60 years of Spontaneously **Broken Symmetries in Quantum Theory**

(From Bogoliubov's theory of Superfluidity to Standard Model)

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Motivation

"Phase transition in Quantum system,

as a rule, is accompanied by the Spontaneous Symmetry Breaking".

XXth Century Folklore

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+ Symmetry of Quantum System

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- Macroscopic vs. Microscopic theories (Bogoliubov 58)

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- Macroscopic vs. Microscopic theories (Bogoliubov 58)
- Macro SuperConduct (Ginzburg+Landau 1950)
- Micro SuperCond (BCS + Bogol 57)
- SuperConductivity as SuperFluidity (of Cooper pairs) (Bogoliubov 1958)

List of items - 2

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- Quantum Symmetries . . .

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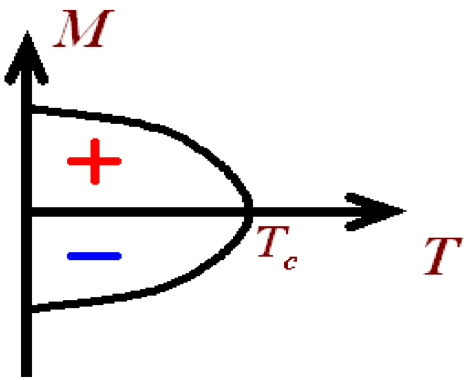
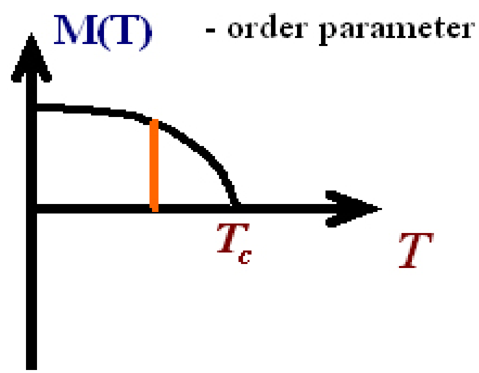
- What Symmetry in SuperConductivity?
- Various levels of Symmetries
- What is the Symmetry ?

Phase transition with Symmetry Breaking

Order parameter [Landau 1937] in magnetics,



- add a weak field δH_+
- tend $V \rightarrow \infty$
- put $\delta H_+ \rightarrow 0$



Ferromagnetism in a finite volume **v**.

In the thermodynamic limit

Correlation function:

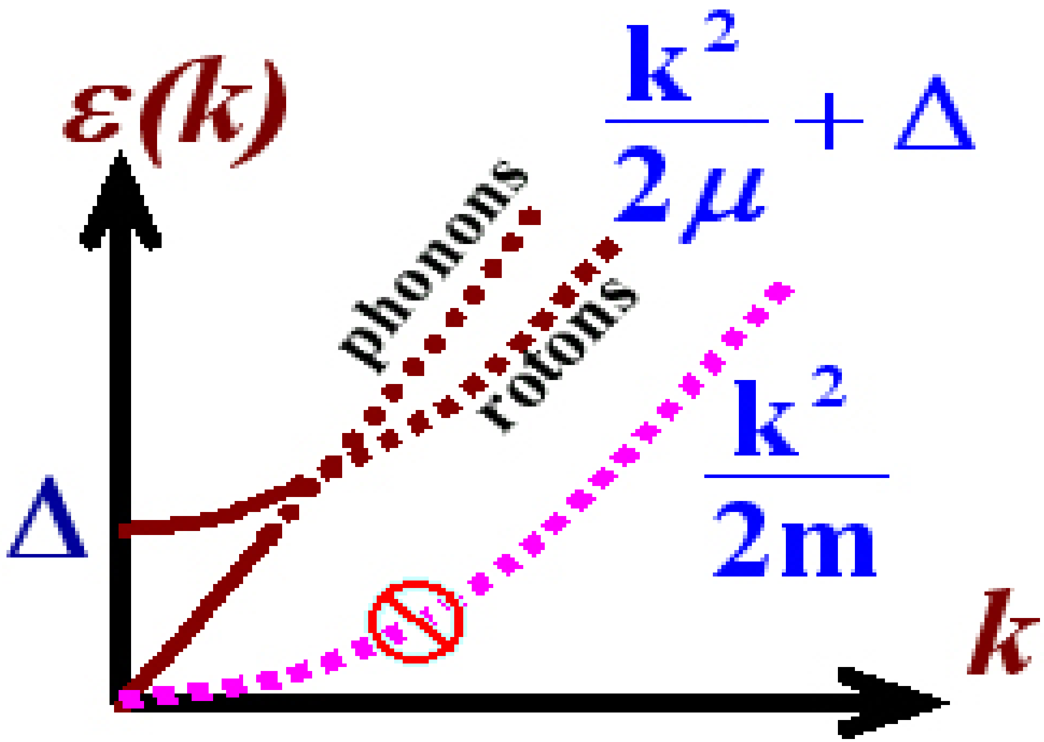
$$K_{\sigma\sigma}(\mathbf{r}) =< \sigma(\mathbf{0})\sigma(\mathbf{r}) > - < \sigma(\mathbf{0}) >< \sigma(\mathbf{r}) >$$

