Endpoint spectra of tritium and rhenium beta decays for massive neutrinos

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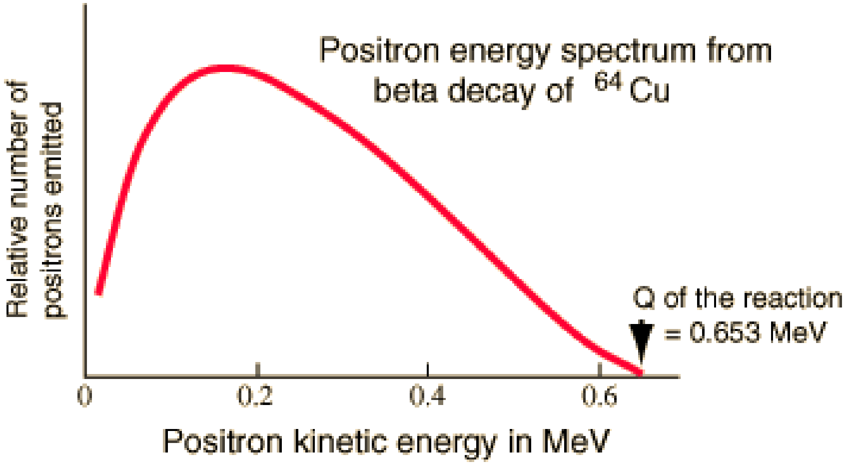
Neutrino

Neutrino was suggested in y. 1930 by Pauli to explain the continuity of β spectrum as a spin 1/2 particle obeying Fermi-Dirac statistics

I have done a terrible thing I invented a particle that cannot be detected

W. Pauli



4th December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^{6} nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Tübingen

Neutrino oscillations

Pontecorvo -Maki-Nakagawa-Sakata matrix

Бруно Понтекорво

Zh.Eksp.Teor .Fiz.,32(1957)

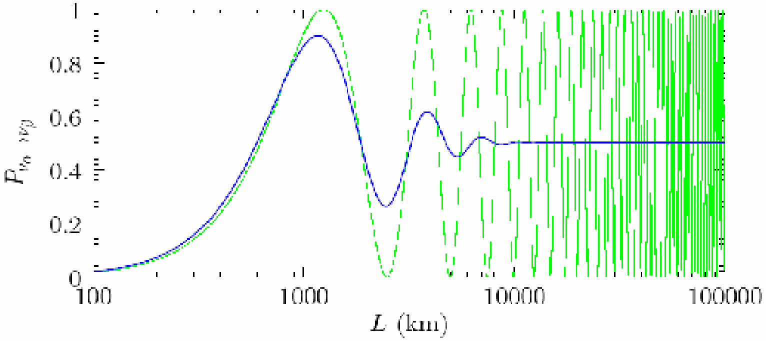
Maki,Nakagawa,Sakata. Prog.Theor.Phys.28(1962)870

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Flavor eigenstates

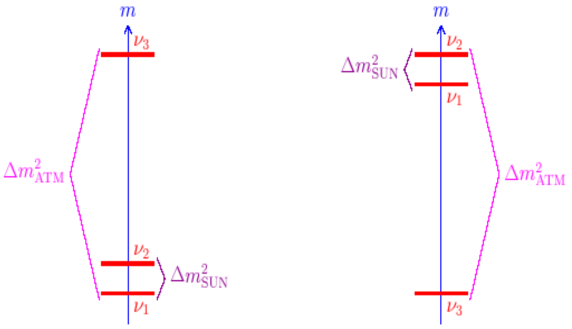
Mass eigenstates

oscillations ⇒ massive neutrinos



$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\vartheta \sin^2 (\frac{\Delta m^2}{4Et})$$

Absolute mass scale of neutrinos ?



"normal" "inverted"

0νββ-decay $m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \; m_i$

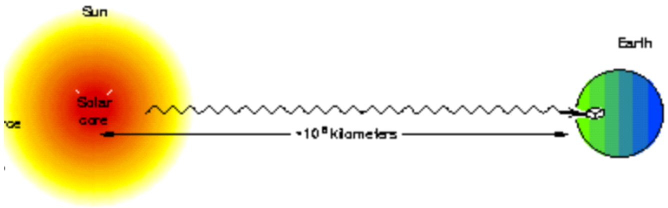
^{3}H decay $m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \; m_i^2}$

Cosmology $\sum_{i=1}^{3} m_i$

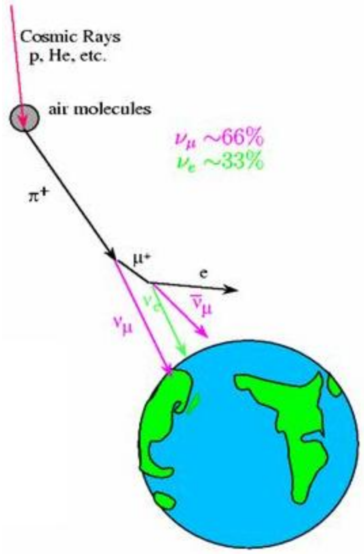
We need 3 mass eigenstates
To explain 2 different Δm^2

$m_2^2 - m_1^2 = \Delta m_{sol}^2 \approx 3.10^{-5}\ \mathrm{eV}^2$
$m_3^2 - m_2^2 = \Delta m_{atm}^2 \approx 2.10^{-3}\ \mathrm{eV}^2$

