Neutrino asymmetry and the growth of cosmological seed magnetic field

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(following paper with J.W.F. Valle, published in JHEP-03 (2008) 067)

BBN constraint on neutrino asymmetry

For small electron chemical potential $\mu_e \ll T$ the electron-positron asymmetry $n_- - n_+ = \mu_e T^2/6$ in hot plasma $T \sim \mathrm{O(MeV)}$ obeys also electroneutrality condition $n_- - n_+ = n_p$. Thus, electron asymmetry is negligible before BBN, $\mu_e/T \sim \eta_B \sim n_B/s \sim 10^{-9}$.

As a result from the nucleosynthesis equilibrium condition $\mu_n = \mu_p + \mu_e - \mu_{\nu_e}$ (e.g. in decay $n \to p + e^- + \bar{\nu}_e$) one finds that only *unknown* electron neutrino asymmetry given by the parameter $\xi_{\nu_e} = \mu_{\nu_e}/T$ influences the measurable uncertainty $\mid n_n/n_p \mid \leq 0.05$ of the neutron-proton ratio $n_n/n_p = \exp(-\Delta m/T + \mu_n/T - \mu_p/T) \simeq \exp(-\Delta m/T - \mu_{\nu_e}/T)$, so that

BBN constraint at $T \simeq T_{BBN} \sim 0.1$ MeV is:

$$\mid \xi_{\nu_e} \mid \leq 0.05.$$

On the other hand, neutrino oscillations provide the equivalence $\mu_{\nu_e} \sim \mu_{\nu_\mu} \sim \mu_{\nu_\tau}$ somewhere at $T \sim 3$ MeV (Dolgov et al, 2002).

Problem: Nobody knows behaviour $\xi_{\nu_a}(T)$ at high $T \gg 1$ MeV...

BOUNDS ON COSMOLOGICAL MAGNETIC FIELD

To separate from the Milky Way contribution→ statistical analysis of Faradey rotation measure (RM) was done for extragalactic radio sources (Ruzmaikin & Sokoloff, 1977; Madsen, 1989).

Upper limit at the present time (t_{now})

$$B < 10^{-9} \text{ - } 10^{-10}\ G$$

at a coherence length $L > L_{Jeans} \gg L_{gal} \sim 100\ kpc$.

CMBR anisotropies for homogeneous magnetic field (Chen et al., 2004)):

$$B < 10^{-8}\ G \text{ - } 10^{-9}\ G$$

PLANK (2007) observation of light polarization $+$ RM $\to B = ???$

Central problem: ***how to produce such fields in early Universe?***

Outline

- Long-range hypercharge field Y_μ in SM at $T \gg T_{EW}$
- Equilibrium relations among chemical potentials
- Vector and pseudovector currents
- Polarization of plasma by a seed field $\mathbf{B}_0^Y$
- Maxwell equations for hypercharge fields
- Faraday equation and α^2 dynamo
- Anomalous MHD with Abelian anomaly
- Summary

Hypercharge massless (long-range) field Y_μ at $T \gg T_{EW}$
(W^3_μ vanishes due to mass gap in hot bath $\sim g^2T$)

This field remains at $T \gg T_{EW}$ instead of massless photon

$$A_\mu = Y_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$

and massive Z–boson,

$$Z_\mu = -Y_\mu \sin\theta_W + W^3_\mu \cos\theta_W,$$

which are relevant only after EWPT, $T < T_{EW}$.

SM Lagrangian for hypercharge field Y_μ

$$L=\sqrt{-g}\,\{-\frac{1}{4}Y_{\mu\nu}Y^{\mu\nu}+$$

$$+\sum_{l=e,\mu,\tau}\frac{g' Y^\mu}{2}\left(-\bar{\nu}_{lL}\gamma_\mu\nu_{lL}-\bar{l}_L\gamma_\mu l_L-2\bar{l}_R\gamma_\mu l_R\right)+$$

$$+\sum_i^N\frac{g' Y^\mu}{2}\left(\frac{1}{3}\bar{U}_{iL}\gamma_\mu U_{iL}+\frac{1}{3}\bar{D}_{iL}\gamma_\mu D_{iL}+\frac{4}{3}\bar{U}_{iR}\gamma_\mu U_{iR}-\right.$$

$$\left.-\frac{2}{3}\bar{D}_{iR}\gamma_\mu D_{iR}\right)\;+i\frac{g' Y^\mu}{2}\left[\varphi^+D_\mu\varphi-(D_\mu\varphi^+)\varphi\right]\}$$

