Production of kink–antikink pair in collisions of high–energy particles

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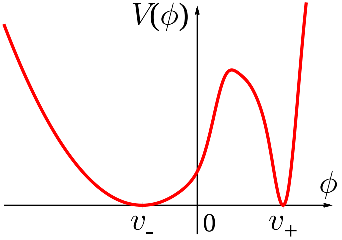
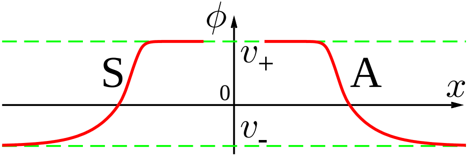
Topological solitons in weakly coupled theories

$\boxed{\phi}$ $(1+1)$ $\hbar = c = 1$

$$S = \frac{1}{g^2}\int dxdt \left[(\partial_\mu \phi)^2/2 - V(\phi)\right]$$

- semiclassical parameter
- coupling constant ($\phi = g\phi'$)

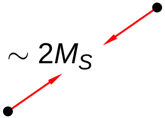


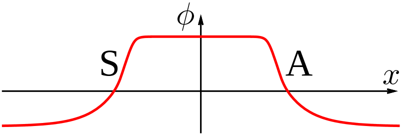


Properties: $L_S \sim m^{-1}$
$M_S \sim m/g^2$

From particles to solitons

$E \sim 2M_S$





$\lambda \sim 1/E \sim g^2/m$ $\Downarrow$

$L_S \sim 1/m$

Exponential suppression!

$$\boxed{\mathcal{P}(E) \approx A \cdot \mathrm{e}^{-F(E)/g^2}}$$

- Coherent–state "estimate"

  $\bar{n}_S \sim M_S/m \sim 1/g^2$

  $\langle 2|SA\rangle \sim \mathrm{e}^{-\bar{n}_S/2} \sim \mathrm{e}^{-\mathrm{c}/g^2}$

  Drukier, Nussinov (1982)

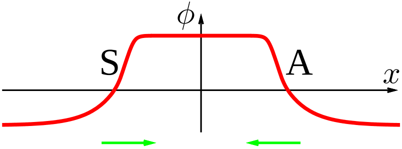
  Banks at al. (1990)

- Unitarity arguments *Zakharov (1991)*

No reliable estimate of $\mathcal{P}(E)$ so far!

Aim: calculate semiclassically $F(E)$.

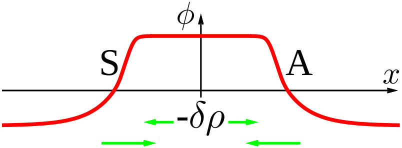
Introducing potential barrier

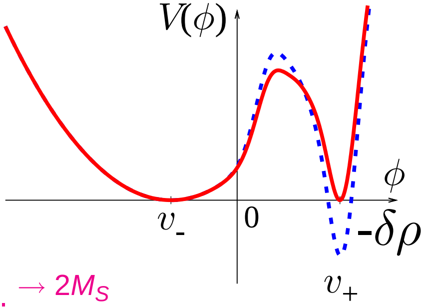


$\Longrightarrow$ Many particles

Attraction!

Not a tunneling process?





Critical bubble $\Leftrightarrow$ Barrier top

$\delta\rho \longrightarrow 0$: cr. bubble $\longrightarrow$ SA; $E_{\text{cr.b.}} \rightarrow 2M_S$

In–state

Rubakov, Son, Tinyakov, 1992

Not semiclassical!

