The muon g-2 discrepancy: errors or new physics?

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The present experimental values:

$$a_e = 1159652180.73\ (28) \times 10^{-12}$$

0.24 parts per <u>billion</u> !! Hanneke et al., PRL100 (2008) 120801

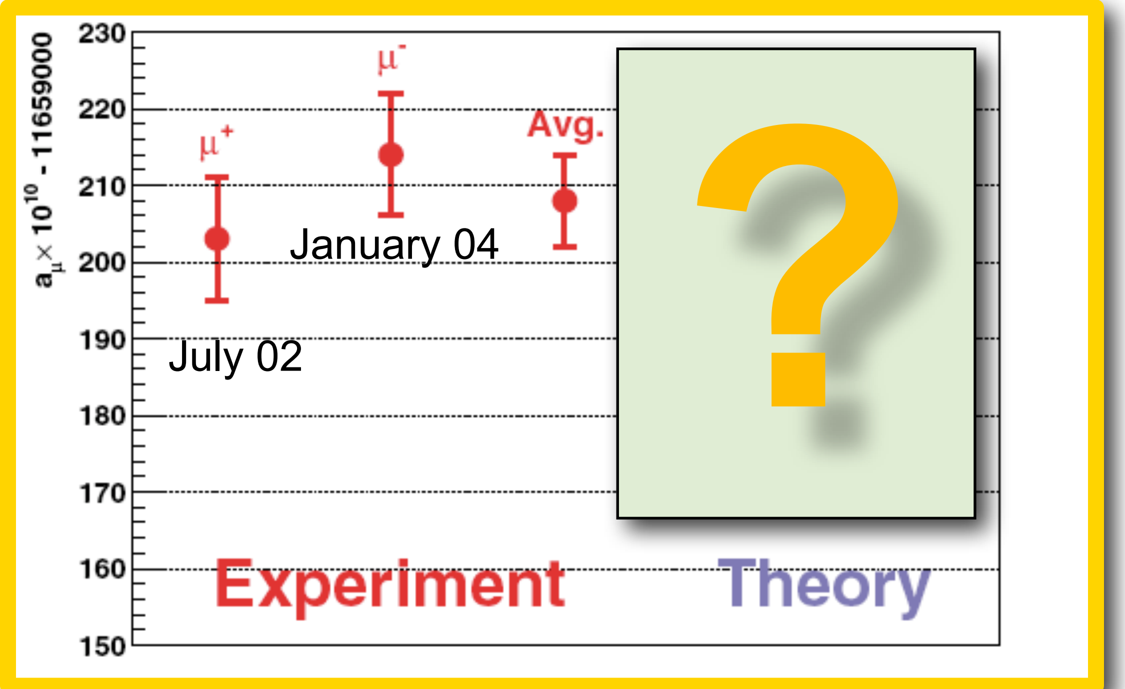
$$a_\mu = 116592080\ (63) \times 10^{-11}$$

0.5 parts per <u>million</u> !! E821 – Final Report: PRD73 (2006) 072003

$$a_\tau = -0.018\ (17)$$

DELPHI - EPJC35 (2004) 159 [$a_\tau^{SM} = 117721(5) \times 10^{-8}$, Eidelman & MP '07]

The muon g-2: experimental result



- Today: $a_\mu^{EXP} = (116592080 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- Future: a new $(g-2)_\mu$ exp aims at 0.14 ppm! [D. Hertzog, KLOE2 Physics Workshop, LNF, April 2009].
- Are theorists ready for this? [not yet]

The anomalous magnetic moment: the basics

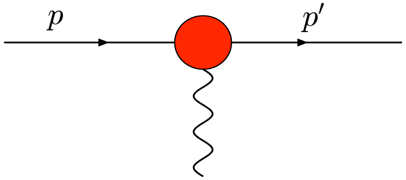
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon-lepton vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

The QED contribution to a_μ

a_μ^{QED} = $(1/2)(\alpha/\pi)$ Schwinger 1948

+ $0.765857408\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

+ $24.05050959\ (42)\ (\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr,Taylor & Newell '08

