Dark matter and dark energy –
two entities in the Universe seen by their gravitational interaction only

DM – non-relativistic
gravitationally clustered

DE – relativistic
unclustered

Definition through equations

I. DM – through the (generalized) Poisson equation

$$\frac{\Delta \Phi}{a^2} = 4\pi G(\rho - \rho_0(t))$$

$\Phi(\vec{r})$ is measured using the motion of 'test particles' in it

a) Stars in galaxies → rotation curves
b) Galaxies → peculiar velocities
c) Hot gas in clusters → X-ray profiles
d) Photons → gravitational lensing (strong and weak)

Resuet: DM is non-relativistic ($p \ll E$), collisionless, and has the same spatial distribution as visible matter for $L \gtrsim 1$ Mpc

$$b \simeq 1, \quad \Omega_{m,tot} = 0.28 \pm 0.03 \qquad (26)$$
$$\Omega_{bar} = 0.046 \pm 0.003$$

II. DE – through relativistic gravitational field equations in the Einsteinian form

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R\right) = \underbrace{T^\nu_{\mu(vis)} + T^\nu_{\mu(DM)}}_{\text{approximately dust-like}} + T^\nu_{\mu(DE)}$$

$G = G_N = const$ – the Newton gravitational constant measured in laboratory

$$T^\nu_{\mu(DE);\nu} = 0$$ (in the absence of decay of DM particles into DE)

Applications of the definition of $T_{\mu(DE)}^{\nu}$

1. To the FRW background

$$\varepsilon_{DE} \equiv T^{0}_{0(DE)} = \varepsilon_{DE}(z) \qquad z = z(t) = \frac{a_0}{a(t)} - 1$$

$$p_{DE} = -T^{\alpha}_{\alpha(DE)} = p_{DE}(z) \qquad \text{(no summation on } \alpha\text{)}$$

$$\dot{\varepsilon}_{DE} + 3H(\varepsilon_{DE} + p_{DE}) = 0 \qquad H \equiv \frac{\dot{a}}{a}$$

1 function

2. To the evolution of density perturbations in the matter component (baryons + DM) at small scales

$\left(\frac{\delta\rho}{\rho}\right)_m (z)$ – second observational function

Main up-to-date result

In the zero approximation (~10% accuracy)

$$\varepsilon_{DE}(z) = \varepsilon_0 = \text{const} \qquad \Omega_{DE} = 1 - \Omega_m$$

$$p_{DE} = -\varepsilon_0, \quad T^{\nu}_{\mu(DE)} = \varepsilon_0 \delta^{\nu}_{\mu}$$

$$\rho_0 = \frac{\varepsilon_0}{c^2} = 6.44 \cdot 10^{-30} \, \frac{\Omega_{DE}}{0.7} \cdot \left(\frac{H_0}{70}\right)^2 \, \text{g cm}^{-3}$$

$$\frac{G^2 \hbar \varepsilon_0}{c^7} = 1.25 \cdot 10^{-123} \cdot (\ldots\downarrow\ldots)$$

Investigation of dark energy

I. From observations to theory

Reconstruction program (1998)

1) $H(z)$, $\varepsilon_{DE}(z)$
2) $q(z)$, $p_{DE}(z)$, $w_{DE}(z)$
3) $r(z)$, $\dfrac{dw_{DE}(z)}{dz}$

1. Inversion of classical cosmological tests $\mathcal{D}_L(z) \rightarrow H(z)$
2. CMB (acoustic peaks spacing, ISW), BAO
3. $\left(\frac{\delta\rho}{\rho}\right)_m(z)$, $\Phi(z)$ from gravitational lensing, correlation of $\frac{\delta\rho}{\rho}$ with CMB

