Unstable-particles pair production in MPT approach in NNLO

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The problem:

A description of processes of production and decays of fundamental unstable particles for needs of colliders of next generation (ILC) must provide
(i) gauge cancellations and unitarity;
(ii) enough high accuracy of calculation of resonant contributions

Modified perturbation theory (MPT) permits direct expansion of the cross-section *in powers of the coupling constant* with the aid of distribution-theory methods

$\Downarrow$

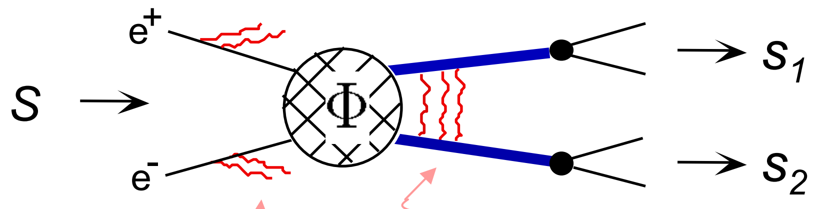
The gauge invariance should be maintained

The accuracy of description of resonant contributions = ?

To clear up this question, I do numerical simulation in the MPT up to the NNLO

(with taking into consideration universal massless-particles contributions)

Double-resonant contributions to pair production and decay of unstable particles:



$$\sigma(s) = \int_{s_{\min}}^{s} \frac{\mathrm{d}s'}{s}\, \phi(s'/s;\, s)\, \hat{\sigma}(s')\,, \qquad \hat{\sigma}(s) = \iint \mathrm{d}s_1\, \mathrm{d}s_2\;\; \hat{\sigma}(s\,;s_1,s_2)\,(1+\delta_c)$$

$$\hat{\sigma}(s\,;s_1,s_2) = \frac{1}{s^2}\,\theta(\sqrt{s}-\sqrt{s_1}-\sqrt{s_2}\,)\,\sqrt{\lambda(s,s_1,s_2)}\;\Phi(s;s_1,s_2)\,\rho(s_1)\,\rho(s_2)$$

$$\rho(s_i) = \frac{M\Gamma}{\pi}\,\frac{1}{|s_i - M^2 + \alpha\Sigma(s_i)|^2}$$

MPT $\Rightarrow$ expansion of the cross-section in powers of α

Ingredients of MPT:

- Asymptotic expansion of BW factors in powers of α / F.Tkachov,1998 /

$$\rho(s) = \frac{M\Gamma_0}{\pi}\,\frac{1}{|s - M^2 + \Sigma(s)|^2} = \delta(s-M^2) + PV\,\mathcal{T}\,[\,\rho(s)\,] + \sum_n c_n(\alpha)\,\delta^{(n)}(s-M^2)$$

Taylor in α

Polinomial in α

NNLO :

3-loop

$$= \delta(s-M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\,\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s-M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s-M^2) + O(\alpha^3)$$

- Analytic regularization of the kinematic factor / M.Nekrasov,2007 /

$$\sqrt{\lambda(s,s_1,s_2)} \longrightarrow \lim_{\nu\to 1/2}\underbrace{\left\{\,\lambda(s,s_1,s_2)\,\right\}^{\nu}}$$

analytic calculation of "singular" integrals

- Conventional-perturbation-theory definition of the "test" function Φ

Model for testing MPT :

- Test function Φ :

  tree-level for $e^+e^- \to \gamma, Z \to t\,\bar{t} \to W^+b\,W^-\bar{b}$

- Breigt-Wigner factors :

  three-loop contributions to self-energy $(\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^3\Sigma_3)$

- Universal soft massless-particles contributions :

  Flux function in leading-log approximation:

$$\phi(z;s) = \beta_e(1-z)^{(\beta_e-1)} - \frac{1}{2}\beta_e(1+z)\,, \qquad \beta_e = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

  Coulomb singularities through one-gluon exchanges:

$$\delta_c = \kappa\,\frac{\alpha_s\pi}{2\beta}\left[1 - \frac{2}{\pi}\,\arctan\left(\frac{|\beta_M|^2 - \beta^2}{2\beta\,\mathrm{Im}\beta_M}\right)\right]$$

