Cosmic neutrino flavor ratios with broken $\nu_\mu - \nu_\tau$ symmetry

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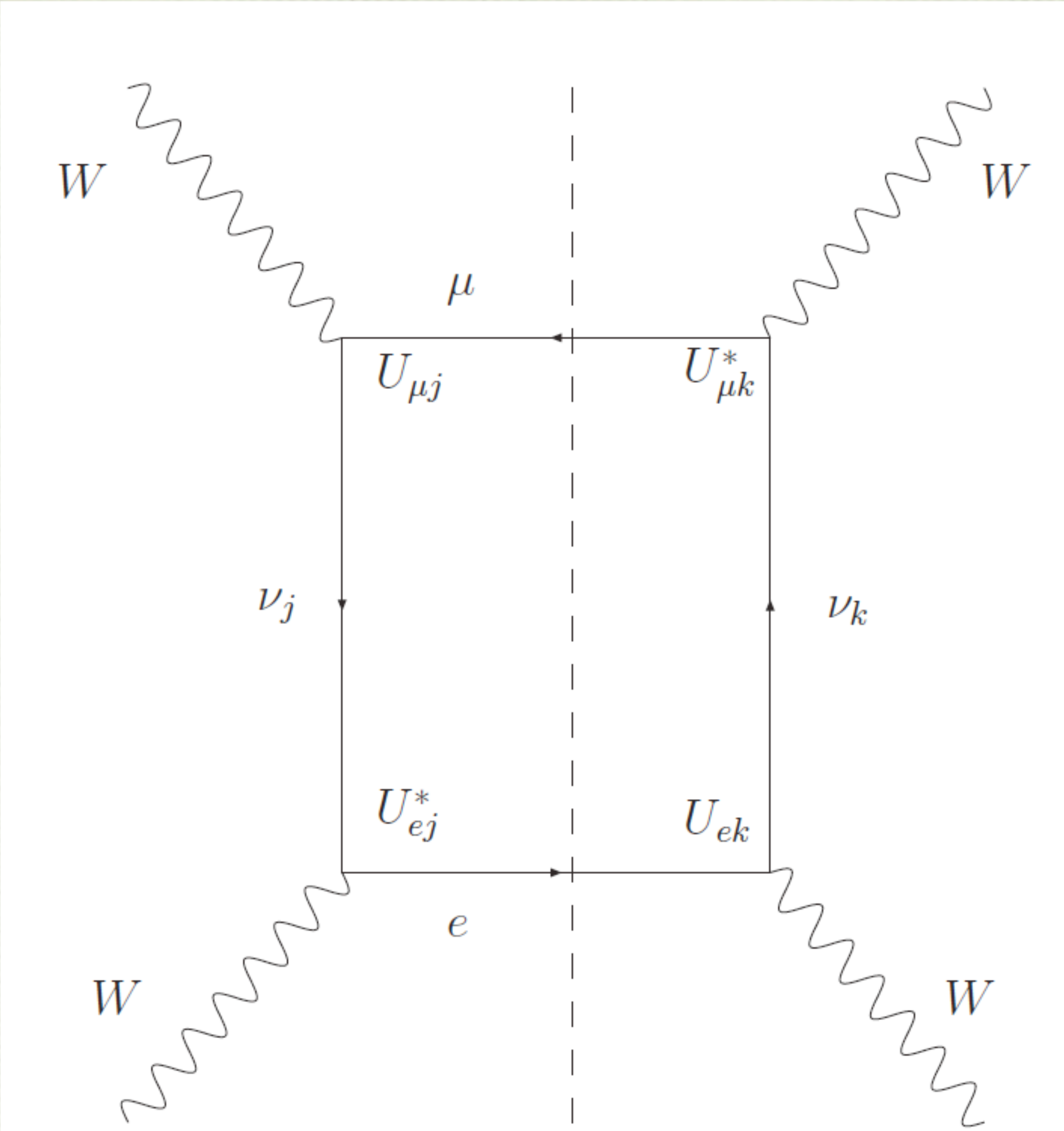


Oscillation phase is $\phi = \frac{L\delta m^2}{4E}$,

so, $\delta\phi = \phi\,\delta \ln \phi = \phi \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta E}{E}\right)^2}$

$$= 2\pi N_{\rm osc} \sqrt{\left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta E}{E}\right)^2}$$

averages out !

