Complexification as alternative to higher dimensions: N = 2 string as a complex source of Kerr geometry.

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Based on:

A.B., Stringlike structures in the Kerr-Schild geometry: N=2 string, twistors and Calabi-Yau twofold [arXiv:1307.5021].

A.B., Complex Structure of the Four-Dimensional Kerr Geometry Adv. High Energy Phys. v.2013, ID 509749, [arXiv:1211.6021].

A.B., String-like Structures in Complex Kerr Geometry, [arXiv:gr-qc/9303003]

BLACK HOLES - STRINGS - PARTICLES

STRING THEORY: *"... realistic model of elementary particles still appears to be a distant dream." (J. Schwarz, arXiv:1201.0981)*

KERR GEOMETRY corresponds to background of an electron!
Measurable parameters of an electron (mass, spin, charge, magnetic moment) indicate that its gravitational and electromagnetic field correspond to Kerr-Newman solution.(Carter 1968, Israel 1970, AB 1974, López 1984,...et al.)

Kerr's gravity as a BRIDGE:

Spinning Particles ↔ Kerr's Gravity ↔ String theory

SPIN of particles is extreme high: over-rotating geometry without horizons!
$a / m = 10^{44}$

NAKED SINGULAR RING, which was interpreted as closed string
(Ivanenko& AB 1975). Fundamental string solutions to low-energy string theory (Witten 1985, Horowitz & Steif 1990, Sen 1992, A. Tseytlin 1993, AB 1995.) Strings as Solitons & Black Holes as Strings, (Dabholkar at al 1995).

KERR's STRINGY SYSTEM

Second *complex* string appears in complex structure of Kerr geometry, (AB 1993).

TWISTOR STRUCTURE OF THE KERR GEOMETRY. *Inherent Calabi-Yau space appears as a quartic in the projective twistor space* CP^3 .

The closed Kerr string and open complex string form together 4D string-membrane system , which is parallel with string/M-theory unification (AB, arXiv:1211.6021).

PROPOSITION: Emergence of this similarity is the N = 2 superstring, structure of which is remarkable similar to structure of COMPLEX SOURCE OF KERR GEOMETRY!

Recently, these stringy structures were independently discussed by Adamo and Newman: *"...It would have been a cruel god to have layed down such a pretty scheme and not have it mean something deep."* (Adamo&Newman, PRD 2011).

N=2 superstring is one of the three consistent critical string theories: D=4, D=10, and D=26. N=2 string (D=4) is complex.

M.Geen, J.Schwarz and E.Witten, *Superstring Theory* V.1.

"...N = 2 extension of the superstring construction gives a highly symmetric two-dimensional (complex) theory an interesting generalization of the super-Virasoro algebra. It seemingly cannot be given the usual interpretation of a string theory... Perhaps it enters physics in some other and yet unknown way... ... crucial subtleties in this theory have not yet been unraveled."

$$Z^\mu = X^\mu + iY^\mu, \quad \mu = 0, 1$$

$$S = -\frac{1}{2\pi} \int d^2\sigma \{\partial_\alpha Z \partial^\alpha \bar{Z} - i\bar{\psi}\gamma^\alpha \partial_\alpha \psi\}$$

The global N = 2 supergauge transformations $\delta Z = \bar{\epsilon}\psi, \quad \delta\psi = -i\gamma^\alpha \epsilon \partial_\alpha Z.$

"...there are no transverse oscillations at all... the massless scalar ground state is the only propagating degree of freedom...(at least for this sector). However, subtleties in the quantization... have been pointed out recently, and this statement may require revision."

"Subtleties" of the N=2 string could be connected with problem of boundary conditions which requires *orientifold projection* (invented later by L.Dixon,J.A.Harvey, C.Vafa, and E.Witten, *Nucl.Phys.*1987).

Complex Structure of the Kerr geometry. Complex shift. Appel 1887!

