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Electromagnetic structure functions of nucleons in the region of very small x

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Electromagnetic structure functions of nucleons and the total photoabsorption cross sections

The electromagnetic structure functions of nucleons $F_1(Q^2,\nu)$ and $F_2(Q^2,\nu)$ are connected with the total absorption cross sections of a virtual photon with transverse or longitudinal polarization by the simple relations:

$$\sigma_T(Q^2,\nu) = \frac{4\pi^2\alpha}{K} \mathrm{F}_1(Q^2,\nu), \quad \sigma_T(Q^2,\nu) + \sigma_L(Q^2,\nu) = \frac{4\pi^2\alpha}{K}\frac{Q^2+\nu^2}{Q^2} \mathrm{F}_2(Q^2,\nu),$$

Where Q^2 is squared transferred momentum, ν is lab system energy of the photon,

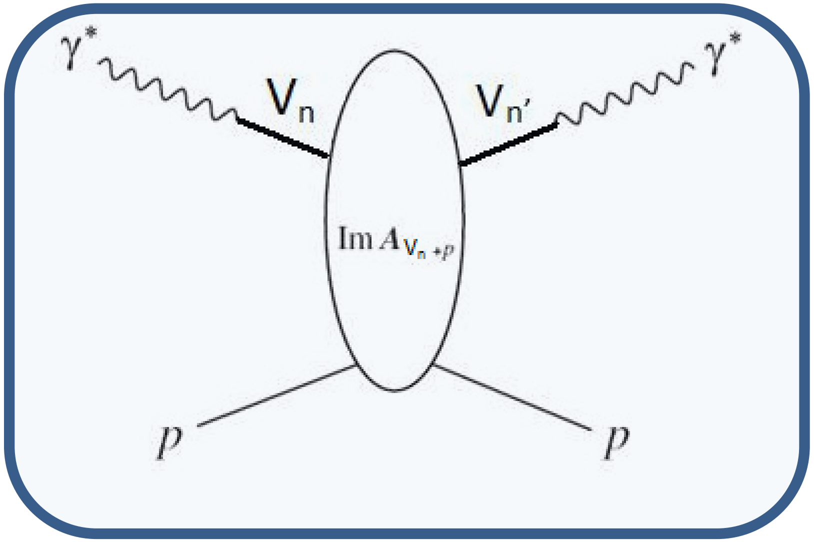
$$\nu = \frac{Q^2}{2M_p} x_{Bj}^{-1} = \frac{s+Q^2-M_p^2}{2M_p} \approx \frac{s}{2M_p}. \qquad K = \nu - \frac{Q^2}{2M_p} \approx \nu.$$

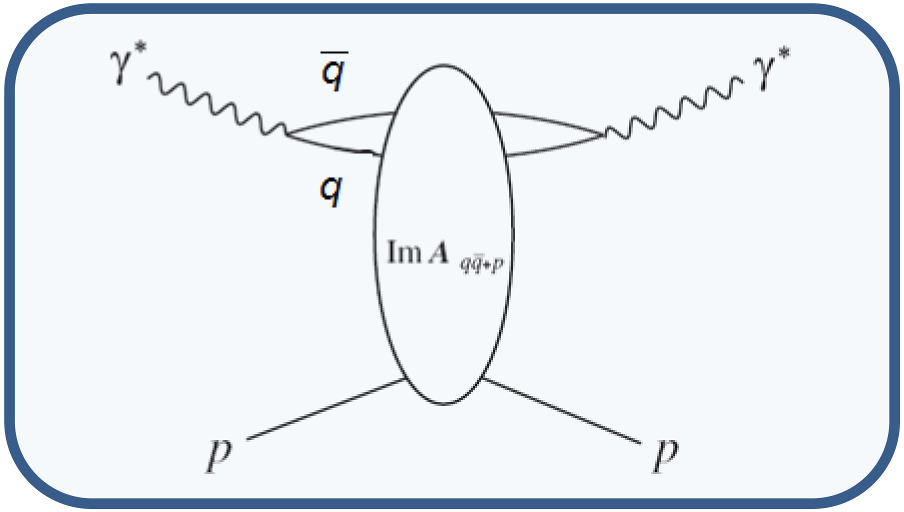
In turn the absorption of the virtual photon by the nucleon $\gamma^* + N \to (hadrons),$

by the optical theorem is connected with the Compton forward scattering of a virtual photon

$$\gamma^* + N \to \gamma^* + N.$$

Compton scattering amplitude





This is GVDM (General Vector Dominance Model) approach for Compton scattering amplitude.

