Neutrino oscillations and oscillation coherence

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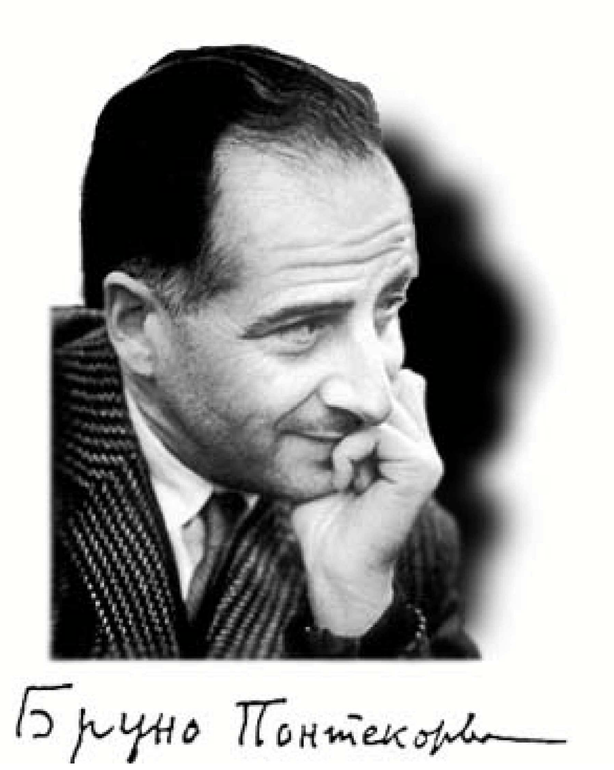
Pontecorvo's centennial

Idea of neutrino oscillations:

First put forward by Pontecorvo in 1957



Bruno Pontecorvo
1913 - 1993

Oscillation probability in vacuum

Master formula:

◊ $$\boxed{P(\nu_\alpha \to \nu_\beta; L) = \left|\sum_i U_{\beta i}\, e^{-i\frac{\Delta m_{i1}^2}{2p}L}\, U_{\alpha i}^*\right|^2}$$

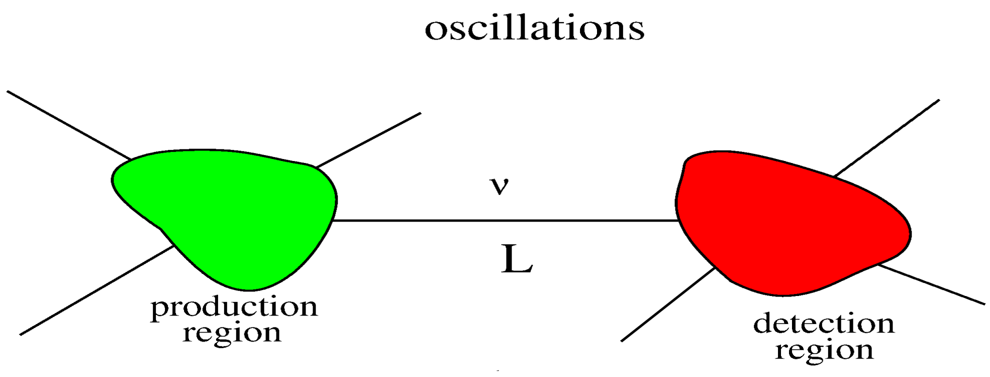
- What are the applicability conditions for this formula?
- When are the oscillations observable?

– One answer to both questions!

Main assumption: Neutrinos produced and detected in weak interaction processes are flavour eigenstates – coherent linear superpositions of mass eigenstates:

$$\diamondsuit \qquad |\nu_a^{\rm fl}\rangle = \sum_i U_{ai}^* \, |\nu_i^{\rm mass}\rangle$$

$$(\alpha \;=\; e\,,\; \mu\,,\; \tau, \qquad i \;=\; 1\,,\; 2\,,\; 3)$$



Overall production – propagation – detection process: amplitudes with different $\nu_i^{\rm mass}$ in the intermediate state contribute coherently (cannot be distinguished)

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E = \sqrt{p^2 + m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Production and detection coherence $\Leftrightarrow$ localization cond.:

$$l_{\rm prod} \ll l_{\rm osc}\,, \qquad\qquad l_{\rm det} \ll l_{\rm osc}$$

Usually satisfied with large margins.