$U_i = u, c, t$ $\quad$ $D_i = d, s, b$ - quarks,

$\varphi = \left(\phi^{(+)}\phi^{(0)}\right)^T$ - Higgs doublet

Equilibrium relations among the chemical potentials

$\mu_W = \mu_- + \mu_0$	$(W^- \leftrightarrow \phi^- + \phi^0)$,
$\mu_{D_L} = \mu_{U_L} + \mu_W$	$(W^- \leftrightarrow \bar{U}_L + D_L)$,
$\mu_L^{(l)} = \mu_{\nu_L}^{(l)} + \mu_W$	$(W^- \leftrightarrow \bar{\nu}_L^{(l)} + l_L)$,
$\mu_{U_R} = \mu_0 + \mu_{U_L}$	$(\phi^0 \leftrightarrow \bar{U}_L + U_R)$,
$\mu_{D_R} = -\mu_0 + \mu_{D_L}$	$(\phi^0 \leftrightarrow D_L + \bar{D}_R)$,
$\mu_R^{(l)} = -\mu_0 + \mu_L^{(l)}$	$(\phi^0 \leftrightarrow l_L + \bar{l}_R)$,

where $l = e, \mu, \tau \qquad U = u, c, t, \qquad D = d, s, b,$

Equilibrium relations among the chemical potentials (continuation)

Plasma neutrality

From global $< Q >= 0$, $< Q_3 >= \mu_W = 0$, and

hypercharge neutrality $< Y >= 0$,

$$Y = 2(Q - Q_3) = 2\left[-2\sum_l \mu_L^{(l)} + 6\mu_{uL} + 14\mu_0\right] = 0,$$

one finds the chemical potential for the neutral Higgs boson:

$$\mu_0 = \frac{\sum_l \mu_L^{(l)} - 3\mu_{uL}}{7}.$$

Sphaleron equilibrium condition valid above EWPT:

$$\sum_l \mu_L^{(l)} = -9\mu_{uL}.$$

Maxwell equations for hypercharge fields $\mathbf{E}_Y$ and $\mathbf{B}_Y$

$$\nabla \cdot \mathbf{B}_Y = 0,$$

$$\nabla \cdot \mathbf{E}_Y = 4\pi \left[J_0^Y(\mathbf{x}, t) + J_{05}^Y(\mathbf{x}, t) \right],$$

$$\frac{\partial \mathbf{B}_Y}{\partial t} = -\nabla \times \mathbf{E}_Y,$$

$$-\frac{\partial \mathbf{E}_Y}{\partial t} + \nabla \times \mathbf{B}_Y = 4\pi \left[\mathbf{J}^Y(\mathbf{x}, t) + \mathbf{J}_5^Y(\mathbf{x}, t) \right].$$

Vector currents

Total current:

$$J_\mu^Y = \sum_l J_{l\mu}^Y + 3N J_{(q)\mu}^Y + J_{(\phi)\mu}^Y.$$

In particular, lepton vector current:

$$J_{l\mu}^Y(\mathbf{x}, t) = -\frac{g^{'}}{4}\left[2\delta j_\mu^{l_R}(\mathbf{x}, t) + \delta j_\mu^{l_L}(\mathbf{x}, t) + \delta j_\mu^{\nu_{lL}}(\mathbf{x}, t)\right],$$

where

$$\delta j_\mu^{(a)} = j_\mu^{(a)} - j_\mu^{(\bar{a})} = \int \frac{d^3p}{(2\pi)^3}\frac{p_\mu}{p}\left[f^{(a)}(\mathbf{p}, \mathbf{x}, t) - f^{(\bar{a})}(\mathbf{p}, \mathbf{x}, t)\right].$$

Hypercharge density:

$$J_0^Y = -\gamma n_{eq}\left(\frac{2\pi^2}{9\zeta(3)}\right)\left(\frac{g^{'}}{4T}\right)\left[-2\sum_l \mu_L^l + 6\mu_{uL} + 14\mu_0\right] = 0 \qquad since \quad < Y >= 0,$$

