Pole and running lepton masses in QED and "maximal transcendentality" hypothesis

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Abstract

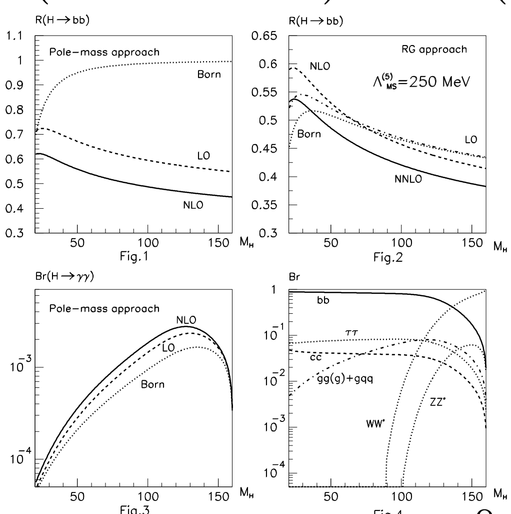
Three discovered charged leptons have pole masses
m_e=0.510998910$\pm$13$\times$10^{-9} MeV (CODATA-06),
m_μ=105.6583668$\pm$38$\times$10^{-9} MeV (CODATA-06) and
m_τ=1776.69$\pm$0.19$\pm$15 MeV from e^+e^- Novosibirsk data for threshold $e^+e^- \to \tau^+\tau^-$ (KEDR- collab (07)). We study questions:

1. Definitions of running lepton masses in the MS-scheme- especially important for τ-lepton mass due to interest in $\Gamma(H \to \tau^+\tau^-)$- how many order $O(\alpha)$-corrections important?
2. QED structure of RG-function $\gamma_m(\alpha)$= -$\gamma_{\bar{\psi}\psi}$, comparison with structure of QED $\beta(\alpha)$. May it give argument **PRO** correctness of Baikov-Chetyrkin-Kuhn (08) results ?
3. Conclusions.

Why running lepton masses in QED ?

a) Not often considered. Interesting to use say for τ-lepton.

b) $H \to \tau^+\tau^-$- important decay mode: $H \to \tau^+\tau^- \to l + jets$ in W-boson fusion may be detectable at LHC if $M_H = 115$-135 GeV and L=60 fb^{-1} (CMS-collab note referred by **N.Krasnikov (07)**- at 13th Lomonosov Conf. talk). Up to $M_H \approx 150$GeV $Br(H \to \tau^+\tau^-) > Br(H \to c\bar{c})$ (see e.g. **Kataev,Kim (96)**:)



We used $\Gamma_{H\tau^-\tau^+} = \Gamma^0_\tau = \sqrt{2}/8\pi \mathrm{M_H} \mathrm{m}^2_\tau$ (m_τ-pole) like in some other codes $\Gamma_{Hc\bar{c}} = 3\sqrt{2}/8\pi \mathrm{M_H} \overline{\mathrm{m}}^2_{\mathrm{c}}(\mathrm{M_H})\mathrm{C_H}(\alpha_{\mathrm{s}})$ with running quark mass.

At present $C_H(\alpha_s)$ is known up to $O(\alpha_s^3)$-terms (**Baikov, Chetyrkin, Kuhn (06)**. As was shown by **Kataev, Kim (07)**- 13 Lomonosov Conf, **Kataev, Kim (08)**- ACAT08- Erice and **Bakulev, Mikhaiklov, Stefanis (08-09-work in progress)** Tevatron and LHC experimental precision do not need at present the inclusion of $O(\alpha_s^4)$- they may be important in case of finding Higgs-boson.

Therefore it is appropriate to study the similar approximation for $\Gamma_{H\tau^-\tau^+}$. In the $\overline{\mathrm{MS}}$-scheme its $O(\alpha^3)$-approximation takes the following form

$\Gamma_{H\tau^-\tau^+} = \Gamma_\tau^0(\frac{\overline{m}_\tau(M_H)}{m_\tau})^2[1 + a(M_H)s_1 + a(M_H)^2 s_2 + a(M_H)^3 s_3 + a(M_H)^2 a_s(M_H)\delta^{QCD}] \quad ,$

$a(M_H) = \frac{\alpha_{\overline{\mathrm{MS}}}(M_H)}{\pi}$, $\overline{m}_\tau(M_H)$ - are QED parameters (they are related to the on-shell α and m_τ- see below), $a_s(M_H) = \frac{\alpha_s^{\overline{\mathrm{MS}}}(M_H)}{\pi}$ is the QCD parameter.