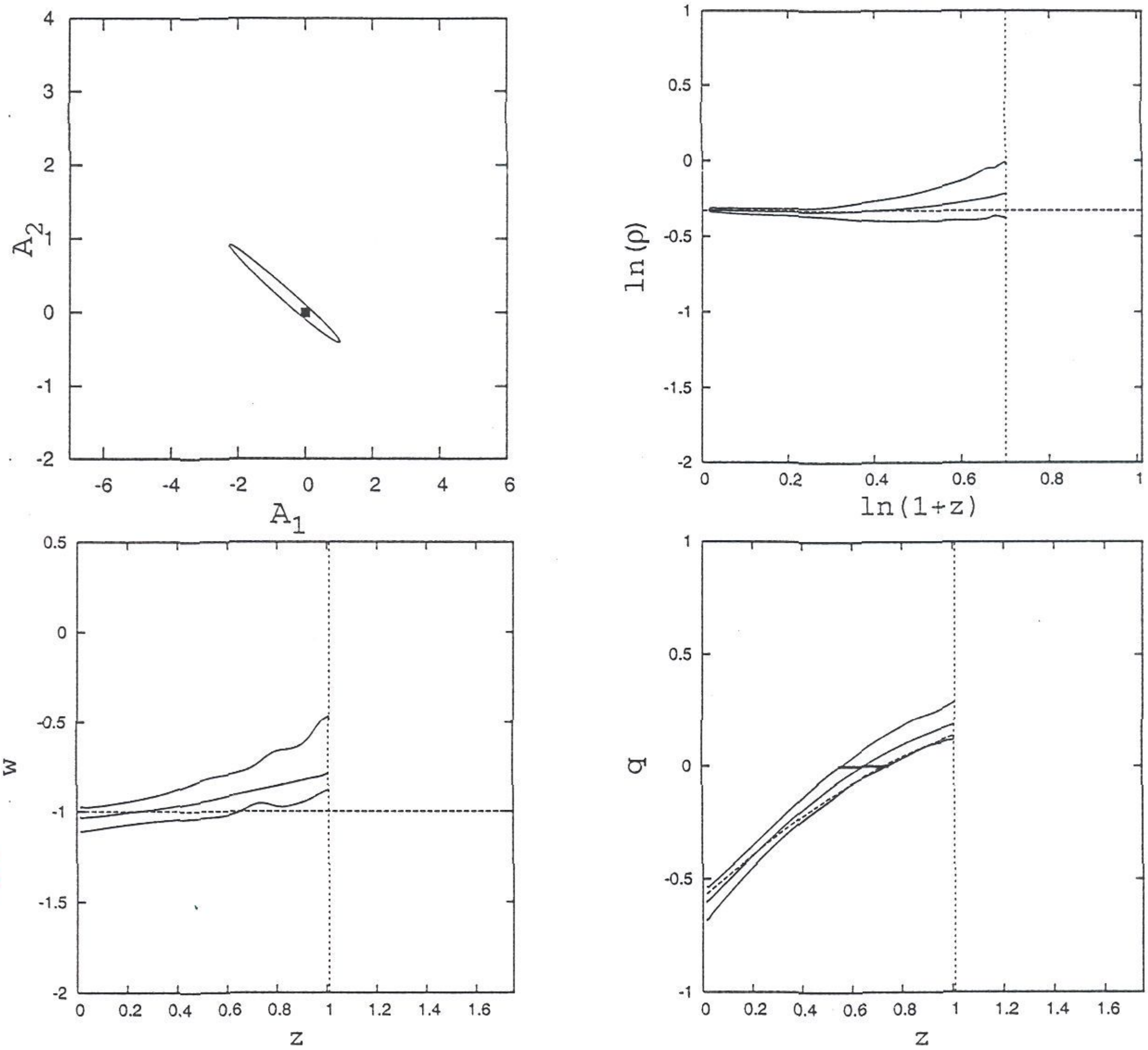
II. From theory to observations

Models (many of them!)

(qualitatively – the same as for inflation)

1. Fundamental constant Λ
2. Scalar field (with $m \sim 10^{-33}$ eV, $w_{DE} \geqslant -1$)
3. Geometrical DE = modified gravity (e.g., scalar-tensor and $f(R)$ DE models)

U. Alam, V. Sahni, A. S., JCAP 0702, 011 (2007)
(astro-ph/0612381)



$w = \frac{p}{\rho}$

FIG. 7: 2σ confidence levels for the SNLS+CMB+BAO dataset using $\Omega_{0m} = 0.28 \pm 0.03$. The upper left hand panel shows the confidence levels in $A_1 - A_2$, with the black dot representing ΛCDM. The upper right hand panel shows the logarithmic 2σ variation of the DE density in terms of redshift. The dashed line represents ΛCDM. The lower left and right hand panels represent the variation of the equation of state and deceleration parameter respectively. The dashed lines in both panels represent ΛCDM. The thick solid line in the lower right hand panel shows the acceleration epoch, i.e. the redshift at which the universe started accelerating. Results are shown upto redshift $z = 1.01$

