$\mu-\tau$ Symmetry, Nonzero θ_{13}, and CP Violation

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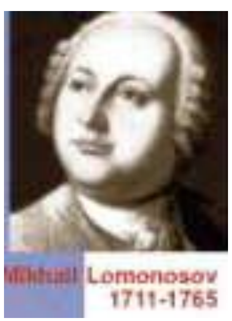

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- $\mu-\tau$ Symmetry and J_{CP}
- Conclusions

Introduction

- Since the confirmation of neutrino oscillations (1988), neutrino oscillations phenomena have been observed in various neutrinos coming from the Sun, Accelerators, and Reactors.
- Neutrino oscillation is interpreted in the term of mixing of the three flavors of neutrinos: ν_e, ν_μ, ν_τ related to three neutrino eigenstates mass basis: ν_1, ν_2, ν_3 by mixing angle *V:*

$$\nu_i = V_{ij}\nu_j \;, \quad i = e, \mu, \tau, \text{and } j = 1,2,3$$

- Observed in experiments: three mixing angles and two squared-mass differences:

$$\theta_{12}, \theta_{23}, \theta_{13}, \Delta m_{21}^2, \Delta m_{32}^2$$

Neutrino oscillation → neutrino eigenstates in flavor basis and neutrino eigenstates in mass basis which is related by neutrino mixing matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{3\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$

Eigenstates in flavor basis

Mixing Matrix

Eigenstates in mass basis

TBM BM DC

Neutrino mixing matrix can be parameterized as follow:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\phi} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\phi} & c_{12}c_{23} - s_{12}s_{23}e^{i\phi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\phi} & -c_{12}s_{23} - s_{12}c_{23}e^{i\phi} & c_{23}c_{13} \end{pmatrix} \quad (2)$$

From the three well-known mixing matrices (TBM, BM, DC), the special mixing matrix that can be related to simple underlying symmetry is TBM:

$$V_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (3)$$

which can be related to $\mu - \tau$ symmetry. But, TBM predicts mixing angle $\theta_{13} = 0$

and mixing angle θ_{23} is maximal.

As one can see from Eq. (3) that the entry $V_{e3} = 0$ which imply that the mixing angle θ_{13} must be zero in the TBM. However, the latest result from long baseline neutrino oscillation experiment T2K indicates that θ_{13} is nonzero and relatively large. For a vanishing Dirac CP-violating phase ($\delta = 0$), the T2K collaboration reported that the values of θ_{13} for neutrino mass in normal hierarchy (NH) are [3]:

$$5.0^o \leq \theta_{13} \leq 16.0^o, \tag{4}$$

and for neutrino mass in inverted hierarchy (IH):

$$5.8^o \leq \theta_{13} \leq 17.8^o, \tag{5}$$

The current combined world data for neutrino squared-mass differences [36,37]:

$$\Delta m^2_{21} = 7.59 \pm 0.20(^{+0.61}_{-0.69}) \times 10^{-5}\ \text{eV}^2, \tag{6}$$

$$\Delta m^2_{32} = 2.46 \pm 0.12(\pm 0.37) \times 10^{-3}\ \text{eV}^2,\ \text{(for NH)} \tag{7}$$

$$\Delta m^2_{32} = -2.36 \pm 0.11(\pm 0.37) \times 10^{-3}\ \text{eV}^2,\ \text{(for IH)} \tag{8}$$

$$\theta_{12} = 34.5 \pm 1.0(^{3.2}_{-2.8})^o,\ \ \theta_{23} = 42.8^{+4.5}_{-2.9}(^{+10.7}_{-7.3})^o,\ \ \theta_{13} = 5.1^{+3.0}_{-3.3}(\leq 12.0)^o, \tag{9}$$

at 1σ (3σ) level. The latest experimental result for the value of θ_{13} is reported by Daya Bay Collaboration which gives [4]:

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat.}) \pm 0.005(\text{syst.}), \tag{10}$$

and RENO Collaboration reported that [5]:

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat.}) \pm 0.014(\text{syst.}). \tag{11}$$

Modified TBM

In this talk, the modified TBM (V'_{TBM}) is obtained by introducing a perturbation matrix such that:

$$V_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_y & s_y e^{-i\delta} \\ 0 & -s_y e^{i\delta} & c_y \end{pmatrix}. \quad (13)$$

where c_y is the $\cos y$, s_y is the $\sin y$, and δ is the Dirac CP phase.