We know that GVDM is inefficient for large Q^2.

And this diagram represents schematically the contribution of $q\bar{q}$ - channel in Compton scattering amplitude.

Perturbative QCD models are not sufficient for a description of electromagnetic structure functions in the region of small Bjorken x and Q^2.

Therefore two component approach is needed (GVDM /soft component/+QCD /hard component/).

Modern approaches for a description of hadrons

For GVDM approach (which is developed below) we need some model of vector mesons.

The following two modern approaches for a description of hadrons are typical:

1) holographic dual of QCD ("hQCD"),

e.g., T.Sakai, S.Sugimoto, hep-th/0507073; J.Hirn, V.Sanz, hep-ph/0507049; A.Karch *et al*, hep-ph/0602229.

2) dimensionally deconstructed QCD ("ddQCD"),

e.g., D.Son, M.Stephanov, Phys.Rev.D69:065020,2004.

The important predictions of the modern QCD models are followings:

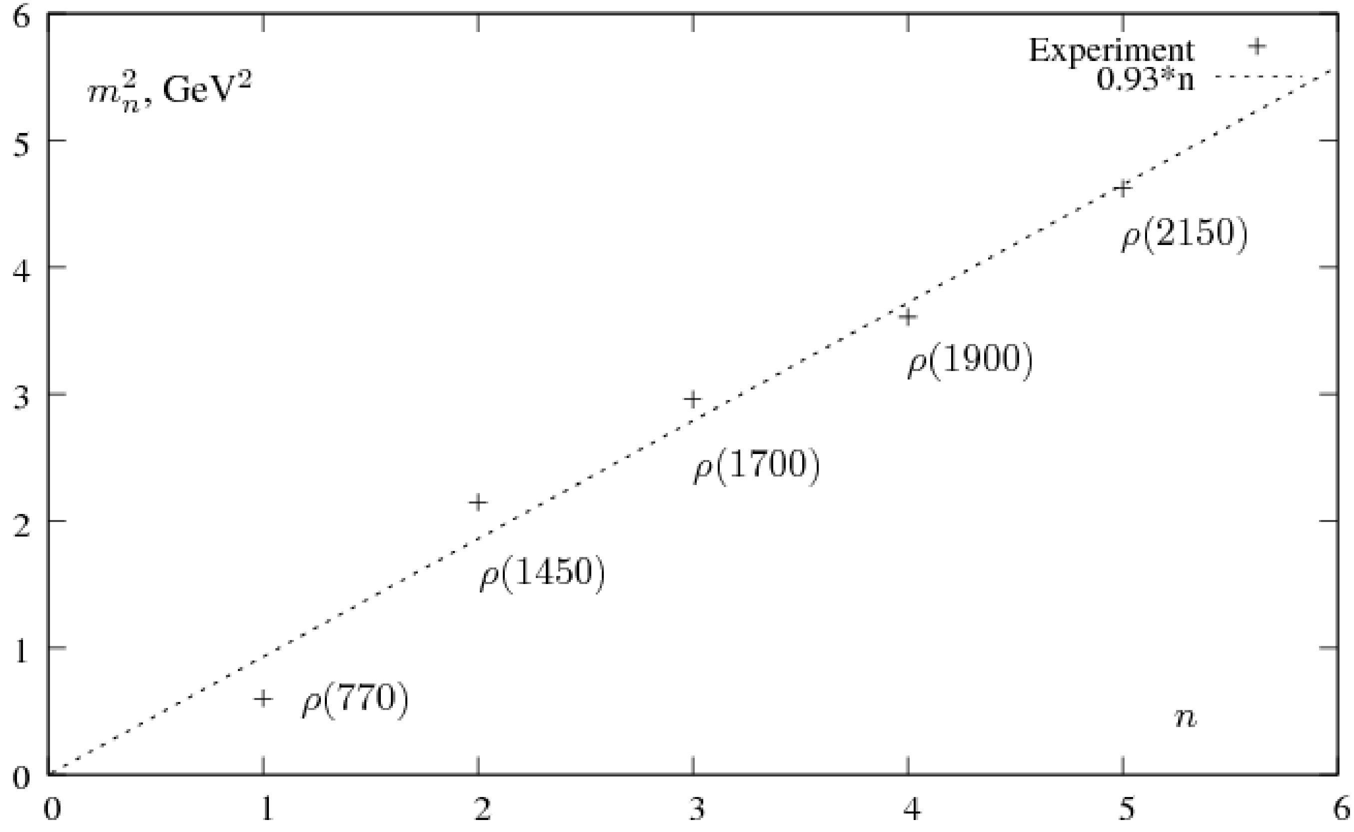
1)The family of vector mesons with infinite numbers of particles: ρ, ρ_1, ρ_2, …, ρ_n, … , etc.

