Radiative and hadronic decays of vector mesons in the gauge model of quark-meson interactions

V.Beylin, V.Kuksa, G.Vereshkov

Southern Federal University

An exact low energy hadron theory should be nonperturbative → effective Lagrangian approach

- Effective Lagrangians from fundamental theory (QCD)
- Phenomenological Lagrangians from dynamical symmetries → LσM

meson-meson and quark-meson interactions

- VDM can be added

QCD → $SU_L(2) \times SU_R(2)$

- Bosonization procedure (see Volkov, Radzhabov, Phys-USP, 2006)
- Current quarks → constituent quarks,

$m_q \approx 300$ MeV, gluon substructures are included

- EM and strong interactions are described by the gauge (vector) fields
- Quark level σ-model (QσM) → hadron-level (NσM)

One of the simplest effective gauge approach is based on

$U_0(1) \times U(1) \times SU(2)$

- γ, ρ, ω – gauge fields
- EM and strong interactions are insensitive to the chirality, it should be localized diagonal sum of the global chiral

$$SU_L(2) \times SU_R(2)$$

- Remained Higgs degrees of freedom can be associated with scalar mesons (a_0, f_0)
- VDM is naturally realized in the gauge way

The model Lagrangian

$$L = i\bar{q}\hat{D}q - \varkappa\bar{q}(\sigma + i\pi^a\tau_a\gamma_5)q + \frac{1}{2}(D_\mu\pi^a)^+(D_\mu\pi^a) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\mu^2(\sigma^2 + \pi^a\pi^a)$$
$$- \frac{1}{4}\lambda(\sigma^2 + \pi^a\pi^a)^2 + (D_\mu H_A)^+(D_\mu H_A) + \mu_A^2(H_A^+H_A) - \lambda_1(H_A^+H_A)^2$$
$$- \lambda_2(H_A^+H_B)(H_B^+H_A) - h(H_A^+H_A)(\sigma^2 + \pi^a\pi^a) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{4}V^a_{\mu\nu}V_a^{\mu\nu}.$$

Here $q = (u, d)$ - is the first generation quark doublet; $H_{1,2}$ - two scalar fields doublets with hypercharges $Y_{1,2} = \pm 1/2$, $a = 1, 2, 3$ and $A = 1, 2$.

In the model vector fields couplings are universal (see also R.Delbourgo, D.Liu, M.D.Scadron,1999)

- Diagonalization of V and S mass forms ⟹ mass spectrum,

ρ(770), ω(782), σ, a_0, f_0 masses and decay properties can be described in the model due to free parameters in the S sector, see

$$L_{\pi h} = (\pi^0\pi^0 + 2\pi^+\pi^-)(g_{\sigma\pi}\sigma_0 + g_{f\pi}f_0 + g_{a\pi}a_0)$$

- In a tree approximation the vector boson physical states are

$$A_\mu = \cos\theta \cdot B_\mu + \sin\theta \cdot V_\mu^3,$$
$$\omega_\mu = \cos\phi \cdot V_\mu + \sin\phi \cdot (\sin\theta \cdot B_\mu - \cos\theta \cdot V_\mu^3),$$
$$\rho_\mu^0 = \sin\phi \cdot V_\mu + \cos\phi \cdot (-\sin\theta \cdot B_\mu + \cos\theta \cdot V_\mu^3)$$

Real tree level mass matrix and real gauge couplings → zero relative phase in $\rho\pi\pi$ and $\omega\pi\pi$

- Phenomenologically, the complexity is resulted from superposition of pure isospin states $|\rho_I>$ and $|\omega_I>$
- In the model the relative phase occurs due to couplings renormalization: tree parameters $\sin\varphi$, $\cos\varphi$ correspond to abs values of renormalized mixings
- The phase value can be fixed due to free parameters (in the S sector)