Complex shift of the Coulomb solution $\phi(\vec{x})$ in Cartesian coordinates $\vec{x} = (x, y, z)$

$\mathbf{T_a}\phi(\tilde{\mathbf{x}}) = \phi_{\mathbf{a}}(\tilde{\mathbf{x}} + \mathbf{i}\tilde{\mathbf{a}})$, where $\tilde{\mathbf{a}} = (\mathbf{0}, \mathbf{0}, -\mathbf{ia})$, creates in the real slice Appel solution $\phi_{\mathbf{a}} = \mathbf{Re}\frac{\mathbf{e}}{\mathbf{r} + \mathbf{ia}\cos\theta}$, in oblate spheroidal coordinates r and θ.
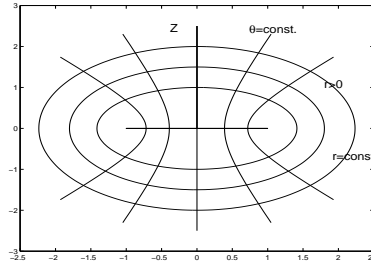
- Complex shift $T_a : (x, y, z) \to (x, y, z - ia)$ of the Schwarzschild solution in the Kerr-Schild form $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, creates in the real slice the Kerr solution.

- Complex shift $T_a : (x, y, z) \to (x, y, z - ia)$ of the Reisner-Nordström solution in the Kerr-Schild form $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, creates the Kerr-Newman solution.

- Global N=2 super-translation $ST_{a,\theta}$: of the Kerr-Newman solution in the Kerr-Schild form creates **in the real slice** the Super-Kerr-Newman solution to broken N=2 Supergravity (AB, arXiv:hep-th/9903032).

Trivial operation of the shift is combined with **nonlinear operation of the real slice.**

Complex retarded-time construction. (Lind & Newman, 1974)

Kerr's source is considered as a mysterious "particle" propagating along a *complex world-line* $x_0^\mu(\tau)$ parametrized by complex time $\tau = t + i\sigma$.
System of the complex light cones emanating by *complex world-line*
$(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$ splits into families of the "Left" and "Right" complex null planes:

$$\mathcal{K}_\mathbf{L} = \{\mathbf{x} : \mathbf{x} = \mathbf{x}_0^\mathbf{i}(\tau_\mathbf{L}) + \psi_\mathbf{L}^\mathbf{A} \sigma_{\mathbf{A}\dot{\mathbf{A}}}^\mathbf{i} \tilde{\psi}_\mathbf{R}^{\dot{\mathbf{A}}}\}. \tag{1}$$

"**Left**" ($\psi_\mathbf{L}$ =const; $\tilde{\psi}_\mathbf{R}$ -var.) and "**Right**"($\tilde{\psi}_\mathbf{R}$ =const; $\psi_\mathbf{L}$ -var.).
The Kerr congruence $\mathcal{K}$ emerges as real slice of the "Left" null planes ($Y = const.$).

Complex Kerr String. (AB, gr-qc/9303003)

Complex world line $\mathbf{x}_0^\mu(\mathbf{t} + \mathbf{i}\sigma)$ is really a world sheet of a complex string parametrized by $\mathbf{t}$ and σ. Real slice fixes $\sigma = \mathbf{a}\cos\theta \Rightarrow \sigma \in [-\mathbf{a}, \mathbf{a}]$. String is open with the end points $\sigma = \pm\mathbf{a}$. Boundary conditions require orientifolding the world-sheet.
The orientifold parity $\sigma \to -\sigma$ reverses orientation of the world sheet, and covers it second time in mirror direction. Two oriented copies of the interval $\Sigma = [-\mathbf{a}, \mathbf{a}]$, are joined, forming world-sheet of a closed but folded string.

REAL structure of the Kerr-Newman solution: Metric

$$\mathbf{g}_{\mu\nu} = \eta_{\mu\nu} + 2\mathbf{H}\mathbf{k}_\mu\mathbf{k}_\nu, \quad \mathbf{H} = \frac{\mathbf{mr} - \mathbf{e}^2/2}{\mathbf{r}^2 + \mathbf{a}^2\cos^2\theta}, \tag{2}$$