Hypercharge 3-current:

$$\mathbf{J}^Y = \sum_a \frac{f^{(a)}(g^{'})}{2}\gamma n_{eq}[\mathbf{V}^{(a)} - \mathbf{V}^{\bar{a}}].$$

Pseudovector currents

$$J^{Y}_{\mu 5} = \sum_{l} J^{Y}_{l\mu 5}(\mathbf{x},t) + 3N J^{Y}_{(q)\mu 5}(\mathbf{x},t)$$

In particular, the lepton pseudovector current:

$$J^{Y}_{l\mu 5}(\mathbf{x},t) = -\frac{g^{'}}{2}\int \frac{d^3p}{p(2\pi)^3}\delta A^{(l_R)}_{\mu}(\mathbf{p},\mathbf{x},t) + \frac{g^{'}}{4}\int \frac{d^3p}{p(2\pi)^3}\delta A^{(l_L)}_{\mu}(\mathbf{p},\mathbf{x},t) +$$

$$+\frac{g^{'}}{4}\int \frac{d^3p}{p(2\pi)^3}\delta A^{(\nu_{lL})}_{\mu}(\mathbf{p},\mathbf{x},t),$$

where $\delta A^{(a)}_{\mu}(\mathbf{p},\mathbf{x},t) = A^{(a)}_{\mu}(\mathbf{p},\mathbf{x},t) - A^{(\bar{a})}_{\mu}(\mathbf{p},\mathbf{x},t)$ and

$$A^{(a)}_{\mu}(\mathbf{p},\mathbf{x},t) = \left[(\mathbf{p}\cdot\mathbf{S}^{(a)}(\mathbf{p},\mathbf{x},t)); \frac{\mathbf{p}(\mathbf{p}\cdot\mathbf{S}^{(a)}(\mathbf{p},\mathbf{x},t))}{p}\right]$$

Pseudovector currents (continuation)

$$J^Y_{\mu 5} = \sum_l J^Y_{l\mu 5}(\mathbf{x},t) + 3N J^Y_{(q)\mu 5}(\mathbf{x},t)$$

Substituting equilibrium spin distributions:

$$\mathbf{S}_0^{(a,\bar{a})}(p) = -\frac{\mid f_a(g^{'}) \mid \mathbf{B}_0^Y}{2p}\frac{\mathrm{d}f_0^{(a,\bar{a})}(p)}{\mathrm{d}p},$$

one gets

$J^Y_{05} = 0$ since odd integrand : $\displaystyle\int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}(\mathbf{p}\mathbf{S}_0^{(\mathrm{a},\bar{\mathrm{a}})}(\mathrm{p})) = 0,$

while 3-vector part differs from zero in equilibrium plasma:

$$(\mathbf{J}_5^Y)_{eq} = \frac{g^{'2}}{96\pi^2}\left[-2\sum_l \mu_L^l + 10\mu_{uL} + 32\mu_0\right]\mathbf{B}_0^Y = \frac{47}{1512\pi^2}g^{'2}\mu_\nu \mathbf{B_0^Y} \neq \mathbf{0}$$

Dirac equation for massless fermions in a seed large-scale hypermagnetic field, $\mathbf{B}_0 = \nabla \times \mathbf{Y}^{(0)} = (0, 0, B_0^Y)$

$$\left[\hat{p} - f^{(a)}(g^{'})\hat{Y}^{(0)}\right]\Psi^{(a)} = 0, \qquad f^{(a)}(g^{'}) = \frac{g^{'}y_a}{2},$$

where y_a is the hypercharge.

Landau spectrum in JWKB approximation, $g^{'}B_0^Y \ll T^2$:

$$\varepsilon(p_z, n, \lambda) = \sqrt{p_z^2 + \mid f^{(a)}(g^{'}) \mid B_0^Y(2n + 1 \mp \lambda)} \approx p \mp \mid f^{(a)}(g^{'}) \mid B_0^Y \frac{\lambda}{2p}$$

where $p = \sqrt{p_z^2 + p_\perp^2}$ and $p_\perp^2 = \mid f^{(a)}(g^{'}) \mid B_0^Y(2n + 1)$.

