Propagation of neutrinos in rapidly rotating neutron stars*

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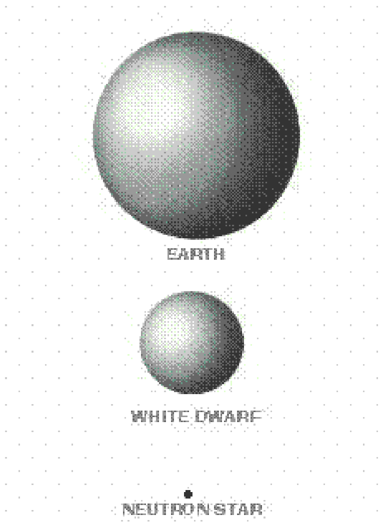
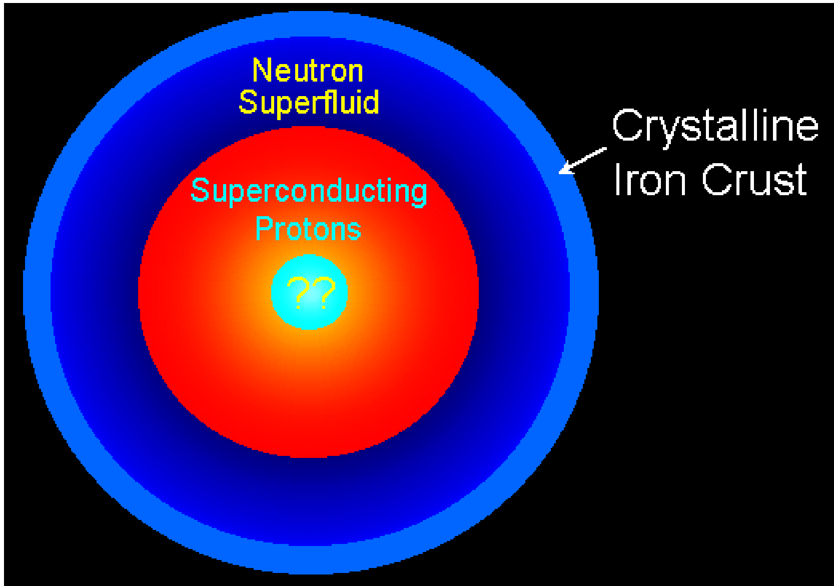
Neutron stars: an overview

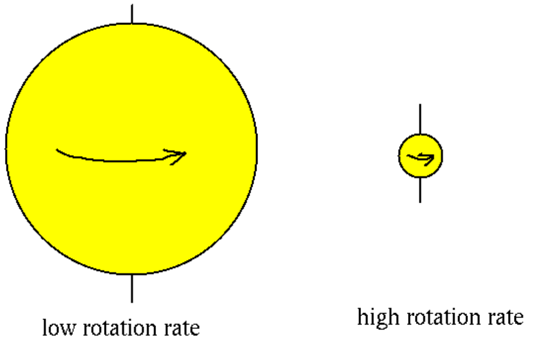
- If a degenerate core (or white dwarf) exceeds the Chandrasekhar mass limit ($\mathbf{1.4}M_\odot$) it must collapse until neutron degeneracy pressure takes over.
- Typical parameters: $M \sim$ several $M_\odot$; $R \sim \mathbf{10\ km}$; $\rho \sim \rho_{\mathbf{nuclear}}$; B up to $\mathbf{10^{15}\ G}$
- These characteristics convert a neutron star a perfect “laboratory” for neutrino physics
[G. G. Raffelt,
Stars as Laboratories for Fundamental Physics, 1996]





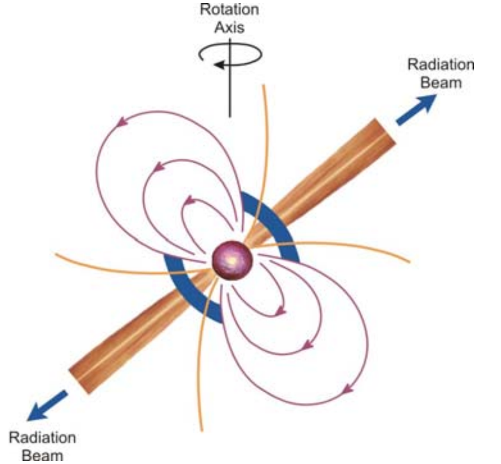
Rotation of a neutron star

- Conservation of angular momentum led to the prediction that neutron stars must be rotating very rapidly.
- The minimum period, ***P***, (or Keplerian angular velocity) of a star is that for which the surface layers are “in orbit”
- The actual angular velocities of radio pulsars are typically smaller **~10^2-10^3 s^{-1}**



