Lepton Flavor Mixing

Neutrino-Antineutrino Oscillation

Zhi-zhong Xing, IHEP, Beijing

Can we ultimately determine the Majorana ν properties?

@the 16th Lomonosov Conference on Elementary Particle Physics@
22 — 28/08/2013, Moscow

Centenary of Pontecorvo

★ Theory of the Symmetry of Electrons and Positrons

Ettore Majorana

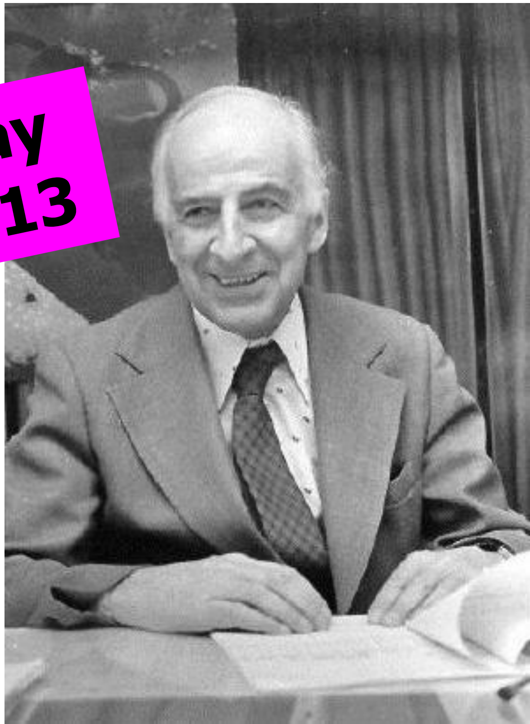
Nuovo Cim. 14 (1937) 171

Are massive neutrinos and antineutrinos identical or different —— a fundamental puzzling question in particle physics.

★ Mesonium and Anti-mesonium

Bruno Pontecorvo

100^{th} birthday = 22/08/2013

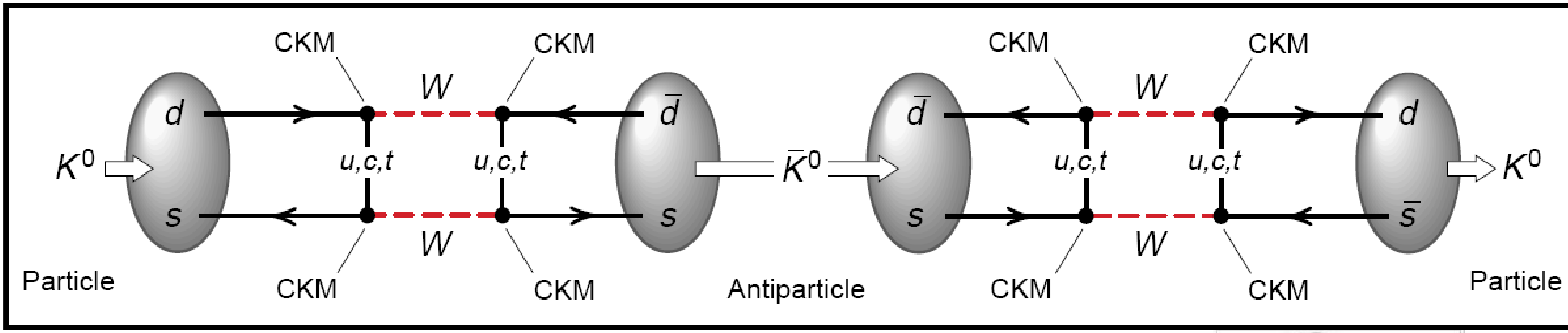


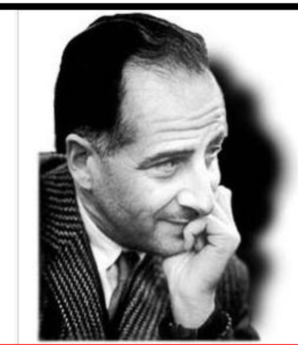
Zh. Eksp. Teor. Fiz. 33 (1957) 549
Sov. Phys. JETP 6 (1957) 429

If the two-component neutrino theory turned out to be incorrect and if the conservation law of neutrino charge didn't apply, then neutrino-antineutrino transitions would in principle be possible to take place in vacuum.

Great Imagination

A lesson learnt from the meson-antimeson oscillations:

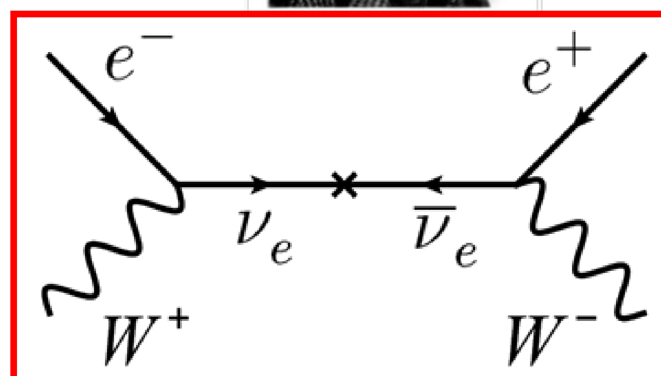




- **Neutrino and antineutrino are massive.**
- **The lepton number is not conserved.**
- **A single family without flavor mixing.**

Pontecorvo's original idea for the neutrino-antineutrino transitions

$$P(\nu_e \to \overline{\nu}_e) = \frac{m_\nu^2}{E^2}|K|^2$$



Flavor Mixing: Road Behind / Ahead

Quark mixing:

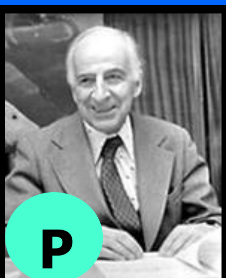
$$V=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}\\0&e^{-i\delta}&0\\-s_{13}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}$$

$\theta_{12} \rightarrow \theta_{23} \rightarrow \theta_{13} \rightarrow \delta$ new physics ?

Experimental time scales??

$\sim 13^\circ$ $\sim 2^\circ$ $\sim 0.2^\circ$ $\sim 65^\circ$ ~ 40 years !

Lepton mixing: (in general, we consider three Majorana neutrinos)



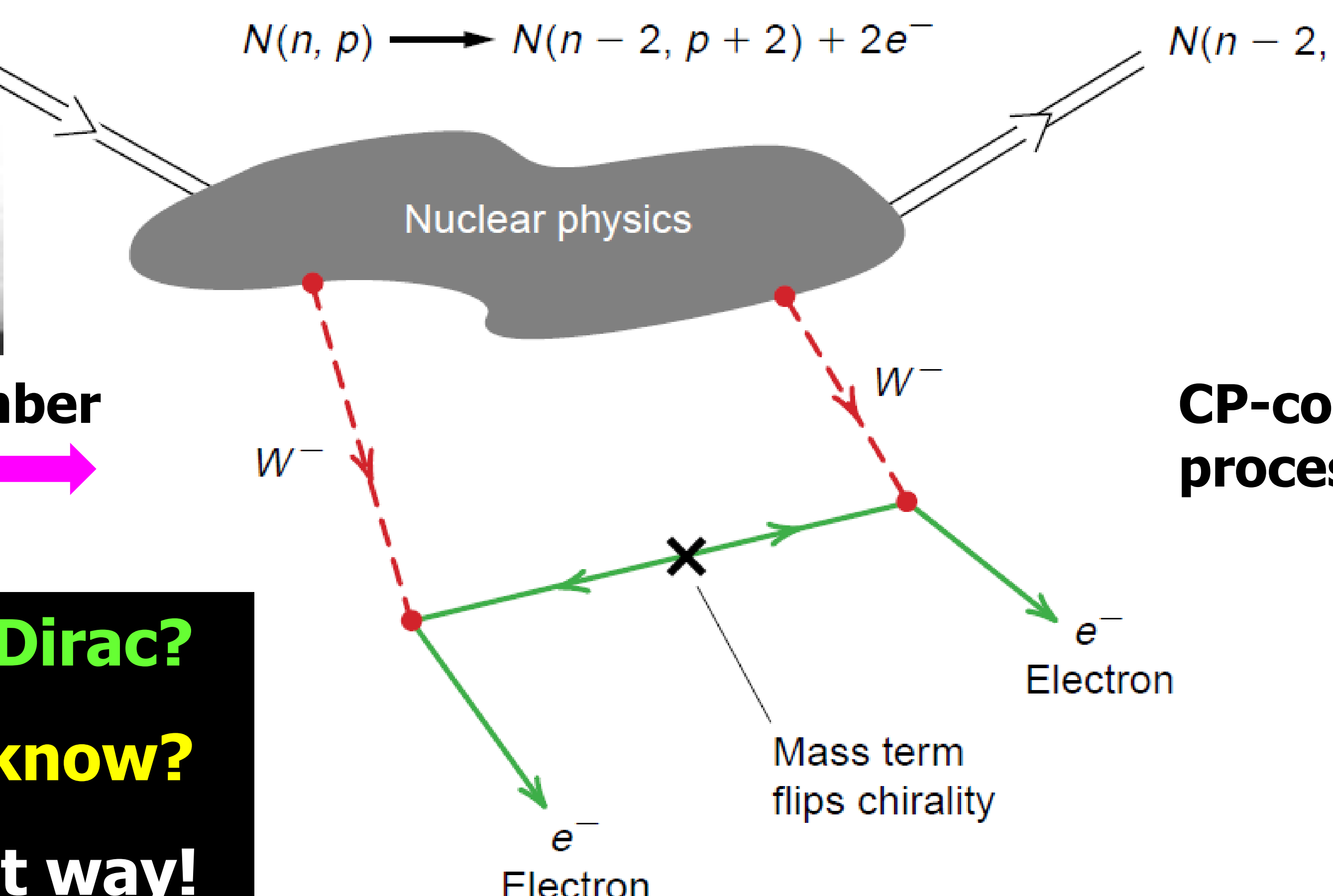
$$U=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}\\0&e^{-i\delta}&0\\-s_{13}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}e^{i\rho}&0&0\\0&e^{i\sigma}&0\\0&0&1\end{pmatrix}$$

$\theta_{23} \rightarrow \theta_{12} \rightarrow \theta_{13} \rightarrow \delta/\rho/\sigma$ new physics ?

$\sim 45^\circ$ $\sim 34^\circ$ $\sim 9^\circ$ $\sim$? ? ? ~ 100 years ?

A Crucial Question

Question: **what can we do to determine all the CP-violating phases in the PMNS matrix if the massive ν's are someday identified to be the Majorana particles via a convincing measurement of the $0\nu\beta\beta$ decay?**



Lepton number violation

CP-conserving process

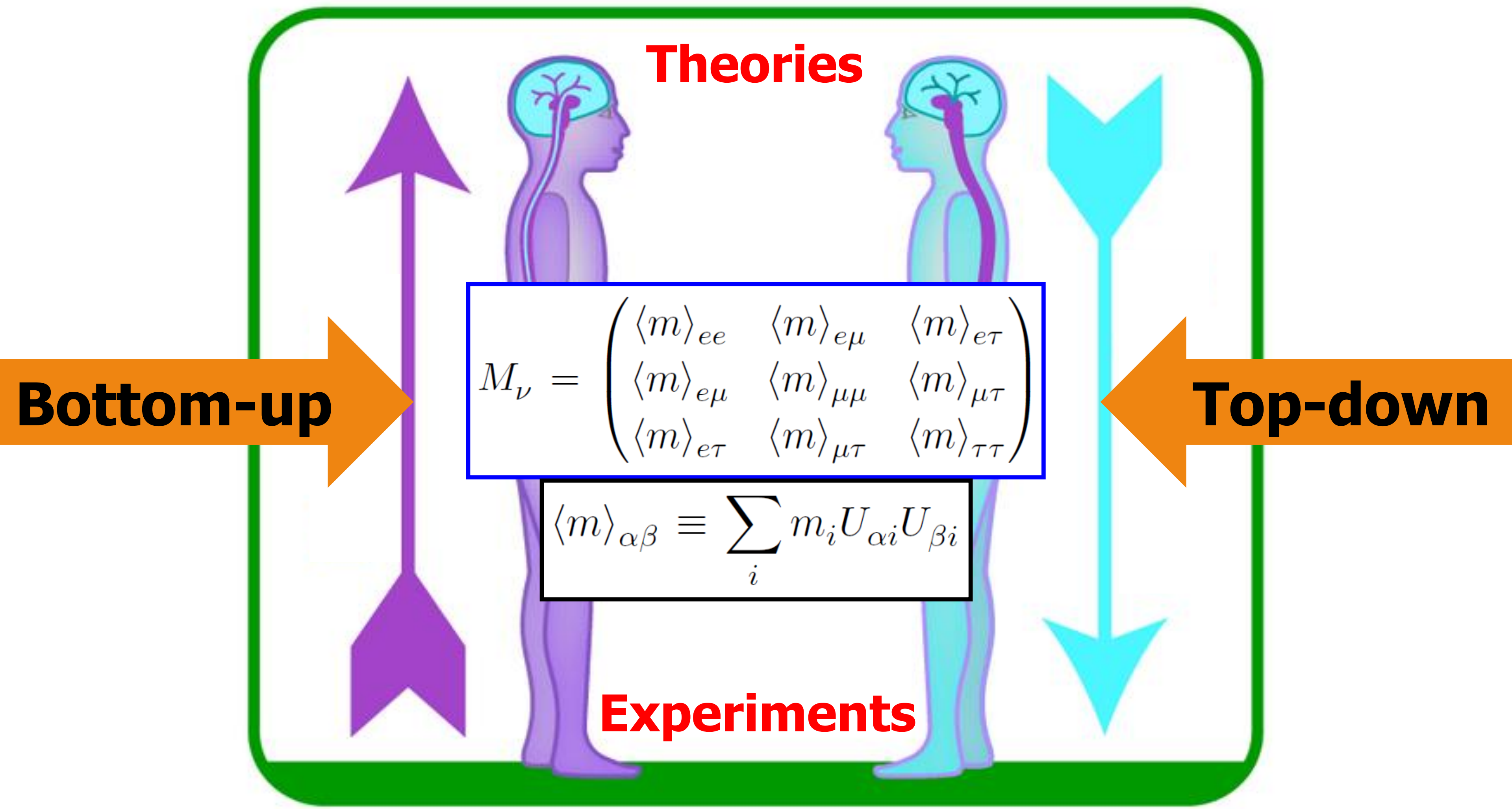
OR just Dirac?

