No black holes LHC?

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ADD model

The simplest model with large extra dimensions was suggested by Arkani-Hamed, Dimopoulos and Dvali (ADD) as solution to the hierarchy problem. The weakness of gravity in four dimensions is explained by the presence of n extra dimensions compactified on a torus large compared to the electroweak scale, while the fundamental mass parameter M_* (higher-dimensional Planck mass) is taken of the order of 1 TeV. Assuming that the extra space is flat, the D-dimensional Einstein action can be related to the 4-dimensional one by

$$\frac{1}{G_D}\int R_D\sqrt{-g_D}\,d^Dx = \frac{V}{G_D}\int R_4\sqrt{-g_4}\,d^4x = \frac{1}{G_4}\int R_4\sqrt{-g_4}\,d^Dx,$$

where V is the volume of the n-torus, so $G_D = V\,G_4$, or in terms of masses,

$$M_{\rm Pl}^2 = M^{*(n+2)}\,V.$$

Denoting $V = (2\pi R)^n$ one finds the compactification radius R for fixed value of $M_* = 1\ TeV$:

$$l_* = V^{1/n} \sim 10^{30/n-17}\ \mathrm{cm}$$

n	R
1	1.5×10^{13} cm
2	$0.5\,\mathrm{mm} = 1/(10^{-4}\mathrm{eV})$
4	$3 \times 10^{-8}\,\mathrm{mm} = 3/(20\mathrm{KeV})$
6	$10^{-10}\,\mathrm{mm} = 1/(1\mathrm{MeV})$

The case $n = 1$ is excluded, but $n = 2$ gives $l_* \sim 1mm$. This is enormous with respect not only to electroweak scale, but also to atomic scale, so such large extra dimensions have to be seen in the low energy processes not to say about direct mechanical tests of the Newton law.

Static force

Two test masses m_1, m_2 at a distance small with respect to the compactification radius $r \ll R$ feel a gravitational potential following from Gauss's law in $(4+n)$ dimensions:

$$V(r) \sim \frac{m_1 m_2}{M_*^{n+2}} \frac{1}{r^{n+1}}, \qquad (r \ll R)$$

with the Tev-scale gravitational constant. Gravity is strong in this case. But being placed at large distances $r \gg R$, they feel four-dimensional potential, since the gravitational flux lines joining them do not penetrate into the extra dimensions:

$$V(r) \sim \frac{m_1 m_2}{M_*^{n+2} R^n} \frac{1}{r}, \qquad (r \gg R)$$

Gravity is weakened by the ratio $(R/r)^n$, so our effective 4 dimensional gravitational coupling is reproduced in view of the ratio $M_{Pl}^2 \sim M_*^{2+n} R^n$. Therefore an observed weakness of gravity is explained by the Gauss law: only a small fraction of the gravitational flux lines propagates along the brane.

Linearized theory

Expanding the D-dimensional metric $g_{MN} = \eta_{MN} + \kappa_D h_{MN}$ we get Fierz-Pauli lagrangian in D-dimensional Minkowski space, with $n \equiv D - 4$ of spatial dimensions forming a torus T^n with equal radii R

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}h^{MN}\Box h_{MN} + \frac{1}{4}h\Box h - \frac{1}{2}h^{MN}\partial_M\partial_N h + \frac{1}{2}h^{MN}\partial_M\partial_P h^P_N \\ &- \frac{\varkappa_D}{2}h^{MN}T_{MN},\end{aligned}$$

where $M, N, .. = 0, 1, 2, ..., D-1$. The Minkowski metric is $\eta_{MN} = \mathrm{diag}(1, -1, -1, ...)$, $h \equiv \eta^{MN}h_{MN} \equiv h^M_M$ and $\Box \equiv \eta^{MN}\partial_M\partial_N$. The gravitational field h_{MN} is coupled in a universal way to a conserved matter stress-tensor T_{MN} ($\partial_N T^{MN} = 0$). The latter is further assumed to be localized on the brane by some confinement mechanism

Momentum quantization

Consider first vacuum case $T_{MN} = 0$. Metric functions depend on

$$x^M = (x^\mu, y^i), \quad \mu = 0, \ldots, 3, \quad i = 1, \ldots n,$$

and must be periodic under translations $y_j \to y_j + 2\pi R$, which leads to quantization of the momentum in compact directions

$$h_{MN}(x^P) = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_n=-\infty}^{+\infty} \frac{h^n_{MN}(x)}{\sqrt{V}} \exp\left(i \frac{n_i y^i}{R} \right)$$