$$\frac{v_{\mathrm{rot}}^2}{R} < \frac{GM}{R^2}$$

$$\frac{4\pi^2 R}{P} < \frac{GM}{R^2}$$



Dirac equation for neutrino mass eigenstates in rotating medium

$$\mathcal{L} = \sum_{\lambda=\alpha,\beta} \bar{\nu}_\lambda \left(i\gamma^\mu \partial_\mu - f_\lambda^\mu \gamma_\mu P_\mathrm{L} \right) \nu_\lambda$$

$$- \sum_{\lambda\lambda'=\alpha,\beta} m_{\lambda\lambda'} \bar{\nu}_{\lambda'} \nu_\lambda, \quad \nu_\lambda(\mathbf{r}, t=0) = \nu_\lambda^{(0)}(\mathbf{r})$$

$$\nu_\lambda = \sum_{a=1,2} U_{\lambda a} \psi_a, \quad (U_{\lambda a}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathrm{i}\frac{\partial \psi_a}{\partial t} = \left(\boldsymbol{\alpha}\mathbf{p} + \beta m_a + (\beta g_{aa}^0 - \boldsymbol{\alpha}\mathbf{g}_{aa}) P_\mathrm{L} \right) \psi_a$$

$$+(\beta g_{ab}^0 - \boldsymbol{\alpha}\mathbf{g}_{ab}) P_\mathrm{L} \psi_b, \quad a, b = 1, 2$$

$$(g_{ab}^\mu) = \frac{G_\mathrm{F}}{\sqrt{2}} \begin{pmatrix} (2 j_e^\mu \sin^2\theta - j_n^\mu) & j_e^\mu \sin^2\theta \\ j_e^\mu \sin^2\theta & (2 j_e^\mu \cos^2\theta - j_n^\mu) \end{pmatrix}$$

$$j_{n,e}^\mu = (n_{n,e}, n_{n,e}\mathbf{v}), \quad \mathbf{v} = (\boldsymbol{\Omega} \times \mathbf{r})$$

- We study the initial condition for the system of two mixed flavor neutrinos interacting with moving matter [MD, Phys. Lett. B **610**, 262 (2005)]

- The evolution equation for mass eigenstates

Neutrino quantum states in a rotating neutron star

- For neutrinos with relatively small energies (several **eV**) a bound state is possible [A. V. Grigoriev, *et al.*, Russ. Phys. J. **50**, 845 (2007)]

$$\eta_a(r,\phi,t) = \sum_{n,s=0}^{\infty} \left(a_{ns}^{(a)}(t) u_{a,ns}^{+}(r,\phi) \exp[-iE_n^{(a)+}t] + b_{ns}^{(a)}(t) u_{a,ns}^{-}(r,\phi) \exp[-iE_n^{(a)-}t] \right)$$

$$E_n^{(a)\pm} = -V_a \pm \sqrt{4V_a\Omega n + m_a^2},$$

$$V_1 = \frac{G_F}{\sqrt{2}}(n_n - 2n_e \sin^2\theta),$$

$$V_2 = \frac{G_F}{\sqrt{2}}(n_n - 2n_e \cos^2\theta)$$

$$u_{a,ns}^{\pm}(r,\phi) = \sqrt{\frac{V_a\Omega}{2\pi}} \begin{pmatrix} I_{n-1,s}(\rho_a) e^{\mathrm{i}(l-1)\phi} \\ \mp \mathrm{i} I_{n,s}(\rho_a) e^{\mathrm{i}l\phi} \end{pmatrix},$$

$$\rho_a = V_a \Omega r^2$$

- Energy of antineutrino $\boldsymbol{E = -E_n^{(a)-} > 0}$. For an antineutrino a bound state is not possible!

