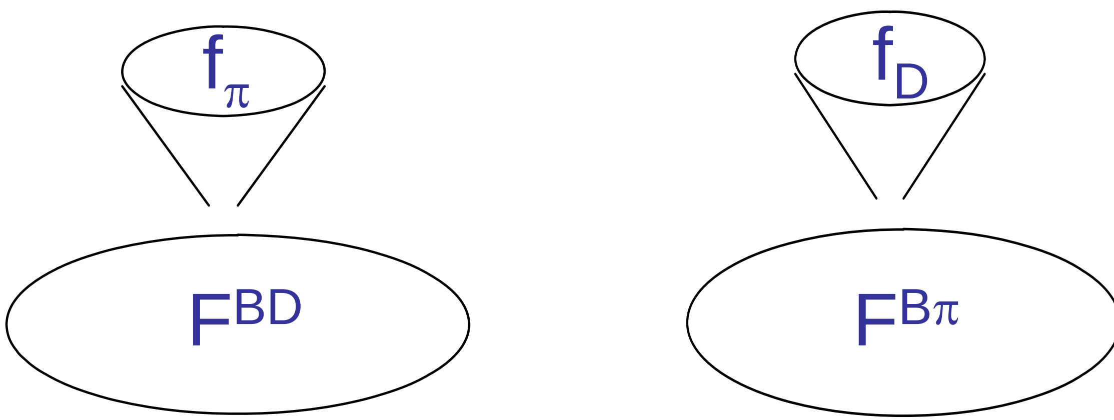
(d) (e)

box
also contributing
to B-Bbar mixing

Factorization assumption

$$A = \langle D\pi|H_{eff}|B\rangle \sim C(\mu)\,\langle D\pi|O(\mu)|B\rangle$$

FA was proposed to deal with the hadronic matrix element (Bauer, Stech, Wirbel 85).



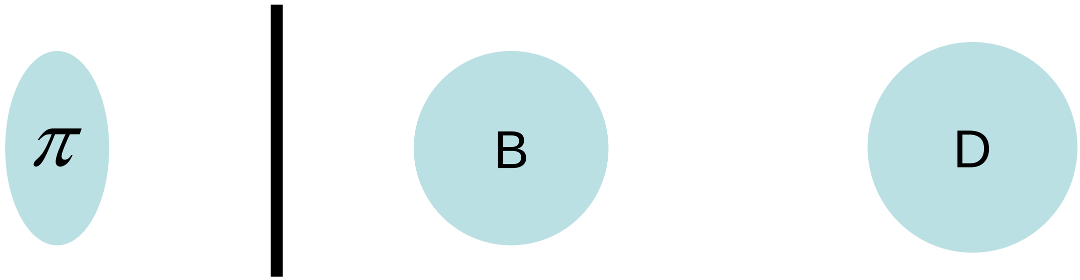
form factor
not calculable

Color-allowed | Color-suppressed

$$A(B \to D\pi) \propto a_1 f_\pi F^{BD} + a_2 f_D F^{B\pi}$$

$$a_1 = C_2 + C_1/N_C, \quad a_2 = C_1 + C_2/N_C \qquad C_1 \sim -0.3,\ C_2 \sim 1$$

Color transparency



Lorentz contraction
small color dipole

decoupling in space-time
from the BD system

Nonfactorizable corrections negligible for
color-allowed amplitudes.

Expect large corrections in color-suppressed
modes due to heavy D, large color dipole.
No strong phase.
No systematic improvement of precision

Incompleteness of FA

- FA cannot be complete: form factor and decay constant are physical, independent of μ. Predictions depend on μ via $C(\mu)$.
- Nonfactorizable contributions must exist, especially in color-suppressed modes. They may be small in color-allowed decays, which are insensitive to μ.
- Power corrections, like strong phases, are crucial for CP violation.
- FA was used for decades due to slow experimental and theoretical progress.

Go beyond FA

- Great experimental (Babar, Belle) and theoretical (factorization approaches) progress around 2000.
- Rare (including color-suppressed) decays with BR down to 10^{-6} and CP asymmetries can be measured precisely at B factories.
- Explore nonfactorizable and power corrections, or even new physics.
- Theorists need to go beyond FA.

Theoretical approaches

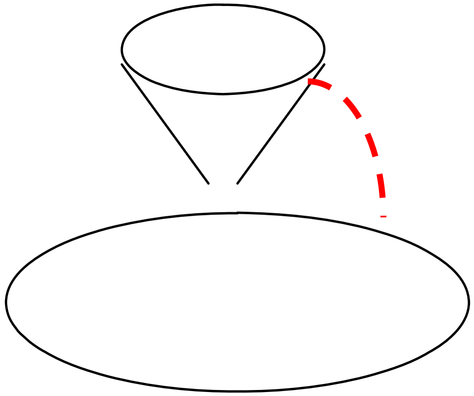
- Hadronic B decays involve abundant QCD dynamics of heavy quarks.
- Complexity of B decays dragged theoretical progress till year 2000, when we could really go beyond naïve factorization.
- Different approaches developed: PQCD, QCDF, SCET.
- Predictive power and data discriminate different approaches.

QCD-improved factorization

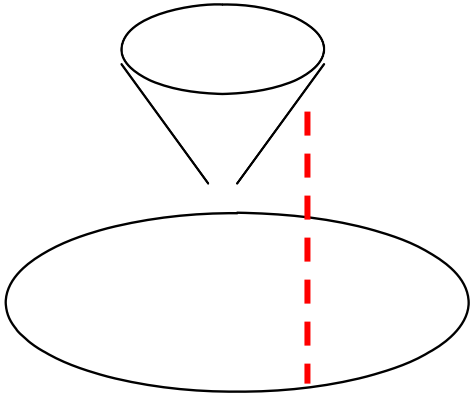
Beneke, Buchalla, Neubert,
Sachrajda, 1999