2)The concrete mass spectrum of vector mesons. In the models with "soft-wall" square of meson mass is proportional to the nu $m^2 \sim n$ eson .

Mass spectrum of vector mesons

In models with "soft-wall" square of meson mass is proportional to number of meson

$$m^2 \sim n$$



This prediction is satisfactorily agree with experiment (A.Karch et al, hep-ph/0602229)

Model of the hadronic amplitudes (Bugaev at al,hep-ph/9912384)
(3D-reduction of BS equation)

We used the simplest model of VN-scattering: two-gluon exchange approximation. For a calculation of the corresponding diagrams one must know, in particular, − qq -wave functions of the mesons.

In a relativistic constituent quark model these wave functions are obtained from the Bethe-Salpeter (BS) equation (we consider the case of scalar identical quarks):

$$i(2\pi)^4\Phi(P,q) = \frac{1}{\Delta_1\Delta_2}\int d^4q' K(q-q')\Phi(P,q').$$

Here, $\Phi(P,q)$ is the BS wave function, $P = k_1 + k_2$, $q = \frac{1}{2}(k_1 - k_2)$, $k_{1,2}$ are quark 4-momenta, $\Delta_{1,2} = k_{1,2}^2 - m_q^2$, $K(q-q')$ is the $q\bar{q}$-interaction kernel.

For the utilization of this equation it is convenient to use the quasipotential formalism in a light-front form (see, e.g., W.Jaus, Phys.Rev. D41(1990)3394, S.Chakrabarty et al., Progr.Part.Nucl.Phys. 22(1989)43.).

The corresponding reduction of the BS-equation leads to the three-dimensional equation

$$(\vec{\tilde{q}}^{\,2} + m_q^2 - \frac{M^2}{4})\Phi(\vec{\tilde{q}}) = \frac{1}{16\pi^3 M}\int d\vec{\tilde{q}}\, K(\vec{\tilde{q}} - \vec{\tilde{q}}')\Phi(\vec{\tilde{q}}').$$

Where $\vec{\tilde{q}}$ $(\tilde{q}_\perp = q_\perp;\ \ \tilde{q}_3 \equiv My)$ is three-dimensional so called "inner momentum .

Model of the hadronic amplitudes
(harmonic oscillator kernel)

To solve this equation we assume that the kernel K has only the long range confining term of the hadronic oscillator type (S.Chakrabarty et al., Progr.Part.Nucl.Phys. 22(1989)43):

$$K(\vec{\tilde{q}} - \vec{\tilde{q}}') = (2\pi)^3 \omega^2_{q\bar{q}} (\vec{\nabla}^2_{\tilde{q}} + \omega_0^{-2}) \delta^3(\vec{\tilde{q}} - \vec{\tilde{q}}'),$$

with two parameters: $\omega^2_{q\bar{q}}$ (a "spring constant") and a zero-point energy ω_0.

Then the equation formally coincides with the equation for a quantum-mechanical 3D-oscillator:

$$(\vec{\tilde{q}}^{\,2} + m_q^2 - \frac{M^2}{4}) \Phi(\vec{\tilde{q}}) = \frac{\omega^2_{q\bar{q}}}{2M} (\vec{\nabla}^2_{\tilde{q}} + \omega_0^{-2}) \Phi(\vec{\tilde{q}}).$$

Solutions of this equation, its eigenfunctions and eigenvalues are well known. We will use them for a description of the ρ-family.

Model of the hadronic amplitudes
(mass spectrum)

The mass spectrum of radial excitations is given by the ratio :

$$\frac{1}{2\beta^2}\left(\frac{M^2}{4} - m_q^2 + \beta^4\omega_0^2\right) = N + \frac{3}{2}\ ; \quad N = 0, 2, 4, \ldots\ .$$

and has the linear form $m_n^2 = a + bn$

For $m_{\rho'} = 1.33 GeV$ we obtain $m_n^2 \cong m_\rho^2(1 + 2n)\ ; \quad n = 0, 1, 2, \ldots\ .$

(We assume that $\beta^2 \equiv \omega_{q\bar{q}} / \sqrt{2M}$ is a constant (i.e., is independent on M). In this case the meson mass spectrum has this form.)