Propagation coherence: $L < l_{\rm coh} \simeq \frac{v_g}{\Delta v_g}\sigma_x = \frac{2E^2}{\Delta m^2}\,v_g\sigma_x$

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates (Kayser, 1981)
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation (Nussinov, 1976)

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p.

When are neutrino oscillations observable?

Production and detection coherence:

$$\Delta E \ll \sigma_E\,, \qquad\qquad \Delta p \ll \sigma_p$$

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If coherence is lost: Flavour transition can still occur, but in a non-oscillatory way. E.g. for $\pi \to \mu\nu_i$ decay with a subsequent detection of ν_i with the emission of e:

$$P \,\propto\, \sum_i P_{\rm prod}(\mu\,\nu_i)P_{\rm det}(e\,\nu_i) \,\propto\, \sum_i |U_{\mu i}|^2|U_{ei}|^2$$

– the same result as for averaged oscillations.

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How can the oscillations destroyed by precise measurements of neutrino energy and momentum? Suppose we can find the neutrino energy E and momentum p with uncertainties σ_E and σ_p. From $E_i^2 = p_i^2 + m_i^2$:

$$\sigma_{m^2} \;=\; \left[(2E\sigma_E)^2 \;+\; (2p\sigma_p)^2\right]^{1/2}$$

When are neutrino oscillations observable?

If $\sigma_{m^2} < \Delta m^2 \; = \; |m_i^2 - m_k^2|$ – one can tell which mass eigenstate is emitted.

$\sigma_{m^2} < \Delta m^2$ implies $2p\sigma_p < \Delta m^2$, or $\sigma_p < \Delta m^2/2p \; \simeq \; l_{\rm osc}^{-1}$.

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<u>But:</u> To measure p with the accuracy σ_p one needs to measure the momenta of particles at production with (at least) the same accuracy $\Rightarrow$ uncertainty of their coordinates (and the coordinate of ν production point) will be

$$\sigma_{\rm x,\,prod} \gtrsim \sigma_p^{-1} > l_{\rm osc}$$

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Natural necessary condition for coherence (observability of oscillations):

$$L_{\rm source} \ll l_{\rm osc}\,, \qquad\qquad L_{\rm det} \ll l_{\rm osc}$$

No averaging of oscillations in the source and detector. For usual neutrinos:

Satisfied with very large margins in most cases of practical interest

Wave packet separation

Wave packets representing different mass eigenstate components ν_i have different group velocities v_{gi} $\Rightarrow$ after time $t_{\rm coh}$ (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v_g \cdot t_{\rm coh} \simeq \sigma_x\,; \qquad\qquad l_{\rm coh} \simeq v_g t_{\rm coh}$$

$$\Delta v_g = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$\boxed{l_{\rm coh} \simeq \frac{v_g}{\Delta v_g}\sigma_x = \frac{2E^2}{\Delta m^2}\,v_g\sigma_x}$$

The standard formula for $P_{\rm osc}$ is obtained when the decoherence effects are negligible.

Neutrino oscillations: *Coherence at macroscopic distances –*

$L > 10{,}000$ km in atmospheric neutrino experiments!

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim U_{e1} = \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim U_{e2} = \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) = \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

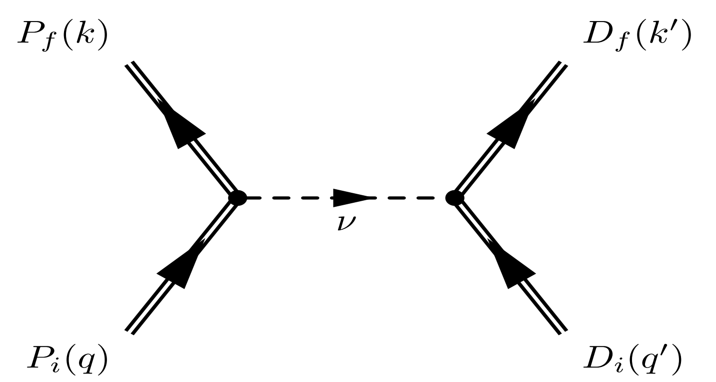
Phase difference $\Delta\phi$ vanishes at short L $\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently) $\Rightarrow$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \;<\; 1$$

A universal oscillation probability?