Equilibrium density matrix and spin distribution in JWKB

$$f_{\lambda' \lambda}^{(a,\bar{a})} = \frac{\delta_{\lambda' \lambda}}{\exp[(\varepsilon(p_z, n, \lambda) \mp \mu_a)/T] + 1} \approx \frac{\delta_{\lambda' \lambda}}{2} f_0^{(a,\bar{a})}(p) + \frac{\sigma^j_{\lambda' \lambda}}{2} S_0^{(a,\bar{a})j}(p),$$

POLARIZATION MECHANISM

Equilibrium density matrix and spin distribution in JWKB

$$f_{\lambda'\lambda}^{(a,\bar{a})} = \frac{\delta_{\lambda'\lambda}}{\exp[(\varepsilon(p_z, n, \lambda) \mp \mu_a)/T] + 1} \approx \frac{\delta_{\lambda'\lambda}}{2} f_0^{(a,\bar{a})}(p) + \frac{\sigma^j_{\lambda'\lambda}}{2} S_0^{(a,\bar{a})j}(p),$$

where equilibrium spin distribution

$$\mathbf{S}_0^{(a,\bar{a})}(p) = -\mathbf{B}_0^Y \frac{\mid f_a(g') \mid}{2\mathrm{p}} \frac{\mathrm{d} f_0^{(a,\bar{a})}(\mathrm{p})}{\mathrm{dp}} = \frac{\mathbf{B}_0^Y}{B_0^Y} S_0^{(a,\bar{a})}(p).$$

PSEUDOVECTOR CURRENTS IN POLARIZED MEDIUM

$$J_{\mu 5}^Y = \frac{g'}{4} \sum_{\lambda,\lambda'} < \hat{\bar{\Psi}}_{\mathbf{p}\lambda} \gamma_\mu \gamma_5 \hat{\Psi}_{\mathbf{p}'\lambda'} >, \qquad m_f \to 0.$$

$$J_{05}^Y \sim \int \frac{d^3p}{(2\pi)^3} \frac{(\mathbf{p} \cdot \mathbf{S}^{(a)}(\mathbf{p}, \mathbf{x}, t))}{p} \to 0, \quad \text{if} \;\; \mathbf{S}^{(a)}(\mathbf{p}, \mathbf{x}, t) \to \mathbf{S}_0^{(a)}(p),$$

$$\mathbf{J}_5^Y \sim \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}(\mathbf{p} \cdot \mathbf{S}^{(a)}(\mathbf{p}, \mathbf{x}, t))}{p^2} \sim \mathbf{B}_0^Y \neq 0, \quad \text{if} \;\; \mathbf{S}^{(a)}(\mathbf{p}, \mathbf{x}, t) \to \mathbf{S}_0^{(a)}(p).$$

Maxwell equations for hypercharge fields $\mathbf{E}_Y$ and $\mathbf{B}_Y$ in hot plasma at $T \gg T_{EW}$

$$\nabla \cdot \mathbf{B}_Y = 0,$$

$$\nabla \cdot \mathbf{E}_Y = 0,$$

$$\frac{\partial \mathbf{B}_Y}{\partial t} = -\nabla \times \mathbf{E}_Y,$$

$$-\frac{\partial \mathbf{E}_Y}{\partial t} + \nabla \times \mathbf{B}_Y = 4\pi \left[\mathbf{J}^Y(\mathbf{x}, t) + \frac{47}{378} \times \frac{g'^2 \mu_\nu}{\pi} \mathbf{B}_Y \right],$$

where $\quad \mu_\nu = \sum_l \mu_{\nu_L}^{(l)}, \quad l = e, \mu, \tau.$

Faraday equation and α^2 dynamo

$$\frac{\partial \mathbf{B}_Y}{\partial t} = \nabla \times \alpha \mathbf{B}_Y + \eta \nabla^2 \mathbf{B}_Y,$$

where $\alpha(t) = \dfrac{47 g^{'2} \mu_\nu(T)}{1512 \pi^2 \sigma_{cond}(T)}$, $\eta(t) = \dfrac{1}{4\pi \sigma_{cond}(T)}$.