How to know?

No direct way!

Theorist's Concern

Warning: **without information on all the CP-violating phases, one will have no way to establish a full theory of ν masses and flavor mixing.**

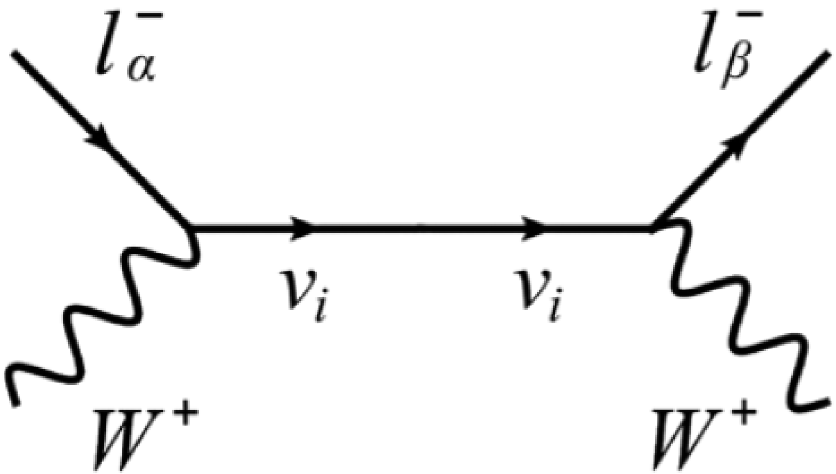


$$M_\nu = \begin{pmatrix} \langle m\rangle_{ee} & \langle m\rangle_{e\mu} & \langle m\rangle_{e\tau} \\ \langle m\rangle_{e\mu} & \langle m\rangle_{\mu\mu} & \langle m\rangle_{\mu\tau} \\ \langle m\rangle_{e\tau} & \langle m\rangle_{\mu\tau} & \langle m\rangle_{\tau\tau} \end{pmatrix}$$

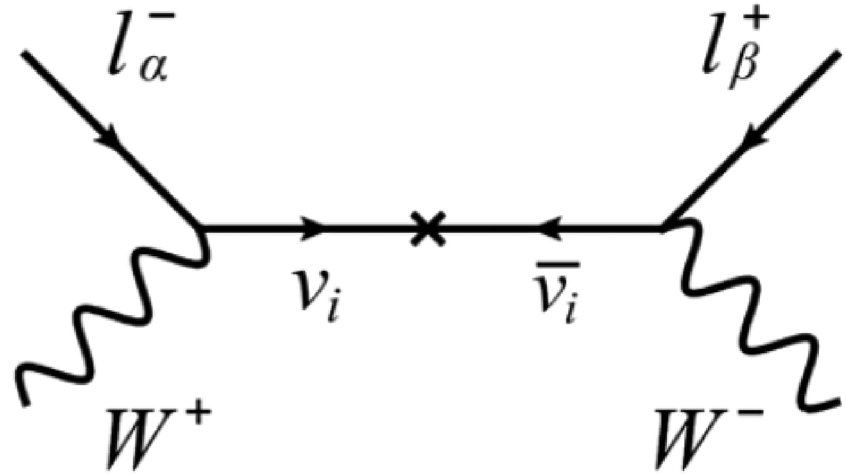
$$\langle m\rangle_{\alpha\beta} \equiv \sum_i m_i U_{\alpha i} U_{\beta i}$$

What to do (conceptually)?

The best approach **(in principle): neutrino ↔ antineutrino oscillations.**



neutrino → neutrino



neutrino → antineutrino

Feasible and successful today!

δ

Unfeasible, a hope tomorrow?

δ ρ σ

In this talk, let's see how much neutrino-antineutrino oscillations can teach us about the fundamental properties of Majorana particles.

References

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[On the Dirac case: C. Giunti, A. Studenikin, PAN 72 (2009) 2089]

The Majorana ν phenomenology

Z.Z. Xing, PRD 87 (2013) 053019

Z.Z. Xing, Y.L. Zhou, PRD 88 (2013) 033002

systematic 3-flavor

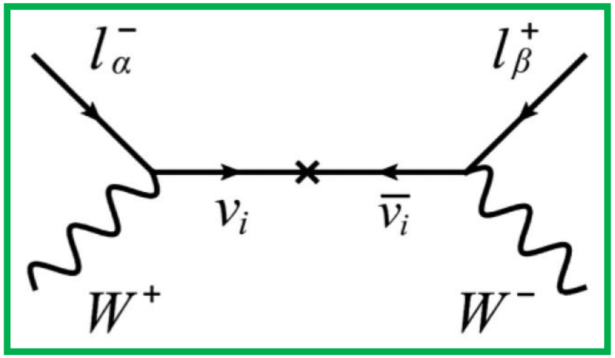
This talk

Neutrino $\leftrightarrow$ Antineutrino

Neutrino-Antineutrino Oscillations:

$$A(\nu_\alpha \to \overline{\nu}_\beta) = \sum_i \left[U^*_{\alpha i} U^*_{\beta i} \frac{m_i}{E} \exp\left(-\mathrm{i} \frac{m_i^2}{2E} L \right) \right] K$$

$$A(\overline{\nu}_\alpha \to \nu_\beta) = \sum_i \left[U_{\alpha i} U_{\beta i} \frac{m_i}{E} \exp\left(-\mathrm{i} \frac{m_i^2}{2E} L \right) \right] \overline{K}$$



$$\phi_{ji} \equiv \Delta m^2_{ji} L/(4E)$$

$$P(\nu_\alpha \to \overline{\nu}_\beta) = \frac{|K|^2}{E^2} \left[|\langle m \rangle_{\alpha\beta}|^2 - 4 \sum_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta} \sin^2 \phi_{ji} + 2 \sum_{i<j} m_i m_j \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji} \right]$$

$$P(\overline{\nu}_\alpha \to \nu_\beta) = \frac{|\overline{K}|^2}{E^2} \left[|\langle m \rangle_{\alpha\beta}|^2 - 4 \sum_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta} \sin^2 \phi_{ji} - 2 \sum_{i<j} m_i m_j \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji} \right]$$

mass **CP-conserving** **CP-violating**

Effective mass terms:

$$\langle m \rangle_{\alpha\beta} \equiv \sum_i m_i U_{\alpha i} U_{\beta i}$$

Jarlskog-like parameters:

$$\mathcal{C}^{ij}_{\alpha\beta} \equiv \mathrm{Re}\left(U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j} \right)$$

$$\mathcal{V}^{ij}_{\alpha\beta} \equiv \mathrm{Im}\left(U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j} \right)$$

Relations

The effective mass terms:

$$\left|\langle m\rangle_{\alpha\beta}\right|^2 = \sum_i m_i^2 \mathcal{C}^{ii}_{\alpha\beta} + 2\sum_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta}$$

$$\left|\langle m\rangle_{\alpha}\right|^2 \equiv \sum_i m_i^2 \left|U_{\alpha i}\right|^2$$

The sum rule (for arbitrary L):

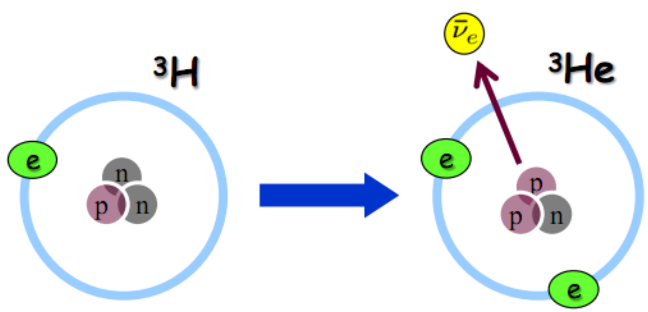
$$\sum_\beta P(\nu_\alpha \to \overline{\nu}_\beta) = \sum_\beta P(\overline{\nu}_\alpha \to \nu_\beta) = \frac{|K|^2}{E^2}\left|\langle m\rangle_\alpha\right|^2$$

Mössbauer ν's

(E = 18.6 keV)

The zero-distance effect (at L = 0):

$$P(\nu_\alpha \to \overline{\nu}_\beta) = P(\overline{\nu}_\alpha \to \nu_\beta) = \frac{|K|^2}{E^2}\left|\langle m\rangle_{\alpha\beta}\right|^2$$



The typical oscillation lengths:

$$\Delta m^2_{21} \simeq 7.5 \times 10^{-5}\ \mathrm{eV}^2$$

$$|\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.4 \times 10^{-3}\ \mathrm{eV}^2$$

$$(1)\ L^{\rm osc}_{31} \simeq L^{\rm osc}_{32} \simeq \frac{E}{10\ \mathrm{keV}} \times 10\ \mathrm{m}$$

$$(2)\ L^{\rm osc}_{21} \simeq \frac{E}{10\ \mathrm{keV}} \times 330\ \mathrm{m}\ ,$$

CP Asymmetries

The CP-violating asymmetry:
(to cancel the |K|/E factors)

$$\mathcal{A}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \overline{\nu}_\beta) - P(\overline{\nu}_\alpha \to \nu_\beta)}{P(\nu_\alpha \to \overline{\nu}_\beta) + P(\overline{\nu}_\alpha \to \nu_\beta)}$$

The explicit expressions:

$$\mathcal{A}_{\alpha\beta} = \frac{2\sum_{i<j} m_i m_j \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji}}{\sum_i m_i^2 \mathcal{C}^{ii}_{\alpha\beta} + 2\sum_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta} \cos 2\phi_{ji}}$$

$$= \frac{2\sum_{i<j} m_i m_j \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji}}{\left|\langle m\rangle_{\alpha\beta}\right|^2 - 4\sum_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta} \sin^2 \phi_{ji}} \; .$$

$$\mathcal{C}^{ij}_{\alpha\beta} \equiv \mathrm{Re}\left(U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j}\right)$$

$$\mathcal{V}^{ij}_{\alpha\beta} \equiv \mathrm{Im}\left(U_{\alpha i} U_{\beta i} U^*_{\alpha j} U^*_{\beta j}\right)$$

sensitive to all 3 CP phases
sensitive to the mass scale.

$$\mathrm{Im}\left(U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}\right) = \mathcal{J} \sum_\gamma \epsilon_{\alpha\beta\gamma} \sum_k \epsilon_{ijk}$$

CP-/T-violating asymmetries in normal neutrino-neutrino oscillations:

$$P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) = 16\mathcal{J} \sin\phi_{12} \sin\phi_{23} \sin\phi_{31}$$

Jarlskog-like Parameters

In the standard parametrization of U:

$$\boxed{\mathcal{J} = c_{12} s_{12} c_{13}^2 s_{13} c_{23} s_{23} \sin\delta}$$

$$\mathcal{V}_{ee}^{12} = c_{12}^2 s_{12}^2 c_{13}^4 \sin 2\left(\rho - \sigma\right)$$

$$\mathcal{V}_{ee}^{13} = c_{12}^2 c_{13}^2 s_{13}^2 \sin 2\left(\delta + \rho\right)$$

$$\mathcal{V}_{ee}^{23} = s_{12}^2 c_{13}^2 s_{13}^2 \sin 2\left(\delta + \sigma\right)$$

Nine-independent Jarlskog-like parameters:

Each one depends on two CP-violating phases or phase combinations.