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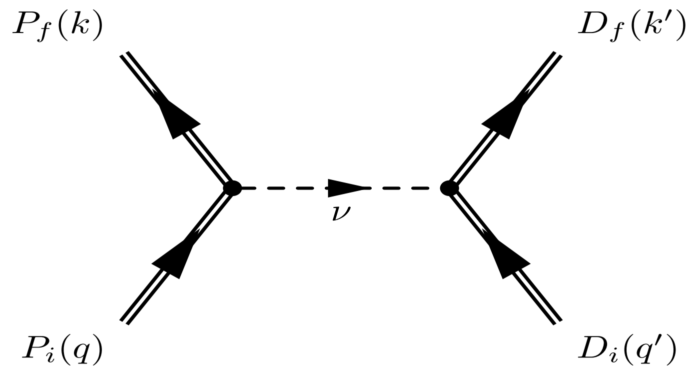

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$$\Delta E \ll \sigma_E, \qquad \Delta p \ll \sigma_p$$

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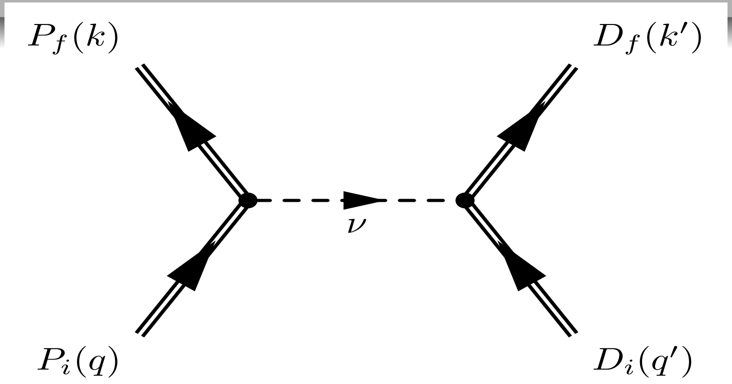

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Under these conditions the rate of the overall neutrino production-propagation-detection process can be factorized into the production rate $d\Gamma_\alpha^{\rm prod}(E)/dE$, propagation (oscillation) probability $P_{\alpha\beta}(E, L)$ and detection cross section $\sigma_\beta(E)$ $\Rightarrow$ $P_{\alpha\beta}(E, L)$ can be extracted

(EA & J. Kopp, arXiv:1001.4815)

Are coherence constraints compatible?

Observability conditions for ν oscillations:

- Coherence of ν production and detection
- Coherence of ν propagation

Both conditions put upper limits on neutrino mass squared differences Δm^2 :

$$(1) \quad \Delta E_{jk} \sim \frac{\Delta m_{jk}^2}{2E} \ll \sigma_E; \qquad\qquad (2) \quad \frac{\Delta m_{jk}^2}{2E^2} L \ll \sigma_x \simeq v_g/\sigma_E$$

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Are they compatible? – Yes, if LHS $\ll$ RHS $\Rightarrow$

$$\boxed{2\pi \frac{L}{l_{\rm osc}} \ll \frac{v_g}{\Delta v_g} \; (\gg 1)}$$

– fulfilled in all cases of practical interest

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r-process nucleosynthesis, unconventional contributions to $2\beta 0\nu$ decay ...

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Production/detection coherence has to be re-checked – important implications for some neutrino experiments!