QCD corrections to FA

- Add gluons to FA

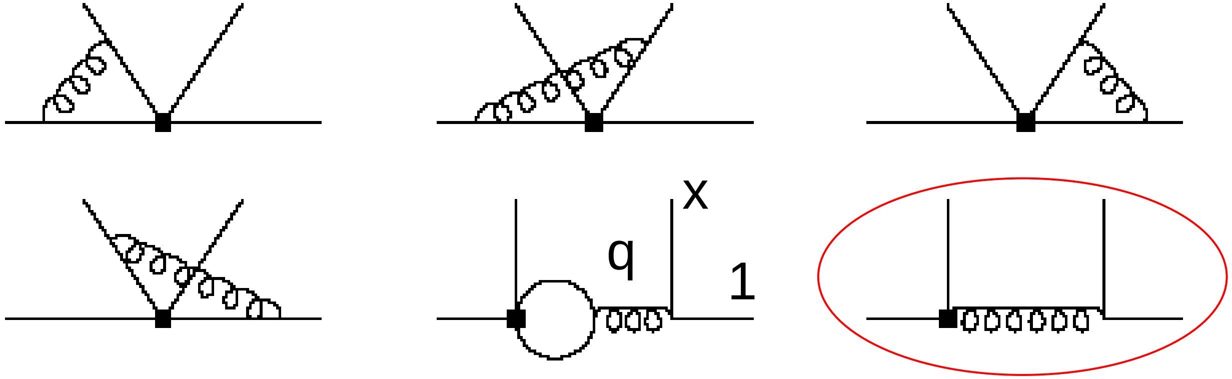


- Nonfactorizable corrections appear at this order



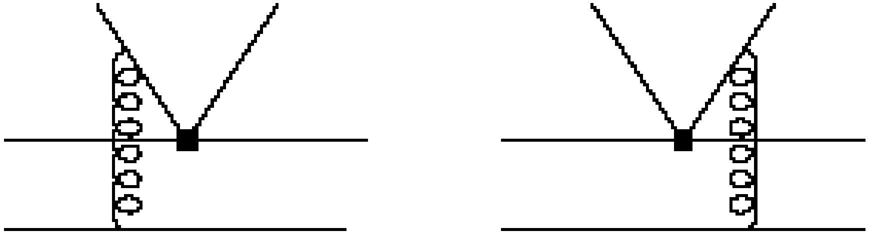
Hard kernels

- T^I comes from vertex corrections



Magnetic penguin O_{8g}

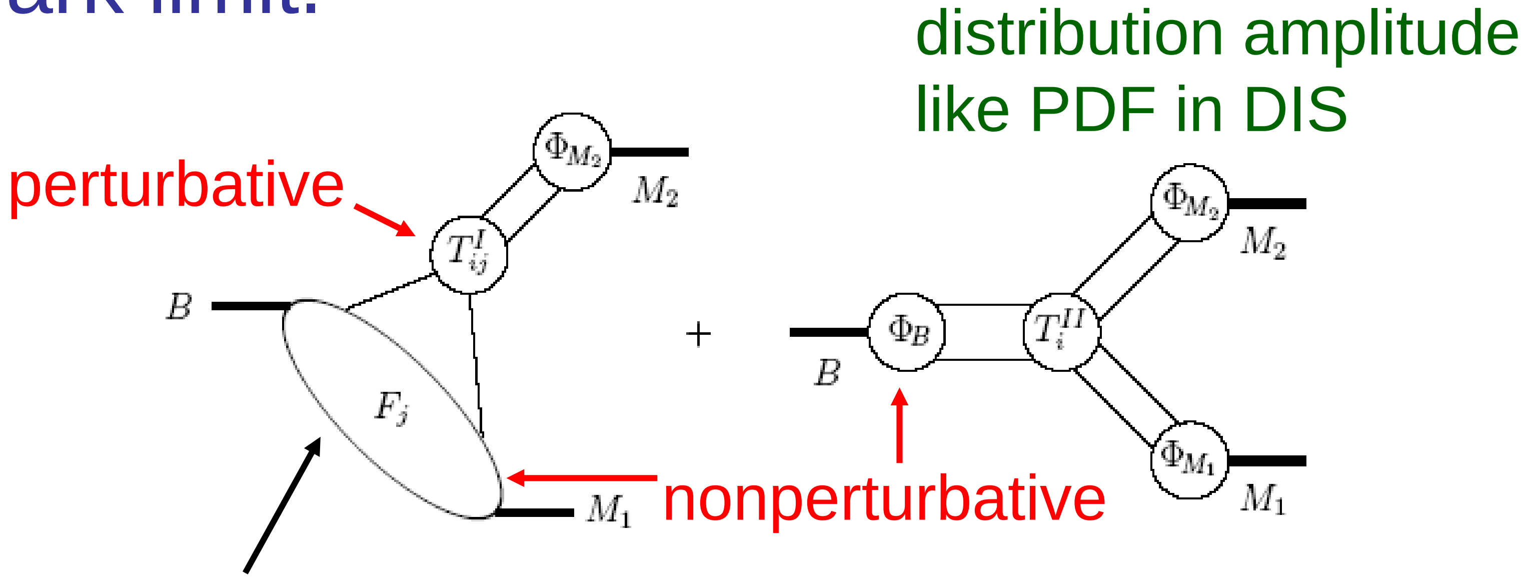
- T^{II} comes from spectator diagrams



soft divergences cancel in pairs

QCDF

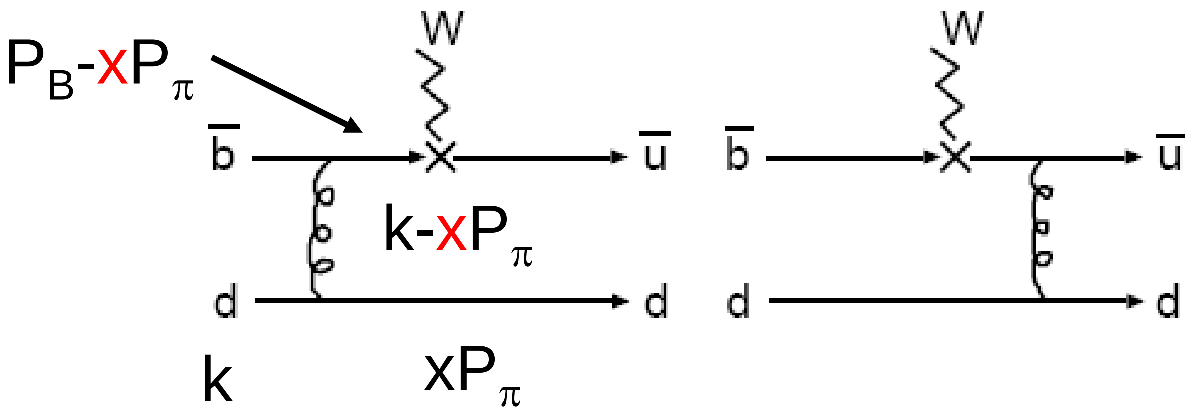
- Based on collinear factorization
- Compute corrections to FA, ie., the heavy-quark limit.



- Due to end-point singularity in collinear factorization

B -> π form factors

- x runs from 0 to 1. The end-point region is unavoidable.
- Collinear factorization gives end-point singularity. Form factors not calculable



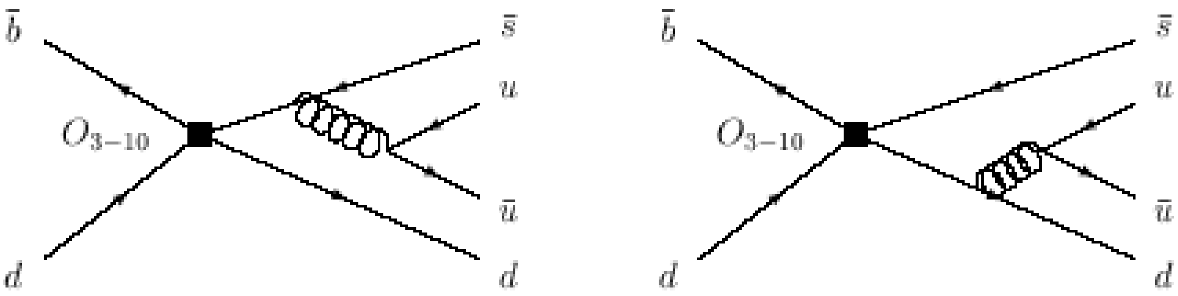
$$\int_0^1 dx \frac{\phi_\pi(x)}{x^2} \to \infty, \qquad \phi_\pi(x) \propto x(1-x)$$

Breakthrough and challenge

- It is amazing that higher-order corrections to FA are calculable!
- Study of two-body hadrnic B decays can be improved systematically
- But, when going to higher powers, challenge appeared….
- End-point singularity avoided by absorbing it into form factors, but reappeared at higher powers

End-point singularity

- Singularity appears at $O(1/m_b)$, twist-3 spectator and annihilation amplitudes, parameterized as $X=(1+\rho\, e^{i\phi})\ln(m_b/\Lambda)$



- For QCDF to be predictive, $O(1/m_b)$ corrections are better to be small ~ FA.
- Data show important $O(1/m_b)$. Different free (ρ,ϕ) must be chosen for B-> PP, PV, VP.

Perturbative QCD approach

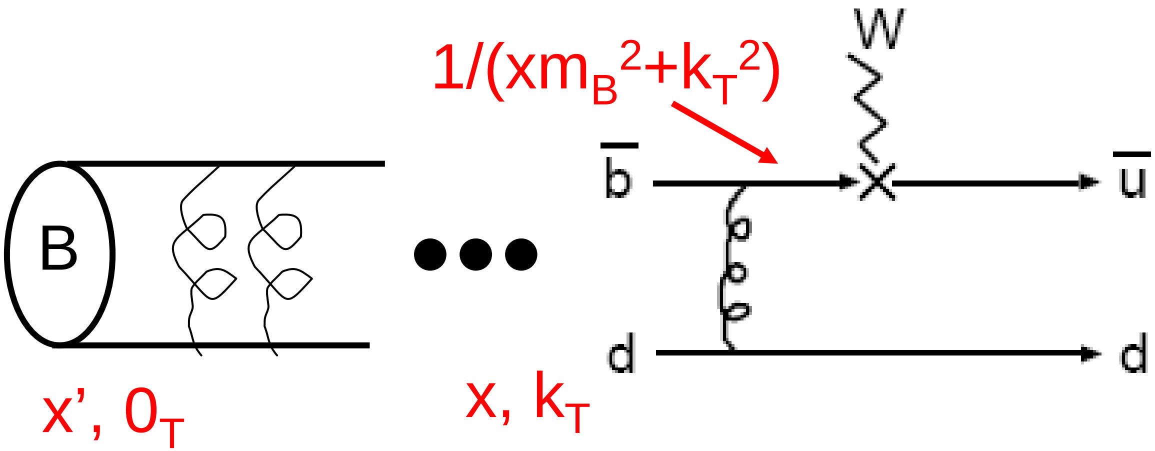
Keum, Li, Sanda, 2000

kT factorization

- If no end-point singularity, collinear factorization works. If yes, small x region is important, and k_T factorization is more appropriate.
- k_T factorization for exclusive processes is basically the same as for inclusive process (small x physics).
- Based on collinear and k_T factorizations, QCDF and PQCD have been developed for exclusive B decays.