With the numerical value $m_{\rho'} = 1.33\ GeV \quad m_q = 0.3\ GeV$

one has $\beta^2 = 0.094\ Gev^2\ , \quad \omega_0^2 = 0.04\ Gev^2$

Model of the hadronic amplitudes
(wave functions and amplitudes)

This is ρ-meson wave function $\Phi_0(\vec{\tilde{q}}) = \Phi_0(\vec{q}_\perp, y) = N_0 exp[-(q_\perp^2 + m_\rho^2 y^2)/2\beta^2],$

From the transverse momentum $q_\perp$ we turn to the dual variable (transverse distance between quarks $\vec{r}_\perp$). In this representation one has

the expression for wave function $\Phi_0(\vec{r}_\perp, y) = N_0 exp[-r_\perp^2 \beta^2/2] exp[-m_\rho^2 y^2/2\beta^2].$

The V_nN-scattering amplitude (diagonal amplitude $V_n N \to V_n N$) has the form

$$F_{nn}(s,t) = \int d^2 r_\perp dy F_{\vec{r}_\perp}(s,t) \Phi_n^2(\vec{r}_\perp, y) \equiv < n \mid F_{\vec{r}_\perp}(s,t) \mid n >,$$

$$F_{\vec{r}_\perp}(s,t) = i\frac{16}{3}\alpha_s^2 s \int \frac{d^2 k_\perp V(\vec{k}_\perp, \vec{Q})}{(\frac{\vec{Q}}{2} - \vec{k}_\perp)^2 (\frac{\vec{Q}}{2} + \vec{k}_\perp)^2} \{e^{-i\frac{\vec{Q}}{2}\vec{r}_\perp} - e^{-i\vec{k}_\perp \vec{r}_\perp}\}.$$

Here, $F_{\vec{r}_\perp}$ is an amplitude for the scattering of the qq -pair with a fixed $\vec{r}_\perp$

on the nucleon (in the two-gluon exchange approximation).
The V-factor describes the ggNN-vertex.

Nondiagonal contributions and cross sections

Similarly nondiagonal amplitude is given by the expression

$$F_{nn'}(s,t) = < n \mid F_{\vec{r}_\perp}(s,t) \mid n' >$$

According to the GVDM total absorption cross sections of a virtual photon with transverse or longitudinal polarization has the form

$$\sigma_{T,L}(Q^2,s) = \sum \frac{e^2}{f_n f_n'} \frac{M_n^2}{M_n^2+Q^2} \frac{M_{n'}^2}{M_{n'}^2+Q^2} \frac{1}{s} Im \mathbf{F}^{T,L}_{n,n'}(s).$$

We assume, as usual, that the coupling constants f_n are proportional to the masses of mesons

$$f_n \sim m_n$$

Now we can calculate the contributions of diagonal and nondiagonal transitions in cross-sections.

The contributions of the diagonal and nondiagonal transitions in a cross-section of the real photon

$V_n + N \to V_{n'} + N$

(mkbn)

n \ n'	0	1	2	3	4	5	6	7	8
0	70.9	20.3	-6.7	2.4	-1.5	0.7	-0.5	0.3	-0.2
1	20.3	40.8	10.0	-4.2	1.4	-1.1	0.4	-0.4	0.2
2	-6.7	10.0	29.8	6.8	-3.2	1.2	-0.9	0.3	-0.4
3	2.4	-4.2	6.8	23.9	5.2	-2.6	1.0	-0.8	0.2
4	-1.5	1.4	-3.2	5.2	19.9	4.1	-2.3	0.9	-0.8
5	0.7	-1.1	1.2	-2.6	4.1	17.2	3.4	-2.0	0.8
6	-0.5	0.4	-0.9	1.0	-2.3	3.4	15.2	2.8	-1.8
7	0.3	-0.4	0.3	-0.8	0.9	-2.0	2.8	13.6	2.4
8	-0.2	0.2	-0.4	0.2	-0.8	0.8	-1.8	2.4	12.3

$$\sigma_{T\ Calculated} = 317\ mkbn$$
$$\sigma_{T\ Experimental} \approx 114\ mkbn$$

The results of calculations for real photon are presented in this table. As the table shows the calculated cross section significantly exceeds the values known from experiment.