+ $130.805\ (8)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

+ $663\ (20)\ (\alpha/\pi)^5$ In progress

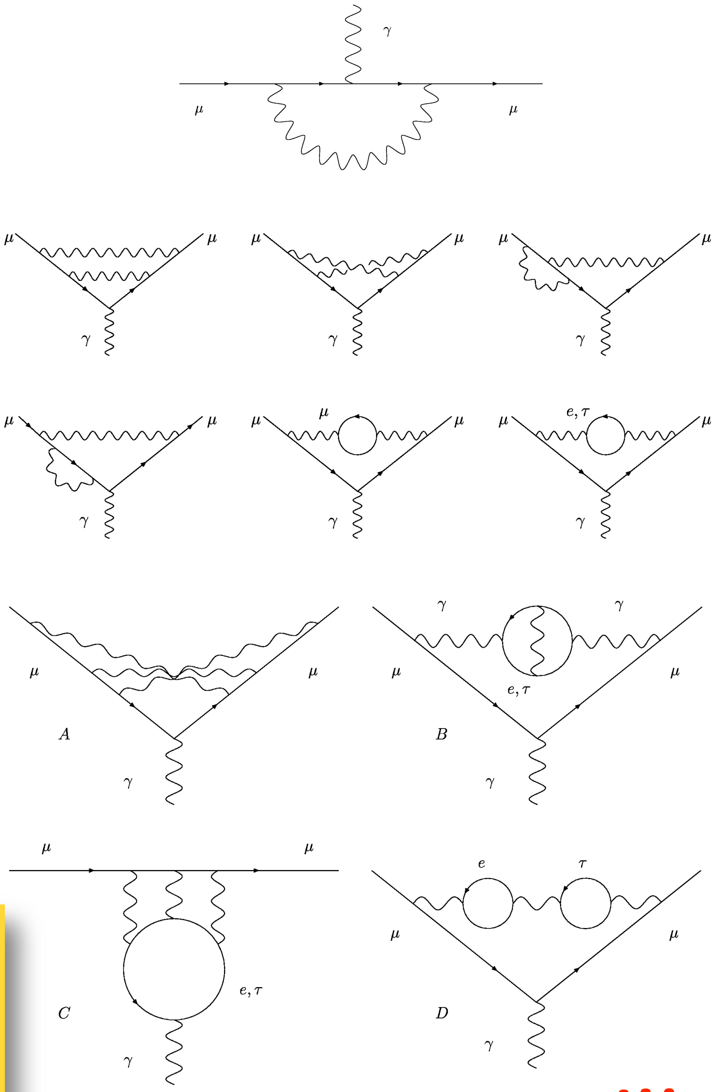
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,…, Kataev, Kinoshita & Nio March '06.

Adding up, I get:

a_μ^{QED} = $116584718.08\ (14)(04) \times 10^{-11}$

from coeffs, mainly from 5-loop unc ↲ ↳ from new $\delta\alpha$('08)

with α=1/137.035999084(51) [0.37 ppb]



…

[A parenthesis on the electron g-2…

a_e^{SM} = $(1/2)(\alpha/\pi)$ - $0.328\,478\,444\,002\,89(60)\,(\alpha/\pi)^2$

Schwinger 1948 Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '06

$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,78\,(26) \times 10^{-7}$

$A_2^{(4)}(m_e/m_\tau) = 1.837\,62\,(60) \times 10^{-9}$

$+\ 1.181\,234\,016\,827\,(19)\,(\alpha/\pi)^3$

Kinoshita, Barbieri, Laporta, Remiddi, … , Li, Samuel; Mohr,Taylor & Newell '08, MP '06

$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,73\,(27) \times 10^{-6}$

$A_2^{(6)}(m_e/m_\tau) = -6.5819\,(19) \times 10^{-8}$

$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,45\,(62) \times 10^{-13}$

$-\ 1.9144\,(35)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio, June '07

$+\ 0.0\,(4.6)\,(\alpha/\pi)^5$ In progress (12672 mass ind. diagrams!)

Aoyama, Hayakawa, Kinoshita, Nio '07; Aoyama, Hayakawa, Kinoshita, Nio & Watanabe 06/2008.

$+\ 1.676\,(20) \times 10^{-12}$ Hadronic

Krause 1997, Jegerlehner & Nyffeler 2009

$+\ 0.02973\,(52) \times 10^{-12}$ Electroweak

Mohr,Taylor & Newell, '08; Czarnecki, Krause, Marciano '96

… and the best determination of alpha]

- The new measurement of the electron g-2 is:

  a_e^{exp} = 1159652180.73 (28) × 10^{-12} Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  a_e^{exp} = 1159652188.3 (4.2) × 10^{-12} Van Dyck et al, PRL59 (1987) 26