Smearing of end-point singularity

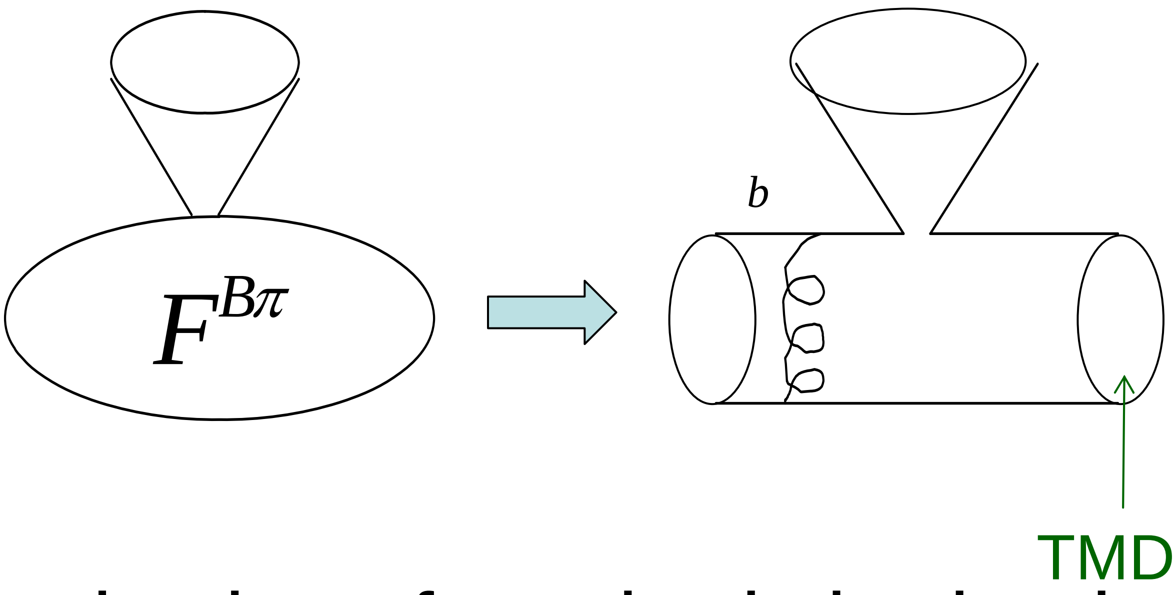
- Including parton k_T accumulated through gluon emissions



$$\int_0^1 dx \frac{1}{x + k_T^2 / m_B^2}$$ is finite

Factorization of form factor

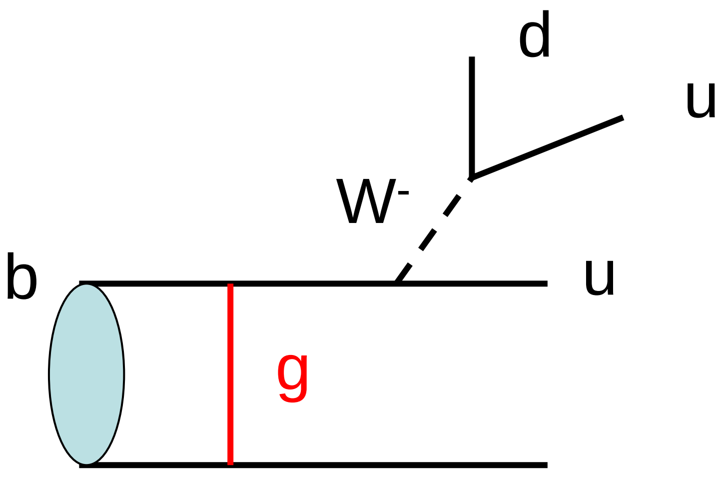
- Form factors are calculable, if TMD is known



- Factorization of two-body hadronic B decays changed!

Three scales

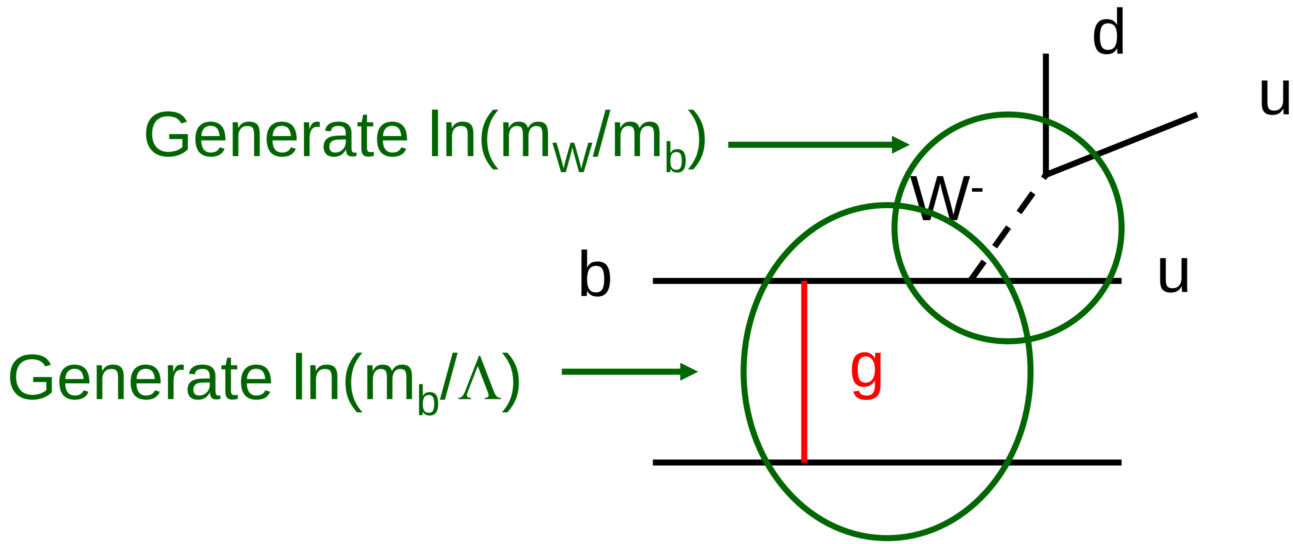
- Start with lowest order diagram for B -> Dπ
- This amplitude involves three dramatically different scales: m_W, m_b, and Λ



- Radiative corrections generate large logs $\ln(m_W/m_b)$, $\ln(m_b/\Lambda)$, which must be summed to all orders to improve perturbation.

Two clusters of corrections

- Two clusters

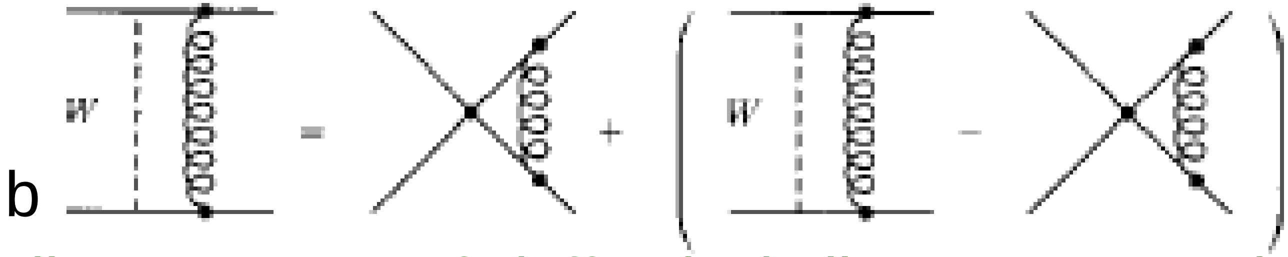


- Sum ln(m_W/m_b) first (integrating out W boson), giving effective weak Hamiltonian, and then sum ln(m_b/Λ), giving Sudakov and RG evolutions.
- PQCD approach incorporates factorizations of effective weak Hamiltonian and IR divergences.

Effective Hamiltonian

- also a kind of factorization theorem.