$$\langle e^{i\phi} \rangle = 0$$

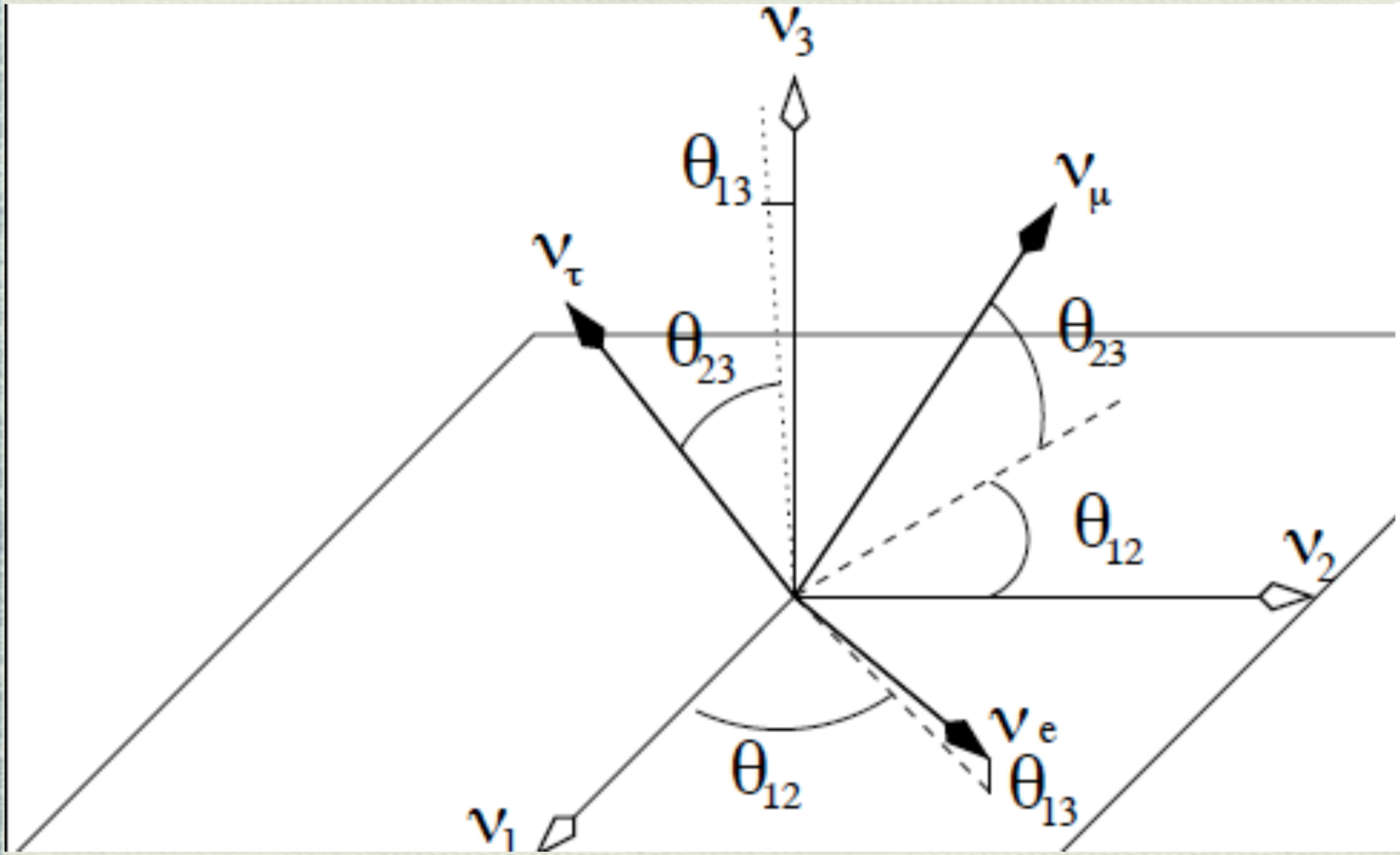
$$P_{\mu\to e} = \sum_j |U_{ej}|^2\, |U_{j\mu}|^2$$

$$P_{\alpha\to\beta} = \sum_j |U_{\beta j}|^2\, |U_{j\alpha}|^2$$

$$= \tilde{P}\, \tilde{P}^{\rm T}$$

$$\stackrel{\rm sym}{=} P_{\beta\to\alpha} \stackrel{\rm CP}{=} P_{\bar\alpha\to\bar\beta}$$