QM wave packet approach

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The evolved produced state:

$$|\nu_\alpha^{\rm fl}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^*\,|\nu_i^{\rm mass}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^*\,\Psi_i^S(\vec{x},t)|\nu_i^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\boxed{\Psi_i^S(\vec{x},t) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f_i^S(\vec{p})\, e^{i\vec{p}\vec{x}-iE_i(p)t}}$$

Momentum distribution function $f_i^S(\vec{p})$: sharp maximum at $\vec{p}=\vec{P}$ (width of the peak $\sigma_{pP} \ll P$).

$$E_i(p) \;=\; E_i(P) + \left.\frac{\partial E_i(p)}{\partial \vec{p}}\right|_{\vec{P}}(\vec{p}-\vec{P}) + \frac{1}{2}\left.\frac{\partial^2 E_i(p)}{\partial \vec{p}^2}\right|_{\vec{p}_0}(\vec{p}-\vec{P})^2 + \,\ldots$$

$$\vec{v}_i \;=\; \frac{\partial E_i(p)}{\partial \vec{p}} \;=\; \frac{\vec{p}}{E_i}\,, \qquad\qquad \alpha \;\equiv\; \frac{\partial^2 E_i(p)}{\partial \vec{p}^2} \;=\; \frac{m_i^2}{E_i^2}$$

Evolved neutrino state

$$\boxed{\Psi_i^S(\vec{x},\, t)\ \simeq\ e^{-iE_i(P)t+i\vec{P}\vec{x}}\, g_i^S(\vec{x}-\vec{v}_i t)} \qquad (\alpha\ \to\ 0)$$

$$g_i^S(\vec{x}-\vec{v}_i t)\ \equiv\ \int \frac{d^3 p_1}{(2\pi)^3}\, f_i^S(\vec{p}_1)\, e^{i\vec{p}_1(\vec{x}-\vec{v}_g t)}$$

Center of the wave packet: $\vec{x}-\vec{v}_i t = 0$. Spatial length: $\sigma_{xP}\ \sim\ 1/\sigma_{pP}$ (g_i^S decreases quickly for $|\vec{x}-\vec{v}_i t| \gtrsim \sigma_{xP}$).

Detected state (centered at $\vec{x}=\vec{L}$):

$$|\nu_\beta^{\rm fl}(\vec{x})\rangle\ =\ \sum_k U_{\beta k}^*\, \Psi_k^D(\vec{x})|\nu_i^{\rm mass}\rangle$$

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Oscillation probability

Transition amplitude:

$$\mathcal{A}_{\alpha\beta}(T,\vec{L}) = \langle \nu_\beta^{\text{fl}} | \nu_\alpha^{\text{fl}}(T,\vec{L})\rangle = \sum_i U_{\alpha i}^* U_{\beta i}\, \mathcal{A}_i(T,\vec{L})$$

$$\boxed{\mathcal{A}_i(T,\vec{L}) = \int \frac{d^3p}{(2\pi)^3}\, f_i^S(\vec{p})\, f_i^{D*}(\vec{p})\, e^{-iE_i(p)T+i\vec{p}\vec{L}}}$$

Strongly suppressed unless $|\vec{L}-\vec{v}_i T| \lesssim \sigma_x$. E.g., for Gaussian wave packets:

$$\mathcal{A}_i(T,\vec{L}) \propto \exp\left[-\frac{(\vec{L}-\vec{v}_i T)^2}{4\sigma_x^2}\right], \qquad \sigma_x^2 \equiv \sigma_{xP}^2 + \sigma_{xD}^2$$

Oscillation probability:

$\diamondsuit$ $$P(\nu_\alpha \to \nu_\beta; T,\vec{L}) = |\mathcal{A}_{\alpha\beta}|^2 = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, \mathcal{A}_i(T,\vec{L}) \mathcal{A}_k^*(T,\vec{L})$$

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) \;=\; \int dT\, P(\nu_\alpha \to \nu_\beta; T, L) \;=\; \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, e^{-i\frac{\Delta m_{ik}^2}{2\bar{P}}L}\, \tilde{I}_{ik}$$