$$K_{\sigma\sigma}(\mathbf{r} \rightarrow \infty) = \begin{cases} 0, & T > T_c \\ M^2(T), & T < T_c \end{cases}$$

Landau Phenomenology of Superfluidity

[Expt'l Discovery], Kapitsa (1937)

$\boxed{\text{Landau 1941}}$ phenomen. phonons-rotons theory



Energy loss at velocities $v < v_{crit}$ forbidden

Bogoliubov model for SuperFluid He II

$\boxed{\text{Bogoliubov, Oct 1946}}$ microscopic theory

$$H = \frac{\hbar^2}{2m} \int d\,x\,\Psi^*(x)\Delta\Psi(x) +$$
$$+ \int d\,x \int d\,y\,\Psi^*(x)\Psi(x)\,V(x-y)\,\Psi^*(y)\Psi(y).$$

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Shift by constant C, to single out condensate

$$\Psi(x) = C + \phi(x) \qquad \Psi^*(x) = C + \phi^*(x) \tag{2}$$

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Transition to momentum p-picture

$$\Psi(x) = \frac{1}{\sqrt{V}}\sum_k a_k\,e^{\frac{i\,(q\,x)}{\hbar}}\,,\ \ \phi(x) = \frac{1}{\sqrt{V}}\sum_{p\neq 0} b_p e^{\frac{i\,(p\,x)}{\hbar}}\,, \qquad C = \frac{1}{\sqrt{V}}a_0$$

yields $\qquad a_k = a_0\,\delta_{k,0}\,\frac{C}{\sqrt{V}} + \left[1-\delta_{k,0}\right]\,\delta_{k,p}\,b_p\,.$

Bogoliubov model for SF He II

Bogoliubov 1946 microscopic theory – non-ideal

$$H_{\mathrm{B-gas}} = \sum_{\vec{p}} \frac{p^2}{2m} a_p^+ a_p + \frac{1}{2V} \sum v(p_1 - p_2) a_{p_1}^+ a_{p_2}^+ a_{p_2} a_{p_1} \;;$$

Bose gas with weak repulsion $v(p) > 0$.

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The Hamiltonian has $\boxed{a \longrightarrow e^{-i\alpha} a,\; a^* \longrightarrow e^{i\alpha} a^* \;=}$

phase symmetry –> No of particles conservation,

as $H_{\mathrm{B-gas}}$ commutes with $\mathbf{N} = \sum_{\vec{p}} a_p^+ a_p$

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Bogoliubov's physical hypothesis:

<u>"macroscopic condensate"</u> $\boxed{\mathbf{N_{p=0}} = \mathbf{a_0^+ \, a_0} \sim \mathrm{N_A}}$

Corollary: condensate operators $a_0^+, a_0 \sim \sqrt{N_0} =$ c-numbers

Bogoliubov 1946 SuperFlu model

Shift $\boxed{\psi(\mathbf{x}) = \mathbf{\Psi_0} + \phi(\mathbf{x})}$ by “big” constant $\Psi_0 \sim \sqrt{N_0}$
results in bilinear approximate Hamiltonian

$$\mathbf{H_{Bog}} = \sum_{\mathbf{p}\neq\mathbf{0}} \left(\frac{\mathbf{p^2}}{\mathbf{2m}} + \frac{\mathbf{N_0}}{\mathbf{V}} \mathbf{v}(\mathbf{p}) \right) \mathbf{b}^+_{\mathbf{p}} \mathbf{b}_{\mathbf{p}}, \; + \; \frac{N_0}{2V} \sum_{p\neq 0} v(p) [b^+_p b^+_{-p} + b_p b_{-p}]$$

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with b_p^+, b_p – "above-condensate" Bose-operators.