Some neutrino flavor physics



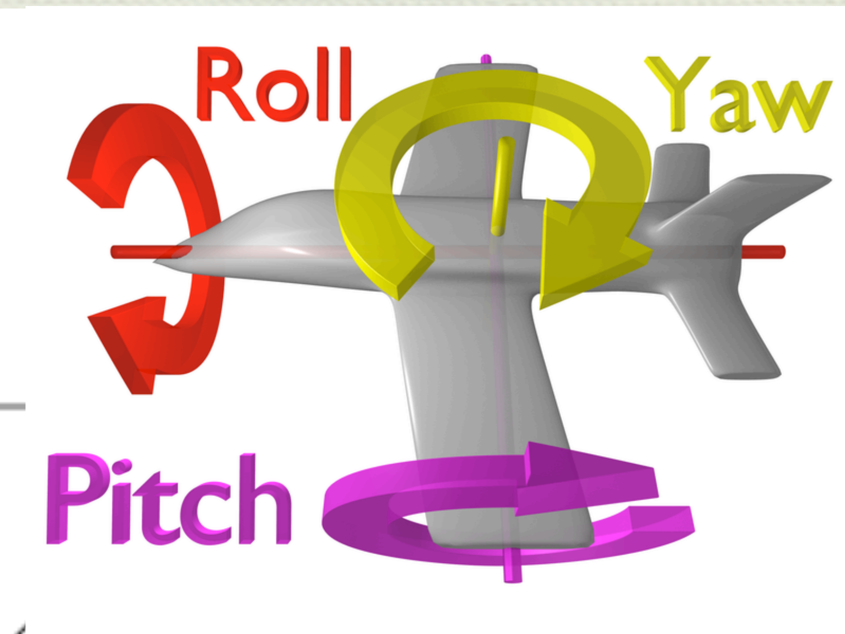
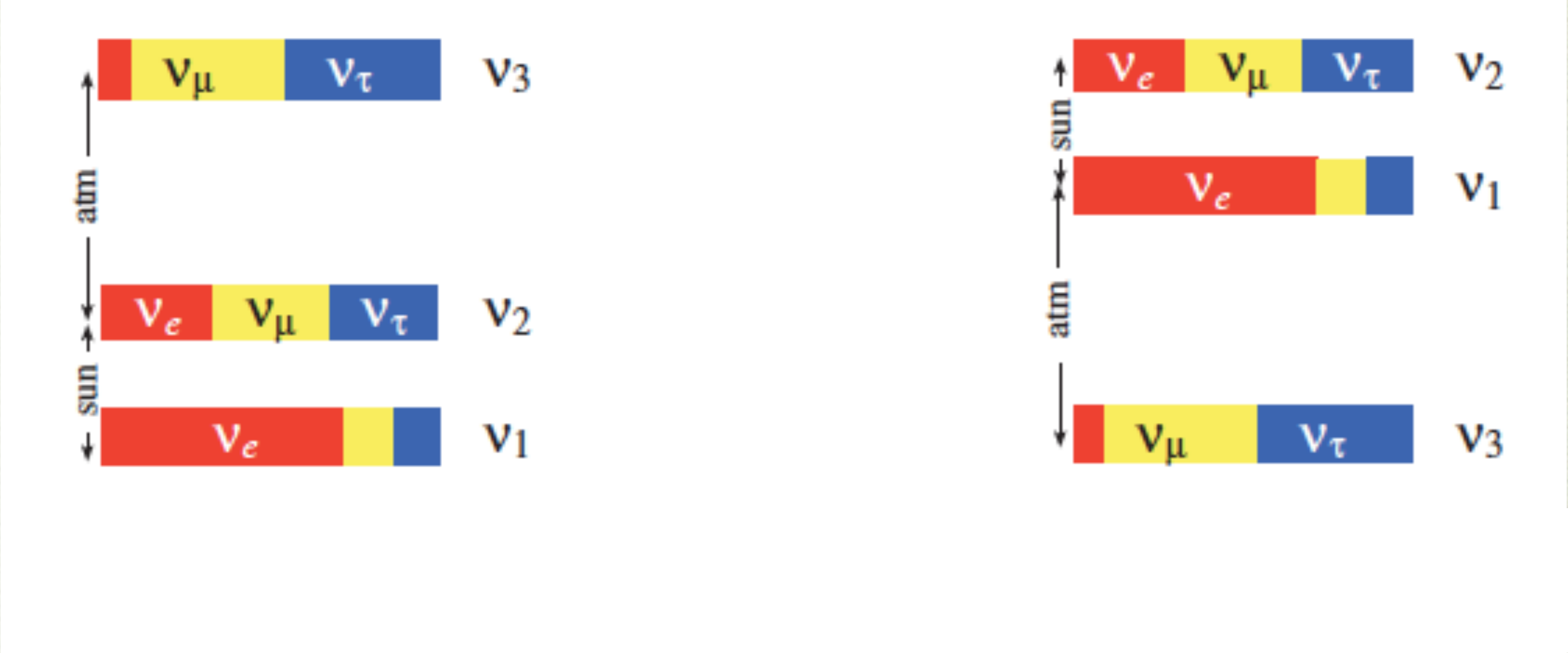