We see that the destructive interference effects and corresponding cancellations of $V_n + N \to V_{n'} + N$ amplitudes inside of VDM sums are small. In other words the approach of non-diagonal VDM alone cannot describe the data.

As a result some modification of the standard VDM scheme is needed: cut-off factors reducing the probability of initial *γ-V* transitions must be introduced. (Bugaev, Mangazeev, Shlepin, hep-ph/9912384, Bugaev, Shlepin, Phys.Rev.D67:034027,2003)

Cut-off factors

The first stage of the photoabsorption process is the gamma-qq -transition. The differential probability of this transition is given by this expression

$$dP_{q\bar{q}} \cong C\frac{1}{M^2_{q\bar{q}}}[x^2+(1-x)^2]dxdM^2_{q\bar{q}}.$$

x is the fraction of the photon 3-momentum carried by the quark. An invariant mass of the qq -pair is this

$$M^2_{q\bar{q}} = \sqrt{p_\perp^2+m_q^2}\,/x(1-x).$$

$p_\perp$ Is the transverse momentum.

It follows from here that, at fixed invariant mass of the qq -pair $M^2_{q\bar{q}}$ the relative part of pair's phase volume havir $p_\perp$ smaller then $p_\perp^{max}$ is given by this expression

$$\eta \approx 3\left(\frac{p_\perp^{max}}{M_{q\bar{q}}}\right)^2, \quad for\ M^2_{q\bar{q}} \gg (p_\perp^{max})^2.$$

Where η is cut-off factor, which is proportional to the part of pair's phase volume

More accurate expressions are given in the paper

Bugaev, Shlepin, Phys.Rev.D67:034027,2003)

We assume that vector meson forms if (and only if) the value $p_\perp$ of the quark is smaller than $p_\perp^{max}$. The value of $p_\perp^{max}$ is the model parameter.

GVDM formulas modification

To take the cut-off into account in GVDM formulas the cut-off factors must be introduced: (Bugaev, Mangazeev, Shlepin, hep-ph/9912384)

$$\sigma_{T,L}(Q^2,s) = \sum \frac{e^2}{f_n f_n'} \frac{M_n^2}{M_n^2+Q^2} \frac{M_{n'}^2}{M_{n'}^2+Q^2} \eta_{T,L}(M_n^2, M_{n'}^2, p_\perp^{max}) \frac{1}{s} Im F_{n,n'}^{T,L}(s).$$

Here $\eta_{T,L}(M_n^2, M_{n'}^2, p_\perp^{max})$ are cut-off factors.
(Bugaev, Shlepin, Phys.Rev.D67:034027,2003)

For the magnitude of $p_\perp^{max}$ we have chosen the value ρ-meson mass divided by two

$$p_\perp^{\max} = M_\rho / 2 = 0.385\ \text{GeV}.$$

The contributions transitions $V_n + N \to V_{n'} + N$ in a cross-section of the real photon (mkbn) with accounting of cut-off factors

n \ n'	0	1	2	3	4	5	6	7	8
0	70.94	10.16	-2.61	0.78	-0.44	0.18	-0.13	0.07	-0.04
1	10.16	10.27	1.94	-0.68	0.21	-0.14	0.05	-0.04	0.02
2	-2.61	1.94	4.48	0.86	-0.36	0.12	-0.09	0.03	-0.03
3	0.78	-0.68	0.86	2.56	0.49	-0.23	0.08	-0.06	0.02
4	-0.44	0.21	-0.36	0.49	1.66	0.31	-0.16	0.06	-0.05
5	0.18	-0.14	0.12	-0.23	0.31	1.17	0.21	-0.12	0.04
6	-0.13	0.05	-0.09	0.08	-0.16	0.21	0.88	0.15	-0.09
7	0.07	-0.04	0.03	-0.06	0.06	-0.12	0.15	0.68	0.11
8	-0.04	0.02	-0.03	0.02	-0.05	0.04	-0.09	0.11	0.54

$$\sigma_{T\ Calculated} = 114\ mkbn$$

$$\sigma_{T\ Experimental} \approx 114\ mkbn$$

We see that the introduction of cut-offs motivated by QCD leads to the correct typical value of the calculated photoabsorption cross section.

