Particle Accretion onto Miniholes

Michael L. Fil'chenkov

Yuri P. Laptev

Institute of Gravitation and Cosmology,
Peoples' Friendship University of Russia, Moscow

Outline

- **Introduction**
- **Classical Accretion onto Black Holes**
- **Quantum Hydrodynamics**
- **Quantum Accretion on Miniholes**
- **Conclusion**

Introduction

A quantum accretion of baryons onto miniholes has been considered in the framework of E. Madelung's quantum hydrodynamics. The total luminosity of the accretion disc and the energy of quanta being emitted have been calculated. The results obtained may be of importance for interpreting the electromagnetic radiation of graviatoms comprising particles captured by miniholes.

Classical Accretion onto Black Holes

A classical model of disc accretion onto black holes at the final stage of close binary evolution may serve as the starting point.

Due to a differential rotation of the disc there arise shear stresses between adjacent layers resulting in angular momentum transfer outward. The matter in the interior approaches the disc centre. The gas is being heated up to elevated temperatures because of friction emitting thermal radiation.

The total luminosity of the accretion disc

$$L_d = \frac{GM\dot{m}}{2R_i}, \qquad (1)$$

where M is the black hole mass, $\dot{m}$ is the accretion rate, R_i is the last stable orbit radius. Since

$$R_i = \frac{6GM}{c^2}, \qquad (2)$$

we obtain

$$L_d = \frac{1}{12}\dot{m}c^2. \qquad (3)$$

Quantum Hydrodynamics

Calculation of a quantum accretion of baryons onto the miniholes may be considered in the framework of E. Madelung's quantum hydrodynamics.

The wave function is written in the form

$$\Psi = \sqrt{\rho} e^{i\frac{S}{\hbar}}, \qquad (4)$$

where ρ is the probability density, S is the action. From Schrödinger's equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + U\Psi \qquad (5)$$

follows the equations of quantum hydrodynamics:

$$\frac{\partial\rho}{\partial t}+\frac{1}{m}\nabla\left(\rho\nabla S\right)=0, \qquad (6)$$

$$\frac{\partial S}{\partial t}+\frac{1}{2m}\left(\nabla S\right)^2+U-\frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}=0, \qquad (7)$$

where the last term

$$Q=-\frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} \qquad (8)$$

is Bohm’s potential, m is the particle mass.

From (7) we have

$$\frac{\partial S}{\partial t}=-E,\ \nabla S=\sqrt{2m\left(E-U-Q\right)}\ . \qquad (9)$$

Quantum Accretion on Miniholes

The stationary quantum accretion rate defined in terms of a sub-stantial derivative reads

$$\dot{m}=\int \nabla S \nabla \rho dV\,, \tag{10}$$

which reduces to

$$\dot{m}=4\pi \sum_{n=2}^{\infty} \int_{3r_g}^{\infty} \frac{dS_n}{dr} \frac{d\rho_n}{dr} r^2 dr\,, \tag{11}$$

where

$$\frac{dS_n}{dr} = \sqrt{2m[E_n - U(r) - Q_n]}, \tag{12}$$

$$U(r) = -\frac{GMm}{r}, \tag{13}$$

$$r_g = \frac{2GM}{c^2}, \tag{14}$$

$$E_n = Q_n, \tag{15}$$

$$\rho_n = \frac{\exp\left(-\frac{2r}{na_B^g}\right)}{2\pi n a_B^g r^2}, \tag{16}$$

providing

$$r \gg n a_B^g,\ r \gg l^2 a_B^g, \tag{17}$$

where

$$a_B^g = \frac{\hbar^2}{GMm^2}\,. \tag{18}$$

Thus we consider asymptotic values of the baryon location probabilities on higher hydrogen-like levels.

The quantum accretion intensity

$$I = \frac{\sqrt{\pi}}{3}\alpha_g^2 A(\alpha_g)\frac{m^2c^4}{\hbar}, \tag{19}$$

where

$$A(\alpha_g) = \sum_{n=2}^{\infty}\frac{1}{n^{3/2}}\left[1-\Phi\left(\alpha_g\sqrt{\frac{12}{n}}\right)\right], \tag{20}$$

$$\alpha_g = \frac{GMm}{\hbar c} \tag{21}$$

$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt. \qquad (22)$$

The thermal radiation energy

$$\hbar\omega=0.8424A^{1/4}(\alpha_g)mc^2. \qquad (23)$$

So, we have

$$I=5.75\cdot 10^{19}\,erg\,s^{-1} \text{ and } \hbar\omega=515\,MeV$$

for $\alpha_g=\frac{1}{2}$, $m=m_p$.

Conclusion

The baryons do not satisfy the geometrical condition of graviatom existence, since their sizes exceed the gravitational radii of miniholes. There occurs a quantum accretion of baryons onto mini-holes which we have considered in the framework of E. Madelung's quantum hydrodynamics. The energies and intensities of the thermal radiation being due to the accretion of baryons onto mini-holes exceed the corresponding values for the dipole radiation of the graviatoms providing the baryons' sizes to be neglected.