Angular Momentum of Spin Light Radiation

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Introduction

- ***Spin light*** [1] corresponds an electromagnetic radiation of a relativistic spin particle with a proper ***magnetic momentum***.
- Such radiation appears only from the electrically neutral particles (neutron [2], neutrino [3]).
- The phenomenon of ***interference*** of radiations from the accelerated charge and from the proper magnetic momentum arises.
- This phenomenon is the most noticeably observed just by extreme electron's energies in the synchrotron accelerators.
- Actually the problem of identification of the spin light against the background of powerful synchrotron radiation and of other relativistic phenomena was studied in details.

  - [1] V. A. Bordovitsyn, I. M. Ternov, V. G. Bagrov, *Usp. Fiz. Nauk* 165, 1083 (1995).
  - [2] I. M. Ternov, V. G. Bagrov, A. M. Khapaev, *JETP* 48, 961 (1965).
  - [3] A. E. Lobanov, A. I. Studenikin, *Phys. Lett* 564B, 27 (2003).

Motivation

- ***Exploration of spin light properties associated with the proper angular momentum of radiation of electromagnetic field.***
- Obtaining the densities of energy-momentum tensor and angular momentum tensor of an arbitrary moving relativistic spin particle with the proper magnetic momentum.
- As an example we consider the angular momentum of spin light of a magneton moving with constant velocity.
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    - It should be noted that the properties of radiation of an arbitrary moving relativistic ***charge*** was discussed in a previous report (author Nemchenko E.A.)

Starting relations

- Four-dimensional vector-potentials

$$A^{\mu}{}_{m} = \bar{A}^{\mu}{}_{m} + \tilde{A}^{\mu}{}_{m},$$

$$A^{\mu}{}_{e} = -e\frac{\upsilon^{\mu}}{\tilde{r}_{\rho}\upsilon^{\rho}},$$

- Stress tensor of electromagnetic field

$$H^{\mu\nu} = D^{\mu}A^{\nu} - D^{\nu}A^{\mu}$$

- Symbols ~ and — correspond to the type of degree falling of field subject to the distance (1|r² for — and 1|r³ for ~)

- Symmetrical stress tensor of energy of spin light

$$\varepsilon^{\mu\lambda} = -\frac{1}{4\pi}\left(\frac{1}{4}g^{\mu\lambda}H_{\alpha\beta}H^{\alpha\beta} + H^{\mu\rho}H_{\rho}{}^{\lambda}\right)$$

Form of the field's strengths

- For the charge

$$H^{\mu\nu}_{e} = \overline{H}^{\mu\nu}_{e} + \widetilde{H}^{\mu\nu}_{e},$$

- Unlike to the charge strength, the tensor of the magnetic momentum consists of three parts:

- For the magnetic momentum

$$H^{\mu\nu}_{m} = \overline{H}^{\mu\nu}_{m} + \overset{\approx}{H}^{\mu\nu}_{m} + \widetilde{H}^{\mu\nu}_{m},$$

  - convection field
  - mixed field
  - radiation field

$$\overline{H}^{\mu\nu}_{m} \sim 1/\bar{r}^{3}$$

$$\overset{\approx}{H}^{\mu\nu}_{m} \sim 1/\bar{r}^{2}.$$

$$\widetilde{H}^{\mu\nu}_{m} \sim 1/\bar{r}$$

• Since the fields caused by the electrical charge are well-known in the literature, in the following we'll pay more attention to the tensor of the magnetic momentum

Explicit form of the magnetic momentum tensor

$$\overline{H}^{\mu\nu}{}_{m} = \frac{c^3}{\left(\tilde{r}_\rho v^\rho\right)^3}\left\{-2\Pi^{\mu\nu} + 3\frac{\Pi^{[\mu\lambda}\tilde{r}_\lambda v^{\nu]}}{\tilde{r}_\rho v^\rho} + 3c^2\frac{\Pi^{[\mu\lambda}\tilde{r}_\lambda \tilde{r}^{\nu]}}{\left(\tilde{r}_\rho v^\rho\right)^2}\right\},$$

$$\widetilde{H}^{\mu\nu}{}_{m} = \frac{c}{\tilde{r}_\rho v^\rho}\left\{\frac{\ddot{\Pi}^{[\mu\lambda}\tilde{r}_\lambda \tilde{r}^{\nu]}\tilde{r}_\sigma \omega^\sigma}{\left(\tilde{r}_\rho v^\rho\right)^2} - \frac{\Pi^{[\mu\lambda}\tilde{r}_\lambda \tilde{r}^{\nu]}\tilde{r}_\sigma \mathring{\omega}^\sigma + \mathring{\Pi}^{[\mu\lambda}\tilde{r}_\lambda \tilde{r}^{\nu]}\tilde{r}_\sigma \omega^\sigma}{\left(\tilde{r}_\rho v^\rho\right)^3} + 3\frac{\Pi^{[\mu\lambda}\tilde{r}_\lambda \tilde{r}^{\nu]}\left(\tilde{r}_\sigma \omega^\sigma\right)^2}{\left(\tilde{r}_\rho v^\rho\right)^4}\right\}$$

