Casimir Effect within (3+1)D Maxwell-Chern-Simons Electrodynamics

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Standard Model Extension (SME [Kostelecký])

- Elaborated for studying the manifestation of the 'New Physics' (Strings, Extra Dimensions, Quantum Gravity,...) at low energies $E \ll m_{\mathrm{Pl}} \sim 10^{19}\mathrm{GeV}$
- Axiomatically introduces a set of correction terms to the Lagrangian of SM(no new fields!), that maintain some 'natural' features of SM:
  - observer Lorentz invariance (although the vacuum is not Lorentz-invariant)
  - unitarity
  - microcausality
  - $SU(3)_C \times SU(2)_I \times U(1)_Y$ gauge invariance
  - power-counting renormalizablilty (for the minimal SME)
- When $E \ll m_W \sim 10^2\mathrm{GeV}$, the SME leads to the extended QED with $U(1)_{em}$ gauge invariance which is typical for SM

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(3+1)D Maxwell-Chern-Simons electrodynamics [1]

A particular case of extended QED with the Chern-Simons term:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \eta^{\mu} \epsilon_{\mu\nu\alpha\beta} A^{\nu} F^{\alpha\beta} + \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi.$$

$D_\mu = \partial_\mu + ieA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita symbol,
η^μ is a constant axial 4-vector, breaks CPT and Lorentz invariance.

This vector coupling η^μ could arise from:

- an axion condensation [Carroll,Field,Jackiw,1992]
- background torsion [Dobado,Maroto,1996]
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$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \eta^{\mu} \epsilon_{\mu\nu\alpha\beta} A^{\nu} F^{\alpha\beta}$$

Features of the theory:

- The action is $U(1)$-invariant, though $\mathcal{L}_A$ is not
- When $\eta^2 > 0$, the theory is unstable
- $T_{\mu\nu}$ cannot be made either symmetric or gauge-invariant

However, in the model we use for considering the Casimir effect, these difficulties can be overcome!

Widely studied (2+1)D case: η is a pseudo-scalar and generates massive photons with unbroken gauge invariance. In the (3+1)D case, the photon dispersion relation is much more complicated.

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1 Introduction

2 Casimir effect within extended QED

- One-photon eigenstates
- Vacuum energy

3 Discussion and conclusion

- Conclusion

The model we use

Photon sector of (3+1)D Maxwell-Chern-Simons electrodynamics

- $\eta^\mu = \{\eta, \mathbf{0}\}$
- Two infinite parallel superconductor plates separated by $D = 2a$
- Gauge: $A^0 = 0$, $\operatorname{div} \mathbf{A} = 0$ (possible to fix for the chosen η^μ)
- Equations of motion: $\Box \mathbf{A} = 2\eta \operatorname{rot} \mathbf{A}$
- $T^{00} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{H}^2) - \eta \mathbf{A} \cdot \mathbf{H}$

In our paper [1], we have shown that:

- When $a < \pi/4|\eta|$, the theory is stable!
- Vacuum energy (per unit plate area) is half the sum over all one-photon mode energies: $E_{vac} = \int \frac{d^3x}{L^2} \langle T^{00}(x) \rangle = \sum_n \frac{\omega_n(D)}{2L^2}$
  n is a complete set of quantum numbers, $L \to \infty$ is the linear plate size
- The Casimir force $f_{\text{Casimir}} = \partial E_{vac}/\partial D$ is gauge-invariant, although the energy is not

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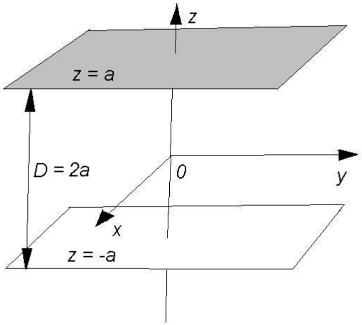
One-photon eigenstates [1]



$$\mathbf{A}(\mathbf{x}, t) = N\, e^{-i\epsilon\omega t + i\mathbf{k}\mathbf{x}}\, \mathbf{f}(z), \quad \mathbf{k} = \{k_x, k_y, 0\}, \quad \epsilon = \pm 1.$$

$$(\nabla^2 + 2\eta\, \mathrm{rot} + \omega^2)\mathbf{A} = 0, \quad \mathrm{div}\, \mathbf{A} = 0,$$

$A_x = A_y = 0$ at $z = \pm a$ (boundary conditions on the conductor)

One-photon eigenstates [2]