Solar neutrinos



Atmospheric neutrinos



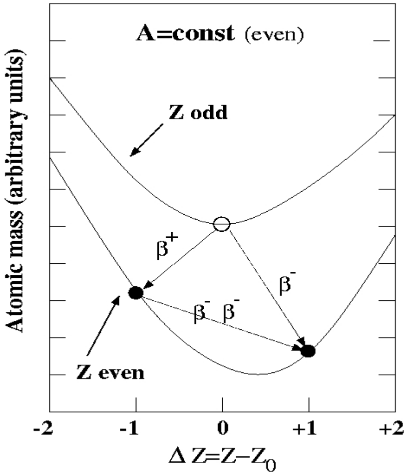
1968
Homestake

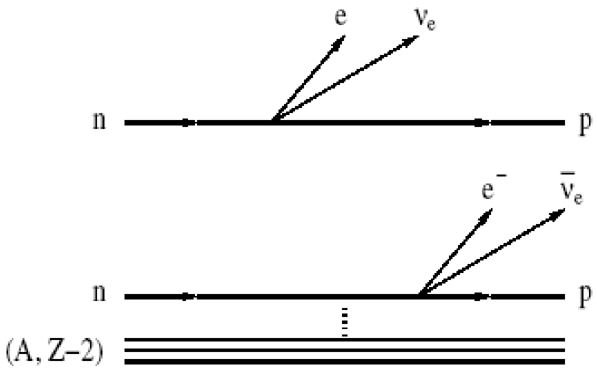
1998
SuperKamiokande

Double beta decay

Enrico Fermi
1934

$$H_\beta = \frac{G_\beta}{\sqrt{2}}\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)j_\mu(x)+h.c.$$





Maria Göppert
Mayer 1935

$$(Z-2, A) \rightarrow (Z, A) + 2e^- + 2\tilde{\nu}_e$$

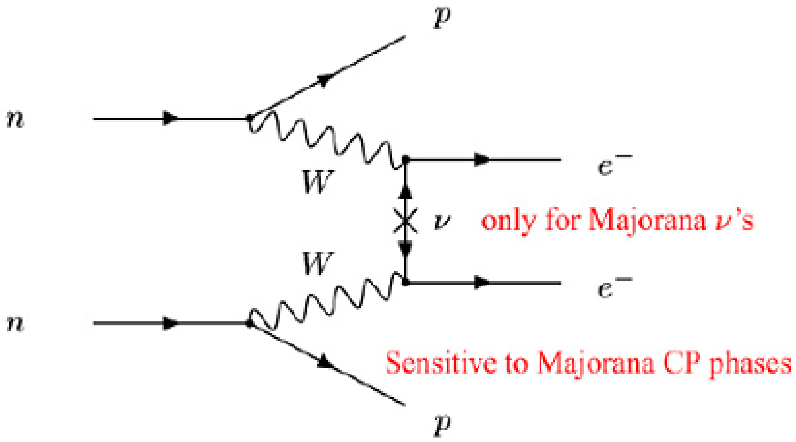
$$(T^{2\nu}_{1/2})^{-1} = G^{2\nu}|M^{2\nu}_{GT}|^2$$

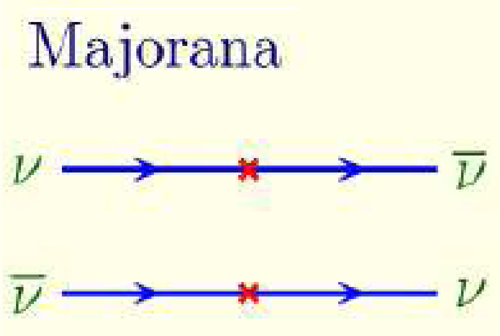
Observed for 10 isotopes: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo. ^{116}Cd. ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, $T_{1/2}\approx 10^{18}$-10^{24} years

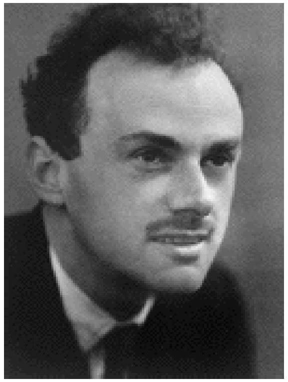
0νββ decay

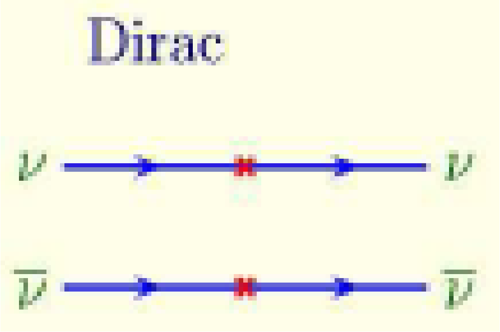
$$|\Delta L| = 2$$

$$(Z-2, A) \rightarrow (Z, A) + 2e^{-}$$









neutrino origin

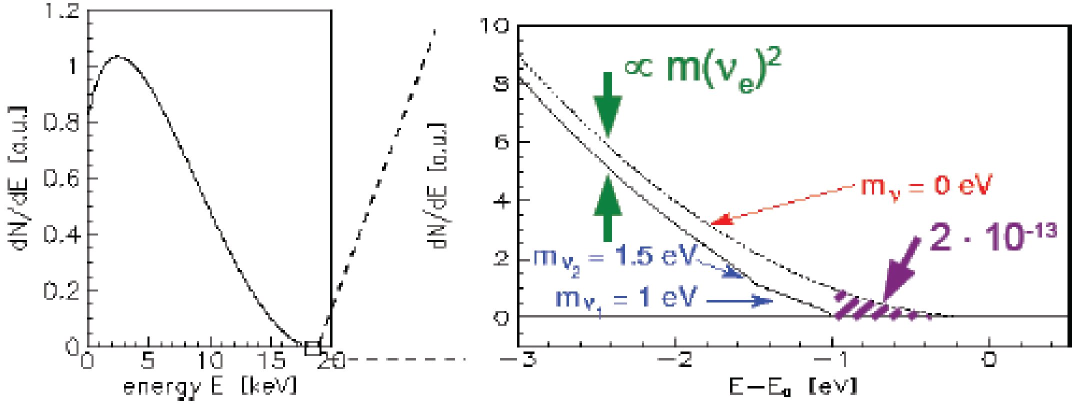
$$\Gamma_{0\nu\beta\beta} \approx |\langle m \rangle|^2 \qquad |\langle m \rangle| = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3 \right|$$