$$\widetilde{\overline{H}}^{\mu\nu}{}_{m} = \frac{c}{\left(\tilde{r}_\rho v^\rho\right)^2}\left\{2\mathring{\Pi}^{\mu\nu} - 2\Pi^{\mu\nu}\frac{\tilde{r}_\sigma \omega^\sigma}{\tilde{r}_\rho v^\rho} + 3\frac{\mathring{\Pi}^{[\mu\lambda} v_\lambda \tilde{r}^{\nu]} + 2\mathring{\Pi}^{[\mu\lambda} v_\lambda v^{\nu]} + \Pi^{[\mu\lambda}\tilde{r}_\lambda \omega^{\nu]}}{\tilde{r}_\rho v^\rho} - \right.$$

$$\left. - 3c^2\frac{\mathring{\Pi}^{[\mu\lambda}\tilde{r}_\lambda \tilde{r}^{\nu]} - \Pi^{[\mu\lambda}\tilde{r}_\lambda v^{\nu]}\tilde{r}_\sigma \omega^\sigma}{\left(\tilde{r}_\rho v^\rho\right)^2} + 6c^2\frac{\Pi^{[\mu\lambda}\tilde{r}_\lambda v^{\nu]}\tilde{r}_\sigma \omega^\sigma}{\left(\tilde{r}_\rho v^\rho\right)^3}\right\},$$

Important properties

- In spite with the complexity of these fields, it can be verified to satisfy the first pair of Maxwell's equations. In the retarded form these equations are:

$$D^{\mu} H^{\rho\nu}_{\ \ \ m} + D^{\nu} H^{\mu\rho}_{\ \ \ m} + D^{\rho} H^{\nu\mu}_{\ \ \ m} = 0.$$

- The second pair of Maxwell's equations are, again, valid on the world line of the particle

$$D_{\nu} H^{\mu\nu}_{\ \ \ m} = 0.$$

- It should be noted that both of the invariants of magneton electromagnetic field ***in the wave zone*** are vanished (as expected in the case of the radiation field of charge)

$$\tilde{H}_{\alpha\beta m} \tilde{H}^{\alpha\beta}_{\ \ \ m} = \tilde{E}_{\alpha\beta m} \tilde{H}^{\alpha\beta}_{\ \ \ m} = 0,$$

Approximations

1. In the following we will consider the pure spin light and the charge radiation will be taken into account only in the mixed fields not far from the wave zone of radiation

2. All the calculations will be held in the case of the uniform motion of the charged magneton

Consequences

1. With respect to this assumption the components of the stress tensor of the spin light we need are the follows

$$_{(-3)}\mathcal{E}^{\mu\nu} = -\frac{1}{4\pi}\left[\widetilde{H}^{\mu\rho}{}_{m}\widetilde{\widetilde{H}}_{\rho m}{}^{\nu} + \overline{H}^{\mu\rho}{}_{e}\widetilde{H}_{\rho m}{}^{\nu} + \widetilde{H}^{\mu\rho}{}_{e}\widetilde{\widetilde{H}}_{\rho m}{}^{\nu} + (\nu,\mu)\right],$$

$$_{(-2)}\mathcal{E}^{\mu\nu} = -\frac{1}{4\pi}\left[\widetilde{H}^{\mu\rho}{}_{m}\widetilde{H}_{\rho m}{}^{\nu} + \widetilde{H}^{\mu\rho}{}_{e}\widetilde{H}_{\rho m}{}^{\nu} + (\nu,\mu)\right],$$

2. All the invariants of the electromagnetic field

$$\widetilde{H}_{\alpha\beta m}\widetilde{\overline{H}}^{\alpha\beta}{}_{m},\ \overline{H}_{\alpha\beta e}\widetilde{H}^{\alpha\beta}{}_{m} + \widetilde{H}_{\alpha\beta e}\widetilde{H}^{\alpha\beta}{}_{m},\ \widetilde{H}_{\alpha\beta e}\widetilde{\overline{H}}^{\alpha\beta}{}_{m},\ \widetilde{H}_{\alpha\beta m}\widetilde{H}^{\alpha\beta}{}_{m},\ \widetilde{H}_{\alpha\beta e}\widetilde{H}^{\alpha\beta}{}_{m}$$

are zeros according to our hypothesis.

