Recent progress in determination of fundamental constants (CODATA 2010)

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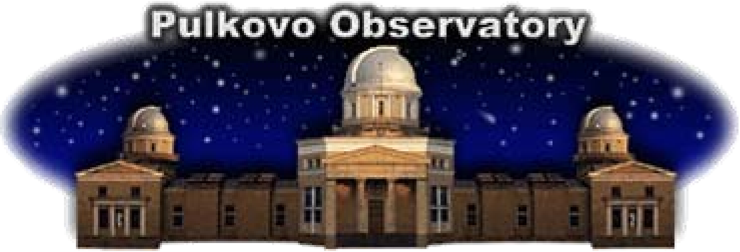
GARCHING

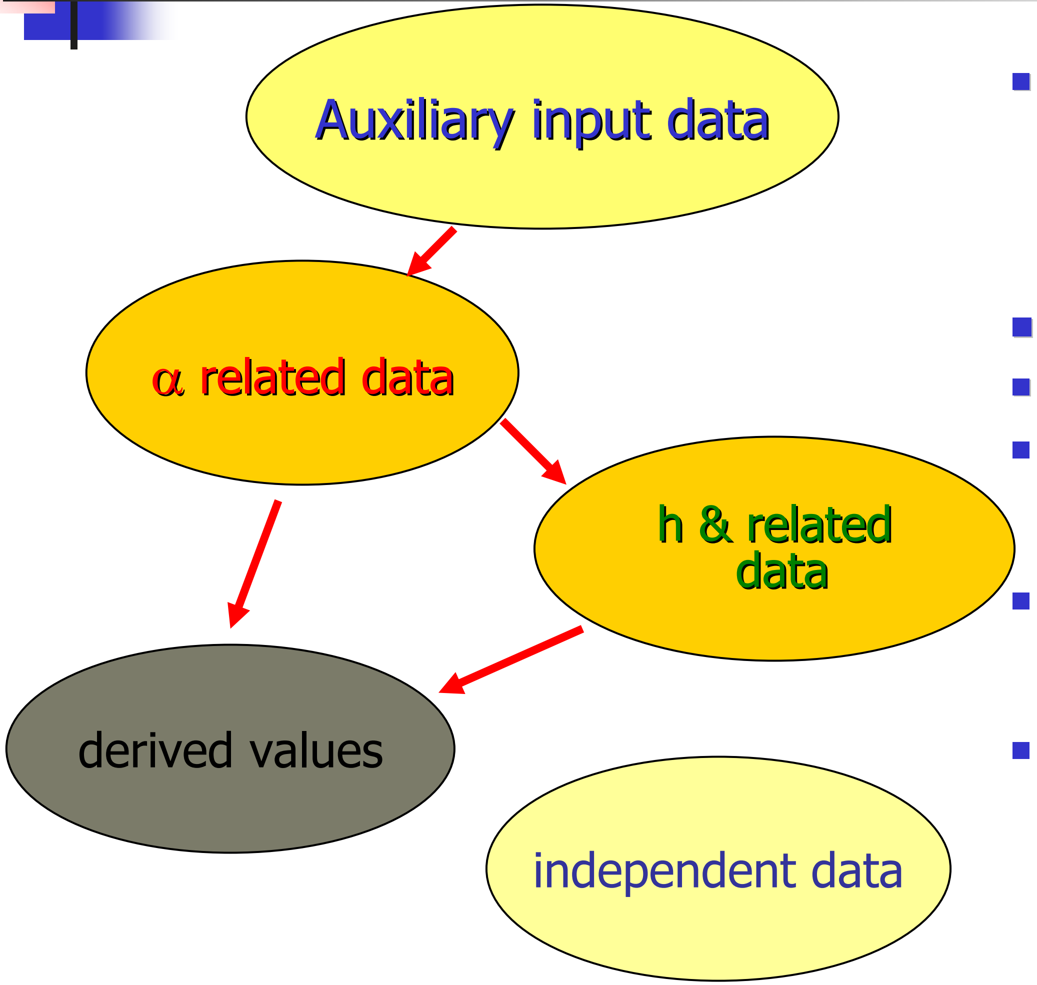
MPQ



Outline

- *structure of input and output*
- *auxiliary data*
  - Rydberg and R_p
  - m_e/m_p
- α
- h
- *mass of a particle*
- *independent constants*
  - G
  - k
- *progress: 2006 vs. 2010*
- *problems*

Structure of the input data and output values



Auxiliary input data

α related data

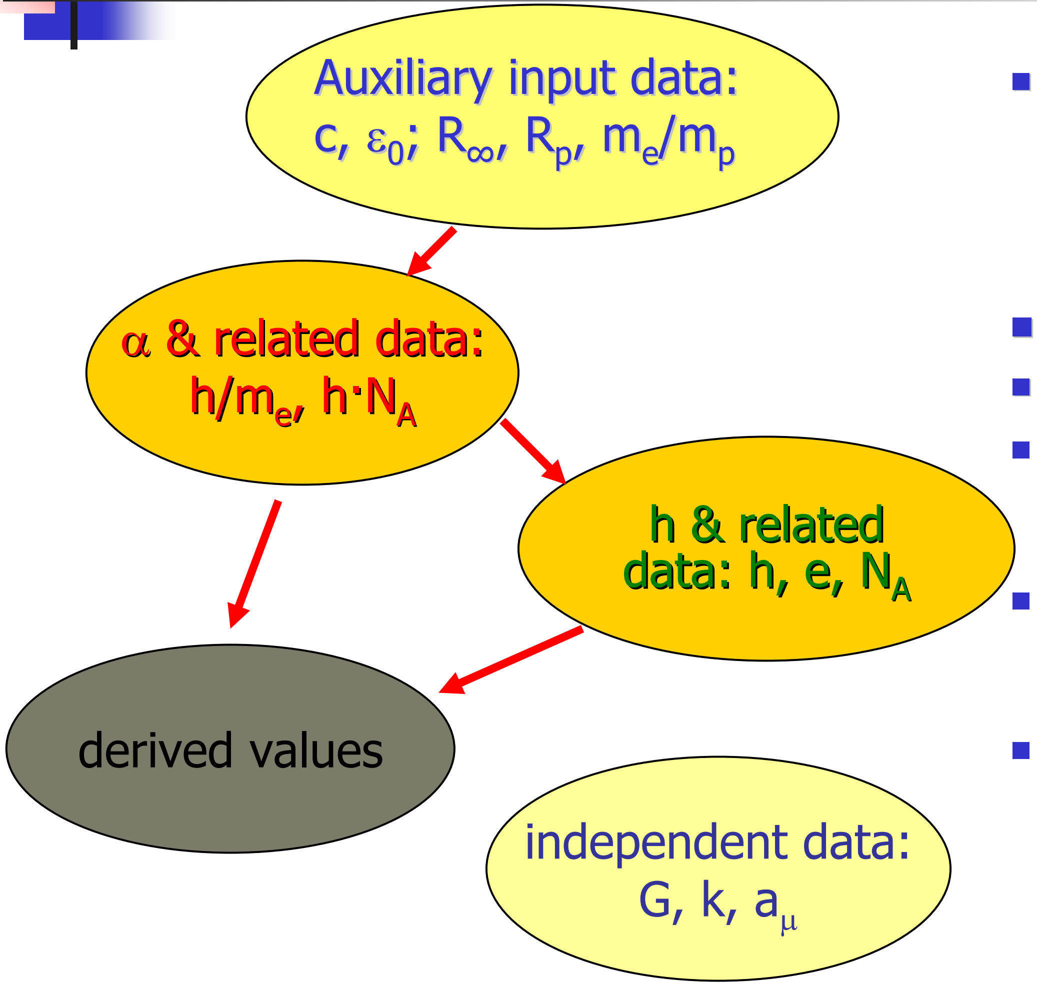
h & related data

derived values

independent data

- Auxiliary data = **exact** + the most accurate data which are to be evaluated prior the adjustment: R_∞, m_e/m_p, atomic masses.
- α related data: h/m, hN_A ...
- h related data: e, e/h, ...
- The lines (→) are equations: e.g., theoretical expressions for h/M, the Lamb shift, ...
- Some data are measured, a lot are derived: m_p [kg], m_e [Mev/c^2], ...
- G is uncorrelated,...

Structure of the input data and output values



Auxiliary input data: c, ε_0; R_∞, R_p, m_e/m_p

α & related data: h/m_e, $h \cdot N_A$

h & related data: h, e, N_A

derived values

independent data: G, k, a_μ

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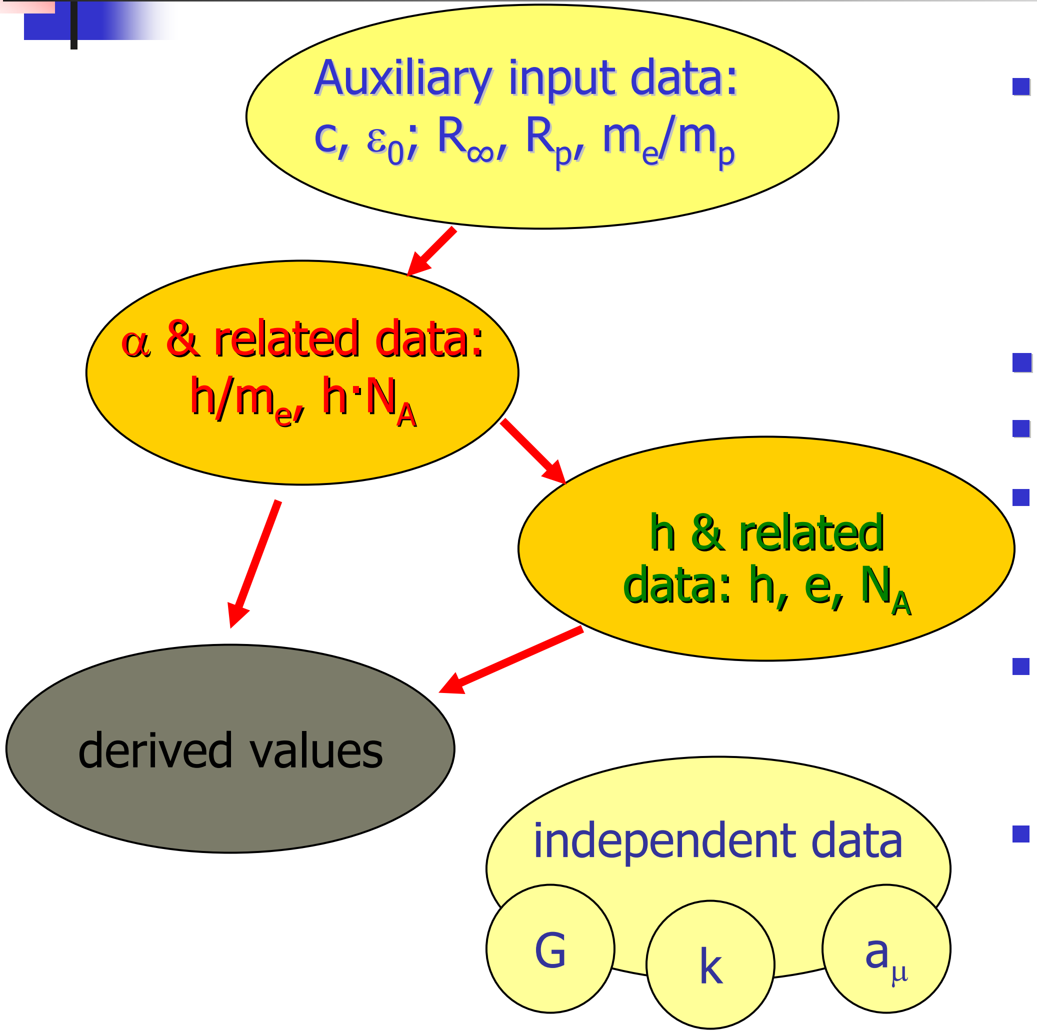

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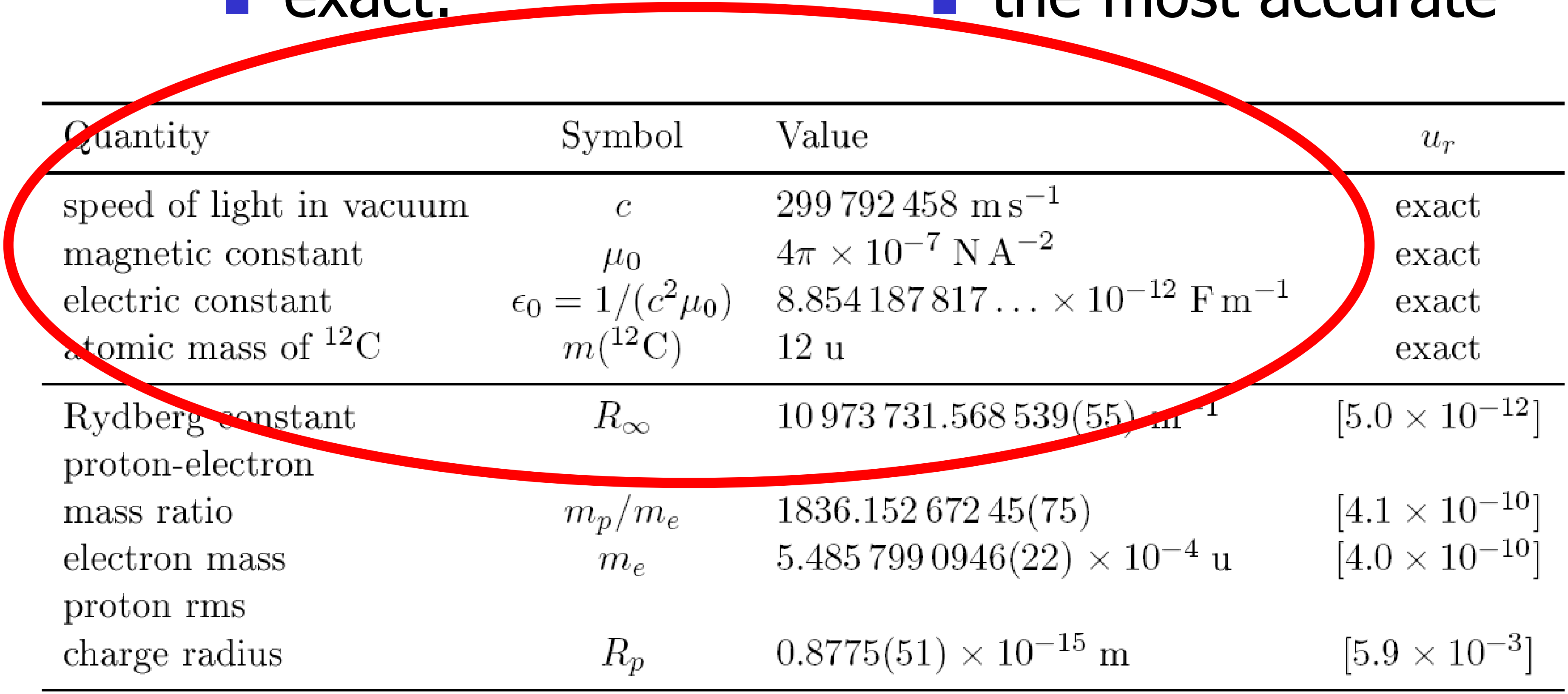
Auxiliary data

- exact:
- the most accurate



Quantity	Symbol	Value	u_r
speed of light in vacuum	c	$299\,792\,458\ \mathrm{m\,s^{-1}}$	exact
magnetic constant	μ_0	$4\pi \times 10^{-7}\ \mathrm{N\,A^{-2}}$	exact
electric constant	$\epsilon_0 = 1/(c^2\mu_0)$	$8.854\,187\,817\ldots \times 10^{-12}\ \mathrm{F\,m^{-1}}$	exact
atomic mass of $^{12}\mathrm{C}$	$m(^{12}\mathrm{C})$	$12\ \mathrm{u}$	exact
Rydberg constant	R_∞	$10\,973\,731.568\,539(55)\ \mathrm{m^{-1}}$	$[5.0 \times 10^{-12}]$
proton-electron mass ratio	m_p/m_e	$1836.152\,672\,45(75)$	$[4.1 \times 10^{-10}]$
electron mass	m_e	$5.485\,799\,0946(22) \times 10^{-4}\ \mathrm{u}$	$[4.0 \times 10^{-10}]$
proton rms charge radius	R_p	$0.8775(51) \times 10^{-15}\ \mathrm{m}$	$[5.9 \times 10^{-3}]$

Example: multiplicative vs. additive: R_∞ vs. α

- equations:

$$1/2\,\alpha^2 = R_\infty \frac{h}{m_e c}$$

$$c_1 R_\infty c + c_2 \alpha^2 R_\infty c = \nu$$

- uncertainties:
  - $R_\infty \sim 10^{-11}$
  - $\alpha \sim 10^{-9} - 10^{-10}$

  - $\alpha^2 \to 10^{-4} \times 10^{-9}$

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`almost´

exact

Auxiliary data

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- the most accurate:

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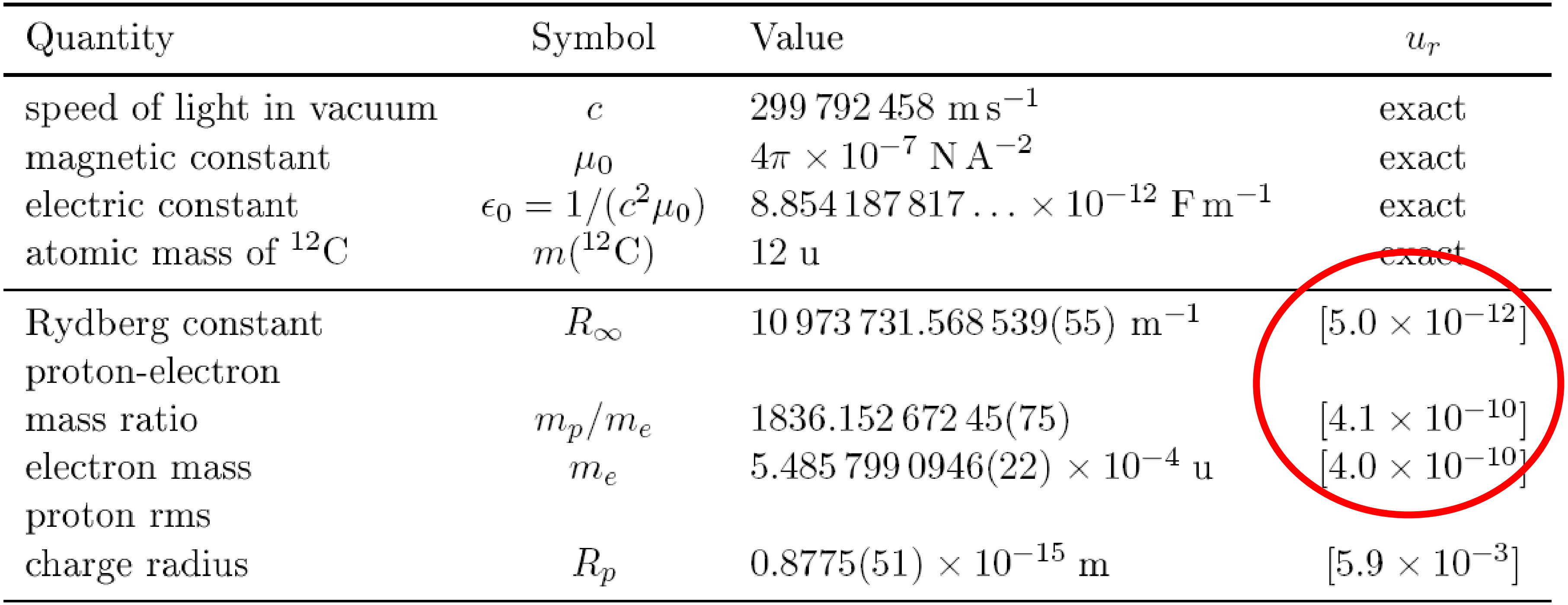