The final physical Lagrangian

$$
\begin{aligned}
L_{Phys} = {} & \bar{u}\gamma^{\mu}u(\frac{2}{3}eA_{\mu} + g_{u\omega}\omega_{\mu} + g_{u\rho}\rho^{0}_{\mu}) + \bar{d}\gamma^{\mu}d(-\frac{1}{3}eA_{\mu} + g_{d\omega}\omega_{\mu} + g_{d\rho}\rho^{0}_{\mu}) \\
& + ig_2(\pi^{-}\pi^{+}_{,\mu} - \pi^{+}\pi^{-}_{,\mu})(\sin\theta\ A^{\mu} - \cos\theta s_{\phi}\ \omega^{\mu} + \cos\theta c_{\phi}\ \rho^{0\mu}) \\
& - \sqrt{2}i\varkappa\pi^{+}\bar{u}\gamma_5 d - \sqrt{2}i\varkappa\pi^{-}\bar{d}\gamma_5 u - i\varkappa\pi^{0}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \\
& + 2g_2 e\cos\theta c_{\phi}\ \rho^{0}_{\mu}A^{\mu}\pi^{+}\pi^{-} - 2g_2 e\cos\theta s_{\phi}\ \omega_{\mu}A^{\mu}\pi^{+}\pi^{-} \\
& + \frac{1}{\sqrt{2}}g_2\rho^{+}_{\mu}\bar{u}\gamma^{\mu}d + \frac{1}{\sqrt{2}}g_2\rho^{-}_{\mu}\bar{d}\gamma^{\mu}u + ig_2\rho^{+\mu}(\pi^{0}\pi^{-}_{,\mu} - \pi^{-}\pi^{0}_{,\mu}) \\
& + ig_2\rho^{-\mu}(\pi^{+}\pi^{0}_{,\mu} - \pi^{0}\pi^{+}_{,\mu}).
\end{aligned}
$$

Here

$$
\begin{aligned}
g_{u\omega} &= \frac{1}{2}g_1 c_\phi + \frac{1}{2}s_\phi\,(\frac{1}{3}g_0\sin\theta - g_2\cos\theta)\,,\\
g_{u\rho} &= \frac{1}{2}g_1 s_\phi - \frac{1}{2}c_\phi\,(\frac{1}{3}g_0\sin\theta - g_2\cos\theta)\,,\\
g_{d\omega} &= \frac{1}{2}g_1 c_\phi + \frac{1}{2}s_\phi\,(\frac{1}{3}g_0\sin\theta + g_2\cos\theta)\,,\\
g_{d\rho} &= \frac{1}{2}g_1 s_\phi - \frac{1}{2}c_\phi\,(\frac{1}{3}g_0\sin\theta + g_2\cos\theta)\,,
\end{aligned}
$$

where s_ϕ and c_ψ are the complex (renormalized) parameters of the $V-B$ mixing with account of the self-energy insertion to the mass matrix.

There are relations in the model

$$\sin\theta = \frac{g_0}{\sqrt{g_0^2+g_2^2}}, \quad e = g_0\cos\theta, \quad v_1^2+v_2^2 = 4\frac{m_{\rho^\pm}^2}{g_2^2}, \quad |s_\phi| = \frac{g_1}{g_2}\sqrt{\frac{m_{\rho^\pm}^2 - m_\omega^2(g_2^2/g_1^2)}{m_\omega^2 - m_{\rho^0}^2}}$$

From $\Gamma(\rho^+ \to \pi^+\pi^0)$ and $\Gamma(\omega \to \pi^+\pi^-)$ ⟶ g_2 and $|s_\varphi|$

$$g_0^2/4\pi = 7.32\cdot 10^{-3}, \quad g_1^2/4\pi = 2.86, \quad g_2^2/4\pi = 2.81,$$
$$|s_\phi| = 0.031, \ \sin\theta = 0.051, \ v_1^2+v_2^2 \approx (250.7\,\mathrm{MeV})^2.$$

These values were applied for calculation of the vector meson radiative decay widths to verify the gauge vector dominance approach. In our strategy of calculations the strong couplings are extracted from the above mentioned processes as the effective final values. So, we do not need in loop corrections to these couplings. At the same time, electromagnetic vertices should be renormalized by the strong interactions

