Modified Gravity and Gravitational Repulsion

E.V. Arbuzova[a,b], **A.D. Dolgov**[b,c,d,e]

[a]Dubna University, [b]Novosibirsk State University
[c]ITEP, Moscow, Russia
[d]Università degli Studi di Ferrara, [e]INFN, Italy

16th Lomonosov Conference
on Elementary Particle Physics
MSU, Moscow, Russia, August 22 - 28, 2013

Outline

- Dark Energy or Modifed Gravity **?**
- Curvature Oscillations in $F(R)$ theories in systems with rising energy density
- Spherically Symmetric Solutions in $F(R)$ Gravity and Gravitational Repulsion
- Conclusions

Cosmological Acceleration

The data in favor of accelerated expansion:

- observation of the large scale structure of the universe
- measurements of the angular fluctuations of the CMBR
- determination of the universe age
- discovery of the dimming of distant Supernovae

With cosmological inflation, at the very beginning, the picture would be:

- first acceleration (initial push)
- then normal deceleration
- and lastly (today) surprising acceleration again

Phenomenological Explanations

Dark Energy: $P < -\varrho/3$

- small vacuum energy, which is identical to cosmological constant
- energy density associated with an unknown, presumably scalar field, which slowly varies in the course of the cosmological evolution

Modification of Gravity:

$$S_{grav} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left[R + F(R)\right]$$

Here $m_{Pl} = 1.22 \cdot 10^{19}\,\mathrm{GeV}$ is the Planck mass.

Non-linear $F(R)$-function: the modified GR equations have a solution $R = \mathrm{const}$ in absence of any matter source.

Pioneering Works

The pioneering suggestion:

- *S. Capozziello, S. Carloni, A. Troisi,* Recent Res. Develop. Astron. Astrophys.1(2003)625; astro-ph/0303041.
- *S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner,* Phys. Rev. D70 (2004) 043528, astro-ph/0306438.

$$\mathbf{F(R)} = -\mu^4/\mathbf{R}$$

$\mu^2 \sim R_c \sim 1/t_u^2$ is a small parameter with dimension of mass squared.

- Agreement with Newtonian limit for sufficiently small μ.
- Strong instability in the presence of matter.

A.D. Dolgov, M. Kawasaki, Phys.Lett. B573 (2003) 1.

Modified modified gravity: free from exponential instability

W.Hu, I. Sawicki, Phys. Rev. D **76**, 064004 (2007).

$$\mathbf{F_{HS}(R) = -\frac{R_{vac}}{2} \frac{c\left(\frac{R}{R_{vac}}\right)^{2n}}{1 + c\left(\frac{R}{R_{vac}}\right)^{2n}}},$$

A.Appleby, R. Battye, Phys. Lett. B **654**, 7 (2007).

$$\mathbf{F_{AB}(R) = \frac{\epsilon}{2} \log\left[\frac{\cosh\left(\frac{R}{\epsilon} - b\right)}{\cosh b}\right] - \frac{R}{2}},$$

A.A. Starobinsky, JETP Lett. **86**, 157 (2007).

$$\mathbf{F_S(R) = \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^2}\right)^{-n} - 1\right]}.$$

S.A. Appleby, R.A. Battye, A.A. Starobinsky, JCAP 1006 (2010) 005.
S.Nojiri, S.Odintsov, Phys.Lett.**B657**(2007)238; Phys.Rept.505(2011)59.

Another Problems

The suggested modifications, however, may lead to infinite-R singularities in the past cosmological history:

- *S.A. Appleby, R.A. Battye, A.A. Starobinsky,* JCAP 1006 (2010) 005.

In the future in astronomical systems with rising energy/matter density:

- *A.V. Frolov,* Phys. Rev. Lett. **101**, 061103 (2008)
- *I. Thongkool, M. Sami, R. Gannouji, S. Jhingan*, Phys. Rev. D **80** 043523 (2009); I. Thongkool, M. Sami, S. Rai Choudhury, Phys. Rev. D **80** 127501 (2009).
- *E.V. Arbuzova, A.D. Dolgov*, Phys.Lett.**B**700, 289(2011).