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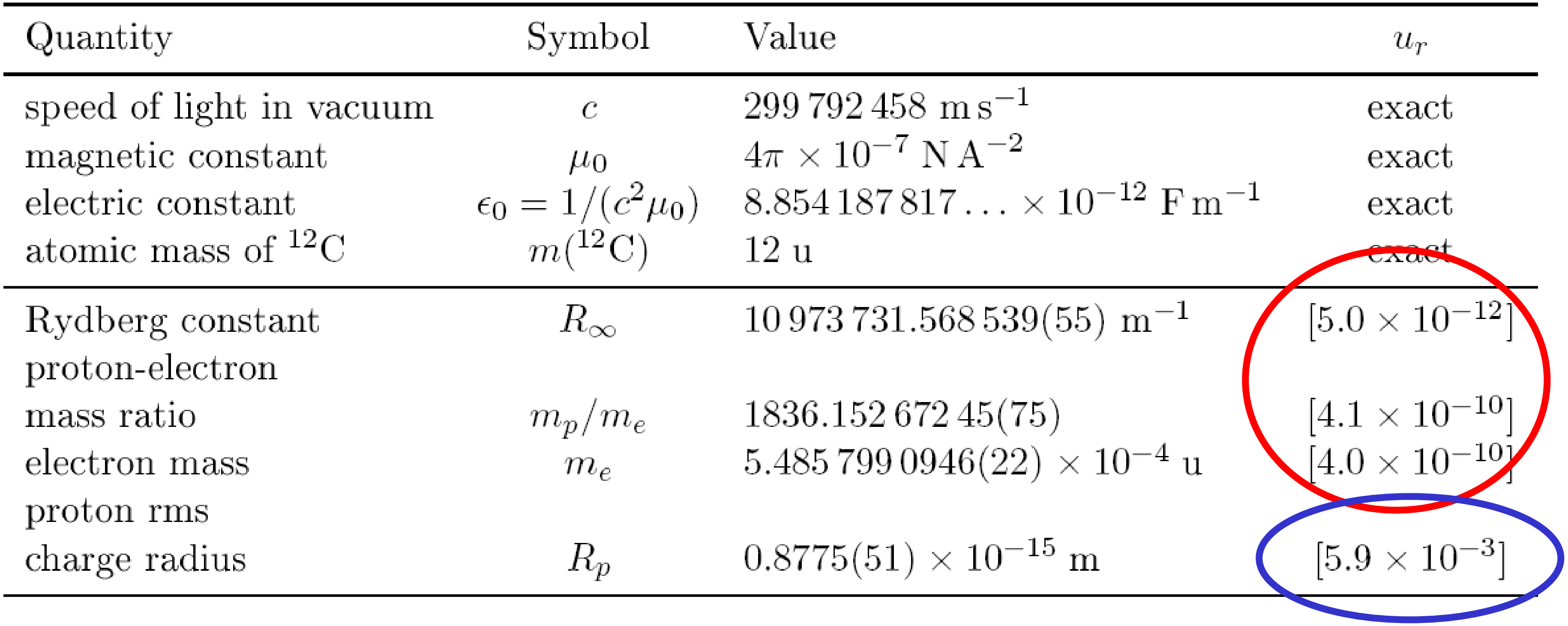

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Atomic & nuclear masses

Quantity	Symbol	Value	u_r
atomic mass of ^{16}O	$m(^{16}\mathrm{O})$	15.994 914 619 57(18) u	$[1.1 \times 10^{-11}]$
atomic mass of ^{28}Si	$m(^{28}\mathrm{Si})$	27.976 926 534 96(62) u	$[2.2 \times 10^{-11}]$
atomic mass of ^{87}Rb	$m(^{87}\mathrm{Rb})$	86.909 180 535(10) u	$[1.2 \times 10^{-10}]$

Quantity	Symbol	Value	u_r
proton mass	m_p	1.007 276 466 812(90) u	$[8.9 \times 10^{-11}]$
deuteron mass	m_d	2.013 553 212 712(77) u	$[3.8 \times 10^{-11}]$
triton mass	m_t	3.015 500 7134(25) u	$[8.2 \times 10^{-10}]$
helion mass	m_h	3.014 932 2468(25) u	$[8.3 \times 10^{-10}]$
alpha particle mass	m_α	4.001 506 179 125(62) u	$[1.5 \times 10^{-11}]$

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen

- LKP (Paris), MPQ (Garching),...

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen
- MAMI = Mainzer Mikrotron
- old world data

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen

- CREMA collaboration @ PSI

Spectroscopy of hydrogen (and deuterium)

Two-photon spectroscopy involves a number of levels strongly affected by QED.

In “old good time” we had to deal only with 2s Lamb shift.

Theory for p

simple sin

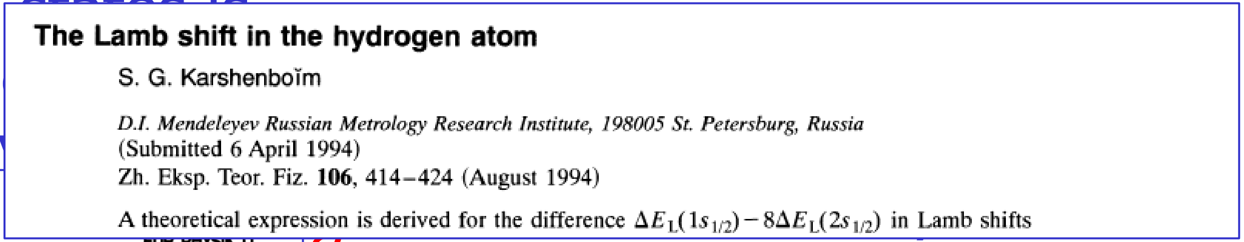
functions

The idea is based on theoretical study of

$$\Delta(2) = L_{1s} - 2^3 \times L_{2s}$$

which we understand much better since any short distance effect vanishes for $\Delta(2)$.

variables to determine: the 1s Lamb shift L_{1s} & R_∞.

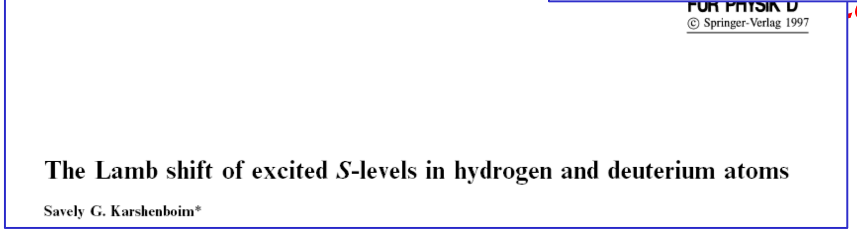


The Lamb shift in the hydrogen atom

S. G. Karshenboĭm

D.I. Mendeleyev Russian Metrology Research Institute, 198005 St. Petersburg, Russia
(Submitted 6 April 1994)
Zh. Eksp. Teor. Fiz. **106**, 414–424 (August 1994)

A theoretical expression is derived for the difference $\Delta E_L(1s_{1/2}) - 8\Delta E_L(2s_{1/2})$ in Lamb shifts



Z. Phys. D 39, 109–113 (1997)

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The Lamb shift of excited *S*-levels in hydrogen and deuterium atoms

Savely G. Karshenboim*

The Lamb shift in muonic hydrogen: experiment

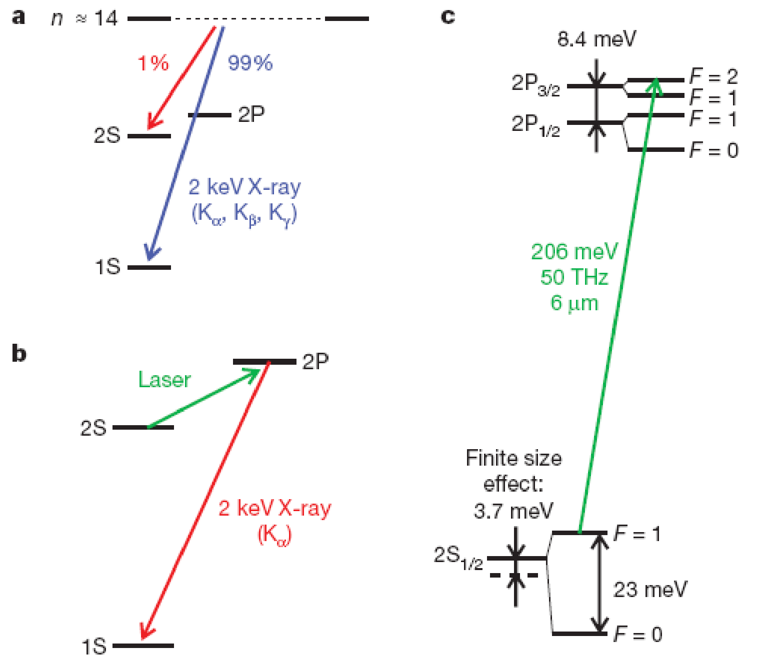


Figure 1 | Energy levels, cascade and experimental principle in muonic hydrogen. a, About 99% of the muons proceed directly to the 1S ground state during the muonic cascade, emitting 'prompt' K-series X-rays (blue). 1% remain in the metastable 2S state (red). **b**, The μp(2S) atoms are illuminated by a laser pulse (green) at 'delayed' times. If the laser is on resonance, delayed K_α X-rays are observed (red). **c**, Vacuum polarization dominates the Lamb shift in μp. The proton's finite size effect on the 2S state is large. The green arrow indicates the observed laser transition at $\lambda = 6$ μm.

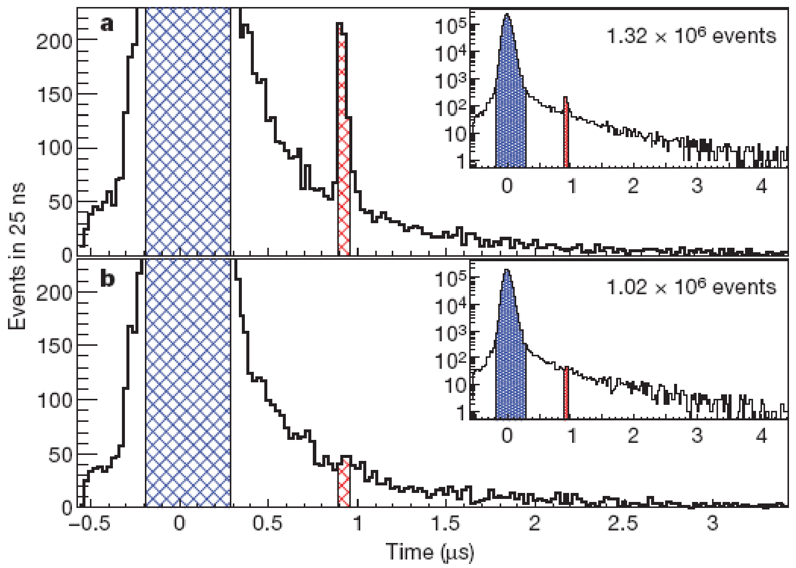


Figure 4 | Summed X-ray time spectra. Spectra were recorded on resonance (**a**) and off resonance (**b**). The laser light illuminates the muonic atoms in the laser time window $t \in [0.887, 0.962]$ μs indicated in red. The 'prompt' X-rays are marked in blue (see text and Fig. 1). Inset, plots showing complete data; total number of events are shown.

The size of the proton

Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

The Lamb shift in muonic hydrogen: experiment

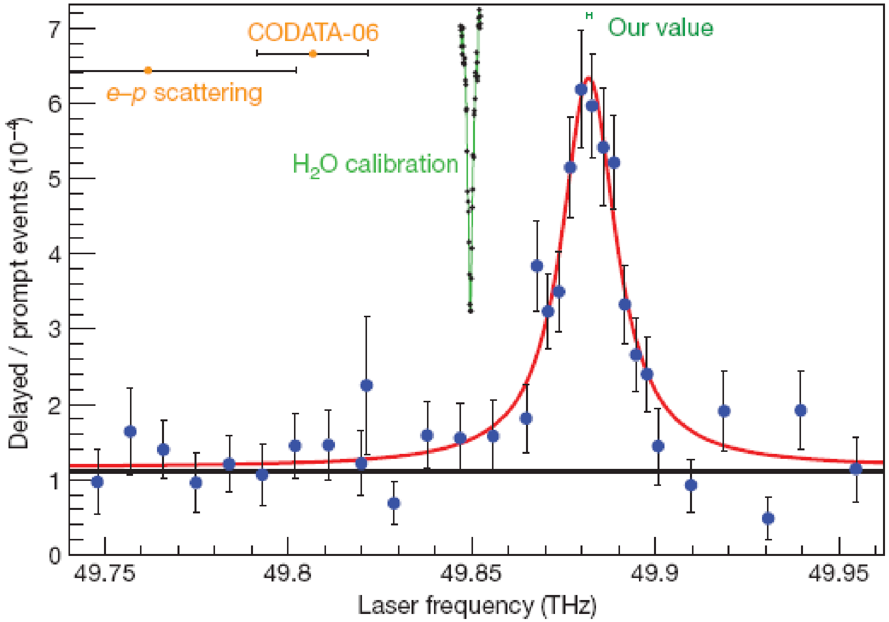
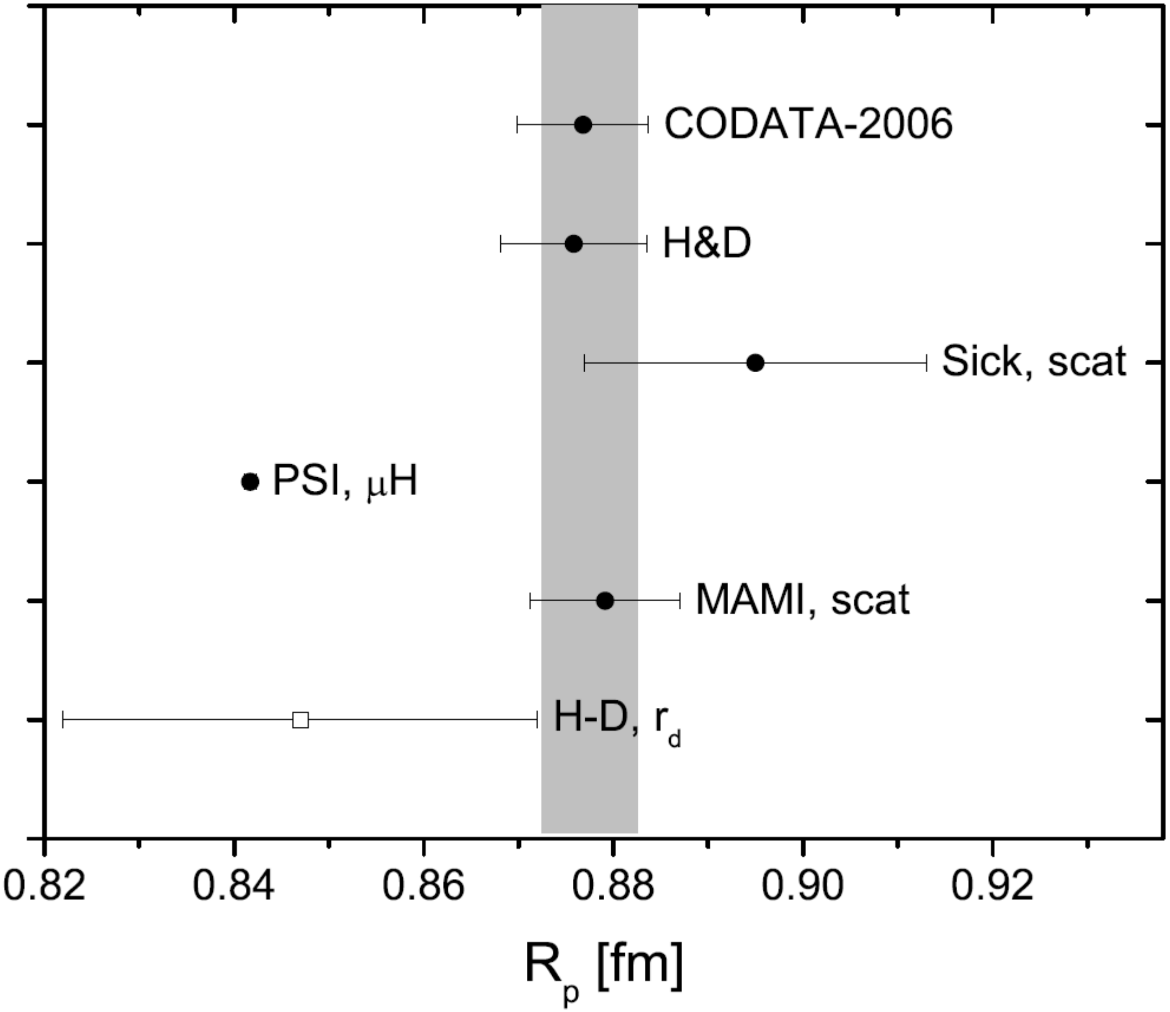
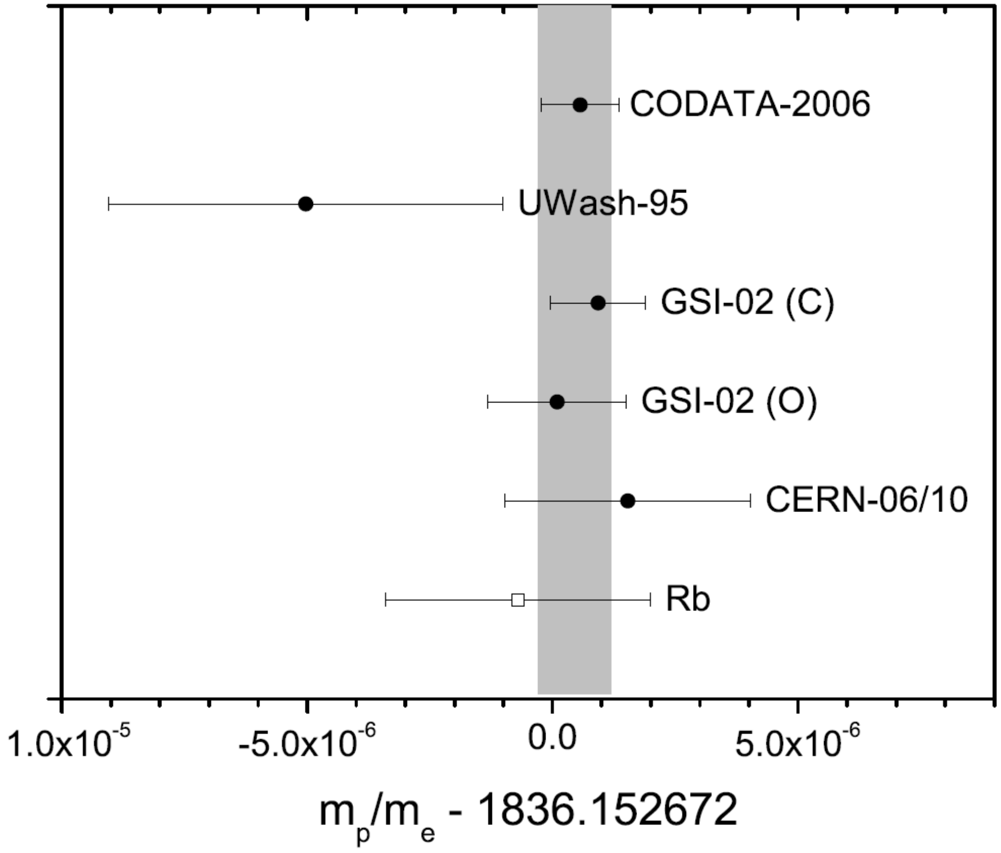


Figure 5 | Resonance. Filled blue circles, number of events in the laser time window normalized to the number of 'prompt' events as a function of the laser frequency. The fit (red) is a Lorentzian on top of a flat background, and gives a χ^2/d.f. of 28.1/28. The predictions for the line position using the proton radius from CODATA[3] or electron scattering[1,2] are indicated (yellow data points, top left). Our result is also shown ('our value'). All error bars are the ±1 s.d. regions. One of the calibration measurements using water absorption is also shown (black filled circles, green line).