$$\begin{aligned}
\mathcal{V}_{\mu\mu}^{12} = \; & c_{12}^2 s_{12}^2 \left(c_{23}^4 - 4 s_{13}^2 c_{23}^2 s_{23}^2 + s_{13}^4 s_{23}^4\right) \sin 2\left(\rho - \sigma\right) \\
& + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \left(c_{23}^2 - s_{13}^2 s_{23}^2\right) \left[c_{12}^2 \sin\left(2\rho - 2\sigma + \delta\right) - s_{12}^2 \sin\left(2\rho - 2\sigma - \delta\right)\right] \\
& + s_{13}^2 c_{23}^2 s_{23}^2 \left[c_{12}^4 \sin 2\left(\rho - \sigma + \delta\right) + s_{12}^4 \sin 2\left(\rho - \sigma - \delta\right)\right]
\end{aligned}$$

$$\mathcal{V}_{\mu\mu}^{13} = c_{13}^2 s_{23}^2 \left[s_{12}^2 c_{23}^2 \sin 2\rho + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \sin\left(\delta + 2\rho\right) + c_{12}^2 s_{13}^2 s_{23}^2 \sin 2\left(\delta + \rho\right)\right]$$

$$\mathcal{V}_{\mu\mu}^{23} = c_{13}^2 s_{23}^2 \left[c_{12}^2 c_{23}^2 \sin 2\sigma - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \sin\left(\delta + 2\sigma\right) + s_{12}^2 s_{13}^2 s_{23}^2 \sin 2\left(\delta + \sigma\right)\right]$$

$$\begin{aligned}
\mathcal{V}_{\tau\tau}^{12} = \; & c_{12}^2 s_{12}^2 \left(s_{23}^4 - 4 s_{13}^2 c_{23}^2 s_{23}^2 + s_{13}^4 c_{23}^4\right) \sin 2\left(\rho - \sigma\right) \\
& - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \left(s_{23}^2 - s_{13}^2 c_{23}^2\right) \left[c_{12}^2 \sin\left(2\rho - 2\sigma + \delta\right) - s_{12}^2 \sin\left(2\rho - 2\sigma - \delta\right)\right] \\
& + s_{13}^2 c_{23}^2 s_{23}^2 \left[c_{12}^4 \sin 2\left(\rho - \sigma + \delta\right) + s_{12}^4 \sin 2\left(\rho - \sigma - \delta\right)\right]
\end{aligned}$$

$$\mathcal{V}_{\tau\tau}^{13} = c_{13}^2 c_{23}^2 \left[s_{12}^2 s_{23}^2 \sin 2\rho - 2 c_{12} s_{12} s_{13} c_{23} s_{23} \sin\left(\delta + 2\rho\right) + c_{12}^2 s_{13}^2 c_{23}^2 \sin 2\left(\delta + \rho\right)\right]$$

$$\mathcal{V}_{\tau\tau}^{23} = c_{13}^2 c_{23}^2 \left[c_{12}^2 s_{23}^2 \sin 2\sigma + 2 c_{12} s_{12} s_{13} c_{23} s_{23} \sin\left(\delta + 2\sigma\right) + s_{12}^2 s_{13}^2 c_{23}^2 \sin 2\left(\delta + \sigma\right)\right]$$

Pseudo-Dirac ($\rho=\sigma=0$)

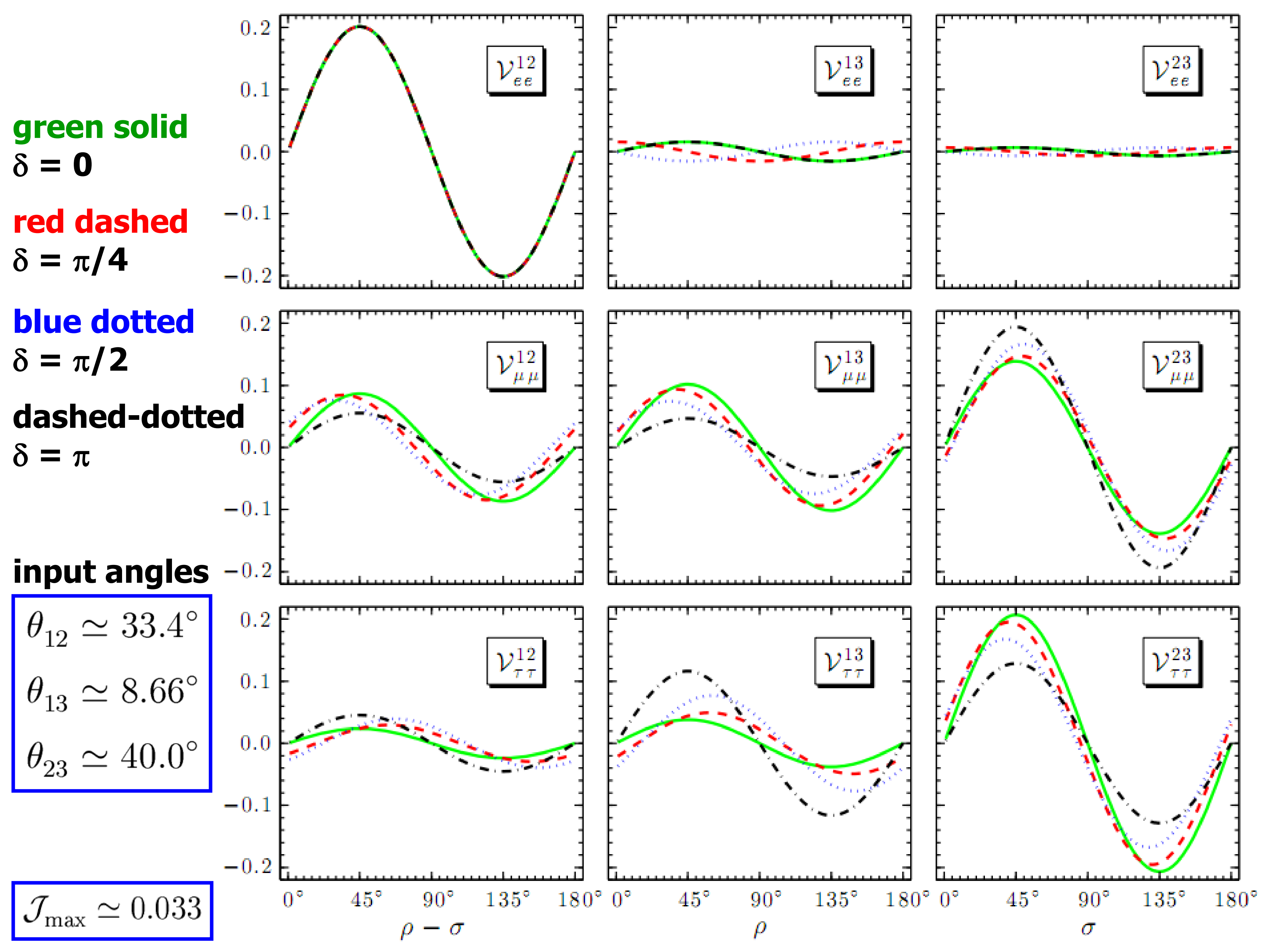
Jarlskog-like parameter	$\delta = 45^\circ$	$\delta = 90^\circ$
$\mathcal{V}_{ee}^{12} = 0$	0	0
$\mathcal{V}_{ee}^{13} = c_{12}^2 c_{13}^2 s_{13}^2 \sin 2\delta$	+0.016	0
$\mathcal{V}_{ee}^{23} = s_{12}^2 c_{13}^2 s_{13}^2 \sin 2\delta$	+0.0067	0
$\mathcal{V}_{\mu\mu}^{12} = \left[2\mathcal{J}\left(c_{23}^2 - s_{13}^2 s_{23}^2\right) + \left(\mathcal{V}_{ee}^{13} - \mathcal{V}_{ee}^{23}\right) c_{23}^2 s_{23}^2\right] / c_{13}^2$	+0.030	+0.039
$\mathcal{V}_{\mu\mu}^{13} = 2\mathcal{J} s_{23}^2 + \mathcal{V}_{ee}^{13} s_{23}^4$	+0.022	+0.028
$\mathcal{V}_{\mu\mu}^{23} = -2\mathcal{J} s_{23}^2 + \mathcal{V}_{ee}^{23} s_{23}^4$	−0.018	−0.028
$\mathcal{V}_{\tau\tau}^{12} = \left[2\mathcal{J}\left(s_{13}^2 c_{23}^2 - s_{23}^2\right) + \left(\mathcal{V}_{ee}^{13} - \mathcal{V}_{ee}^{23}\right) c_{23}^2 s_{23}^2\right] / c_{13}^2$	−0.017	−0.027
$\mathcal{V}_{\tau\tau}^{13} = -2\mathcal{J} c_{23}^2 + \mathcal{V}_{ee}^{13} c_{23}^4$	−0.022	−0.039
$\mathcal{V}_{\tau\tau}^{23} = 2\mathcal{J} c_{23}^2 + \mathcal{V}_{ee}^{23} c_{23}^4$	+0.030	+0.039
$\mathcal{V}_{e\mu}^{12} = -\mathcal{J}$	−0.024	−0.033
$\mathcal{V}_{e\mu}^{13} = -\mathcal{J} + \mathcal{V}_{ee}^{13} s_{23}^2$	−0.030	−0.033
$\mathcal{V}_{e\mu}^{23} = \mathcal{J} - \mathcal{V}_{ee}^{13} s_{23}^2$	+0.021	+0.033
$\mathcal{V}_{e\tau}^{12} = \mathcal{J}$	+0.024	+0.033
$\mathcal{V}_{e\tau}^{13} = \mathcal{J} - \mathcal{V}_{ee}^{13} c_{23}^2$	+0.014	+0.033
$\mathcal{V}_{e\tau}^{23} = -\mathcal{J} - \mathcal{V}_{ee}^{23} c_{23}^2$	−0.028	−0.033
$\mathcal{V}_{\mu\tau}^{12} = \left[\mathcal{J}\left(1 + s_{13}^2\right)\left(s_{23}^2 - c_{23}^2\right) - \left(\mathcal{V}_{ee}^{13} - \mathcal{V}_{ee}^{23}\right) c_{23}^2 s_{23}^2\right] / c_{13}^2$	−0.0064	−0.0060
$\mathcal{V}_{\mu\tau}^{13} = \mathcal{J}\left(c_{23}^2 - s_{23}^2\right) + \mathcal{V}_{ee}^{13} c_{23}^2 s_{23}^2$	+0.0078	+0.0058
$\mathcal{V}_{\mu\tau}^{23} = \mathcal{J}\left(s_{23}^2 - c_{23}^2\right) + \mathcal{V}_{ee}^{23} c_{23}^2 s_{23}^2$	−0.0025	−0.0058