Tritium – ^3H

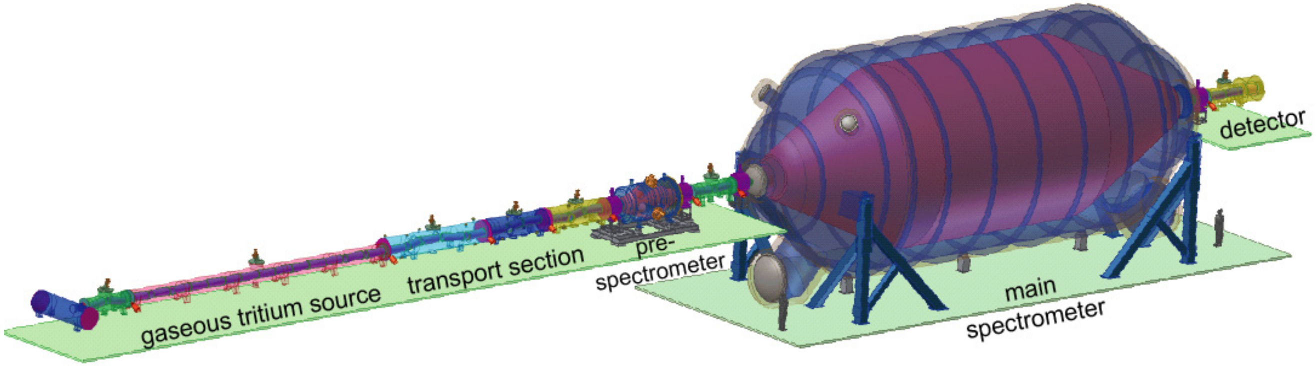
1. low endpoint – Q=18.6 keV
2. super-allowed nuclear transition (spectrum shape)
3. short half-live $T_{1/2} = 12.32$ y
4. simple molecular structure

KATRIN experiment

Direct measurement of neutrino mass



Measuring last
300 eV of endpoint



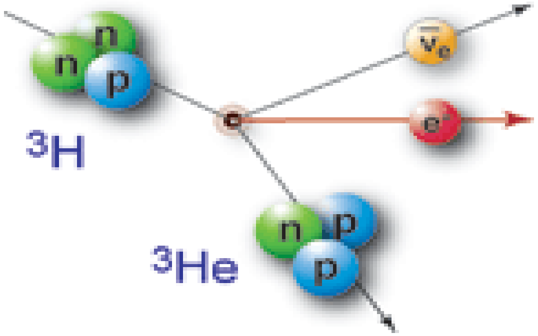
KArlsruhe TRItium Neutrino experiment

2010 – start
data taking

$$m_\beta = \sqrt{\Sigma_{i=1}^{3} |U_{ei}|^2 m_i^2} < 0.2\ eV$$

Weinheimer, Nucl.Phys. Proc.Suppl. **168**,5(2007)

Tritium beta decay



$$^3H \rightarrow {}^3He + e^- + \tilde{\nu}$$

Q=18.6 keV

Differential spectrum

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |M|^2 F(E) pE (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |M|^2 F(E) pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$

Both – Fermi & Gamow-Teller transitions ($T_{1/2}$=12.32 y)

$$|M|^2 = g_V^2 |M_F|^2 + g_A^2 |M_{GT}|^2$$

Tritium beta decay

Spin – isospin properties are identical

$$^{3}H \rightarrow {}^{3}He + e^{-} + \tilde{\nu}$$

$$1/2^{+} \rightarrow 1/2^{+}$$

$$n \rightarrow p + e^{-} + \tilde{\nu}$$

Elementary particle treatment (Kim & Primakoff,
Phys.Rev. 139, B 1447(1965))

Exact relativistic treatment of tritium β decay. Recoil effect taken into account (3.4 eV less than standard Q value)

$$E_e^{\max} = \frac{1}{2M_f}\left[M_i^2 + m_e^2 - \left(M_f^2 - m_\nu^2\right)\right]$$

$$E_e^{\max} \cong M_i - M_f - m_\nu$$

Nucleus as elementary particle

Beta decay on free nucleons.

Relativistic calculation.

No nuclear matrix elements.

Šimkovic, Dvornický, Faessler:

Phys. Rev. C77,055502(2008)

$$
\begin{aligned}
\frac{d\Gamma}{dE_e} &= \frac{1}{(\pi)^3}(G_F\cos\theta_c)^2 F(Z,E_e)p_e \\
&\times\frac{M_i^2}{(m_{12})^2}\sqrt{y\left(y+2m_\nu\frac{M_f}{M_i}\right)} \\
&\times\Bigg[(g_V+g_A)^2 y\left(y+m_\nu\frac{M_f}{M_i}\right)\frac{M_i^2(E_e^2-m_e^2)}{3(m_{12})^4} \\
&\quad (g_V+g_A)^2(y+m_\nu\frac{M_f+m_\nu}{M_i})\frac{(M_iE_e-m_e^2)}{m_{12}^2} \\
&\qquad\qquad \times(y+M_f\frac{M_f+m_\nu}{M_i})\frac{(M_i^2-M_iE_e)}{m_{12}^2} \\
&\quad -(g_V^2-g_A^2)M_f\left(y+m_\nu\frac{(M_f+M_\nu)}{M_i}\right) \\
&\qquad\qquad \times\frac{(M_iE_e-m_e^2)}{(m_{12})^2} \\
&\quad +(g_V-g_A)^2E_e\left(y+m_\nu\frac{M_f}{M_i}\right)\Bigg]
\end{aligned}
$$