Fig courtesy Steve King

	Quarks (CKM)	Leptons (PMNS)
θ_{12}	13°	35°
θ_{23}	2°	43°
θ_{13}	0.2°	9°
δ	68°	unknown



From classical probabilities $\tilde{P} \equiv \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix}$

and $P \equiv \tilde{P}\tilde{P}^T$, get

Zeroth Order (TBM)
Flavor Evolution Matrix: P

$$P = \frac{1}{18}\begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix}$$

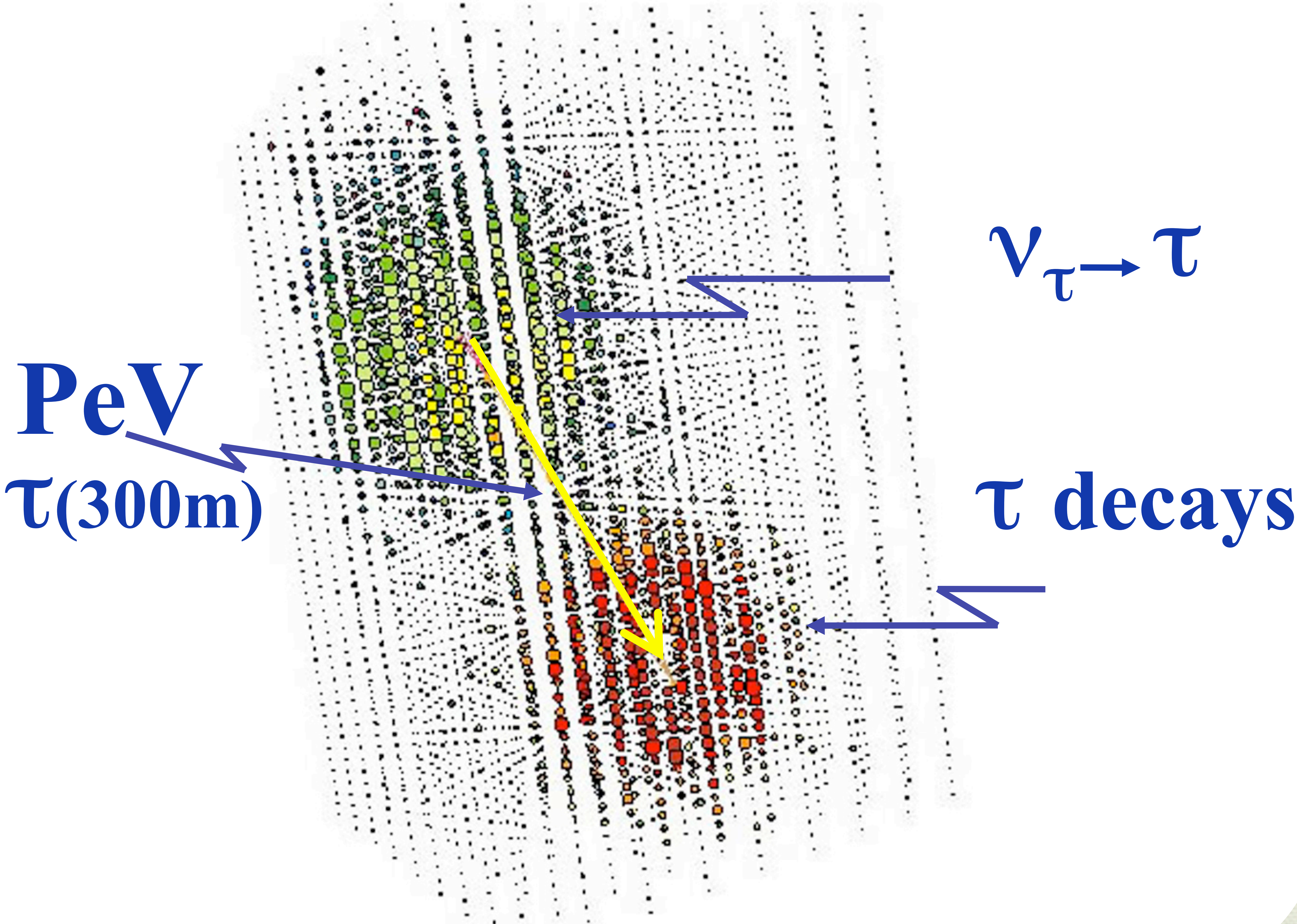
Working forward - guessing injection models, have:

Flavor Mix at Earth, $\sin\theta_{13} = 0$:

Beam type	Initial (input)	Final (output)
Conventional (pp,pγ)	1:2:0	1:1:1
Damped Muon	0:1:0	4:7:7
Beta Beam(n decay)	1:0:0	5:2:2
Prompt	1:1:0	14:11:11

Now we know that $\sin\theta_{13} = 0.16$

And spacetime foam/virtual black holes democratize neutrino flavors to (1,1,1).



Double Bang signature: Learned and Pakvasa (1995)

More neutrino flavor (astro) physics

[Fu, Ho, TJW, 1209.52382 and PLB (2013)]

$$(\tilde{P}\tilde{P}^T)_{\rm TBM} = \tfrac{1}{18}\begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix}$$

repeated rows => TBM Determinant = 0; ergo, not invertible:

$$\begin{pmatrix} w_e \\ w_\mu \\ w_\tau \end{pmatrix} = (\tilde{P}\tilde{P}^T)\begin{pmatrix} W_e \\ W_\mu \\ W_\tau \end{pmatrix} \quad \text{but not} \quad \begin{pmatrix} W_e \\ W_\mu \\ W_\tau \end{pmatrix} = (\tilde{P}\tilde{P}^T)^{-1}\begin{pmatrix} w_e \\ w_\mu \\ w_\tau \end{pmatrix}$$

where $\vec{W}$ and $\vec{w}$ are the flavor-ratio vectors at injection, and at Earth (after processing), respectively.

e.g., pion chain W=(1,2,0)/3 --> w=(1,1,1)/3;
& beta beam W=(1,0,0) --> w=(5,2,2)/9.

Reducing 3 Flavors to 2 within the TBM paradigm:

We may obtain the propagation matrix in this $(e, \not e)$ basis by adding the identical ν_μ and ν_τ rows in Eq. (6), and omitting the now redundant third column. One gets

$$P^{\rm eff}_{\rm TBM} = \frac{1}{9}\begin{pmatrix} 5 & 2 \\ 4 & 7 \end{pmatrix}. \tag{7}$$

The propagation equation

$$\begin{pmatrix} w_e \\ w_{\not e} \end{pmatrix} = P^{\rm eff}_{\rm TBM} \begin{pmatrix} W_e \\ W_{\not e} \end{pmatrix} \tag{8}$$

$$(P^{\rm eff}_{\rm TBM})^{-1} = \frac{1}{3}\begin{pmatrix} 7 & -2 \\ -4 & 5 \end{pmatrix}, \begin{pmatrix} W_e \\ W_{\not e} \end{pmatrix} = (P^{\rm eff}_{\rm TBM})^{-1} \begin{pmatrix} w_e \\ w_{\not e} \end{pmatrix} \tag{9}$$

From Eq. (9) we derive an interesting expression relating flavor ratios at the source to the same ratio observed at Earth:

$$\frac{W_e}{W_{\not e}} = \frac{7 - 11\left(\frac{w_\mu}{w_{sh}}\right)}{14\left(\frac{w_\mu}{w_{sh}}\right) - 4} \tag{11}$$

E.g., positivity implies $\frac{2}{7} \le \frac{w_\mu}{w_{sh}} \le \frac{7}{11}$

Note: IceCube data at low end right now

Some 3-flavor neutrino physics

Constructing inverse propagation matrix, and assuming $W_\tau = 0$, get, e.g.,

$$\frac{W_\mu}{W_e} = \frac{P_{e\mu} - (P_{ee} + P_{e\tau})\left(\frac{w_\mu}{w_{sh}}\right)}{(P_{e\mu} + P_{\mu\tau})\left(\frac{w_\mu}{w_{sh}}\right) - P_{\mu\mu}} .$$

This equation generalizes Eq. (11) to the condition of broken ν_μ-ν_τ symmetry. It is independent of any injection model.