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$$\boxed{\begin{aligned}\tilde{I}_{ik} = N \int \frac{dq}{2\pi} f_i^S(r_k q - \Delta E_{ik}/2v + P_i) f_i^{D*}(r_k q - \Delta E_{ik}/2v + P_i) \\ \times f_k^{S*}(r_i q + \Delta E_{ik}/2v + P_k) f_k^D(r_i q + \Delta E_{ik}/2v + P_k)\, e^{i\frac{\Delta v}{v} qL}\end{aligned}}$$

Here: $v \equiv \frac{v_i + v_k}{2}\,, \quad \Delta v \equiv v_k - v_i\,, \quad r_{i,k} \equiv \frac{v_{i,k}}{v}\,, \quad N \equiv 1/[2E_i(P) 2E_k(P) v]$

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- $\tilde{I}_{ik}$ is also strongly suppressed unless $\Delta E_{ik}/v \ll \sigma_p$, i.e. $\Delta E_{ik} \ll \sigma_E$ – coherent production/detection condition

The standard osc. probability?

The standard formula for the oscillation probability corresponds to $\tilde{I}_{ik} = 1$.

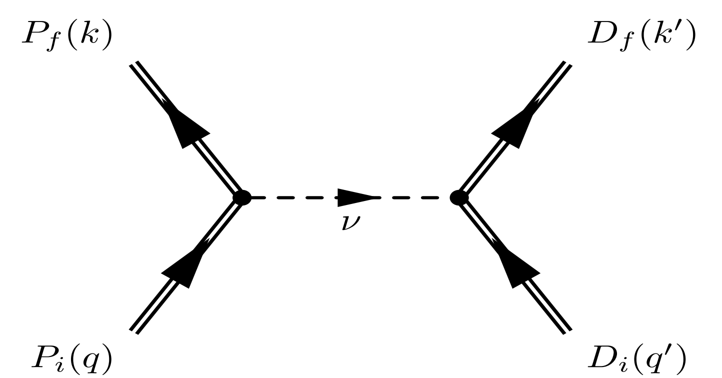
If the two above conditions are satisfied, $\tilde{I}_{ik}$ is not suppressed and is L-, E- and i,k-independent (i.e. a constant).

The standard probability is obtained when this constant is 1 (normalization necessary!)

Normaliz. condition:

$$\int \frac{d^3p}{(2\pi)^3} |f_i^S(\vec{p})|^2 |f_i^D(\vec{p})|^2 = 1$$

How can neutrino wave packet be derived?



Amplitude of the production process (neutrino ν_j with a momentum $\vec{p}$):

$$\Phi_{jP}(\vec{p}) = \int d^4x_1' e^{ipx_1'} \int [dq] \int [dk]\, f_{Pi}(\vec{q},\vec{Q})\, f_{Pf}^*(\vec{k},\vec{K})\, e^{-i(q-k)x_1'} M_{jP}(q,k)$$

Detection amplitude:

$$\Phi_{jD}(\vec{p}) = \int d^4x_2' e^{-ipx_2'} \int [dq'] \int [dk']\, f_{Di}(\vec{q}',\vec{Q}')\, f_{Df}^*(\vec{k}',\vec{K}')\, e^{-i(q'-k')x_2'} M_{jD}(q',k')$$

<u>But:</u> The probability amplitude that ν_j is produced with momentum $\vec{p}$ is its momentum-space w.p.! $\Rightarrow$

$$\boxed{f_j^S(\vec{p}) = \Phi_{jP}(\vec{p})\,, \qquad f_j^D(\vec{p}) = \Phi_{jD}^*(\vec{p})}$$