Fit:

$$\frac{H^2(z)}{H_0^2} = A_0 + A_1(1+z) + A_2(1+z)^2 + \Omega_m(1+z)^3$$

$$A_0 + A_1 + A_2 + \Omega_m = 1$$

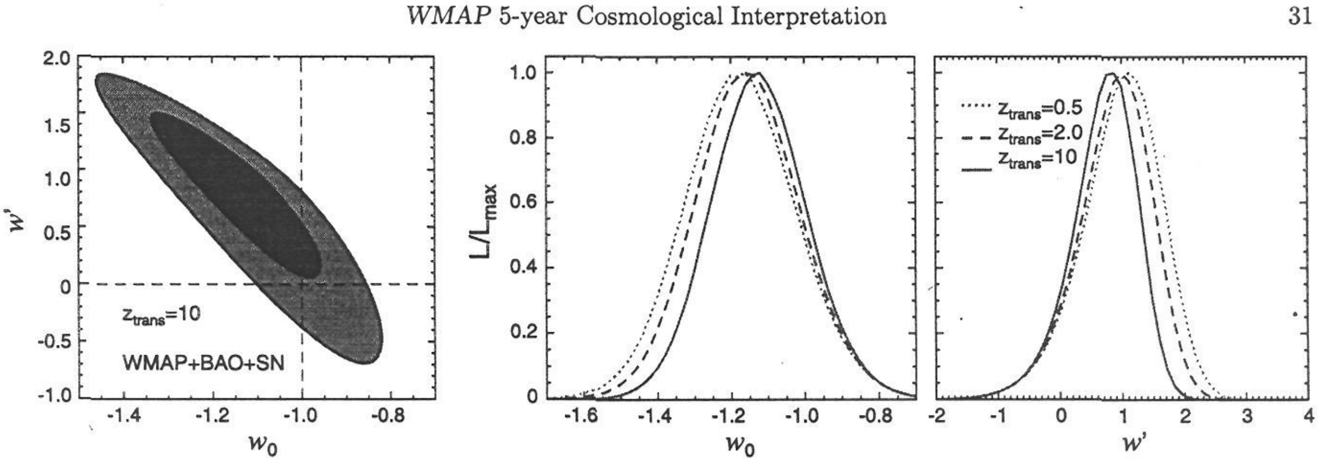


FIG. 14.— Constraint on models of time-dependent dark energy equation of state, $w(z)$ (Eq. [70]), derived from the *WMAP* distance priors (l_A, R, and z_*) combined with the BAO and SN distance data (§ 5.4.2). There are three parameters: w_0 is the value of w at the present epoch, $w_0 \equiv w(z=0)$, w' is the first derivative of w with respect to z at $z=0$, $w' \equiv dw/dz|_{z=0}$, and $z_{\rm trans}$ is the transition redshift above which $w(z)$ approaches to -1. Here, we assume flatness of the universe, $\Omega_k = 0$. (*Left*) Joint two-dimensional marginalized distribution of w_0 and w' for $z_{\rm trans} = 10$. The constraints are similar for the other values of $z_{\rm trans}$. The contours show the $\Delta\chi^2_{\rm total} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\rm total} = 6.17$ (95.4% CL). (*Middle*) One-dimensional marginalized distribution of w_0 for $z_{\rm trans} = 0.5$ (dotted), 2 (dashed), and 10 (solid). (*Middle*) One-dimensional marginalized distribution of w' for $z_{\rm trans} = 0.5$ (dotted), 2 (dashed), and 10 (solid). The constraints are similar for all $z_{\rm trans}$. We do not find evidence for the evolution of dark energy.