Ansatz:

$$\mathbf{A}_{\epsilon,\mathbf{k},\Pi,n_z}(\mathbf{x},t) = N\, e^{-i\epsilon\omega t + i\mathbf{kx}}(f_z \mathbf{e}_z + f_k \hat{\mathbf{k}} + f_{zk}[\mathbf{e}_z \hat{\mathbf{k}}]), \quad \mathbf{k} = \{k_x, k_y, 0\}.$$

Transversality implies: $f_k = \frac{i}{k}\partial_z f_z,$

Parity $\Pi = \pm 1$: $f_k(-z) = -\Pi f_k(z),\ f_{zk,z}(-z) = \Pi f_{zk,z}(z).$

Equations for f_z and f_{zk}:

$$\begin{aligned}(\omega^2 - k^2 + \partial_z^2) k f_{zk} &= 2i\eta(k^2 - \partial_z^2) f_z,\\ (\omega^2 - k^2 + \partial_z^2) f_z &= -2i\eta k f_{zk},\\ f_{zk}(a) = 0, &\qquad \partial_z f_z(a) = 0\end{aligned}$$

The existence of nontrivial solutions implies that:

$$g_\Pi(\omega^2) \equiv \varphi_\Pi(\varkappa_+ a)\varphi_{-\Pi}(\varkappa_- a)\sin\theta_- + \varphi_\Pi(\varkappa_- a)\varphi_{-\Pi}(\varkappa_+ a)\sin\theta_+ = 0,$$

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Vacuum energy

In our theory, like in the conventional QED, the vacuum energy (per unit plate area) is

$$E_{\text{vac}} = \frac{1}{L^2}\int \langle T^{00}\rangle d^3x = \frac{1}{L^2}\sum_n \frac{\omega_n(D)}{2} = \int_0^\infty \frac{kdk}{2\pi} D(S_+ + S_-),$$

where $n = \{k_x, k_y, \Pi, n_z\}$ is the full set of quantum numbers.

Smooth cutoff regularization:

$$S_\Pi = \frac{1}{D}\sum_{\omega_{k,\Pi,n_z}\in\mathbb{R}^+} \omega_{k,\Pi,n_z} e^{-\omega_{k,\Pi,n_z}/\sqrt{k\Lambda}}, \quad \Lambda \to +\infty.$$

The spectrum is determined with the equation $\quad g_\Pi(\omega^2) = 0$

Sum $\longrightarrow$ complex plane integral [1]

Let us now transform discrete sums S_Π into complex plane integrals, using the residue theorem.

Instead of $g_\Pi(\omega^2)$ whose zeros are the one-photon energy eigenvalues, we will use the meromorphic (analytical, except for the numerable set of poles; in particular, with no branch points) function

$$\tilde{g}_\Pi(K_+) \equiv \frac{g_\Pi(\omega)}{\varphi_\Pi(\varkappa_+ a)\varphi_\Pi(\varkappa_- a)} = \tan^\Pi \varkappa_+ a \sin\theta_+ + \tan^\Pi \varkappa_- a \sin\theta_-,$$

$$\omega^2 = K_+ K_-, \quad K_- = K_+ + 2\eta, \quad \varkappa_\pm = \sqrt{K_\pm^2 - k^2}, \quad \sin\theta_\pm = \varkappa_\pm / K_\pm .$$

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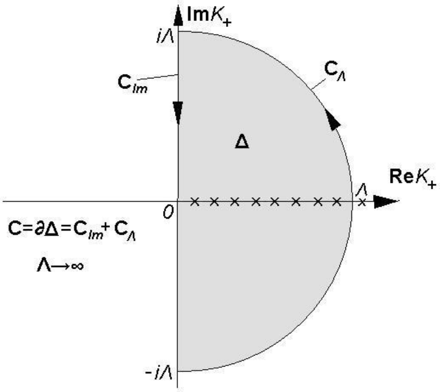

Residue theorem (we assume $\eta \geq 0$, since the spectrum depends on $|\eta|$):

$$\oint\limits_C \frac{dK_+}{2\pi i}\omega e^{-\frac{\omega}{\sqrt{k\Lambda}}}\frac{\partial\tilde{g}_\Pi/\partial K_+}{\tilde{g}_\Pi} = S_\Pi D + \sum_{\bar{\omega}_n}\bar{\omega}_n e^{-\frac{\bar{\omega}_n}{\sqrt{k\Lambda}}}\frac{\mathrm{Res}\left[\partial\tilde{g}_\Pi/\partial K_+, K_+ = \bar{\omega}_n\right]}{\tilde{g}_\Pi(\bar{\omega}_n)},$$

where $\bar{\omega}_n$ are the poles of function $\partial\tilde{g}_\Pi/\partial K_+$ within Δ.