Energy dependence of cross section (soft component) and hard component

For GVDM (soft) component the energy dependence was chosen in the Regge-type form

$$\sigma_{\gamma p}(s) = 114 * \left(\frac{1.29}{\sqrt{s}} + \left(\frac{s}{1600} \right)^{0.06} \right).$$

(σ in μbn, s in GeV²)

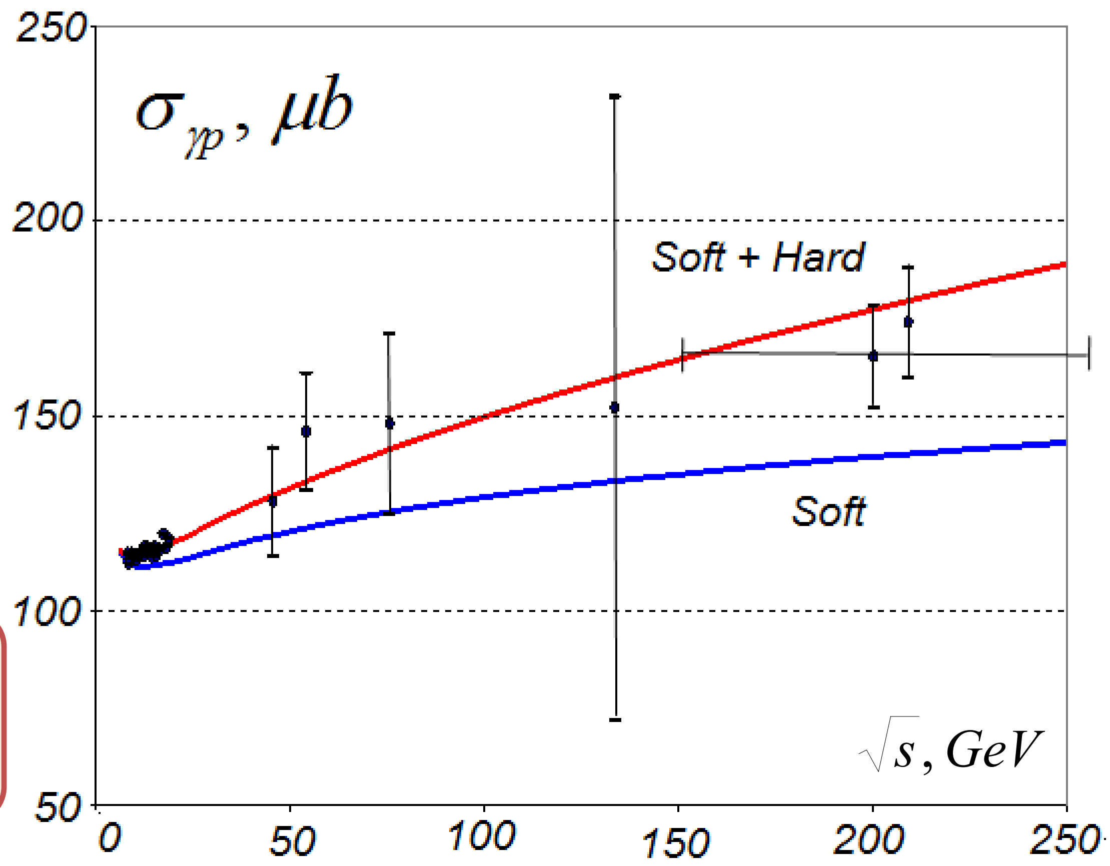
For the hard component, we used the color dipole model and the parameterization of the dipole cross section $\sigma(r_{\perp}, s)$ (p(QCD part) from the work by J.Forshaw, G.Kerley and G.Shaw, Phys.Rev.D60, 074012 (1999).

Result of the calculations for the photoabsorption cross section of real photon

The result of the calculations for the photoabsorption cross section of real photon is shown on the picture:

Blue line is the soft (GVDM) component.

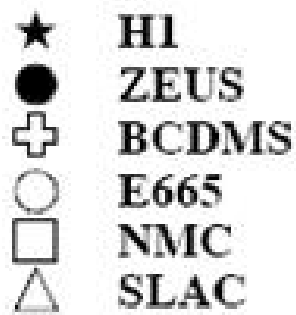
Red line is the total (soft + hard).