$E \sim m/g^2$

RST conjecture: $F(E)$ universal

Does not depend on details of the in–state

$$\mathcal{P}(E,N) = \sum_{i,f} \left| \langle i | \underbrace{\hat{P}_E \hat{P}_N}_{\text{Projectors}} \hat{S} | f \rangle \right|^2$$

- $N \gg 1 \Rightarrow$ semiclassical in–states
- $N \ll 1/g^2 \Rightarrow F(E,N) \approx F(E)$

Checks of universality:

- Field theory

  Tinyakov, 1991

  Mueller, 1992

- Toy QM models

  Bonini et al, 1999

  Levkov et al, 2009

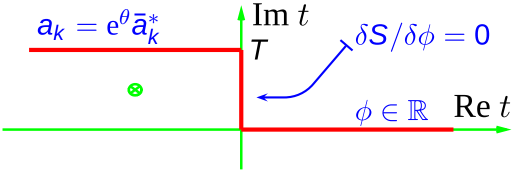
$$\boxed{F(E) = \lim_{g^2 N \to 0} F(E,N)}$$

Semiclassical method

Rubakov, Son, Tinyakov, 1992

$$\mathcal{P}(E,N) = \sum_{i,f} \left| \langle i | \hat{P}_E \hat{P}_N \hat{S} | f \rangle \right|^2 = \int [d\phi][d\phi'] \, \mathrm{e}^{iW}$$

$$W \propto 1/g^2 \Rightarrow \text{Saddle–point method!}$$

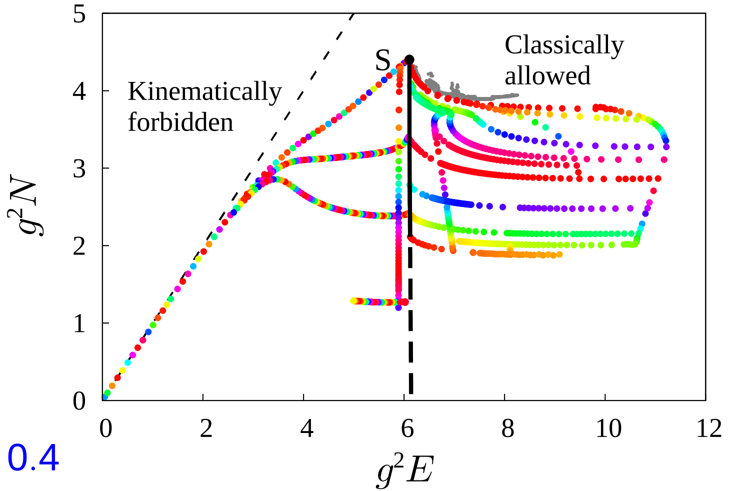


$$\boxed{\phi(x,t) \in \mathbb{C}}$$

$$\mathcal{P} = A \cdot \mathrm{e}^{-F/g^2} \qquad \frac{1}{g^2} F(E,N) = 2\,\mathrm{Im}\, S[\phi] - TE - \theta N$$

Numerical solutions

$$V(\phi) = \frac{1}{2}(\phi+1)^2\left[1 - v\cdot f\left(\frac{\phi-1}{a}\right)\right], \qquad f(x) = e^{-x^2}\left(1+x^3+x^5\right)$$

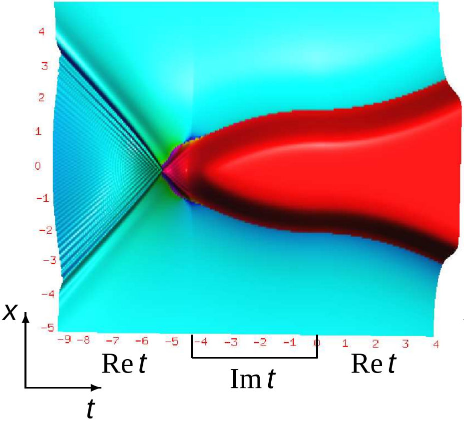


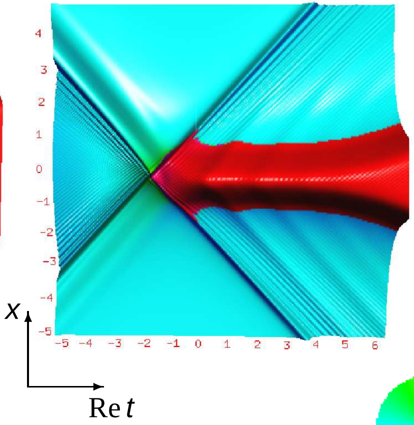
$\delta\rho = 0.4$

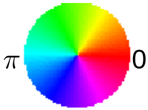
Going to $E > 2M_S$

$E \approx 5.48$ $(2M_S \approx 6.23)$ $E \approx 9.06$

$N \approx 2.39,\ \delta\rho = 0.4$ $N \approx 2.47,\ \delta\rho = 0.4$







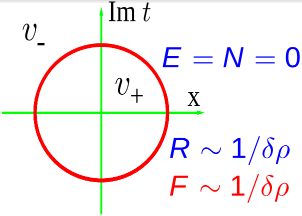
$\delta\rho \to 0$: thin–wall limit!

