A Field Theory approach to important Cosmological Issues including Dark Energy and the gravitational collapse of Dark Energy

-Anupam Singh,

L.N.M. I.I.T.

Outline

- Introduction and Motivation
    - Field Theory: Fundamental Tool of Physics
- Some Important Applications of Field Theory to Cosmology including Dark Energy
- Summary & Conclusions

Introduction & Motivation

- Field Theory is a fundamental tool of Physics
- Provides a Theoretical Framework that is a starting point across the sub-disciplines of Physics including Nuclear and Particle Physics, Phase Transitions, Statistical Mechanics, Condensed Matter, Cosmology and Astrophysics.

Introduction & Motivation

- For most cosmological applications Field Theory in curved space time is sufficient to explain the observations. It has a track record of success.
- Thus for example for understanding and quantitatively addressing issues related to Inflation and Reheating after Inflation, Field Theory in curved space time is sufficient (we don't need to work with quantum gravity).
- The phenomenon of Dark Energy involves energy scales much lower than those involved in Inflation so we certainly expect Field Theory in curved space time to be sufficient to address this issue.

Introduction & Motivation

- Indeed since the energy scales involved in the physics of Dark Energy is so low one has to worry about protecting the energy scales involved. This can be done by using Pseudo Nambu Goldstone Bosons and using symmetry to protect the energy scales involved in Dark Energy.
- Approach we will take is to move from the familiar terrain into the unfamiliar terrain.

Plan

- Will quickly describe the fundamentals of Field Theory – in a form useful across sub-disciplines of Physics for numerous applications.
- Will describe selected applications:
  - Reheating after Inflationary Phase Transition
  - The Cosmological Constant Problem in the light of modern observations and the Dark Energy Solution
  - Gravitational Collapse of Dark Energy field configurations

Field Theory

Consider field Φ in space – time metric:

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

Action and Lagrange density:

$$\begin{aligned} S &= \int d^4x \mathcal{L} \\ \mathcal{L} &= a^3(t)\left[\frac{1}{2}\dot{\Phi}^2(\vec{x},t) - \frac{1}{2}\frac{(\vec{\nabla}\Phi(\vec{x},t))^2}{a(t)^2} - V(\Phi(\vec{x},t))\right] \end{aligned}$$

Momentum conjugate to Φ :

$$\Pi(\vec{x},t) = a^3(t)\dot{\Phi}(\vec{x},t)$$

Hamiltonian :

$$H(t) = \int d^3x \left\{\frac{\Pi^2}{2a^3(t)} + \frac{a(t)}{2}(\vec{\nabla}\Phi)^2 + a^3(t)V(\Phi)\right\}$$

Time Evolution

In general, $$i\hbar\frac{\partial\hat{\rho}}{\partial t} = [H(t), \hat{\rho}]$$

where, $\hat{\rho}$ is the density matrix

Study the time evolution of the mean field:

$$\phi(t) = \frac{1}{\Omega}\int d^3x\langle\Phi(\vec{x},t)\rangle = \frac{1}{\Omega}\int d^3x Tr\left[\hat{\rho}(t)\Phi(\vec{x})\right]$$

where, Ω is the comoving volume enclosing system

The time evolution of the mean field is then given by:

$$\begin{aligned}
\frac{d\phi(t)}{dt} &= \frac{1}{a^3(t)\Omega}\int d^3x\langle\Pi(\vec{x},t)\rangle = \frac{1}{a^3(t)\Omega}\int d^3x Tr\,\hat{\rho}(t)\Pi(\vec{x}) = \frac{\pi(t)}{a^3(t)} \\
\frac{d\pi(t)}{dt} &= -\frac{1}{\Omega}\int d^3x a^3(t)\langle\frac{\delta V(\Phi)}{\delta\Phi(\vec{x})}\rangle
\end{aligned}$$

Evolution Equations

Do a split: $\Phi(x,t) = \phi(t) + \psi(x,t)$

Mean Field   Fluctuations

For: $V(\Phi) \;=\; \frac{1}{2}\; m^2\Phi^2(\vec{x},t) + \frac{\lambda}{4!}\Phi^4(\vec{x},t)$

The complete set of evolution equations is given by:

$$\ddot{\phi} + m^2\phi + \frac{\lambda}{6}\phi^3 + \frac{\lambda}{2}\phi\langle\psi^2(t)\rangle = 0$$

$$\langle\psi^2(t)\rangle = \int \frac{d^3k}{(2\pi)^3}\left[-iG_k^<(t,t)\right] = \int \frac{d^3k}{(2\pi)^3}\frac{\left|U_k^+(t)\right|^2}{2\omega_k(0)}$$

$$\left[\frac{d^2}{dt^2} + \omega_k^2(t)\right]U_k^+(t) = 0 \quad ; \qquad \omega_k^2(t) = \vec{k}^2 + \mathcal{M}^2(t)$$

$$\mathcal{M}^2(t) = V''(\phi) + \frac{\lambda}{2}\langle\psi^2(t)\rangle = m^2 + \frac{\lambda}{2}\phi^2(t) + \frac{\lambda}{2}\langle\psi^2(t)\rangle$$

The evolution equations can be solved numerically and will display plots of solutions later for the applications.

Application: Reheating after Inflation

- Inflation solves long standing problems of cosmology (isotropy of observed Microwave Background Radiation)
- Universe: Cold, Dark Place after Inflation (Rapid Expansion $\longrightarrow$ Energy Densities $\longrightarrow$ 0)
- Need to recover Hot Big Bang (in order for example for Nucleosynthesis to give us the heavier elements observed in Nature).
- Achieved by dissipational dynamics of fields and particle production

Reference:
D. Boyanovsky , D. Cormier , H.J. de Vega, R. Holman , A. Singh, M. Srednicki .
Phys.Rev.D56 (1997).

Reheating after Inflation

The formalism and evolution equations were described earlier.

For this application also want to keep track of the expectation value of the particle number operator:

$$\mathcal{N}(t) = \int \frac{d^3k}{(2\pi)^3} \frac{Tr\left[a_k^\dagger(0) a_k(0) \rho(t)\right]}{Tr\,\rho(0)}$$