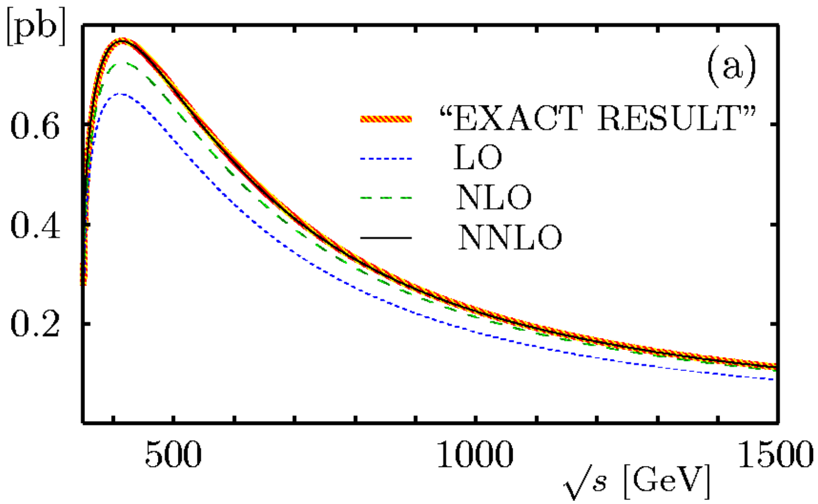
$$\beta = s^{-1}\sqrt{\lambda(s, s_1, s_2)}$$

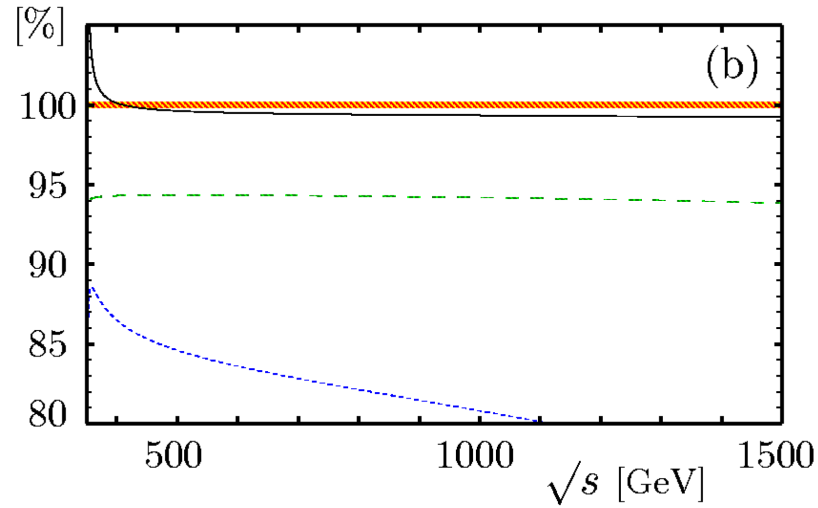
$$\beta_M = \sqrt{1 - 4(M^2 - \mathrm{i}M\Gamma)/s}$$

Results :

Total cross-section $\sigma(s)$:

$M_t = 175$ GeV
$M_W = 80.4$ GeV $\quad M_b = 0$





$\sqrt{s}$ [GeV]	σ [pb]	σ_{LO}	σ_{NLO}	σ_{NNLO}
500	0.6724 100%	0.5687 84.6%	0.6344 94.3%	0.6698(7) 99.6(1)%
1000	0.2255 100%	0.1821 80.8%	0.2124 94.2%	0.2240(2) 99.3(1)%

Conclusion

In the case of pair production and decays of unstable particles:

- MPT stably works at the energies near the maximum of the cross-section and higher

- At 500-1000 GeV (the energies of ILC) MPT in NNLO provides approximately 0.5% accuracy

Comment:

- The higher precision is possible if proceeding to NNNLO or if using NNLO of MPT for calculation of loop contributions only (as, for instance, it is made at applying DPA)