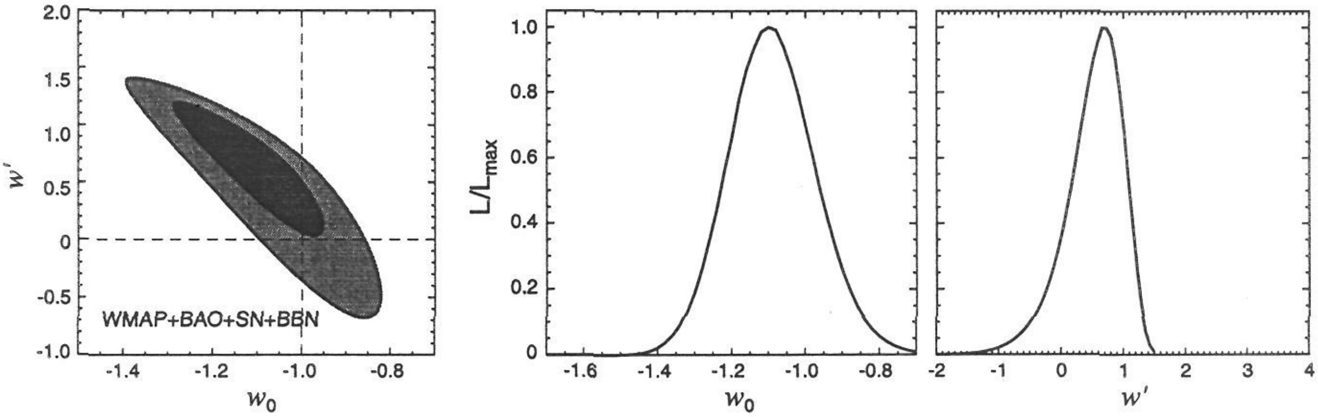


FIG. 15.— Constraint on the linear evolution model of dark energy equation of state, $w(z) = w_0 + w'z/(1+z)$, derived from the *WMAP* distance priors (l_A, R, and z_*) combined with the BAO and SN distance data as well as the Big Bang Nucleosynthesis (BBN) prior (Eq. [71]). Here, we assume flatness of the universe, $\Omega_k = 0$. (*Left*) Joint two-dimensional marginalized distribution of w_0 and w'. The contours show the $\Delta\chi^2_{\rm total} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\rm total} = 6.17$ (95.4% CL). (*Middle*) One-dimensional marginalized distribution of w_0. (*Middle*) One-dimensional marginalized distribution of w'. We do not find evidence for the evolution of dark energy. Note that Linder (2003) defines w' as the derivative of w at $z=1$, whereas we define it as the derivative at $z=0$. They are related by $w'_{\rm linder} = 0.5w'_{WMAP}$.

The 95% limit on w_0 for $z_{\rm trans} = 10$ is $-0.38 < 1 + w_0 < 0.14$[46], whose upper limit is surprisingly close to the limit for a constant w model in a flat universe, $-0.097 < 1 + w < 0.142$ (95% CL). Therefore, we have a fairly robust upper limit on the present-day value of the equation of state, $1 + w_0 < 0.14$. (This statement is, however, true only for a flat universe.) On the other hand, the lower limit has weakened significantly – by a factor of about four. Our results are consistent with the previous work using the *WMAP* 3-year data (see Wang & Mukherjee 2007; Wright 2007, for recent work and references therein). We find that our upper limit on w_0 and lower limit on w' are better than the previous work by a factor of ~ 2.

Alternatively, one may take the linear form, $w(a) = w_0 + (1-a)w_a$, literally and extend it to an arbitrarily high redshift. This can result in an undesirable situation in which the dark energy is as important as the radiation density at the epoch of the Big Bang Nucleosynthesis (BBN); however, one can constrain such a scenario severely using the limit on the expansion rate from BBN (Steigman 2007). We follow Wright (2007) to adopt a Gaussian prior on

$$\sqrt{1 + \frac{\Omega_\Lambda (1+z_{\rm BBN})^{3[1+w_{\rm eff}(z_{BBN})]}}{\Omega_m(1+z_{\rm BBN})^3 + \Omega_r(1+z_{\rm BBN})^4 + \Omega_k(1+z_{\rm BBN})^2}} = 0.942 \pm 0.030, \quad (71)$$

where we have kept Ω_m and Ω_k for definiteness, but they are entirely negligible compared to the radiation density at the redshift of BBN, $z_{\rm BBN} = 10^9$. Figure 15 shows the constraint on w_0 and w' for the linear evolution model derived from the *WMAP* distance priors, the BAO and SN data, and the BBN prior. The 95% limit on w_0 is

[46] The 68% intervals are $w_0 = -1.12 \pm 0.13$ and $w' = 0.70 \pm 0.53$ (WMAP+BAO+SN; $\Omega_k = 0$).