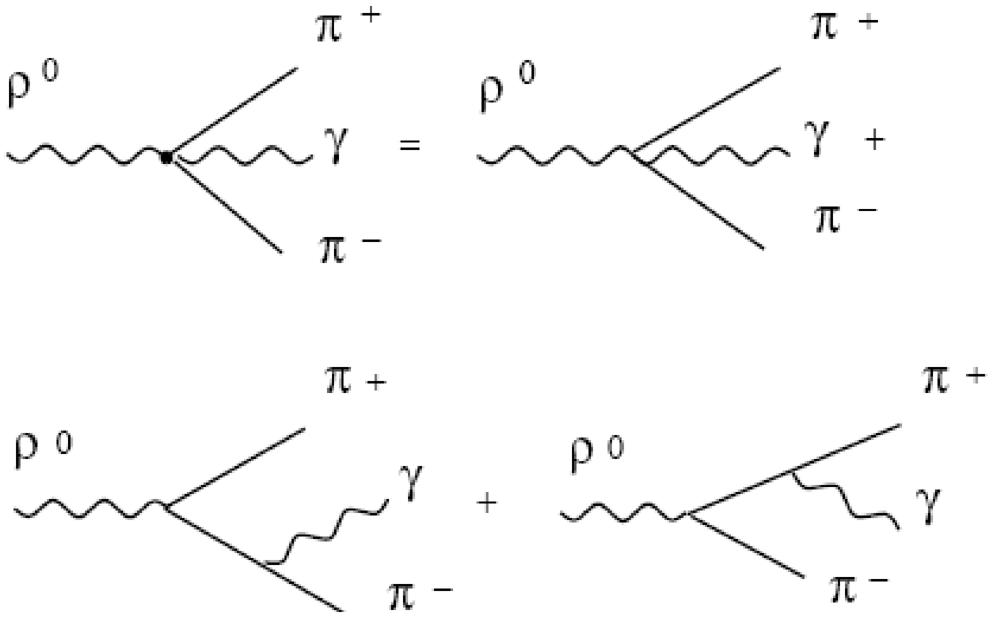
The gauge scheme describes:

- meson-meson interactions at the tree and loop levels $V \longrightarrow \pi\pi\gamma$
- quark-meson interactions at the loop level

$\omega,\ \rho^0 \rightarrow \pi^0\gamma$ $\omega, \rho \longrightarrow 3\pi$

Radiative decays

$\rho^0 \to \pi^+\pi^-\gamma$ and $\omega \to \pi^+\pi^-\gamma$



Differential width has the form

$$d\Gamma(E_\gamma)/dE_\gamma = \frac{G}{\kappa}\left(F_1(\kappa) + F_2(\kappa)\ln F_3(\kappa)\right),$$

where:

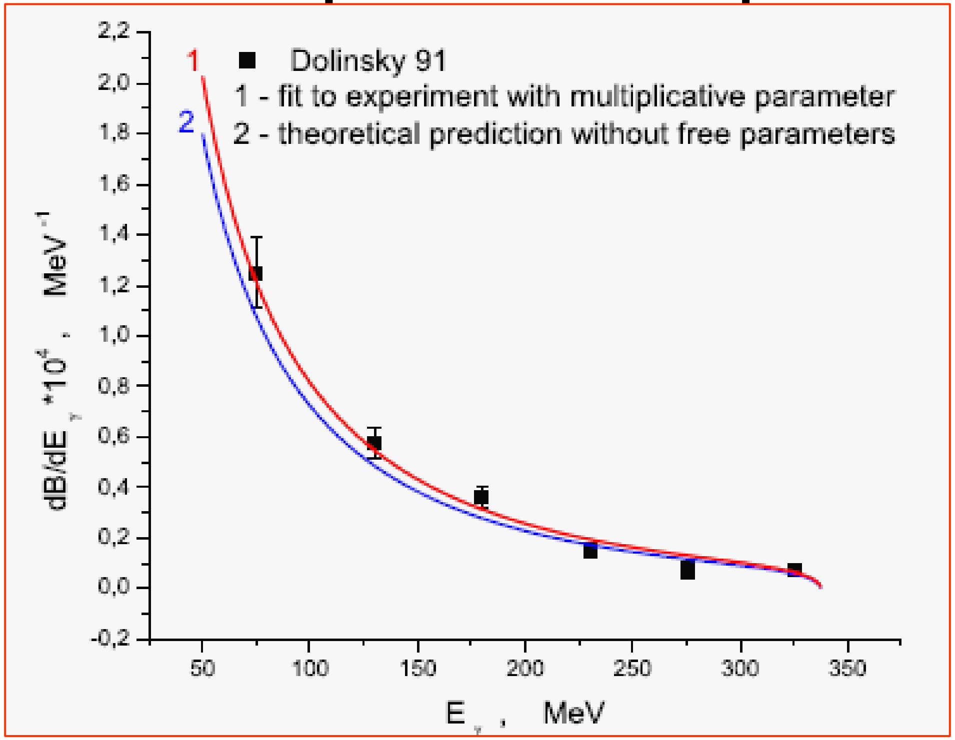
$$\kappa = E_\gamma/m_\rho,\ \ G = \alpha_{em}\cdot g_2^2\cos^2\theta\cdot|c_\phi|^2/24\pi^2,\ \ \mu = m_\pi^2/m_\rho^2,$$

$$F_1(\kappa) = \left(\frac{1-2\kappa-4\mu}{1-2\kappa}\right)^{1/2}\left(-1+2\kappa+4\kappa^2+4\mu(1-2\kappa)\right);$$

$$F_2(\kappa) = 1-2\kappa-2\mu(3-4\kappa-4\mu);$$

$$F_3(\kappa) = \frac{1}{2\mu}\left[1-2\kappa-2\mu+((1-2\kappa)\cdot(1-2\kappa-4\mu))^{1/2}\right].$$

Spectrum of photons in ρ ⟶2πγ



$$B(\rho^0 \rightarrow \pi^+\pi^-\gamma) = 1.17 \cdot 10^{-2}$$

$$B^{exp}(\rho^0 \rightarrow \pi^+\pi^-\gamma) = (0.99 \pm 0.16) \cdot 10^{-2}$$

$$B^{phen}(\rho^0 \rightarrow \pi^+\pi^-\gamma) = (1.22 \pm 0.02) \cdot 10^{-2}$$

$\omega \longrightarrow \pi^+\pi^-\gamma$: with replacement $c_\phi \longrightarrow s_\phi$

$m_\rho \longrightarrow m_\omega$

$$B(\omega \to \pi^+\pi^-\gamma) = 4.0 \cdot 10^{-4}$$

$$B(\omega \to \pi^+\pi^-\gamma) = 2.6 \cdot 10^{-4}$$

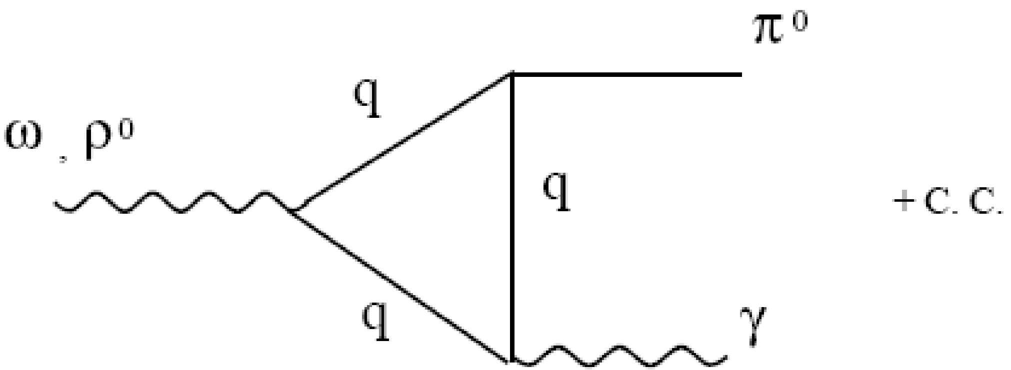
$$B^{exp}(\omega \to \pi^+\pi^-\gamma) \leq 3.6 \cdot 10^{-3}$$