Some properties of such singularities were further studied in:

- *K. Bamba, S. Nojiri, S.D. Odintsov*, Phys. Lett. **B** 698, 451 (2011).

Cure of Sigularities: $\mathbf{R^2}$-term

Additional $\mathbf{R^2}$-term in the action: naturally appears as a result of quantum corrections due to matter loops in curved space-time.

- V.Ts. Gurovich, A.A. Starobinsky, *Sov. Phys. JETP* **50** (1979) 844; [Zh. Eksp. Teor. Fiz. 77 (1979) 1683];
- A.A. Starobinsky, *JETP Lett.* **30** (1979) 682; [Pisma Zh. Eksp. Teor. Fiz. 30 (1979) 719]; *Phys. Lett.* **B91**, 99 (1980).
- A.A. Starobinsky, *Proc. of the Second Seminar "Quantum Theory of Gravity"* (Moscow, 13-15 Oct. 1981), INR Press, Moscow , 1982, pp. 58-72; reprinted in: Quantum Gravity, eds. M. A. Markov and P. C. West. Plenum Publ. Co., N.Y., 1984, pp. 103-128.

Renewed interest in possible effects of additional ultraviolet terms, $\sim \mathbf{R^2}$, in infrared-modified $\mathbf{F(R)}$ gravity models.

- E.V. Arbuzova, A.D. Dolgov, L. Reverberi, *JCAP* **02** *(2012) 049.*
- *H. Motohashi, A. Nishizawa, Phys.Rev.* **D86** *(2012) 083514.*

Model

We consider the version of the modified gravity suggested by Starobinsky:

$$\mathbf{F(R) = -\lambda R_0 \left[1 - \left(1 + \frac{R^2}{R_0^2}\right)^{-n}\right] - \frac{R^2}{6m^2}}$$

- $\mathbf{n}$ is an integer, $\boldsymbol{\lambda > 0}$, $|\mathbf{R_0}| \sim \mathbf{1/t_U^2}$, $\mathbf{t_U \approx 13}$ Gyr is the universe age.
- Parameter $\mathbf{m}$ is bounded by $\mathbf{m \gtrsim 10^5}$ GeV to preserve successful predictions of BBN.
- $\mathbf{R^2}$-term is included to prevent curvature singularities in the presence of contracting bodies and is relevant at very large curvatures.

Basic Equations

The evolution of $\mathbf{R}$ is determined from the equation:

$$3\mathcal{D}^2 F'_R - R + RF'_R - 2F = T$$

- $\mathcal{D}^2$ is the covariant D'Alambertian operator, $F'_R \equiv dF/dR$
- $T \equiv 8\pi T^\mu_\mu / m^2_{Pl}$ and $T_{\mu\nu}$ is energy-momentum tensor of matter.

We are interested in the regime $|R_0| \ll |R| \ll m^2$, in which:

$$F(R) \simeq -\lambda R_0 \left[1 - \left(\frac{R_0}{R}\right)^{2n}\right] - \frac{R^2}{6m^2}\,.$$

We study the evolution of R in a contracting astrophysical system with rising energy density:

$$\varrho_m(t) = \varrho_{m0}(1 + t/t_{contr})$$

We assume that the gravity of matter is not strong and thus the background metric is flat: $3\partial_t^2 F'_R - R - T = 0.$

New Notations: Oscillator Equation

With the dimensionless quantities:

$$z \equiv \frac{T(t)}{T(t_{in})} \equiv \frac{T}{T_0} = \frac{\varrho_m(t)}{\varrho_{m0}}, \quad y \equiv -\frac{R}{T_0}, \quad \tau \equiv m\sqrt{g}\, t$$

$$g = \frac{1}{6\lambda n (m t_U)^2} \left(\frac{\varrho_{m0}}{\varrho_c}\right)^{2n+2}, \quad \varrho_c \approx 10^{-29} g/cm^3$$

and new function, proportional to $F'(R)$:

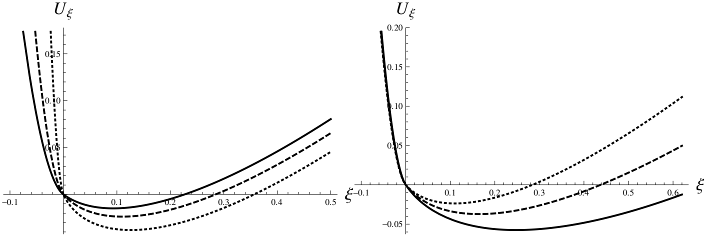
$$\xi \equiv \frac{1}{2\lambda n} \left(\frac{T_0}{R_0}\right)^{2n+1} F'_R = \frac{1}{y^{2n+1}} - gy$$

The equation of motion for ξ takes the simple oscillator form:

$$\xi'' + dU/d\xi = 0, \quad \text{where} \quad dU/d\xi = z - y(\xi).$$

y cannot be expressed through ξ analytically so we have to use different approximate expressions in different ranges of ξ.

Potential: $\mathbf{U}(\xi) = \mathbf{U}_+(\xi)\Theta(\xi) + \mathbf{U}_-(\xi)\Theta(-\xi)$



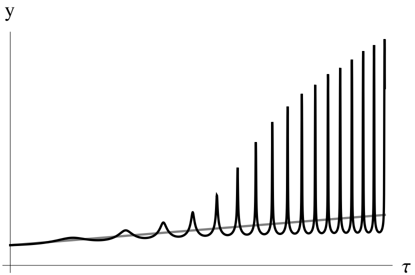
- *Left panel* $(\mathbf{n}=\mathbf{2},\ \mathbf{z}=\mathbf{1.5})$: solid line: $\mathbf{g}=\mathbf{0.02}$, dashed line: $\mathbf{g}=\mathbf{0.01}$, dotted line: $\mathbf{g}=\mathbf{0.002}$. *Right panel* $(\mathbf{n}=\mathbf{2},\ \mathbf{g}=\mathbf{0.01})$: solid line: $\mathbf{z}=\mathbf{1.3}$, dashed line: $\mathbf{z}=\mathbf{1.4}$, dotted line: $\mathbf{z}=\mathbf{1.5}$.

$$\mathbf{U}_+(\xi) = \mathbf{z}\xi - \frac{\mathbf{2n+1}}{\mathbf{2n}}\left[\left(\xi + \mathbf{g}^{(\mathbf{2n+1})/(\mathbf{2n+2})}\right)^{\mathbf{2n}/(\mathbf{2n+1})} - \mathbf{g}^{\mathbf{2n}/(\mathbf{2n+2})}\right],$$

$$\mathbf{U}_-(\xi) = \left(\mathbf{z} - \mathbf{g}^{-1/(\mathbf{2n+2})}\right)\xi + \frac{\xi^2}{\mathbf{2g}}.$$

Oscillations of $\mathbf{y}$

The oscillations of $\mathbf{y}$ are strongly unharmonic.



"Spikes" in the solutions. $\mathbf{n = 2}$, $\mathbf{g = 0.001}$

If the energy density rises with time, fast oscillations of the scalar curvature are induced, with an amplitude possibly much larger the the usual GR value $\mathbf{R = -\tilde{T}}$.

Spike-like Solutions

The maximum value of **y** in the spike region is:

$$\mathbf{y(t) \sim 6n(2n+1)mt_u \left(\frac{t_u}{t_{contr}}\right) \left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{(n+1)/2} \left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2}}$$

- *E. Arbuzova, A. Dolgov, L. Reverberi*, Eur.Phys.J.**C**(2012) 72:2247, arX:1211.5011; Phys.Rev.**D** 88, 024035 (2013), arX:1305.5668.

Solution of modified gravity equations for finite-size astronomical objects with rising energy density:

$$\mathbf{R = R_{GR}(r)y(t), \quad R_{GR} = -\tilde{T}(r) = -\frac{8\pi T^{\mu}_{\mu}(r)}{m_{Pl}^2}}$$

- $\mathbf{R_{GR}}$ is the would-be solution in the limit of GR
- quickly oscillating function $\mathbf{y(t)}$ is much larger than unity

Spherically Symmetric Solutions in $F(R)$ Gravity

- *E.V. Arbuzova, A.D. Dolgov, L. Reverberi*, arXiv: 1306.5694.