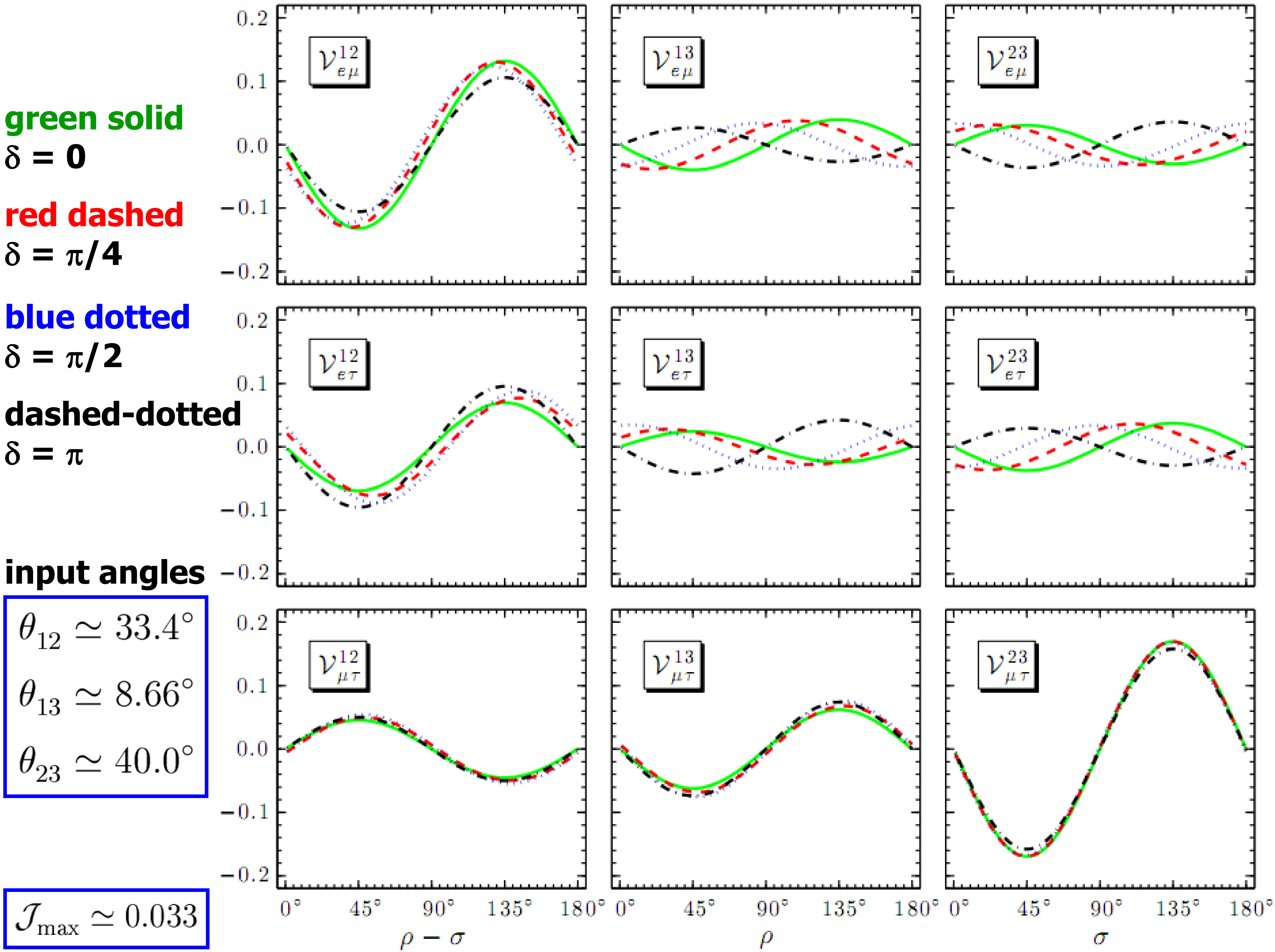
input angles

$$\theta_{12} \simeq 33.4^\circ$$

$$\theta_{13} \simeq 8.66^\circ$$

$$\theta_{23} \simeq 40.0^\circ$$


green solid
δ = 0
red dashed
δ = π/4
blue dotted
δ = π/2
dashed-dotted
δ = π
input angles
$\theta_{12} \simeq 33.4^\circ$
$\theta_{13} \simeq 8.66^\circ$
$\theta_{23} \simeq 40.0^\circ$
$\mathcal{J}_{\max} \simeq 0.033$
$\mathcal{V}^{12}_{ee}$
$\mathcal{V}^{13}_{ee}$
$\mathcal{V}^{23}_{ee}$
$\mathcal{V}^{12}_{\mu\mu}$
$\mathcal{V}^{13}_{\mu\mu}$
$\mathcal{V}^{23}_{\mu\mu}$
$\mathcal{V}^{12}_{\tau\tau}$
$\mathcal{V}^{13}_{\tau\tau}$
$\mathcal{V}^{23}_{\tau\tau}$
0.2
0.1
0.0
−0.1
−0.2
0° 45° 90° 135° 180°
$\rho - \sigma$
ρ
σ



green solid
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$\mathcal{V}^{12}_{e\mu}$
$\mathcal{V}^{13}_{e\mu}$
$\mathcal{V}^{23}_{e\mu}$
$\mathcal{V}^{12}_{e\tau}$
$\mathcal{V}^{13}_{e\tau}$
$\mathcal{V}^{23}_{e\tau}$
$\mathcal{V}^{12}_{\mu\tau}$
$\mathcal{V}^{13}_{\mu\tau}$
$\mathcal{V}^{23}_{\mu\tau}$
0.2
0.1
0.0
−0.1
−0.2
0° 45° 90° 135° 180°
$\rho - \sigma$
ρ
σ

Effective Mass Terms

To reconstruct the Majorana neutrino mass matrix, we have to know all the 3 CP phases and the absolute scale of neutrino masses.

$$M_\nu = \begin{pmatrix} \langle m \rangle_{ee} & \langle m \rangle_{e\mu} & \langle m \rangle_{e\tau} \\ \langle m \rangle_{e\mu} & \langle m \rangle_{\mu\mu} & \langle m \rangle_{\mu\tau} \\ \langle m \rangle_{e\tau} & \langle m \rangle_{\mu\tau} & \langle m \rangle_{\tau\tau} \end{pmatrix}$$

$$\langle m \rangle_{ee} = m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2 e^{-2i\delta} ,$$

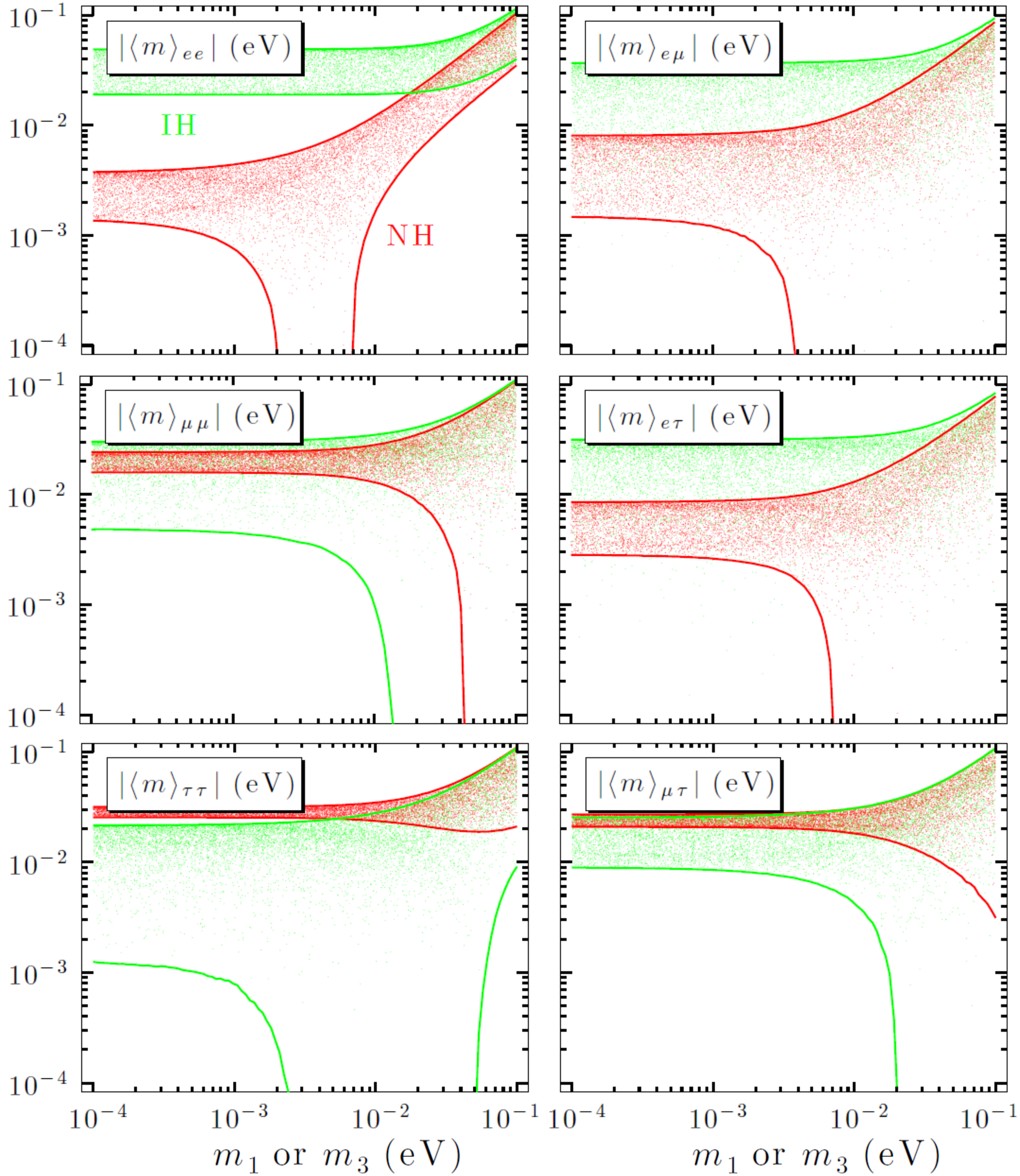
$$\langle m \rangle_{\mu\mu} = m_1 \left(s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta} \right)^2 e^{2i\rho} + m_2 \left(c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} \right)^2 e^{2i\sigma} + m_3 c_{13}^2 s_{23}^2 ,$$

$$\langle m \rangle_{\tau\tau} = m_1 \left(s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta} \right)^2 e^{2i\rho} + m_2 \left(c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta} \right)^2 e^{2i\sigma} + m_3 c_{13}^2 c_{23}^2 ,$$

$$\begin{aligned} \langle m \rangle_{e\mu} = & -m_1 c_{12} c_{13} \left(s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta} \right) e^{2i\rho} + m_2 s_{12} c_{13} \left(c_{23} c_{12} - s_{12} s_{13} s_{23} e^{i\delta} \right) e^{2i\sigma} \\ & + m_3 c_{13} s_{13} s_{23} e^{-i\delta} , \end{aligned}$$

$$\begin{aligned} \langle m \rangle_{e\tau} = & +m_1 c_{12} c_{13} \left(s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta} \right) e^{2i\rho} - m_2 s_{12} c_{13} \left(c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta} \right) e^{2i\sigma} \\ & + m_3 c_{13} s_{13} c_{23} e^{-i\delta} , \end{aligned}$$

$$\begin{aligned} \langle m \rangle_{\mu\tau} = & -m_1 \left(s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta} \right) \left(c_{23} s_{12} + c_{12} s_{13} s_{23} e^{i\delta} \right) e^{2i\rho} \\ & - m_2 \left(c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta} \right) \left(c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} \right) e^{2i\sigma} + m_3 c_{13}^2 c_{23} s_{23} . \end{aligned}$$



NH = normal hierarchy
the smallest mass: m1

IH = inverted hierarchy
the smallest mass: m3

WMAP / PLANCK:
m1+m2+m3 < 0.23 eV
(95% C.L., 1303.5076)

input angles

$$\theta_{12} \simeq 33.4^\circ$$
$$\theta_{13} \simeq 8.66^\circ$$
$$\theta_{23} \simeq 40.0^\circ$$

$$\Delta m^2_{21} \simeq +7.50 \times 10^{-5}\ \mathrm{eV}^2$$
$$\Delta m^2_{31} \simeq +2.473 \times 10^{-3}\ \mathrm{eV}^2\ \text{(NH)}$$
$$\Delta m^2_{32} \simeq -2.427 \times 10^{-3}\ \mathrm{eV}^2\ \text{(IH)}$$

Arbitrary values of 3 CP phases: δ, ρ, σ.

Lessons for Model Building

Flavor Symmetry

Texture zeros | **Element correlations** | **GUT relations**

They reduce the number of free parameters, and thus lead to predictions for 3 flavor mixing angles in terms of either the mass ratios or constant numbers.

Example (Continuous symmetries)

$$\boxed{M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}}$$

Dependent on mass ratios

Example (Discrete symmetries)

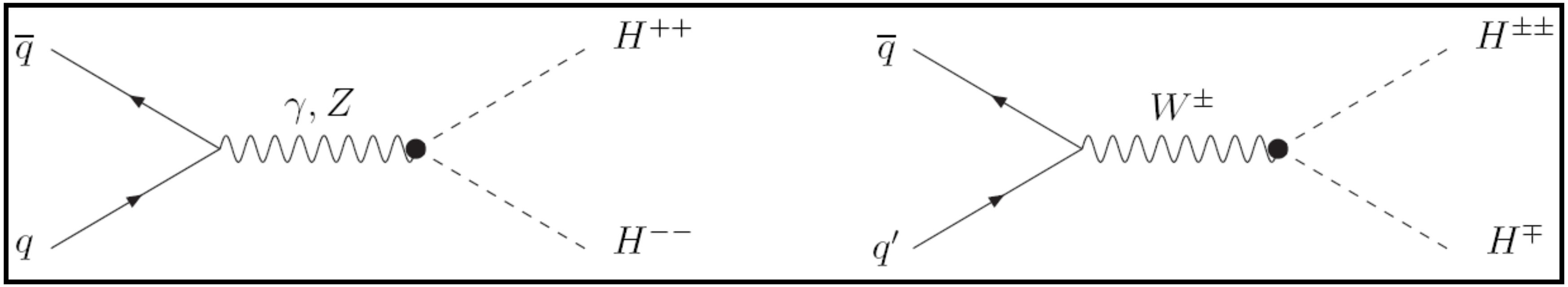
$$\boxed{M_{\nu} = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}}$$

Dependent on simple numbers

PREDICTIONS

Type-Two Seesaw

Salient features: 1) **U** is exactly unitary; 2) singly- or doubly-charged Higgs bosons can be produced at colliders independently of Yukawa couplings; 3) **LNV** processes are directly related with neutrino masses and flavor mixing parameters.