$\mathrm{H_{Bog}}$ describes creation of pairs of Helium atoms with opposite momenta from condensate and their "annihilation" into condensate.

Interaction btwn pairs is small $\sim N_0^{-1/2}$ and omitted.

Total No of these correlated pairs is <u>not fixed</u>.

Bogoliubov-1946 SuperFlu, 3

One gets in the leading order :

$$H_{B1} = E_0 + H_{B2}\,(b) + \ldots$$

(E_0 = condensate energy), $H_2 -$ bilinear operator form

$$H_{B2}\,(b) = \tfrac{N_0}{2V} \sum_{p \neq 0} v(p)\,[b_p^+ b_{-p}^+ + b_p b_{-p}] + \sum_{p \neq 0} \left(T(p) + \tfrac{N_0}{V} v(p)\right) b_p^+ b_p, \quad (1$$

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diagonalized by Bogoliubov (u, v) transformation

$$\boxed{\xi_p = u_p b_p + v_p b_{-p}^+; \quad \xi_p^+ = u_p b_p^+ + v_p b_{-p}}$$

with real coefficients $u_p^2 - v_p^2 = 1;\; u_{-p} = u_p;\; v_{-p} = v_p$.

Also by unitary transformation

$$\xi_p = U_\alpha^{-1} b_p\, U_\alpha = u_p b_p + v_p b_{-p}^+ \,, \qquad U_\alpha = e^{\sum_p \alpha(p)\,[b_p^+ b_{-p}^+ - b_p b_{-p}]} \,.$$

Bogoliubov explanation for Landau spectrum

The (u,v) transformation $b_p \to \xi_p$ correlates pairs of particles with opposite momenta. New Hamiltonian

$$H_{B2}(b) = H_{Bog3}(\xi)\,; \quad H_{Bog3} = \sum_{p\neq 0} E(p)\, \xi_p^+\, \xi_p\,,$$

$$E(p) = \sqrt{(T(p))^2 + T(p)\,v(p)}; \qquad T(p) = \frac{p^2}{2m}$$

describes new collective excitations.

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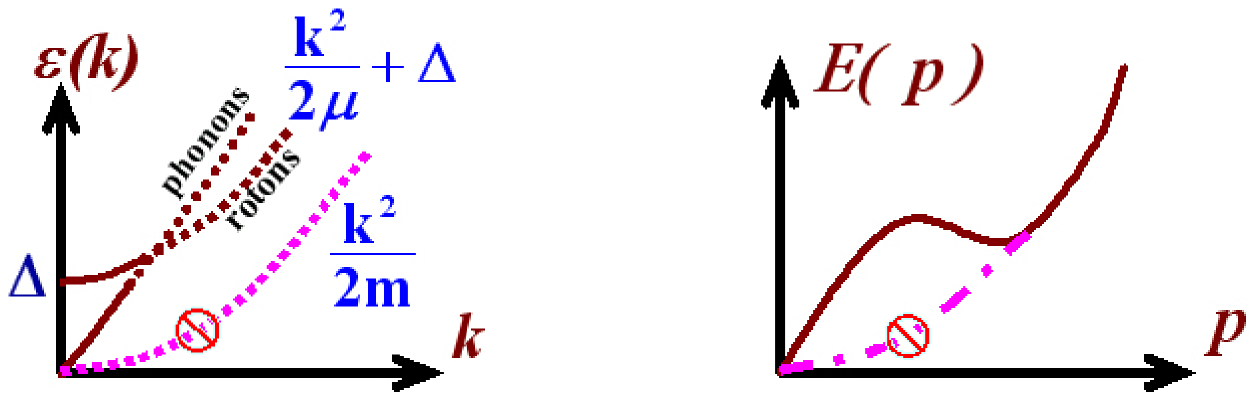

Figure 2: (a) Phonon + roton spectra – Landau phenomenology; (b) Bogoliubov spectrum from non-ideal Bose-gas microscopical model.