We consider a spherically symmetric bubble of matter of radius $\mathbf{r_m}$, and study spherically symmetric solution of modified EoM:

$$\mathbf{R_{00} - R/2} = \frac{\mathbf{\tilde{T}_{00} + \Delta F'_R + F/2 - RF'_R/2}}{\mathbf{1 + F'_R}}$$

$$\mathbf{R_{rr} + R/2} = \frac{\mathbf{\tilde{T}_{rr} + (\partial_t^2 + \partial_r^2 - \Delta)F'_R - F/2 + RF'_R/2}}{\mathbf{1 + F'_R}}$$

We use the Schwarzschild metric:

$$\mathbf{ds^2 = A(r,t)dt^2 - B(r,t)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)}$$

$$\mathbf{A_1 = A - 1 \ll 1 \text{ and } B_1 = B - 1 \ll 1}$$

If the energy density of matter inside the the cloud, i.e. for $\mathbf{r} < \mathbf{r_m}$, is much larger than the cosmological energy density, then:

$$\mathbf{F'_R \ll 1 \quad \text{and} \quad F \ll R}$$

Equations of Motion and General Solutions

In the weak field limit:

$$R_{00} \approx \frac{A'' - \ddot{B}}{2} + \frac{A'}{r}, \quad R_{rr} \approx \frac{\ddot{B} - A''}{2} + \frac{B'}{r}$$

$$R \approx A'' - \ddot{B} + \frac{2A'}{r} - \frac{2B'}{r} + \frac{2(1-B)}{r^2}$$

We assume that spatial derivatives of F'_R are small and we find:

$$B_1' + \frac{B_1}{r} = r\tilde{T}_{00}$$

$$A_1'' - \frac{A_1'}{r} = -\frac{3B_1}{r^2} + \ddot{B}_1 + \tilde{T}_{00} - 2\tilde{T}_{rr} + \frac{\tilde{T}_{\theta\theta}}{r^2} + \frac{\tilde{T}_{\varphi\varphi}}{r^2 \sin^2\theta} \equiv S_A$$

General solutions:

$$B_1(r,t) = \frac{C_B(t)}{r} + \frac{1}{r}\int_0^r dr' r'^2 \tilde{T}_{00}(r',t)$$

$$A_1(r,t) = C_{1A}(t) r^2 + C_{2A}(t) + \int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2} S_A(r_2,t)$$

The Schwarzschild Limit

The mass of matter inside a radius $\mathbf{r}$ is defined in the usual way:

$$\mathbf{M(r,t) = \int_0^r d^3r\, T_{00}(r,t) = 4\pi \int_0^r dr\, r^2\, T_{00}(r,t)}$$

If all matter is confined inside a radius $\mathbf{r_m}$, the total mass is $\mathbf{M \equiv M(r_m)}$ and it does not depend on time.

Since $\mathbf{\tilde{T}_{00} = 8\pi T_{00}/m_{Pl}^2}$, we obtain for $\mathbf{r > r_m}$:

$$\mathbf{B_1 = r_g/r}, \text{ where } \mathbf{r_g = 2M/m_{Pl}^2}$$

The metric coefficient $\mathbf{A_1}$ outside the source is:

$$\mathbf{A_1 = -\frac{r_g}{r} + \left[C_{1A}(t) - \frac{r_g}{2r_m^3}\right] r^2 + \left[C_{2A}(t) + \frac{3r_g}{2r_m}\right]}$$

Metric Functions inside a Cloud

The metric functions inside the cloud are equal to:

$$\mathbf{B(r,t)} = 1 + \frac{2M(r,t)}{m_{Pl}^2 r} \equiv 1 + B_1^{(Sch)}\,,$$
$$\mathbf{A(r,t)} = 1 + \frac{R(t)\, r^2}{6} + A_1^{(Sch)}(r,t)\,.$$

For the Schwarzschild part of the solution we find:

$$A_1^{(Sch)}(r,t) = \frac{r_g r^2}{2r_m^3} - \frac{3r_g}{2r_m} + \frac{\pi \ddot{\varrho}_m}{3m_{Pl}^2}\,(r_m^2 - r^2)^2$$

The oscillating part $\mathbf{R(t) r^2/6}$ gives the dominant contribution into $\mathbf{A_1}$:

- $r^2 R(t) \sim r^2 y(t)\, R_{GR}$ with $y \gg 1$, $|R_{GR}| = 8\pi \varrho_m / m_{Pl}^2$.
- the canonical Schwarzschild terms: $r_g/r_m \sim \varrho_m r_m^2/m_{Pl}^2 \sim r_m^2 R_{GR}$.