Typical LNV modes and their branching ratios:

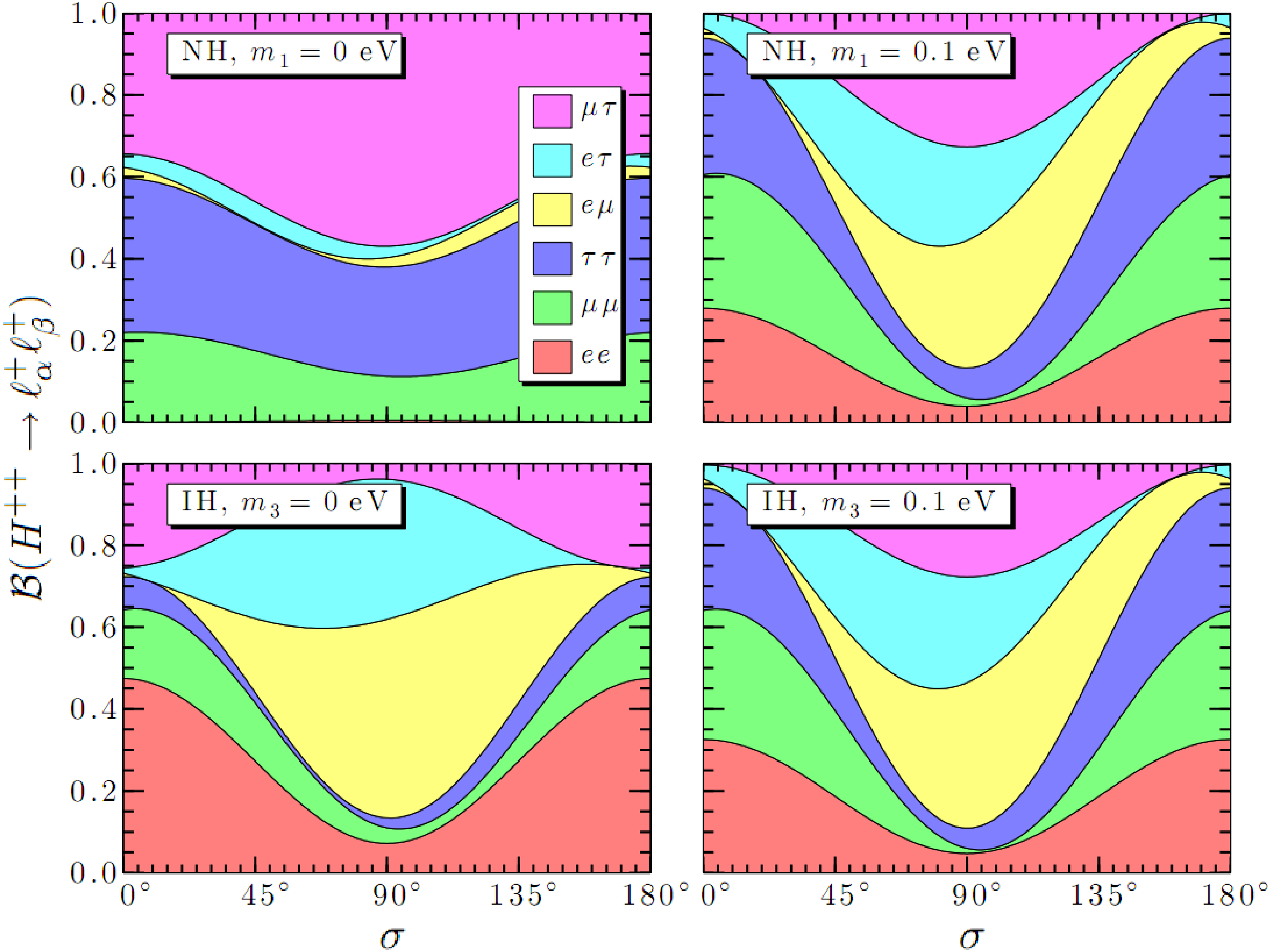
$$\mathcal{B}(H^+ \to \ell_\alpha^+ \overline{\nu}) \equiv \frac{\sum_\beta \Gamma(H^+ \to \ell_\alpha^+ \overline{\nu}_\beta)}{\sum_\alpha \sum_\beta \Gamma(H^+ \to \ell_\alpha^+ \overline{\nu}_\beta)} = \frac{\left|\langle m\rangle_\alpha\right|^2}{\sum_i m_i^2}$$

Insensitive to 3 phases

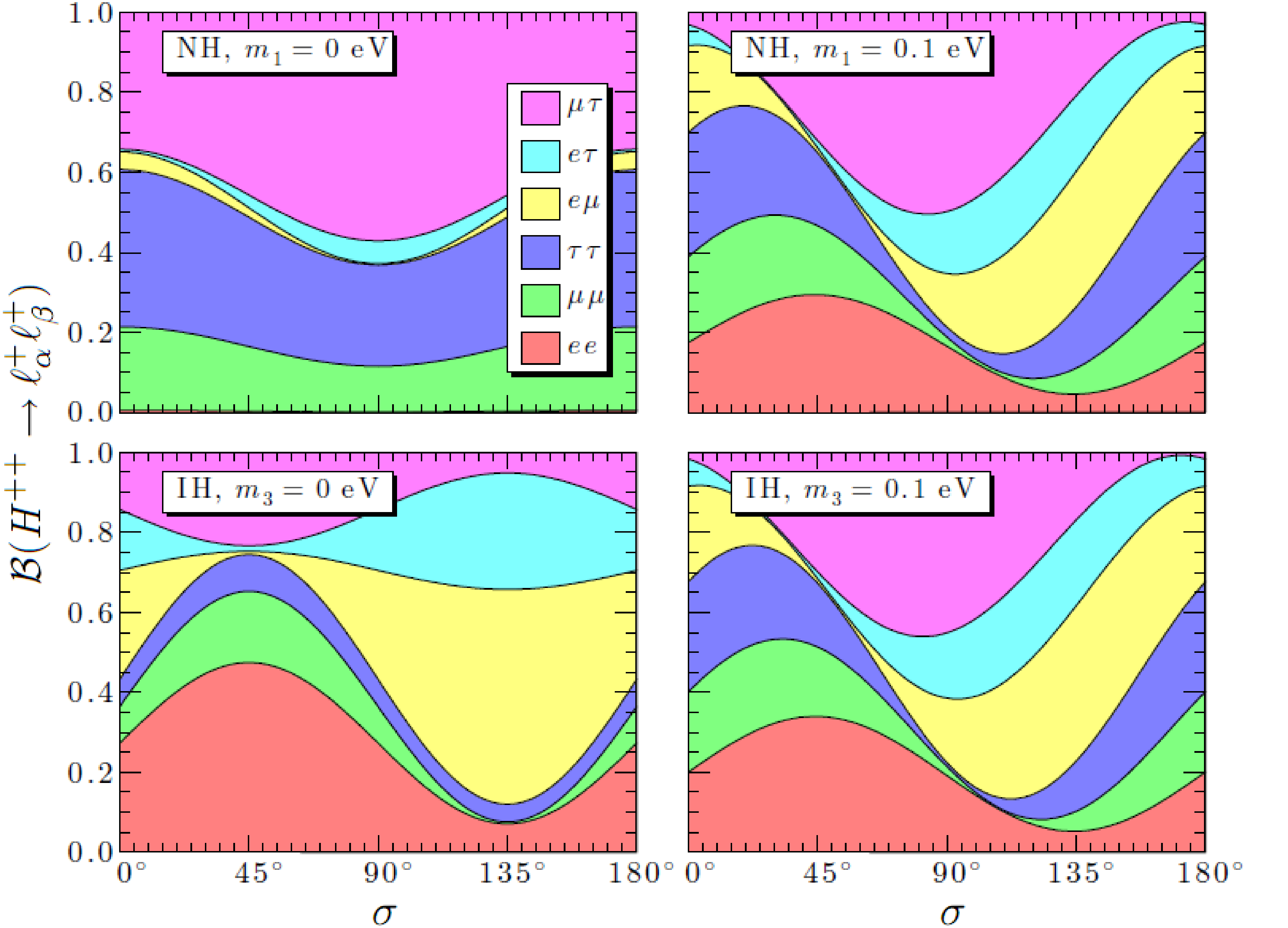
$$\begin{aligned}\left|\langle m\rangle_\alpha\right|^2 &= \sum_i m_i^2 |U_{\alpha i}|^2 \\ &= m_1^2 \left(1 - |U_{\alpha 3}|^2\right) \\ &\quad + m_3^2 |U_{\alpha 3}|^2 \\ &\quad + \Delta m_{21}^2 |U_{\alpha 2}|^2\end{aligned}$$

$$\mathcal{B}(H^{++} \to \ell_\alpha^+ \ell_\beta^+) \equiv \frac{\Gamma(H^{++} \to \ell_\alpha^+ \ell_\beta^+)}{\sum_\alpha \sum_\beta \Gamma(H^{++} \to \ell_\alpha^+ \ell_\beta^+)} = \frac{2}{(1+\delta_{\alpha\beta})} \cdot \frac{\left|\langle m\rangle_{\alpha\beta}\right|^2}{\sum_i m_i^2}$$

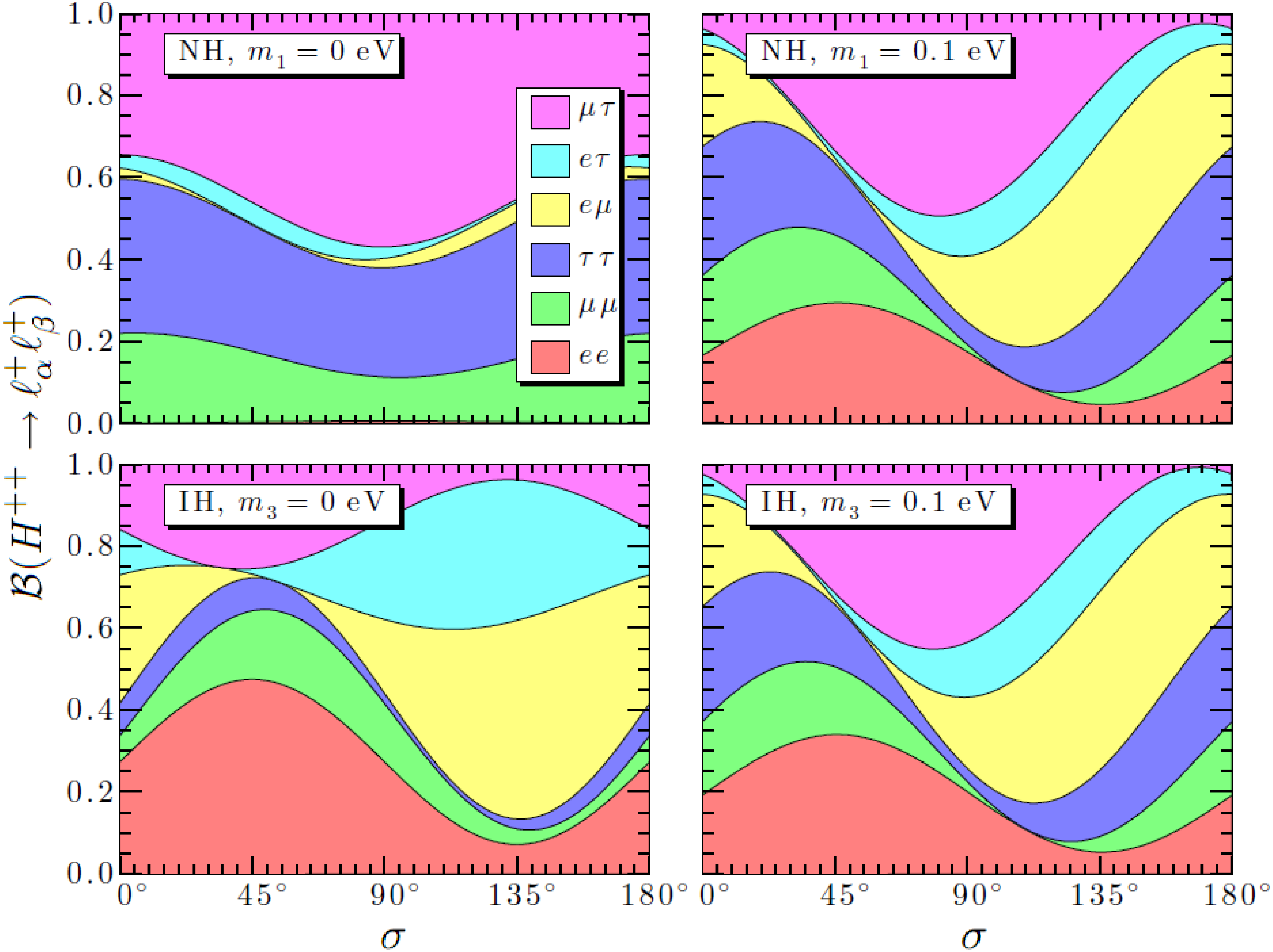
δ ρ σ



Taking $\rho = 0^\circ$ and $\delta = 90^\circ$. Other inputs are the same as taken before.