If this relation is violated, then ν_τ's are produced at the source!

Some more neutrino flavor physics

Flavor fractions must sum to one: $w_e + w_\mu + w_\tau = 1$ at Earth

(and $W_e + W_\mu + W_\tau = 1$ at source)

Restricts 3D space to a tri-symmetric bounded plane, a triangle:

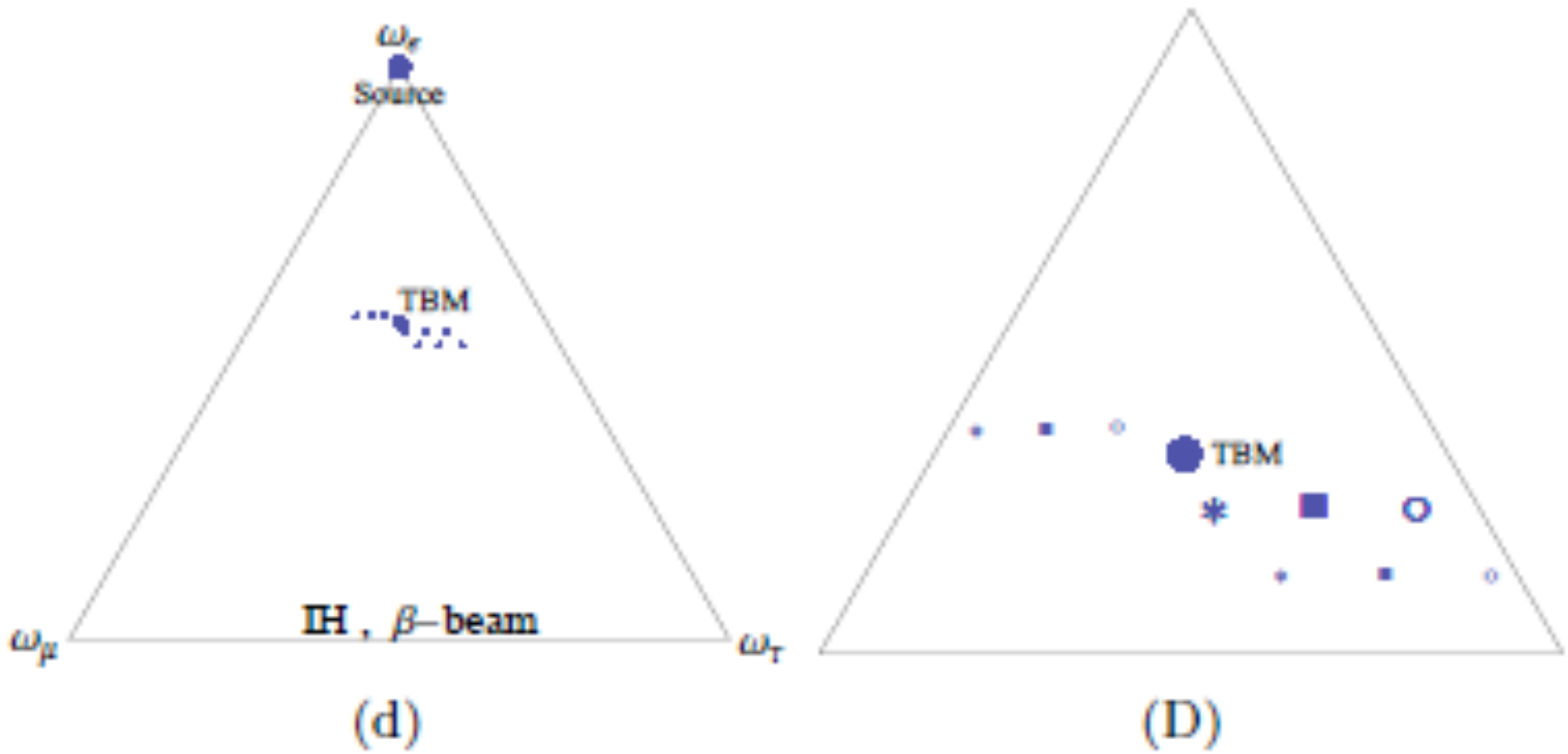


FIG. 1: Triangle plots of (left) entire $\vec{w}$-parameter space, (right) un-normalized blow-up of left panel