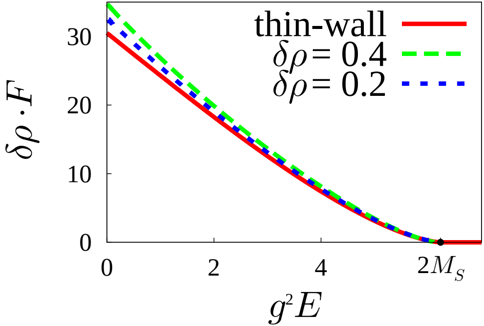
$$F(\delta\rho) = F_{-1}/\delta\rho + F_0 + O(\delta\rho)$$

$$F_{-1}(E, N) = E_S^2 \left(\pi - 2\arcsin\frac{E}{2E_S} - \frac{E}{E_S}\sqrt{1 - \frac{E^2}{4E_S^2}} \right)$$

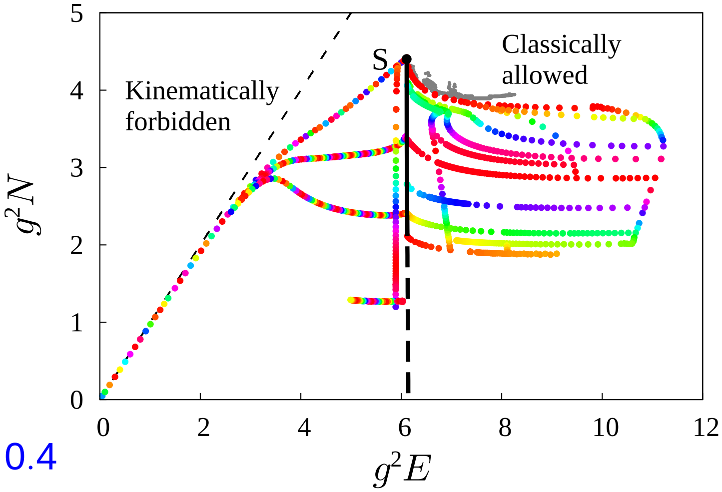
Voloshin, Selivanov, 1986

Rubakov et al, 1991



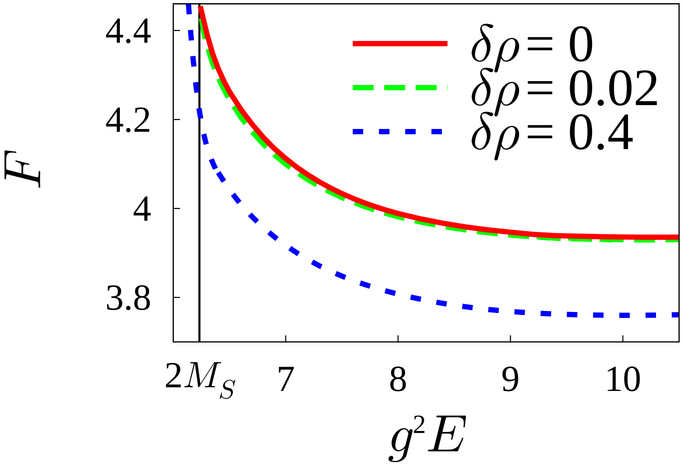


Extrapolating to $N \to 0$



$\delta\rho = 0.4$

Result



Conclusions

- Method is applicable in $2D$ scalar field models.
- The probability of SA creation in high–energy collisions is

$$\mathcal{P}(E) \approx \mathrm{e}^{-F(E)/g^2}$$