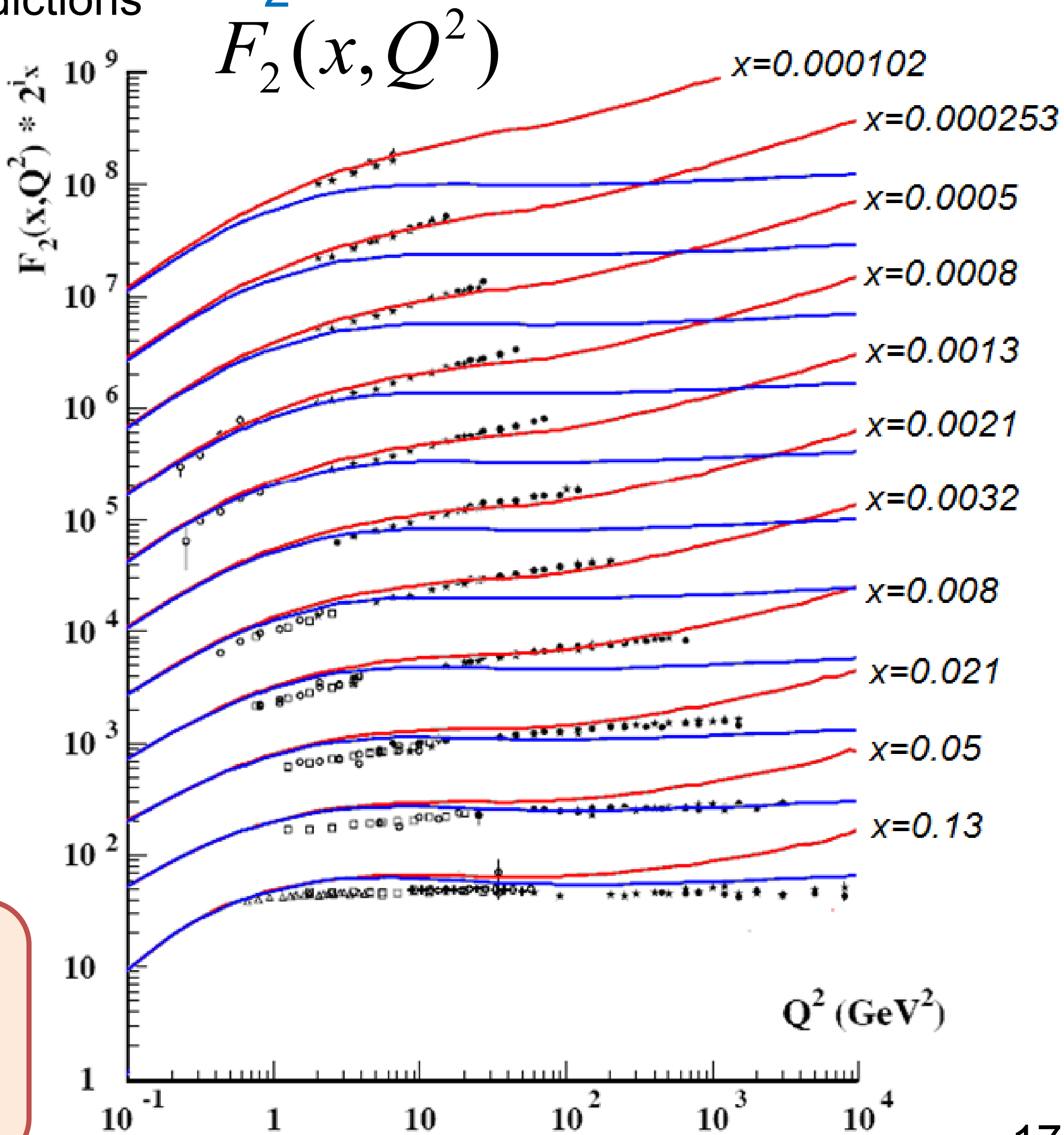
The sum of two components agrees with the experimental data.

Result of the calculations for the structurefunction F_2

This picture presents our predictions for the structure function F_2.



Particle Data Group, 2007



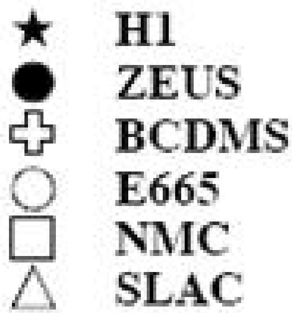
Blue line is the soft (GVDM) component.

Red line is the total (soft + hard).

To achieve a good agreement with the experimental data accounting for the hard contribution is required.