μ is renormalization scale



Full diagram

μ–independent

Soft (effective) diagram
4-fermion operator O_2

μ–dependent

Hard part
Wilson coefficient

μ–dependent

- Radiative corrections give new operators (operator mixing) O_1 -> O_1,O_2
- Effective Hamiltonian

Different color flows

$$H=\frac{G_F}{\sqrt{2}}V_{ij}V^*_{kl}(\overline{q}_l q_k)(\overline{q}_j q_i) \quad \text{->} \quad H_{\text{eff}}=\frac{G_F}{\sqrt{2}}V_{ij}V^*_{kl}[c_1(\mu)O_1+c_2(\mu)O_2]$$

CKM matrix elements

QCD and EM penguins

- Radiative corrections also introduce QCD and electroweak penguins

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qs}^* V_{qb} \left[C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right]$$

$$O_1^{(q)} = (\bar{s}_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A} \ , \qquad O_2^{(q)} = (\bar{s}_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A} \ ,$$

$$O_3 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} \ , \quad O_4 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \ ,$$

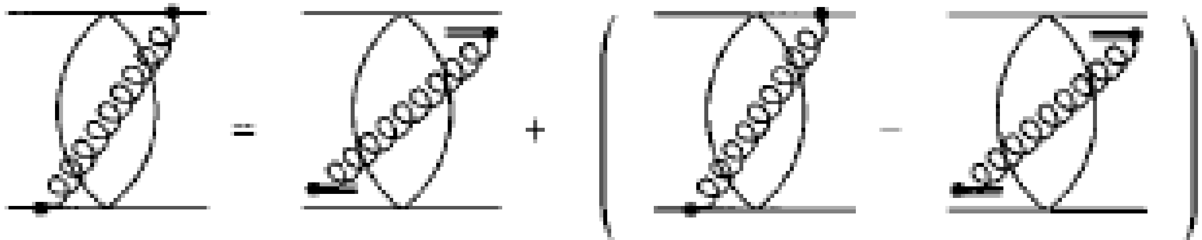
$$O_5 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A} \ , \quad O_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \ ,$$

$$O_7 = \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A} \ , \quad O_8 = \frac{3}{2} (\bar{s}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A} \ ,$$

$$O_9 = \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A} \ , \quad O_{10} = \frac{3}{2} (\bar{s}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A} \ ,$$

Factorization of IR divergence

- Use soft divergence for demonstration.



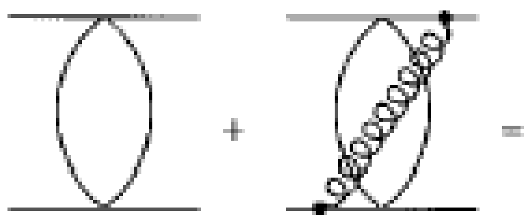
full diagram
current conservation
μ_f-independent

soft diagram
eikonalized
μ_f-dependent

hard part
μ_f-dependent

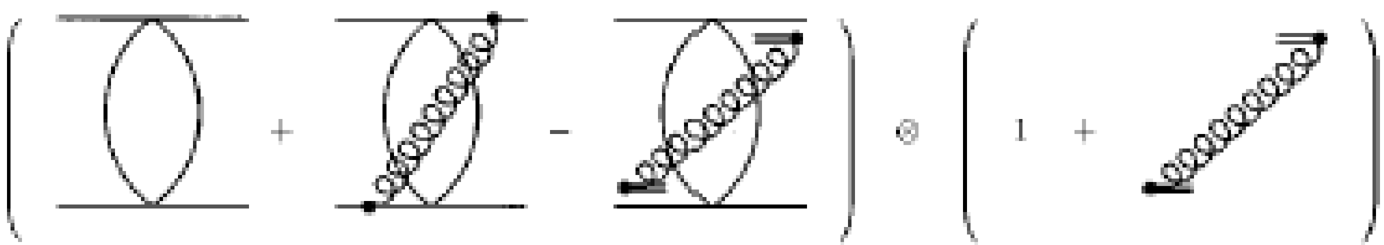
μ_f is factorization scale

- Factorization formula

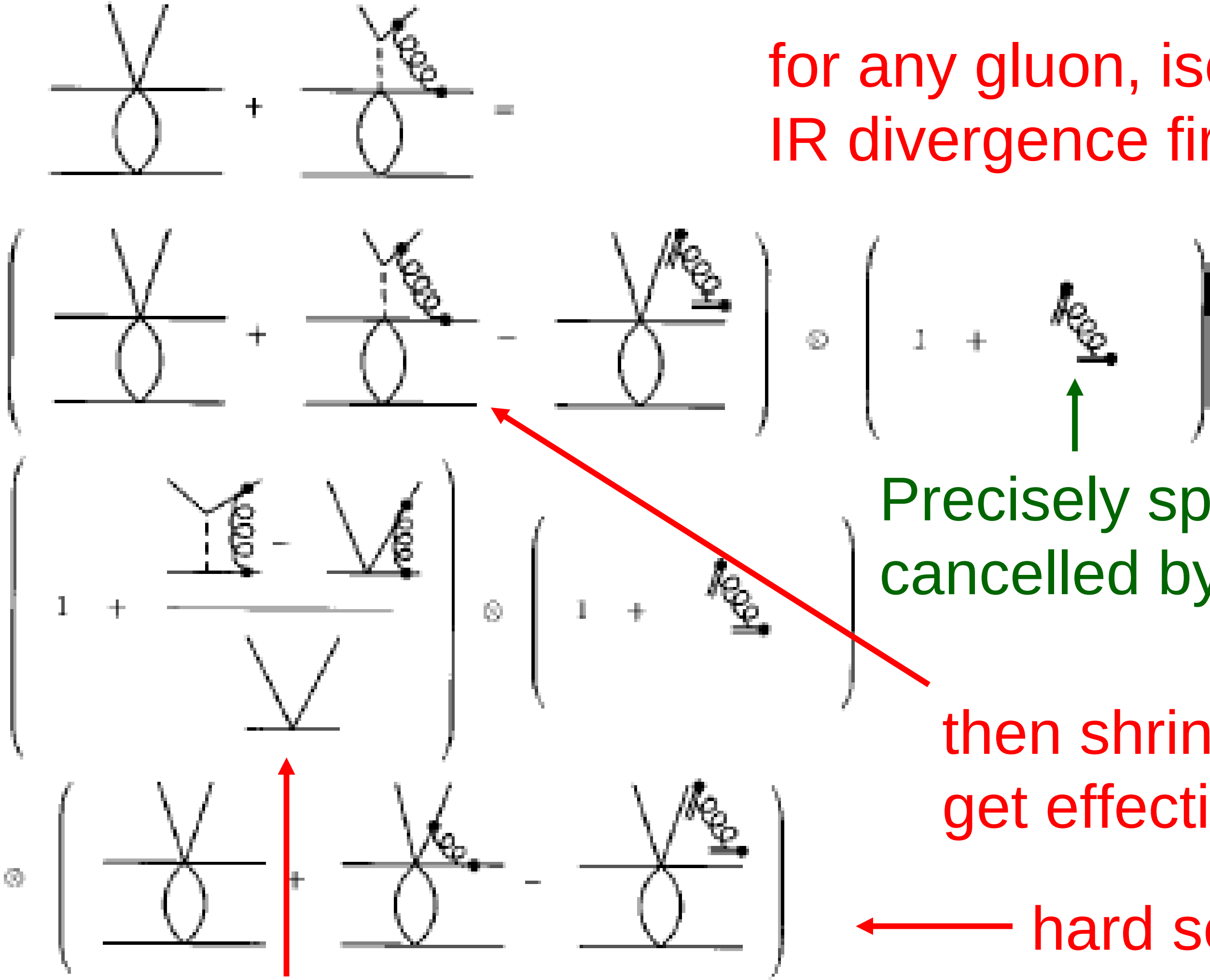


TMD wave function

hard
kernel



Three-scale factorization



for any gluon, isolate
IR divergence first

Precisely speaking, it is
cancelled by another gluon

then shrink W line to
get effective Hamiltonian

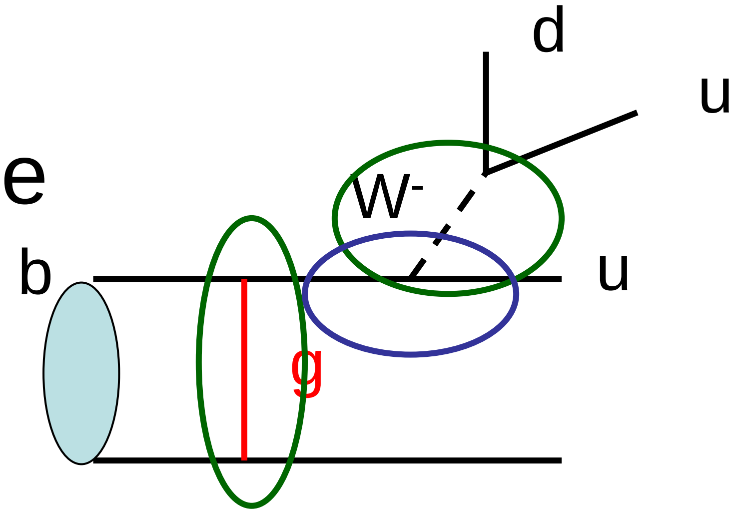
hard scale $t^2 \sim \Lambda m_b$

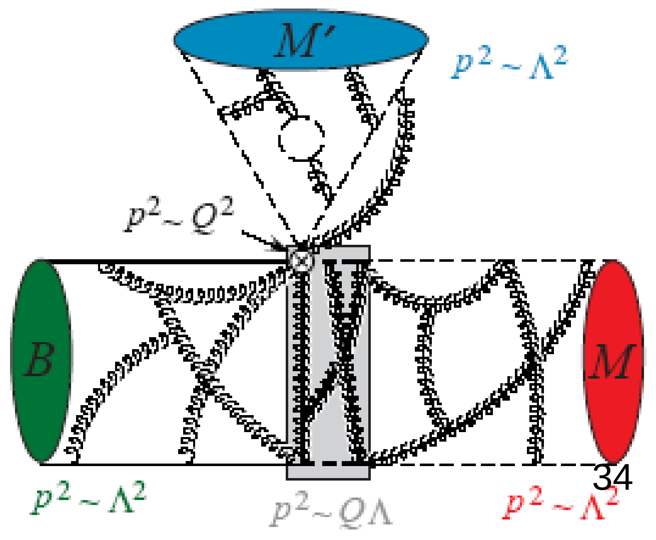
- $A = C(m_W,\mu) * H(t,\mu,\mu_f) * \phi(\mu_f,1/b)$

