Long quantum transitions due to unstable semiclassical dynamics

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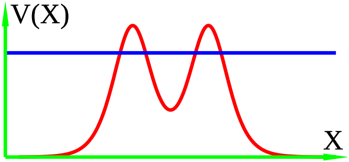
Why transition time?

Experimental signature!

Long transitions $\Rightarrow$ long–living intermediate “states”:

Quasistationary state

- *Yamamoto, Miyamoto, Hayashi, 1998*
  Peskin, Galperin, Nitzan, 2002



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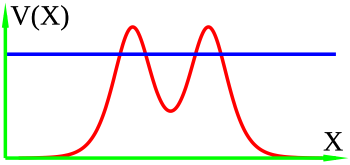
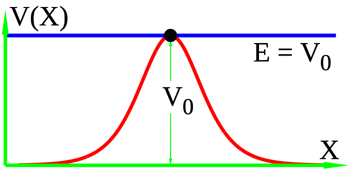


- Classically unstable "state"

  (Not a quantum state at all)

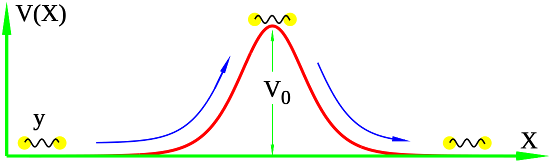
  Takahashi, Ikeda, 2006



Surprise

Classically unstable “states”

form in the generic processes of multidimensional tunneling



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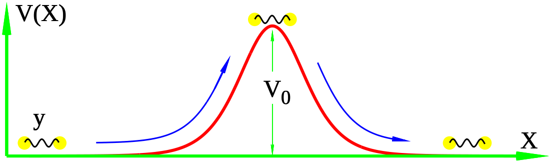
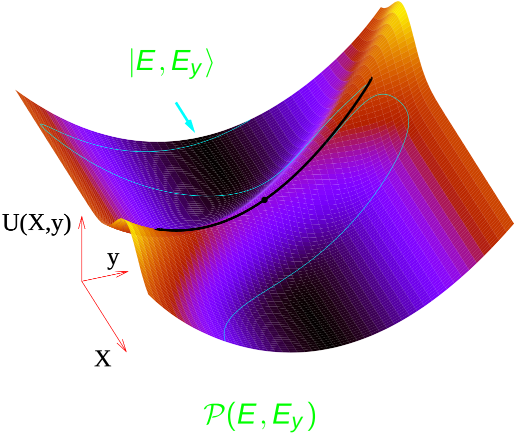


Examples — tunneling in

- regular quantum systems ($N = 2$) *Bonini, Cohen, Rebbi, Rubakov, 1999*
- systems with dynamical chaos ($N = 1.5,\ \ 2$) *Takahashi, Ikeda, 2003*
- electroweak theory ($N = \infty$) *Bezrukov, DL, Rebbi, Rubakov, Tinyakov, 2003*
- scalar field theory ($N = \infty$) *DL, Sibiryakov, 2005*

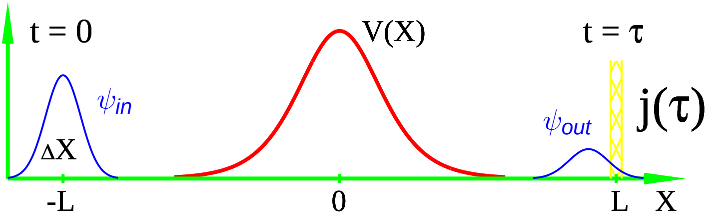
The model

$$S = \int dt \left[(\dot{X}^2 + \dot{y}^2 - \omega^2 y^2)/2 - \exp(-(X+y)^2/2) \right]$$



$\hbar \to 0 \qquad \Rightarrow$ Semiclassical description

Traversal time definition



Normalized probability distribution:

$$\rho(\tau) = j(\tau)/\mathcal{P}_{tot}$$

$$\Downarrow$$

- Average time of passing $\langle\tau\rangle$
- Time dispersion σ_τ^2

Semiclassical method

Counts time spent by trajectory in the region $X < L$:

$$T_{int}[X] = \int_0^{\tau} dt\, \theta(L - X(t)) \,,$$

Faddeev–Popov unity

$$\underbrace{1 = \int_0^{\tau} d\tau\, \delta(\tau - T_{int}[X(t)])}$$

$$\psi_{out} = \int [dX(t)\, dy(t)] \cdot e^{iS/\hbar} \cdot \psi_{in}(X_{in}, y_{in})$$

$\hbar \to 0$ $\Rightarrow$ saddle–point approximation for all integrals!

Semiclassical expression

$$\mathcal{P}_{tot} = A \cdot \mathrm{e}^{-F/\hbar}$$

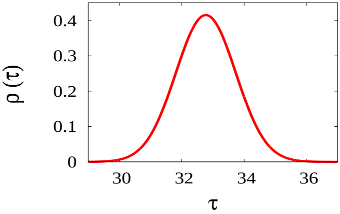
$$\Downarrow$$

$$\rho(\tau) = A(\tau) \cdot \mathrm{e}^{-F(\tau)/\hbar}$$

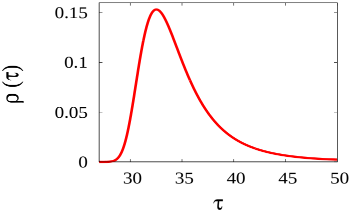
Semiclassical trajectory with additional constraint $T_{int}[X] = \tau$

Results

direct



via unstable “state”



Gaussian

$$\rho = \mathcal{N} \cdot \mathrm{e}^{-(\tau - \langle\tau\rangle)^2/2\sigma_t^2}$$

$$\langle\tau\rangle \sim O(\hbar^0)$$

$$\sigma_\tau^2 \sim O(\hbar)$$

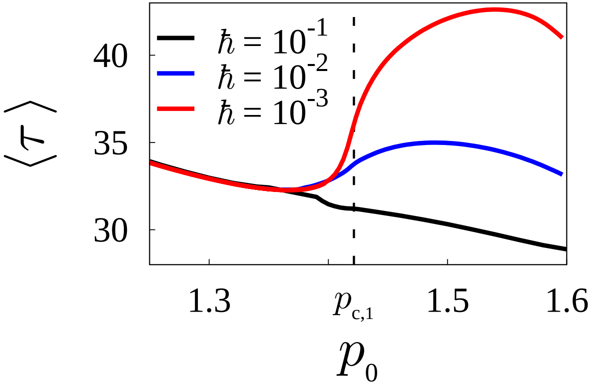
Gumbel type I distribution

$$\rho \sim \alpha \cdot \exp\left\{-\beta\tau - \alpha \cdot \mathrm{e}^{-\beta\tau}\right\}$$

$$\langle\tau\rangle \sim -\log\hbar$$

$$\sigma_\tau^2 \sim O(\hbar^0)$$

Results



$\hbar = 10^{-1}$
$\hbar = 10^{-2}$
$\hbar = 10^{-3}$
40
35
30
$\langle \tau \rangle$
1.3
$p_{c,1}$
1.5
1.6
p_0

Conclusions

- Quantum traversal time is an observable of quantum transition.
- Unstable semiclassical motions correspond to long quantum transitions.

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Thank you for attention!