Solution for any scale k^{-1}

$$B_Y(k,t) = B_0^Y \exp\left[\int_{t_0}^{t} [\alpha(t^{'})k - \eta(t^{'})k^2)] \mathrm{d}t^{'}\right],$$

Fastest growth, $k = \alpha/2\eta$; $\xi_\nu(T) = \mu_\nu(T)/T$, $x = T/T_{EW} \geq 1$:

$$B_Y(t) = B_0^Y \exp\left[\int_{t_0}^{t} \frac{\alpha^2(t^{'})}{4\eta(t^{'})} \mathrm{d}t^{'}\right] = B_0^Y \exp\left[32 \int_{x}^{x_0} \frac{\mathrm{d}x^{'}}{x^{'2}} \left(\frac{\xi_\nu(x^{'})}{0.001}\right)^2\right].$$

Survival against ohmic dissipation and change of spatial scales

$$\Lambda > l_{diff},$$

$$l_{diff} = \sqrt{\eta l_H}, \qquad \eta = \frac{1}{4\pi\sigma_{cond}},$$

for $\Lambda \sim \eta/\alpha$, $\alpha = 47g'^2\mu_\nu(T)/1512\pi^2\sigma_{cond}$ leads to the bound on the neutrino chemical potential $\xi_\nu(T) = \mu_\nu(T)/T$, $\mu_\nu(T) = \Sigma_{l=e,\mu,\tau}\,\mu^{(l)}_{\nu_L}(T)$:

$$\frac{\xi_\nu(x)}{0.001} < A(\Lambda)\sqrt{x}, \qquad x = \frac{T}{T_{EW}} > 1,$$

which depends on chosen scale Λ. E.g. for the fastest growth $\Lambda = 2\eta/\alpha$ one gets $A(\Lambda) = 0.23$ and factor ~ 32 in the exponential amplification of $B_Y(t)$.

Anomalous MHD (AMHD) with right electrons at $T \gg T_{EW}$

(Giovannini + Shaposhnikov, 1997)

$$\frac{\partial \vec{B}_Y}{\partial t} = -\frac{g'^2}{4\pi^3 \sigma_{cond} a(\tau)} \nabla \times \left(\mu_R \vec{B}_Y \right) + \frac{1}{4\pi\sigma_{cond}} \nabla^2 \vec{B}_Y$$

In FRW metric

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = a^2(\tau)\left(d\tau^2 - d\vec{x}^2\right)$$

conformal time τ and cosmic time t are related as $dt = a(\tau) d\tau$

- **Abelian anomaly and evolution of chemical potential**

$$\partial_\mu j_R^\mu = -\frac{g'^2 y_R^2}{64\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu},$$

- The product in r.h.s (hypercharge field and its dual) $\sim \vec{B}_Y \cdot \vec{E}_Y$ enters the kinetic equation of the right electron chemical potential (Giovannini, Shaposhnikov, 1997)

$$\frac{\partial \mu_R}{\partial t} = -\frac{g'^2}{4\pi T^2}\frac{783}{88}\vec{B}_Y \cdot \vec{E}_Y - \Gamma\mu_R$$

Γ is the perturbative chirality-changing rate ,

$$\vec{E}_Y \sim \nabla \times \vec{B}_Y / 4\pi\sigma_{cond}$$

Scenarios by Shaposhnikov et al (1997-2000). (continuation)

- **Abelian anomaly**

$$\partial_\mu j^\mu_R = -\frac{g'^2 y_R^2}{64\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu},$$

- The product in r.h.s (hypercharge field and its dual) $\sim \vec{B}_Y \cdot \vec{E}_Y$ is governed by anomalous MHD for hypercharge fields.