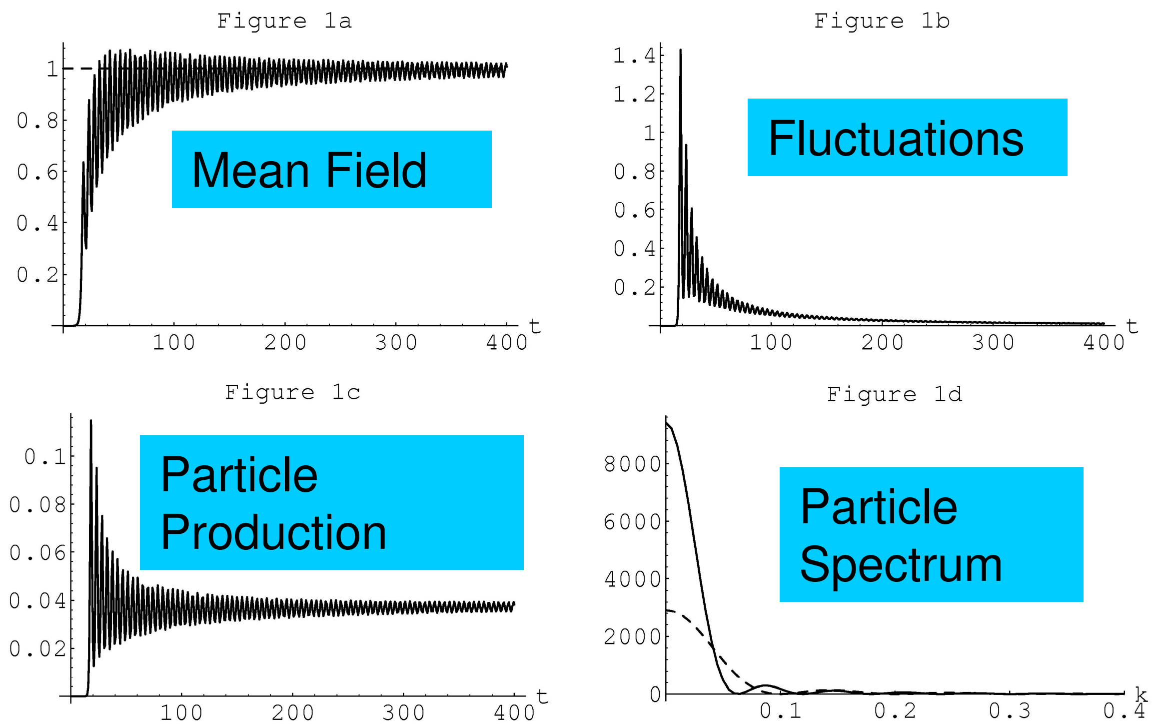
This can be expressed in terms of the mode functions given earlier

$$\mathcal{N}_k(t) = \left(2\left|\mathcal{F}_{+,k}(t)\right|^2 - 1\right)\mathcal{N}_k(0) + \left(\left|\mathcal{F}_{+,k}(t)\right|^2 - 1\right)$$

with $$\left|\mathcal{F}_{+,k}(t)\right|^2 = \frac{1}{4}\left|U_k^+(t)\right|^2\left[1 + \frac{\left|\dot{U}_k^+(t)\right|^2}{\omega_k^2(0)\left|U_k^+(t)\right|^2}\right] + \frac{1}{2}$$

Will show solutions to the time evolution equations as plots on the next slide.

Solutions to the Evolution Equations



From:
D. Boyanovsky , D. Cormier , H.J. de Vega, R. Holman , A. Singh, M. Srednicki .
Phys.Rev.D56 (1997).

Results for Reheating after Inflation

- Instabilities due to parametric resonance and spinodal instabilities → very short timescale for particle production (much shorter than thermalization timescales) → preheating.

- Two stage picture of reheating after inflation:
  1. Rapid Particle Production: Preheating
  2. Thermalization: Slower process happens on a much longer timescale.

Cosmological Constant Application

PHYSICAL REVIEW D VOLUME 52, NUMBER 12 15 DECEMBER 1995

Small nonvanishing cosmological constant from vacuum energy: Physically and observationally desirable

Anupam Singh
Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 2 December 1994)

Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small nonvanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density $\rho_v \sim (10^{-3}\ \mathrm{eV})^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this paper that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age versus Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

PACS number(s): 98.80.Cq, 12.15.Ff, 14.60.Pq, 98.80.Hw

The Cosmological Constant

- Cosmological Constant was first introduced by Albert Einstein in his General Theory of Relativity
- The exact magnitude of the Cosmological Constant and its Physical Role have been unclear until recently
- Observations in the mid 1990s shed new light on the magnitude of the Cosmological Constant and its Physical Role.

The Observations

- Hubble Constant: Measures how fast the Universe is expanding. Was measured in mid 1990s to be: $H_o = 80 \pm 17km\ s^{-1}\ Mpc^{-1}$
- Age of the Universe: Universe has to be at least as old as its contents. In particular, universe has to be at least as old as the globular clusters. Age has now been measured to be: $t_o = 14 \pm 2\, Gyr$

The Implications

- Einstein's Equations can be solved for Cosmological Space Time to obtain the following relationship:

$$t_o = \frac{2}{3} H_o^{-1} \Omega_{vac}^{-1/2} \ln \left[\frac{1 + \Omega_{vac}^{1/2}}{(1 - \Omega_{vac})^{1/2}} \right]$$

Where t_o is the age of the Universe, H_o is the Hubble Constant and Ω_{vac} is the Vacuum Energy Density of the Universe.

The Vacuum Energy Density

- From the relationship between the Age of the Universe, the Hubble Constant and the Vacuum Energy Density together with the observed values of the Age and Hubble Constant it can be inferred that

$$\Omega_{vac} \sim \quad 0.7$$

- In Units of eV the Vacuum Energy Density is then given by $\rho_v \sim (10^{-3} eV)^4$

Physical Origin of Vacuum Energy

- The Energy Density of quantum fields propagating in curved space time can be re-interpreted as the cosmological constant (vacuum energy).

- To compute its value one has to calculate the Effective Potential from first principles.

Magnitude of the Effective Potential

- The detailed expression for the Effective Potential incorporating non-vanishing neutrino masses has been calculated by us from first principles.
- For the current cosmological evolution only the magnitude of the Effective Potential matters.
- This magnitude of the Effective Potential calculated by us is exactly right to explain the modern observations.

Current Status of Dark Energy

Dark Energy and the Accelerating Universe

Annu. Rev. Astron. Astrophys. 2008 46

JOSHUA A. FRIEMAN
Center for Particle Astrophysics, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, IL 60510;
Kavli Institute for Cosmological Physics, The University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637; email: frieman@fnal.gov

MICHAEL S. TURNER
Kavli Institute for Cosmological Physics, The University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637; email: mturner@uchicago.edu

DRAGAN HUTERER
Department of Physics, University of Michigan, 450 Church St., Ann Arbor, MI, 48109; email: huterer@umich.edu

Key Words cosmology, cosmological constant, supernovae, galaxy clusters, large-scale structure, weak gravitational lensing

Abstract
The discovery ten years ago that the expansion of the Universe is accelerating put in place the last major building block of the present cosmological model, in which the Universe is composed of 4% baryons, 20% dark matter, and 76% dark energy. At the same time, it posed one of the

Dark Energy Time Evolution and Masses of Objects formed by Gravitational Collapse

Outline

- Introduction and Motivation
- Evolution Equations
- Solution to Evolution Equations
- Energy Density and Masses of Collapsed Objects
- Summary and Conclusions

Dark Energy: An invention driven by necessity (observations)

PHYSICAL REVIEW D VOLUME 52, NUMBER 12 15 DECEMBER 1995

Small nonvanishing cosmological constant from vacuum energy: Physically and observationally desirable

Anupam Singh
Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 2 December 1994)

Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small nonvanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density $\rho_v \sim (10^{-3}\ \mathrm{eV})^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this paper that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age versus Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

Introduction and Motivation

- Dark Energy is the dominant component of the Energy Density of the Universe.
- Most Natural Candidate for Dark Energy is the Energy Density due to fields in Curved Space-time
- Specific Particle Physics candidates exist which can be characterized as Pseudo Nambu Goldstone Bosons with a well-defined potential [Singh, Holman & Singh, Gupta, Hill, Holman & Kolb]

Introduction & Motivation

- Need to understand the dynamics of these Dark Energy fields
- For cosmology: need to understand the gravitational dynamics of the Dark Energy fields
- Now we will describe the formalism for understanding the gravitational dynamics of the Dark Energy fields for any general potential for the Dark Energy fields.