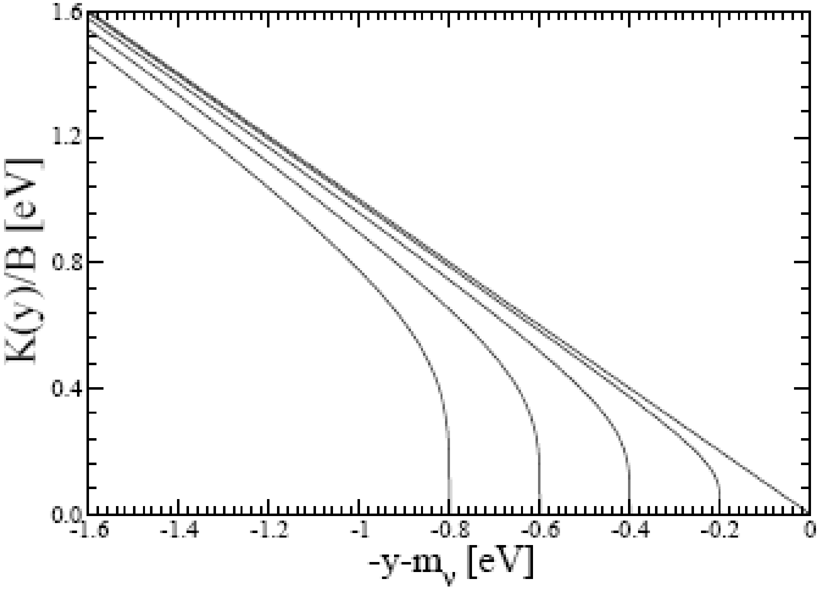
$$
\begin{aligned}
y &= E_e^{max}-E_e \\
(m_{12})^2 &= M_i^2-2M_iE_e+m_e^2
\end{aligned}
$$

Tritium beta decay

Approximation with keeping dominant terms

$$\frac{d\Gamma}{dE_e} \simeq \frac{1}{2\pi^3}(G_F V_{ud})^2 F(Z,E_e) p_e E_e (g_V^2 + 3g_A^2) \times \sqrt{y\,(y+2m_\nu)}\,(y+m_\nu)\,.$$

Relativistic Kurie plot



$$K(y) = B_T\left(\sqrt{y(y+2m_\nu)}(y+m_\nu)\right)^{1/2}$$

$$B_T = \frac{G_F V_{ud}}{\sqrt{2\pi^3}}\sqrt{g_V^2 + 3g_A^2}$$

$$T_{1/2}{}^{\exp} = 12.32y \Rightarrow g_A = 1.247$$

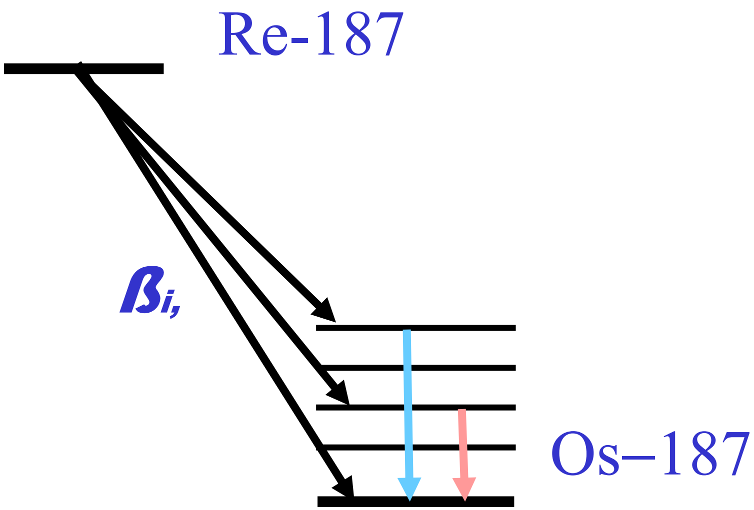
Rhenium beta decay

1. the lowest known Q value 2.47 keV
2. $T_{1/2}$= 4.35 x 10^{10} y ~ age of universe
3. natural abundance ^{187}Re is 63%

MARE experiment

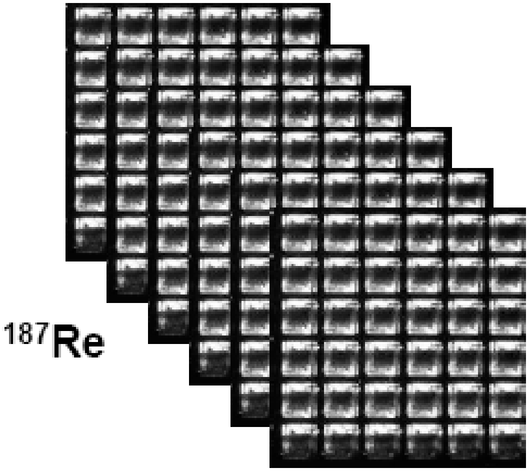
calorimeter
source=detector

-we don’t bother with energy loses
-no corrections for atomic or molecular structure