By inserting Eqs. (3) and (13) into Eqs. (12), we then have the modified neutrino mixing matrix as follow:

$$V'_{\rm TBM} = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3}c_y & \frac{\sqrt{3}}{3}s_y e^{-i\delta} \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3}c_y - \frac{\sqrt{2}}{2}s_y e^{i\delta} & \frac{\sqrt{3}}{3}s_y e^{-i\delta} + \frac{\sqrt{2}}{2}c_y \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3}c_y + \frac{\sqrt{2}}{2}s_y e^{i\delta} & \frac{\sqrt{3}}{3}s_y e^{-i\delta} - \frac{\sqrt{2}}{2}c_y \end{pmatrix}, \quad (14)$$

If we compare this modified TBM to Standard neutrino mixing matrix V in Eq. (2), then we have:

$$\tan\theta_{12} = \left|\frac{\sqrt{2}c_y}{2}\right|, \quad \tan\theta_{23} = \left|\frac{\frac{\sqrt{3}}{3}s_y e^{-i\delta} + \frac{\sqrt{2}}{2}c_y}{\frac{\sqrt{3}}{3}s_y e^{-i\delta} - \frac{\sqrt{2}}{2}c_y}\right|, \quad \sin\theta_{13} = \left|\frac{\sqrt{3}}{3}s_y\right|. \tag{15}$$

and for $\delta = 0$ [10]:

$$\tan\theta_{12} = \left|\frac{\sqrt{2}c_y}{2}\right|, \quad \tan\theta_{23} = \left|\frac{\frac{\sqrt{3}}{3}s_y + \frac{\sqrt{2}}{2}c_y}{\frac{\sqrt{3}}{3}s_y - \frac{\sqrt{2}}{2}c_y}\right|, \quad \sin\theta_{13} = \left|\frac{\sqrt{3}}{3}s_y\right|. \tag{16}$$

From Eq. (16) we have the relation between the three mixing angles as follow:

$$\tan\theta_{23} = \left|\frac{\sin\theta_{13} + \tan\theta_{12}}{\sin\theta_{13} - \tan\theta_{12}}\right| \tag{17}$$

From Eq. (17) we can determine mixing angle θ_{13} by using the advantage of experimental values of mixing angles θ_{12} and θ_{23} from Eq. (9) with its mean value indeed, then we have:

$$\theta_{13} = 7.89^0 \tag{18}$$

which is in agreement with the T2K [3] and Daya Bay [4] experimental results.

The three equations in (15) can also combined to one equation as follow:

$$\tan\theta_{23} = \left| \frac{\sin\theta_{13} e^{-i\delta} + \tan\theta_{12}}{\sin\theta_{13} e^{-i\delta} - \tan\theta_{12}} \right| \tag{19}$$

If we insert the values of mixing angles from experimental results as shown in (9) especially for the values of mixing angles θ_{12}, θ_{23}, and θ_{13} from (18), then we have:

$$\delta = 77.20^o \tag{20}$$

$\mu-\tau$ Symmetry and J_{CP}

Concerning the $\mu-\tau$ symmetry, a lot of papers have discussed it together with its relation to mixing angle (reactor angle, θ_{13}) [20] and its implication to the origin of matter via leptogenesis [21]. The effect of $\mu-\tau$ symmetry broken in the neutrino mass matrix that can arise the CP violation have been proposed in Ref [26].

In this talk, we construct a neutrino mass matrix with the assumption that the charged lepton mass matrix is diagonal in flavor basis, then in this basis we have neutrino mass matrix:

$$M_\nu = VMV^T \tag{21}$$

where

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{22}$$

and V is the modified TBM (V'_{TBM}) in Eq. (14). Neutrino mass matrix in this scheme is given by:

$$M_\nu = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix}, \tag{23}$$

where:

$$A = \frac{2m_1}{3} + \frac{m_2}{3}c_y^2 + \frac{m_3}{3}s_y^2 e^{-2i\delta}, \tag{24}$$