Introduction & Motivation

- The set of evolution equations describing the time evolution of the Dark Energy fields coupled with gravity is a set of coupled Partial Differential Equations.
- These equations can be solved numerically and this has been done by us.
- Our results demonstrate the gravitational collapse of Dark Energy field configurations.

Evolution Equations

- Interested in studying gravitational dynamics of Dark Energy field configurations.
- In addition to the time evolution of the Field we need to study the time evolution of space-time which is described by the metric:

$$ds^2 = dt^2 - U(r,t)dr^2 - V(r,t)\left[d\theta^2 + \sin^2\theta d\phi^2\right]$$

Evolution Equations

metric, is of course a generalization of the usual FRW metric used to study cosmological space-times. Note that the functions $U(r,t)$ and $V(r,t)$ are functions of both space and time and can capture both homogeneous cosmological expansion as well as inhomogeneous gravitational collapse under appropriate circumstances.

We of course also want to study the time evolution of the field for which we need the Lagrangian for the field.

Lagrangian L given by

$$L = \frac{1}{2}\partial^{\mu}\Phi\partial_{\mu}\Phi - \mathcal{V}(\Phi) \tag{2}$$

where $\mathcal{V}$ is the potential for the field Φ and is for now a general function. Later, when we consider the physically motivated PNGB models this potential will take on a specific functional form.

Evolution Equations

$$\ddot{V} = 2\left[-1 + \frac{V''}{2U} - \frac{V'U'}{4U^2} - \frac{\dot{V}\dot{U}}{4U} + 8\pi G V\left(\frac{\rho}{2} - \frac{P}{2} - \frac{(\Phi')^2}{3U}\right)\right] \qquad (3)$$

$$\ddot{U} = 2U\left[-\frac{\ddot{V}}{V} + \frac{\dot{U}^2}{4U^2} + \frac{\dot{V}^2}{2V^2} - 4\pi G(\rho + 3P)\right] \qquad (4)$$

$$\ddot{\Phi} = \frac{\Phi''}{U} - \dot{\Phi}\left[\frac{\dot{V}}{V} + \frac{\dot{U}}{2U}\right] + \frac{\Phi'}{U}\left[\frac{V'}{V} - \frac{U'}{2U}\right] - \frac{\partial \mathcal{V}(\Phi)}{\partial \Phi} \qquad (5)$$

where a dot represents a partial derivative w.r.t. t and a prime represents a partial derivative w.r.t. r. Further,

$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U} \qquad (6)$$

and

$$P = \frac{1}{2}\dot{\Phi}^2 - \mathcal{V}(\Phi) + \frac{(\Phi')^2}{6U}. \qquad (7)$$

Evolution Equations

The above equations are true for any general potential $\mathcal{V}(\Phi)$. One can of course write down the corresponding equations for PNGB fields. The simplest potential one can write down for the physically motivated PNGB fields [6] can be written in the form:

$$\mathcal{V}(\Phi) = m^4 \left[K - \cos(\frac{\Phi}{f}) \right] \tag{8}$$

- The above potential can thus be substituted in the general equations given on the previous slide to get the full system of evolution equations.

- These are coupled Partial Differential Equations which can be solved numerically to obtain the results of interest to us.

Solutions to the Evolution Equations

- Key issue we want to understand is the *timescale for the gravitational collapse* for dark energy fields. If this timescale is larger than the age of the Universe then this gravitational collapse has no significance today. On the other hand, *if gravitational collapse occurs on timescales less than the age of the Universe then the gravitational collapse of Dark Energy fields must be considered.*

Solution to the Evolution Equations

Guided by the evolution equations given in the previous section we define dimensionless quantitites such that the field is measured in units of f and time and space are measured in units of $\frac{f}{m^2}$.