Evaluation of running τ-lepton mass: $\overline{m}_\tau(M_H) \to \overline{m}_\tau(m_\tau) \to m_\tau$:

$$\begin{aligned} \overline{m}_l(M_H) &= \overline{m}_l(m_l) \exp\Big[-\int_{a(M_H)}^{a(m_l)} \frac{\gamma_m^{QED}(x)}{\beta^{QED}(x)} dx \Big] \\ &= \overline{m}_l(m_l) \bigg(\frac{a(M_H)}{a(m_l)} \bigg)^{2\gamma_0/\beta_0} \bigg(\frac{AD(a(M_H))}{AD(a(m_l))} \bigg) \qquad (1) \end{aligned}$$

where $AD(a)$ are defined up to $O(\alpha^3)$-corrections, which depend from first 4 terms of the RG-functions

$\frac{\partial \alpha}{\partial \ln \mu^2} = \beta_{\bar{M}S}^{QED}(a) = \beta_0 a^2 + \beta_1 a^3 + \beta_2 a^4 + \beta_3 a^5 + \beta_4 a^6$

$\frac{\partial \ln \bar{m}_l}{\partial \ln \mu^2} = \gamma_{\bar{m}}^{QED}(a) = -\gamma_0 a - \gamma_1 a^2 - \gamma_2 a^3 - \gamma_3 a^4 - \gamma_4 a^5$

β_3 known from **Gorishny et al (91)**; 1-fermion loop contribution to β_4 from **Baikov, Chetyrkin and Kuhn (07)**; QED γ_3 **Chetyrkin (97)** agrees with QED limit of **Vermaseren, Larin, van Ritbergen (97)** QCD γ_3; β_4, γ_4 are added for theoretical reasons. Next- relation $\overline{m}_\tau(m_\tau) \to m_\tau$

$\overline{m}_\tau(m_\tau) = m_\tau[1 - (\frac{\alpha}{\pi}) + (\frac{\alpha}{\pi})^2(-0.51 + (N_L = 2) * 1.56 - 0.15) + (\frac{\alpha}{\pi})^3(2.06 + (N_L = 2) * 0.84 + 2.49 - (N_L = 2) - 0.07 - 0.19 - (N_L = 2)^2 * 1.96)]$ Obtained from analytical result of **Melnikov, van Ritbergen (OO)**, which agrees with semi-analytical result of **Chetyrkin, Steinhauser**.

$\overline{m}_\tau(m_\tau) = m_\tau[1 - (\frac{\alpha}{\pi}) + (\frac{\alpha}{\pi})^2 2.46 - (\frac{\alpha}{\pi})^3 1.94)]$

Conclusion: a) for τ-lepton sign-alternating structure of QED PT series seems to manifest itself.electron ($N_L = 0$)- unclear - ($-$, $-$, $+$).

b) It is enough to take into account one term in PT (others are rather small). In the coefficient function- the same:

Indeed, in the $\overline{MS}$-scheme QED coefficients of the coefficient function read

$s_1 = d_1^E = \frac{17}{4} \approx 4.25 \quad\quad s_2 = d_2^E - \gamma_0(\beta_0 + 2\gamma_0)\pi^2/3$

$d_2^E = \frac{691}{64} - \frac{9}{4}\zeta_3 - [\frac{65}{16} - \zeta_3](N_L + 1 + 3\sum_F Q_F^2)$ (at $Q_F = 0$

$d_2^E = -.49$ (in case of τ) 2.37 (in case of μ)

$s_3 = d_3^E - [d_1(\beta_0 + \gamma_0)(\beta_0 + 2\gamma_0) + \beta_1\gamma_0 + 2\gamma_1(\beta_0 + 2\gamma_0)]\pi^2/3$

$\delta^{QCD} = \delta^E - \beta_1^{QED-QCD}\gamma_0\pi^2/3$

$d_3^E = \frac{23443}{768} - \frac{239}{16}\zeta_3 + \frac{45}{8}\zeta_5) - [\frac{88}{3} - \frac{65}{4}\zeta_3 - \frac{3}{4}\zeta_4 + 5\zeta_5](N_L + 1 + 3\sum_F Q_F^4)$

$+[\frac{15511}{3888} - \zeta_3](N_L + 1 + 3\sum_F Q_F^2)^2 \quad (d_3^E = -2.03 \ \ for \ \ \tau)$

$\delta^E = [\frac{15511}{2916} - \frac{4}{3}\zeta_3]4\sum_F Q_F^2 \approx 12.95$ (In Euclidean region $(\alpha/\pi)^2\alpha_s/pi$-coefficient is larger- but overall contribution is smaller than 1-loop term.