Graviton polarizations can be split in the Kaluza-Klein spirit

$$h_{MN} \; = \; V_n^{-1/2} \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu j} \\ A_{i\nu} & 2\phi_{ij} \end{pmatrix} ,$$

$\phi \equiv \phi_{ii}$, $\mu, \nu = 0, 1, 2, 3$ and $i, j = 5, 6, \cdots, 4 + n$

Massless modes

Different polarization states for zero modes ($\vec{n} = 0$) give rise to massless fields: 4D graviton , n massless graviphotons $A^0_{\mu i}$, massless radion ϕ^0 and $n(n+1)/2$ massless moduli ϕ^0_{ij}. These fields which have no momentum in the compactified dimensions are confined to the brane. The trace of the scalar matrix, the radion ϕ^0 describes fluctuations of the torus. It is supposed that there must exist a mechanism giving it the mass, which stabilize the volume of the compact space. The lagrangian for the massless modes reads

$$\begin{aligned}\mathcal{L}_{n_i=0} &= \frac{1}{4}\left(\partial^\mu h^{\nu\rho}\partial_\mu h_{\nu\rho} - \partial^\mu h \partial_\mu h - 2h^\mu h_\mu + 2h^\mu \partial_\mu h\right) \\ &\quad - \sum_{i=1}^{n} \frac{1}{4} F_i^{\mu\nu} F_{\mu\nu i} + \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + \sum_{(ij)=1}^{n(n+1)/2} \partial^\mu \phi_{ij} \partial_\mu \phi_{ij} \ ,\end{aligned}$$

where $F_{\mu\nu i} = \partial_\mu A_{\nu i} - \partial_\nu A_{\mu i}$

Massive modes

Fourier components with $\vec{n} \neq 0$ can be suitably rearranged to form an infinite tower of massive 4D spin two fields $h^{\vec{n}}_{\mu\nu}$, $n-1$ towers of massive vector fields $A^{\vec{n}}_{\mu i}$ and $n(n-1)$ towers of massive scalars $\phi^{\vec{n}}_{ij}$, with the remaining 'could be' massive vectors and scalars eliminated by D-dimensional coordinate transformations. This rearrangement is similar to spontaneous symmetry breaking. Like in the Higgs mechanism, the massless spin-2 graviton fields absorb the spin-1 and spin-0 fields and become massive. The massive fields obey the equations

$$
\begin{aligned}
\left(\Box + m^2_{\vec{n}}\right)\left(h^{\vec{n}}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{\vec{n}}\right) &= 0, \\
\left(\Box + m^2_{\vec{n}}\right) A^{\vec{n}}_{\mu i} = 0, \quad \left(\Box + m^2_{\vec{n}}\right)\phi^{\vec{n}}_{ij} &= 0,
\end{aligned}
$$

where $\Box$ is 4D d'Alembert operator and $m^2_{\vec{n}} = \frac{4\pi^2 \vec{n}^2}{R^2}$.

Coupling to matter

D-dimensional gravity strongly couples to bulk fields in a universal way. However, for the matter occupying only the brane one gets the four-dimensional Newton coupling. In 4d terms the lowest order interaction lagrangian reads

$$-\frac{\kappa_4}{2}\int d^4x(h^{\mu\nu}T_{\mu\nu}+\phi T^{\mu}_{\ \mu})\ ,$$

where $T_{\mu\nu}$ is the 4d stress-tensor. Rewriting this in terms of physical massive modes one finds

$$-\frac{\kappa_4}{2}\sum_{\vec{n}}\int d^4x(h^{\mu\nu,\vec{n}}T_{\mu\nu}+\omega\phi^{\vec{n}}T^{\mu}_{\ \mu})\ .$$

Note that the vector KK modes $A^{\vec{n}}_{\mu i}$ fully decouple and the scalar KK modes $\phi^{\vec{n}}_{ij}$ only couple through their trace $\phi^{\vec{n}}$, the dilaton mode. Although each individual graviton couples very weakly to ordinary matter, their large number may enhance to an observable scale the effects due to both the virtual graviton exchange and emission of real KK gravitons.