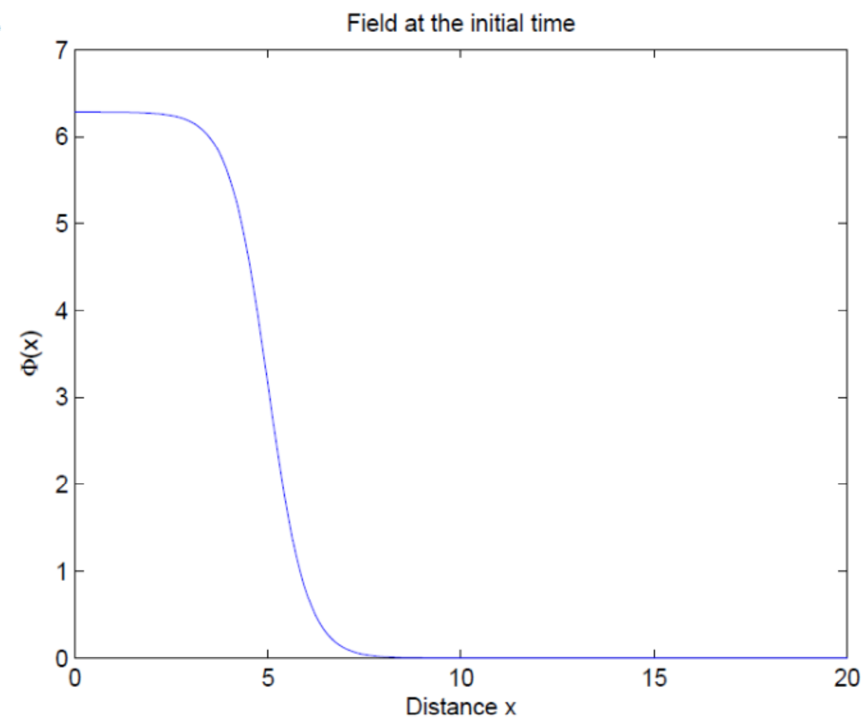


Figure 1: Initial Field configuration

Solution to the Evolution Equations

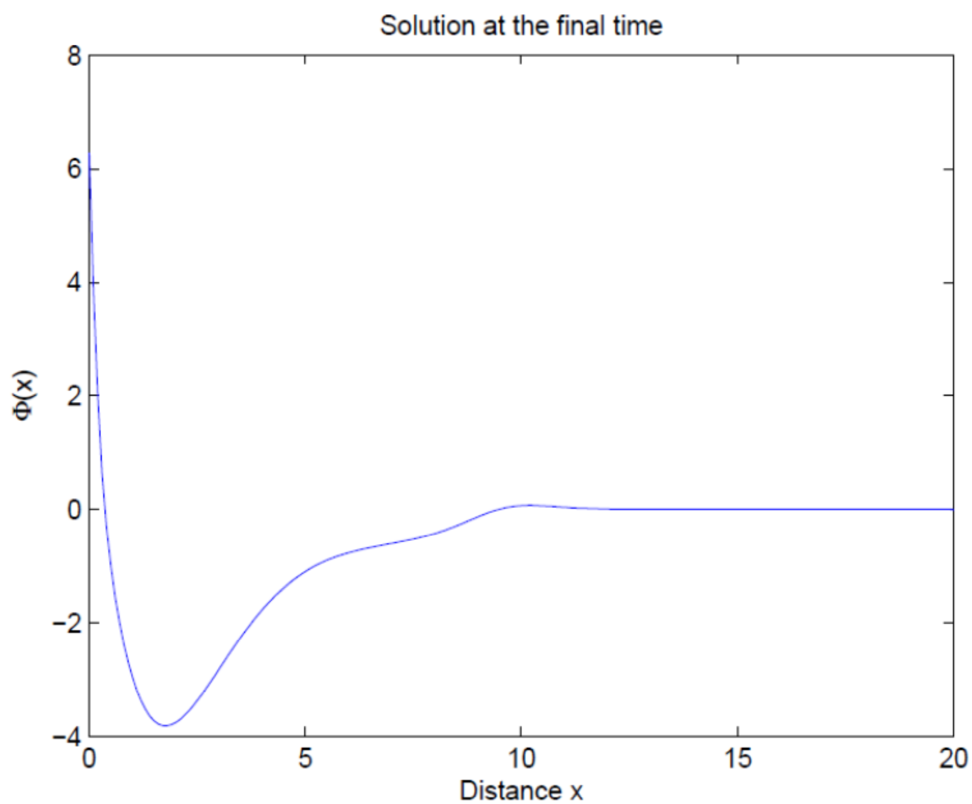


Figure 2: Final Field configuration

Gravitational Collapse of Field Configuration has occurred.

Solution to the Evolution Equation

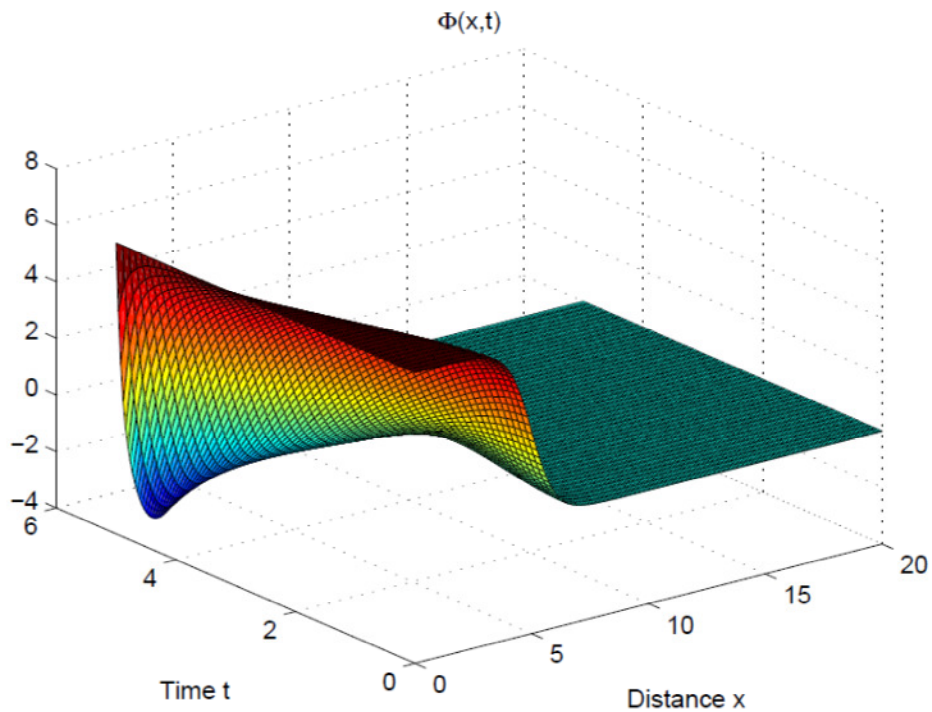


Figure 3: Field configuration in space-time

From this it can be clearly seen that field configuration has collapsed and the timescale for collapse can be seen by studying the figure 3. Since the units of time are given by $\frac{f}{m^2}$, we note that gravitational collapse happens on timescales of $\sim \frac{f}{m^2}$. This timescale is much shorter than the age of the Universe.

Time Evolution of the Energy Density and the Masses of Collapsed Objects

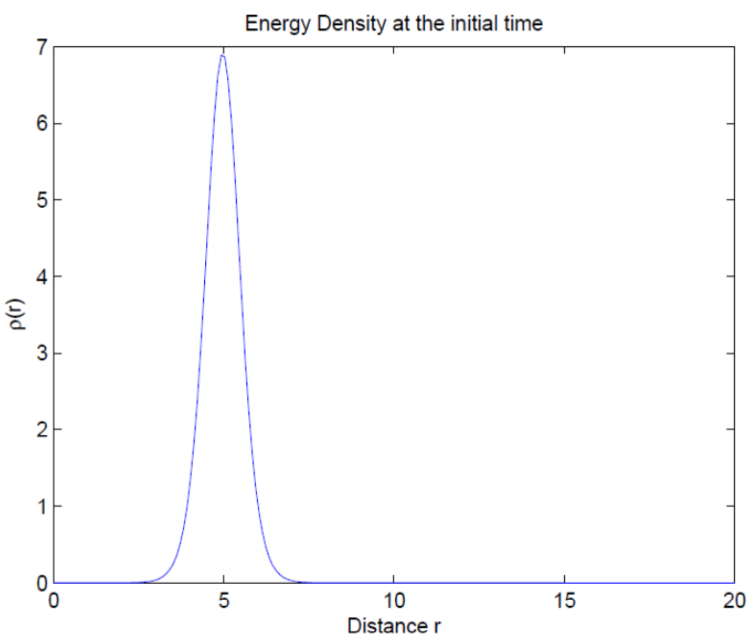
The Energy Density is given by:

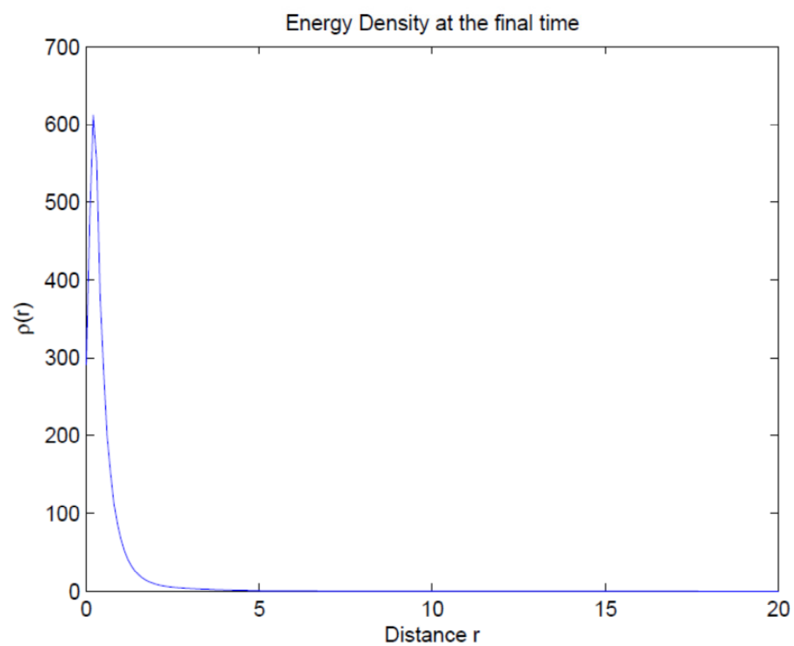
$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U}$$

This can be plotted as a function of space and time and can also be integrated to obtain the Masses of Collapsed Objects.