Bogoliubov SF collective modes*

Diagonalization of Bogoliubov bilinear Hamiltonian

$$H_{B2} = \sum_p \epsilon(p)\, b_p^+ b_p + \sum_p v(p)[b_p^+ b_{-p}^+ + b_p b_{-p}] = \sum_p E(p)\, \xi_p^+\, \xi_p$$

by unitary transformation $\xi_p = u_p b_p + v_p b_{-p}^+ = U_\alpha^{-1} b_p\, U_\alpha\,,$

$$U_\alpha = e^{\sum_p \alpha(p)\, [b_p^+ b_{-p}^+ - b_p b_{-p}]}\ ; \qquad \alpha(p) = f[E(p), v(p)]\ .$$

New ground state $\Psi_0(\alpha) = U_\alpha^{-1}\, \Phi_0 \sim \boxed{\sim e^{\sum_p \alpha(p)\, [b_p^+ b_{-p}^+]}\ \Phi_0}$ is

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Bogoliubov (u, v) transformation and new ground state

$$\Psi_0(q) \sim e^{\sum_k c(k,q)\, b_k^+ \, b_{q-k}^+}\, \Phi_0$$

of the same pair-correlated nature, is used now

in quantum optics to describe “squeezed states”.

Phase symmetry breaking in SF state

Initial Hamiltonian $H_{B1}(a_p^+ , a_p)$ for normal states $< a_p >= 0$ is invariant with respect to the

$\boxed{\text{Phase (Gauge) transformation } \; a_p \longrightarrow a_p \, e^{i\,\varphi} \text{ (GT)}}$

related to conservation of particles number $< a_p^+ a_p >= n_p$.

Bilinear Bogoliubov model Hamiltonian $H_{B2}(b)$ – as well as the (u, v) canonical transformation and $H_{Bog3}(\xi)$ – is not compatible with GT,.

Physically, this corresponds to non-conservation the number of particles with non-zero momenta

Macroscopical vs Dynamic Eqs.

The goal of the macroscopic theory is the derivation of equations of the type of classical eqs. of mathematical physics that would reflect the whole set of experimental facts entering into the treatment of macroscopic objects.

[Bogoliubov 1958]

1950

Microscopical vs Macroscopic Models

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In this case ... one should derive relations between quantities which result in eqs of the macroscopic theory

[Bogoliubov 1958]

Phenomenological vs Dynamic Models

- From 4-fermion Fermi (1932) interaction to
  EW W, Z_0 Gauge Dynamics (1964) (with Higgs ...?)
- Landau SF (1940) phonon-roton model of He II
  vs Bogoliubov non-ideal Bose gas (1946)
- Ginzburg-Landau SC (1950) order parameter Ψ
  via Cooper pairs condensate ψ BSC- (1957);
  to Bogoliubov–SC by electron-phonon H_{Fr}
- Low-energy chiral models (Nambu,JL - 1961) via
  quark-meson model (Eguchi,Kikkawa 1976) ? vs ?
  QCD quark-gluon Gauge Dynamics
  <confinement, hadronization (2???)>

Phase transition and broken symmetry

Connection btwn Phase transition and symmetry breaking was evident before the QM creation –> e.g., from physics of crystals

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Landau 1937 theory of phase transitions :

- starts with Introduction in Symmetries,
- but, only discrete symmetries :
- on SuperFluid – “He II is not a liquid crystal !”

Meanwhile, Landau’s “Mechanics”(1937/40) is based upon continuous symmetries, invariance and conservation laws.

Symmetries and groups

Symmetries and groups : discrete and continuous.

Continuous group –> Lie group of transformations.

Lagrangian –> Invariance –> Nöther theorem –> current –> conservation law.

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Quantum Symmetries :

- Non-relativistic 2nd-quantized neutral field
  Phase transformation= $a \longrightarrow e^{-i\alpha}\, a,\;\; a^* \longrightarrow e^{i\alpha}\, a^*$
  –> N = const. Conserving Number of particles
- Charged (2-, 3-component) field; Gauge=phase transformation –> Current; Charge conservation

Quantum Symmetries

Qu-Symmetries: Phase, Gauge, Chiral, SuSy,

Qu-Symmetries are quite different from "Classical" ones, like spatial (boosts, rotations, Lorentz) and internal (isospin, flavor) ones.

For their formulation and understanding one has to use quantum notions :

* nonobservability of the ψ-function phase;
* spin, chirality ;
* distinction btwn Bose– and Fermi–statistics.