PRESENT STAGE OF DARK ENERGY RECONSTRUCTION

1. In the first approximation, DE is well described by a cosmological constant

$$w_{DE} \simeq -1$$

2. $w_{DE} = -1$ is inside 2σ error bars for all data

3. If $w_{DE} = \text{const} \neq -1$, then

$$|w_{DE} + 1| \lesssim 0.1$$

E.g. W.J. Percival et al., arXiv: 0705.3323

$$w_{DE} = -1.004 \pm 0.089$$

WMAP5 + BAO + SN: $w_{DE} = -0.97 \pm 0.06$

4. No evidence for permanent phantom DE.
No evidence for the Big Rip in future ($\Delta T > 50$ byl.y.)

$a(t) \propto (t_1 - t)^{-p}$, $p > 0$

However

5. SNe: small discrepancy between Gold and SNLS samples

BAO: comparison of $D_V(0.2)$ with $D_V(0.35)$ slightly favors $w_{DE} < -1$ for $z < 0.35$

6. If the assumption $w_{DE} = const$ is omitted, some place for 'temporary phantom' DE still exists for $z \lesssim 0.3$

But $\overline{w}_{DE}\,(0 < z \lesssim 0.5) \simeq -1$

WMAP5 + BAO + SN: $-0.38 < 1 + w(0) < 0.14$

$-0.7 < w'(0) < 1.5$

7. Place for dynamical dark energy (especially, a geometric one) still exists!

Recent review on the reconstruction approach:

V. Sahni, A.A. Starobinsky, IJMPD 15, 2105 (2006) [astro-ph/0610026]

What if recent phantom behaviour ($w<-1$) of dark energy will be confirmed by observations?

Ghost phantom models of dark energy are bad.

1. Quantum instability

2. At the classical level: does not explain homogeneity and isotropy of the Universe

E.g.: for a given $\bar{H} = \frac{1}{3}\frac{d}{dt}\ln abc$, it is much more probable to have very different $\frac{\dot a}{a}, \frac{\dot b}{b}, \frac{\dot c}{c}$ compensated by the negative energy density of the ghost field.

Scalar-tensor models of dark energy do not have this problem.

Reconstruction of dark energy in scalar-tensor gravity

B. Boisseau, G. Esposito-Farese, D. Polarski, A. S.
Phys. Rev. Lett. 85, 2236 (2000)

$\varepsilon_{DE} + p_{DE} < 0$ is permitted

$$\mathcal{L} = \frac{1}{2}\left(F(\varphi)R + Z(\varphi)\varphi_{,\mu}\varphi^{,\mu}\right) - V(\varphi) + \mathcal{L}_m$$

Includes $R + f(R)$ theory for $Z(\varphi) = 0$.

$Z(\varphi) = 1$ $\qquad$ $\omega^{-1}(\varphi) = F^{-1}\left(\frac{dF}{d\varphi}\right)^2$

Two independent observable cosmological functions are required for reconstruction of $F(\varphi)$ and $V(\varphi)$

$$\begin{array}{ccccc} D_L(z), & & \delta(z) & & \\ \downarrow & & \downarrow & & \\ H(z) & \longrightarrow & F(z) & \longrightarrow & V(z) \\ & & & \longrightarrow & \varphi(z) \end{array}$$

Background equations

$$3FH^2 = \rho_m + \frac{\dot\varphi^2}{2} + V - 3H\dot F$$

$$-2F\dot H = \rho_m + \dot\varphi^2 + \ddot F - H\dot F$$

$\rho_m \propto a^{-3}$

Their consequence:

$$\ddot\varphi + 3H\dot\varphi + \frac{dV}{d\varphi} - 3(\dot H + 2H^2)\frac{dF}{d\varphi} = 0$$

In terms of redshift:

$$F'' + \left[(\ln H)' - \frac{4}{1+z}\right]F' + \left[\frac{6}{(1+z)^2} - \frac{2(\ln H)'}{1+z}\right]F$$

$$= \frac{2V}{(1+z)^2 H^2} + 3(1+z)\left(\frac{H_0}{H}\right)^2 F_0\,\Omega_{m,0}$$

$$\varphi'^2 = -F'' - \left[(\ln H)' + \frac{2}{1+z}\right]F' + \frac{2(\ln H)'}{1+z}F$$

$$-3(1+z)\frac{H_0^2}{H^2}F_0\,\Omega_{m,0}$$

I. First step → as in GR

$$H(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}$$

II. Equation for sufficiently small-scale inhomogeneities

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m\,\delta = 0$$

where $\delta \equiv \left(\frac{\delta\rho}{\rho}\right)_{CDM+bar}$ at a fixed comoving scale $\lambda = a(t)/k$ and

$$\frac{k^2}{a^2} \gg \max\left(\frac{d^2V}{d\varphi^2},\ H^2 \cdot \max\left(1, \frac{d^2F}{d\varphi^2}\right)\right);$$

$$G_{eff} = \frac{1}{8\pi F} \cdot \frac{F + 2\left(\frac{dF}{d\varphi}\right)^2}{F + \frac{3}{2}\left(\frac{dF}{d\varphi}\right)^2}$$

From this, excluding d_L:

a second-order differential equation for $F(z)$.