Orbital and proper angular momentum of radiation

- With respect to Taitelboim's assumption [13] the full spin stress tensor is defined like

$$\mathcal{M}^{\mu\nu\lambda} = \mathcal{A}^{\mu\nu\lambda} + \mathcal{T}^{\mu\nu\lambda}$$

- ***Orbital*** and ***spin*** stress tensors of angular momentum are:

$$\mathcal{A}^{\mu\nu\lambda} = \frac{1}{c}\left(r^{\mu}\mathcal{E}^{\nu\lambda} - r^{\nu}\mathcal{E}^{\mu\lambda}\right) \quad \mathcal{T}^{\mu\nu\lambda} = \frac{1}{c}\left(\tilde{r}^{\mu}\mathcal{E}^{\nu\lambda} - \tilde{r}^{\nu}\mathcal{E}^{\mu\lambda}\right)$$

- [13] C. Teitelboim, D. Villarroel, Ch. G. van Weert, *Riv. Nuovo Cim*. 3, 1 (1980).

Obtaining the total angular momentum power

- ***With respect to our assumptions***

1. Then we will consider only wave zone, where

1.
$$\tilde{\mathcal{A}}^{\mu\nu\lambda} = \frac{1}{c}\left(r^{\mu} \underset{(-2)}{\mathcal{E}}^{\nu\lambda} - r^{\nu} \underset{(-3)}{\mathcal{E}}^{\mu\lambda} \right),$$
$$\tilde{\mathcal{T}}^{\mu\nu\lambda} = \frac{1}{c}\left(\bar{r}^{\mu} \underset{(-3)}{\mathcal{E}}^{\nu\lambda} - \bar{r}^{\nu} \underset{(-3)}{\mathcal{E}}^{\mu\lambda} \right).$$

2. Moreover, according to relations

2.
$$\upsilon^{\mu} = const, \omega^{\mu} = \mathring{\omega}^{\mu} = 0$$

may be written

$$\Pi^{\mu\nu} \upsilon_{\nu} = \mathring{\Pi}^{\mu\nu} \upsilon_{\nu} = \overset{\circ\circ}{\Pi}^{\mu\nu} \upsilon_{\nu} = 0,$$

Integration

- Now we can find the stress tensor of the ***orbital*** momentum

$$\tilde{\lambda}^{\mu\nu\lambda}{}_{m} = -\frac{c}{4\pi}\frac{\tilde{r}_{\alpha}\overset{\circ\circ}{\Pi}{}^{\alpha\rho}\,\overset{\circ\circ}{\Pi}{}_{\rho\beta}\,\tilde{r}^{\beta}}{\left(\tilde{r}_{\rho}v^{\rho}\right)^{6}}\tilde{r}^{\mu}\tilde{r}^{\nu}.$$

- Integrating over the spacelike plane, we can obtain the ***orbital tensor*** of the magnetic momentum

$$\tilde{L}^{\mu\nu}{}_{m} = \int \tilde{\lambda}^{\mu\nu\lambda}{}_{m} e_{\lambda}\varepsilon^{2} c\,d\tau\, d\Omega_{0},$$

- In a similar fasion we can consider also the ***spin*** angular momentum
- Stress tensor is defined in accordance with

$$\tilde{\pi}^{\mu\nu\lambda}{}_{m} = -\frac{c}{2\pi}\left(\frac{1}{\tilde{r}_{\rho}v^{\rho}}\tilde{r}^{\mu}\mathring{\Pi}^{\nu\alpha} - \tilde{r}^{\nu}\mathring{\Pi}^{\mu\alpha} + \left(\tilde{r}^{\mu}v^{\nu} - \tilde{r}^{\nu}v^{\mu}\right)\frac{\Pi^{\alpha\rho}\tilde{r}_{\rho}}{\tilde{r}_{\rho}v^{\rho}}\right)$$

- Similarly we can construct the stress tensor for ***mixed*** radiation

Total power

- The total power of angular momentum radiation contains ***3 terms***

1. Radiation power of the ***orbital*** angular momentum
2. The total power of ***spin*** light
3. And the total power of ***interferential*** radiation

1. $$\frac{d\tilde{L}^{\mu\nu}_{m}}{d\tau} = \frac{1}{3c^5}\ddot{\Pi}_{\alpha\beta}\,\ddot{\Pi}^{\alpha\beta}\left(\tilde{r}^{\mu}\upsilon^{\nu} - \tilde{r}^{\nu}\upsilon^{\mu}\right).$$

2. $$\frac{d\tilde{\Pi}^{\mu\nu}_{m}}{d\tau} = -\frac{2}{3c^2}\left(\mathring{\Pi}^{\mu\rho}\,\ddot{\Pi}^{\nu}{}_{\rho} - \mathring{\Pi}^{\nu\rho}\,\ddot{\Pi}^{\mu}{}_{\rho}\right).$$

3. $$\frac{d\tilde{\Pi}^{\mu\nu}_{em}}{d\tau} = \frac{2e^2}{3c^3}\ddot{\Pi}^{\mu\nu}.$$

Conclusion

- Thus, in this paper we demonstrate such new phenomenon of nature as proper angular momentum of electromagnetic field of spin light radiation.
- The main properties of spin light radiation are explored to define the way to following investigations.

Thanks for your attention!