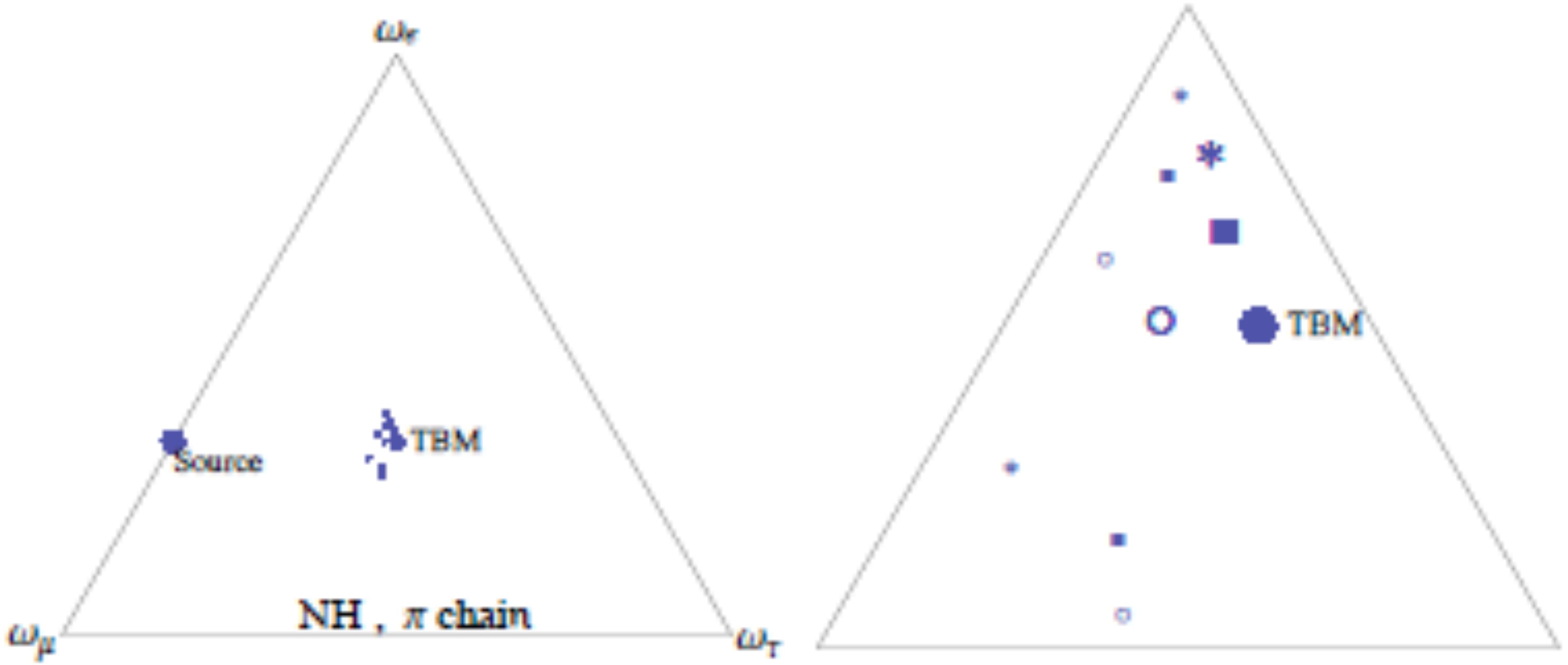
$\delta = \frac{\pi}{2}$ (square)
$\delta = 0$ (star)
$\delta = \pi$ (open circle)

