Nonhydrogen-like Graviatom Radiation

Michael L. Fil'chenkov[1]
Yuri P. Laptev[2]

[1] *Institute of Gravitation and Cosmology, Peoples' Friendship University of Russia, Moscow*

[2] *Physics Department, Moscow Bauman Technical University*

Bound quantum systems via

electromagnetic interaction

- atoms
- molecules

gravitational interaction

?

strong interaction

- hadrons
- nuclei

Quantum systems bound by gravity

Graviatoms are bound quantum systems maintaining particles in orbit around mini-holes (primordial black holes).
Particles: mesons, leptons.

Theoretical solution to the graviatom problem

Schrödinger's equation for the graviatom

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{pl}}{dr}\right)-\frac{l(l+1)}{r^2}R_{pl}+\frac{2m}{\hbar^2}\left(E+\frac{mc^2r_g}{2r}-\frac{mc^2r_gr_q}{4r^2}\right)R_{pl}=0 \qquad (1)$$

describes a radial motion of a particle with the mass m and the charge q in the mini-hole potential, taking account of DeWitt's self-interaction, where $r_g=\frac{2GM}{c^2}$ and $r_q=\frac{q^2}{mc^2}$ are the mini-hole gravitational radius and the classical radius of a charged particle respectively.

Graviatom existence conditions

- the geometrical condition $L > r_g + R$, where L is the characteristic size of the graviatom and R is the characteristic size of a particle;
- the stability condition given by
  (a) $\tau_{gr} < \tau_H$, where τ_{gr} is the graviatom lifetime and τ_H is the mini-hole lifetime, and
  (b) $\tau_{gr} < \tau_p$, where τ_p is the particle lifetime (for unstable particles);
- the indestructibility condition (due to tidal forces and the Hawking effect) $E_d < E_b$, where E_d is the destructive energy and E_b is the binding energy.

The charged particles satisfying these conditions are the *electron*, *muon*, *taon*, *wino*, *pion* and *kaon*.

Dependence of mini-hole masses on charged-particle ones

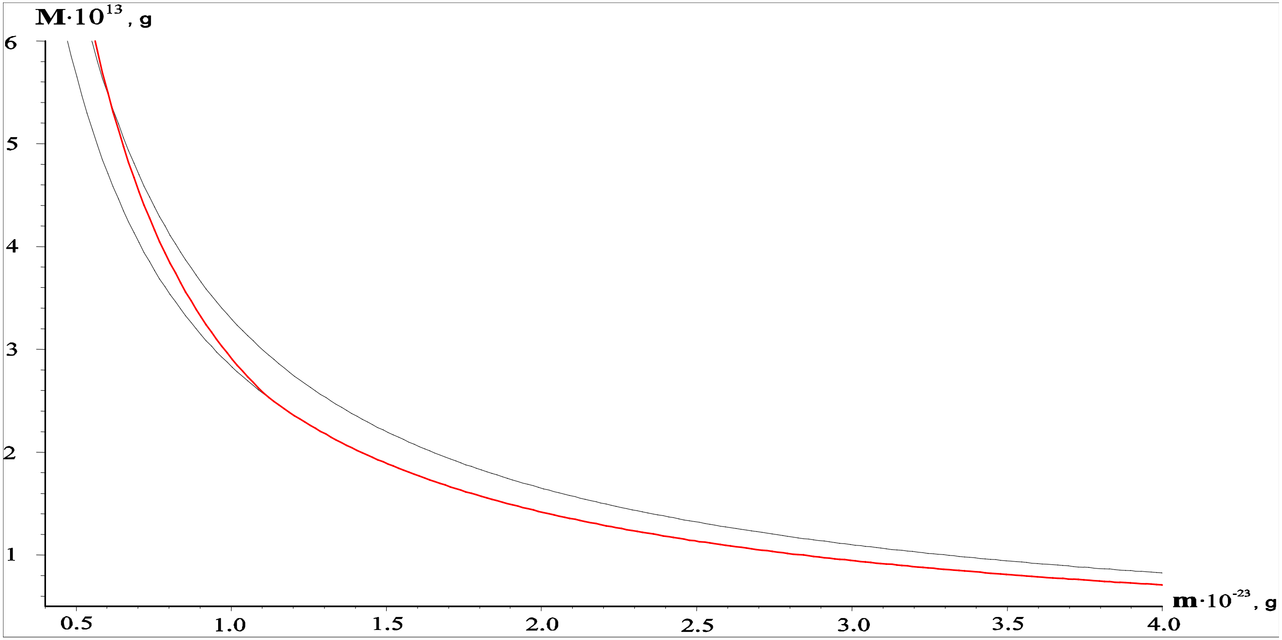

Figure 1. The black curves indicate the range of values related to the geometrical condition (the upper curve) and to Hawking's effect "ionization" (the lower curve). The red curve is related to the particle stability condition ($\tau_p = 10^{-22}$ s).

DeWitt's self-action corrections to the hydrogen-like graviatom

The energy spectrum of a hydrogen-like atom has the form

$$E_n^{(0)} = -\frac{mc^2\alpha_g^2}{2n^2}, \tag{2}$$

where $\alpha_g = \frac{GMm}{\hbar c}$ is the fine-structure constant gravity equivalent.
The perturbation being due to DeWitt's self-interaction

$$V_q = \frac{mc^2 r_g r_q}{4r^2}. \tag{3}$$

Corrections to the hydrogen-like spectrum

$$E_{nl}^{(1)} = \frac{mc^2 \alpha_{eg} \alpha_g^2}{n^3 \left(l + \frac{1}{2} \right)}, \tag{4}$$

where $\alpha_{eg} = \frac{e^2}{\hbar c} \alpha_g$.