Wave functions of high energy neutrinos

- Neutrino wave functions inside the star

$$u_{a,\kappa}^{\mathrm{in}}(r,\phi)=\sqrt{\frac{V_a\Omega}{\pi}}\begin{pmatrix} C_1^{\mathrm{in}}e^{-\rho_a/2}\rho_a^{(l-1)/2}F(l-\kappa,l,\rho_a)e^{\mathrm{i}(l-1)\phi}/\{(l-1)!\sqrt{\kappa}\} \\ \mathrm{i}C_2^{\mathrm{in}}e^{-\rho_a/2}\rho_a^{l}F(l-\kappa,l+1,\rho_a)e^{\mathrm{i}l\phi}/l! \end{pmatrix}$$

- Neutrino wave functions outside the star

$$u_{a,\kappa}^{\mathrm{out}}(r,\phi)=\frac{1}{\sqrt{2\pi}}\begin{pmatrix} C_1^{\mathrm{out}}H_{l-1}^{(1)}(p_\perp r)e^{\mathrm{i}(l-1)\phi} \\ \mathrm{i}C_2^{\mathrm{out}}H_{l}^{(1)}(p_\perp r)e^{\mathrm{i}l\phi} \end{pmatrix}$$

- Wave functions should be equal at the star surface: $\boldsymbol{u_a^{(\mathrm{in})}(R,\varphi)=u_a^{(\mathrm{out})}(R,\varphi)}$
- Energy has a continuous value: $E_\kappa^{(a)}=-V_a+\sqrt{4V_a\Omega\kappa+m_a^2}$

Evolution equation for the mass eigenstates

- The ordinary differential equations for the coefficients $\boldsymbol{a_{ns}}$

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}a_{ns}^{(a)}(t)=\sum_{n's'=0}^{\infty}\left\{\left\{\int u_{a,ns}^{+\dagger}(\mathbf{r})\left(g_{ab}^{\mu}\tilde{\sigma}_{\mu}\right)u_{b,n's'}^{+}(\mathbf{r})\,\mathrm{d}^2\mathbf{r}\right\}\exp\left[\mathrm{i}\left(E_n^{(a)+}-E_{n'}^{(b)+}\right)t\right]a_{n's'}^{(b)}(t)\right.$$
$$\left.+\left\{\int u_{a,ns}^{+\dagger}(\mathbf{r})\left(g_{ab}^{\mu}\tilde{\sigma}_{\mu}\right)u_{b,n's'}^{-}(\mathbf{r})\,\mathrm{d}^2\mathbf{r}\right\}\exp\left[\mathrm{i}\left(E_n^{(a)+}-E_{n'}^{(b)-}\right)t\right]b_{n's'}^{(b)}(t)\right\},\quad a\neq b$$

- In general the transitions $\boldsymbol{(l,s)\rightarrow(l,s\pm 1)}$ are possible
- If $\boldsymbol{s>>l}$ (neutrino emission from the center of the star)

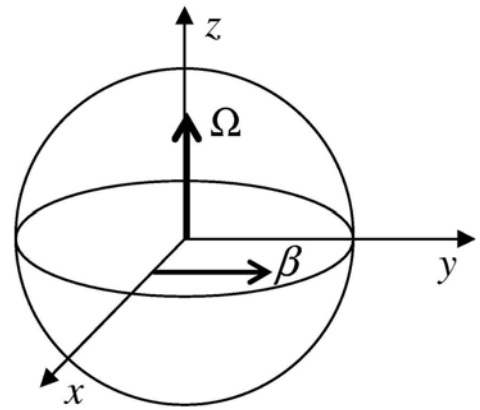
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\tilde{a}_s^1\\ \tilde{a}_s^2\end{pmatrix}=\begin{pmatrix}\omega/2 & \Delta\\ \Delta & -\omega/2\end{pmatrix}\begin{pmatrix}\tilde{a}_s^1\\ \tilde{a}_s^2\end{pmatrix},\quad \Delta=\frac{G_F}{\sqrt{2}}n_e\sin 2\theta,\quad \frac{\omega}{2}=\frac{\delta m^2}{4k}-\frac{G_F}{\sqrt{2}}n_e\cos 2\theta$$

- The rotation causes almost no effect on the neutrinos emitted from the center of the star

Neutrinos with big angular momentum

- We can use the conventional Schrodinger equation based approach to describe neutrino flavor oscillations in moving matter [A. Grigoriev, *et al.*, Phys. Lett. B **535**, 187 (2002)]

$$H = \frac{\delta m^2}{4E_\nu}\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} - \frac{G_\mathrm{F}}{\sqrt{2}} n_e (1 - \boldsymbol{\beta}\mathbf{v}) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