Ginzburg-Landau [1950] SuperConductivity

$\Psi(r)$ $\sim$a system (of SC electrons) eff. function = 2-component order parameter for (SC) transition

$$\Psi(r) = |\Psi(r)|e^{i\Phi(r)}$$

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Free energy functional

$$F = F_n + \int \left(\frac{\hbar}{2m^*} |\vec{\nabla}\Psi(r)|^2 + a|\Psi(r)|^2 + b|\Psi(r)|^4 \right) dV$$

with $a \sim T - T_c\,,$ $\quad b \approx$ const , m^* – effective mass

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SC current $j_\alpha = \frac{e^* \hbar}{m^*} |\Psi|^2 \nabla_\alpha \Phi,\ e*$ – effectve charge

Gor'kov (1959) : $m^* = 2m,\ e^* = 2e,\quad |\Psi|^2 = n_s/2\,.$

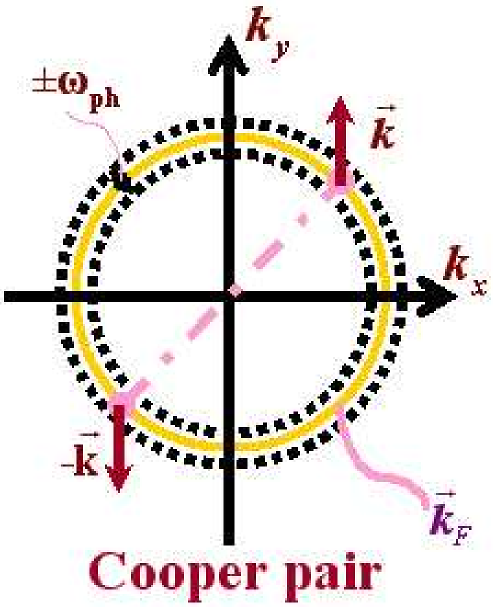
BSC SuperConductivity

BCS model:

$$H = \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}}\, c^{+}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{k}'} V_{\vec{k},\vec{k}'}\, c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow} c_{-\vec{k}'\downarrow} c_{\vec{k}'\uparrow}\,,$$

- eff. Cooper pairs (antipodes) attraction

$\varepsilon_{\vec{k}} = \frac{\vec{k}^2}{2m} - \varepsilon_F$ - electron energy above ε_F



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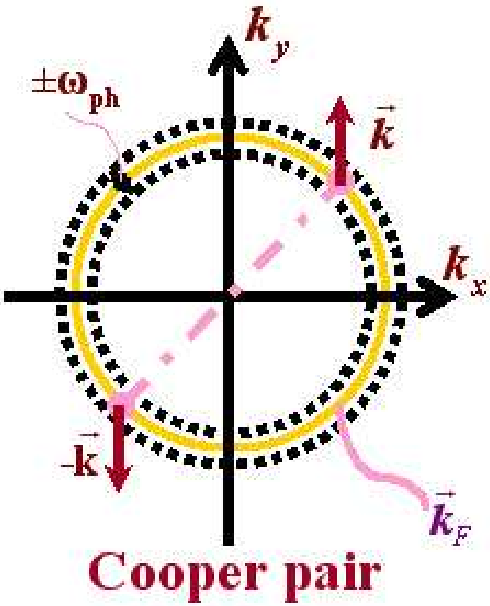
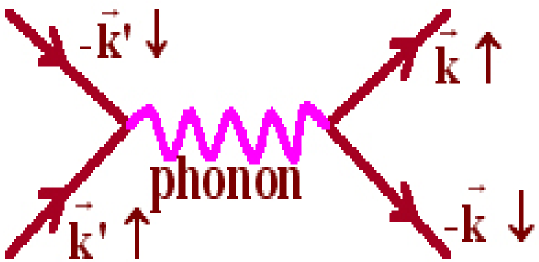


Effective electron-electron attraction in the vicinity of Fermi surface



$$V(\vec{k},\vec{k}') = \begin{cases} -V_C, & |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| < \omega_{ph} \\ 0, & |\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}| > \omega_{ph} \end{cases}$$

Semi-Phenomenological BSC theory, 2

Variational BCS wave function

$$|\Psi_{BCS}> = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c^{+}_{\vec{k}\uparrow} c^{+}_{-\vec{k}\downarrow}) |0>; \qquad c_{\vec{k}\sigma}|0> = 0.$$