can choose $\mu=\mu_f$

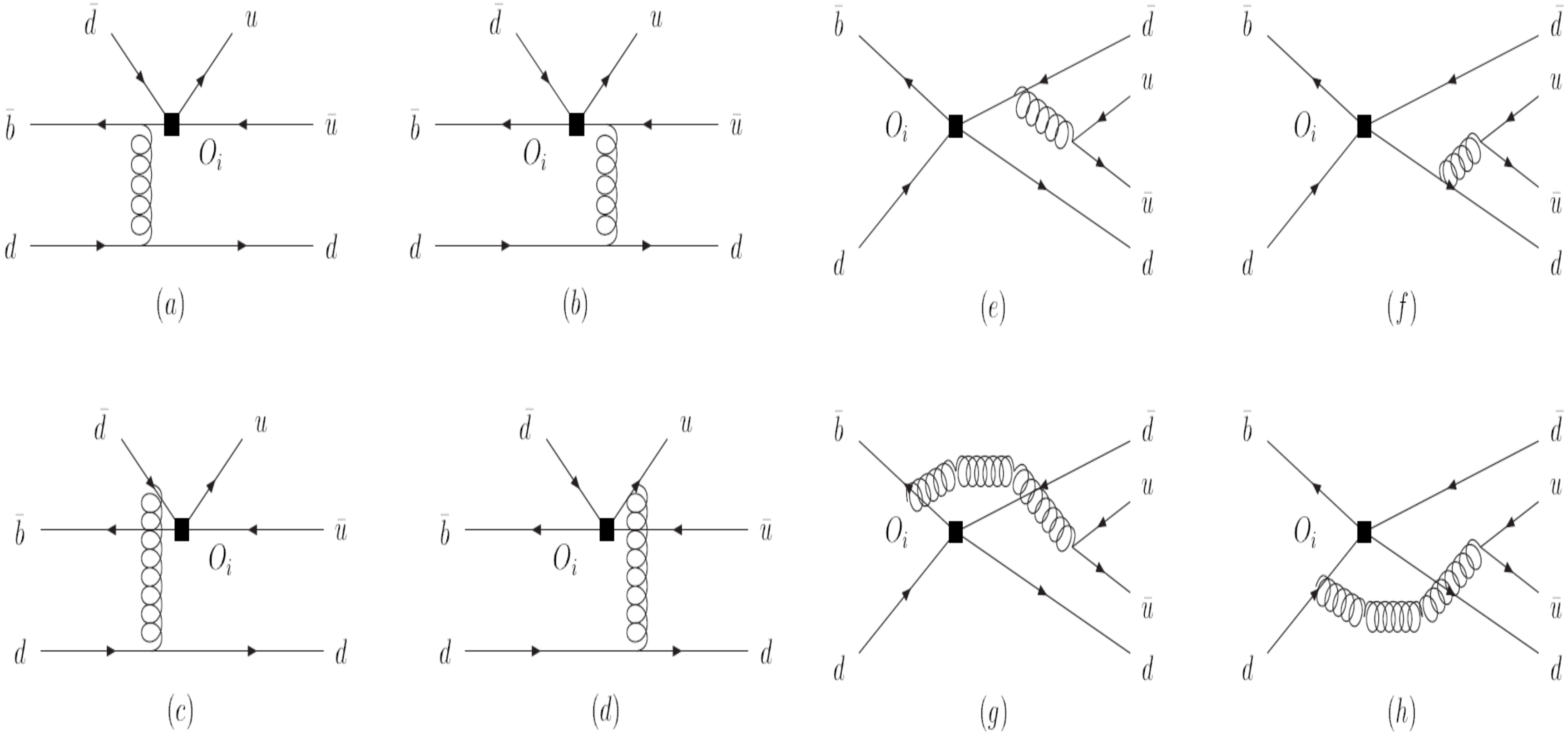
Four scales actually

- Strictly speaking, there are 4 scales: m_W, m_b, $t \sim (\Lambda m_b)$^(1/2), Λ.
- There are 3 clusters. The cluster of hard kernel splits into one for weak b ->u vertex, and another for hard gluon.
- The above are scales in Soft-Collinear Effective Theory





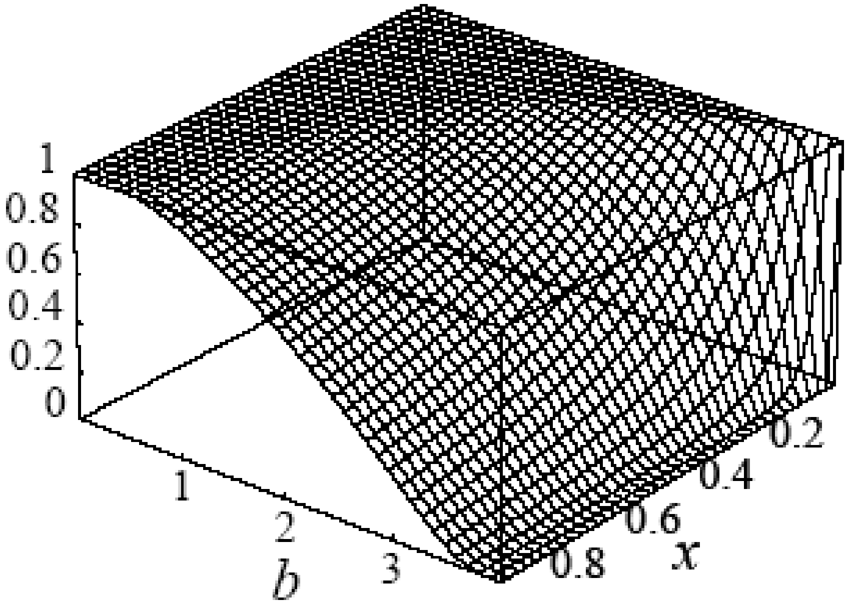
Feynman diagrams



emission diagrams

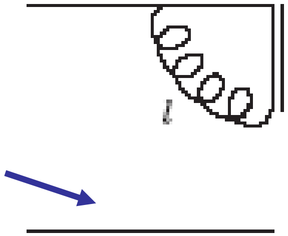
annihilation diagrams

Sudakov effect increases kT



suppression at large b becomes stronger at larger energy

Small x



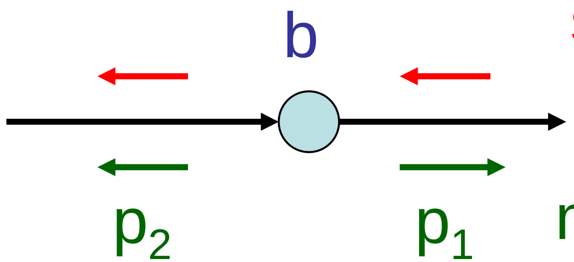
- Physical picture: large b means large color dipole. Large dipole tends to radiate during hard scattering. No radiation in exclusive processes. Small b configuration preferred.

Strong phase and CP violation

annihilation marks most crucial
difference between
QCD and PQCD

(V-A) and (V+A) currents

- For (V-A)(V-A), left-handed current

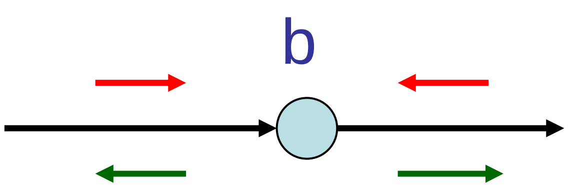


spin (this configuration is not allowed)

fermion flow

momentum

- Pseudo-scalar B requires spins in opposite directions, namely, helicity conservation $\lambda_1 = s_1 .\ p_1 = (-s_2).(-p_2) = \lambda_2$.
- For (V-A)(V+A)=(S-P)(S+P), scalar current



Survive helicity conservation,

Small annihilation?