Emission of KK gravitons

Radiation is the most efficient tool to probe extra dimensions. Emission of light KK gravitons may lead to drastic and inadmissible modification of certain astrophysical and cosmological patterns thus providing restrictions to LED scenarios. First are the effects of KK graviton emission in hot stars such as the Sun, red giants and supernova SN1987A. Excessive energy losses in the stars can alter the stellar evolution. Emission of the KK gravitons (g_{KK}) is due to:

- $\gamma + \gamma \to g_{KK}$, photon-photon annihilation;
- $e^- + e^+ \to g_{KK}$, electron-positron annihilation;
- $\gamma + e^- \, (Ze) \to e^- \, (Ze) + \, g_{KK}$, gravi-Compton scattering;
- $e^- + Ze \to e^- + Ze + \, g_{KK}$, gravi-bremsstrahlung in a static electric field of the nuclei;
- $N + N \to N + N + \, g_{KK}$, nucleon-nucleon bremsstrahlung.

SN cooling and related effects

The dominant graviton emission process from a SN core is nucleon-nucleon bremsstrahlung. The requirement that KK gravitons do not carry away more than half of the energy emitted by the supernova SN1987A gives the bounds $M_* > 13\,TeV$ for $n = 2$ and $M_* > 1\,TeV$ for $n = 3$. Further evolution of these gravitons leads to more stringent bounds. After being created, KK gravitons are quasi-stable except for their slow, gravitational-strength, decay into photons, neutrinos, and other standard particles. Therefore, the decays of KK gravitons produced in all cosmic SNe will contribute to the measured diffuse cosmic γ-ray background, providing more restrictive limits than the SN 1987A energy-loss argument. Measurements by the EGRET satellite imply $M_* > 34\,TeV$ for $n = 2$ and $M_* > 3\,TeV$ for $n = 3$. Limits on gamma-rays from all the neutron-star sources imply $M_* > 180\,TeV$ for $n = 2$ and $M_* > 10\,TeV$ for $n = 3$.

Even more stringent restriction follow from the secondary effects due to KK gravitons. The decay products of the gravitons forming the halo can hit the surface of the neutron star, providing a heat source. The low measured luminosities of some pulsars imply $M_* > 670\, TeV$ for $n = 2$ and $M_* > 22\, TeV$ for $n = 3$. These are the most restrictive bounds which probably make $n = 2$ case uninteresting as a solution of the hierarchy problem.

Astrophysical constraints set very strong bounds on M_* for $n < 4$ in some cases even ruling out the possibility to observe any signature of KK gravitons at the LHC. But it has to be kept in mind, however, that these constraints refer to soft KK gravitons lighter than 100 MeV. They disappear in more sophisticated models in which the graviton spectrum is bounded from below at this value.

Emission of gravitons at colliders

KK gravitons may be produced at colliders both in leptonic and hadronic collisions. Since the produced gravitons interact with matter only on 4D gravitational scale, they will remain undetected leaving a “missing energy” signature. Such events were searched for in the processes

$$e^+ + e^- \to \gamma + \text{missing}, \quad e^+ + e^- \to Z + \text{missing}$$

at LEP and

$$p + \bar{p} \to \gamma + \text{missing}, \quad p + \bar{p} \to \text{jet} + \text{missing}$$

at Tevatron. The combined LEP limits are $M_* > 1.4\,TeV$ for $n = 2$, $M_* > .8\,TeV$ for $n = 3$, $M_* > .5\,TeV$ for $n = 4$, $M_* > .3\,TeV$ for $n = 5$ and $M_* > .2\,TeV$ for $n = 6$. Experiments at LHC will improve this sensitivity. Theoretical predictions for hadron machines have uncertainties, and can be applied only at subplanckian energies $\sqrt{s} \ll M_*$, where $s = (p_1 + p_2)^2$

Transplanckian regime

Physics at $\sqrt{s} \sim M_*$ can be described only by the full underlying quantum gravity or string theory. However, for the transplanckian energies $\sqrt{s} \gg M_*$ the semiclassical description is possible. Since an effective gravitational coupling grows with energy, gravity becomes dominant. On the other hand, it can be argued that at ultrahigh energies, particle scattering not only becomes dominated by gravity, but in addition it involves only *classical* gravitational dynamics. Indeed, in the usual 4D theory quantum gravity effects should not, by definition, be important in the classical limit $\hbar \to 0$. This, in terms of the two relevant lengths, i.e. the Planck length $l_{\rm Pl} = (\hbar G_4/c^3)^{1/2} = \hbar/M_{\rm Pl}c$ and the gravitational radius associated with the energy of the collision $r_g = G_4\sqrt{s}/c^4$, implies that the classicality condition $l_{\rm Pl} \ll r_g$, is equivalent to the condition $\sqrt{s} \gg M_{\rm Pl}c^2$ of transplanckian energies.