New SC ground state: $\boxed{c_{\vec{k}\sigma}|\Psi_{BCS}> \neq 0}$

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New SC ground state: $\boxed{c_{\vec{k}\sigma}|\Psi_{BCS}>\neq 0}$

- SC order parameter $=$ Cooper pair condensate:
  $< c^{+}_{\vec{k}\uparrow}c^{+}_{-\vec{k}\downarrow} >= \Psi(\vec{k}) = |\Psi(\vec{k})|exp[i\Phi(\vec{k})]$
- Phase symm breaking: $\tilde{c}^{+}_{\vec{k}\sigma} = e^{i\phi}c^{+}_{\vec{k}\sigma} \Rightarrow \boxed{\tilde{\Psi}(\vec{k}) = e^{2i\phi}\Psi(\vec{k})}$
- Energy gap: $\Psi(\vec{k}) = \frac{\triangle_{\vec{k}}}{2E_{\vec{k}}}$ $\quad \boxed{\triangle_0 \approx exp\left(-\frac{1}{\lambda}\right);} \quad \lambda = N_0 V_C$
- SC temperature $\quad T_c = 1.14\,\omega_{ph}\, exp\left(-\frac{1}{\lambda}\right); \quad 2\triangle_0 = 3.52 T_c$

Bogoliubov SuperCond theory

Fr$\ddot{o}$hlich electron-phonon model: $H_{Fr} =$

$$= \sum_{\vec{k},\sigma} \varepsilon_{\vec{k}} c^+_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{q}} \omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} + g_{Fr} \sum_{\vec{k},\vec{k}',\sigma} \sqrt{\frac{\omega(\vec{q})}{2V}} c^+_{\vec{k}\sigma} c_{\vec{k}'\sigma} (b^+_{\vec{q}} + b_{-\vec{q}})$$

Bogoliubov (u,v) transformation:

$$\alpha_{\vec{k}\uparrow} = u_{\vec{k}} c_{\vec{k}\uparrow} - v_{\vec{k}} c^+_{-\vec{k}\downarrow}; \quad \alpha^+_{\vec{k}\uparrow} = u_{\vec{k}} c^+_{\vec{k}\uparrow} + v_{\vec{k}} c_{-\vec{k}\downarrow}$$

$$u^2_{\vec{k}} = 1 - v^2_{\vec{k}} = \frac{1}{2}\left(1 + \frac{\varepsilon_{\vec{k}}}{E_{\vec{k}}}\right)$$

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Gap solution : $$\boxed{\triangle_B = \widetilde{\omega}\, exp\left(-\frac{1}{\rho_B}\right)}$$

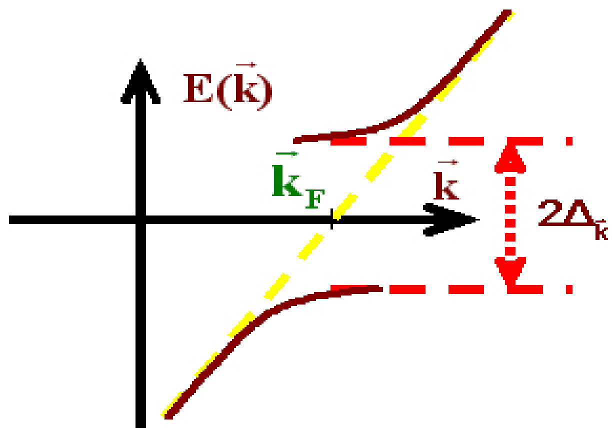
Microscopical $\rho_B = g^2_{Fr} N_0$ *vs* BCS phen $\lambda = V_{\mathrm{C}} N_0$

Excitation spectrum of

quasiparticles (“Bogolons”)

$$H_{Fr} \rightarrow H_B = \sum E_{\vec{k}} \alpha^{+}_{\vec{k},\sigma} \alpha_{\vec{k},\sigma}$$

$$E_{\vec{k}} = \sqrt{\varepsilon^2_{\vec{k}} + |\triangle_{\vec{k}}|^2}$$



Bogoliubov SuperCond, 2

Spectrum with gap; Bogolon dissociation

To elucidate Bogolon's physical content, take spectral function of quasiparticle excitations in SC phase

$$A_{sc}(\mathbf{k},\omega) = u_{\mathbf{k}}^2\,\delta(\omega - E_{\mathbf{k}}) + v_{\mathbf{k}}^2\,\delta(\omega + E_{\mathbf{k}}), \tag{4}$$