- Equating $a_e^{SM}(\alpha) = a_e^{exp}$ → best determination of alpha to date:

$$\alpha^{-1} = 137.035\,999\,084\,\underbrace{(12)}_{\delta C_4^{qed}}\underbrace{(37)}_{\delta C_5^{qed}}\underbrace{(2)}_{\delta a_e^{had}}\underbrace{(33)}_{\delta a_e^{exp}}\ [0.37\text{ppb}]$$

Hanneke et al, '08

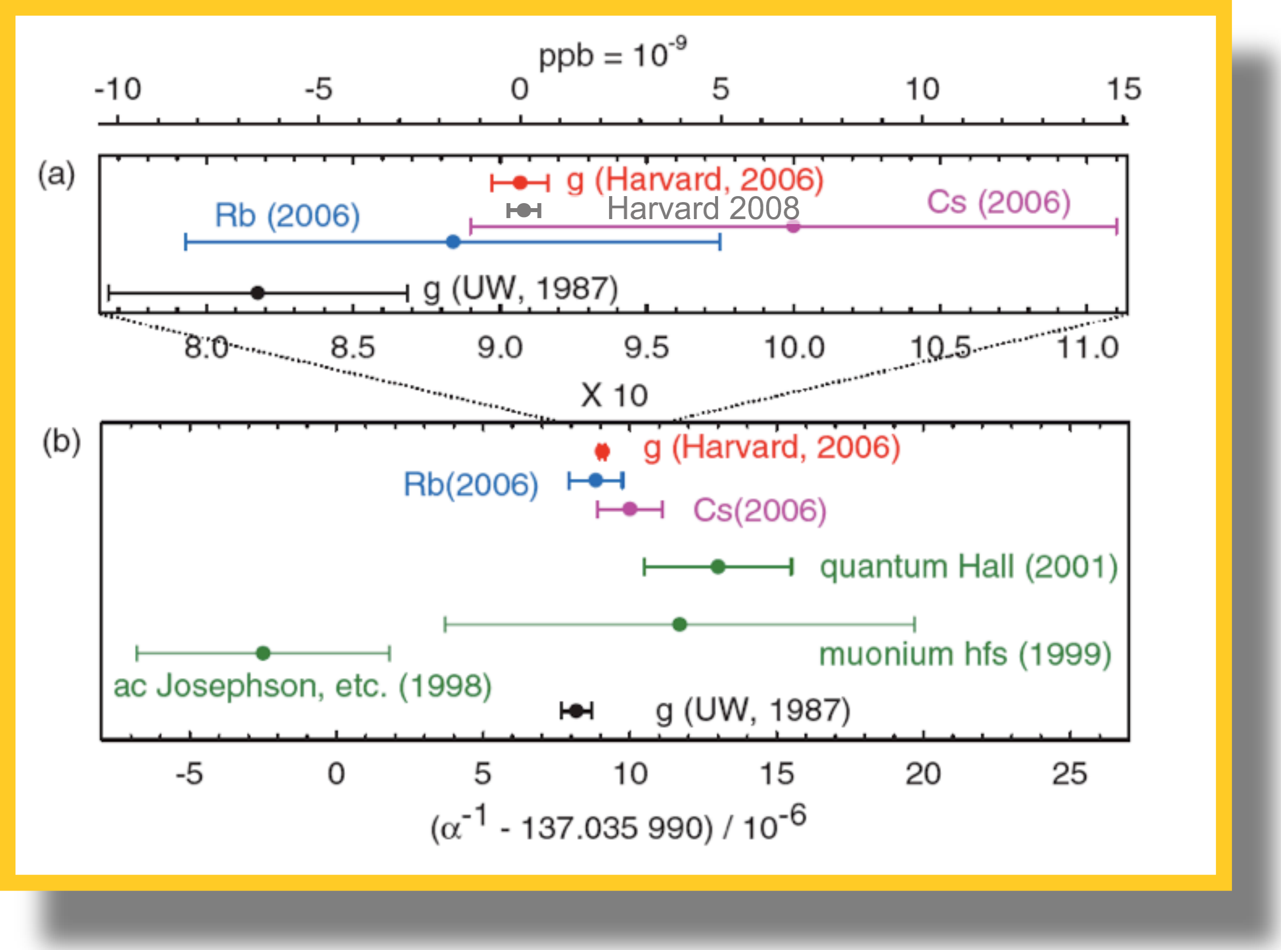
δa_e^{exp} (smaller than th!)