- Annihilation is of higher power
- But what is the power suppression factor?
- If Λ/m_B , annihilation is negligible
- But it turns out to be $2m_0/m_B \sim 0.6$ for

$$m_0 = \frac{m_\pi^2}{m_u + m_d} \approx 1.5\, GeV$$

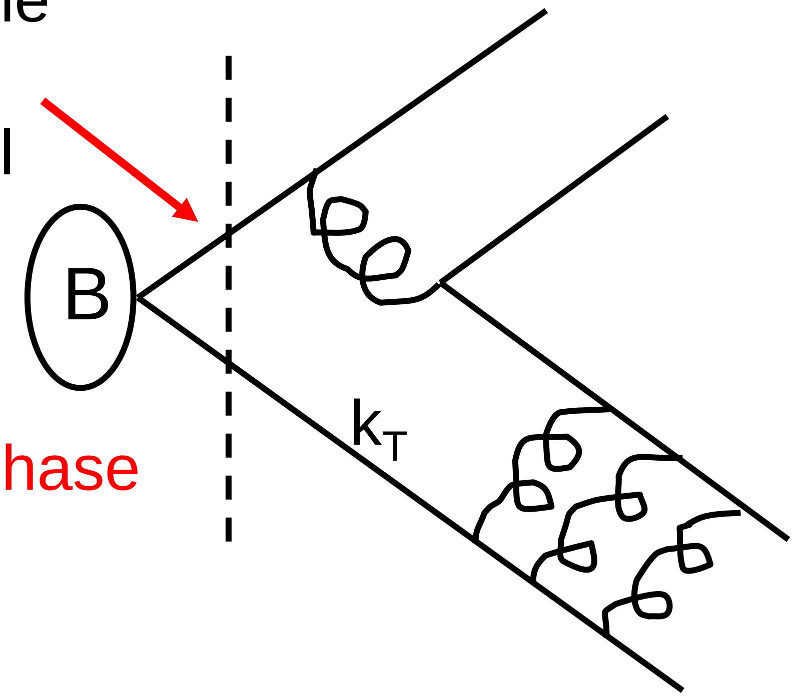
- Introduced by pseudoscalar-current matrix element (for twist-3 DA ϕ^S)
- What is impact of sizable annihilation?

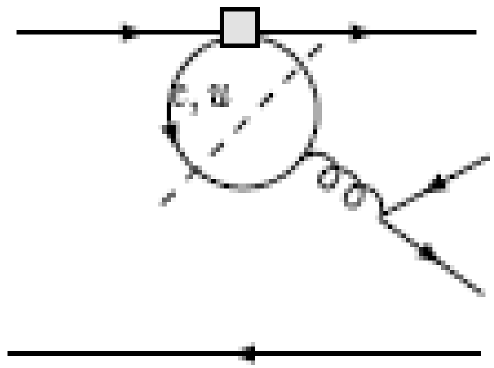
Principle value

$$\frac{1}{xm_B^2 - k_T^2 + i\epsilon} = \frac{P}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2).$$

Loop line
can go
on-shell

Strong phase

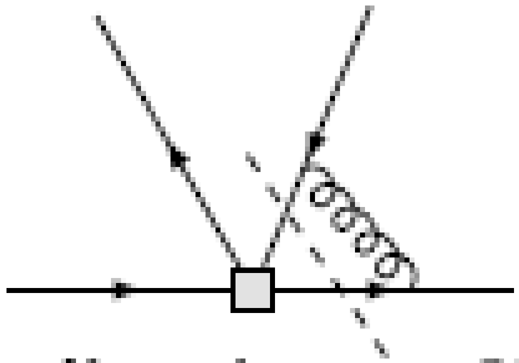




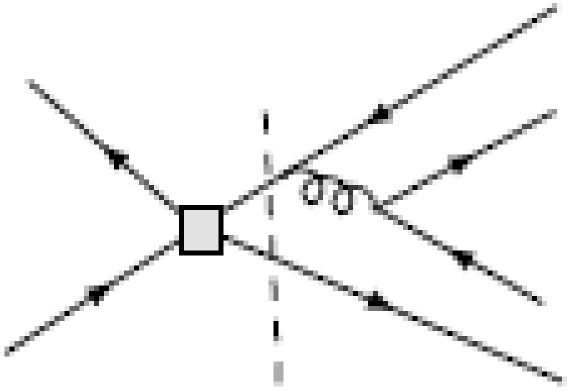
Bander-Silverman-Soni
mechanism, strong
phase source in FA

loop with the weight factor from
k_T distribution function (TMD)

Sources of strong phase



Vertex diagram in QCDF

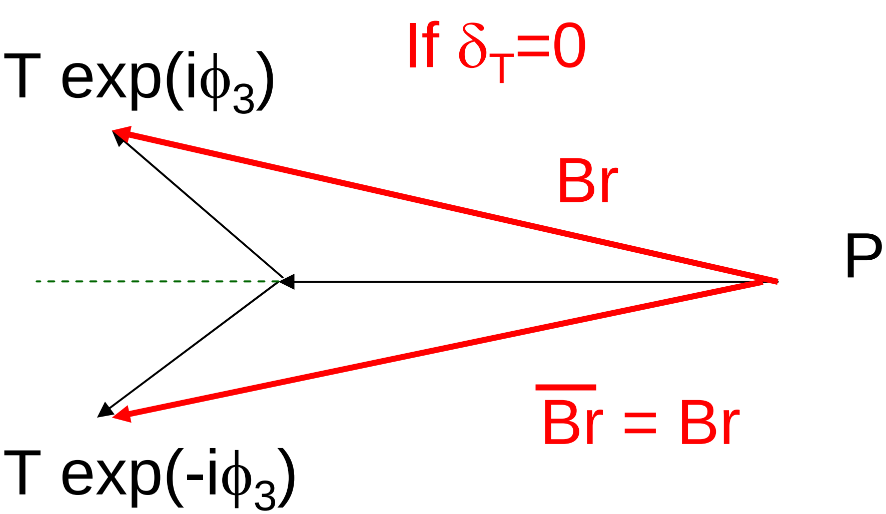


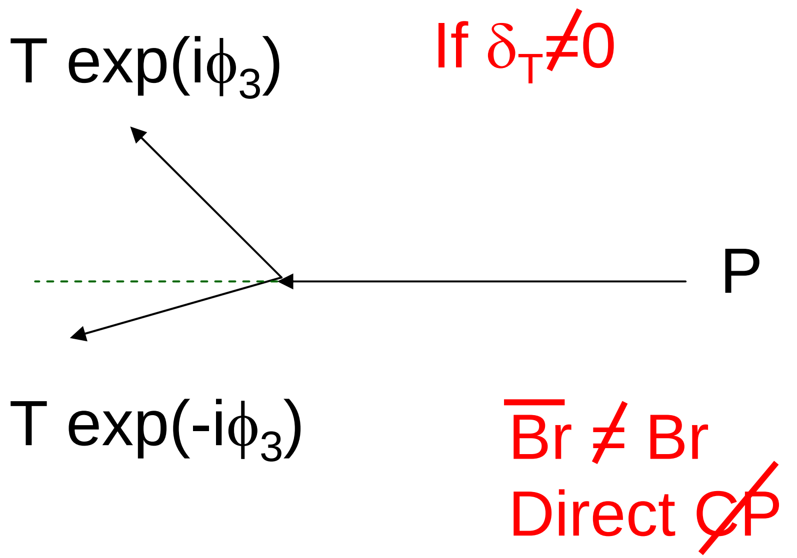
Annihilation diagram in PQCD

Annihilation is assumed to be small, down by α_s/m_b in QCDF,
but calculable and sizable in PQCD
Different sources lead to different direct CP asy.

Large strong phase

- $A_{CP}(K^+\pi^-) \sim -0.1$ implies sizable $\delta_T \sim 15^o$ between T and P from annihilation (PQCD, 00)

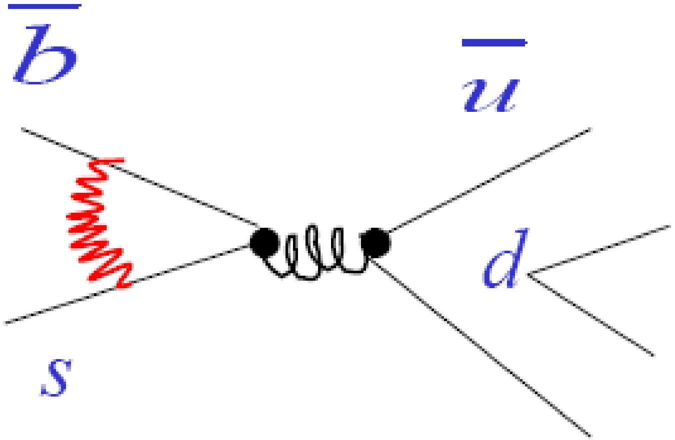




CP Violation in $B \to \pi\,\pi\,(K)$ (real prediction before exp.)