as in the Figure

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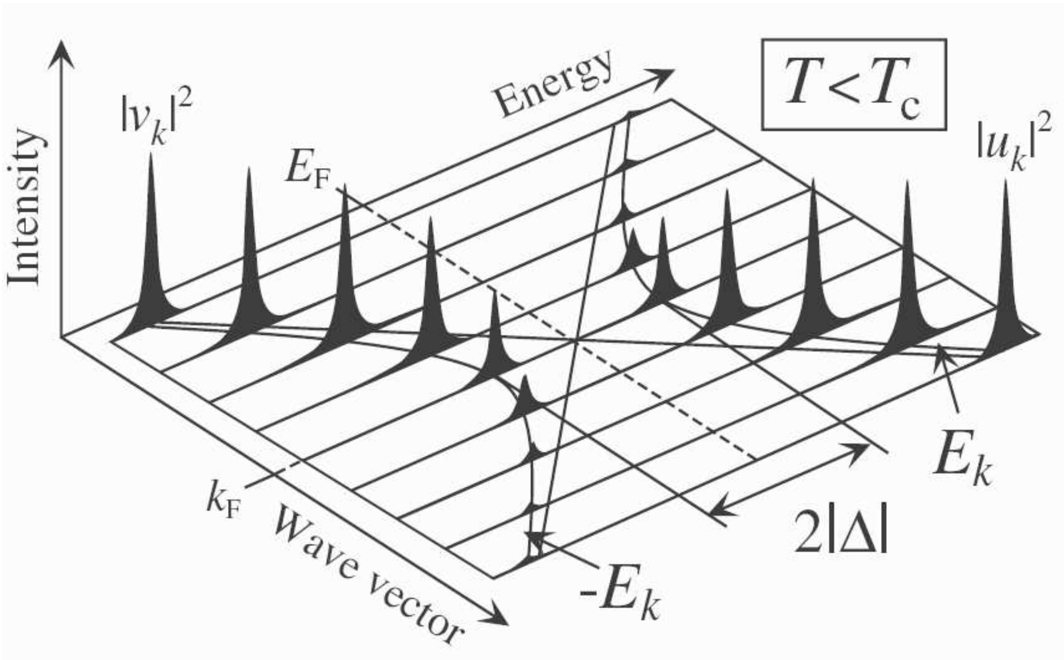
$$A_{sc}(\mathbf{k},\omega) = u_{\mathbf{k}}^2\,\delta(\omega - E_{\mathbf{k}}) + v_{\mathbf{k}}^2\,\delta(\omega + E_{\mathbf{k}}), \tag{5}$$

as in the Figure



Spectral function of quasi-partical exitations in Bog's theory

Conclusion to Quantum Statistics (SF and SC)

1. The superfluid and the superconducting phase transitions are accompanied by the Spontaneous Symmetry Breaking

2. At these SSBs, the phase (gauge) symmetry, related to number of particles conservation, is broken

3. In the Symmetry Broken state both the amplitude and the phase of the order parameter are fixed

“$1/q^2$ theorem” $\sim$ Goldstone mode in QFT

Bogoliubov theorem on $1/q^2$ singularity (1961), (i.e., on long range forces) of Green function for systems with degenerate ground state was proven in context of “method of quasi-averages” for SSB.

Analog of this in QFT was proposed by Goldstone [1960]; massless excitations in QFT are known as the Goldstone (bosonic) modes.

The proof of Goldstone theorem analogous to Bogoliubov theorem, was given half a year later [1962]

SSB Transition to QFT; Early 60s

Spontaneous Breaking of Chiral (γ_5) Invariance

2-dim models with cutoff Λ

- Vaks + Larkin I, II [August 1960]
- Tavkhelidze [Aug 1960] {ref: Bogoliubov, Sept '60}
- Nambu [? 1960 Purdue Conf] {ref: Nobel Comm '08 doc }
- Nambu, Jona-Lasinio I [Oct 1960]

2-dim, + cutoff Λ

- Nambu, Jona-Lasinio II [May 1961]

2-dim without cutoff

- Arbuzov, Tavkhelidze, Faustov [Nov 1961]