$$B = -\frac{m_1}{3} + m_2\left(\frac{1}{3}c_y^2 - \frac{\sqrt{6}}{6}c_y s_y e^{i\delta}\right) + m_3\left(\frac{1}{3}s_y^2 e^{-2i\delta} + \frac{\sqrt{6}}{6}s_y c_y e^{-i\delta}\right), \tag{25}$$

$$C = -\frac{m_1}{3} + m_2\left(\frac{1}{3}c_y^2 + \frac{\sqrt{6}}{6}c_y s_y e^{i\delta}\right) + m_3\left(\frac{1}{3}s_y^2 e^{-2i\delta} - \frac{\sqrt{6}}{6}s_y c_y e^{-i\delta}\right), \tag{26}$$

$$D = \frac{m_1}{6} + m_2\left(\frac{\sqrt{3}}{3}c_y - \frac{\sqrt{2}}{2}s_y e^{i\delta}\right)^2 + m_3\left(\frac{\sqrt{3}}{3}s_y e^{-i\delta} + \frac{\sqrt{2}}{2}c_y\right)^2, \tag{27}$$

$$E = \frac{m_1}{6} + m_2\left(\frac{1}{3}c_y^2 - \frac{1}{2}s_y^2 e^{2i\delta}\right) + m_3\left(\frac{1}{3}s_y^2 e^{-2i\delta} - \frac{1}{2}c_y^2\right), \tag{28}$$

$$F = \frac{m_1}{6} + m_2\left(\frac{\sqrt{3}}{3}c_y + \frac{\sqrt{2}}{2}s_y e^{i\delta}\right)^2 + m_3\left(\frac{\sqrt{3}}{3}s_y e^{-i\delta} - \frac{\sqrt{2}}{2}c_y\right)^2, \tag{29}$$

The Jarlskog rephasing invariant J_{Cp} can be determined from relation [38]:

$$J_{\mathrm{CP}} = -\frac{\mathrm{Im}\left[(M_\nu^{'})_{e\mu}(M_\nu^{'})_{\mu\tau}(M_\nu^{'})_{\tau e}\right]}{\Delta m_{21}^2 \Delta m_{32}^2 \Delta m_{31}^2} \tag{30}$$

which gives:

$$J_{CP} \neq 0. \tag{31}$$

But, if we impose the $\mu - \tau$ symmetry as a constraint to neutrino mass matrix in (23) we must put : $B = C$ and $D = F$ that give:

$$\frac{m_2}{m_3} = e^{-2i\delta}. \tag{32}$$

The neutrino mass matrix in this symmetry read:

$$M_\nu = \begin{pmatrix} P & Q & Q \\ Q & R & S \\ Q & S & R \end{pmatrix}, \tag{33}$$

where:

$$P = \frac{1}{3}\left(2m_1 + m_2\right), \tag{34}$$

$$Q = \frac{1}{3}(m_2 - m_1), \tag{35}$$

$$R = \frac{1}{6}\left(m_1 + m_2(2 + 3e^{2i\delta})\right), \tag{36}$$

$$S = \frac{1}{6}\left(m_1 + m_2(2 - 3e^{2i\delta})\right). \tag{37}$$

which give:

$$J_{CP} = 0. \tag{38}$$

Conclusions

The nonzero and relatively large θ_{13} from the latest experimental results have a serious implication on the well-known neutrino mixing matrix. One of the well-known mixing matrix is tribimaximal (TBM) neutrino mixing matrix which predict $\theta_{13} = 0$. In order to accommodate nonzero θ_{13} and CP violation, we modified TBM by introducing a simple perturbation matrix into TBM matrix that can produces $\theta_{13} = 7.89$ which is in agreement with the present experimental results. The Dirac phase $\delta = 77.20^o$ and the Jarlskog rephasing invariant: $J_{\rm CP} \approx 0.044$ are also obtained. The obtained neutrino mass matrix from the modified TBM with both nonzero θ_{13} and δ is the complex neutrino mass matrix. If we impose the $\mu - \tau$ symmetry, as a constraint into neutrino mass matrix,one find that the Jarlskog rephasing invariant: $J_{\rm CP} = 0$ which implies that CP violation cannot be accommodated in the $\mu - \tau$ symmetry scheme.

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Thank you !!!