Anti-gravity inside a Cloud

For large objects, such that $Rr^2/6 \sim 1$, the approximation used here is not applicable.

If $A_1 \sim 1$, the evolution of $R(t)$ may significantly differ from presented above, but even for small A_1 there would arise interesting new effects.

In the lowest order in the gravitational interaction the motion of a non-relativistic test particle is governed by the equation:

$$\ddot{r} = -\frac{A'}{2} = -\frac{1}{2}\left[\frac{R(t)r}{3} + \frac{r_g r}{r_m^3}\right]$$

- Since $R(t)$ is always negative and large, the modifications of GR considered here lead to anti-gravity inside a cloud with energy density exceeding the cosmological one.

Gravitational repulsion dominates over the usual attraction if:

$$\frac{|R|r_m^3}{3r_g} = \frac{|R|r_m^3 m_{Pl}^2}{6M} = \frac{|R|r_m^3 m_{Pl}^2}{8\pi\varrho\, r_m^3} = \frac{|R|}{\tilde{T}_{00}} > 1$$

Conclusions

We have shown:

- In contracting astrophysical systems with rising energy density powerful oscillations of curvature scalar, **R**, are induced.
- Initially harmonic, these oscillations evolve to strongly unharmonic ones with high frequency and large amplitude, which could be much larger than the value of curvature in the standard **GR.**
- Structure formation in modified gravity would be very much different from that in the standard GR.
- Sufficiently large primordial clouds would not shrink down to smaller and smaller bodies with more or less uniform density but form thin shells empty (or almost empty) inside.
- This anti-gravitating behavior may also be a possible driving force for the creation of cosmic voids.

THE END

THANK YOU FOR THE ATTENTION!

Spherically Symmetric Solutions in $F(R)$ Gravity

Assumption: the background space-time is nearly flat and so the background metric is almost Minkowsky.
Is this approximation valid for large deviation of curvature from its GR value?

- *E.V. Arbuzova, A.D. Dolgov, L. Reverberi*, arXiv: 1306.5694.

We consider a spherically symmetric bubble of matter of radius $\mathbf{r_m}$, and study spherically symmetric solution of modified EoM

$$\left(1+F'_R\right)R_{\mu\nu}-\frac{1}{2}\left(R+F\right)g_{\mu\nu}+\left(g_{\mu\nu}D_\alpha D^\alpha-D_\mu D_\nu\right)F'_R=\tilde{T}_{\mu\nu}$$

$$3D^2F'_R-R+RF'_R-2F=\tilde{T}$$

We use the Schwarzschild metric and assume that the metric is close to the flat one:

$$ds^2=A(r,t)dt^2-B(r,t)dr^2-r^2(d\theta^2+\sin^2\theta\,d\phi^2)$$

$$A_1=A-1\ll 1 \text{ and } B_1=B-1\ll 1$$

Equations of Motion

It is convenient to use EoM in the following form:

$$\mathbf{R_{00} - R/2 \;=\; \frac{\tilde{T}_{00} + \Delta F'_R + F/2 - RF'_R/2}{1 + F'_R}}$$

$$\mathbf{R_{rr} + R/2 \;=\; \frac{\tilde{T}_{rr} + (\partial_t^2 + \partial_r^2 - \Delta)F'_R - F/2 + RF'_R/2}{1 + F'_R}}$$

In the weak field limit:

$$\mathbf{R_{00} \;\approx\; \frac{A'' - \ddot{B}}{2} + \frac{A'}{r}, \quad R_{rr} \approx \frac{\ddot{B} - A''}{2} + \frac{B'}{r}}$$

$$\mathbf{R \;\approx\; A'' - \ddot{B} + \frac{2A'}{r} - \frac{2B'}{r} + \frac{2(1 - B)}{r^2}}$$

If the energy density of matter inside the the cloud, i.e. for $\mathbf{r < r_m}$, is much larger than the cosmological energy density, then:

$$\mathbf{F'_R \ll 1 \quad \text{and} \quad F \ll R}$$

For static solutions the effects of gravity modifications in this limit are weak and the solution is quite close to the standard Schwarzschild one.