Chern-Simons change $\int_{t_1}^{t_2} dt(\partial j^0_R/\partial t) = \Delta N_R = (1/2) y_R^2 \Delta N_{CS}$

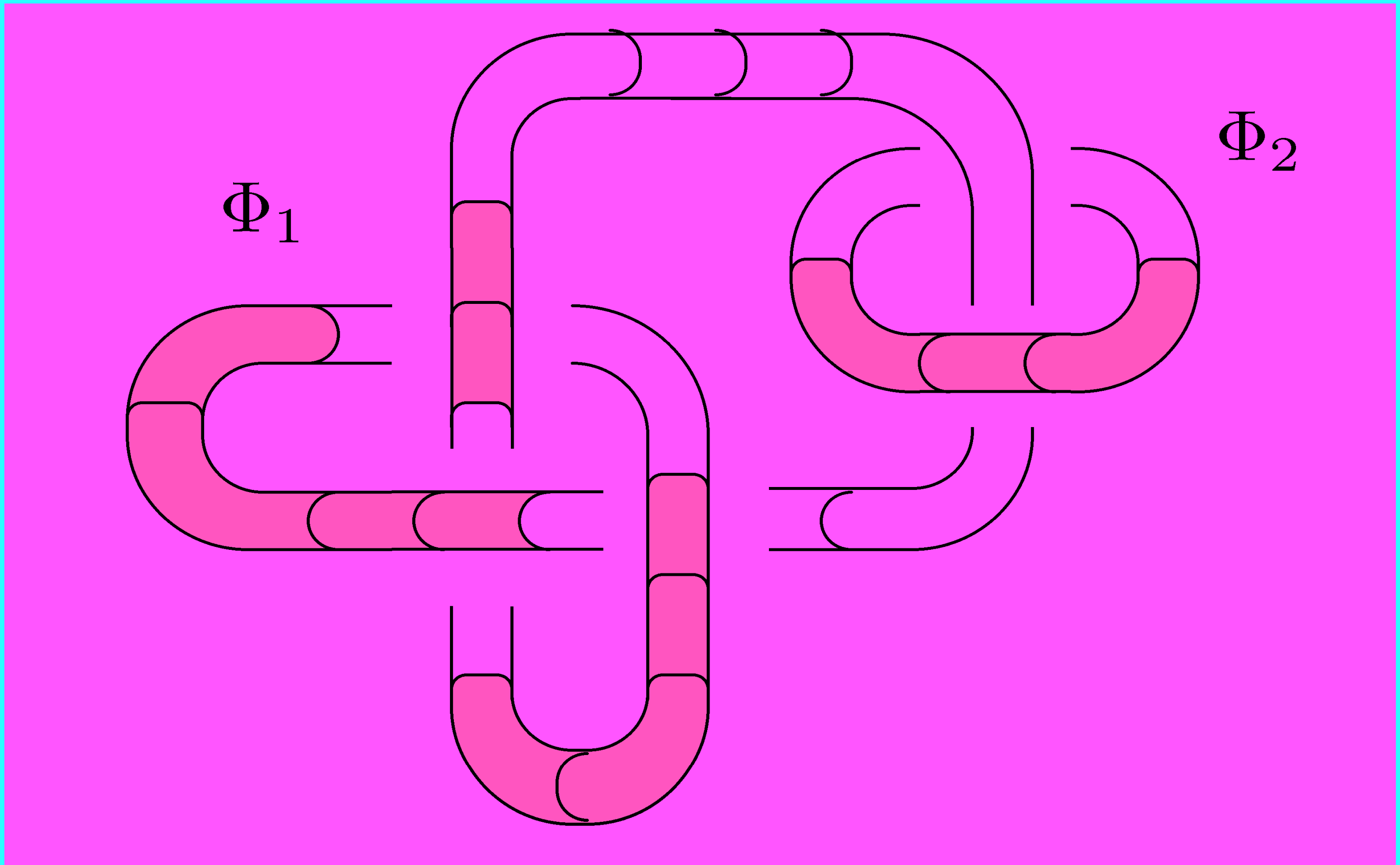
is given by

hypermagnetic helicity

$$N_{CS} = -\frac{g'^2}{32\pi^2} \int d^3x (\vec{B}_Y \cdot \vec{Y})$$

Hypermagnetic fluxes $\Phi = \oint \mathbf{B}_Y \cdot \mathrm{d}\mathbf{S}$ and topology (linkage) number m (Chern-Simons analogue)

$$H = \int_v \mathrm{d}^3 x (\mathbf{B}_Y \cdot \mathbf{Y}) = m \Phi_1 \Phi_2$$



Summary

1) Primordial cosmological hypermagnetic fields **polarize the early Universe plasma** prior EWPT. As a result of the long-range parity violating gauge interaction presented in SM their magnitude $B_Y(t)$ gets amplified opening a new perturbative way of seeding the primordial Maxwellian magnetic field at EWPT.