Taking $\rho = 45^\circ$ and $\delta = 0^\circ$**. Other inputs are the same as taken before.**



Taking $\rho = 45°$ and $\delta = 90°$**. Other inputs are the same as taken before.**

CP Violation

Normal mass hierarchy with m1 = 0

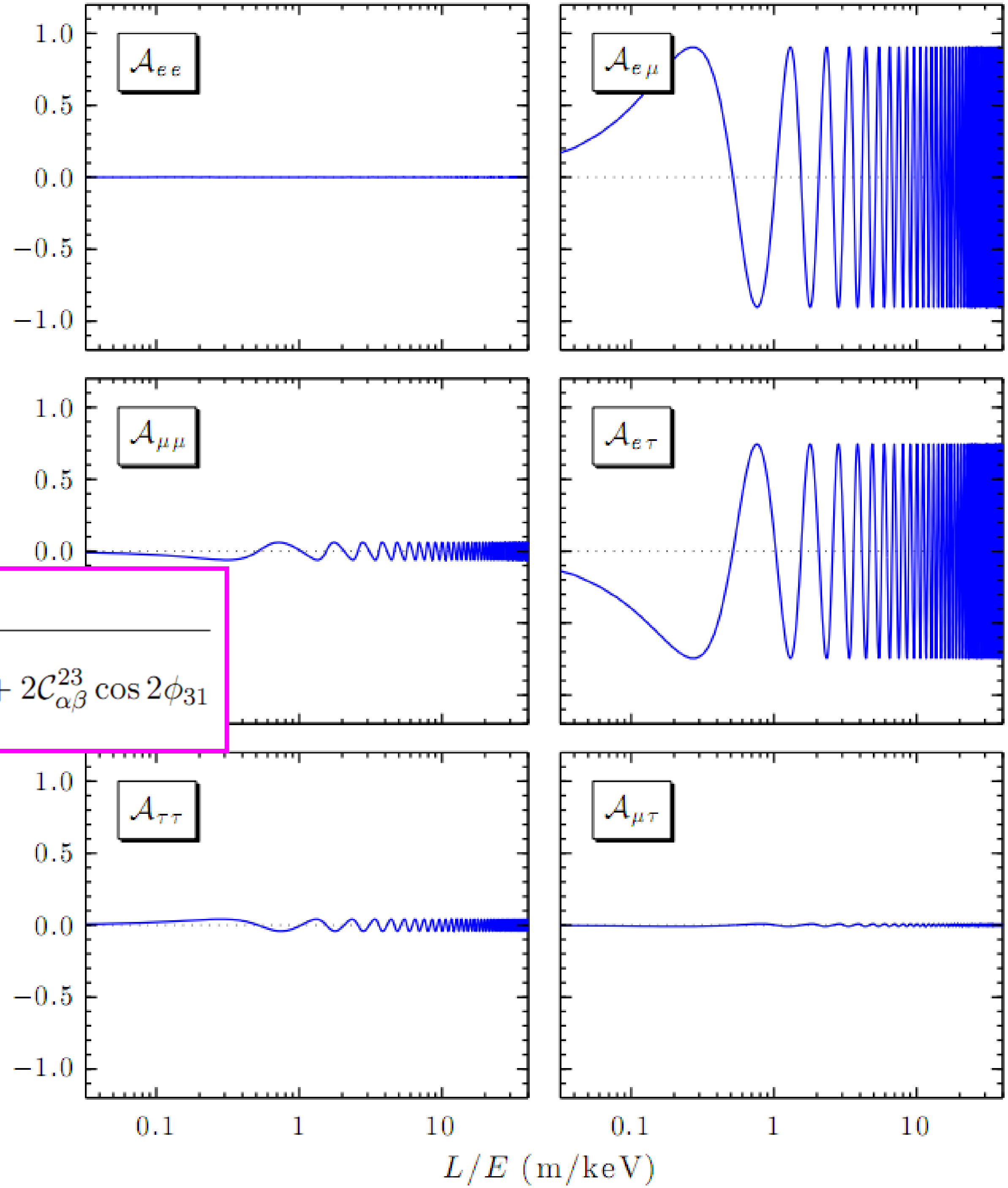
δ ~~ρ~~ σ

$$\mathcal{A}_{\alpha\beta} = \frac{2\mathcal{V}^{23}_{\alpha\beta}\sin 2\phi_{31}}{\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\,\mathcal{C}^{22}_{\alpha\beta} + \sqrt{\frac{\Delta m^2_{31}}{\Delta m^2_{21}}}\,\mathcal{C}^{33}_{\alpha\beta} + 2\mathcal{C}^{23}_{\alpha\beta}\cos 2\phi_{31}}$$

$$\boxed{\delta = 90^\circ \text{ and } \sigma = 0^\circ}$$

input angles

$$\theta_{12} \simeq 33.4^\circ$$
$$\theta_{13} \simeq 8.66^\circ$$
$$\theta_{23} \simeq 40.0^\circ$$



CP Violation

Normal mass hierarchy with m1 = 0

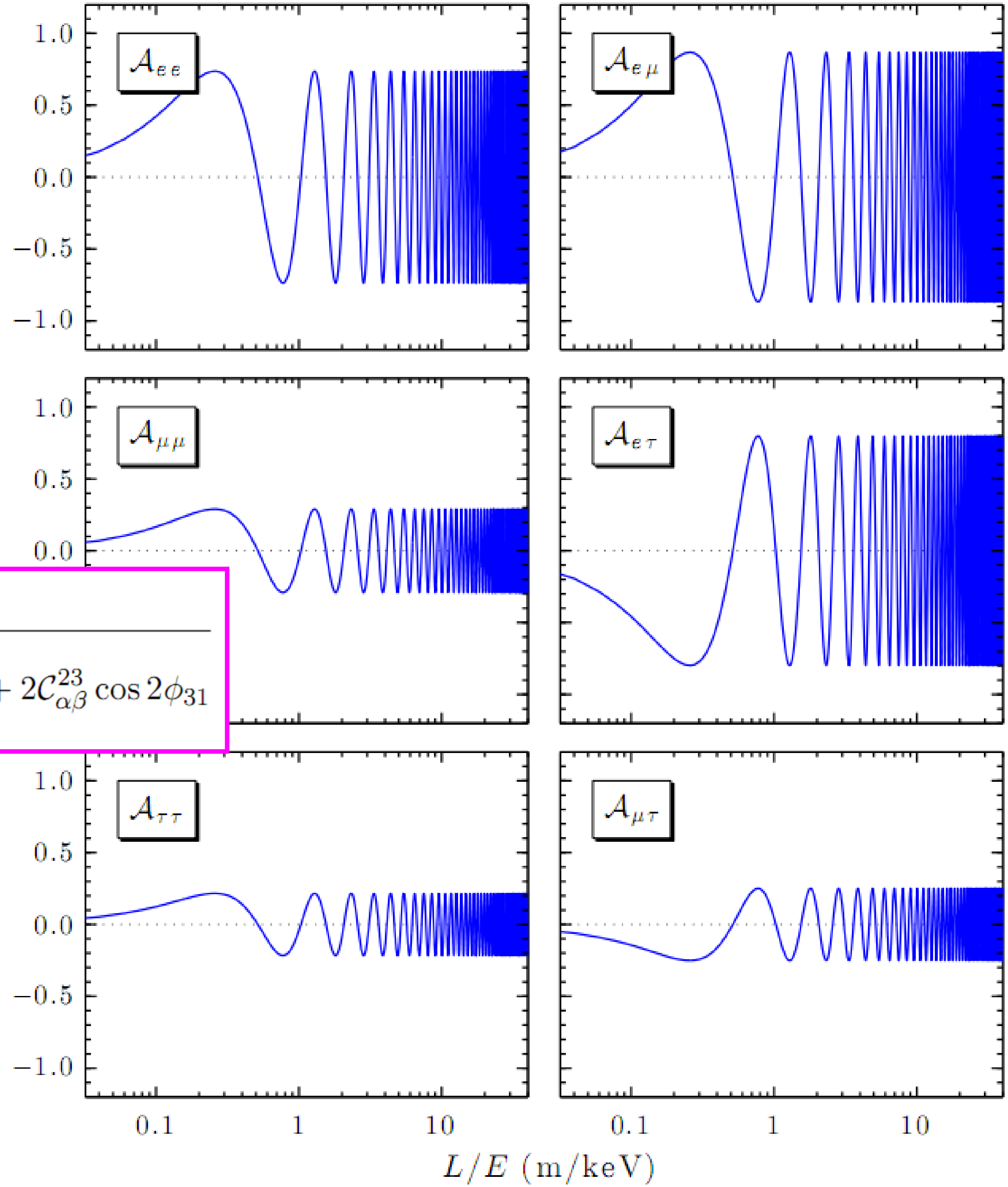
δ ~~ρ~~ σ

$$\mathcal{A}_{\alpha\beta} = \frac{2\mathcal{V}^{23}_{\alpha\beta}\sin 2\phi_{31}}{\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\,\mathcal{C}^{22}_{\alpha\beta} + \sqrt{\frac{\Delta m^2_{31}}{\Delta m^2_{21}}}\,\mathcal{C}^{33}_{\alpha\beta} + 2\mathcal{C}^{23}_{\alpha\beta}\cos 2\phi_{31}}$$

$$\boxed{\delta = 0^\circ \ \text{ and } \sigma = 45^\circ}$$

input angles

$$\theta_{12} \simeq 33.4^\circ$$
$$\theta_{13} \simeq 8.66^\circ$$
$$\theta_{23} \simeq 40.0^\circ$$



CP Violation

Normal mass hierarchy with m1 = 0

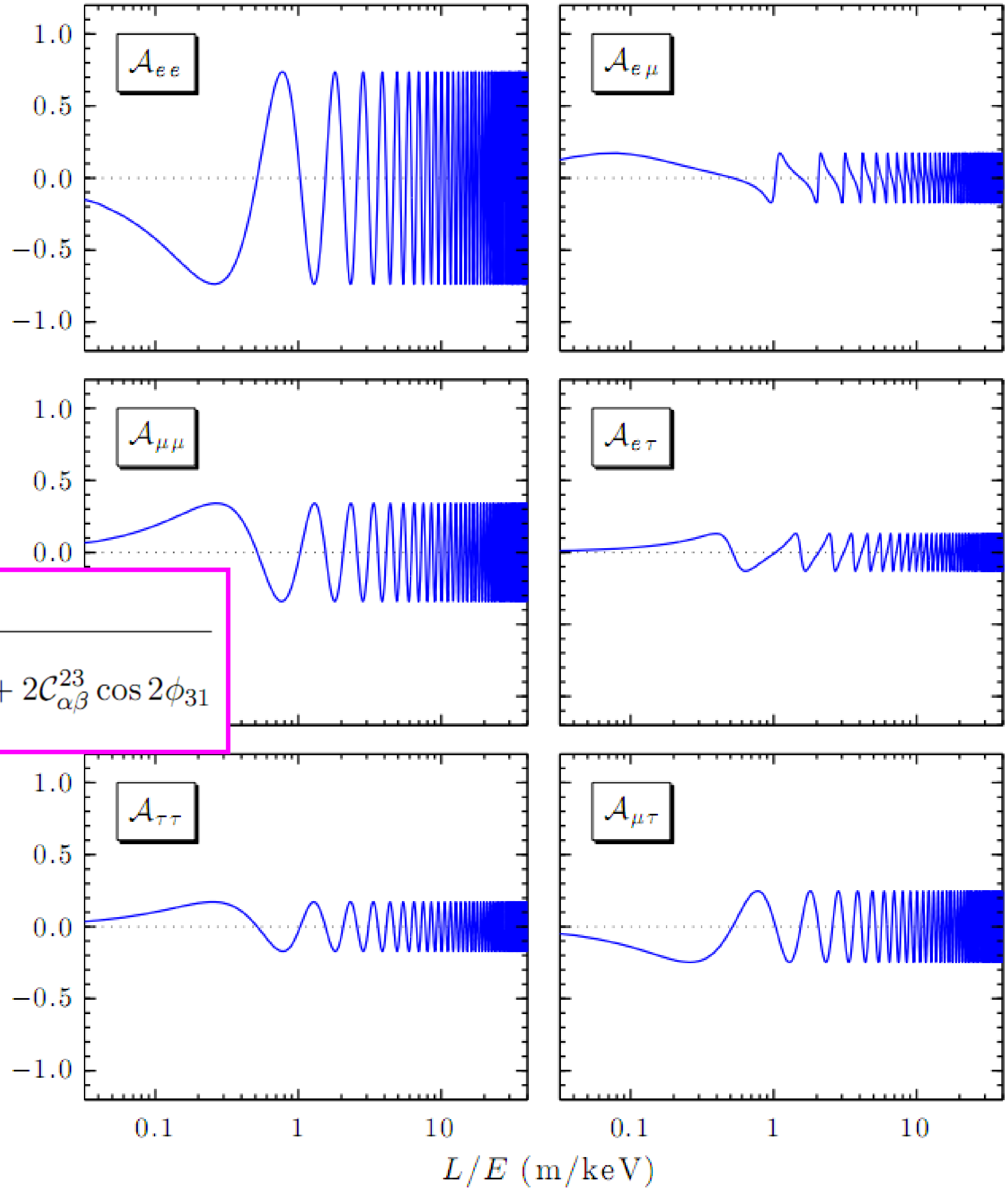
δ $\cancel{\rho}$ σ

$$\mathcal{A}_{\alpha\beta} = \frac{2\mathcal{V}^{23}_{\alpha\beta}\sin 2\phi_{31}}{\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\,\mathcal{C}^{22}_{\alpha\beta} + \sqrt{\frac{\Delta m^2_{31}}{\Delta m^2_{21}}}\,\mathcal{C}^{33}_{\alpha\beta} + 2\mathcal{C}^{23}_{\alpha\beta}\cos 2\phi_{31}}$$

$$\boxed{\delta = 90^\circ \text{ and } \sigma = 45^\circ}$$

input angles

$$\theta_{12} \simeq 33.4^\circ$$
$$\theta_{13} \simeq 8.66^\circ$$
$$\theta_{23} \simeq 40.0^\circ$$