CP(%)	FA	BBNS	PQCD (2001)	Exp (2004)
π^+K^-	$+9 \pm 3$	$+5 \pm 9$	-17 ± 5	-11.5 ± 1.8
π^+K^0	1.7 ± 0.1	1 ± 1	-1.0 ± 0.5	-2 ± 4
π^0K^+	$+8 \pm 2$	7 ± 9	-13 ± 4	$+4 \pm 4$
$\pi^+\pi^-$	-5 ± 3	-6 ± 12	$+30 \pm 10$	$+37 \pm 10$

First evidence: $B^0_s \rightarrow \pi^+\pi^-$



Pure annihilation

$$\mathcal{B}(B_s^0 \to \pi^+\pi^-) = (0.57 \pm 0.15\ (stat.) \pm 0.10\ (syst.)) \times 10^{-6}.$$

CDF Results **arXiv:1111.0485**

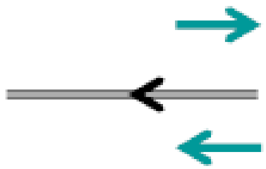
The pQCD prediction given at 2007

$Br(Bs^0 \rightarrow \pi+\ \pi-) = 5.7 \times 10^{-7}$

Ali et al., 2007

QCDF prediction smaller by an order of magnitude

B -> VV polarizations

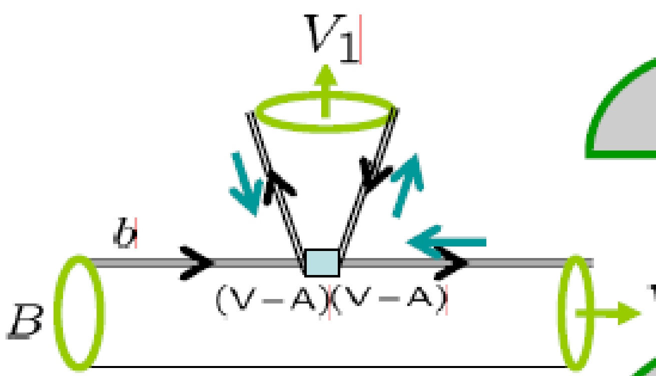


left-handed (-)

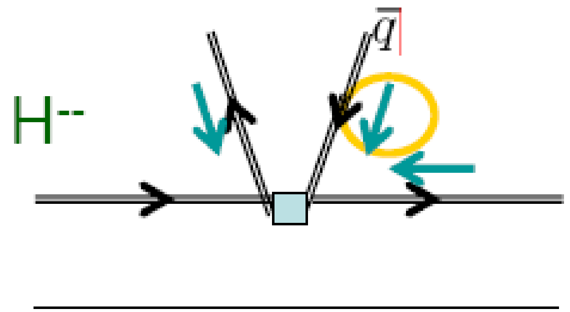
right-handed (+)

Transverse

Longitudinal



A_L

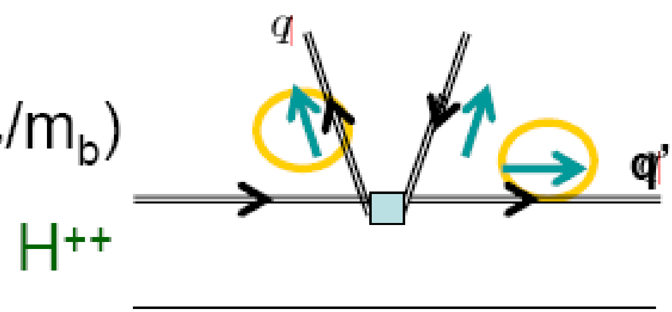


costs m_q/m_b

$+ \to A_{\parallel}, - \to A_{\perp}$

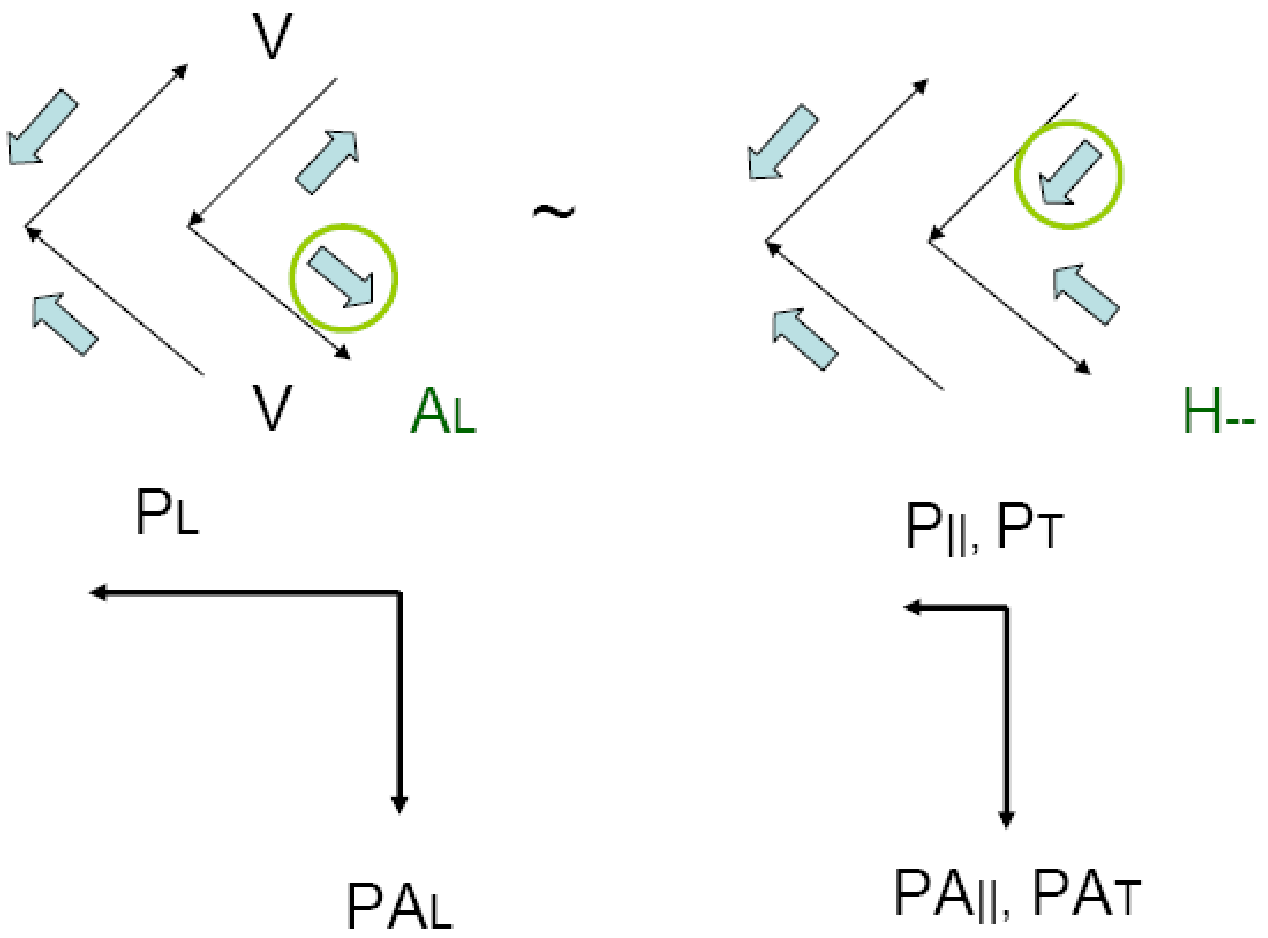
$A_L \gg A_{\parallel} \approx A_{\perp}$

costs $(m_q/m_b)(m_{q'}/m_b)$



The longitudinal component dominates

(S+P) penguin annihilation contributes to L, ||, T at the same power in 1/mb (Chen, Keum, and Li 02)



Polarization of B→VV decays

Table 1 Longitudinal Polarization Fractions

Process	Belle	Babar	QCDF
$B^0 \to \phi K^{*0}$,	$0.45 \pm 0.05 \pm 0.02$	$0.52 \pm 0.05 \pm 0.02$	0.91
$B^+ \to \phi K^{*+}$,	$0.52 \pm 0.8 \pm 0.03$	$0.46 \pm 0.12 \pm 0.03$	0.91
$B^+ \to \rho^0 K^{*+}$,		$0.78 \pm 0.12 \pm 0.03$	0.94
$B^+ \to \rho^+ K^{*0}$,	$0.43 \pm 0.11^{+0.05}_{-0.07}$	$0.52 \pm 0.10 \pm 0.04$	0.95
$B^+ \to \rho^+ \rho^0$,	$0.95 \pm 0.11 \pm 0.02$	$0.97 \pm 0.04^{+0.03}_{-0.07}$	0.94
$B^+ \to \rho^+ \omega$,		$0.88 \pm 0.04^{+0.12}_{-0.15}$	
$B^0 \to \rho^+ \rho^-$,		$0.99 \pm 0.03^{+0.04}_{-0.03}$	0.95