2) **Polarization mechanism** based here on standard plasma physics and SM for particle physics differs from mechanism based on Abelian anomaly for right electrons suggested in works by Shaposhnikov et al.

3) Conversion of amplified hypermagnetic field $\mathbf{B}_Y$ into Maxwellian $\mathbf{B}$ at EWPT is an *open problem to be studied*, especially, accounting for evolution of hypermagnetic helicity $\mathrm{d}\mathcal{H}/\mathrm{dt} \sim \int d^3x \mathbf{Y} \cdot \mathbf{B}_Y$ (Chern-Simons number jump) and leptogenesis (or baryogenesis).

4) *Other problems.* Evolution of primordial Maxwellian field as a seed of galactic field. Inverse cascade, change of scales. Impact on CMBR and LSS.

Motivations to study neutrino asymmetry + hypermagnetic fields

1) At present (z=0) **lepton asymmetry** could exist only **in neutrinos** while it is very difficult to measure and compare densities of relic neutrinos and antineutrinos checking $n_\nu - n_{\bar{\nu}} \sim \xi_\nu = \mu_\nu/T$.

2) We showed in MSM that even without anomalies at the classical level (or $\partial_\mu j_5^\mu = 0$ for massless particles at $T \gg T_{EW}$) a small **neutrino asymmetry** $\xi_\nu \sim 0.001$ **leads to STRONG amplification of hypermagnetic field** $\mathbf{B}_Y(T)$ (Semikoz, Valle, 2008).

3) The presence of **strong primordial hypercharge magnetic fields** at the electroweak epoch may make an EWPT **strongly first order even for large Higgs masses** that removes a main objection against the possibility of baryon asymmetry generation in MSM (Shaposhnikov, 1986; Dolgov, 1992; Cohen, Kaplan, Nelson, 1993).

4) The **hypermagnetic helicity change** $(\sim (\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y)$ can itself lead to **baryogenesis** near $T \sim T_{EW}$ if we add Abelian anomaly (Shaposhnikov, Giovannini, 1998). The point is that the **fermion number can sit** not only in the fermions (and in their associated chemical potentials, like μ_{eR}), but also **in the hypermagnetic field itself**.

5) **Seeding cosmological Maxwellian field by hypermagnetic fields** and impact of primordial fields on CMBR anisotropies and LSS. Some groups continue to study...

Faraday equation in SM (from $\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$)
(V.S. & D.Sokoloff, 2004)

$$\partial_t \mathbf{B} = \nabla \times \alpha \mathbf{B} + \eta \nabla^2 \mathbf{B}$$

$\eta = (4\pi 137 T)^{-1}$ is the diffusion coefficient

The scale of mean field $\Lambda = \eta/\alpha$

$$\frac{\Lambda}{l_H} = 1.6 \times 10^9 \left(\frac{T}{\mathbf{MeV}}\right)^{-5} \left(\frac{\lambda^{(\nu)}_{\mathtt{fluid}}}{l_\nu(T)}\right) \mid \xi_{\nu_e}(T) \mid^{-1}$$

scale $\Lambda \geq l_H$ for $\lambda^{(\nu)}_{\mathtt{fluid}} \sim l_\nu(T)$ at temperature $T \lesssim 100\ \mathbf{MeV}$

Strength of magnetic field in α^2 dynamo

$$B(t) = B_{\max} \exp\left(\int_{t_{\max}}^{t} \frac{\alpha^2(t')}{4\eta(t')} dt' \right)$$

$$B(x) = B_{\max} \exp\left[25 \int_{x}^{1} \left(\frac{\xi_{\nu_e}(x')}{0.07} \right)^2 x'^{10} dx' \right]$$

$$x = T/20\ GeV$$

BBN bound (A.D.Dolgov et al, 2002):

electron neutrino chemical potential $\xi_{\nu_e} = \mu_{\nu_e}/T \lesssim 0.07$

Magnetic helicity

The definition: $H(t) = \int d^3x \mathbf{A}(\mathbf{x},t) \cdot \mathbf{B}(\mathbf{x},t)$ is the magnetic helicity (Gauss was first who calculated the knots number m, $H = 2m\Phi_1\Phi_2$).