CP Violation

Inverted mass hierarchy with m3 = 0

blue solid lines

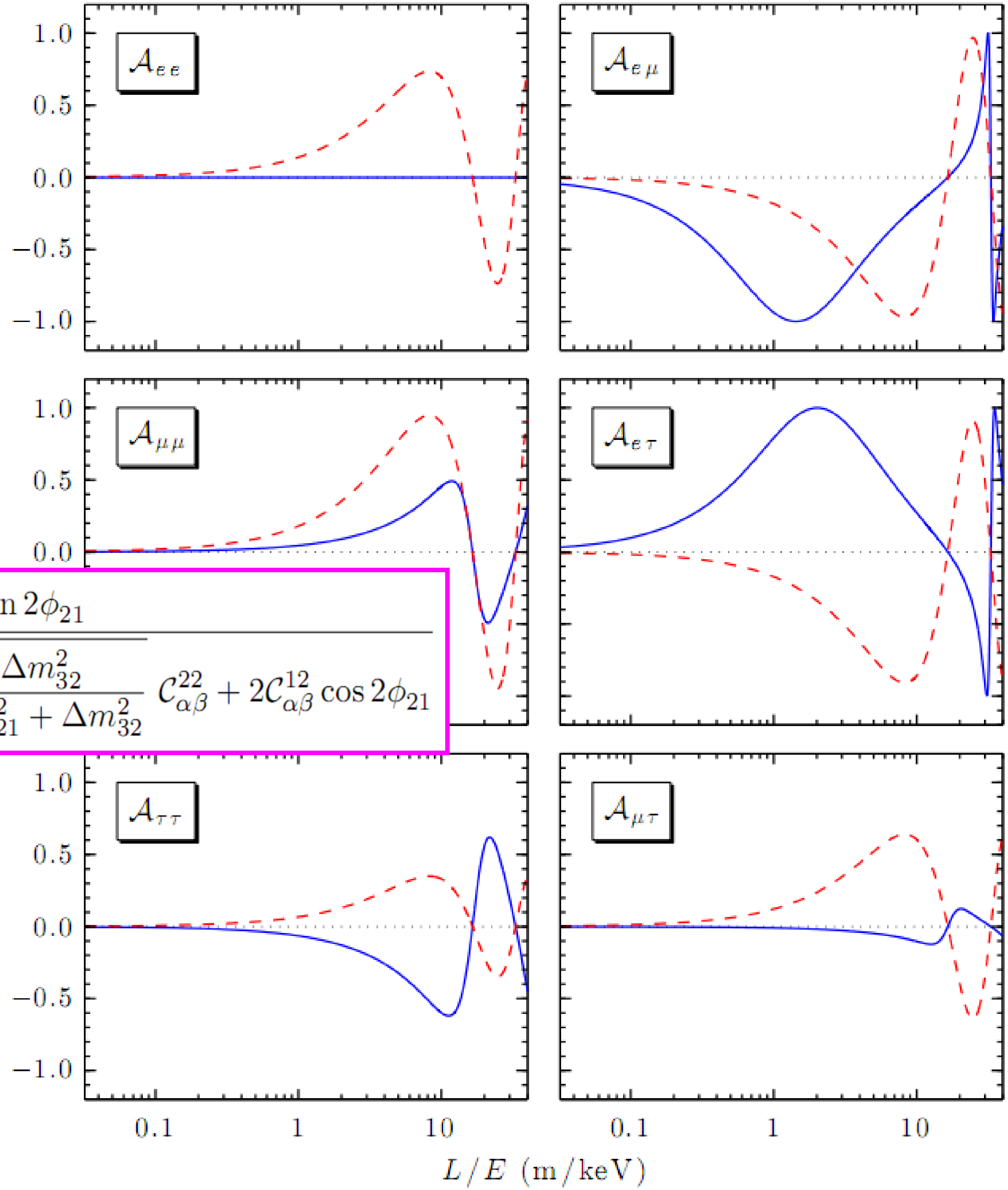
δ = π/2, ρ - σ = 0

red dashed lines

δ = 0, ρ - σ = π/4

$$\mathcal{A}_{\alpha\beta} = \frac{2\mathcal{V}^{12}_{\alpha\beta}\sin 2\phi_{21}}{\sqrt{\frac{\Delta m^2_{21}+\Delta m^2_{32}}{\Delta m^2_{32}}}\,\mathcal{C}^{11}_{\alpha\beta} + \sqrt{\frac{\Delta m^2_{32}}{\Delta m^2_{21}+\Delta m^2_{32}}}\,\mathcal{C}^{22}_{\alpha\beta} + 2\mathcal{C}^{12}_{\alpha\beta}\cos 2\phi_{21}}$$

insensitive to ν masses



input angles

$$\theta_{12} \simeq 33.4^\circ$$
$$\theta_{13} \simeq 8.66^\circ$$
$$\theta_{23} \simeq 40.0^\circ$$

CP Violation

Nearly degenerate mass hierarchy:

blue solid lines

$$\mathcal{A}_{\alpha\beta} \simeq \frac{2\sum_{i<j} \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji}}{\sum_i \mathcal{C}^{ii}_{\alpha\beta} + 2\sum_{i<j} \mathcal{C}^{ij}_{\alpha\beta} \cos 2\phi_{ji}}$$

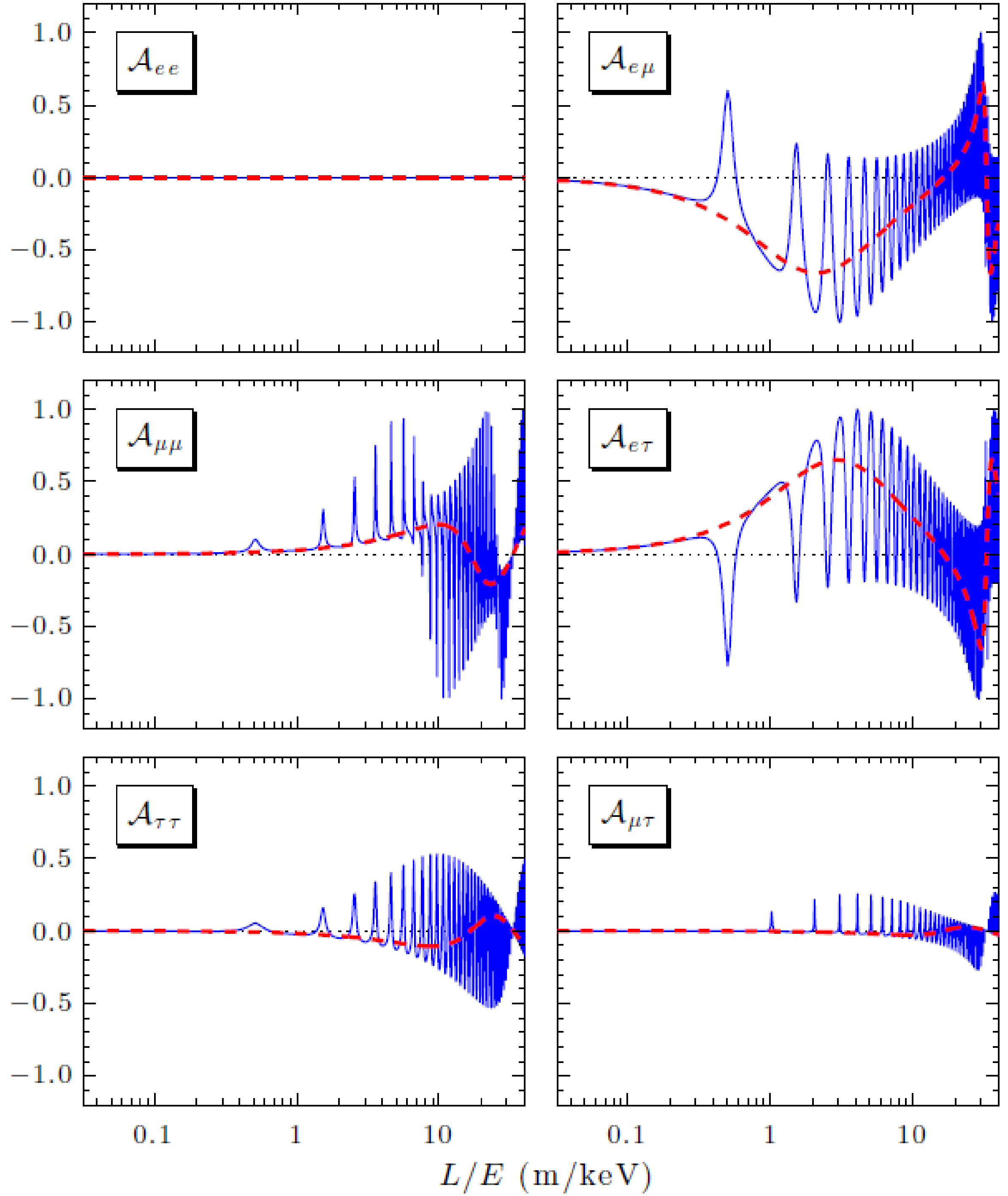
red dashed lines

$$\mathcal{A}^{21}_{\alpha\beta} \simeq \frac{2\mathcal{V}^{12}_{\alpha\beta} \sin 2\phi_{21}}{\sum_i \mathcal{C}^{ii}_{\alpha\beta} + 2\mathcal{C}^{12}_{\alpha\beta} \cos 2\phi_{21}}$$

insensitive to ν masses

input CP-violating phases

$$\boxed{\rho = \sigma = 0^\circ \text{ and } \delta = 90^\circ}$$



CP Violation

Nearly degenerate mass hierarchy:

blue solid lines

$$\mathcal{A}_{\alpha\beta} \simeq \frac{2\sum_{i<j} \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji}}{\sum_i \mathcal{C}^{ii}_{\alpha\beta} + 2\sum_{i<j} \mathcal{C}^{ij}_{\alpha\beta} \cos 2\phi_{ji}}$$