Let us first show the time evolution of the energy density to demonstrate the formation of collapsed objects.

Time Evolution of the Energy Density and the Masses of Collapsed Objects





Energy Density moves radially inwards as collapse occurs.

Time Evolution of the Energy Density and the Masses of Collapsed Objects

Finally, by integrating the Energy Density:

$$\rho = \frac{1}{2}\dot{\Phi}^2 + \mathcal{V}(\Phi) + \frac{(\Phi')^2}{2U}$$

We can obtain the Masses of Collapsed Objects.

This has been done by us for a range of initial radii R and we get the result that the masses of the collapsed objects is given by

$$4\pi R^2 10 m^2 f$$

Summary & Conclusions

- Dark Energy is the dominant component of the Energy Density of the Universe.
- Most natural candidate for Dark Energy motivated by Particle Physics is the Energy Density due to fields in curved space-time.
- We described the formalism for studying the gravitational dynamics of Dark Energy field configurations.

Summary & Conclusions

- After writing down the complete set of evolution equations describing the time evolution for the fields and the metric, we numerically solved these equations.
- We demonstrated the gravitational collapse of Dark Energy fields.
- ***Our results show*** that the timescale for the gravitational collapse of Dark Energy fields is smaller than the age of the Universe and so the ***gravitational collapse of Dark Energy field configurations must be considered in a complete picture of our Universe.***

Summary & Conclusions

- We also looked at the time evolution of the Energy Density of the Field Configurations
- We demonstrated that the Energy Density moves radially inwards as collapse occurs
- Finally, we integrated the energy density and looked at the masses of collapsed objects formed.
- The result we obtained was that the masses of collapsed objects are given by

$$4\pi R^2 10 m^2 f$$

Thank you

Applications to Phase Transitions

PHYSICAL REVIEW D VOLUME 48, NUMBER 2 15 JULY 1993

Phase transitions out of equilibrium: Domain formation and growth

Daniel Boyanovsky and Da-Shin Lee
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

Anupam Singh
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 18 December 1992)

We study the dynamics of phase transitions out of equilibrium in weakly coupled scalar field theories. We consider the case in which there is a rapid supercooling from an initial symmetric phase in thermal equilibrium at temperature $T_i > T_c$ to a final state at low temperature $T_f \approx 0$. In particular we study the formation and growth of correlated domains out of equilibrium. It is shown that the dynamics of the process of domain formation and growth (spinodal decomposition) cannot be studied in perturbation theory, and a nonperturbative self-consistent Hartree approximation is used to study the long time evolution. We find in weakly coupled theories that the size of domains grows at long times as $\xi_D(t) \approx \sqrt{t\xi(0)}$. The size of the domains and the amplitude of the fluctuations grow up to a maximum time t_s which in weakly coupled theories is estimated to be

$$t_s \approx -\xi(0) \ln \left[\left(\frac{3\lambda}{4\pi^3} \right)^{\frac{1}{2}} \left(\frac{(\frac{T_i}{2T_c})^3}{[\frac{T_i^2}{T_c^2} - 1]} \right) \right]$$

with $\xi(0)$ the zero-temperature correlation length. For very weakly coupled theories, their final size is several times the zero-temperature correlation length. For strongly coupled theories the final size of the domains is comparable to the zero-temperature correlation length and the transition proceeds faster.

PACS number(s): 11.10.Ef, 05.70.Fh, 64.90.+b

Evolution Equations

We are now interested in the growth of the correlation function:

$$\begin{aligned}\mathcal{D}^{(HF)}(x,\tau) &= \frac{\lambda_R}{6m_{f,R}^2}\left[\langle\Phi(\vec{r},t)\Phi(\vec{0},t)\rangle - \langle\Phi(\vec{r},0)\Phi(\vec{0},0)\rangle\right] \\ 3\mathcal{D}^{(HF)}(x,\tau) &= g\int_0^1 dp\frac{p^2}{p^2+L_R^2}\frac{\sin(px)}{px}\left[\mathcal{U}_p^+(t)\mathcal{U}_p^-(t)-1\right]\end{aligned}$$

Which can be expressed in terms of the mode functions whose time evolution is given below:

$$\left[\frac{d^2}{d\tau^2}+q^2-1+g\int_0^1 dp\left\{\frac{p^2}{p^2+L_R^2}[\mathcal{U}_p^+(t)\mathcal{U}_p^-(t)-1]\right\}\right]\mathcal{U}_q^\pm(t)=0$$

with:

$$\begin{aligned}\mathcal{U}_q^\pm(t<0) &= \exp[\mp i\omega_<(q)t] \\ \omega_<(q) &= \sqrt{q^2+L_R^2} \\ L_R^2 &= \frac{m_{i,R}^2}{m_{f,R}^2}=\frac{T_i^2-T_c^2}{T_c^2-T_f^2} \\ g &= \frac{\sqrt{24\lambda_R}}{4\pi^2}\frac{T_i}{[T_c^2-T_f^2]^{\frac{1}{2}}}\end{aligned}$$