Properties of scalar-tensor models of dark energy

R. Gannouji, D. Polarski, A. Ranquet, A. S.

JCAP 09, 016 (2006) [astro-ph/0606287]

1. Temporary phantom behaviour and crossing of the phantom boundary $w=-1$ are possible for an open set of $F(\varphi)$ and non-zero and non-constant $V(\varphi)$.

   "Curvature induced phantomness"

2. In the absence of dust-like matter ($\Omega_m = 0$), power-law solutions leading to the Big Rip singularity in future and to $w<-1$ exist if

$$F = \alpha \varphi^2 \,, \quad \varphi \to \infty$$

$$V = V_0 |\varphi|^n \,, \quad 2 < n < 4$$

(Barrow & Maeda 1990)

Then $a(t) \propto (t_0 - t)^q$, $\quad q = \dfrac{2\left(n+2+\frac{1}{\alpha}\right)}{(n-2)(n-4)} < 0$

$\varphi(t) \propto (t_0 - t)^r$, $\quad r = \dfrac{2}{2-n} < 0$

However, for these solutions $|w+1| \le \frac{\alpha}{3} \sim \frac{1}{\omega_{BD}}$ and very small.

Present observational bounds:

$$\gamma_{PN} - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$$

Bertotti et al., 2003 → Cassini mission

$$\beta_{PN} - 1 = (0 \pm 1) \cdot 10^{-4}$$

Pitjeva, 2005 → ephemerides of planets

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = (-0.2 \pm 0.5) \cdot 10^{-13} y^{-1}$$

$$\beta_{PN} - 1 = (1.2 \pm 1.1) \cdot 10^{-4}$$

Williams et al., 2005 → lunar laser ranging

$$\omega_{BD,0} \equiv \left(\frac{F}{\left(\frac{dF}{d\varphi}\right)^2} \right)_0 > 4 \cdot 10^4$$

3. Small z expansion.

$$\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + \ldots$$

$$\frac{V(z)}{3 F_0 H_0^2} = \Omega_{V,0} + u_1 z + u_2 z^2 + \ldots$$

$$\frac{H^2(z)}{H_0^2} = 1 + h_1 z + h_2 z^2 + \ldots$$

$$F_0^{-1/2} \varphi'(z) = \varphi_0' + \varphi_1' z + \varphi_2' z^2 + \ldots$$

$$w_{DE}(z) = w_0 + w_1 z + w_2 z^2 + \ldots$$

$$|F_1| < 10^{-2}$$

$$\varphi_0'^2 = 6 (1 - \Omega_{m,0} - \Omega_{V,0} - F_1) \geq 0$$

What is required to get significant phantomness ($|w+1| \gg \frac{1}{\omega_{BD,0}}$) ?

$$F_2 < 0 \,, \quad |F_2| \sim 1 \gg |F_1| \qquad (|F_1| < 10^{-2})$$

$$|F_2| > 3\,(\underbrace{\Omega_{DE,0}}_{1-\Omega_{m,0}} - \Omega_{V,0}) > 0$$

$$w_0 + 1 = \frac{2F_2 + 6(\Omega_{DE,0} - \Omega_{V,0})}{3\Omega_{DE,0}} < 0$$

k_1 can be negative, too

4. Connection with post-Newtonian parameters in the significantly phantom case.

$$\gamma_{PN} - 1 = -\frac{F_1^2}{6(\Omega_{DE,0} - \Omega_{V,0})} < 0$$

$$\beta_{PN} - 1 = -\frac{F_1^2 F_2}{72(\Omega_{DE,0} - \Omega_{V,0})} > 0$$

$$-4 < \frac{\gamma_{PN} - 1}{\beta_{PN} - 1} = \frac{12(\Omega_{DE,0} - \Omega_{V,0})}{F_2} < 0$$

However, $|\gamma_{PN} - 1|$ and $|\beta_{PN} - 1|$ may be much smaller than $|1+w|$ if F_1 is very small

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = H_0 F_1 \left(1 - \frac{F_2}{3(\Omega_{DE,0} - \Omega_{V,0})}\right)$$

Positive detection of $\gamma_{PN} < 1$, $\beta_{PN} > 1$ may be a strong argument for significant phantomness of present DE.