In the ADD scenario the D-dimensional Planck length l_* (marking quantum gravity effects) and the gravitational radius r_g^* (classical) are, correspondingly

$$l_* = \left(\frac{\hbar G_D}{c^3}\right)^{\frac{1}{n+2}} \sim \frac{\hbar}{M_* c}, \quad r_g^* = \left(\frac{G_D\sqrt{s}}{c^4}\right)^{\frac{1}{n+1}}.$$

The above reasoning remains essentially the same and shows that in the transplanckian regime

$$\sqrt{s} \gg M_* c^2$$

scattering is also classical, at least for some range of momentum transfer. Indeed, from this condition it follows

$$\lambda_{DB} \ll l_* \ll r_g^*$$

where

$$\lambda_{DB} = \frac{\hbar c}{\sqrt{s}}$$

is De Broglie wavelength of collision, marking QFT effects.

The novel feature which is due to extra dimensions is the existence of one more scale parameter

$$b_c \sim \left(\frac{G_D s}{\hbar c^5}\right)^{\frac{1}{n}} \sim r_g^* \left(\frac{r_g^*}{\lambda_{DB}}\right)^{\frac{1}{n}},$$

which does not exists for $n = 0$. In the limit of vanishing Planck constant $\hbar \to 0$ this quantity does to infinity, so the classical region is bounded from above by $b < b_c$. For $b > b_c$, the ordinary QFT become important, while quantum gravity effects are still negligible.

Another restriction on the feasibility of calculations is the weak gravitational field approximation $b \gg r_g^*$, otherwise one has to use the fully non-linear Einstein theory. This, however, poses not only technical problems, but also the problem of the overall consistency of the ADD model beyond the linearized level. Fortunately, the weak field condition $b \gg r_g^*$ automatically imply the classicality condition $b \ll b_c$ since in the transplanckian region $r_g^* \ll \lambda_{DB}$.

Black hole production

One of the most amazing predictions of theories with LEDs is that one could actually form black holes from particle collisions at the LHC. Black holes are formed when the mass of an object is within the horizon size corresponding to the mass of the object. If the center-mass energy and the impact parameter of the collision are such that the $D = 4 + n$ dimensional gravitational radius is larger than the impact parameter

$$ds^2 = (1 - \frac{M}{M_*^{2+n} r^{1+n}})dt^2 - \frac{dr^2}{(1 - \frac{M}{M_*^{2+n} r^{1+n}})} + r^2 d^2\Omega,$$

the horizon size is given by

$$r_H \sim \left(\frac{M}{M_*}\right)^{\frac{1}{1+n}} \frac{1}{M_*}.$$

.

Then the collided particles will form a black hole with mass $M_{BH} = \sqrt{s}$, and the cross section as we have seen is roughly the geometric cross section corresponding to the horizon size of a given collision energy

$$\sigma \sim \pi r_H^2 \sim \frac{1}{M_{Pl}^2} \left(\frac{M_{BH}}{M_*}\right)^{\frac{2}{n+1}}$$

The cross section would thus be of order $1/\mathrm{TeV}^2 \sim 400$ pb, and the LHC would produce about 10^7 black holes per year! These black holes would not be stable, but decay via Hawking radiation. This has the features that every particle would be produced with an equal probability in a spherical distribution. In the SM there are 60 particles, out of which there are 6 leptons, and one photon. Thus about 10 percent of the time the black hole would decay into leptons, 2 percent of time into photons, and 5 percent into neutrinos, which would be observed as missing energy. These would be very specific signatures of black hole production at the LHC.

The widely accepted picture consists in the four-stage process of formation and evaporation of BHs in colliders,

- formation of a closed trapped surface (CTS) in the collision of shock waves modeling the head-on particle collision,
- the balding phase, during which the BH emits gravitational waves and relaxes to the Myers-Perry BH,
- Hawking evaporation and superradiance phase in which the experimental signatures are supposed to be produced, and
- the quantum gravity stage, where more fundamental theory like superstrings is important. This scenario was implemented in computer codes to simulate the BH events in LHC, where they are expected to be produced at a rate of several per second, and in ultra high energy cosmic rays.