Growth of Correlations

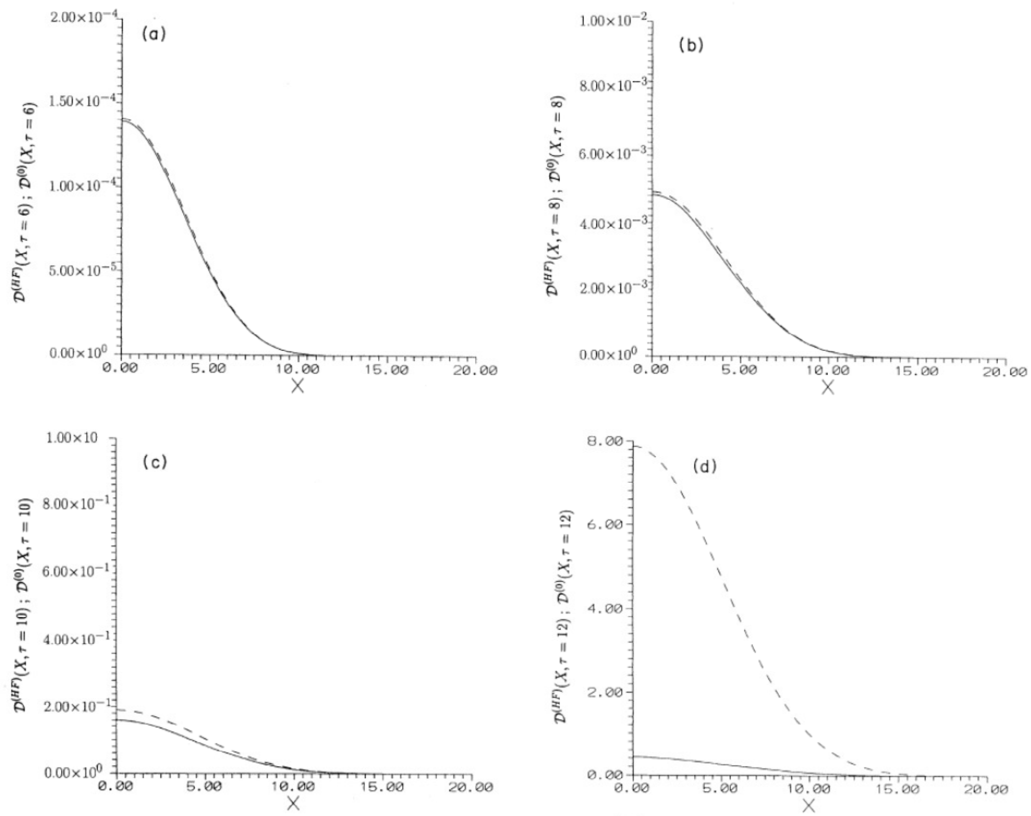


FIG. 5. (a) Scaled correlation functions for $\tau = 6$, as function of x, $\mathcal{D}^{(\mathrm{HF})}(x,\tau)$ (solid line), and $\mathcal{D}^{(0)}(x,\tau)$ (dashed line). $\lambda = 10^{-12}$, $\frac{T_i}{T_c} = 2$. (b) Scaled correlation functions for $\tau = 8$, as function of x, $\mathcal{D}^{(\mathrm{HF})}(x,\tau)$ (solid line), and $\mathcal{D}^{(0)}(x,\tau)$ (dashed line). $\lambda = 10^{-12}$, $\frac{T_i}{T_c} = 2$. (c) Scaled correlation functions for $\tau = 10$, as function of x, $\mathcal{D}^{(\mathrm{HF})}(x,\tau)$ (solid line), and $\mathcal{D}^{(0)}(x,\tau)$ (dashed line). $\lambda = 10^{-12}$, $\frac{T_i}{T_c} = 2$. (d) Scaled correlation functions for $\tau = 12$, as function of x, $\mathcal{D}^{(\mathrm{HF})}(x,\tau)$ (solid line), and $\mathcal{D}^{(0)}(x,\tau)$ (dashed line). $\lambda = 10^{-12}$, $\frac{T_i}{T_c} = 2$.

Results

- Computed growth of correlations and domain sizes.
- For weakly coupled theories, we got the result that the size of domains grows as:

  $$\xi_D(t) \approx \sqrt{t\xi(0)}$$

  and the size of domains and the fluctuations grow up to a maximum time of:

  $$t_s \approx -\xi(0)\ln\left[\left(\frac{3\lambda}{4\pi^3}\right)^{\frac{1}{2}}\left(\frac{(\frac{T_i}{2T_c})^3}{[\frac{T_i^2}{T_c^2}-1]}\right)\right]$$

- Of course, we obtained results also for strongly coupled theories and more importantly the formalism we laid down has subsequently been used by us and numerous authors to study phase transitions in a number of different contexts from the early universe and Nuclear Matter to Condensed Matter Systems.

Extensions of our work on Phase Transitions ranging from Early Universe to Nuclear Matter and Condensed Matter Systems

DYNAMICS OF SYMMETRY BREAKING OUT OF EQUILIBRIUM: FROM CONDENSED MATTER TO QCD AND THE EARLY UNIVERSE*

D. Boyanovsky[(a,b)] **and H.J. de Vega**[(b,a)]
(a)Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260 USA
(b) LPTHE[†] *Université Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII), Tour 16, 1er. étage, 4, Place Jussieu 75252 Paris, Cedex 05, France*
(February 1, 2008)

Abstract

The dynamics of symmetry breaking during out of equilibrium phase transitions is a topic of great importance in many disciplines, from condensed matter to particle physics and early Universe cosmology with definite experimental impact. In these notes we provide a summary of the relevant aspects of the dynamics of symmetry breaking in many different fields with emphasis on the experimental realizations. In condensed matter we address the dynamics of phase ordering, the emergence of condensates, coarsening and dynamical scaling. In QCD the possibility of disoriented chiral condensates of pions emerging during a strongly out of equilibrium phase transition is discussed. We elaborate on the dynamics of phase ordering in phase transitions in the Early Universe, in particular the emergence of condensates and scaling in FRW cosmologies. We mention some experimental efforts in different fields that study this wide ranging phenomena and offer a quantitative theoretical description both at the phenomenological level in condensed matter, introducing the scaling hypothesis as well as at a microscopic level in quantum field theories. The emergence of semiclassical condensates and a dynamical length scale is shown in detail, in quantum field theory this length scale is constrained by causality.

Extensions of our work to Bose Einstein Condensation

Microscopic Evolution of a Weakly Interacting Homogeneous Bose Gas

D. G. Barci[1,2] *, E. S. Fraga[3] †, Rudnei O. Ramos[2,4] ‡

[1] *Department of Physics, University of Illinois at Urbana-Champaign, 1110W. Green St., Urbana, IL 61801-3080, USA*

[2] *Universidade do Estado do Rio de Janeiro, Instituto de Física, Departamento de Física Teórica, 20550-013 Rio de Janeiro, RJ, Brazil*

[3] *Department of Physics, Brookhaven National Laboratory, Upton, NY 11973-5000, USA*

[4] *Department of Physics and Astronomy, Dartmouth College, Hanover, New Hampshire 03755-3528, USA*

We provide a detailed description of the nonequilibrium time evolution of an interacting homogeneous Bose-Einstein condensate. We use a nonperturbative in-medium quantum field theory approach as a microscopic model for the Bose gas. The real-time dynamics of the condensate is encoded in a set of self-consistent equations which corresponds to an infinite sum of ladder Feynman diagrams. The crucial role played by the interaction between fluctuations for the instability generation is thoroughly described.

To appear in Laser Physics (2001), Special Issue on Bose-Einstein Condensation of Trapped Atoms

really intends to describe the actual evolution of the condensate formation, then finite-density, non-zero temperature, and nonequilibrium dynamics effects will have to be taken into consideration. This may be accomplished quite naturally by in-medium nonequilibrium quantum field theory methods. This set of characteristics makes BEC one of the most attractive and promising systems in which one can use models and approximations that could also prove useful in very different environments such as neutron stars or heavy-ion collisions [4].