The topology number m shows the ***linkage and tangling*** of magnetic force lines and this is a good integral of motion in MHD: it is conserved much better (decays much slower) than the magnetic energy in viscous matter. It is also ***GAUGE-INVARIANT*** under transformation $\mathbf{A}(\mathbf{x},t) \to \mathbf{A}(\mathbf{x},t) + \nabla\chi$ and supports the evolution of magnetic field (via inverse cascade) to large-scale fields from the small-scale ones.

The change of helicity (in gauge $\mathbf{E} = -\partial\mathbf{A}/\partial t$) using also $\partial\mathbf{B}/\partial t = -\nabla \times \mathbf{E}$ is given by

$$\frac{dH}{dt} = -2\int d^3x \mathbf{E} \cdot \mathbf{B},$$

and in ideal plasma ($\sigma_{cond} = \infty$, standard MHD with Lorentz force only, $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$) helicity is conserved.

Magnetic helicity change in SM with neutrinos

the standard $\frac{dH}{dt} = -2\int_{\mathbf{v}} (\mathbf{E}\cdot\mathbf{B})d^3x$ gives

$$\frac{dH}{dt} = -2\eta\int_{\mathbf{v}} d^3x(\nabla\times\mathbf{B})\cdot\mathbf{B} + 2\alpha\int_{\mathbf{v}} d^3xB^2$$

$$H(t) = 2B^2_{\max}\int_{\mathbf{v}} d^3x\int_{t_{\max}}^{t} dt'\,\alpha(t')\exp\left[2\int_{t_{\max}}^{t'}\left(\frac{\alpha^2(t'')}{4\eta(t'')}\right)dt''\right] + H_{\max}$$

Left-handed ($H<0$) or right-handed ($H>0$) magnetic helicity? Sign of $\alpha(t)$?

Magnetic helicity density in early Universe

(V.S. & D. Sokoloff, 2005)

$$h(x) = 2.4 \times 10^3 B_{\max}^2 m_e^{-1} J(x)$$

$$J(x) = \int_x^1 \left(\frac{\xi_{\nu_e}(x')}{0.07} \right) x'^3 dx' \exp\left[50 \int_{x'}^1 x''^{10} \left(\frac{\xi_{\nu_e}(x'')}{0.07} \right)^2 dx'' \right] \sim 10$$

where $B_{\max} = \kappa (T_{\max})^2/e = \kappa \times 7 \times 10^{22}\ G \ll B_{EW} = T_{EW}^2/e \sim 10^{24}\ G$

WKB parameter $\kappa \ll 1$, e.g. $\kappa = 0.01$ obeys

BBN limit at $T = 0.1\ MeV:\ B < 10^{11}\ G.$

Finally:

$$h(x) \simeq 4.5 \times 10^{38} \kappa^2 J(x)\ G^2 cm \sim 4.5 \times 10^{39} \kappa^2\ G^2 cm$$

Uncertain sign of α – parameter and magnetic helicity

$$\alpha \sim \frac{\delta n_\nu}{n_\nu} = \sum_a c^{(A)}_{\nu_a}\frac{\delta n_{\nu_a}}{n_{\nu_a}} = \frac{2\pi^2}{9\varsigma(3)}[\xi_{\nu_\mu}(T) + \xi_{\nu_\tau}(T) - \xi_{\nu_e}(T)]$$

$c^{(A)}_{\nu_a} = \pm 0.5$ is the axial coupling for neutrinos in SM (lower sign for electron neutrinos),

$\xi_{\nu_a}(T) = \frac{\mu_{\nu_a}}{T}$ dimensionless chemical potential

BBN bound at $T_{BBN} \simeq 0.1$ MeV on ξ_{ν_e} :

$|\xi_{\nu_e}| \lesssim 0.07$, Dolgov et al., 2002

CMBR/LSS bound: $|\xi_{\nu_{\mu,\tau}}| \lesssim 2.6$, Hansen et al., 2001