Larger symbols = best fit values
smaller symbols = $\pm 2\sigma$ values

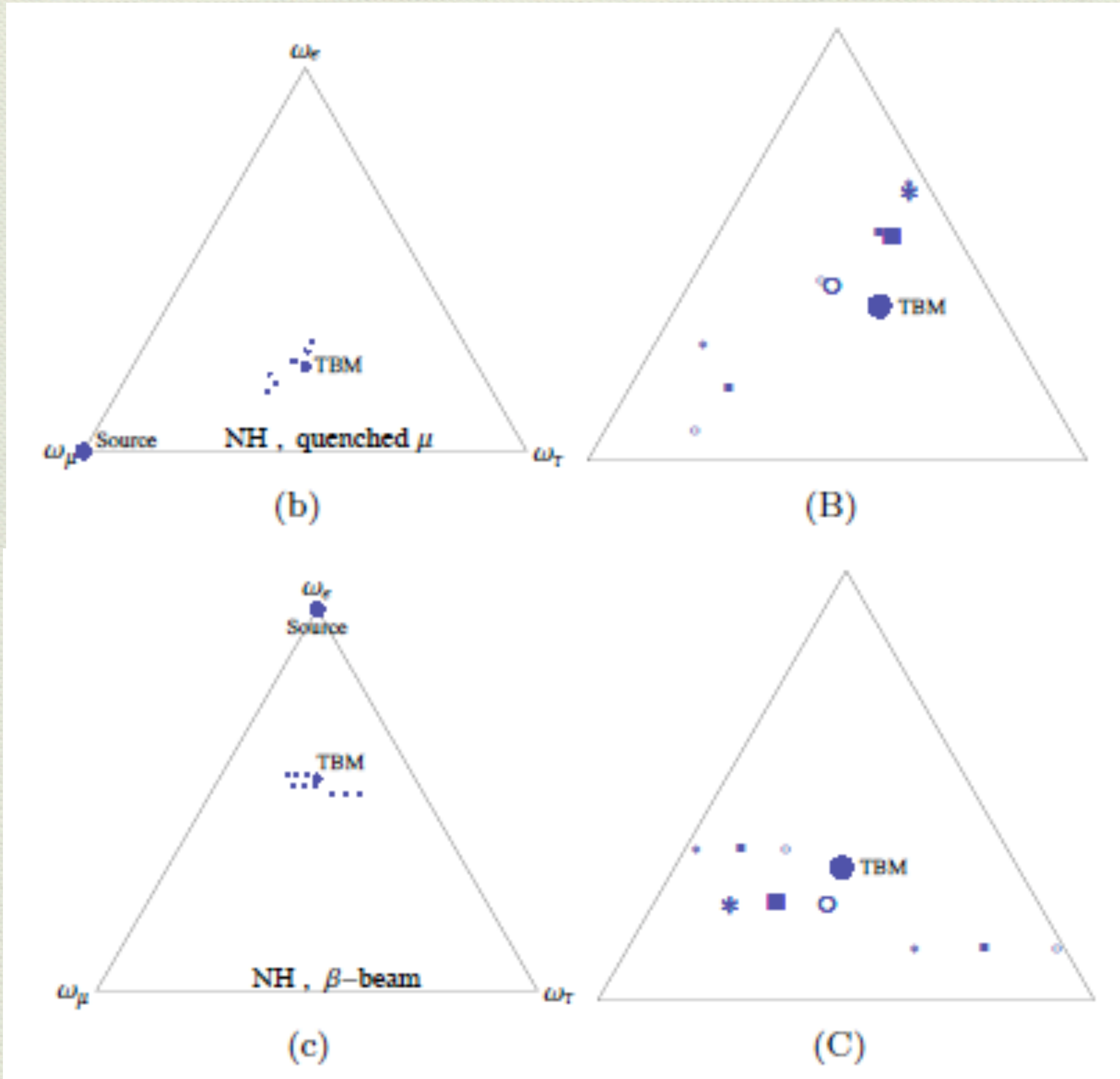
TBM value = large solid dot

and more neutrino flavor physics

Symbolized are three possible values of δ ,
and 2-sigma errors on fitted angles.



(a) (A)


ωe
TBM
Source
NH , quenched μ
ωμ
ωτ
(b)
TBM
(B)
ωe
Source
TBM
NH , β–beam
ωμ
ωτ
(c)
TBM
(C)

Conclusions:

- Flavor ratios, like photon polz'n or cosmic ray A, contain valuable information
- Propagation of the flavor ratios is easily calculated, and corrects TBM
- Due to $\theta_{13} \neq 0$ the propagation matrix P is (likely) invertible, enabling reconstruction of injection ratios from data.
- Presented a general formula for the ν_μ/ν_e injection ratio in terms of track-to-shower observable at Earth.
- Presented a relation among flavor ratios at Earth that determines whether ν_τ's are injected at source.

- Flavor ratios are statistical, requiring N >> 1 Events!