As will be shown below $Mm = \alpha_g m_{pl}^2$, where $\alpha_g = 0.5 \pm 0.6$.

The intensity of the electric dipole radiation of a particle with mass m and charge e in the gravitational field of a mini-hole for the transition $2p \to 1s$ is

$$I_{10,21} \approx I_{10,21}^{(0)} \left(1 - \frac{46}{9}\alpha_{eg}\right)^4 \left(1 - 6\alpha_{eg}\right)^2, \tag{5}$$

where

$$I_{10,21}^{(0)} = \frac{2\hbar e^2 \left[\omega_{12}^{(0)}\right]^3}{mc^3} f_{10,21}^{(0)} \tag{6}$$

is the intensity for a hydrogen-like graviatom, $\omega_{12}^{(0)} = (E_1^{(0)} - E_2^{(0)})/\hbar$ is the frequency and $f_{10,21}^{(0)} = \dfrac{2^{13}}{3^9}$ is the oscillator strength.

Thus, DeWitt's self- interaction diminishes both frequencies and intensities of the hydrogen-like graviatom.

Pauli's corrections to the hydrogen-like graviatom

Taking account both DeWitt's self-action and Pauli's spin corrections, the graviatom energy spectrum takes the form

$$E=-\frac{mc^2\alpha_g^2}{2n^2}+\frac{mc^2\alpha_g^3}{2n^3}\left(\frac{2}{l+\frac{1}{2}}\frac{e^2}{\hbar c}-\frac{\alpha_g}{j+\frac{1}{2}}\right)+\frac{3mc^2\alpha_g^4}{8n^4}, \tag{7}$$

where $j=l+s$, $s=0$ for mesons and $s=\pm\frac{1}{2}$ for leptons.

The first nl_j levels: $1s_{1/2}, 2s_{1/2}, 2p_{1/2}, 2p_{3/2}$. The dipole transition $2p\to 1s$ splits into two transitions: $2p_{1/2}\to 1s_{1/2}$ with $\Delta j=0$ and $2p_{3/2}\to 1s_{1/2}$ with $\Delta j=1$.

The intensity for the transition $2p_{3/2}\to 1s_{1/2}$ is $I_{10,21}=\frac{2}{3}I_{10,21}^{(0)}\left(1+\frac{15}{48}\alpha_g^2\right)^4$

Graviatoms with slowly rotating miniholes

Lense-Thirring's metric for a slowly rotation black hole has the form

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) + \frac{2 r_g a}{r} \sin^2\theta \, d\varphi \, dt, \quad (8)$$

where a is the specific angular momentum.
The perturbation being due to rotation

$$V_a = \frac{r_g a \hbar \sqrt{l\left(l+1\right)}}{r^3} - \frac{r_g^2 a \hbar \sqrt{l\left(l+1\right)}}{r^4}. \quad (9)$$

The intensity for the transition $2p \to 1s$

$$I_{21} = I_{21}^{(0)} \left(1 + 0.0349 \frac{ma}{\hbar}\right)^4 \left(1 - 1.1064 \frac{ma}{\hbar}\right)^2$$

The correction to the hydrogen-like spectrum of the graviatom taking account of DeWitt's self-force, particle spin and a slow rotation of the minihole reads

$$E_{nl}^{(1)} = \frac{mc^2\alpha_g^3}{2n^3}\left(\frac{2}{l+\frac{1}{2}}\,\frac{e^2}{\hbar c} - \frac{\alpha_g}{j+\frac{1}{2}}\right) + \frac{3mc^2\alpha_g^4}{8n^4} + \frac{2\alpha_g^4 m^2 c^2 a}{\hbar n^3\left(l+\frac{1}{2}\right)^2}\left[1 - \frac{3n^2 - \left(l+\frac{1}{2}\right)^2}{n^2\left(l+\frac{1}{2}\right)^2}\alpha_g^2\right] \quad (10)$$

Providing $\frac{am}{\hbar}\alpha_g^2 \ll 1$, we obtain

$$\frac{E_{nl}^{(1)}}{E_n^{(0)}} = \frac{\alpha_g^2}{n}\left[\frac{1}{j+\frac{1}{2}} - \frac{4am}{\hbar}\frac{1}{\left(l+\frac{1}{2}\right)^2}\right]. \quad (11)$$

$$\frac{E_{nl}^{(1)}}{E_n^{(0)}} \ll 1, \text{ if } \quad \alpha_g^2 \ll n, \quad \frac{4am}{\hbar} \ll \left(l+\frac{1}{2}\right)^2, \quad j+\frac{1}{2} \gg 1.$$

As a result, the perturbation theory is valid for a slow rotation of the minihole and higher levels of the particle being constituents of the graviatom.

Conclusion

The graviatoms can contain only leptons and mesons. The hydrogen-like graviatom is perturbed by DeWitt's self-action and minihole rotation diminishing, whereas particle spin enhancing the dipole radiation intensity.