Negative detection tells nothing.

5. Correct asymptotic behaviour for large z ($\varphi'^2 \geq 0$, $w_{DE} \leq 0$) requires non-zero and non-constant $V(\varphi)$

E.g.

$$F(\varphi) \to F_\infty < F_0$$

$$V(\varphi) \propto \exp\left(\sqrt{\frac{3}{2F_0\Omega_{u,\infty}}}\,\varphi\right)$$

$$z, \varphi \to \infty$$

6. In the stable case $F > 0$, $\omega_{BD} > -\frac{3}{2}$, no possibility to construct a stable wormhole (even with an electromagnetic field)

(K. A. Bronnikov & A. S., JETP Lett. 85, 1 (2007) [gr-qc/0612032])

Geometrical $f(R)$ model of DE

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R) \qquad R \equiv R_\mu^\mu$$

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = -8\pi G\left(T_{\mu(m)}^\nu + T_{\mu(DE)}^\nu\right)$$

$$8\pi G\, T_{\mu(DE)}^\nu \equiv F'(R) R_\mu^\nu - \frac{1}{2}F(R)\delta_\mu^\nu$$

$$+ \left(\nabla_\mu \nabla^\nu - \delta_\mu^\nu \nabla_\rho \nabla^\rho\right) F'(R)$$

Particle content: graviton + massive scalar particle $\left(M^2 = \frac{1}{3f''(R)}\right)$ (dubbed "scalaron" in A.S., 1980)

Stability conditions:

1. $f' > 0$ graviton is not a ghost
2. $f'' > 0$ scalaron is not a tachyon

imposed for $R \geq R_{now}$ at least (i.e. during the whole evolution of the Universe)

Violation of these conditions is undesirable from the classical point of view, too!

$f'(R_0) = 0$ – instant loss of homogeneity and isotropy

$f''(R_0) = 0$ – weak singularity

$$R(t) = R_0 + O(\sqrt{t})$$
$$a(t) = a_0 + a_1 t + a_2 t^2 + O(t^{5/2})$$

③ Existense of the Newtonian regime

$$(\Delta \varphi = 4\pi G \rho)$$

$$|F| \ll R, \quad |F'(R)| \ll 1, \quad R|F''(R)| \ll 1$$

for $R_{now} \ll R$ (at up to some very large R)

$$\longleftrightarrow$$

De Sitter regime

$$Rf' = 2f$$

Stable if

$$f'(R_1) > R_1 f''(R_1)$$

$$\longleftrightarrow$$

Equivalent to $\omega_{BD} = 0$ scalar-tensor gravity

Use for inflation

$$f(R) = R + \frac{R^2}{6M^2}$$ (+ small non-local terms)

AS, 1980

Internally self-consistent inflationary model with slow-roll decay, a graceful exit to the subsequent RD FRW stage (through an intermediate matter-dominated stage) and sufficiently effective reheating

$\tau \sim M_{Pl}^2/M^3$ $\quad N \sim 50$

Remains viable

$$M = 3.0 \times 10^{-6} (N/50)^{-1} M_{Pl}$$

$$n_s = 1 - \frac{2}{N} = 0.96 \quad \text{for } N=50$$

$$r = \frac{12}{N^2} = 4.8 \times 10^{-3} (N/50)^{-2}$$

Eksp.: $\overline{n_s} = 0.96 \pm 0.014$, $r < 0.20$

Use for DE

$F(R) \propto R^{-n}$ for $R \to 0$

Does not work for many reasons

Viable model – regular at $R=0$

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1+\frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

AS, JETP Lett. 86, 157 (2007)
arXiv: 0706.2041 [astro-ph]

or even

$$f(R) = R - \lambda R_0 \tanh^2 \frac{R}{R_0}$$

$f(0) = 0$ – 'disappearing' cosmological constant in flat space-time

Induced Λ at high curvatures:

$$\Lambda_\infty \equiv -\frac{1}{2} F(\infty) = \frac{\lambda R_0}{2}$$

Observational restrictions

1. Cosmology

Anomalous growth of non-relativistic matter perturbations in the regime

$k \gg M(R)\,a$

$G_{eff} = 4G/3f'(R) \simeq \frac{4G}{3}$

$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}}$ (instead of $\propto t^{2/3}$)