Proton radius puzzle



electron-to-proton mass ratio



- cyclotron frequencies of e & p (UWash)
- g factor of a bound e in H-like ion (magnetic moment precession vs. ion cyclotron frequency) @ Mainz
- antiprotonic He spectroscopy (ASACUSA @ CERN)

α block

- equations:

$$\boxed{R_\infty = \frac{\alpha^2 m_e c}{2h}}$$

- input data
  - α
  - h/m_e

α block

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$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- m_e/m_p

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α block

- equations:

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- m_e/m_p
- m_p in u
- m_{at} in u

- input data
  - α
  - h/m_e
  - h/m_p
  - h/m_{at}

α block

- equations:

$$\alpha^2 m \quad c$$

$$m(^{12}\mathrm{C})/12 \cdot N_A = 1\,\mathrm{g\,mol}^{-1}$$

- m_e/m_p
- m_p in u
- m_{at} in u

- input data
  - α
  - h/m_e
  - h/m_p
  - h/m_{at}
- output
  - $h{\cdot}N_A$

$$\frac{mc^2}{h} = \frac{1}{(h \cdot N_A)} \times \frac{m}{m(^{12}\mathrm{C})/12} \times c^2 \times \left(m(^{12}\mathrm{C})/12 \cdot N_A\right)$$

α block

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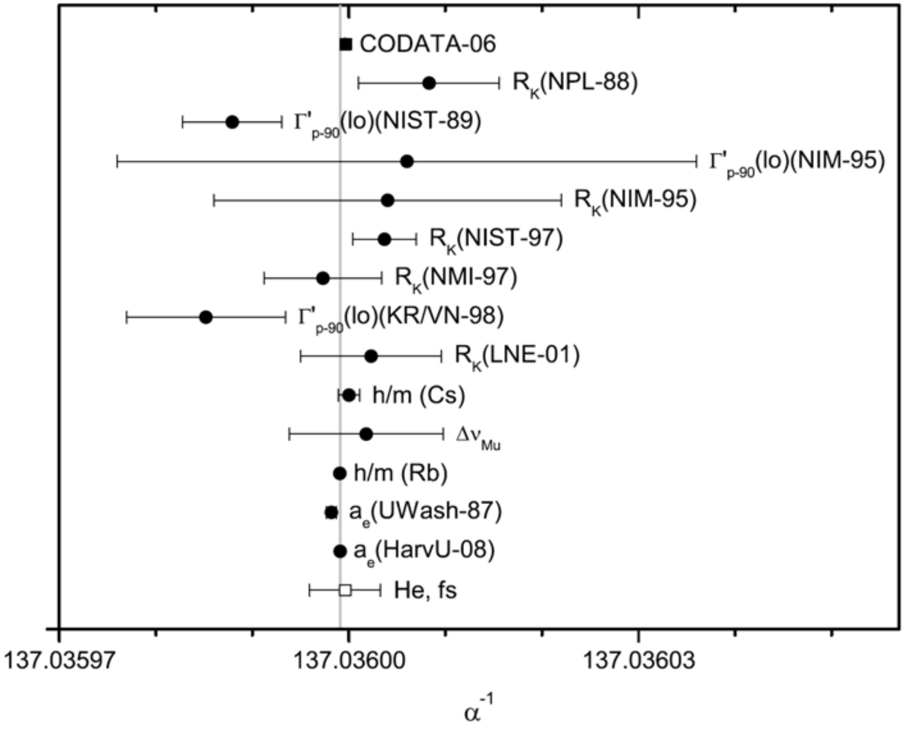
$$\frac{mc^2}{h} = \frac{1}{(h \cdot N_A)} \times \frac{m}{m(^{12}\mathrm{C})/12} \times c^2 \times (m(^{12}\mathrm{C})/12 \cdot N_A)$$

α block

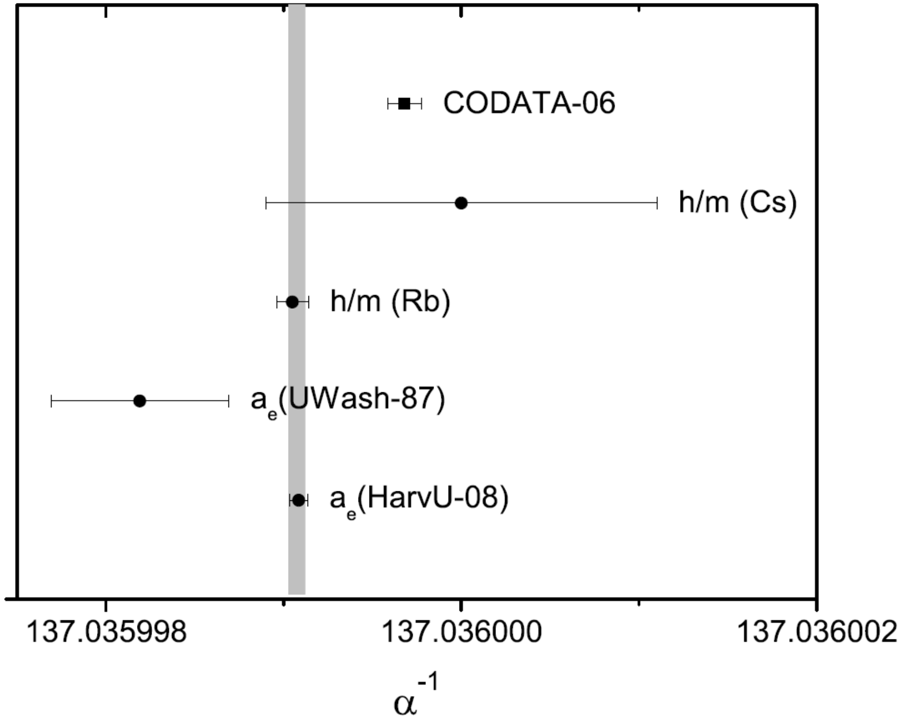
Quantity	Symbol	Value	u_r
inverse fine structure constant	α^{-1}	$137.035\,999\,074(44)$	$[3.2 \times 10^{-10}]$
molar Planck constant	$h \cdot N_A$	$3.990\,312\,7176(28) \times 10^{-10}$ J s mol^{-1}	$[7.0 \times 10^{-10}]$
quantum of circulation	$h/(2m_e)$	$3.636\,947\,5520(24) \times 10^{-4}$ m^2 s^{-1}	$[6.5 \times 10^{-10}]$
Compton wavelength	$\lambda_{\mathrm{C}} = h/(m_e c)$	$2.426\,310\,2389(16) \times 10^{-12}$ m	$[6.5 \times 10^{-10}]$
von Klitzing constant	$R_K = h/e^2$	$25\,812.807\,4434(84)\ \Omega$	$[3.2 \times 10^{-10}]$
muon-electron mass ratio	m_μ/m_e	$206.768\,2843(52)$	$[2.5 \times 10^{-8}]$

α block



- QED vs. Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- quatum Hall standard vs calculable capacitor: R_K

α block



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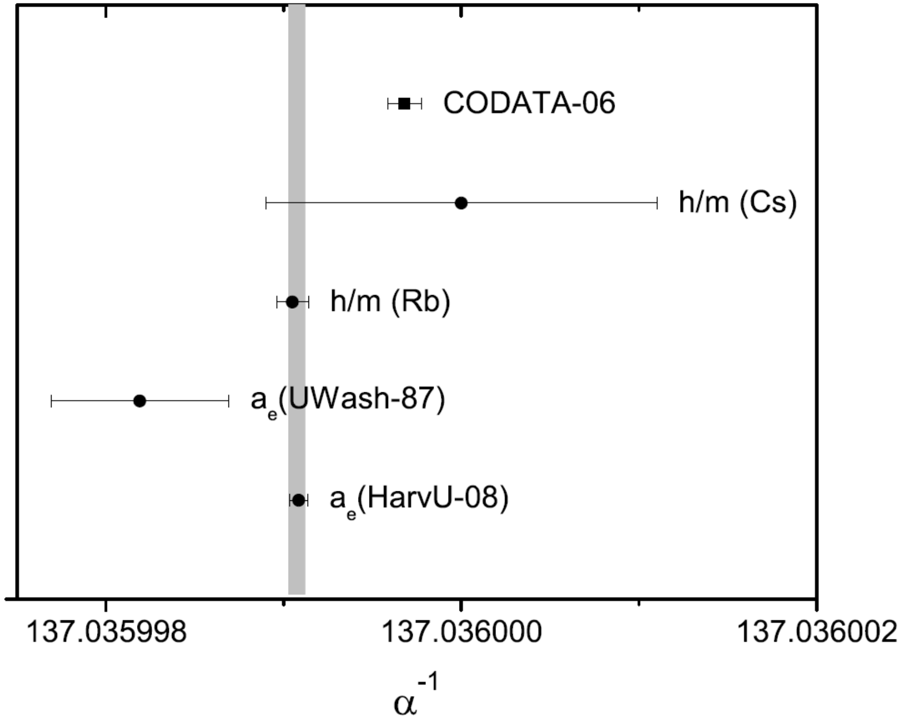
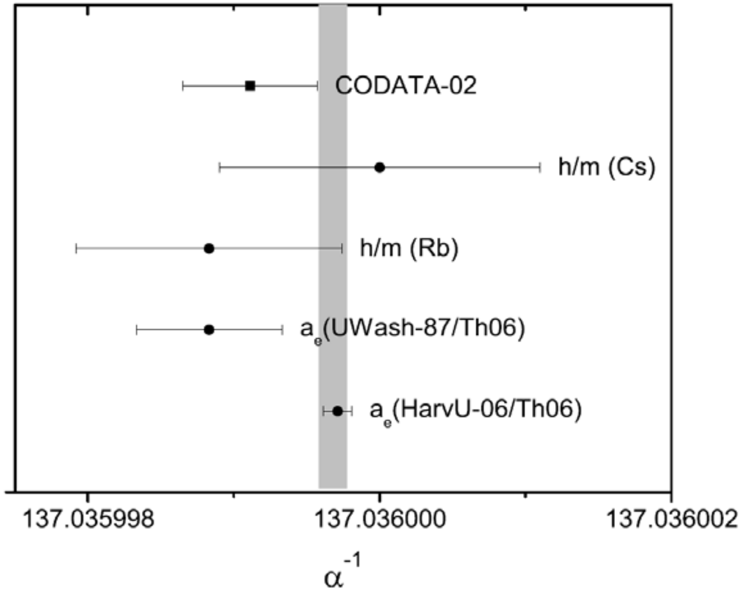
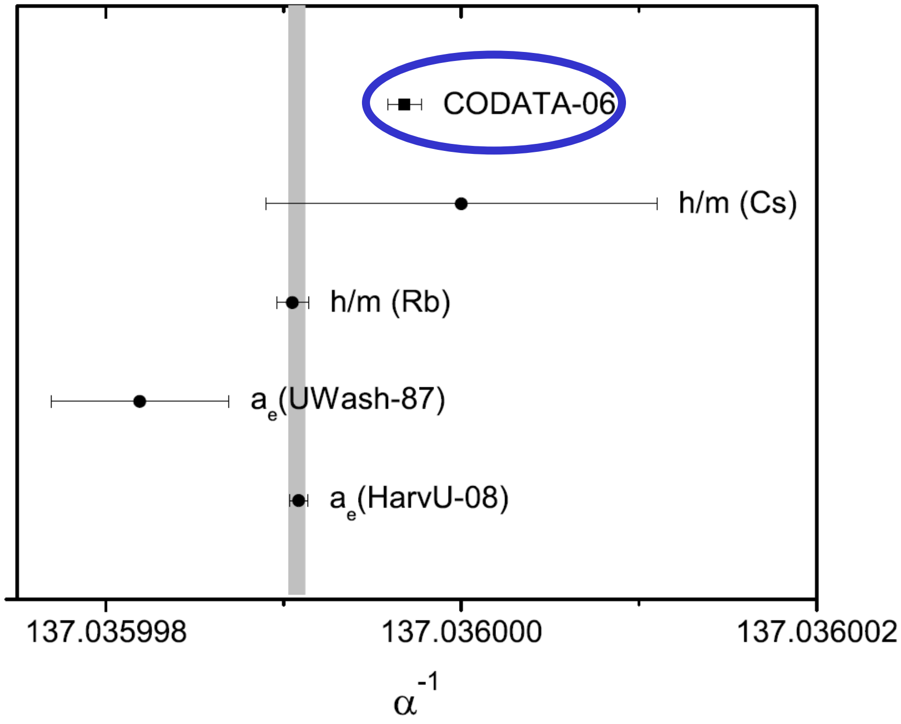

CODATA-06
h/m (Cs)
h/m (Rb)
a_e(UWash-87)
a_e(HarvU-08)
137.035998
137.036000
137.036002
α^{-1}

- 2006:

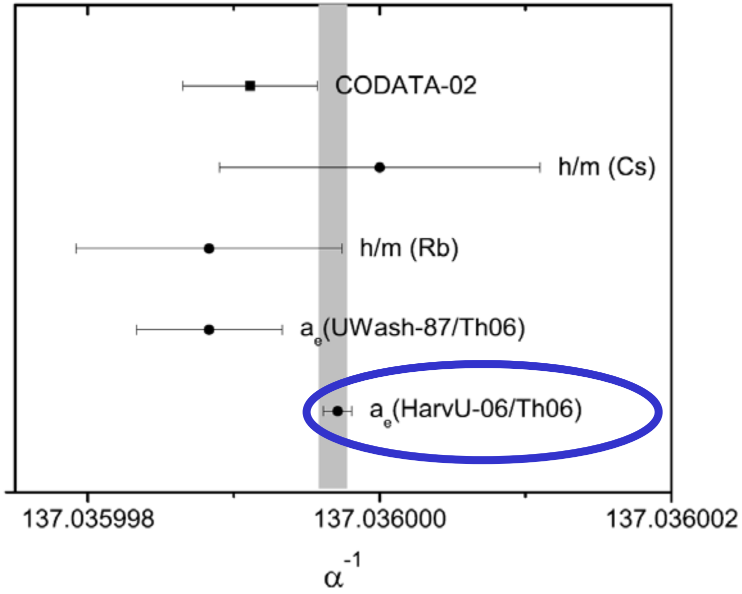

CODATA-02
h/m (Cs)
h/m (Rb)
a_e(UWash-87/Th06)
a_e(HarvU-06/Th06)
137.035998
137.036000
137.036002
α^{-1}

α block

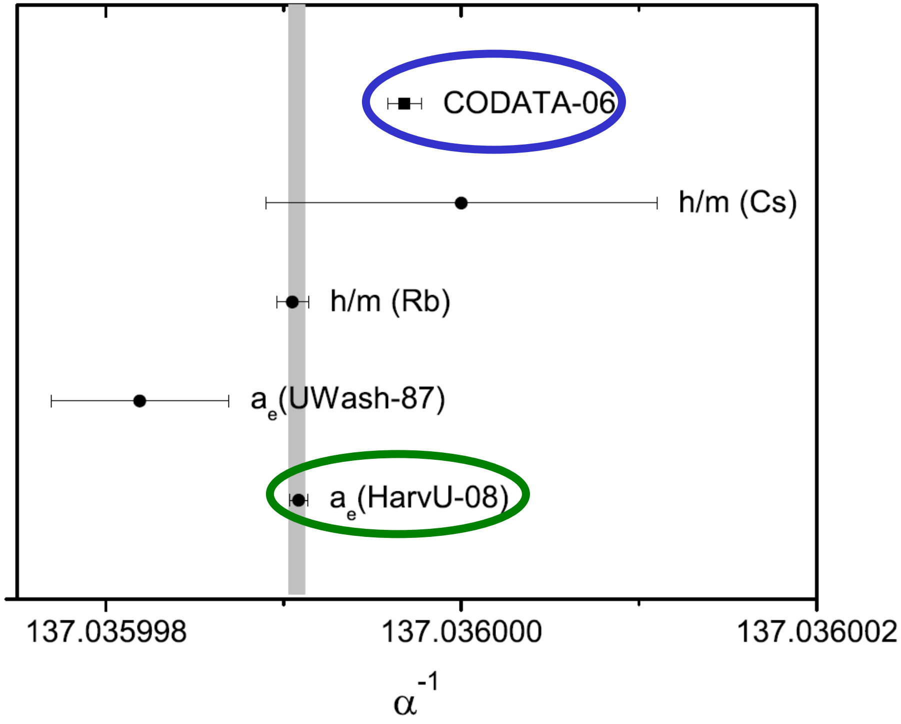


- old CODATA = old a_e

- 2006:

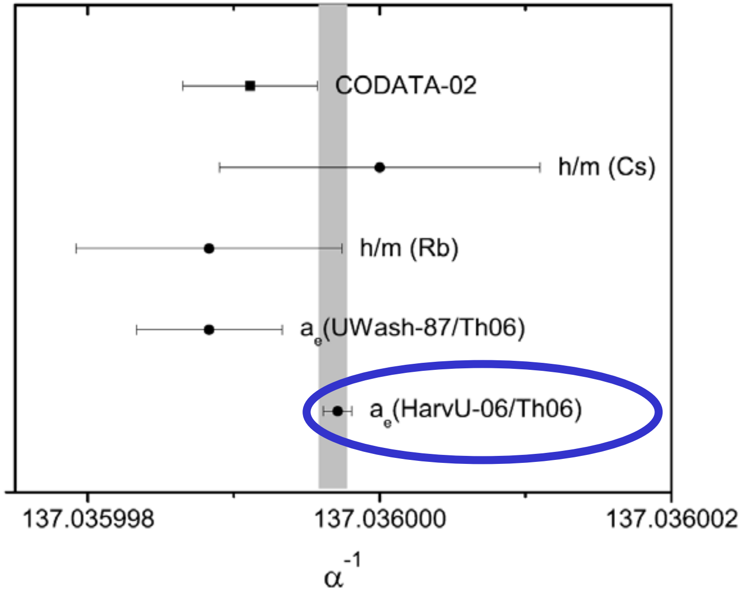


α block



- a_e theory jump

- 2006:



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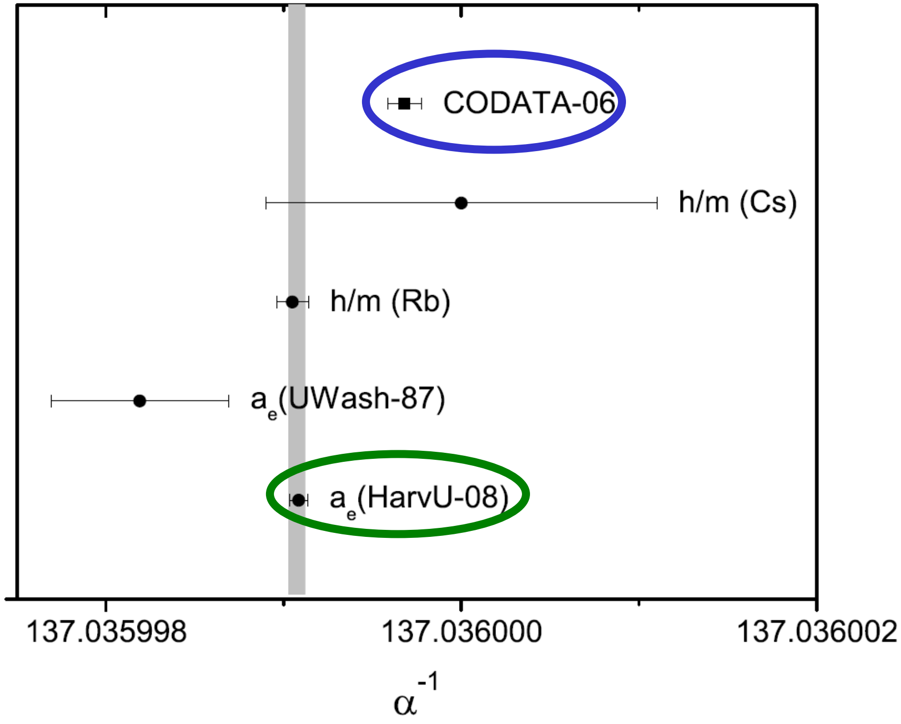
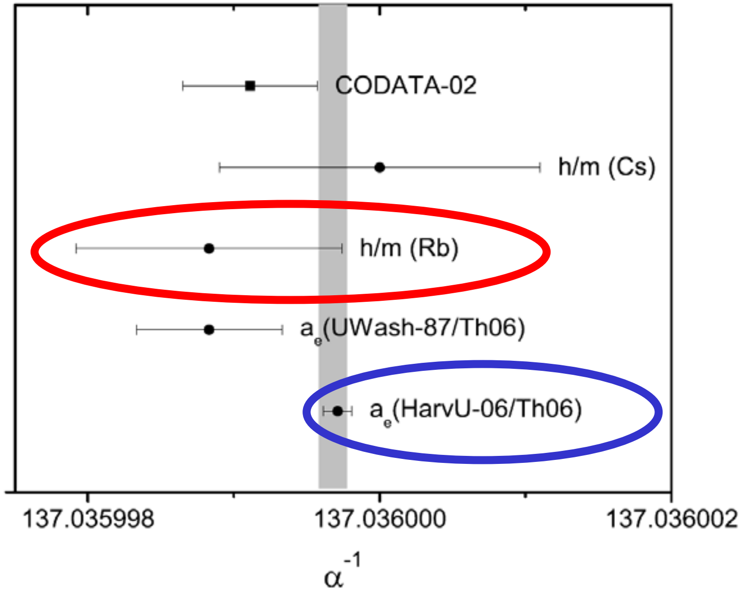
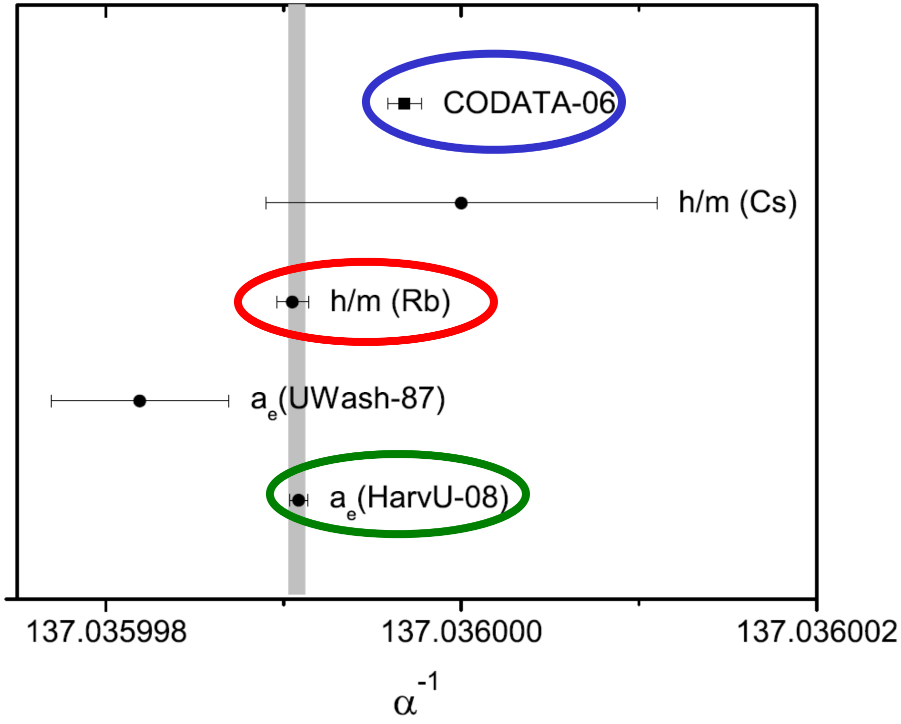


- not sensitive

- 2006:



α block



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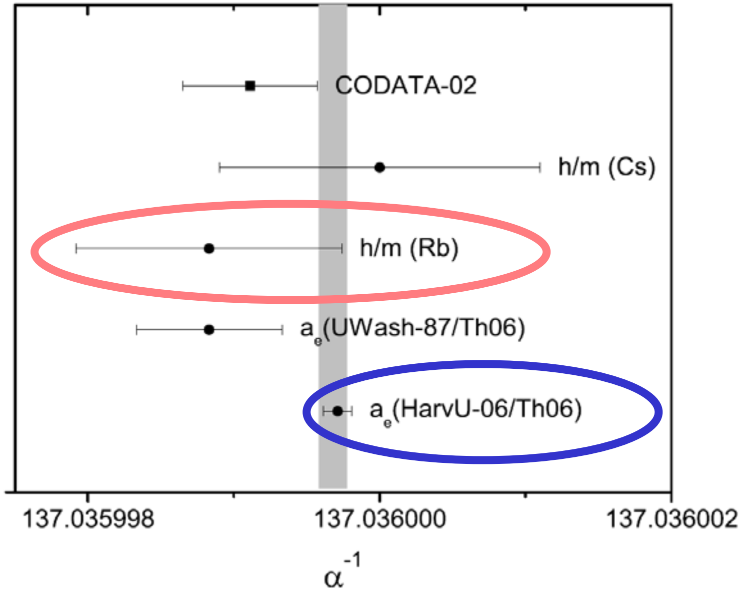
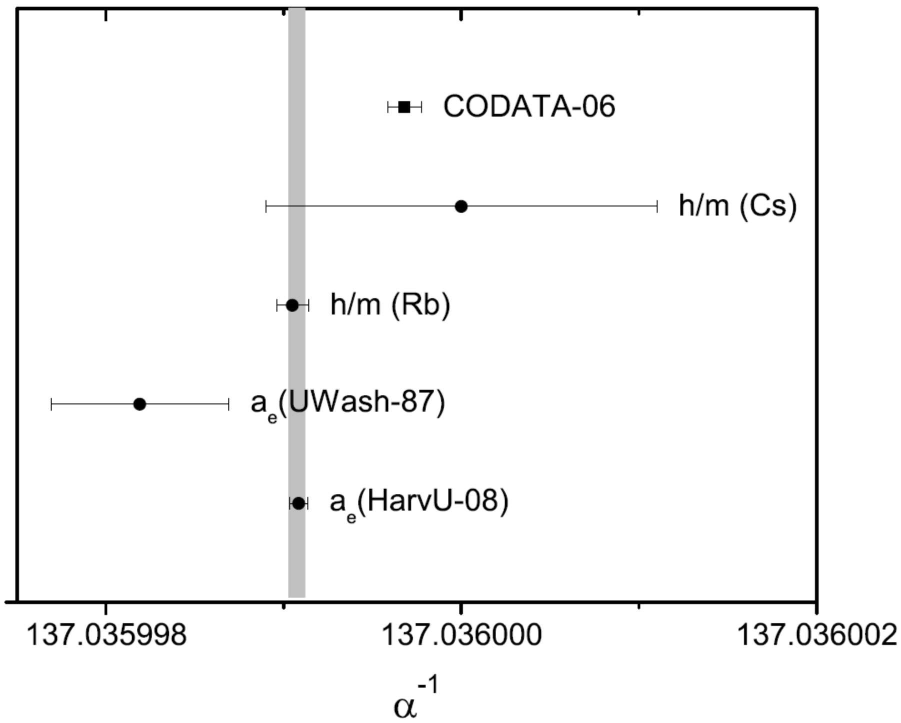
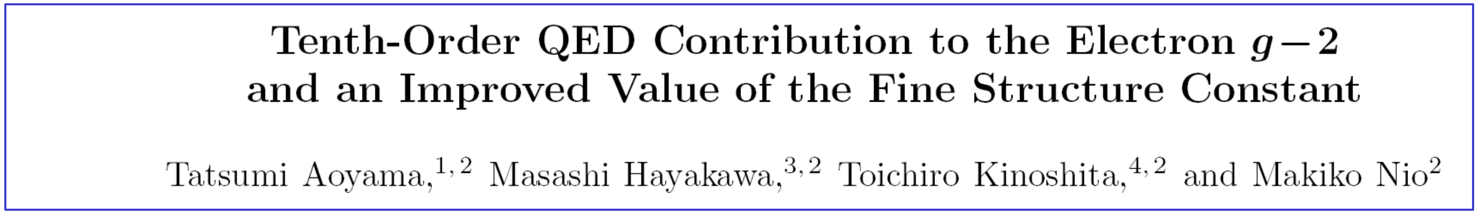


- sensitive

α block



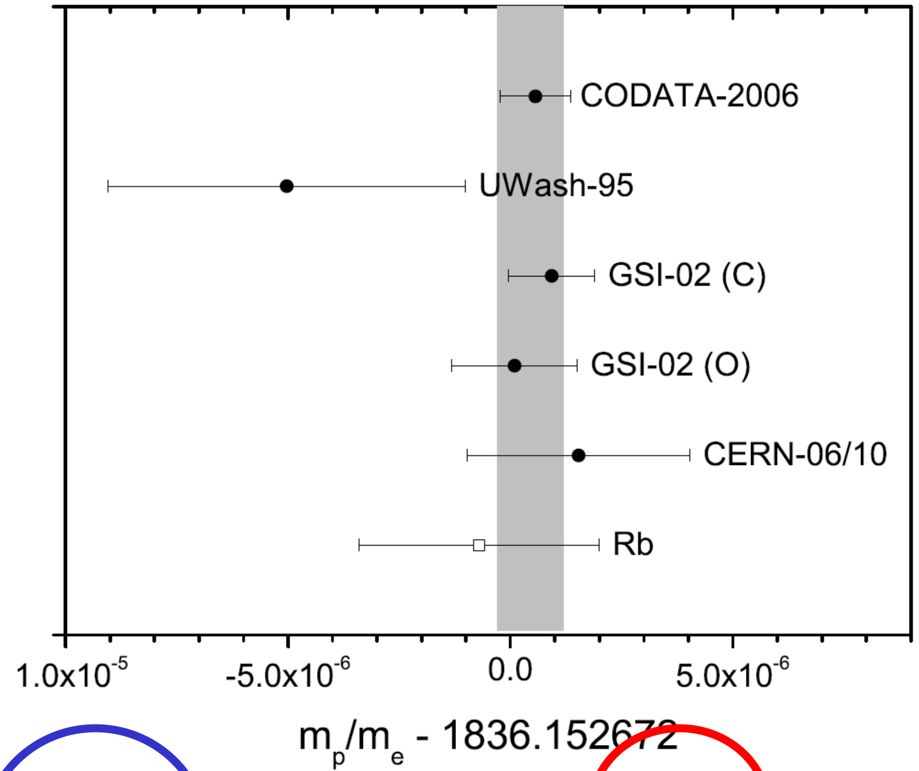
- QED vs Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- 5-loop corrections to $(g\text{-}2)_e$



Tenth-Order QED Contribution to the Electron $g-2$
and an Improved Value of the Fine Structure Constant

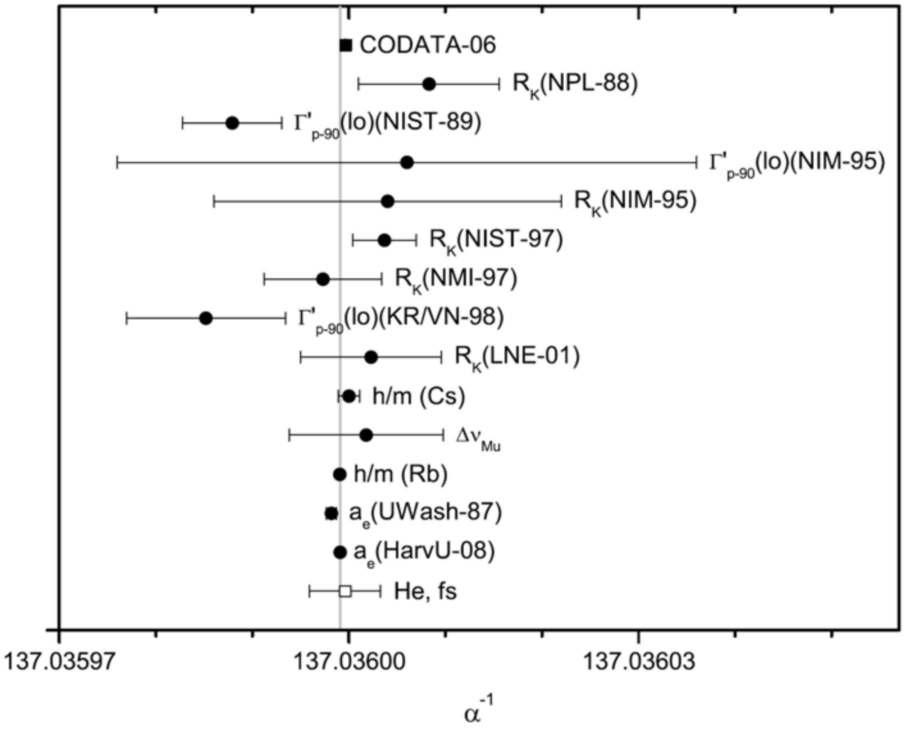
Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]

m_e/m_p vs α: accuracy is close!



$$\frac{m_p}{m_e}[\mathrm{Rb}] = \frac{m_p[\mathrm{u}]}{m_{\mathrm{Rb}}[\mathrm{u}]}\,\frac{1}{h/m_{\mathrm{Rb}}}\,\frac{\alpha^2[a_e]\,c^2}{2R_\infty} = 1836.152\,6713(27)\,, \quad [u_r = 1.5\times 10^{-9}]$$

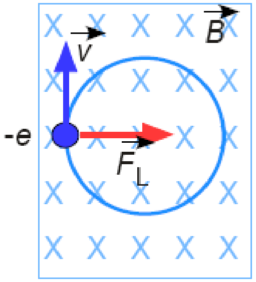
α block



- QED vs. Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- quatum Hall standard vs calculable capacitor: R_K

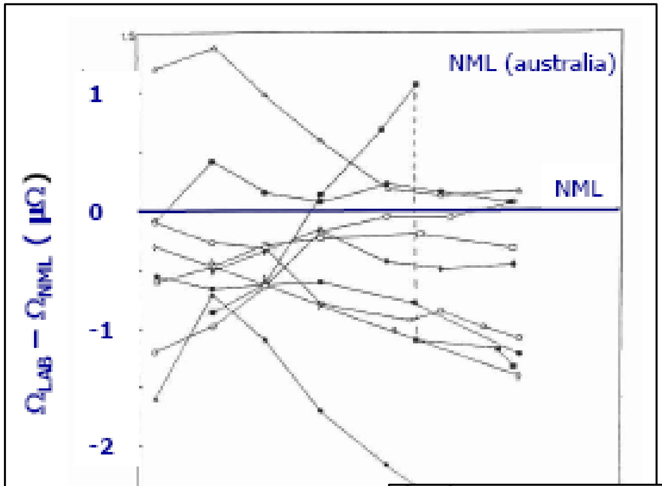
Quantum Hall effect and a standard of resistance

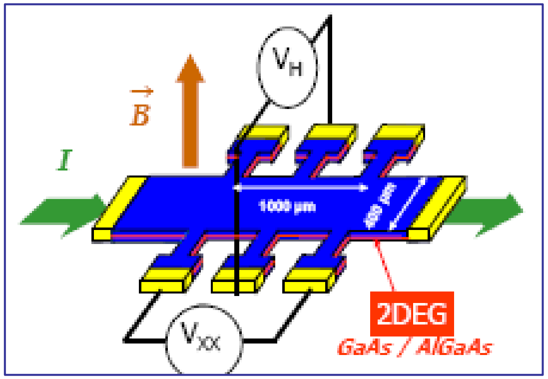
Classical Hall effect (1879)

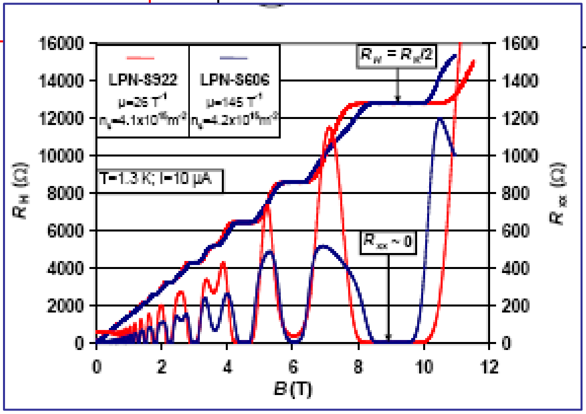


Lorentz Force

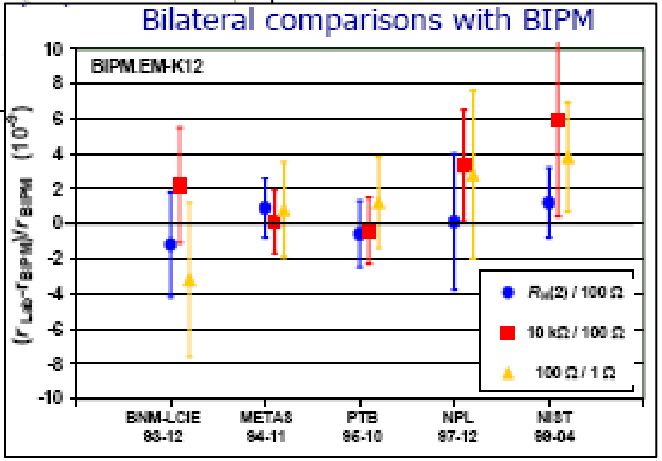
$$\vec{F}_L = q(\vec{v} \times \vec{B})$$