Due to loop contributions

$B(\omega \to \pi^+\pi^-\gamma)$ increase up to $(2-3) \cdot 10^{-3}$

Processes via quark loops

$$\omega,\ \rho^0 \longrightarrow \pi^0\gamma$$



$$\Gamma(\omega \to \pi^0\gamma) = \frac{3\alpha g_1^2}{2^7\pi^4}|c_\phi|^2\ m_q \frac{m_q^3}{m_\omega f_\pi^2}\left(1 - \frac{m_\pi^2}{m_\omega^2}\right)|L_\omega|^2$$

$$L_\omega = Li_2\left(\frac{2}{1+\sqrt{\lambda_1}}\right) + Li_2\left(\frac{2}{1-\sqrt{\lambda_1}}\right) - Li_2\left(\frac{2}{1+\sqrt{\lambda_2}}\right) - Li_2\left(\frac{2}{1-\sqrt{\lambda_2}}\right)$$

$$\lambda_1 = 1 - 4m_q^2/m_\omega^2,\ \lambda_2 = 1 - 4m_q^2/m_\pi^2$$

$$\Gamma(\rho^0 \to \pi^0\gamma) = \frac{\alpha g_1^2}{3 \cdot 2^7 \pi^4} |c_\phi|^2 \cdot \left(\cos\theta \cdot \frac{g_2}{g_1}\right)^2 m_q \frac{m_q^3}{m_\rho f_\pi^2} \left(1 - \frac{m_\pi^2}{m_\rho^2}\right) |L_\rho|^2$$

$$\Gamma^{theor}(\omega \to \pi^0\gamma) = 0.74 \pm 0.02\,\mathrm{MeV}, \qquad \Gamma^{exp}(\omega \to \pi^0\gamma) = 0.76 \pm 0.02\,\mathrm{MeV};$$

$$\Gamma^{theor}(\rho^0 \to \pi^0\gamma) = 0.081 \pm 0.003\,\mathrm{MeV}, \qquad \Gamma^{exp}(\rho^0 \to \pi^0\gamma) = 0.090 \pm 0.012\,\mathrm{MeV}.$$

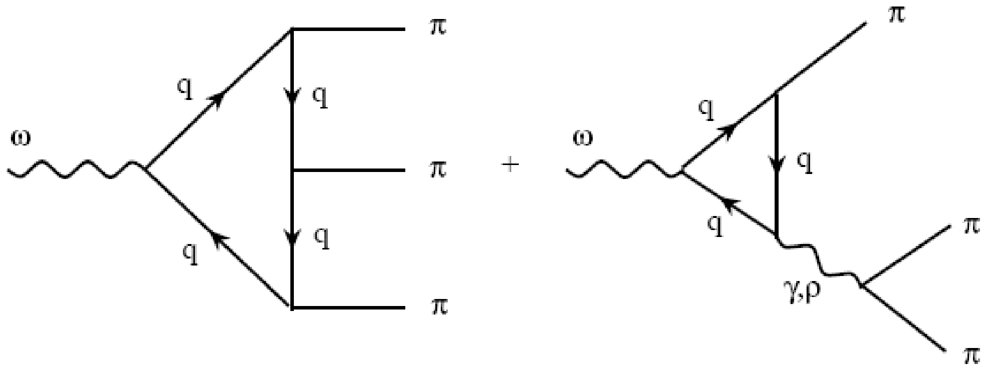
$$m_q = 175 \pm 5\,\mathrm{MeV}$$

$$\Updownarrow$$

$$m_N = 3m_q + m_G; \quad m_\rho = 2m_q + m_G$$

A consequence of a model separation of quark and gluon degrees of freedom in a hadron?