Results in the apparent mismatch

$$\Delta n_s = n_s^{(gal)} - n_s^{(CMB)} = \frac{\sqrt{33}-5}{2(3n+2)}$$

$\Delta n_s < 0.05 \longrightarrow n \geq 2$

$n = 2$ ($F(R) \propto R^{-4}$) $\Rightarrow$ increase of σ_8 from 0.75 to 0.9

2. Laboratory and Solar system tests

$M(R)\,L \gg 1$ with $R = 8\pi G T_m = 8\pi G \rho_m$

Otherwise, $\gamma_{PN} = \frac{1}{2}$ and the 'fifth' force appears

$M(R(\rho_m)) \propto \rho_m^{n+1}$ $\qquad (R = 8\pi G \rho_m)$

$n \geq 2$ is sufficient for all tests

Structure of corrections at the matter-dominated and earlier stages

$$R = R^{(0)} + \delta R_{ind} + \delta R_{osc}$$

$$R^{(0)} = 8\pi G T_m \propto a^{-3}$$

$$\delta R_{ind} = \left(R F'(R) - 2F(R) - 3 \nabla_\mu \nabla^\mu F'(R) \right)_{R=R^{(0)}}$$

$R \gg R_0$: $\delta R_{ind} \simeq const = -2F(\infty) = 4\Lambda(\infty)$

No Dolgov-Kawasaki instability

$$\delta R_{osc} \propto \begin{cases} t^{-3n-4} \sin\left(const \cdot t^{-2n-1}\right) & \text{MD} \\ t^{-\frac{3n}{4}-3} \sin\left(const \cdot t^{-(3n+1)/2}\right) & \text{RD} \end{cases}$$

$\frac{\delta a}{a}$ is small but $\frac{\delta R_{osc}}{R^{(0)}}$ diverges for $t \to 0$

δR_{osc} should be very small just from beginning – problem for those $F(R)$ models which do not let R become negative (due to the crossing of the $F''(R) = 0$ point)

"Scalaron overproduction" problem

"Big Boost" SINGULARITY WITH $R \to \infty$ AND ITS ELIMINATION

If $F(R) \to 0$ at $R \to \infty$, then a new generic "Big Boost" singularity can arise:

$F(R) \propto R^{-2n}$; $F''(\infty) = 0$

$a = a_0 + a_1(t - t_0) + a_2|t - t_0|^k$, $1 < k = \frac{2n+1}{n+1} < 2$

$R \propto |t - t_0|^{k-2} > 0$

Elimination:

add $\frac{R^2}{6M^2}$ to $F(R)$ $\qquad M^2(\infty) = M$

Additional advantages:

1. No unlimited growth of $M(R)$
2. A toy "UV-completion" – further radiative corrections are logarithmic only
3. A possibility to unify inflation and present dark energy in one $F(R)$ model if $M = 3 \cdot 10^{-6} M_{Pl}$

$\tau_{dec} \sim \frac{M_{Pl}^2}{M^3} \longrightarrow$ $M > 10^4$ GeV for scalarons to decay before BBN

CONCLUSIONS FOR $f(R)$ MODELS

1. With a regular $f(R)$ satisfying

$f'(R) > 0$, $f''(R) > 0$ for all R

$|f - R| \ll R$, $|f' - 1| \ll 1$, $R|f''| \ll 1$ for $R_0 \ll R \ll M^2$ with $M \gtrsim 10^4$ GeV

$f(R) \propto R^2$ for $R \to \infty$,

it is possible to construct viable models of DE, satisfying all existing cosmological, Solar system and laboratory data, and distinguishable from ΛCDM

2. Further unification of primordial DE (producing inflation) and present DE is possible for the specific choice of M: $M \approx 3 \cdot 10^{-6} M_{Pl}$

3. The most critical test of these DE models: anomalous growth of scalar perturbations at recent time ($z \sim 1-3$ for $L = 8h^{-1}$ Mpc)

CONCLUSIONS

1. Deviation of dynamical DE from an exact cosmological constant is $\lesssim 10\%$, but still may exist

2. The simplest DE model which can accomodate its possible recent phantom behaviour and crossing of the "phantom boundary" $w_{DE} = -1$ is based on scalar-tensor gravity and does not have ghosts or instabilities

3. Viable models in $f(R)$ gravity, though more restricted, are possible, too

4. However close the present DE may be to Λ, simply by analogy with primordial DE, one should <u>not</u> expect it to be absolutely stable and eternal