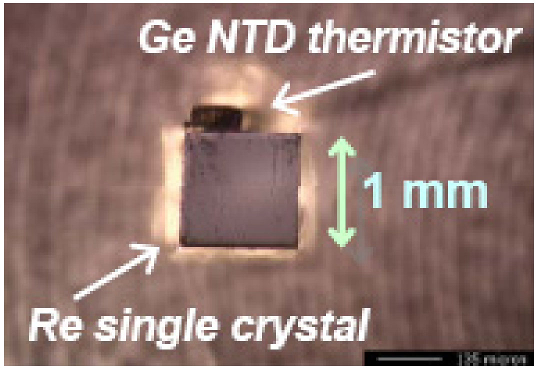


$T_{1/2}$=4.35 × 10^{10} y - low radioactivity

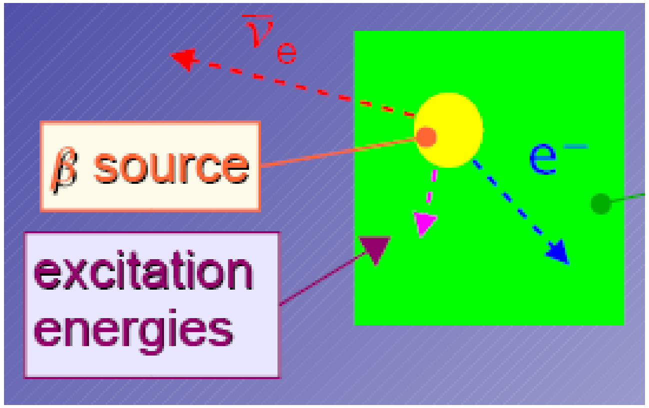
calorimeter
source=detector



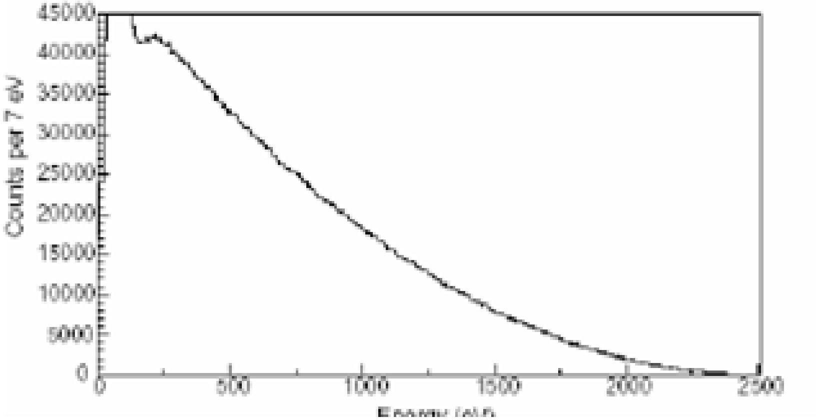
Microcalorimeter
Arrays for a
Rhenium
Experiment



Rhenium beta decay



M. Galeazzi et al., Phys. Rev. C 63, 014302 (2001)
F. Gatti, Nucl. Phys. B 91, 293 (2001)



whole spectrum is measured

^{187}Re $\rightarrow ^{187}$Os$+e^-+\nu_e$ $\Delta J^\pi = 2^-$ first unique transition

$\mathbf{5/2^+ \rightarrow 1/2^-}$ higher partial waves of leptons

Rhenium beta decay

$\Delta J^{\pi} = 2^{-}$ => p-waves have to be taken into account

$$H_{\beta} = \frac{G_{\beta}}{\sqrt{2}} \overline{\psi}_{e}(x) \gamma^{\mu} (1 - \gamma_{5}) \psi_{\nu}(x) j_{\mu}(x) + h.c.$$

$$\psi_{\nu}(x) = (1 + i\vec{k}.\vec{r}) v(k)$$

plane wave expansion - $e^{i\vec{k}.\vec{r}}$

s wave p wave

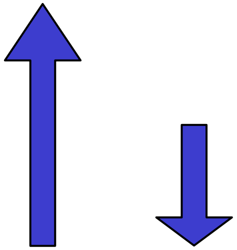
$$\overline{\psi}_{e}(x) = \overline{u}(p) \left(\sqrt{F_{0}(Z,E)} (1 - i \frac{\alpha Z}{2} \gamma^{0} \vec{\gamma}.\hat{\vec{r}}) - i \sqrt{F_{1}(Z,E)} (\vec{p}.\vec{r} + \frac{1}{3} \vec{\gamma}.\vec{p} \vec{\gamma}.\vec{r}) \right)$$

$s_{1/2}$ wave $p_{1/2}$ wave $p_{3/2}$ wave

Rhenium beta decay

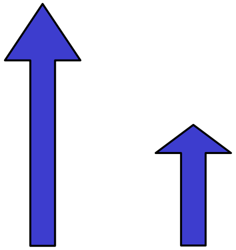
$\Delta J^{\pi} = 2^{-}$ => leptons have to take $\Delta L = 2$

Amplitude = e ($s_{1/2}$) & ν(p) + e($p_{3/2}$) & ν(s)



$$1\hbar - 1/2\hbar$$

+



$$1\hbar + 1/2\hbar$$

e ($p_{1/2}$) & ν(p) => no change of parity

Rhenium beta decay

first unique transition

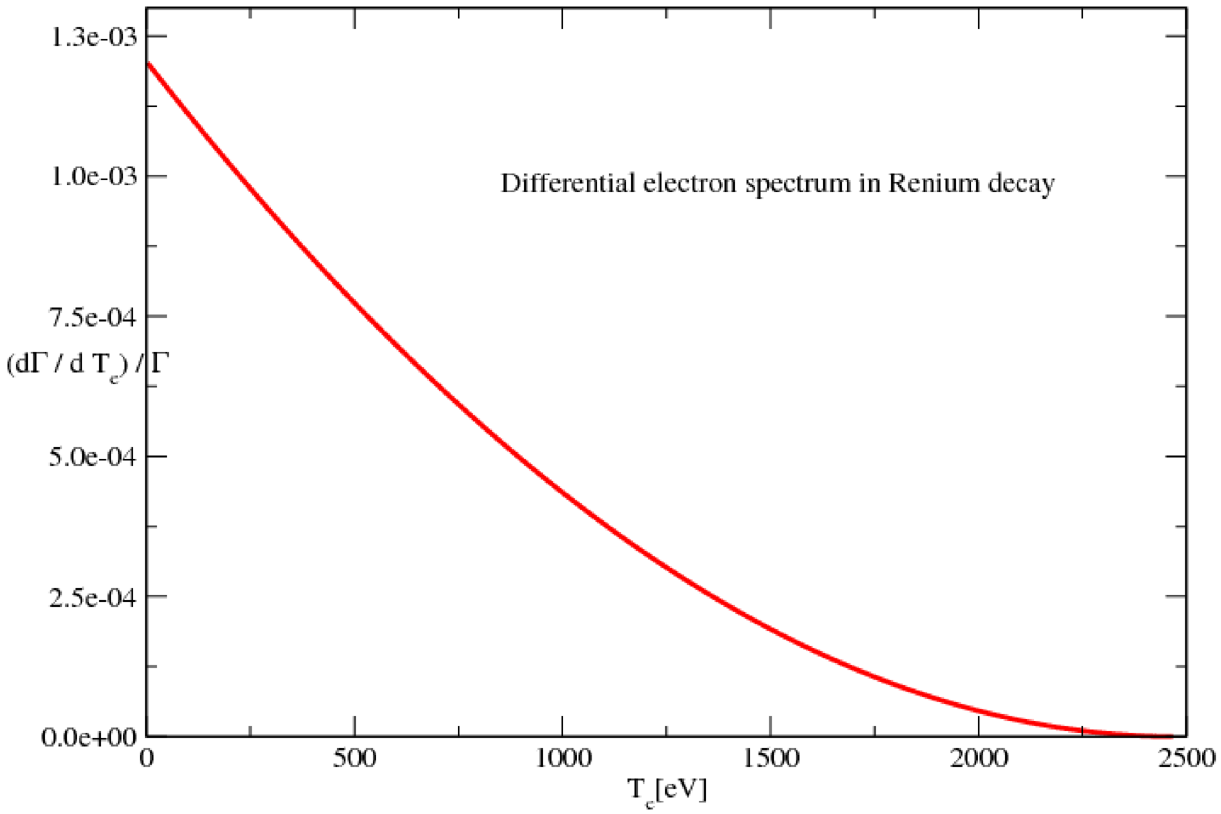
$$\frac{d\Gamma}{dE}=\frac{G_F^2V_{ud}^2}{2\pi^3}\left|M\right|^2 pE(E_0-E)\sqrt{(E_0-E)^2-m_\nu^2}$$
$$\times\frac{1}{3}R^2\left(p^2F_1(Z,E)+k^2F_0(Z,E)\right)$$