Transplankian radiation

Radiation is the main inelastic process which will accompany transplanckian scattering. For $b < b_c$ its main features can be understood classically. During recent years radiation in presence of uncompactified LED was extensively studied within the classical theory (Kosyakov '99; D.G. '01; Kazinski et al. '02; Lemos et al. '03; D.G. and Spirin '04-'09, etc.).Bremsstrahlung in transplanckian collisions in the ADD model was recently considered by D.G., Kofinas, Tomaras and Spirin (0908.0675 [hep-ph]).Bremsstrahlung is substantially enhanced due to exchange of KK modes and emission of light massive gravitons. The cross-section exhibits rapid increase with the number of extra-dimensions and may serve an efficient tool to test theories with large n. Radiation emitted in the KK modes is invisible and provides a new channel of the missing energy processes at colliders.

The radiated energy in the rest frame one one of the particles reads

$$E_{\rm rad} = \tilde{C}_D \frac{m^2 m'^2 \varkappa_D^6}{b^{3d+3}} \gamma^{d+3}$$

with a known dimension-dependent coefficient. Qualitatively the dependence on b and γ can be understood estimating the number of light KK modes participating in interaction and radiation. To pass to the CM frame, one calculates the relative energy loss (radiation efficiency) $\epsilon \equiv E_{\rm rad}/E$, and expresses the result in terms of the Lorentz factor in the CM frame via (for $m = m'$) $\gamma_{\rm cm}^2 = (1+\gamma)/2$:

$$\epsilon = C_d \left(\frac{r_S}{b}\right)^{3(d+1)} \gamma_{\rm cm}^{2d+1}.$$

The two new features of this expression are: (a) the large factor $\gamma_{\rm cm}^{2d+1}$ due to the large number of light KK modes involved both in the gravitational force and in the radiation, and (b) the growing with d coefficient:

d	1	2	3	4	5	6
C_d	10.1	184	3359	$6 \cdot 10^4$	$1.06 \cdot 10^6$	$1.8 \cdot 10^7$
r_S	3.45	1.88	1.46	1.29	1.21	1.17
b_c	196	7.90	3.15	2.11	1.72	1.53

r_S and b_c in TeV^{-1} evaluated for $M_* \simeq 1\mathrm{TeV}$ and $\sqrt{s} \simeq 14\mathrm{TeV}$.
The classical description of small angle ultrarelativistic scattering is valid for impact parameters in the region $r_S < b < b_c$, where $b_c \equiv \pi^{-1/2} \left[\Gamma(d/2) G_D s/\hbar c^5\right]^{1/d} \sim r_S \left(r_S/\lambda_B\right)^{1/d}$ is the scale beyond which (for $d \neq 0$) classical notion of trajectory is lost

Another restriction comes from the quantum bound on the radiation frequency $\hbar\omega_{\rm cr} < m\gamma$, which is equivalent to $b > \lambda_C \equiv \hbar/(mc)$. For $d \neq 0$ the two conditions overlap provided $\lambda_C < b_c$. To estimate ϵ set $b = \lambda_C$ to obtain $\epsilon = B_d(sm/M_*^3)^{d+2}$ ($B_d = 7.4, .8, .6, .9, 1.9, 5.6, 21$ for $d = 0, 1, \ldots, 6$). Thus, a simple condition for strong radiation damping is

$$sm \gtrsim M_*^3 \ , \tag{1}$$

which may well hold for heavy particles and nuclei with LHC energies and cosmic rays. For example, for $\sqrt{s} = 14$ TeV and $m = .2$ TeV all conditions are met for $d = 1, 2$, and at the quantum boundary $b = \lambda_C$ one has $\epsilon_1 \simeq 5 \times 10^4, \ \epsilon_2 \simeq 10^6$. For protons in LHC, $\lambda_C > b_c$, our formula does not apply for $d \neq 0$. For $d = 0$ and $b = \lambda_C$ it gives $\epsilon = .25$.

Conclusions

Our analysis shows that Kaluza-Klein bremsstrahlung may lead to strong radiation damping in transplanckian collisions. Therefore,

- One may have to include the reaction force in the study of BH production, which might even exclude the formation of a CTS. Incidentally, there are indications that gravitational collapse of an oscillating string does not take place, once gravitational radiation is taken into account.
- Our results also imply that bremsstrahlung is a strong process leading to missing energy signatures in transplanckian collisions, which may further constrain the ADD parameters