Coherence of ν production in different points

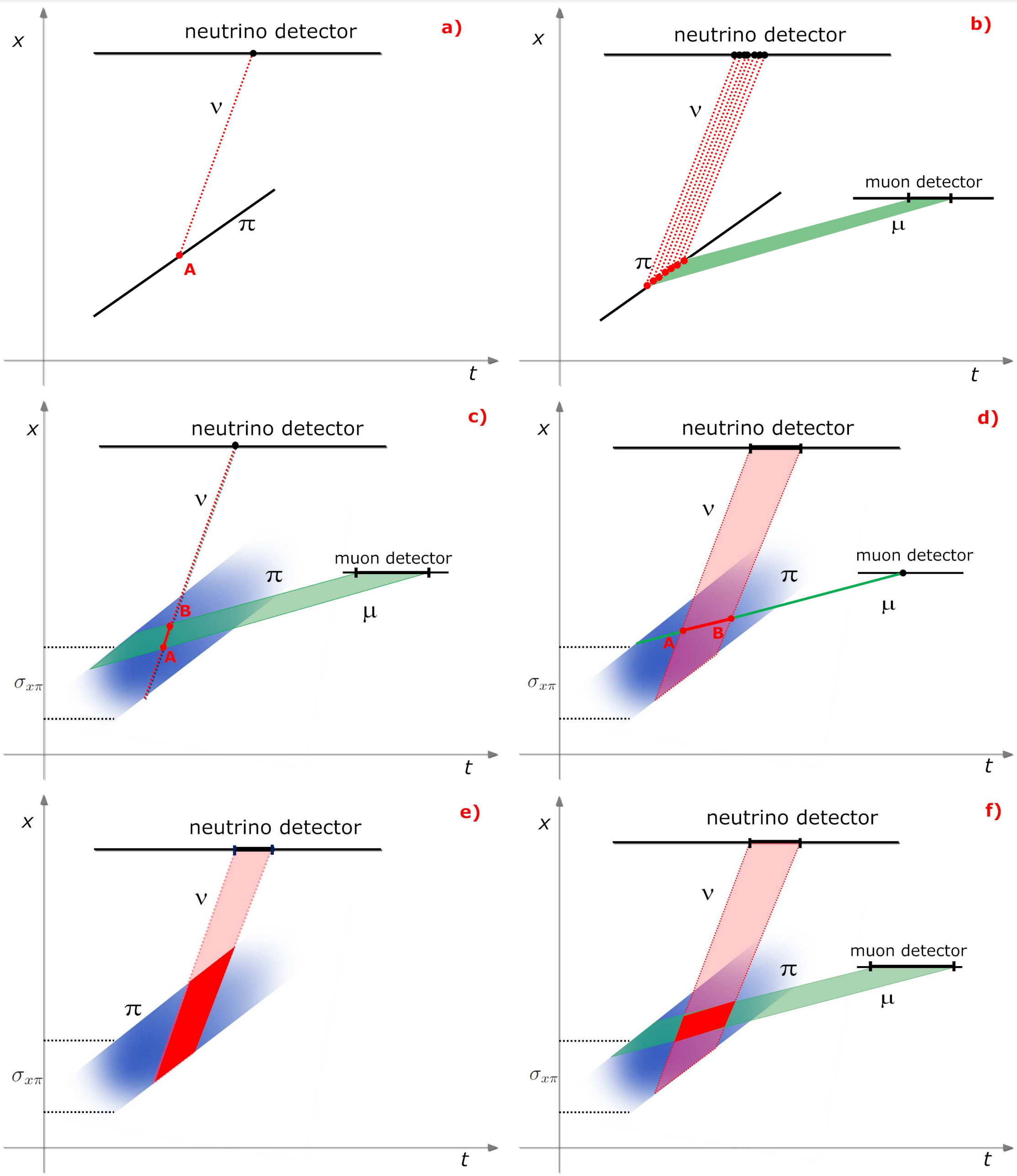
Neutrino production in extended sources: Amplitudes of neutrino emission in different points must be summed – a consistent QM procedure.

The standard approach: calculate the probability that neutrino produced at a fixed point x oscillates, and then integrate over all x in the source (probability summation procedure – classical in nature).