for allowed trans. only F_0 survives = s waves of both leptons

$$\left|M\right|^2=\frac{g_A^2}{2J_i+1}\left|<^{187}Os\,||\sqrt{\frac{4\pi}{3}}\sum_n\tau_n^+\frac{r_n}{R}(\sigma_n\otimes Y_n)_2\,||^{187}\mathrm{Re}>\right|^2$$

Rhenium beta decay

electron kinetic energy spectrum normalized to unity

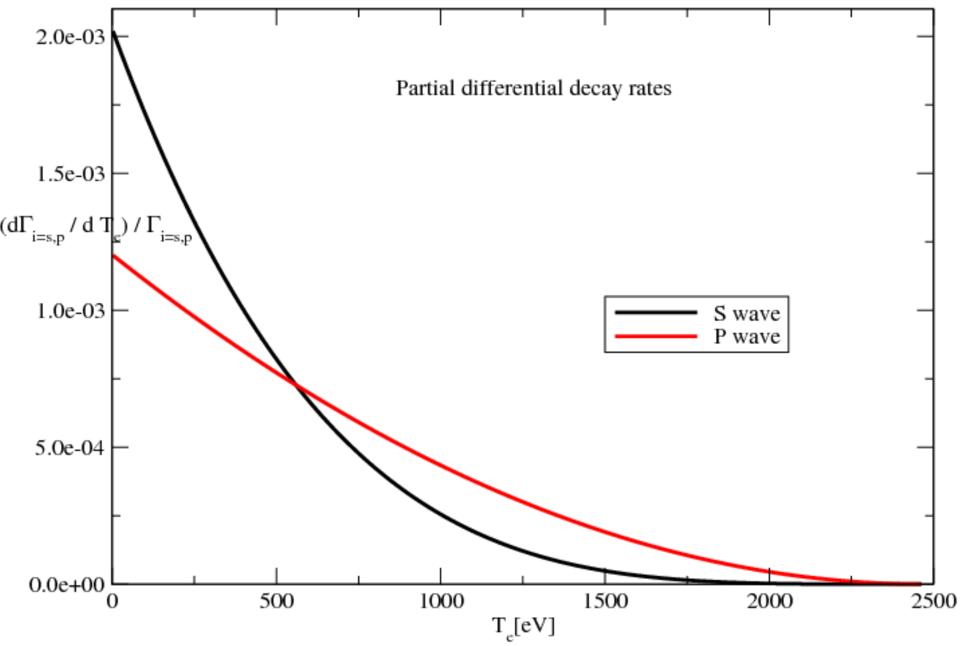


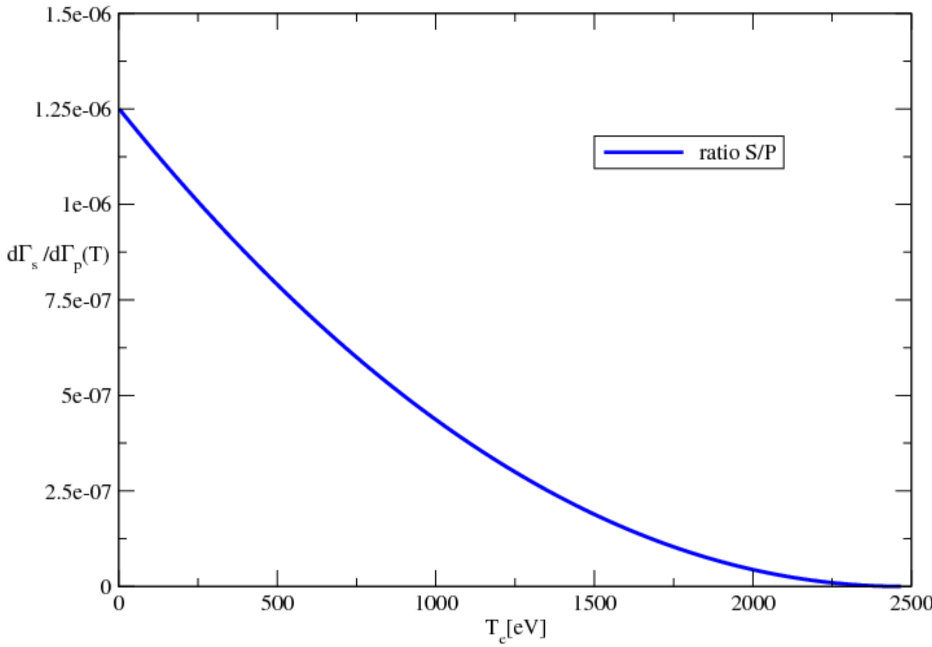
$$T_{1/2} = 4.35 * 10^{10}\, y \Rightarrow \left| < J_f \, \| \sqrt{\frac{4\pi}{3}} \sum_n \frac{r_n}{R} (\sigma_n \otimes Y_n)_2 \, \| \, J_i > \right| = 0.0523$$