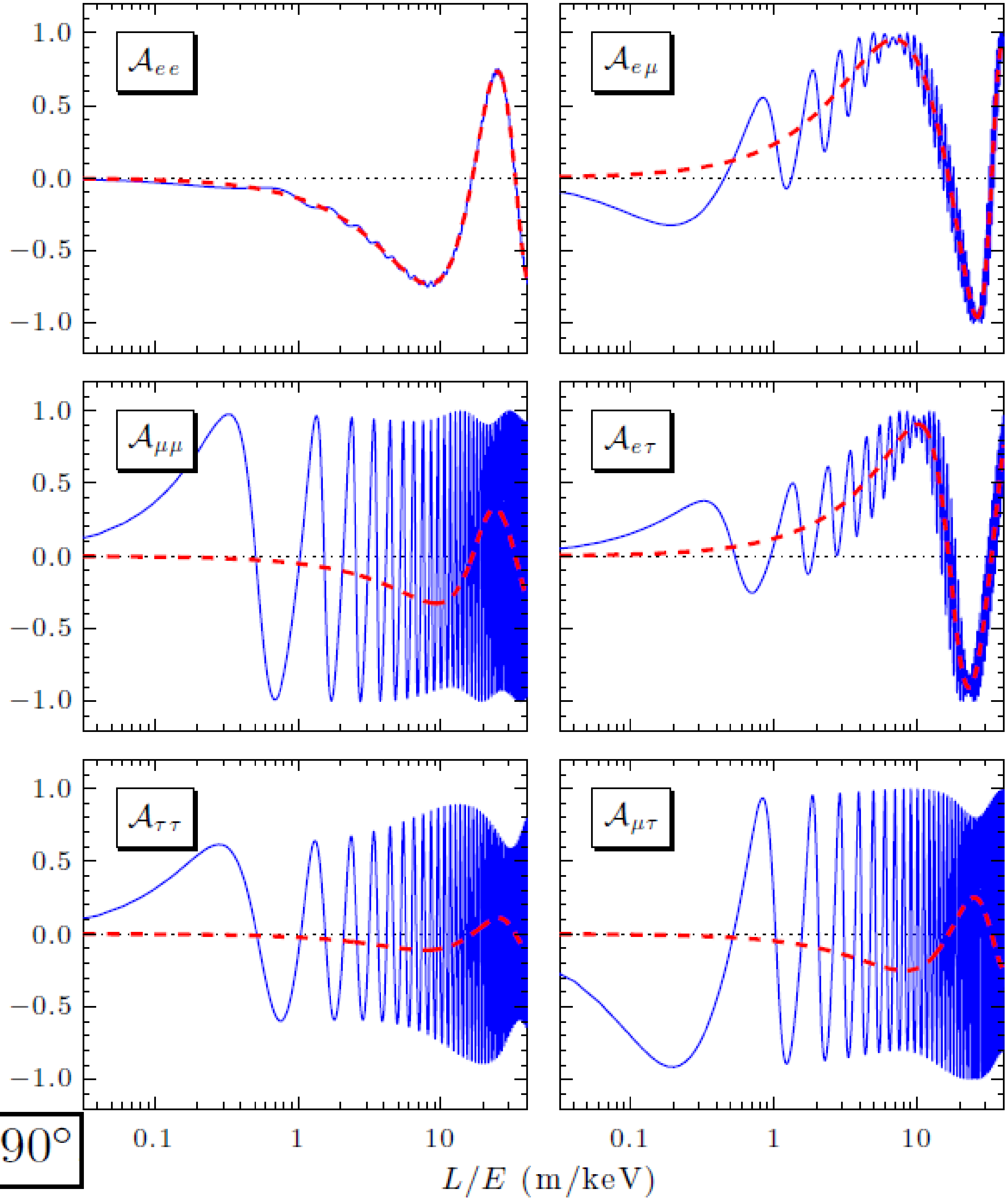
red dashed lines

$$\mathcal{A}^{21}_{\alpha\beta} \simeq \frac{2\mathcal{V}^{12}_{\alpha\beta} \sin 2\phi_{21}}{\sum_i \mathcal{C}^{ii}_{\alpha\beta} + 2\mathcal{C}^{12}_{\alpha\beta} \cos 2\phi_{21}}$$

insensitive to ν masses

input CP-violating phases

$$\rho = 0^\circ,\ \sigma = 45^\circ \ \text{and}\ \delta = 90^\circ$$



CP Violation

Nearly degenerate mass hierarchy:

blue solid lines

$$\mathcal{A}_{\alpha\beta} \simeq \frac{2\sum\limits_{i<j} \mathcal{V}^{ij}_{\alpha\beta} \sin 2\phi_{ji}}{\sum\limits_{i} \mathcal{C}^{ii}_{\alpha\beta} + 2\sum\limits_{i<j} \mathcal{C}^{ij}_{\alpha\beta} \cos 2\phi_{ji}}$$

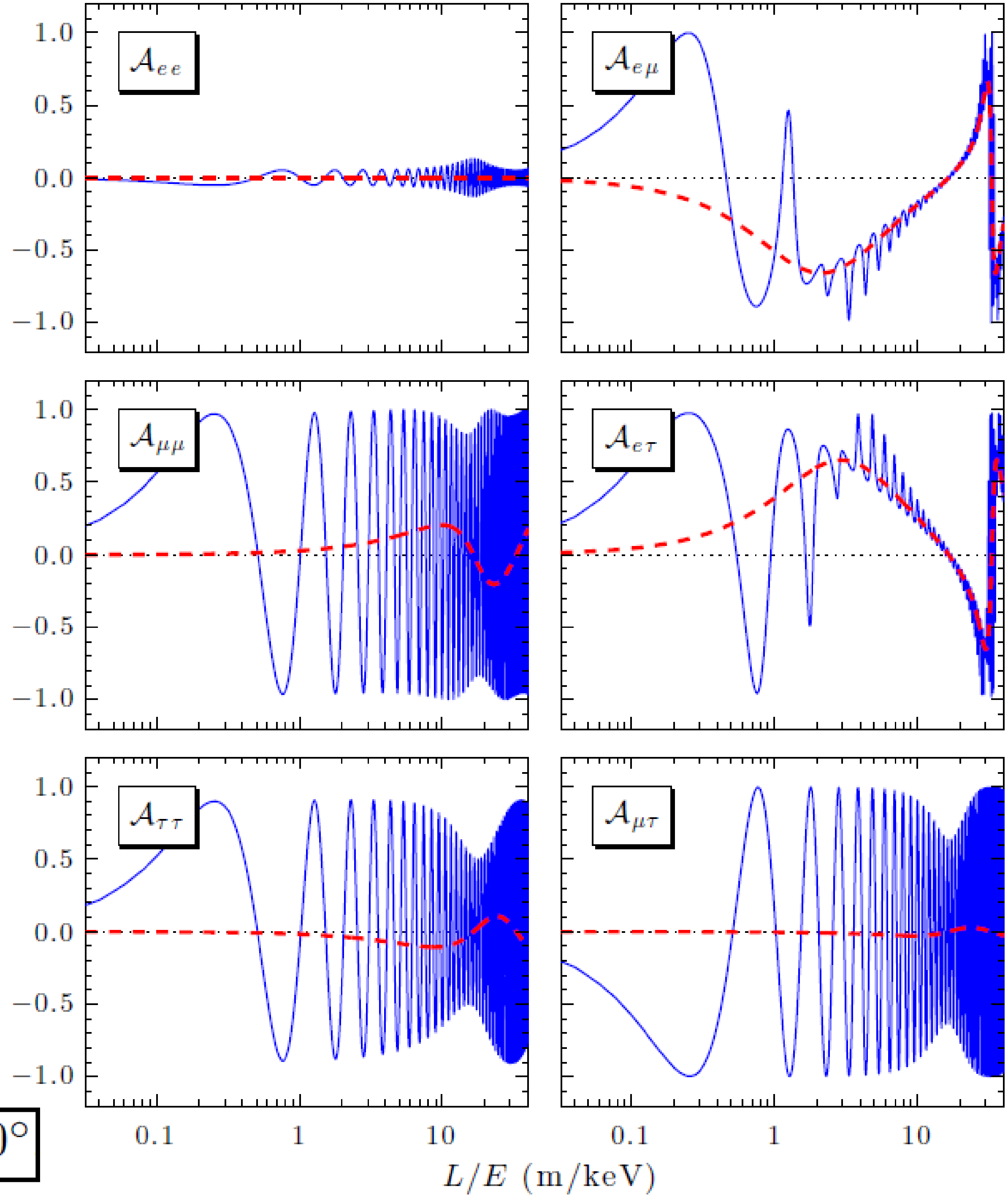
red dashed lines

$$\mathcal{A}^{21}_{\alpha\beta} \simeq \frac{2\mathcal{V}^{12}_{\alpha\beta} \sin 2\phi_{21}}{\sum\limits_{i} \mathcal{C}^{ii}_{\alpha\beta} + 2\mathcal{C}^{12}_{\alpha\beta} \cos 2\phi_{21}}$$

insensitive to ν masses

input CP-violating phases

$$\rho = \sigma = 45^\circ \text{ and } \delta = 90^\circ$$



Summary: YES or NO?

QUESTION: are massive neutrinos the Majorana particles?

One might be able to answer YES through a measurement of the $0\nu\beta\beta$ decay or other LNV processes someday, but how to answer with NO?

YES

or

I don't know!

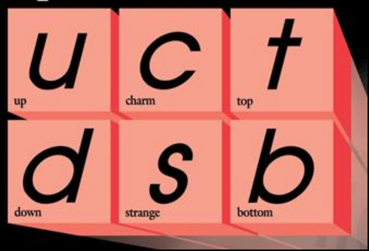
If I don't know, then no way to know which ν mass model to be right?

If YES, then how to determine the two Majorana CP-violating phases?

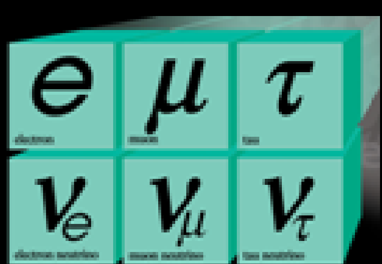
An answer might not be available until the second centenary of Pontecorvo? (if so, be patient!)

SM + ν's is left with CP-violating phases: way out?

Quarks

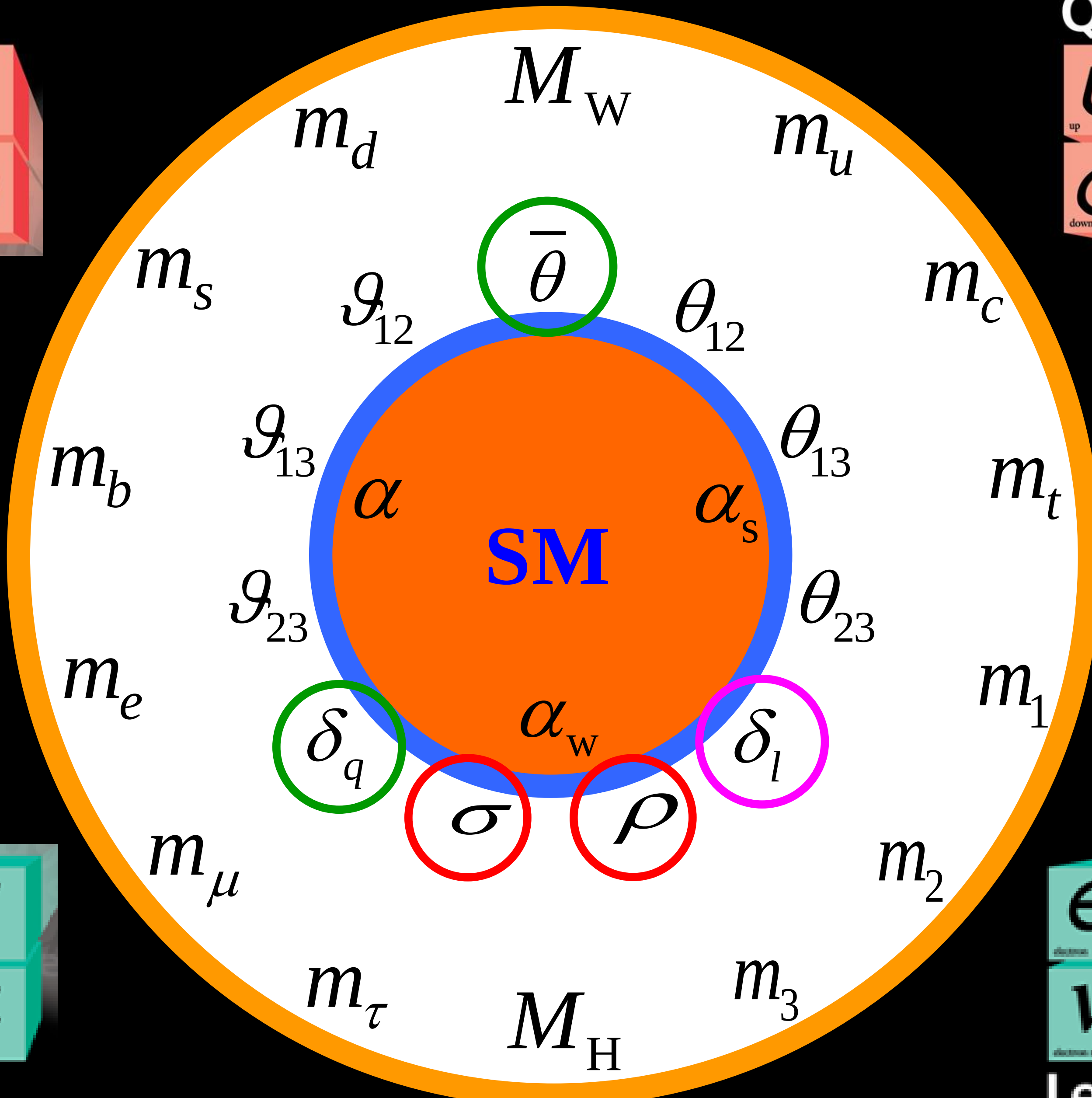


Leptons



1/5

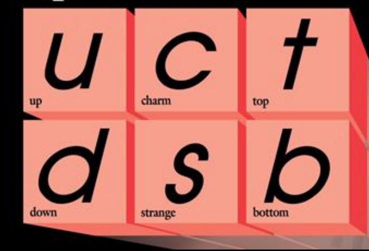
OK!



4/5

NO!

Quarks



Leptons