Recent experiments with dilute atomic gases [5] were able to start probing quantities which are relevant to the understanding of the underlying dynamics of BEC, such as the time scales for the condensate formation and its final size. On the theoretical side, the microscopic be-

Extensions of our work to the Ferromagnetic Phase Transition

PHYSICAL REVIEW B **73**, 214437 (2006)

Domain growth in the Heisenberg ferromagnet: Effective vector theory of the $S=1/2$ model

V. M. Turkowski,* P. D. Sacramento, and V. R. Vieira
CFIF, Instituto Superior Técnico, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal
(Received 11 November 2005; revised manuscript received 19 January 2006; published 21 June 2006)

We derive an effective vector theory of the spin $S=1/2$ Heisenberg ferromagnet in an external magnetic field using the Majorana representation for the spin operators and decoupling the interaction term via a Hubbard-Stratonovich transformation. This theory contains both cubic and quartic bosoniclike field terms. We analyze the problem in the Hartree approximation, similarly to the analysis by Boyanovsky *et al.* [Phys. Rev. E **48**, 767 (1993); Phys. Rev. D **48**, 800 (1993)] for the scalar case. The time dependence of the radius of the stable phase domain (bubble) in this bosonic theory is studied in the cases of different dimensionalities and weak magnetic field H. The role of the cubic terms in the process of domain growth is analyzed. It is shown that the field components perpendicular to H acquire a larger amplitude than the component parallel to H, at early times. The domain radius grows as $\sqrt{t}$ for times smaller than the spinodal time or in the case of a very weakly coupled

Summary & Conclusions

- Field Theory is a Fundamental Tool of Physics
- It is simultaneously an old subject at the base of much of Physics and also an active area of Research providing Fundamental insights at the cutting edge of modern observations.
- Provides a Theoretical Framework that is a starting point across the sub-disciplines of Physics including Nuclear and Particle Physics, Phase Transitions, Statistical Mechanics, Condensed Matter, Cosmology and Astrophysics.
- While I have worked at the cutting edge of Field Theory across these sub-disciplines of Physics, today I focused on describing the overall framework and three specific examples.
- This same framework of Field Theory can be used across the sub-disciplines of Physics and provides a common framework that can be used across all of Physics.

Reference Material on PNGBs in Cosmology and other useful material.

Selected References on Pseudo Nambu Goldstone Boson models from Particle Physics relevant for Cosmology today.

PHYSICAL REVIEW D VOLUME 47, NUMBER 2 15 JANUARY 1993

Non-Abelian soft boson phase transitions and large-scale structure

Richard Holman and Anupam Singh

Institute for Theoretical Physics, University of California, Santa Barbara, California 93016
and Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213

(Received 24 April 1992)

A new class of models with pseudo Nambu-Goldstone bosons is constructed using a *non-Abelian* symmetry in the right-handed Majorana neutrino sector of *seesaw* neutrino mass models. The phase structure of these models is examined both at zero and nonzero temperatures, with particular emphasis on their phase transition characteristics. We find that the vacuum manifold of these models exhibits a rich structure in terms of possible topological defects, and we argue that these models may have applications to late-time phase transition theories of structure formation.

PACS number(s): 98.80.Cq, 05.70.Fh, 12.10.Gq

Selected References on Pseudo Nambu Goldstone Boson models from Particle Physics relevant for Cosmology today.

PHYSICAL REVIEW D VOLUME 52, NUMBER 12 15 DECEMBER 1995

Small nonvanishing cosmological constant from vacuum energy: Physically and observationally desirable

Anupam Singh
Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 2 December 1994)

Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small nonvanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density $\rho_v \sim (10^{-3}\ \mathrm{eV})^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this paper that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age versus Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

Formation of Inhomogeneous Field Configurations.

PHYSICAL REVIEW D VOLUME 48, NUMBER 2 15 JULY 1993

Phase transitions out of equilibrium: Domain formation and growth

Daniel Boyanovsky and Da-Shin Lee
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

Anupam Singh
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213

(Received 18 December 1992)

We study the dynamics of phase transitions out of equilibrium in weakly coupled scalar field theories. We consider the case in which there is a rapid supercooling from an initial symmetric phase in thermal equilibrium at temperature $T_i > T_c$ to a final state at low temperature $T_f \approx 0$. In particular we study the formation and growth of correlated domains out of equilibrium. It is shown that the dynamics of the process of domain formation and growth (spinodal decomposition) cannot be studied in perturbation theory, and a nonperturbative self-consistent Hartree approximation is used to study the long time evolution. We find in weakly coupled theories that the size of domains grows at long times as $\xi_D(t) \approx \sqrt{t\xi(0)}$. The size of the domains and the amplitude of the fluctuations grow up to a maximum time t_s which in weakly coupled theories is estimated to be

$$t_s \approx -\xi(0) \ln \left[\left(\frac{3\lambda}{4\pi^3} \right)^{\frac{1}{2}} \left(\frac{(\frac{T_i}{2T_c})^3}{[\frac{T_i^2}{T_c^2} - 1]} \right) \right]$$

with $\xi(0)$ the zero-temperature correlation length. For very weakly coupled theories, their final size is several times the zero-temperature correlation length. For strongly coupled theories the final size of the domains is comparable to the zero-temperature correlation length and the transition proceeds faster.

PACS number(s): 11.10.Ef, 05.70.Fh, 64.90.+b

Structure Formation due to PNGB fields in the see-saw model of Neutrino Masses.

PHYSICAL REVIEW D VOLUME 50, NUMBER 2 15 JULY 1994

Quasar production: Topological defect formation due to a phase transition linked with massive neutrinos

Anupam Singh
Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 30 August 1993)

Recent observations of the space distribution of quasars indicate a very notable peak in space density at a redshift of 2 to 3. It is pointed out in this article that this may be the result of a phase transition which has a critical temperature of roughly a few meV (in the cosmological units $h=c=k=1$). It is further pointed out that such a phase transition is natural in the context of massive neutrinos. In fact, the neutrino masses required for quasar production and those required to solve the solar neutrino problem by the Mikheyev-Smirnov-Wolfenstein mechanism are consistent with each other.

PACS number(s): 98.80.Cq, 12.15.Ff, 14.60.Pq, 98.54.Aj

Physical Significance of Quantum Fields

- Quantum Fields represent the density of particles.

How real is a quantum field and what is its physical significance? To begin to to get a feeling of its meaning, let us look at some key properties. The transformation from wavefunction, to operator also extends to more directly observable quantities. Let us begin with Born's famous expression for the probability density in first quantization, $\rho(x) = \psi^*(x)\psi(x)$. By elevating the wavefunction to the status of a field operator, we obtain

$$\rho(x) = |\psi(x)|^2 \longrightarrow \hat{\rho}(x) = \hat{\psi}^\dagger(x)\hat{\psi}(x), \tag{3.11}$$

which is now the *operator* that represents the fluctuating particle density in the many body systems, so loosely speaking, the intensity of the quantum field represents the density of particles

Quantum Fields for the Bose-Einstein Case

- Field theory is used to understand Bose Einstein condensation experiments with gases.

$$\mathcal{L} = \phi^* \left(i\frac{d}{dt} + \frac{1}{2m}\nabla^2 \right) \phi + \mu\phi^*\phi - \frac{g}{2}(\phi^*\phi)^2 \quad , \qquad (4)$$

where the complex scalar field $\phi(\mathbf{x}, t)$ represents charged spinless bosons of mass m, and g is the coupling constant defined above. In (4) we have also explicitly introduced a chemical potential μ that guarantees a constant total density of particles

$$\langle \phi^*\phi \rangle = n \quad . \qquad (5)$$

Magnitude of the Effective Potential

- The detailed expression for the Effective Potential has been calculated by us

$$V(\mathcal{M}^2) = \left(V_0 - \frac{7\pi^2 T^4}{90}\right) + (m_r^2 + T^2/6)\mathcal{M}^2$$

$$+\frac{(\mathcal{M}^2)^2}{8\pi^2}\left(n - \ln\frac{T^2}{\mu^2}\right),$$

- For the current cosmological evolution only the magnitude of the Effective Potential matters.