Rhenium beta decay

$$\frac{d\Gamma}{dE}=\frac{G_F^2 V_{ud}^2}{6\pi^3}\left|M\right|^2 R^2 pE(E_0-E)\sqrt{(E_0-E)^2-m_\nu^2}\,\left(p^2F_1(Z,E)+k^2F_0(Z,E)\right)$$

$$\frac{d\Gamma}{dE}=\frac{d\Gamma_P}{dE}+\frac{d\Gamma_S}{dE}\qquad \Gamma_S/\Gamma_P=1.027\times10^{-4}$$





electron P wave is dominant => important

Rhenium beta decay

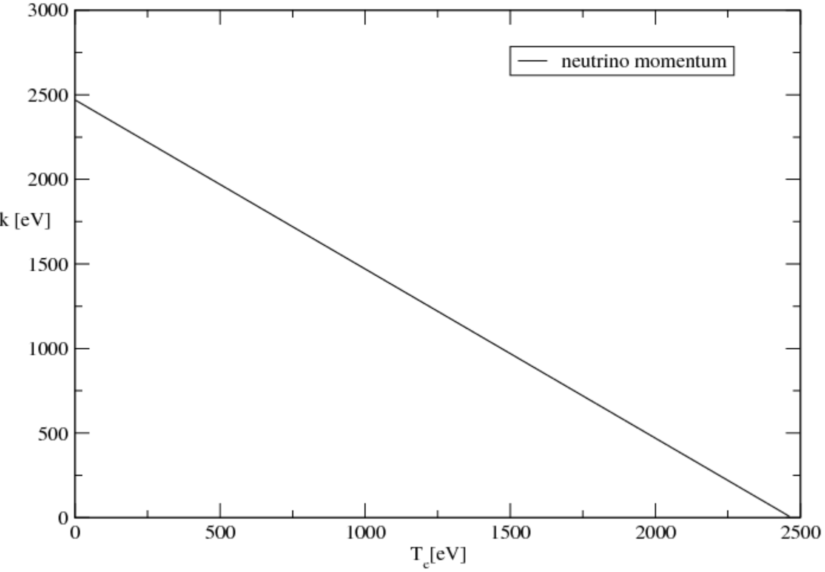
plane wave limit => neglecting the Coulomb interaction