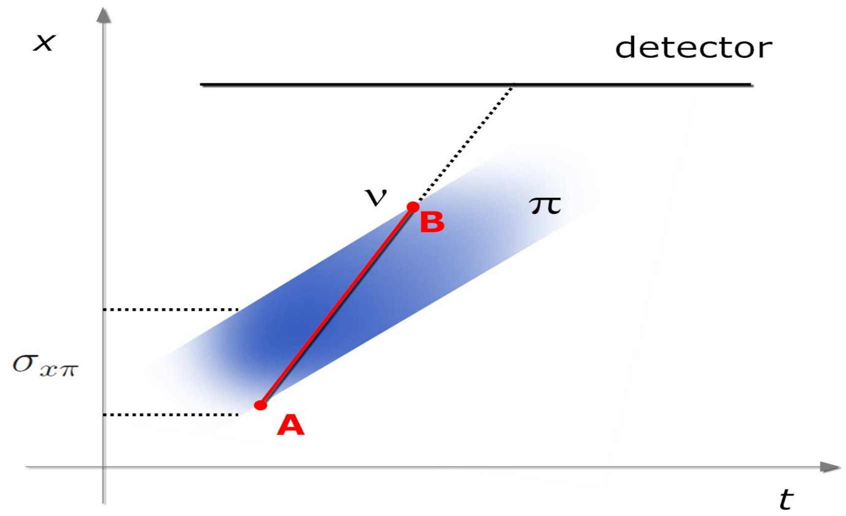
Both procedures give identical answers under realistic conditions!

The two approaches lead to different results whenever the localization properties of the parent particles at neutrino production and of the detection process are such that they prevent the precise localization of the point of neutrino emission – difficult to realize in practice.

Graphical interpretation



Finite-width pion WP



Additional phase for the segment AB:

$$\Delta\phi = -[E_j(P_j) - E_k(P_k)]\Delta t + (P_j - P_k)\Delta x\,.$$

Δt and Δx: projections of AB on the t and x axes. $\Rightarrow$

$$\Delta t = \frac{\sigma_{x\pi}}{v_g - v_\pi}\,, \qquad \Delta x = \sigma_{x\pi}\frac{v_g}{v_g - v_\pi}\,.$$

$$\Delta\phi \simeq -\frac{v_g}{v_g - v_\pi}\cdot\frac{\Delta m^2_{jk}}{2P}\sigma_{x\pi}$$

Finite-width pion WP – contd.

Are deviations between the results of the coherent amplitude summation and incoherent probability summation approaches experimentally observable? Requires extremely high energies of the parent pion:

$$2\,(E_\pi \sigma_{x\pi})\,\frac{\Delta m^2}{m_\pi^2} \gtrsim 1\,.$$

E.g. for $\sigma_{x\pi} \sim 10^{-4}$ cm and $\Delta m^2 \sim 1$ eV^2 $\Delta\phi$ would be ~ 1 for pion energies $E_\pi \gtrsim 10^3$ TeV – not feasible,

Another possibility: increase significantly the spatial width of w. packets of ancestor protons, which would increase the values of $\sigma_{x\pi}$. But: not clear how this could be achieved.

Other possibilities...

Production coherence for some experiments

Unless otherwise specified, $\Delta m^2 = 2$ eV^2. For β-beams $E_0 = 2$ MeV, $\tau_0 = 1$s, $\gamma = 100$.

Experiment	$\langle E_\nu\rangle$(MeV)	L(m)	$l_p(m)$	$l_{\rm dec}$(m)	$l_{\rm osc}$(m)	ϕ	$\Gamma l_p/v_P$	ϕ_p	ξ
LSND	$\sim$40	30	0	0	50	3.8	-	0	0
KARMEN	$\sim$40	17.7	0	0	50	2.24	-	0	0
MiniBooNE	$\sim$800	541	50	89	992	3.43	0.56	0.32	0.56
NOMAD	$2.7\cdot 10^3$	770	290	3009	33480	0.145	0.1	0.054	0.56
(20 eV^2)					3348	1.45	0.1	0.54	5.64
CCFR(10^2 eV^2)	$5\cdot 10^4$	891	352	5570	1240	4.51	0.06	1.78	28.2
CDHS	3000	130	52	334	3720	0.22	0.155	0.088	0.56
(20 eV^2)					372	2.2	0.155	0.878	5.64
K2K	1500	300	200	167	1861	1.01	1.2	0.68	0.56
T2K	600	280	96	66.4	744	2.36	1.45	0.81	0.56
Minos	3300	1040	675	368	4092	1.6	1.84	1.04	0.56
NOνA	2000	1040	675	223	2480	2.64	3.03	1.71	0.56
β-beams	400	$1.3\cdot 10^5$	2500	$3\cdot 10^{10}$	496	1647	$8.3\cdot 10^{-8}$	31.7	$3.8\cdot 10^8$