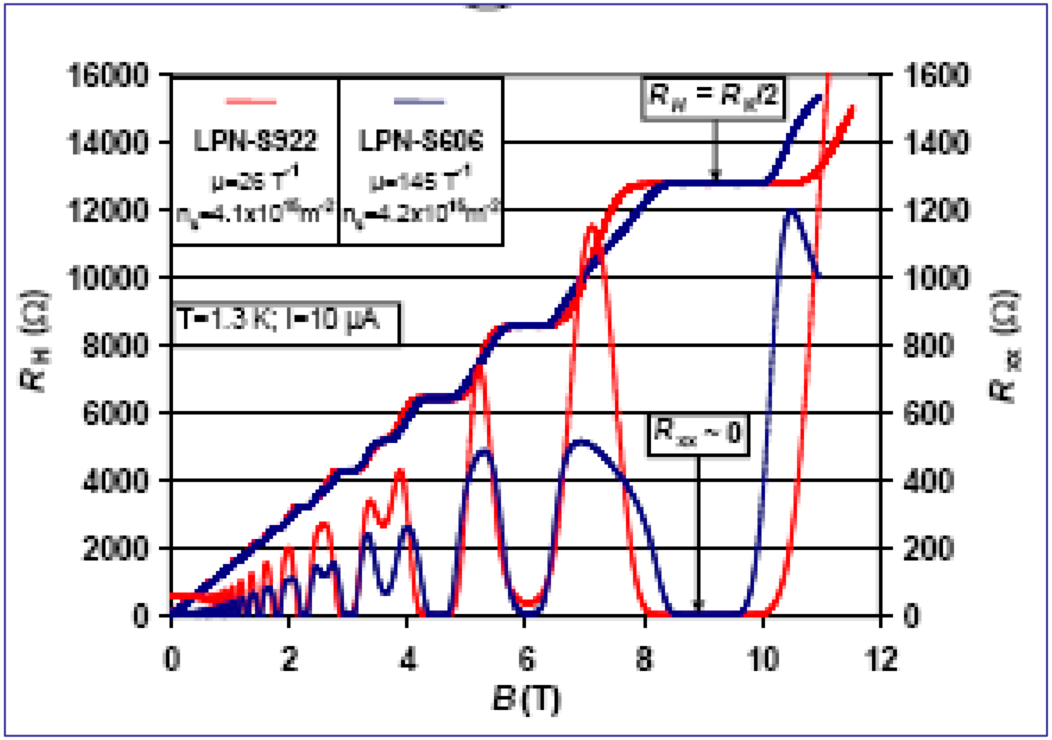


Bilateral comparisons with BIPM



W. Poirier, Les Houches, 2007

Needs for a `theory´ for QHE



- steps R_n
- rational $R_n = R_1/n$
- universal $R_1 = R_H$
- relation to α

$$R_H = R_K \equiv h/e^2$$

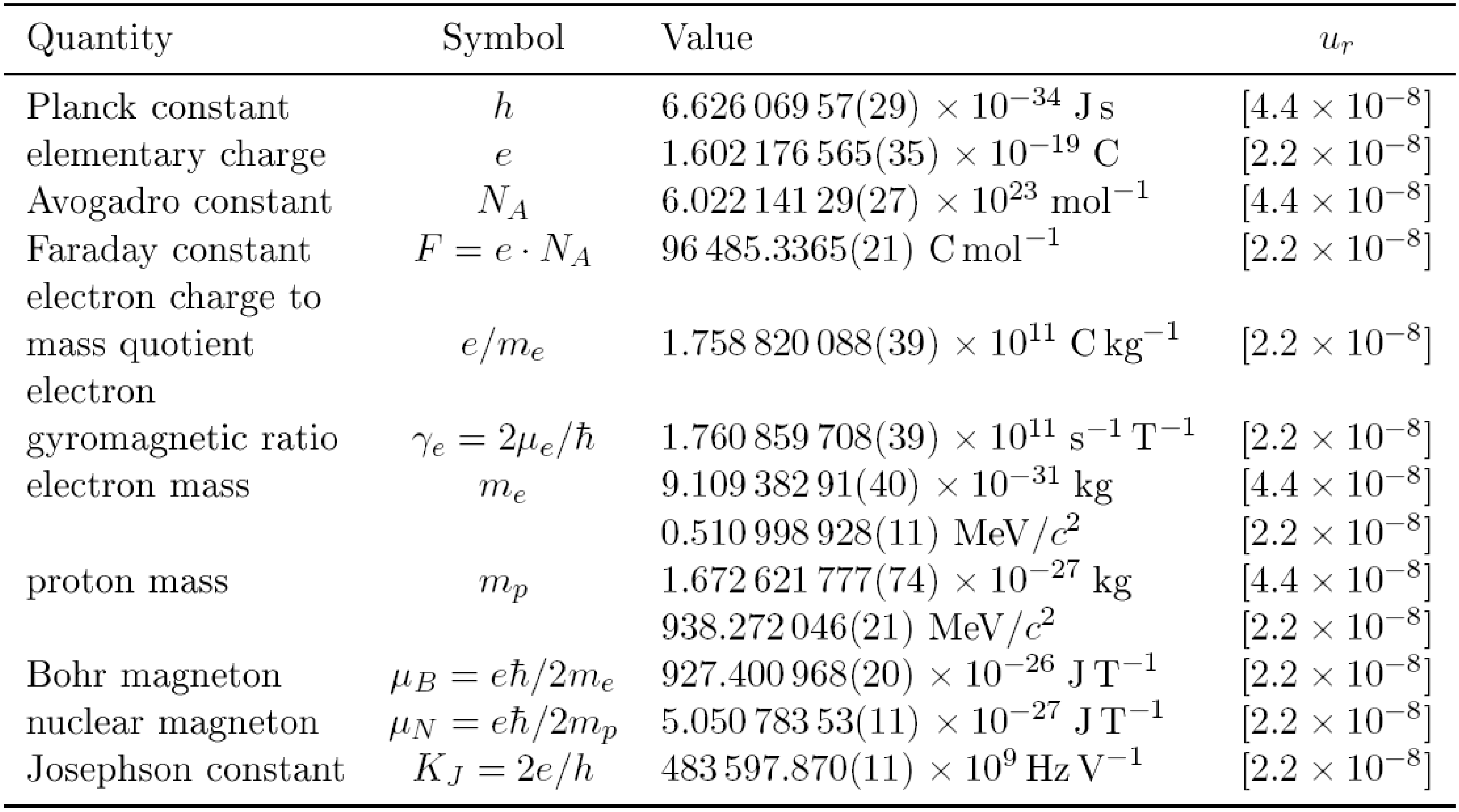
h block

known from α block

- $$\alpha = \frac{e^2}{4\pi\epsilon_0 hc}$$
- $h \cdot N_A$
- h/m_e

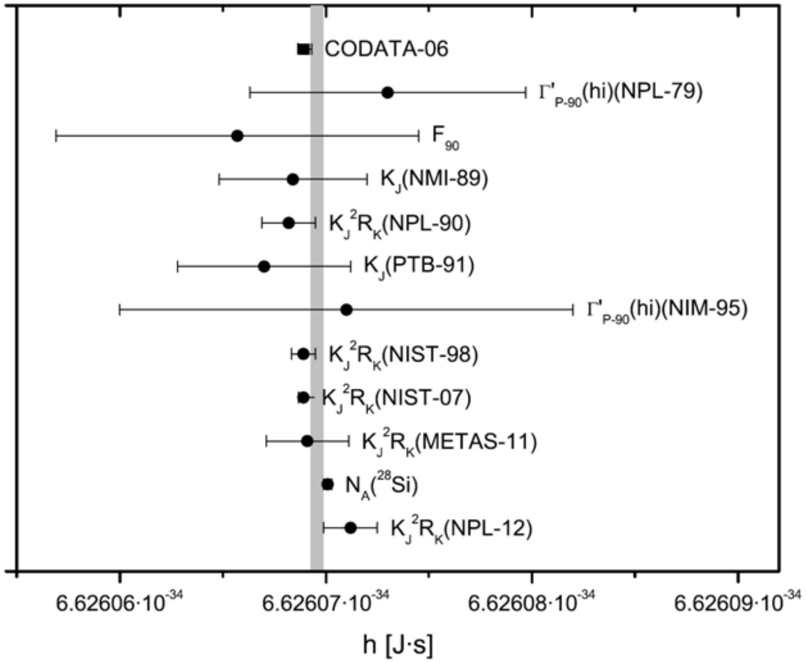
- input:
  - h
  - e
  - N_A
- output
  - m_e
  - μ_B

h block

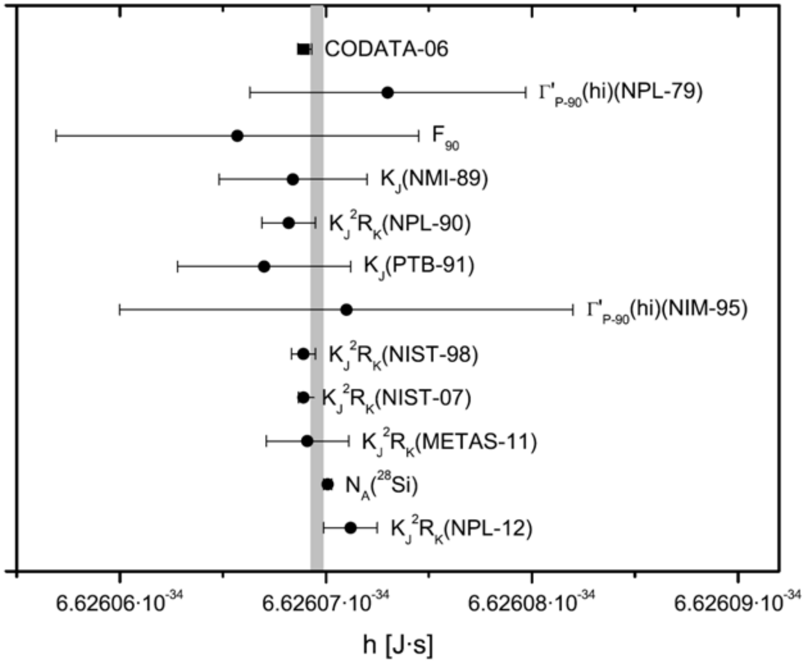


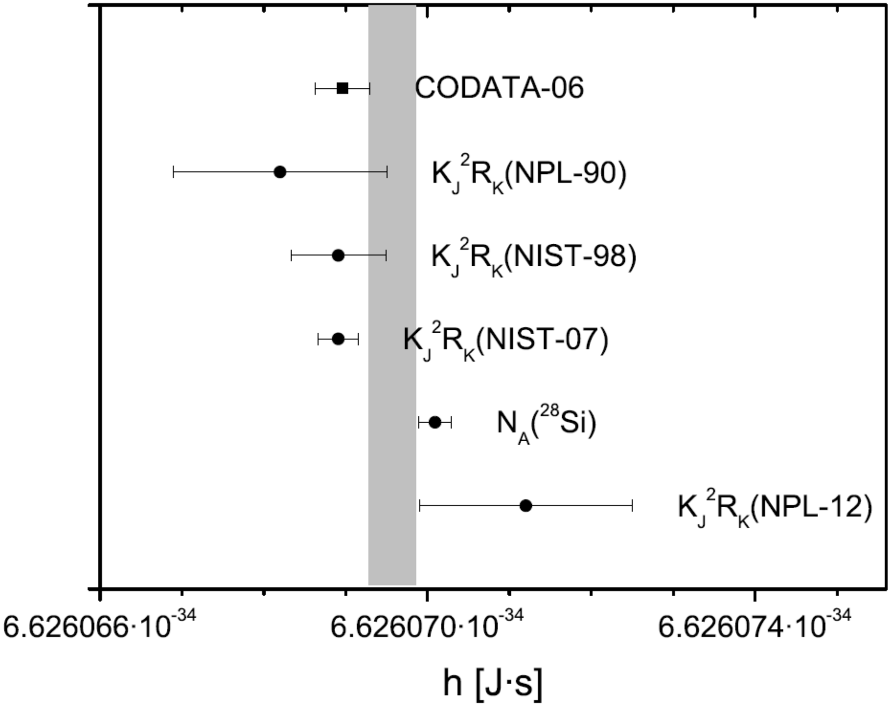
Quantity	Symbol	Value	u_r
Planck constant	h	$6.626\,069\,57(29) \times 10^{-34}$ J s	$[4.4 \times 10^{-8}]$
elementary charge	e	$1.602\,176\,565(35) \times 10^{-19}$ C	$[2.2 \times 10^{-8}]$
Avogadro constant	N_A	$6.022\,141\,29(27) \times 10^{23}$ mol^{-1}	$[4.4 \times 10^{-8}]$
Faraday constant	$F = e \cdot N_A$	$96\,485.3365(21)$ C mol^{-1}	$[2.2 \times 10^{-8}]$
electron charge to mass quotient	e/m_e	$1.758\,820\,088(39) \times 10^{11}$ C kg^{-1}	$[2.2 \times 10^{-8}]$
electron gyromagnetic ratio	$\gamma_e = 2\mu_e/\hbar$	$1.760\,859\,708(39) \times 10^{11}$ s^{-1} T^{-1}	$[2.2 \times 10^{-8}]$
electron mass	m_e	$9.109\,382\,91(40) \times 10^{-31}$ kg	$[4.4 \times 10^{-8}]$
		$0.510\,998\,928(11)$ MeV/c^2	$[2.2 \times 10^{-8}]$
proton mass	m_p	$1.672\,621\,777(74) \times 10^{-27}$ kg	$[4.4 \times 10^{-8}]$
		$938.272\,046(21)$ MeV/c^2	$[2.2 \times 10^{-8}]$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$927.400\,968(20) \times 10^{-26}$ J T^{-1}	$[2.2 \times 10^{-8}]$
nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,53(11) \times 10^{-27}$ J T^{-1}	$[2.2 \times 10^{-8}]$
Josephson constant	$K_J = 2e/h$	$483\,597.870(11) \times 10^{9}$ Hz V^{-1}	$[2.2 \times 10^{-8}]$

h block

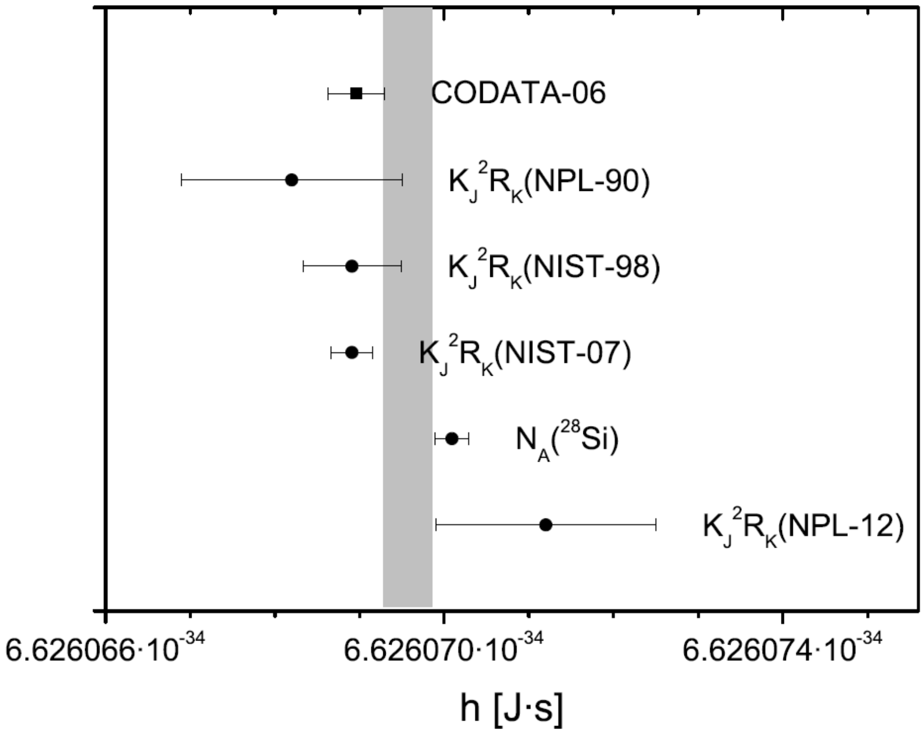

CODATA-06
Γ'P-90(hi)(NPL-79)
F90
KJ(NMI-89)
KJ2RK(NPL-90)
KJ(PTB-91)
Γ'P-90(hi)(NIM-95)
KJ2RK(NIST-98)
KJ2RK(NIST-07)
KJ2RK(METAS-11)
NA(28Si)
KJ2RK(NPL-12)
6.62606·10-34
6.62607·10-34
6.62608·10-34
6.62609·10-34
h [J·s]

h block: the most important data





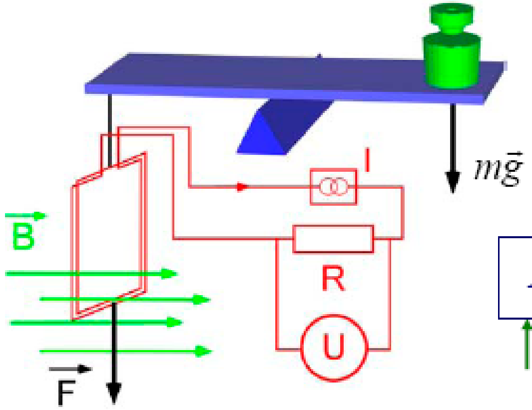
h block: the most important data



- watt ballance
- Avogadro constant from ehrhiched Si

watt-ballance

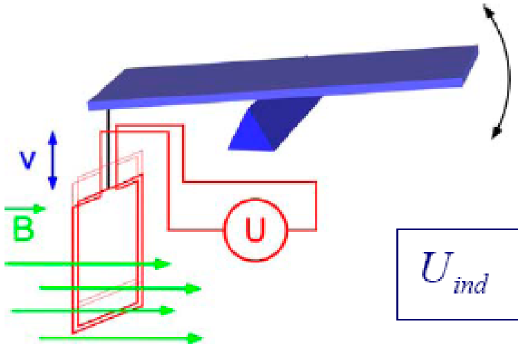
WB Principle (1): static phase / weighing mode



$$I \cdot \int d\ell \times B \;=\; mg$$

$$I = \frac{U_j}{R}$$ (QHE & JVS)

WB Principle (2): dynamic phase / velocity mode



$$U_{ind} \;=\; v \cdot \int d\ell \times B$$

WB Principle (3): combination of modes

Only if $G_m = G_e$

$$G(B,\ell) \;=\; \underbrace{\frac{mg}{I}}_{\text{static}} = \underbrace{\frac{U}{v}}_{\text{dynamic}} \;\Rightarrow\; \boxed{UI = m\,g\,v}$$

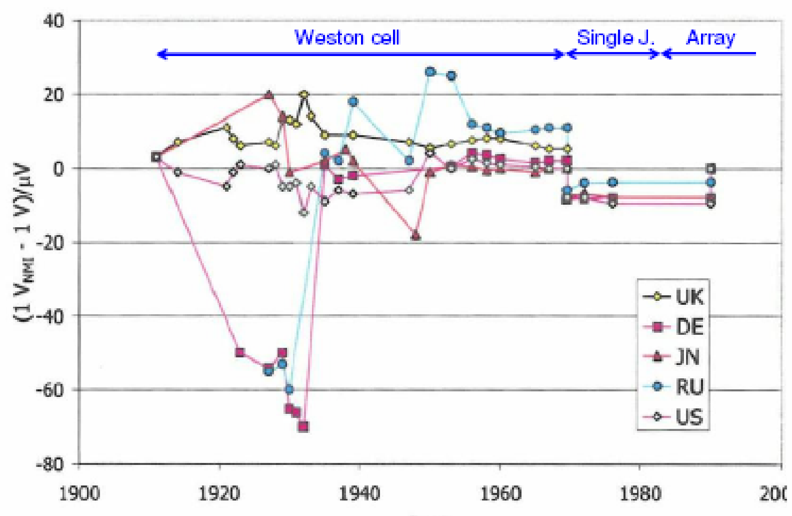
electrical power

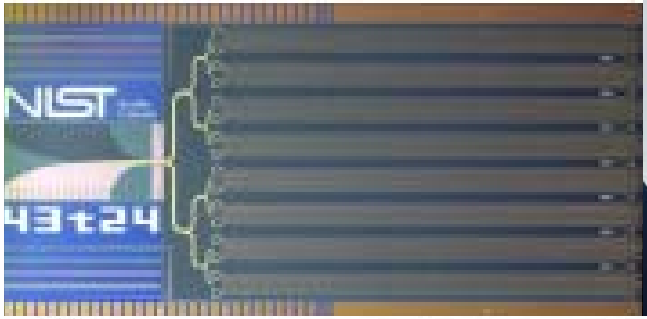
mechanical power

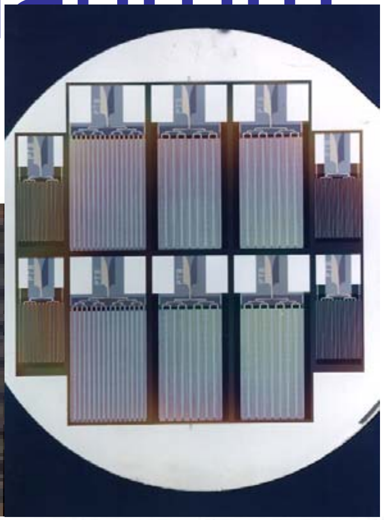
B. Jeanneret, Les Houches, 2007

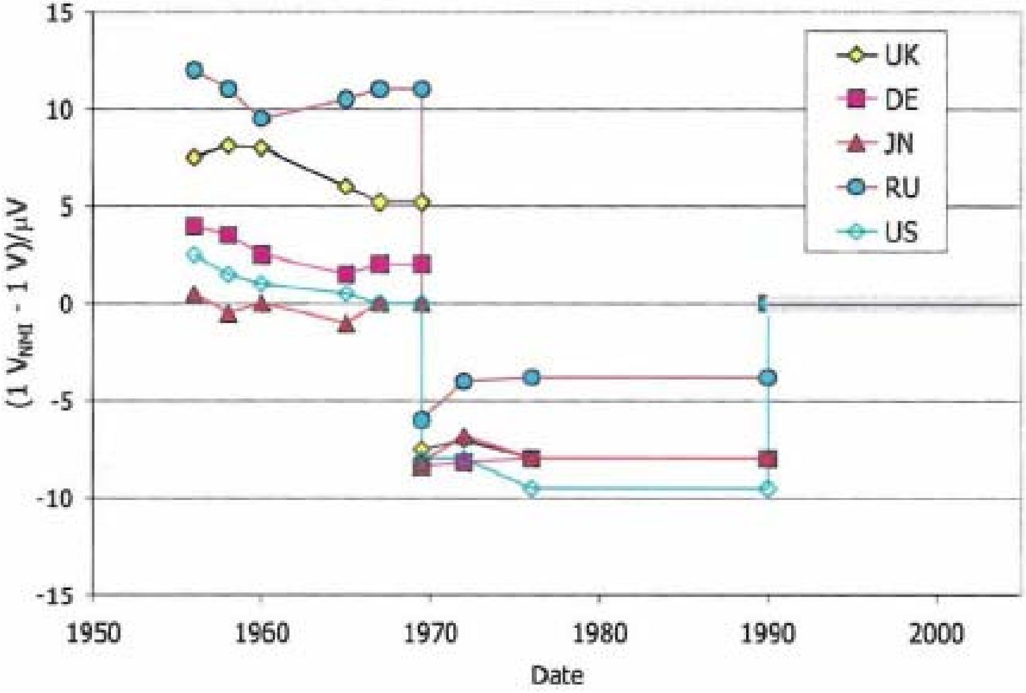
Josephson effect and quantum volt stardard