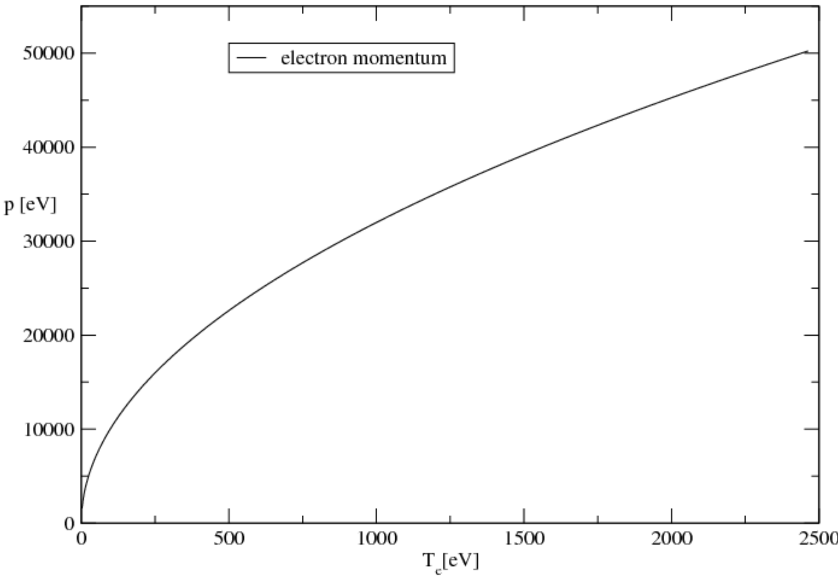
$$F_0(Z,E) \to 1$$

$$F_1(Z,E) \to 1$$

$$k^{\max} = 2.47 keV$$

$$p^{\max} \cong 50 keV$$





kinematics is enhancing the P wave

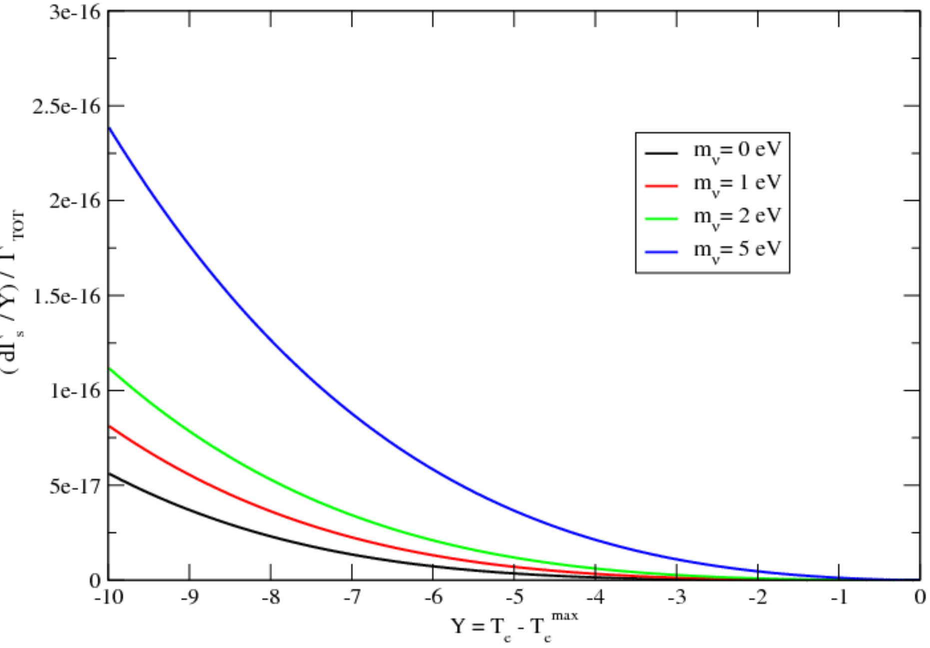
Rhenium beta decay

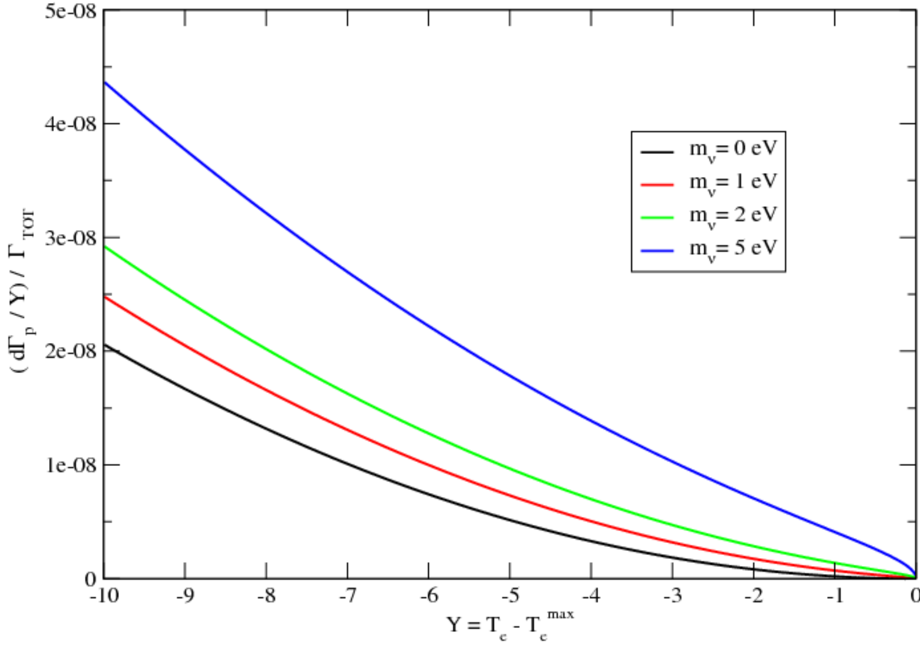
$$T_e^{\max} = Q - m_\nu$$

$$Y = T_e - T_e^{\max}$$

Q - highest electron kinetic energy to obtain in case of zero neutrino mass

S and P wave contributions near the endpoint





Rhenium beta decay

The goal is that we can define (similar as for tritium)

$$B_{Re} = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \frac{g_A}{\sqrt{2J_i+1}} \left| < J_f || \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_n \otimes Y_n\}_2 || J_i > \right|$$

$$\times \sqrt{\frac{1}{3} R^2 \left(p_e^2 \frac{F_1(Z,E_e)}{F_0(Z,E_e)} + ((E_0 - E_e)^2 - m_\nu^2) \right)}$$

$k^2 = (E_0 - E_e)^2 - m_\nu^2$ - negligible

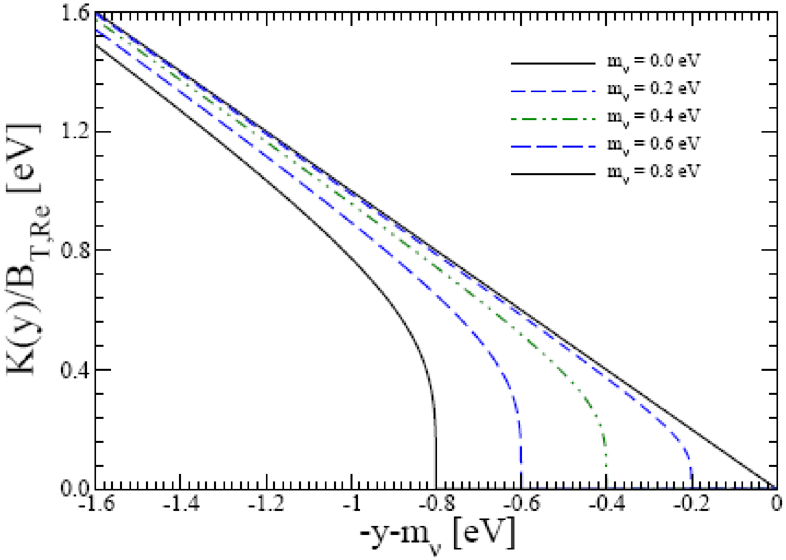
$p^2 \frac{F_1(Z,E_e)}{F_0(Z,E_e)}$ - almost constant due to small Q value in comparison with m_e

Rhenium beta decay

Kurie plot properly scaled is the same for ^{3}H & ^{187}Re

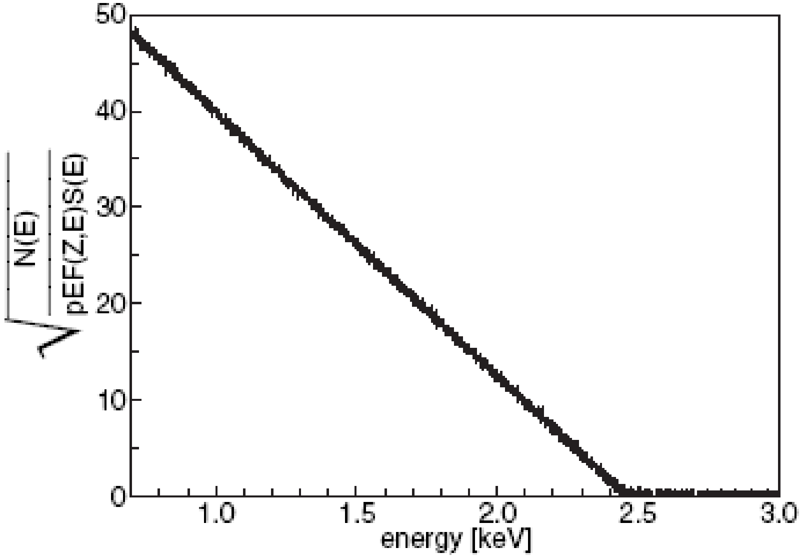
$$K(E_e)/B_{\mathrm{Re}} \cong (E_0 - E_e)\sqrt[4]{1 - \frac{m_\nu^2}{(E_0 - E_e)^2}}$$

theory



$$y = E_e^{\max} - E_e$$

experiment



Arnaboldi, PRL **96**, 042503 (2006)

Conclusions

- Exact relativistic treatment of ^{3}H beta decay including recoil
- Dominance of P wave of electron in ^{187}Re first unique decay
- Linearity of Kurie plot in ^{187}Re decay under discussion with Milano group