- Compare it with other determinations (independent of a_e):

α^{-1} = 137.036 000 00	(110)	[7.7 ppb]	PRA73 (2006) 032504 (Cs)
α^{-1} = 137.035 998 78	(91)	[6.7 ppb]	PRL96 (2006) 033001 (Rb)
α^{-1} = 137.035 999 45	(62)	[4.6 ppb]	PRL101 (2008) 230801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

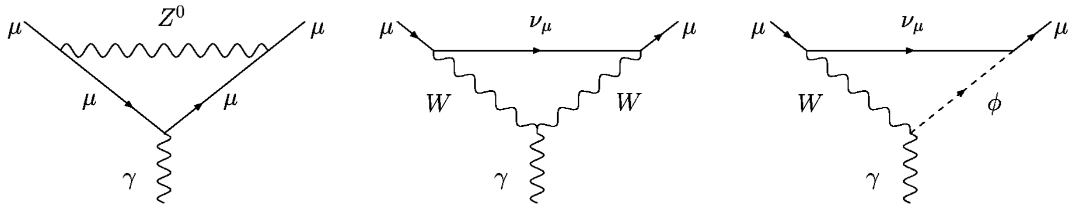
Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801

The Electroweak contribution

- One-loop term:



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

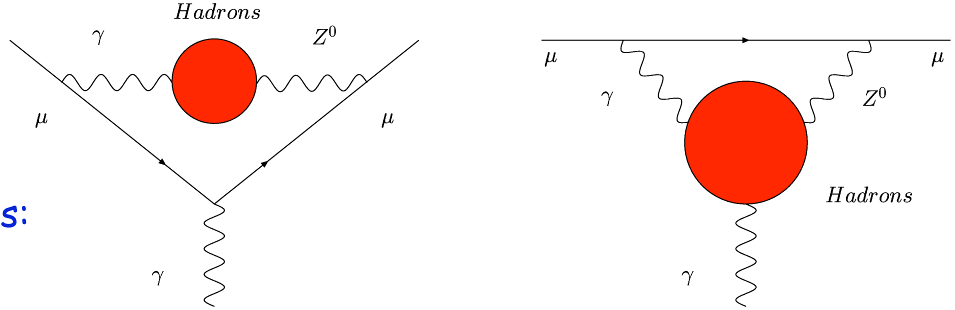
- One-loop plus higher-order terms:

a_μ^{EW} = 154 (2) (1) × 10^{-11}

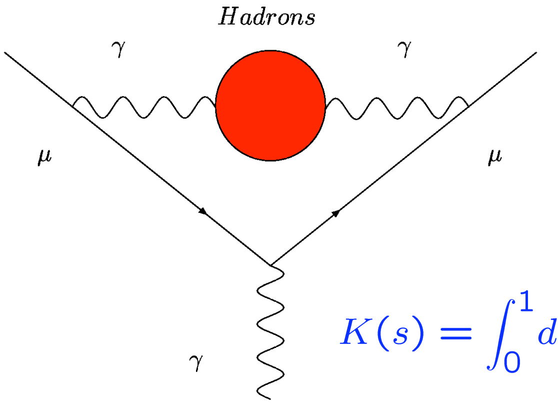
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrassi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

Higgs mass, M_{top} error,
3-loop nonleading logs

Hadronic loop uncertainties:



The hadronic leading-order (HLO) contribution



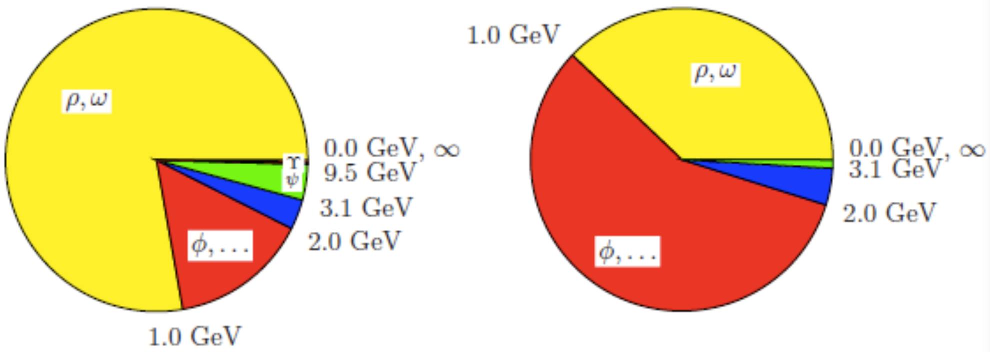
$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\,K(s)\,\sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}K(s)\,R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}$$