These results are obtained from **Gorishny et al (90-91)** and **Chetyrkin (97)**

Structure of analytically known QED terms of RG-functions (it is possible to separate the dependence from the number of leptons $N = N_L + 1$ and quark charges Q_F):

$\beta_0 = \frac{1}{3}(N + 3\sum_F Q_F^2)$ $\beta_1 = \frac{1}{4}(N + \sum_F Q_F^2)$

$$\beta_2 = \frac{1}{32}\Big[- (N + 3\sum_F Q_F^6) + \frac{22}{9}(N + 3\sum_F Q_F^2)^2 \Big]$$

$$\beta_3 = \frac{1}{128}\Big[- 23(N + 3\sum_F Q_F^8) - (N + 3\sum_F Q_F^4)^2 \left(\frac{676}{27} - \frac{352}{9}\zeta_3\right) + \frac{616}{243}(N + 3\sum_F Q_F^2)^3 + (N + 3\sum_F Q_F^4)^2 \left(\frac{352}{9} - \frac{256}{3}\zeta_3\right) \ (light - by - light - scattering\ term) \Big]$$

$\beta_4^{[1]} = \frac{1}{1024}\left(\frac{4157}{6} + \underline{128\zeta_3}\right)(N + 3\sum_F Q_F^{10})$ Notice the appearance of ζ_3-terms in $\beta_4^{[1]}$- scheme-independent part of β_4- did not appear in similar lower terms (in QCD this term has group weight C_F^5)- rather special feature- interesting the check/study (**Kataev (08)**).

In the case of anomalous mass dimension **at present** it us possible to get similar result up to 3-loops $\gamma_0 = \frac{3}{4}$

$\gamma_1 = \frac{1}{16}\left[\frac{3}{2} - \frac{10}{3}(N + 3\sum_F Q_F^2)\right]$

$\gamma_2 = \frac{1}{64}\left[\frac{129}{2} + (-46 + 48\zeta_3)(N + 3\sum_F Q_F^4) - \frac{140}{27}(N + 3\sum_F Q_F^2)^2\right]$

At 4-loops it is not possible to get similar expressions from final result- e.g. the second structure is composed from $(N + 3\sum_F Q_F^6)$ and $(N + 3\sum_F Q_F^4)$-terms. In view of this we neglect the contribution of Q_F and retain N-dependence only:

$$\gamma_3 = \frac{1}{256}\Bigg[(-\frac{1261}{8} - \underline{336\zeta_3}) + (-\frac{280}{3} + 552\zeta_3 - 480\zeta_5)N$$

$$(\frac{304}{27} - 160\zeta_3 + \underline{\underline{96\zeta_4}})N^2 + N^3(-\frac{664}{81} + \frac{128}{9}\zeta_3)N^3$$

$$+ (64 - 480\zeta_3)\, N\ (light - by - light - scattering\ term)\Bigg]$$

Notice the appearance of ζ_3 in $\gamma_3^{[1]}$- in QCD this term is multiplied by C_F^4.

CONCLUSION: For

1) For careful study of $\Gamma_{H\tau^-\tau^+}$ is worth to take into account 2-loop running of mass and 1 -loop coefficient function. At 2-loop we may sum π^2-terms (**Gorishny,Kataev,Larin (84)**, **Bakulev et al- (08-09)**

2) The structure of RG functions γ_m in the previous **THREE** coefficients this ζ_3 did not appear, but appear at the 4-loop level- rather similar to maximal transcendendality property, which appear in conformal-invariant theories.

a) Is this 4-loop property similar to the one, observed in QED at 5-loop level only ? Note, that in Yang-Mills theory with matter these parts of $l - loop$ contributions to the RG functions are proportional th C_F^l. In case of β-function conformal limit.

b) Next question- what is the origin of appearance of ζ_4-term in γ_4?

We hope to clarify these question at Bogolubov Conference (09)