Implication to QFT; Higgs field

Lagrangian for normal quantum scalar field with quartic self-interaction and stable ground state

$$L(\varphi, g) = \frac{1}{2}\,(\partial_\mu \varphi)^2 - V(\varphi), \qquad \boxed{V(\varphi) = \frac{m^2}{2}\,\varphi^2 + g\,\varphi^4;\ \ g > 0}$$

Since 60s, in QFT play with toy models *à la Ginzburg-Landau* with phantom scalar field $\mathbf{\Phi}(x)$, like the Higgs (1964) one

$$V_{\text{Higgs}}(\Phi^2) = \lambda\left(\mathbf{\Phi}(x)^2 - \Phi_0^2\right)^2\,; \qquad \mathbf{\Phi}^2 = \Phi_1^2 + \Phi_2^2\,; \qquad \Phi_0^2 = \text{const.}$$

with imaginary initial mass $\mu_{\text{H}}^2 = -4\lambda\,\Phi_0^2$ and the final one $m_{\text{Higgs}} = 2\sqrt{2\lambda}\,\Phi_0$ obtained after shift of field operator by constant

$$\Phi(x) \to \varphi(x) = \Phi(x) - \Phi_0\,,$$

like in Bogoliubov's SuperFluidity.

QFT; masses of fermions*

In $\Phi(x) = \varphi(x) + \Phi_0\,,$ constant $\Phi_0 \neq 0$ is the Vacuum Expectation Value of the Higgs field $< \Phi(x) >= \Phi_0\,.$

The main purpose of this trick is to attribute mass to particles of some other fields. Besides intermediate vector bosons $W, Z_0\,,$ to leptons and quarks via Yukawa coupling

$$g_i\bar{\psi}\,\Phi(x)\,\psi \longrightarrow g_i\bar{\psi}\varphi(x)\,\psi + m_i\,\bar{\psi}\,\psi;\;\; m_i = g_i\,\Phi_0$$

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Hence, “Higgs mechanism” provides masses to fermions via Yukawa couplings along the rule

“One mass - one coupling constant”

In Standard Model, No of Yukawa couplings = 12
(besides the issue on neutrino masses !)

BS in QFT; Standard Model*

In 60s the SSB mechanism in QFT models with degenerate vacuum formed cornerstone of Glashow-Weinberg-Salam gauge model of Weak and EM interaction with massive W and Z_0 vector mesons. Two Nobel Prizes :

- *EW-theory*; Glashow-Salam-Weinberg (NP-1979);
- W, Z_0-exp'tl; Rubbia+VanDer Meer (NP-1984)

Together with QCD, GWS-model forms Standard Model. Based on the principle **"Dynamics from Symmetry"**, SM contains only 3 **basic running couplings:** $\bar{\alpha}_{i=1,2,3}(E)$ with the Renorm-group evolution.

BS in Standard Model; the Higgs issue

Up to now, so-called Higgs particle escaped of observation. Current window for it possible mass is

$$114 \text{ GeV} < M_{\text{Higgs}} \quad 154 \text{ GeV} .$$

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In such a case, the hopes of its direct observation on LHC look like illusive.

Message to Theorists

1. Theory of phase transitions and macroscopical superfluidity (Landau, 1940) was founded by Microscopic Superfluidity (Bogoliubov, 1946)

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2. Microscopic Superconductivity devised by BCS + Bogoliubov (1957) was understood as a Superfluidity of Cooper pairs (Bogoliubov, 1958)

3. Higgs model in QFT is a replica of Ginzburg – Landau phenomenology for Superconductivity.

Its physical base is an open question ! ??

Different Symmetries in Macro- and Micro-

Thus, the Broken Symmetry of micro-theory of SuperConductivity (like in Bog's SuperFluidity) – is the phase Symmetry.

Nonconservation of the No of Cooper pairs or He II atoms relevant for the phase transition.

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Compare with Champagne bottle Symmetry of macro-phenomenological Ginzburg-Landau theory.

Micro and Macro Symmetries are different

Essentially different !

Symmetries: exact and approximate

What is Symmetry (broken) of physical problem ?

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* Do they relate to Symmetry of physical system ?

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- Quantum Symmetries (hidden), like phase=gauge, Chiral and Super- Symmetry.
- Among Qu-Sym, approximate (in pQCD)

Modern Pilatus vs Critical phenomena

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"What is the Verity ?" = that's the old question (by Pilatus to Jesus). The modern analog :

What is the Symmetry ?

Pilatus

“Quid est symmetria ?”