Voltage Unit: Representation









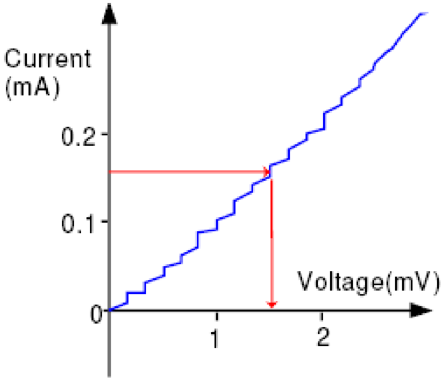
Irradiation with microwave:

- Cooper pairs synchronize with radiation
- Voltage steps appear

$$\boxed{V_n = n\frac{h}{2e}f}$$

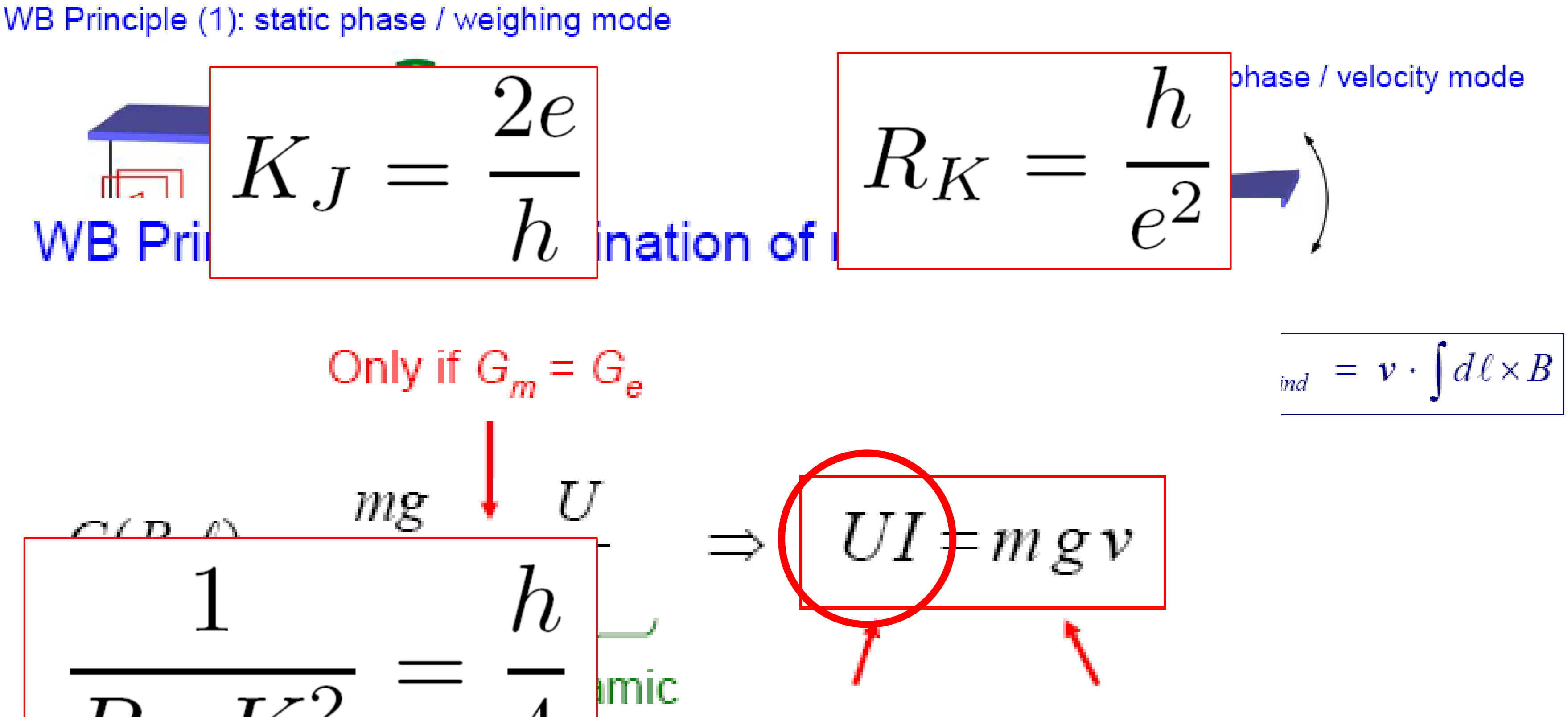
Shapiro step, 1963

V_1 ~ 145 μV @ 70 GHz



B. Jeanneret, Les Houches, 2007

watt-ballance



$$K_J = \frac{2e}{h}$$

$$R_K = \frac{h}{e^2}$$

Only if $G_m = G_e$

$$\Rightarrow UI = mgv$$

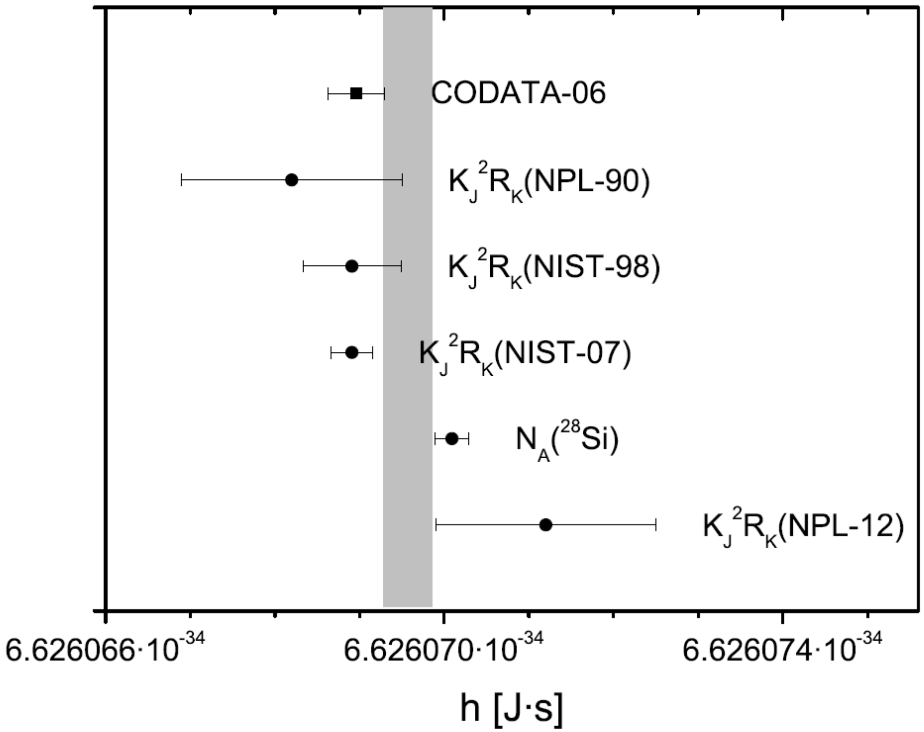
electrical power

mechanical power

$$\frac{1}{R_K K_J^2} = \frac{h}{4}$$

B. Jeanneret, Les Houches, 2007

h block: the most important data



- watt ballance
- Avogadro constant from ehrhiched Si

Monocrystale of ^{28}Si

monocrystale ~ 1 kg

isotopic composition

- ^{28}Si: 92%
- ^{29}Si: 5%
- ^{30}Si: 3%

Monocrystale of ^{28}Si

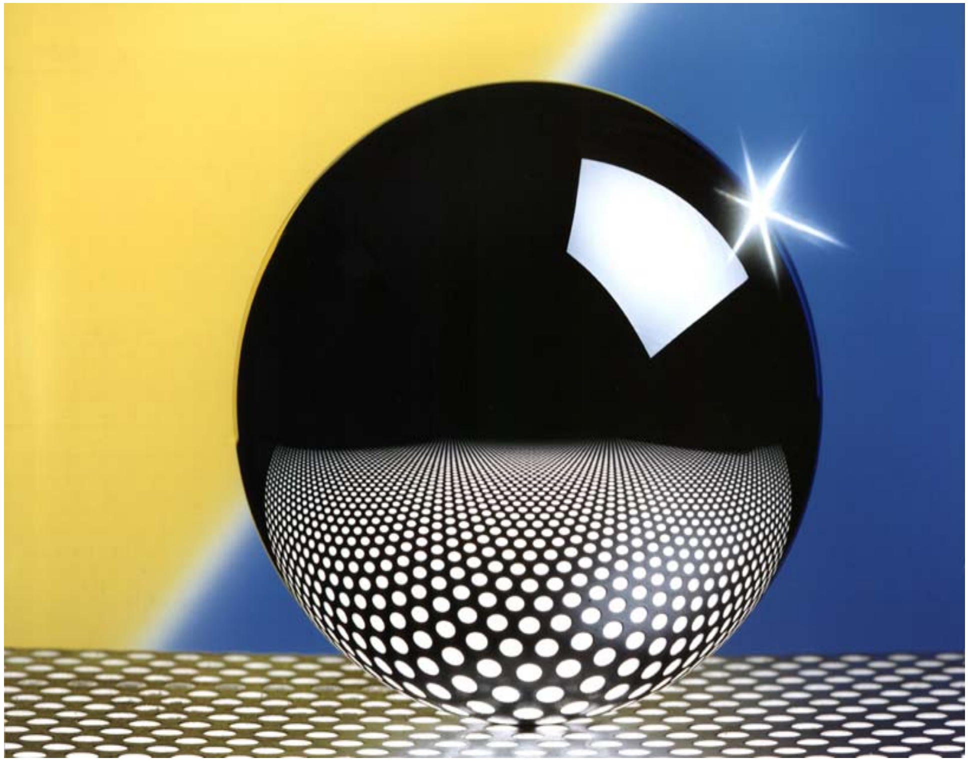
monocrystale ~ 1 kg

isotopic composition

- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: ~~5%~~
- ^{30}Si: ~~3%~~

Monocrystale of ^{28}Si

monocrystale ~ 1 kg



isotopic composition

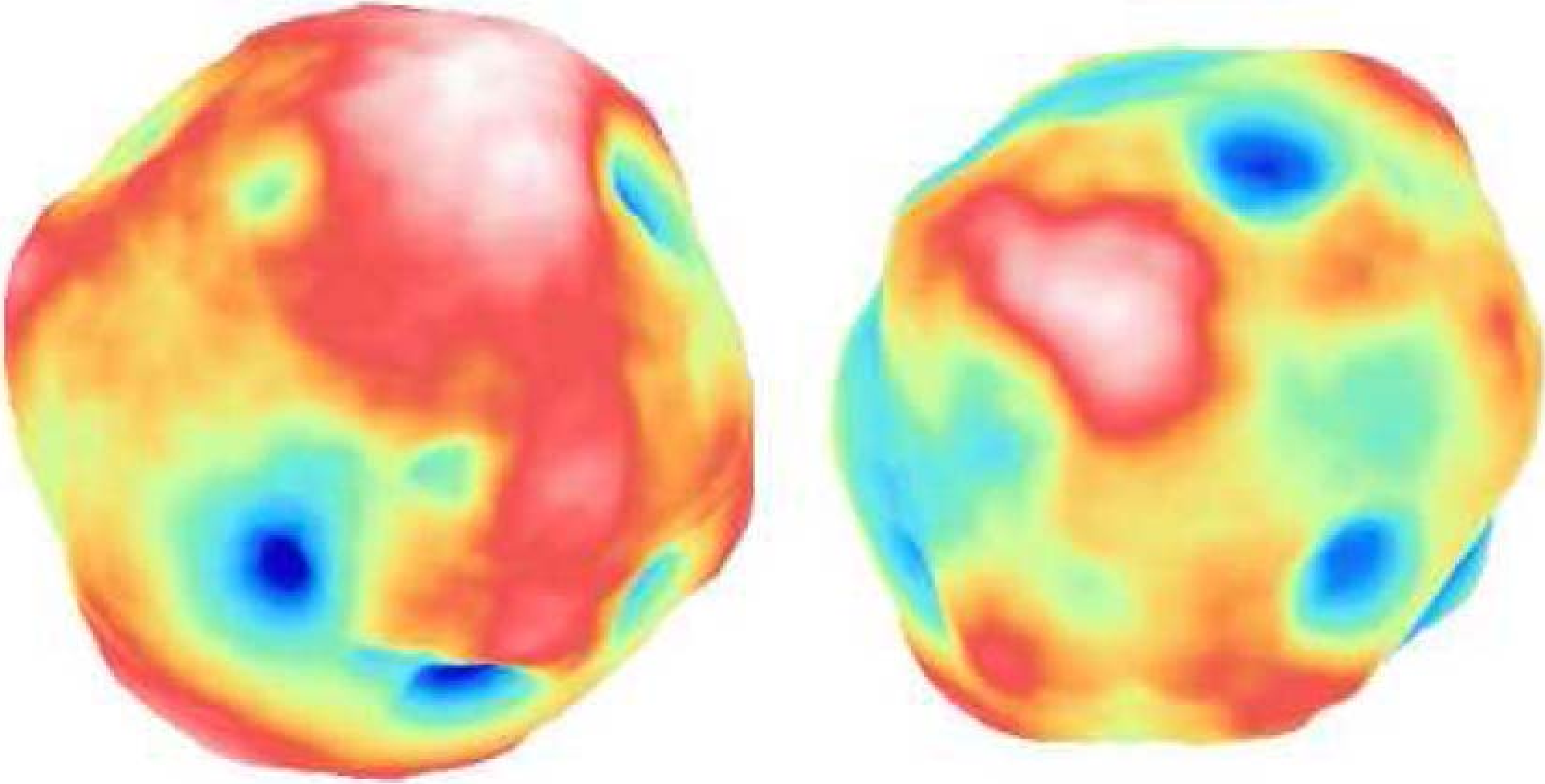
- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: ~~5%~~
- ^{30}Si: ~~3%~~

Monocrystale of ^{28}Si

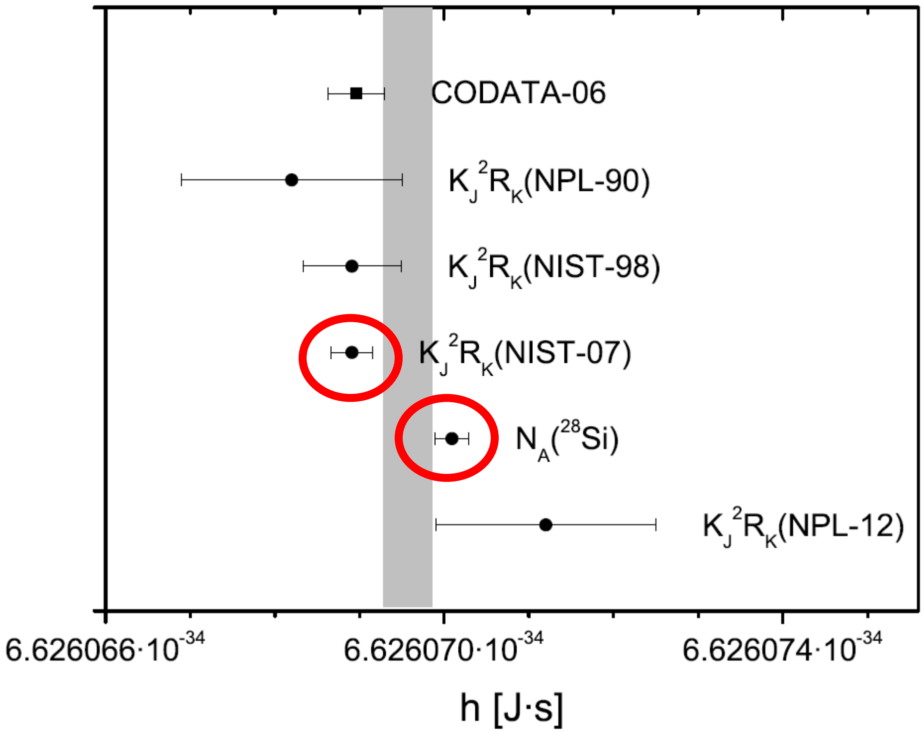
monocrystale ~ 1 kg

isotopic composition

5%

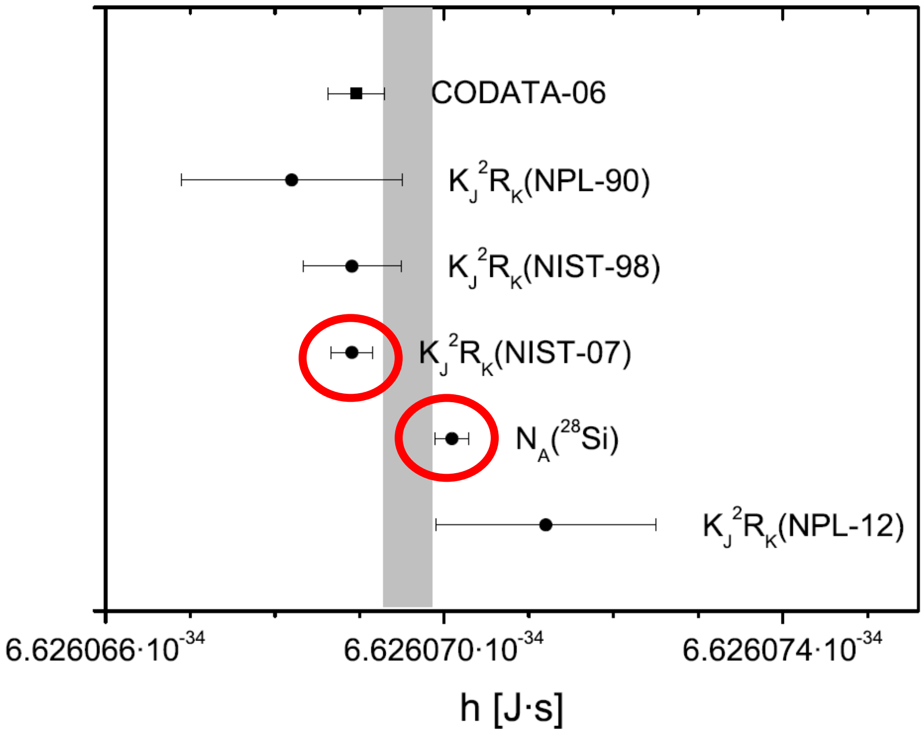


h block: the most important data

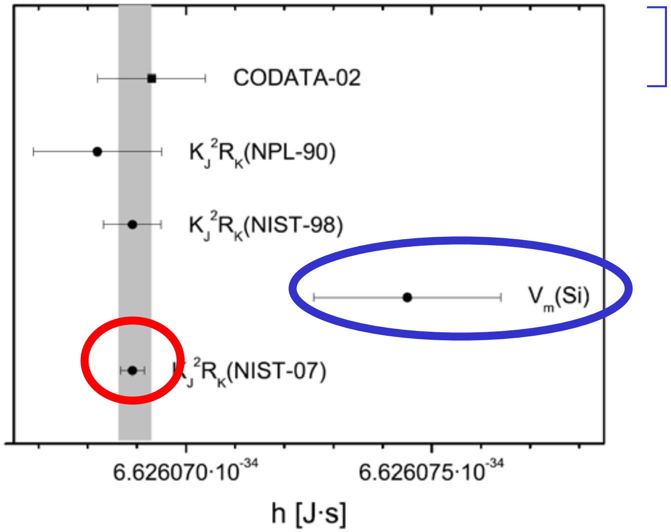


- watt ballance
- Avogadro constant from ehrhiched Si
- problem remains

h block: the most important data



- watt ballance
- Avogadro constant from ehrhiched Si



Mass of a proton in different units

Symbol	Value	u_r
m_p	$1.007\,276\,466\,812(90)$ u	$[8.9 \times 10^{-11}]$
m_p	$1836.152\,672\,45(75)\ m_e$	$[4.1 \times 10^{-10}]$
m_pc^2/h	$2.268\,731\,8139(16) \times 10^{23}$ Hz	$[7.1 \times 10^{-10}]$
m_pc^2	$938.272\,046(21)$ MeV	$[2.2 \times 10^{-8}]$
m_p	$1.672\,621\,777(74) \times 10^{-27}$ kg	$[4.4 \times 10^{-8}]$