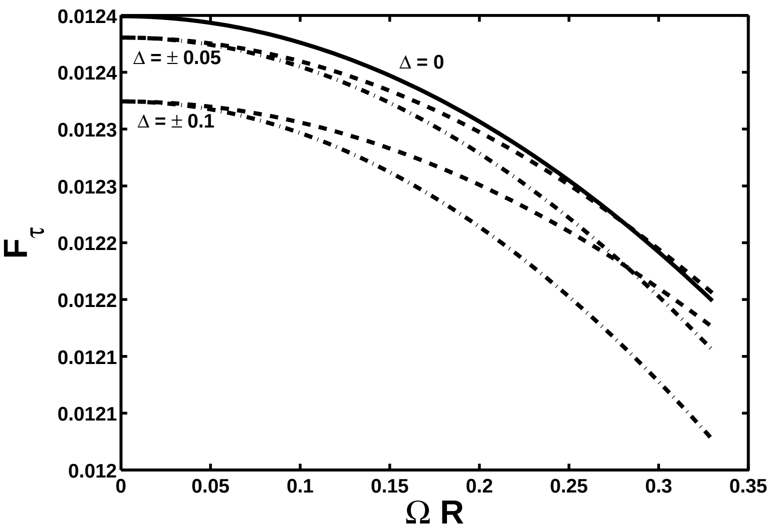


$$P(x,y) = \frac{H_{12}^2}{H_{12}^2 + H_{11}^2} \sin^2\left(\sqrt{H_{12}^2 + H_{11}^2}\, y\right)$$

$$H_{11} = \frac{\delta m^2}{4E_\nu}\cos 2\theta - \frac{G_\mathrm{F}}{\sqrt{2}} n_e (1 - \Omega x),$$

$$H_{12} = \frac{\delta m^2}{4E_\nu}\sin 2\theta$$

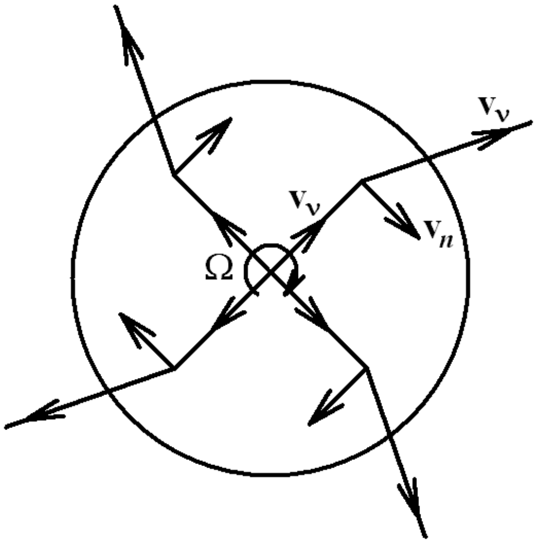
$$\delta j_{\mu,\tau} = F_{\mu,\tau}\, j_e, \quad F = \int\limits_{\substack{\text{outside} \\ \text{neutrinosphere}}} \mathrm{d}x\, \mathrm{d}y\, P(x,y)$$



Neutron star spin-down

- Main mechanisms of the neutron star spin-down at the latest stages of the evolution (thousands of years) are magnetic dipole radiation, for radio pulsars, and gravitational waves radiation (D. R. Lorimer and M. Kramer, *Handbook of Pulsar Astronomy*, 2004)

- Neutrinos may significantly contribute to the neutron star spin-down at the initial stages of the evolution (10 sec)



Collisions with moving matter

- We have demonstrated that rotation causes very small effect on flavor dynamics of neutrinos. Therefore neutrinos keep their initial flavors
- Matrix element for the reaction $\boldsymbol{\nu(k_1) f(p_1) \to \nu(k_2) f(p_2)}$ [L. B. Okun', *Leptons and Quarks*, 1990]

$$M = \frac{G_F}{\sqrt{2}} \bar{f}(p_2)\left[g_L \gamma^\mu (1-\gamma^5) + g_R \gamma^\mu (1+\gamma^5)\right] f(p_1) \cdot \bar{\nu}(k_2)\gamma_\mu (1-\gamma^5)\nu(k_1)$$

$$p_{1,2} = (E_{1,2}, \mathbf{p}_{1,2}), \quad k_{1,2} = (\omega_{1,2}, \mathbf{k}_{1,2})$$