Sum $\to$ complex plane integral [3]

Transforming the pole residue term back into an integral, we obtain:

$$\tilde{g}_\Pi(K_+) \equiv \tan^\Pi \varkappa_+ a \sin\theta_+ + \tan^\Pi \varkappa_- a \sin\theta_-,$$

$$S_\Pi = \frac{\Pi}{2} \oint\limits_C \frac{\omega e^{-\omega/\sqrt{k\Lambda}} dK_+}{2\pi i \tilde{g}_\Pi(K_+)} \left\{ 2 - \tan^\Pi \varkappa_+ a \tan^\Pi \varkappa_- a \left(\frac{\sin\theta_-}{\sin\theta_+} + \right.\right.$$
$$\left.\left. + \frac{\sin\theta_+}{\sin\theta_-} \right) + \frac{\Pi \tan^\Pi \varkappa_+ a}{\varkappa_+ a} + \frac{\Pi \tan^\Pi \varkappa_- a}{\varkappa_- a} \right\}$$

The integral over the semicircle C_Λ does not depend on a when $\Lambda \to \infty$, within any finite order in a, thus it is cancelled when renormalized.

Renormalization:

$$S_\Pi^{ren}(D) = S_\Pi(D) - S_\Pi^{div}(D), \quad S_\Pi^{div}(D) = C_1(\Lambda) + C_2(\Lambda)/D.$$

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After renormalization and $\Lambda \longrightarrow \infty$

Let us redefine $K_+ \to -iK_+$ and make all momentum quantities dimensionless multiplying them by a, then we obtain:

$$f_{Casimir} = \frac{1}{2}\frac{\partial}{\partial D}\left(D\frac{(\tilde{S}_+^{ren} + \tilde{S}_-^{ren})}{a^4} \right),$$

$$\tilde{S}_\Pi^{ren} = -\frac{1}{2}\int_0^\infty \frac{kdk}{2\pi}\int_{-\infty}^{+\infty}\frac{dK_+}{2\pi}\frac{\operatorname{sgn} K_+ \quad \sqrt{K_+K_-}}{\tanh^\Pi \varkappa_+ \cosh\theta_+ + \tanh^\Pi \varkappa_- \cosh\theta_-}\Sigma_\Pi,$$

$$\Sigma_\Pi = 1 + \tanh^\Pi \varkappa_+ \tanh^\Pi \varkappa_- \frac{\cosh\theta_+}{\cosh\theta_-} - \left(1 + \frac{\cosh\theta_+}{\cosh\theta_-}\right)\tan^\Pi \varkappa_+ +$$

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$$K_- = K_+ - i\eta D, \quad \varkappa_\pm = \sqrt{k^2 + K_\pm^2}, \quad \sinh\theta_\pm = k/K_\pm .$$

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The results of the calculation

After the expansion with respect to ηD and taking the integrals, we obtain:

$$\tilde{S}_{+}^{ren} + \tilde{S}_{-}^{ren} = -\frac{\pi^2}{5760} - \frac{5(\eta D)^2}{1152} + \mathcal{O}((\eta D)^4),$$

$$f_{Casimir} = \frac{1}{2}\frac{\partial}{\partial D}\left(D\frac{\tilde{S}_{+}^{ren} + \tilde{S}_{-}^{ren}}{a^4}\right) = \frac{\pi^2}{240 D^4}\left(1 + \frac{25}{3\pi^2}(\eta D)^2 + \mathcal{O}((\eta D)^4)\right),$$

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The correction to the Casimir force

$$f_{Casimir} = \frac{\pi^2}{240 D^4}\left(1 + \frac{25}{3\pi^2}(\eta D)^2 + \mathcal{O}((\eta D)^4)\right), \ |\eta| D \ll 1.$$

Discussion:

- The correction is attractive, contrary to the recent result obtained by [Frank, Turan, 2006]
- The difference from the Maxwell value becomes stronger for comparatively large D
- Experimental data [Mohideen et al., $D \sim 500$nm, $L \sim 1$cm, 1% accuracy] gives the constraint:

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Main results

- The eigenstates and energy eigenvalues for the Maxwell-Chern-Simons photon between the conducting plates
- The vacuum is stable when $|\eta|D < \pi/2$ [1]
- The leading correction to the Casimir force, which is quadratic in η
- The constraint on η

References:

[1] O.G.Kharlanov and V.Ch.Zhukovsky, *Casimir Effect within $D = 3 + 1$ Maxwell-Chern-Simons Electrodynamics*, `arXiv:0905.3680 [hep-th]`.

Thank you for your attention!