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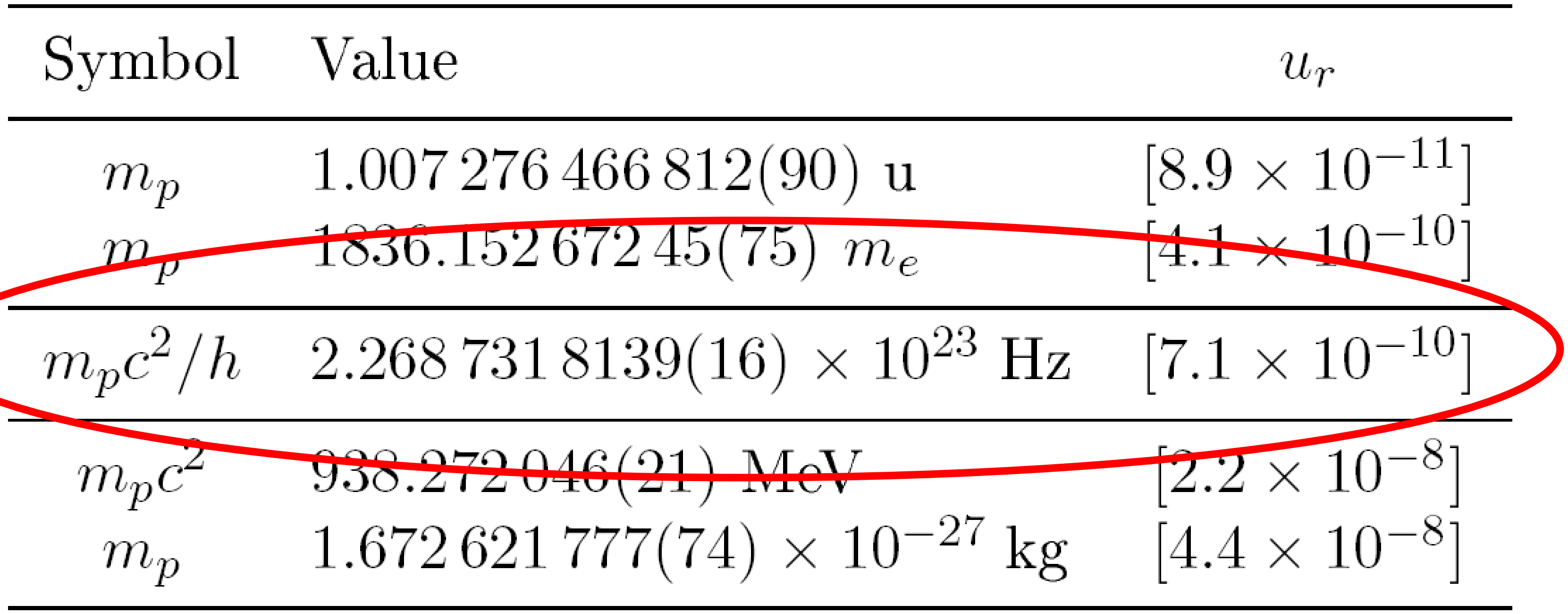
- auxiliary data

Mass of a proton in different units



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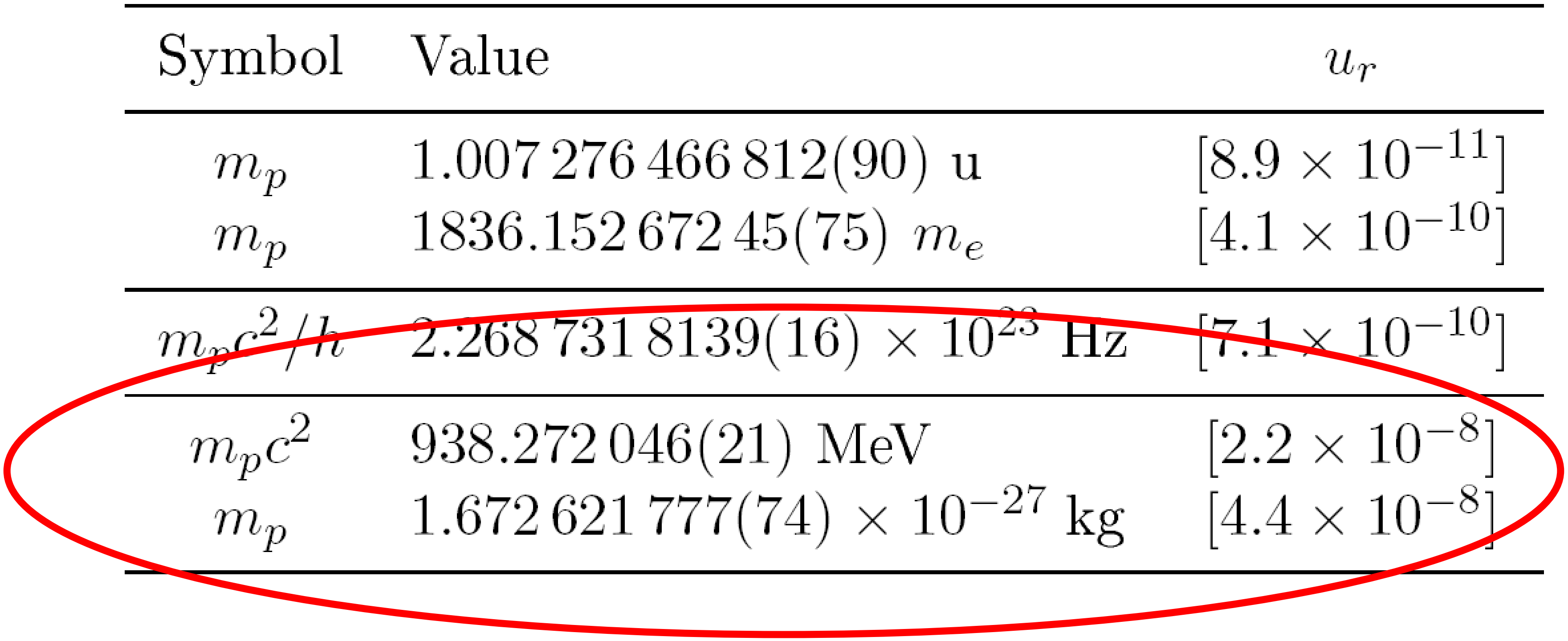
- α block

Mass of a proton in different units



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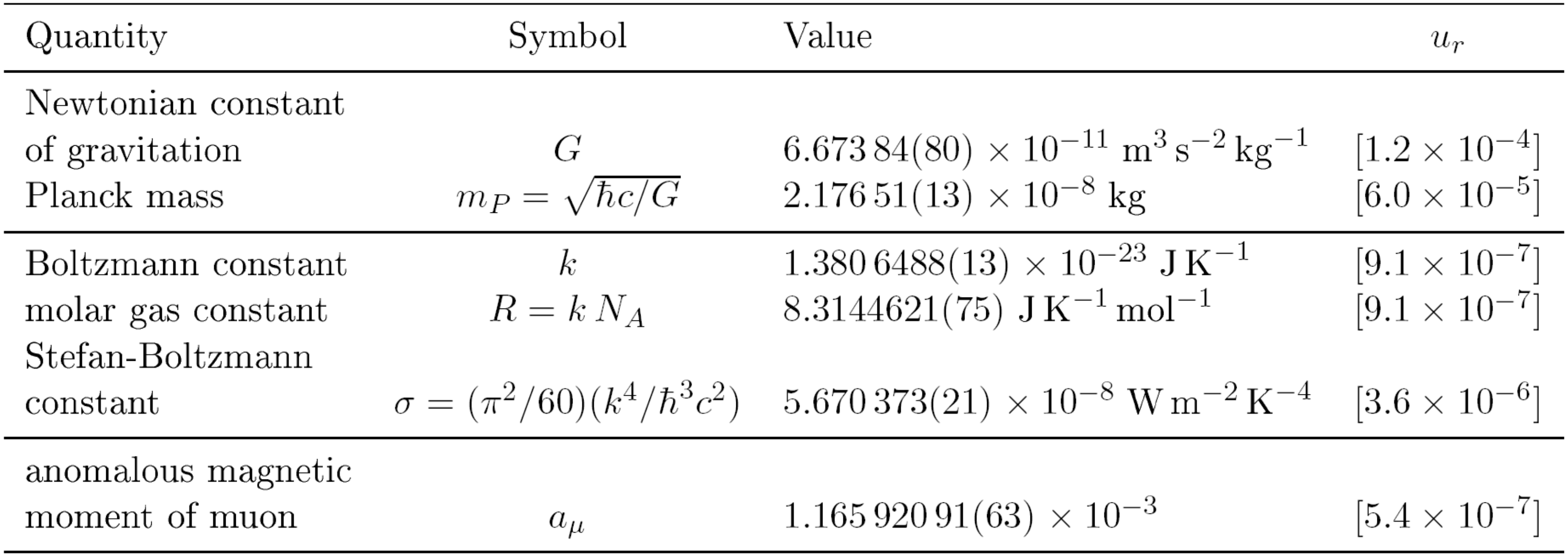
- h block

Independent constants



Quantity	Symbol	Value	u_r
Newtonian constant of gravitation	G	$6.673\,84(80) \times 10^{-11}\ \mathrm{m^3\,s^{-2}\,kg^{-1}}$	$[1.2 \times 10^{-4}]$
Planck mass	$m_P = \sqrt{\hbar c/G}$	$2.176\,51(13) \times 10^{-8}\ \mathrm{kg}$	$[6.0 \times 10^{-5}]$
Boltzmann constant	k	$1.380\,6488(13) \times 10^{-23}\ \mathrm{J\,K^{-1}}$	$[9.1 \times 10^{-7}]$
molar gas constant	$R = k\,N_A$	$8.314\,4621(75)\ \mathrm{J\,K^{-1}\,mol^{-1}}$	$[9.1 \times 10^{-7}]$
Stefan-Boltzmann constant	$\sigma = (\pi^2/60)(k^4/\hbar^3 c^2)$	$5.670\,373(21) \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$	$[3.6 \times 10^{-6}]$
anomalous magnetic moment of muon	a_μ	$1.165\,920\,91(63) \times 10^{-3}$	$[5.4 \times 10^{-7}]$

Independent constants: G

$$GM_\odot = 1.327\,124\,4210(1) \times 10^{20}\ \mathrm{m^3 s^{-2}}$$

$$GM_\oplus = 3.986\,004\,418(8) \times 10^{14}\ \mathrm{m^3 s^{-2}}$$

IESR, 2010

- $\delta G/G \sim 10^{-4}$

$$\begin{aligned} M_\odot &= 1.988\,55(24) \times 10^{30}\ \mathrm{kg} \\ M_\oplus &= 5.972\,58(72) \times 10^{24}\ \mathrm{kg} \end{aligned}$$

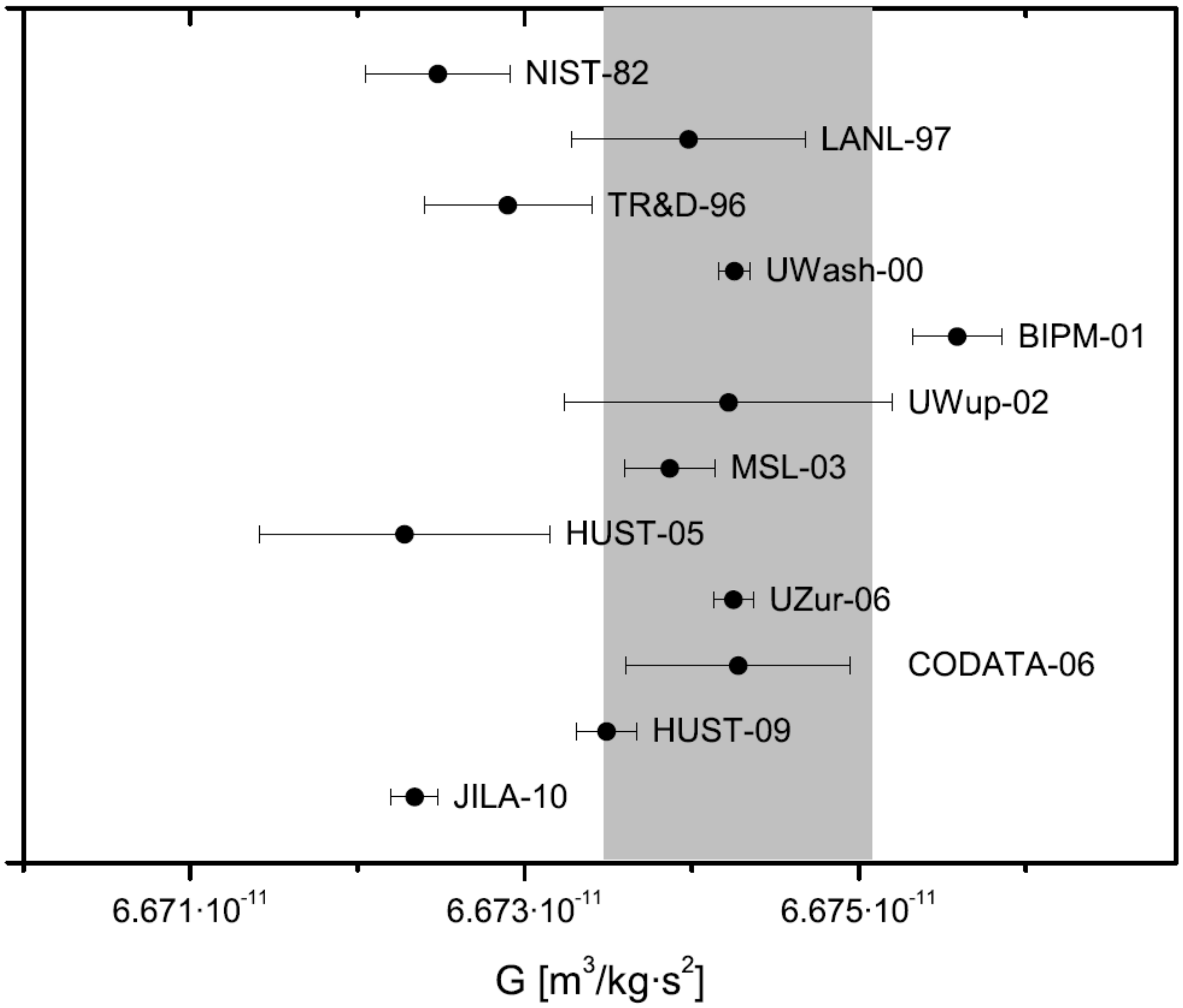
PSR J0737-3039/A/B

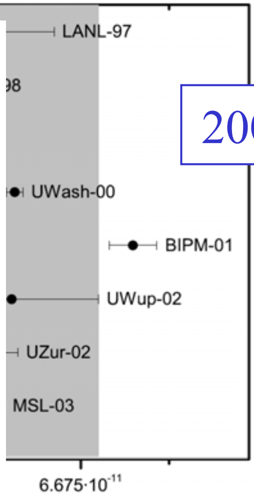
$$\begin{aligned} M_m &= 1.3381(7)\ M_\odot = 2.6609(14) \times 10^{30}\ \mathrm{kg} \\ M_p &= 1.2489(7)\ M_\odot = 2.4835(14) \times 10^{30}\ \mathrm{kg} \end{aligned}$$