Three-pion decays of light vector mesons



$$M_{tot} = M_{box} + M_\rho = 4 \cdot m_q N_c \cdot \varkappa g_2 cos\phi \cdot e^\mu F_\mu \cdot \Big[-\varkappa^2 \sum_{k=1}^{12} \int \frac{d^4q}{(2\pi)^4} \frac{1}{D_k(q)}$$

$$+ \frac{g_3^2 cos\theta^2 cos\phi^2}{d_\rho(k)} \int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{D_{13}(q)} + \frac{1}{D_{14}(q)} \right)$$

$$+ \frac{g_3^2}{d_\rho(k_1)} \int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{D_{15}(q)} + \frac{1}{D_{16}(q)} \right)$$

$$+ \frac{g_3^2}{d_\rho(k_2)} \int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{D_{17}(q)} + \frac{1}{D_{18}(q)} \right) \Big].$$

Differential width is

$$d\Gamma_{\omega\to 3\pi} = \frac{32\pi}{768(2\pi)^8}\frac{\varkappa^2 g_2^6 \cdot m_q^2 \cdot N_c^2}{m_\omega^3} \cdot F(s,t)dsdt.$$

$$\Gamma_{\omega\to 3\pi} \sim \int_{2m_\pi)^2}^{(m_\omega - m_\pi)^2} ds \int_{t_-}^{t_+} dt\, F(s,t)$$

$$t_\pm = \frac{m_\omega^2 + 3m_\pi^2 - s \pm \sqrt{1 - 4m_\pi^2/s} \cdot \lambda^{1/2}(m_\omega, m_\pi, s)}{2}$$

$\Gamma_{theor}(\omega \longrightarrow 3\pi) = (8.2 – 7.4)$ MeV

$\Gamma_{theor}(\rho \longrightarrow 3\pi) = (0.72 – 0.60) \times 10$^(-3) MeV

$\Gamma_{exp}(\omega \longrightarrow 3\pi)$=(7.6 – 7.4) MeV

$\Gamma_{exp}(\rho \longrightarrow 3\pi)$=(0.5 – 2.9)x10^(-3) MeV

Due to $m_q \approx 180\,\mathrm{MeV} \sim m_\pi$

we need in an exact integration procedure

because the ratio m_π^2/m_q^2. is not small

see also

J.L.Lucio M., M.Napsuciale,M.D.Scadron, V.M.Villanueva, 1999 with a “standard” quark mass $\approx 300\,\mathrm{MeV}$

Conclusions

- The gauge generalization of σ-model, including quark degrees of freedom explicitly, is considered.
- VDM occurs at the tree level.
- $\rho^0 \rightarrow \pi^+\pi^-\gamma$ and $\omega \rightarrow \pi^+\pi^-\gamma$ decays have been

calculated in a good agreement with experimental data as and

- $\rho^0 \rightarrow \pi^0\gamma$ and $\omega \rightarrow \pi^0\gamma$ decays.
- The gauge model works well for the vector mesons three-pion decays also.
- So, the gauge quantum field approach can be effectively used for the meson-meson and quark-meson interactions in the radiative and hadronic decay modes.
- In principle, free Higgs degrees of freedom can be associated in the model with some known scalar mesons.

σ-model can be realized in a various representations with 6 independent parameters

Transformations of quark fields

$$q' = q + \frac{i}{2}(\alpha_a \tau^a + \beta_a \tau^a \gamma_5) q$$
$$= q + \frac{i}{2}(\alpha + \beta)\tau^a q_R + \frac{i}{2}(\alpha - \beta)_a \tau^a q_L$$

where α_a are the parameters of the $SU_{L+R}(2)$ group and β_a are the parameters of the $SU_{L-R}(2)$ group. Thus, here we use $SU(2) \times G(3) = SU_{L+R}(2) \times SU_{L-R}(2)$ representation for π and σ fields. Then, the corresponding transformation properties fo (σ, π) are:

$$\pi'_a = \pi_a + \epsilon_{abc}\alpha_b \pi_c + \beta_a \sigma,$$
$$\sigma' = \sigma + \beta_a \pi_a.$$