Result of the calculations for the structure function F_2

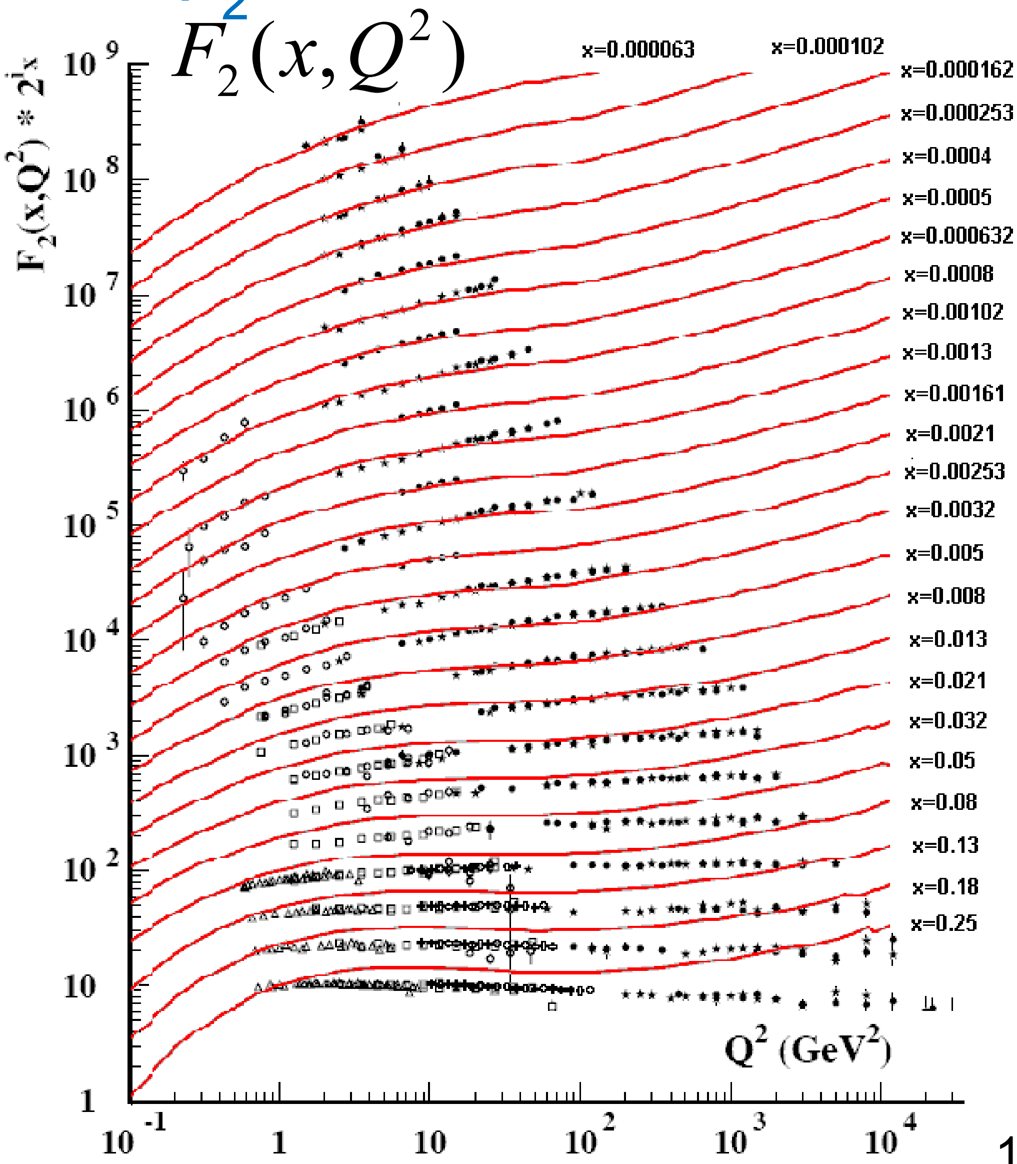
Finally, here only the sum of soft and hard parts is represented.



Particle Data Group, 2007

Red line is the total (soft + hard).

The good agreement with the available data in the region of small x (x < 0.003, Q^2 < 100 GeV^2) is obtained.



CONCLUSIONS

1. If no cut-offs are introduced, GVDM is not able to describe photoabsorption data.

2. The introducing of the cut-off factors motivated by QCD can give the correct predictions.

3. To achieve a good agreement with the experimental data accounting for the hard contribution is required.

4. The present model has, for a description of the soft component, the minimum number of parameters, in fact, only the parameter $p_{\perp}^{max}$.

5. The good agreement with the available data in the region of small x (x < 0.003, $Q^2 < 100$ GeV^2) is obtained.