and **electromagnetic vector potential is** $\mathbf{A}^\mu_{\mathbf{KN}} = \mathbf{Re}\frac{\mathbf{e}}{\mathbf{r}+\mathbf{ia}\cos\theta}\mathbf{k}^\mu$. **The Kerr singular ring is a**
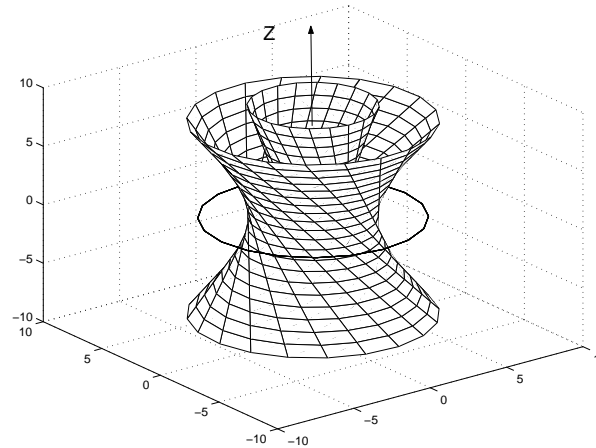
branch line forming TWOSHEETED Kerr space!
Kerr congruence is controlled by **KERR THEOREM**: as analytic solution of the equation $F(T^a) = 0$, where F is a holomorphic function of the ***projective twistor coordinates*** $\mathbf{T^a} = \{\mathbf{Y}, \quad \zeta - \mathbf{Yv}, \quad \mathbf{u} + \mathbf{Y}\bar{\zeta}\} \in \mathbf{CP^3}$. For the Kerr-Newman solution function $\mathbf{F}$ is *quadratic* in $\mathbf{Y}$, which yields TWO roots $\mathbf{Y}^\pm(\mathbf{x})$ resulting in **twosheeted** Kerr background! Functions $\mathbf{F}(\mathbf{T^a})$ of higher degrees in Y correspond to **multi-sheeted geometry** and multi-particle solutions, [AB (2006)]. Orientifold doubles the number of Kerr's sheets, which is described by *quartic* eq. in $\mathbf{CP^3}$, creating inherent **Calabi -Yau twofold**.
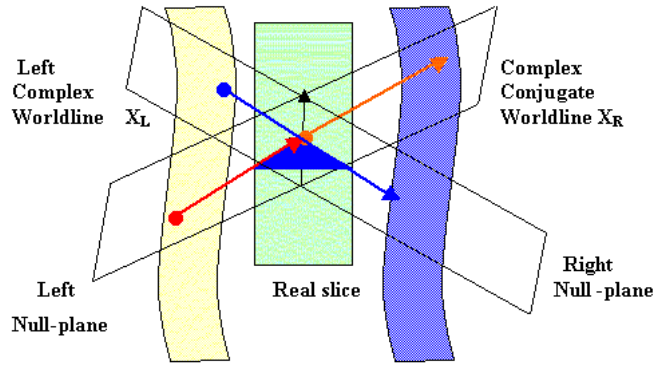
Figure 1: **Four roots for the retarded and advanced times, X_L^{adv}, X_L^{ret} and X_R^{adv}, X_R^{ret} creating the K3 surface.**
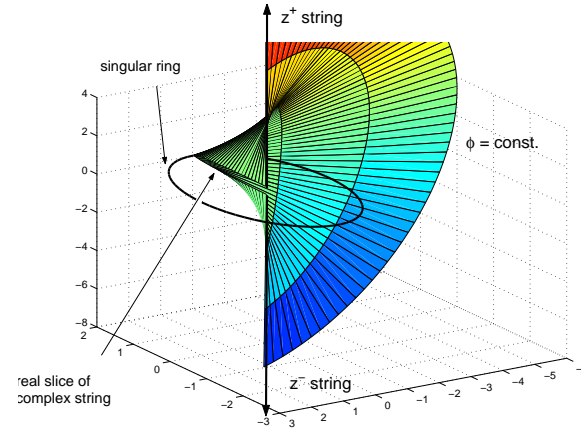

Figure 2: **One sheet of the K3 for $r > 0$ and $\phi = const$. Kerr congruence is tangent to singular ring at $\theta = \pi/2$.**

The real closed string and complex Kerr string form together a 4D string-membrane system is parallel with string/M-theory unification (AB, arXiv:1211.6021)

The inherent Calabi-Yau space (K3 surface) is projected into real 4D Kerr geometry in the form of analytic extension of the Kerr principal null congruence.

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Problem of embedding of the N = 2 string was principal obstacle for its application: embedding in the real minkowskian space-time is only consistent with signatures (2,2) or (4,0).

There is no problem for embedding in the *complex* 4D Kerr geometry, since diverse sections may have different signatures.

Kerr's complex source and supersymmetry? *"...Wess-Zumino formalism may have close connection with the Twistor formalism of Penrose..."*(Salam-Strathdee, 1974) Fermionic part of the N = 2 superstring (the Dirac spinor) plays important role fixing the Left null planes of twistorial structure of the complexified 4d Kerr geometry.

   Conclusion.
I. N = 2 superstring may consistently be embedded in the complex 4D Kerr geometry, playing the role of its complex source.

II. Complexification is alternative to higher dimensions.

III. Planck scale is replaced by Compton scale of Kerr geometry.

THANK YOU FOR ATTENTION!