Kramer et al., 2006

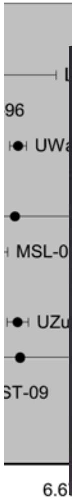
BIPM 1889

Independent constants: G

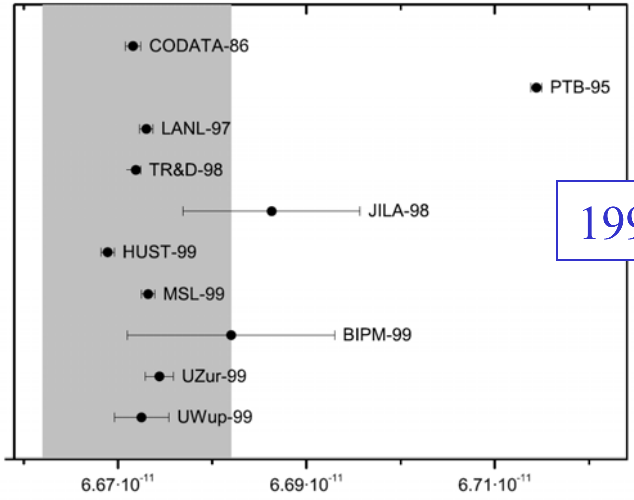


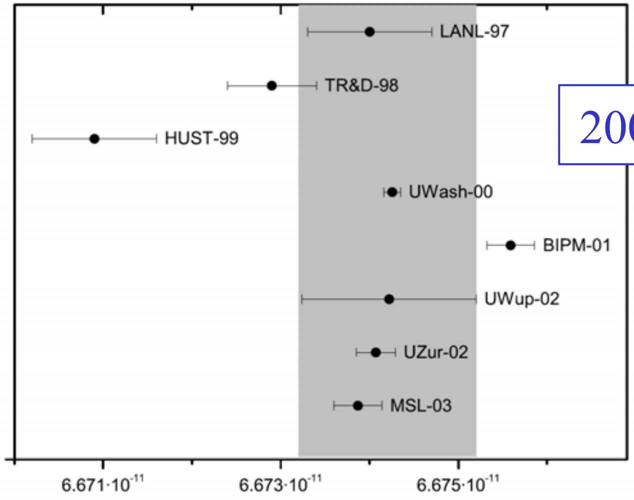


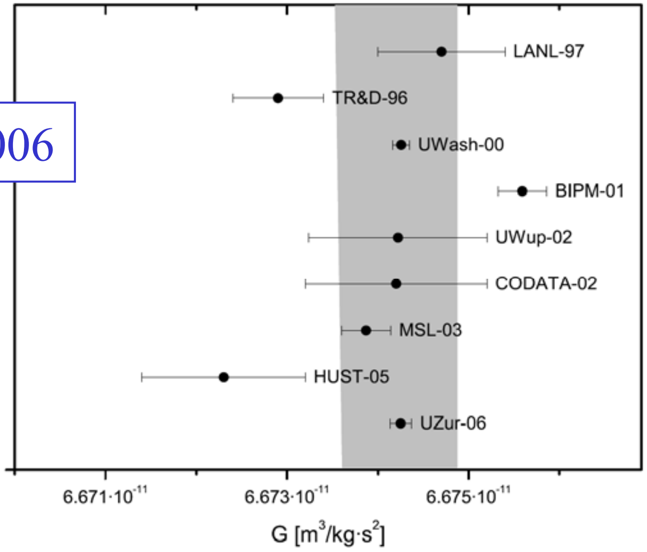
2002

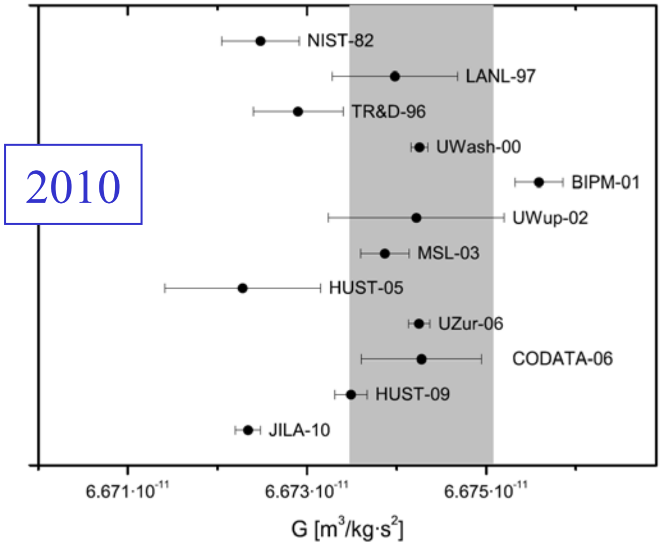


Independent constants: G


1998


2002

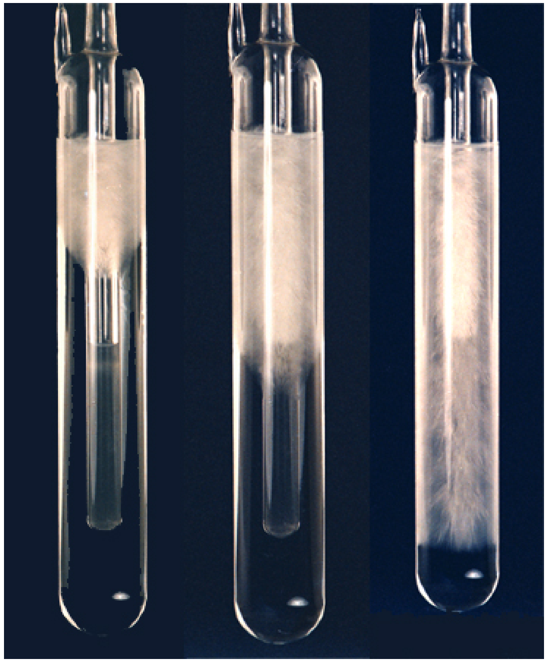

2006


2010

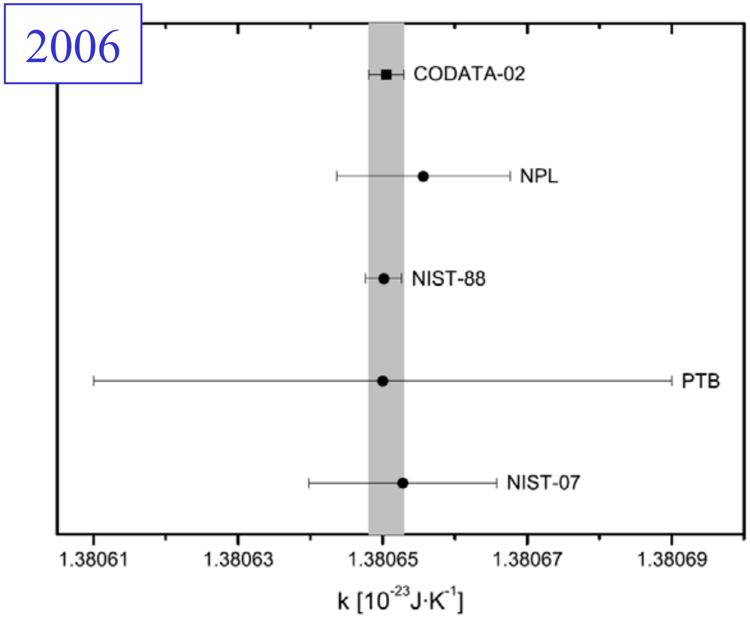
Independent constants: k

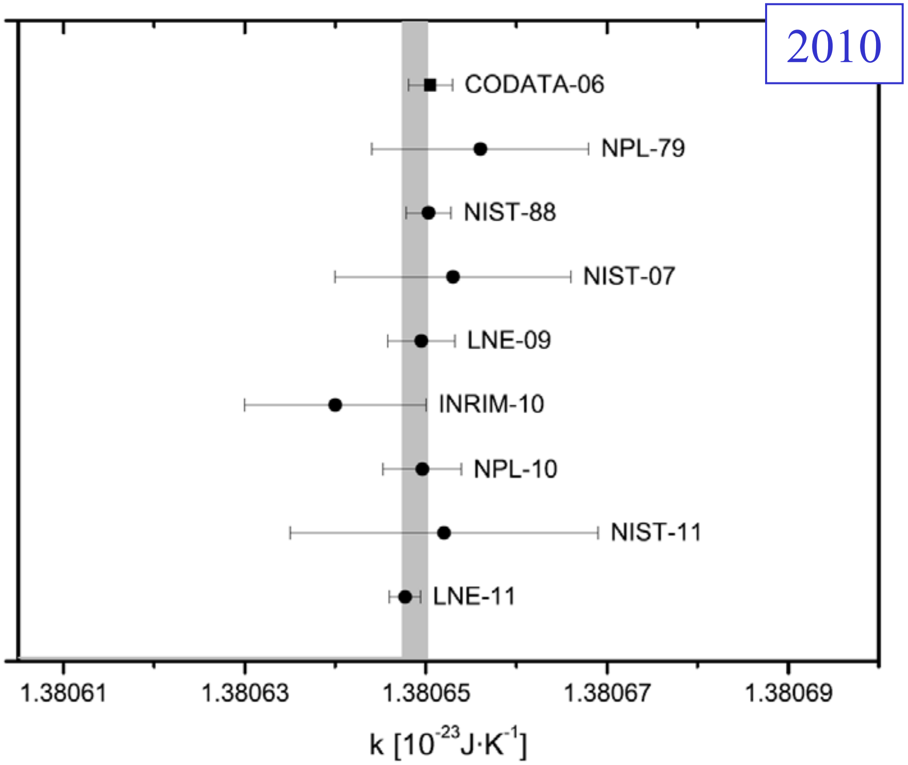
$$T_{\mathrm{CMB}} = 2.725\,48(57)\ K$$

Fixsen, 2009: COBE

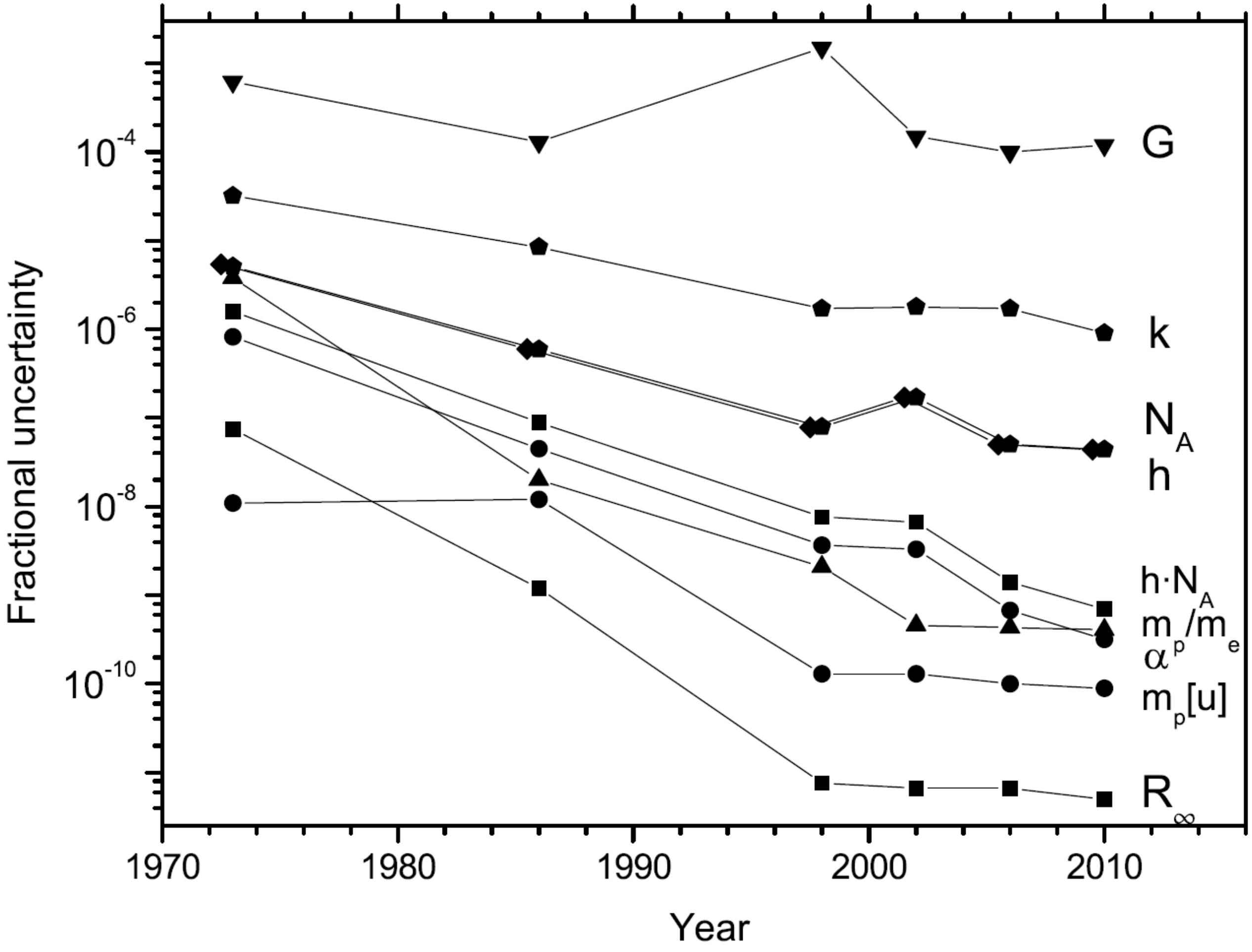


Independent constants: k


2006

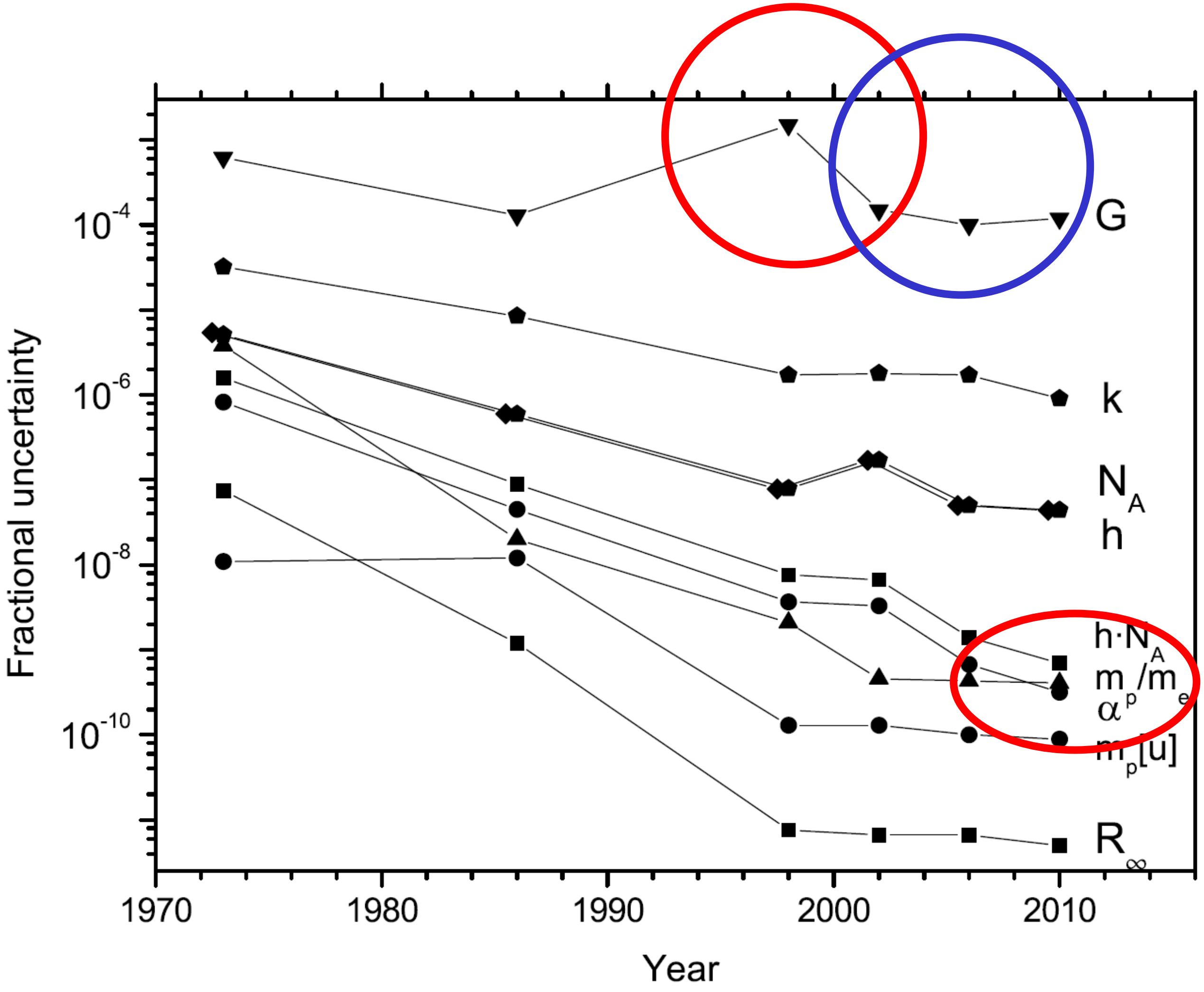

2010

Progress



Fractional uncertainty
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1970
1980
1990
2000
2010
Year
G
k
N_A
h
$h \cdot N_A$
m_p/m_e
α
m_p[u]
R_∞

Progress


Fractional uncertainty
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1970
1980
1990
2000
2010
Year
G
k
N_A
h
$h \cdot N_A$
m_p/m_e
α
$m_p[u]$
R_∞

Progress

Quantity	$u_r(2006)$	Δ	$\Delta/u_r(2006)$	$u_r(2010)$	$u_r(2010)/u_r(2006)$
R_∞	6.6×10^{-12}	1.1×10^{-12}	0.17	5.0×10^{-12}	0.76
m_e/m_p	4.3×10^{-10}	0.1×10^{-10}	0.03	4.1×10^{-10}	0.95
α	6.8×10^{-10}	44.2×10^{-10}	6.50	3.2×10^{-10}	0.47
h	5.0×10^{-8}	9.2×10^{-8}	1.84	4.4×10^{-8}	0.88
k	1.7×10^{-6}	-1.2×10^{-6}	-0.68	9.1×10^{-7}	0.53
G	1.0×10^{-4}	-0.7×10^{-4}	-0.66	1.2×10^{-4}	1.2

Progress

Quantity	$u_r(2006)$	Δ	$\Delta/u_r(2006)$	$u_r(2010)$	$u_r(2010)/u_r(2006)$
R_∞	6.6×10^{-12}	1.1×10^{-12}	0.17	5.0×10^{-12}	0.76
m_e/m_p	4.3×10^{-10}	0.1×10^{-10}	0.03	4.1×10^{-10}	0.95
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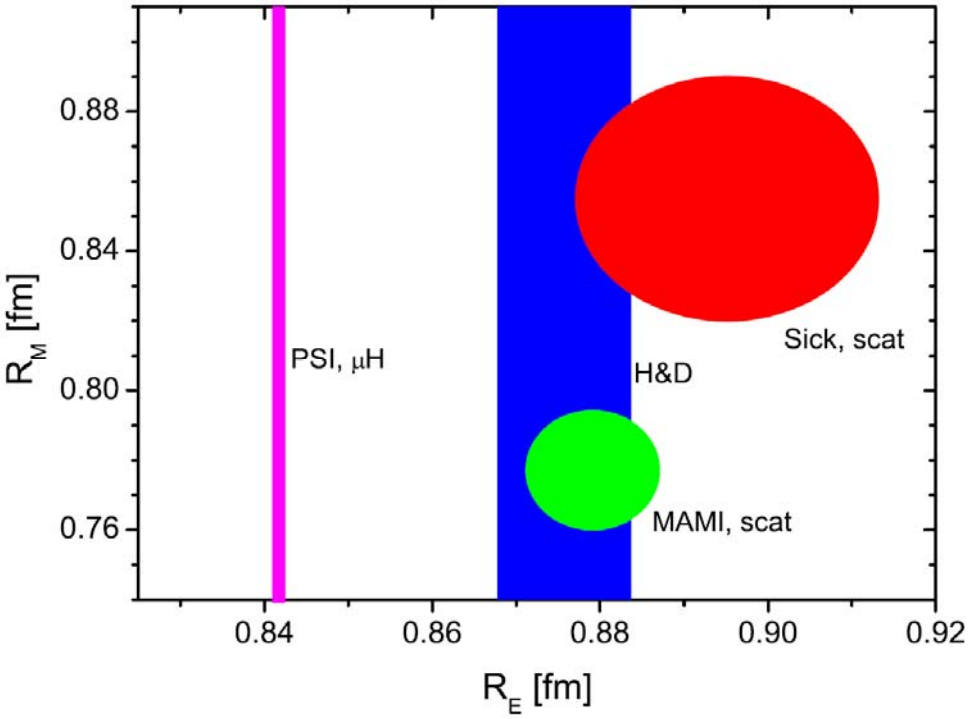
k	1.7×10^{-6}	-1.2×10^{-6}	-0.68	9.1×10^{-7}	0.53
G	1.0×10^{-4}	-0.7×10^{-4}	-0.66	1.2×10^{-4}	1.2

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k

Problems

- R_∞ & R_p
- m/m



+ better accuracy in scattering

+ new method for R_p

- discrepancy in data

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k

+ slow progress in two methods

+ no discrepancies

overlap with α data

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k

+ better accuracy

+ two methods

+ sensitivity to 5 loops

– 6-sigma jump

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k

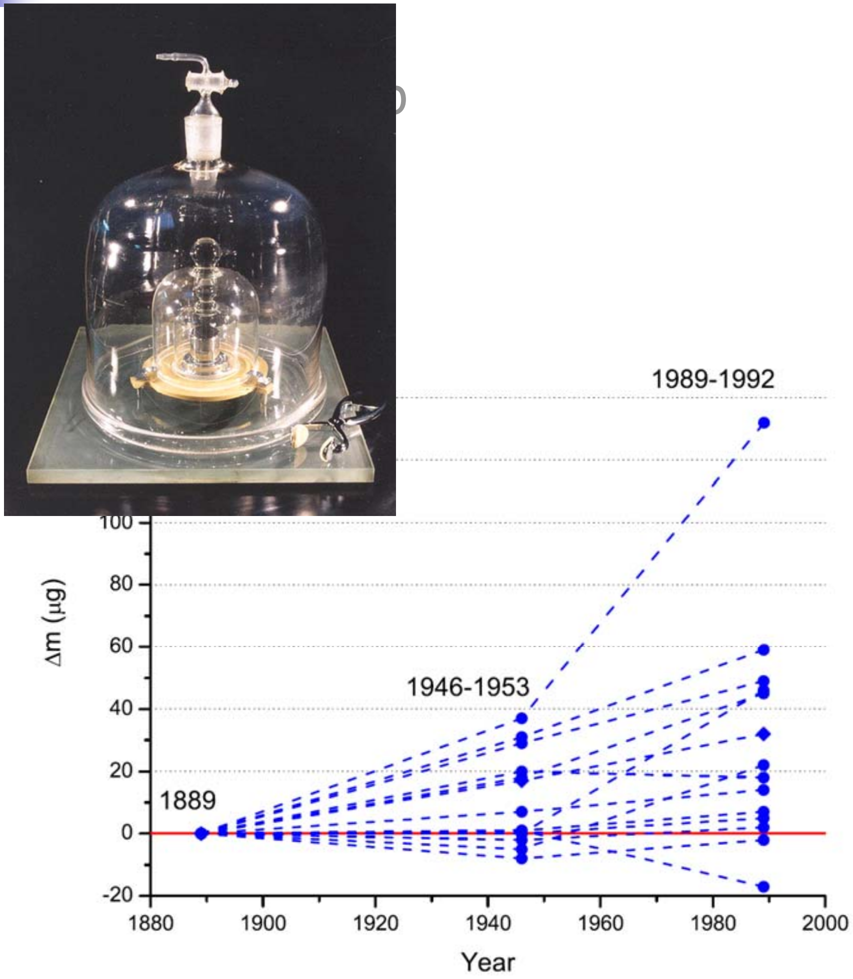
+ natural-silicon discrepacy resolved

+ better accuracy for Avodagro

- new discrepancy

NPL → NRC

Problems



- + natural-silicon discrepancy resolved
- + better accuracy for Avodagro

- - new discrepancy

NPL → NRC

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k

+ more accurate results

– bigger scatter

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k

+ more accurate results

+ more methods

+ efforts for atomic/molecular spectroscopy

CODATA Recommended Values of the Fundamental Physical Constants: 2010*

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