Noticeable effects for MiniBooNE, NOMAD (20 eV^2), CCFR (100 eV^2), CDHS (20 eV^2), K2K, T2K, MINOS, NOνA, very large effects for β-beams

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But: Conditions for partial decoherence are difficult to realize

They may still be realized if relatively heavy sterile neutrinos exist

Backup slides

Coherence production conditions

Coherence production conditions:

$$|\Delta E| \ll \sigma_E\,, \qquad\qquad |\Delta p| \ll \sigma_p\,.$$

On the other hand:

$$\Delta E \simeq v_g \Delta p + \frac{\Delta m^2}{2E}.$$

Constraint $|\Delta E| \ll \sigma_E \quad \Rightarrow$

$$\left|\frac{v_g \Delta p}{\sigma_E} + \frac{\Delta m^2}{2E\sigma_E}\right| \ll 1. \qquad\qquad (*)$$

(a) The two terms in ΔE do not approximately cancel each other. $\Rightarrow$ $v_g|\Delta p| \ll \sigma_E \le \sigma_p$, i.e. for relativistic neutrinos $|\Delta p| \ll \sigma_p$ follows from $|\Delta E| \ll \sigma_E$.

(b1) There is a strong cancellation, but both terms on the l.h.s. of (*) are small – see case (a).

(b2) Strong cancellation, but both terms on the l.h.s. of (*) are $\gtrsim 1$: momentum condition is independent. But: the only known case – Mössbauer neutrinos.

The paradox of σ_E and σ_p

QM uncertainty relations: σ_p is related to the spatial localization of the production (detection) process, while σ_E to its time scale $\Rightarrow$ independent quantities.

On the other hand: Neutrinos propagating macroscopic distances are on the mass shell. For on-shell mass eigenstates $E^2 = p^2 + m_i^2$ means

$$E\sigma_E = p\sigma_p$$

How can this be understood?

The solution: At production, neutrinos are *not* on the mass shell. They go on shell only after they propagate $x \sim$ (a few)$\times$ De Broglie wavelengths. After that their energy and momentum get related by $E^2 = p^2 + m_i^2 \Rightarrow$ the larger uncertainty shrinks towards the smaller one to satisfy $E\sigma_E = p\sigma_p$.

On-shell relation between E and p allows to determine the less certain of the two through the more certain one, reducing the error of the former.

What determines the length of ν w. packets?

The length of ν w. packets: $\sigma_x \sim 1/\sigma_p$. For propagating on-shell neutrinos:

$$\sigma_p \simeq \min\{\sigma_p^{\rm prod}, (E/p)\sigma_E^{\rm prod}\} = \min\{\sigma_p^{\rm prod}, (1/v_g)\sigma_E^{\rm prod}\}$$

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- If $T_S > \tau$ (quasi-free parent particle) $\Rightarrow$ $\sigma_E \simeq \tau^{-1} = \Gamma$.

  $\sigma_p \simeq [(p/E)\tau]^{-1} \simeq [(p/E)\sigma_E]^{-1}$, i.e. $\sigma_E \simeq (p/E)\sigma_p < \sigma_p$.

The length of ν w. packets – contd.

In both cases $\boxed{\sigma_E^{\rm prod} < \sigma_p^{\rm prod}}$ $\Leftarrow$ also when $\nu' s$ are produced in collisions.

$$\Longrightarrow \qquad \sigma_{p\ {\rm eff}} \simeq \frac{\sigma_E}{v_g}\,, \qquad\qquad \boxed{\sigma_x \simeq \frac{v_g}{\sigma_E}}$$

In the stationary limit $(\sigma_E \to 0)$ one has $\sigma_{p\ {\rm eff}} \to 0$ even though σ_p is finite! Therefore $\sigma_x \to \infty$ and so the coherence length $l_{\rm coh} \to \infty$
– a well known result.