General Solutions

We assume that spatial derivatives of $\mathbf{F}'_R$ are small and we find:

$$\begin{aligned} B_1' + \frac{B_1}{r} &= r\tilde{T}_{00} \\ A_1'' - \frac{A_1'}{r} &= -\frac{3B_1}{r^2} + \ddot{B}_1 + \tilde{T}_{00} - 2\tilde{T}_{rr} + \frac{\tilde{T}_{\theta\theta}}{r^2} + \frac{\tilde{T}_{\varphi\varphi}}{r^2\sin^2\theta} \equiv S_A \end{aligned}$$

Equation for B_1 has the solution:

$$B_1(r,t) = \frac{C_B(t)}{r} + \frac{1}{r}\int_0^r dr' r'^2 \tilde{T}_{00}(r',t)$$

To avoid a singularity at $\mathbf{r = 0}$ we have to assume that $\mathbf{C_B(t) \equiv 0}$.

$$A_1(r,t) = C_{1A}(t) r^2 + C_{2A}(t) + \int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2} S_A(r_2,t)\,,$$

$$S_A = -\frac{3}{r^3}\int_0^r dr' r'^2 \tilde{T}_{00}(r',t) + \frac{1}{r}\int_0^r dr' r'^2 \ddot{\tilde{T}}_{00}(r',t) + \tilde{T}_{00} - 2\tilde{T}_{rr} + \frac{\tilde{T}_{\theta\theta}}{r^2} + \frac{\tilde{T}_{\varphi\varphi}}{r^2\sin^2\theta}$$

The Schwarzschild Limit

The mass of matter inside a radius $\mathbf{r}$ is defined in the usual way:

$$\mathbf{M(r,t) = \int_0^r d^3r\, T_{00}(r,t) = 4\pi \int_0^r dr\, r^2\, T_{00}(r,t)}$$

If all matter is confined inside a radius $\mathbf{r_m}$, the total mass is $\mathbf{M \equiv M(r_m)}$ and it does not depend on time.

Since $\mathbf{\tilde{T}_{00} = 8\pi T_{00}/m_{Pl}^2}$, we obtain for $\mathbf{r > r_m}$:

$$\mathbf{B_1 = r_g/r}, \text{ where } \mathbf{r_g = 2M/m_{Pl}^2}$$

In the region $\mathbf{r > r_m}$ we have $\mathbf{T_{\mu\nu} = 0}$ and the integral containing $\mathbf{\ddot{\tilde{T}}_{00}}$ is also zero due to total mass conservation. The remaining integral gives:

$$\mathbf{\int_r^{r_m} dr_1\, r_1 \int_{r_1}^{r_m} \frac{dr_2}{r_2} \frac{3}{r_2^3} \int_0^{r_2} dr' r'^2 \tilde{T}_{00}(r',t) = \frac{r_g}{r} + \frac{r_g\, r^2}{2r_m^3} - \frac{3r_g}{2r_m}}$$

Thus the metric coefficient outside the source is:

$$\mathbf{A_1 = -\frac{r_g}{r} + \left[C_{1A}(t) - \frac{r_g}{2r_m^3}\right] r^2 + \left[C_{2A}(t) + \frac{3r_g}{2r_m}\right]}$$

Modified Gravity Solutions

The coefficient $\mathbf{C_{1A}(t)}$ can be found from equation for $\mathbf{R}$:

$$\mathbf{R \approx A'' + \frac{2A'}{r} - \ddot{B} - \frac{2B'}{r} + \frac{2(1-B)}{r^2}}$$

- In systems with rising energy density the curvature scalar may be much larger than its value in GR.