Captions

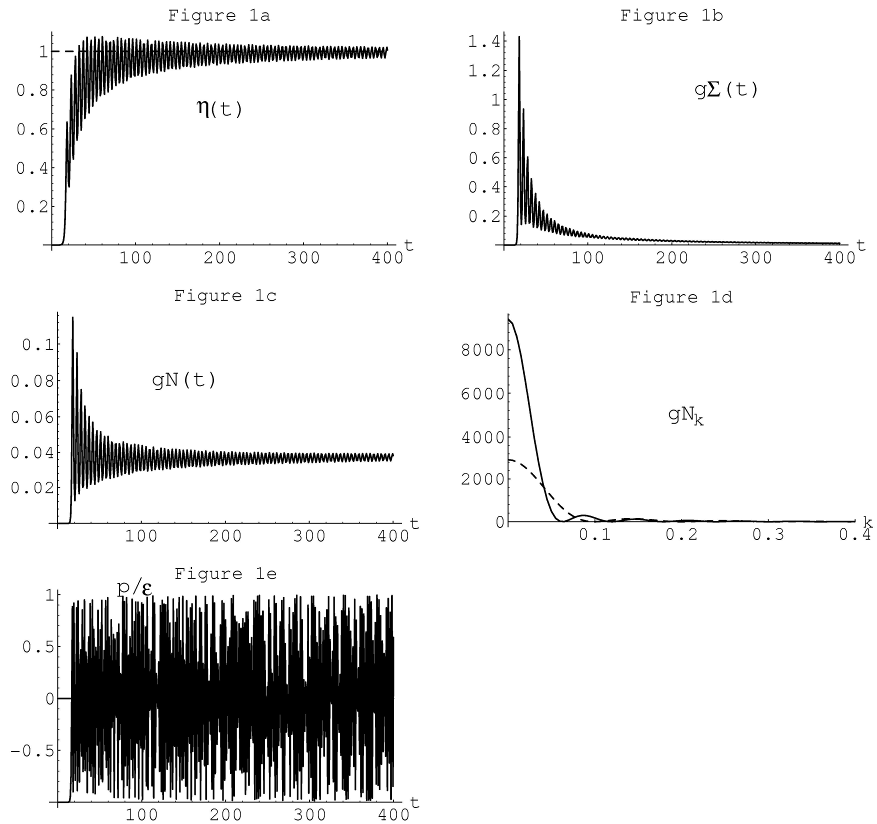


FIG. 1. Figure 1: Symmetry broken, slow roll, large N, matter dominated evolution of (a) the zero mode $\eta(t)$ vs. t, (b) the quantum fluctuation operator $g\Sigma(t)$ vs. t, (c) the number of particles $gN(t)$ vs. t, (d) the particle distribution $gN_k(t)$ vs. k at $t = 149.1$ (dashed line) and $t = 398.2$ (solid line), and (e) the ratio of the pressure and energy density $p(t)/\varepsilon(t)$ vs. t for the parameter values $m^2 = -1$, $\eta(t_0) = 10^{-7}$, $\dot{\eta}(t_0) = 0$, $g = 10^{-12}$, $H(t_0) = 0.1$.

Renormalized Equations

$$\ddot{\phi} + M_R^2\phi + \frac{\lambda_R}{2}\left[1 - \left(\frac{2}{3}\right)\frac{1}{1 - \frac{\lambda_R}{16\pi^2}\ln\left(\frac{\Lambda}{\kappa}\right)}\right]\phi^3 + \frac{\lambda_R}{2}\phi\left(\langle\psi^2(t)\rangle_R - \langle\psi^2(0)\rangle_R\right) = 0$$

$$\left[\frac{d^2}{dt^2} + k^2 + M_R^2 + \frac{\lambda_R}{2}\phi^2(t) + \frac{\lambda_R}{2}\left(\langle\psi^2(t)\rangle_R - \langle\psi^2(0)\rangle_R\right)\right]U_k^+(t) = 0$$

$$\begin{aligned}\left(\langle\psi^2(t)\rangle_R - \langle\psi^2(0)\rangle_R\right) &= \left[\frac{1}{1 - \frac{\lambda_R}{16\pi^2}\ln\left(\frac{\Lambda}{\kappa}\right)}\right] \times \\ &\left\{\int \frac{d^3k}{(2\pi)^3}\frac{\left|U_k^+(t)\right|^2 - 1}{2\omega_k(0)} + \frac{\lambda_R}{16\pi^2}\ln\left(\frac{\Lambda}{\kappa}\right)\left(\phi^2(t) - \phi^2(0)\right)\right\}\end{aligned}$$

Dynamics of Fields in FRW Space-times.

PHYSICAL REVIEW D VOLUME 56, NUMBER 4 15 AUGUST 1997

Scalar field dynamics in Friedmann-Robertson-Walker spacetimes

D. Boyanovsky,[1] D. Cormier,[2] H. J. de Vega,[3] R. Holman,[2] A. Singh,[4] and M. Srednicki[4]
[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260*
[2]*Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213*
[3]*Laboratoire de Physique Théorique et Hautes Energies Université Pierre et Marie Curie (Paris VI), Tour 16, 1er. étage, 4, Place Jussieu 75252 Paris, Cedex 05, France*
[4]*Department of Physics, University of California, Santa Barbara, California 93106*
(Received 14 March 1997)

We study the nonlinear dynamics of quantum fields in matter- and radiation-dominated universes, using the nonequilibrium field theory approach combined with the nonperturbative Hartree and the large N approximations. We examine the phenomenon of explosive particle production due to spinodal instabilities and parametric amplification in expanding universes with and without symmetry breaking. For a variety of initial conditions, we compute the evolution of the inflaton, its quantum fluctuations, and the equation of state. We find explosive growth of quantum fluctuations, although particle production is somewhat sensitive to the expansion of the universe. In the large N limit for symmetry-breaking scenarios, we determine generic late time solutions for any flat Friedmann-Robertson-Walker (FRW) cosmology. We also present a complete and numerically implementable renormalization scheme for the equation of motion and the energy momentum tensor in flat FRW cosmologies. In this scheme the renormalization constants are independent of time and of the initial conditions. [S0556-2821(97)02616-7]

PACS number(s): 98.80.Cq