Direct CP asymmetries, pure annihilation branching ratios, B->VV polarizations all indicate sizable annihilation

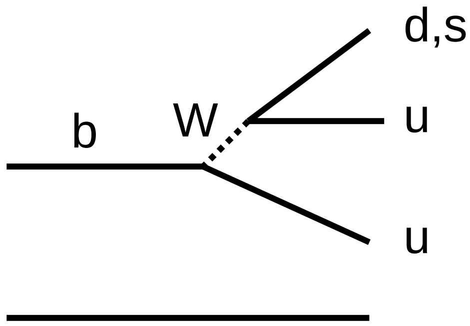
Puzzles in B decays

Puzzles in B physics

- $B(\pi^0\pi^0), B(\pi^0\rho^0)$ much larger than predictions
- $A_{CP}(\pi^0K^\pm)$ much different from $A_{CP}(\pi^\mp K^\pm)$
- $S(\pi^0K_S)$ lower than $S(c\bar{c}s)$
- New physics or QCD effect?
- If new physics, how about $B(\pi^0\pi^0), B(\pi^0\rho^0)$
- If QCD, but $B(\rho^0\rho^0)$ is normal
- How to resolve these puzzles?

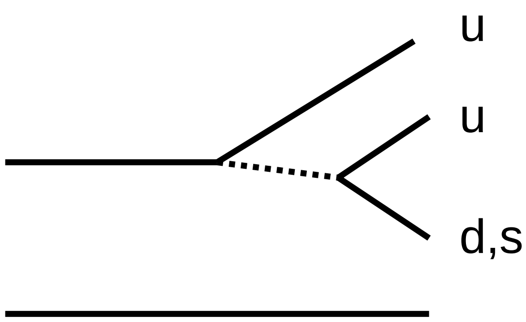
Quark amplitudes

2 color traces $\Rightarrow N_c^2$

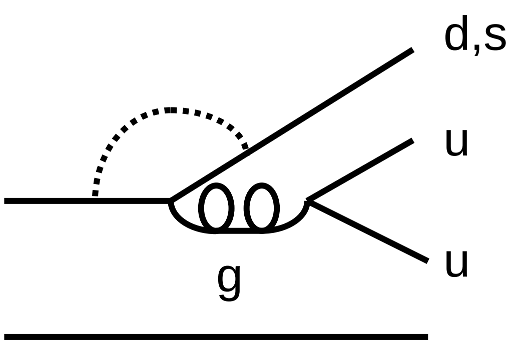


Color-allowed tree T

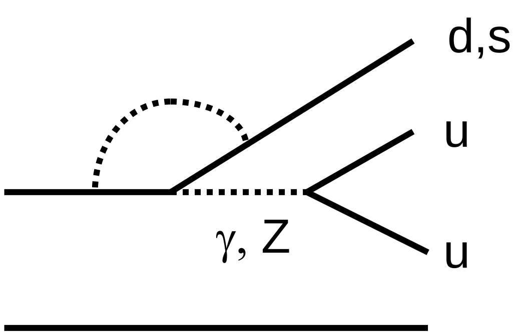
1 color trace $\Rightarrow N_c$



Color-suppressed tree C



QCD penguin P



Electroweak penguin P_{ew}

Kπ parameterization

$$A(B^+ \to K^0\pi^+) = P' ,$$

$$\boxed{\begin{aligned} A(B_d^0 \to K^+\pi^-) &= -P'\left(1 + \frac{T'}{P'}e^{i\phi_3}\right) , \\ \sqrt{2}A(B^+ \to K^+\pi^0) &= -P'\left[1 + \frac{P'_{ew}}{P'} + \left(\frac{T'}{P'} + \frac{C'}{P'}\right)e^{i\phi_3}\right] , \end{aligned}}$$

$$\sqrt{2}A(B_d^0 \to K^0\pi^0) = P'\left(1 - \frac{P'_{ew}}{P'} - \frac{C'}{P'}e^{i\phi_3}\right) ,$$

$$\frac{T'}{P'} \sim \lambda , \quad \frac{P'_{ew}}{P'} \sim \lambda , \quad \frac{C'}{P'} \sim \lambda^2$$

$(C_2/C_4)(V_{us}V_{ub}/V_{ts}V_{tb}) \sim (1/\lambda^2)(\lambda^5/\lambda^2) \sim \lambda$

B→ Kπ puzzle

- **$K^+\pi^-$ and $K^+\pi^0$ differ by sub-leading amplitudes P_{ew} and color suppressed tree (C). Their CP asymmetry are expected to be similar.**
- **Their data differ by 5σ! A puzzle!**

$$A_{CP}(K^+pi^-) = (-9.8 \pm 1.3)\%$$

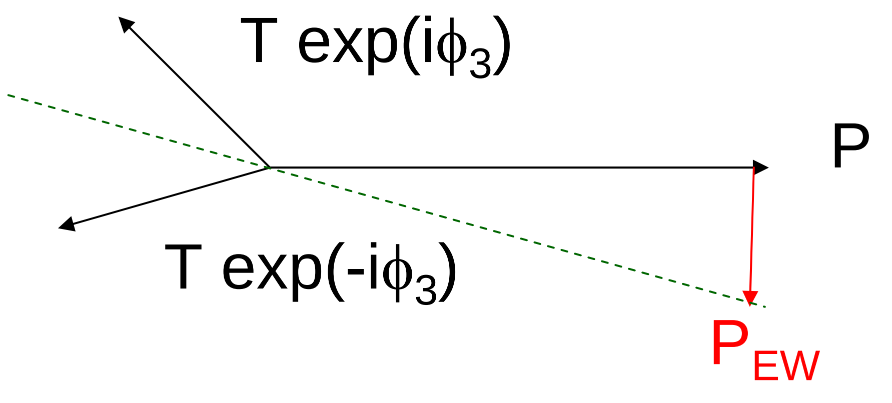
$$A_{CP}(K^+pi^0) = (5.1 \pm 2.5)\%$$

Importance of power corrections

Explanation 1

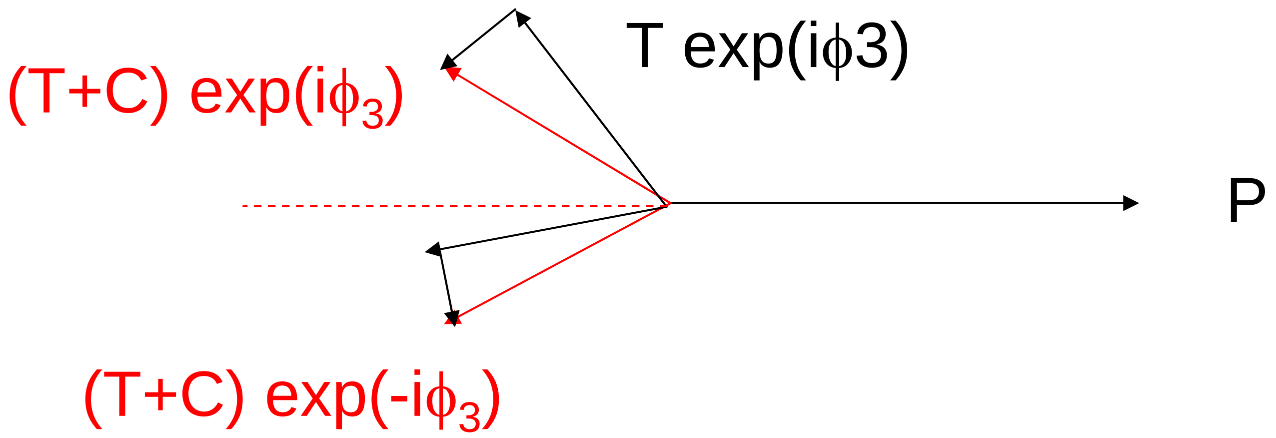
- How to understand positive $A_{CP}(K^+\pi^0)$?
- Large P_{EW} to rotate P (Buras et al.; Yoshikawa; Gronau and Rosner; Ciuchini et al., Kundu and Nandi)

new physics?



Explanation 2

- Large C to rotate T (Charng and Li; He and McKellar). mechanism missed in naïve power counting?
- C is subleading by itself. Try NLO…. An not-yet-finished story



Summary

- Naïve factorization of hadronic B decays relies only on color transparency.
- Fully utilize heavy-quark expansion. Combine effective Hamiltonian (separation of m_W and m_b) and IR divergence factorization (separation of m_b and Λ).
- Factorization theorem can explain many data, but some puzzles remain
- New physics or complicated QCD dynamics? More hard working is needed