Using eqs. for $\mathbf{A_1}$ and $\mathbf{B_1}$ and comparing them to expression for $\mathbf{R}$, we get:

$$\mathbf{C_{1A}(t) = R(t)/6},$$

$$\mathbf{R(t) \sim -6n(2n+1)mt_u \left(\frac{t_u}{t_{contr}}\right) \left[\frac{\varrho_m(t)}{\varrho_{m0}}\right]^{(n+1)/2} \left(\frac{\varrho_c}{\varrho_{m0}}\right)^{2n+2} \tilde{T}}$$

The difference between modified and standard solutions in vacuum:

- In the standard case the term proportional to $\mathbf{r^2}$ appears both at $\mathbf{r < r_m}$ and $\mathbf{r > r_m}$ with the same coefficient and must vanish.
- For modified gravity such condition is not applicable and the $\mathbf{C_{1A} r^2}$-term may be present at $\mathbf{r < r_m}$ and absent at $\mathbf{r \gg r_m}$.

Metric Functions inside a Cloud

The metric functions inside the cloud are equal to:

$$\mathbf{B(r,t)} = 1 + \frac{2M(r,t)}{m_{Pl}^2 r} \equiv 1 + B_1^{(Sch)},$$
$$\mathbf{A(r,t)} = 1 + \frac{R(t)\, r^2}{6} + A_1^{(Sch)}(r,t).$$

- The matter is nonrelativistic, so the space components of $\mathbf{T}_{\mu\nu}$ are negligible in comparison to $\mathbf{T}_{00}$.
- $\mathbf{T}_{00} \equiv \varrho_m(t)$ is spatially constant but may depend on time.

For the Schwarzschild part of the solution we find:

$$A_1^{(Sch)}(r,t) = \frac{r_g r^2}{2r_m^3} - \frac{3r_g}{2r_m} + \frac{\pi \ddot{\varrho}_m}{3m_{Pl}^2}\,(r_m^2 - r^2)^2$$

The oscillating part $\mathbf{R(t)r^2/6}$ gives the dominant contribution into $\mathbf{A_1}$:

- $r^2 R(t) \sim r^2 y\, R_{GR}$ with $y \gg 1$, $|R_{GR}| = 8\pi\varrho_m/m_{Pl}^2$.
- the canonical Schwarzschild terms: $r_g/r_m \sim \varrho_m r_m^2/m_{Pl}^2 \sim r_m^2 R_{GR}$.

Applicability of the Approximation

If the initial energy density of the cloud is of the order of the cosmological energy density

$$R_{GR} \sim 1/t_u^2,$$

the metric would deviate from the Minkowsky one for clouds having radius:

$$r_m > t_u/\sqrt{y}.$$

- Structure formation proceeds at red shifts of order unity when the density fluctuations $\delta\varrho$ became of the same order of the background cosmological energy density.

For large objects, such that $Rr^2/6 \sim 1$, the approximation used here is not applicable.
If $A_1 \sim 1$, the evolution of $R(t)$ may significantly differ from presented above, but even for small A_1 there would arise interesting new effects.

Anti-gravity inside a Cloud

In the lowest order in the gravitational interaction the motion of a non-relativistic test particle is governed by the equation:

$$\ddot{\mathbf{r}} = -\frac{\mathbf{A}'}{2} = -\frac{1}{2}\left[\frac{\mathbf{R(t)r}}{3} + \frac{\mathbf{r_g r}}{\mathbf{r_m^3}}\right]$$

- Since $\mathbf{R(t)}$ is always negative and large, the modifications of GR considered here lead to anti-gravity inside a cloud with energy density exceeding the cosmological one.

Gravitational repulsion dominates over the usual attraction if:

$$\frac{|\mathbf{R}|\mathbf{r_m^3}}{\mathbf{3r_g}} = \frac{|\mathbf{R}|\mathbf{r_m^3 m_{Pl}^2}}{\mathbf{6M}} = \frac{|\mathbf{R}|\mathbf{r_m^3 m_{Pl}^2}}{\mathbf{8\pi\varrho\, r_m^3}} = \frac{|\mathbf{R}|}{\tilde{\mathbf{T}}_{\mathbf{00}}} > 1$$