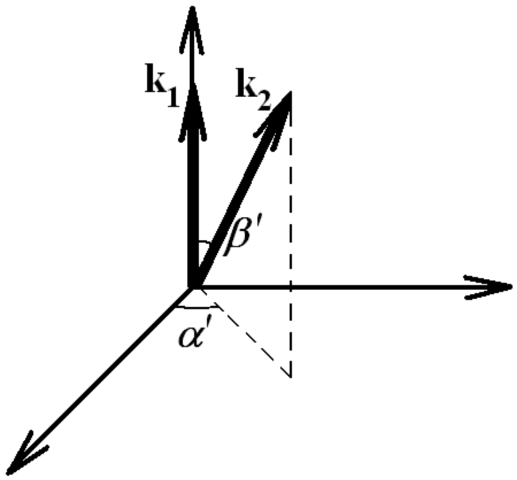
- Differential cross section

$$\frac{d\sigma}{do} = \frac{1}{128\pi^2} \frac{|M|^2}{|\mathcal{F}|} \frac{\omega_2}{\omega_1 E_1 E_2},$$

$$E_2 = \sqrt{E_1^2 - 2\omega_1\omega_2 \cos\alpha' + \omega_2^2 - 2E_1\omega_2 v_f \sin\alpha' \sin\beta'},$$

$$\omega_2 = \frac{E_1\omega_1}{E_1(1 - v_f \sin\alpha' \sin\beta') + \omega_1(1-\cos\alpha')},$$

$$\mathcal{F} = 1 + (\omega_2 - \omega_1 \cos\alpha' - E_1 v_f \sin\alpha' \sin\beta')/E_2$$



Angular momentum carried away by neutrinos

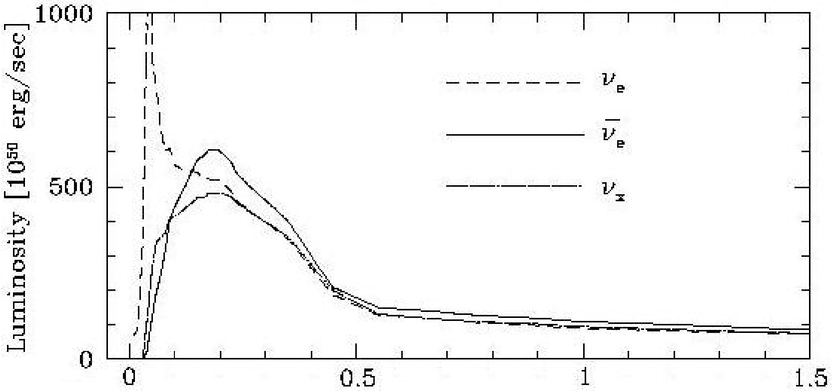
- Angular momentum per unit time

$$\dot{L}_z = \int \langle \dot{k}_\phi \rangle r \sin\vartheta \, n_f(\mathbf{r}) \, \mathrm{d}^3\mathbf{r}, \quad \langle \dot{k}_\phi \rangle = \int \omega_2 \sin\alpha' \sin\beta' J(r) \frac{\mathrm{d}\sigma}{\mathrm{d}o} \mathrm{d}o$$

- Assuming that $\boldsymbol{n_{e,p} \ll n_n}$ as well as $\boldsymbol{v_f}$ and $\boldsymbol{\omega_1/E_1}$ are small parameters we express the final result in the form

$$\frac{\dot{L}_z}{L_0} \approx 0.1 \left(\frac{E_\nu}{10\,\mathrm{MeV}} \right)^3 \left(\frac{R}{10\,\mathrm{km}} \right)^3 \left(\frac{n_n}{10^{38}\,\mathrm{cm}^{-3}} \right) \left(\frac{J_0}{10^{43}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}} \right) \left(\frac{\mathcal{M}_\odot}{\mathcal{M}} \right) \mathrm{s}^{-1}$$

- Neutrino luminosity reaches **10^{52} erg/s** during **$t \sim 1$ s** [T. Totani, *et al.*, Astrophys. J. **496**, 216 (1998)] giving the neutrino flux at the neutron star surface **10^{43} cm^{-2}s^{-1}**. Finally we obtain that neutrinos can carry away **~10%** of the initial angular momentum.



Discussion

- Rotation of the neutron star causes small effect on flavor oscillations of neutrinos
- The trapping of neutrinos by rotating matter is possible. Antineutrinos cannot be trapped.
- Neutrinos can carry away an essential fraction of the initial angular momentum [see also K. Mikaelian, Astrophys. J. **214**, L22 (1977); R. Epstein, Astrophys. J. **219**, L39 (1978)].
- An effort to infer initial angular velocities of neutron stars was made by E. van der Swaluw and Y. Wu [Astrophys. J. **555**, L49 (2001)]. It was revealed that there should be a significant uncertainty in the results. Thus there exists a possibility that neutrino emission can contribute to the spin-down of a neutron star.

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