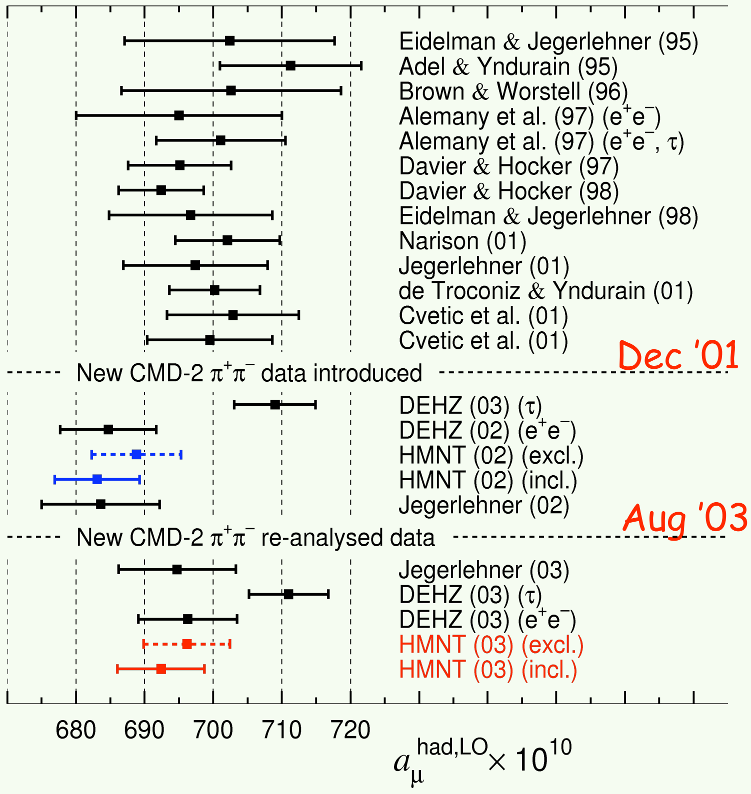
Bouchiat & Michel 1961; Gourdin & de Rafael 1969

Central values

Errors^2



F. Jegerlehner and A. Nyffeler, arXiv:0902.3360



Hagiwara et al., PRD 69 (2004) 093003

The HLO contribution: e^+e^- data

$a_\mu^{HLO} = 6909\,(39)_{exp}\,(19)_{rad}\,(7)_{qcd} \times 10^{-11}$ S. Eidelman, ICHEP06; M. Davier, TAU06

$= 6894\,(42)_{exp}\,(18)_{rad} \times 10^{-11}$ Hagiwara, Martin, Nomura, Teubner, PLB649(2007)173

$= 6903\,(53)_{tot} \times 10^{-11}$ F. Jegerlehner, A. Nyffeler, arXiv:0902.3360

$= 6898\,(43)_{exp+rad}\,(7)_{qcd} \times 10^{-11}$ Davier et al, arXiv:0906.5443v2 (all data)

- Radiative Corrections (Luminosity, ISR, Vacuum Polarization, FSR) are a very delicate issue! Are they all under control?

- CMD2: The 1998 $\pi^+\pi^-$ data in the ρ energy range, published in 2007, agree with their earlier 1995 ones.

- SND's $\pi^+\pi^-$ 2006 data reanalysis in agreement with CMD2.

The HLO contribution: e^+e^- data (ISR Method)

- The RADIATIVE RETURN (ISR) Method: KLOE & BaBar. Collider operates at fixed energy but s_π can vary continuously. Important independent method made possible by beautiful interplay between theory and experiment.

- KLOE: In 2008 KLOE presented an update of their 2005 $\pi^+\pi^-$ analysis plus a new measurement: PLB670 (2009) 285. The new measurement supersedes the 2005 one.

- Agreement between KLOE (2008) and CMD2-SND below the ρ, some discrepancies above. Their contributions to a_μ^{HLO} agree.

- BaBar: $\pi^+\pi$ preliminary results (from 0.5 to 3 GeV) presented at Tau08. Disagreement with CMD2, SND and KLOE. Better agreement with τ results, especially with Belle. We must wait for a publication.

The HLO contribution: Tau-decay data (Aleph, Opal, Cleo & Belle)

- The τ data of ALEPH and CLEO are significantly higher than the CMD2-SND-KLOE ones, particularly above the ρ.
- The 2008 $a_\mu^{\pi\pi}$ τ result of Belle agrees with Aleph-Cleo-Opal. Some deviations from Aleph's spectral functions.
- Yesterday: $\boxed{a_\mu^{HLO} = 7053\,(45) \times 10^{-11}}$ Davier et al, arXiv:0906.5443v2

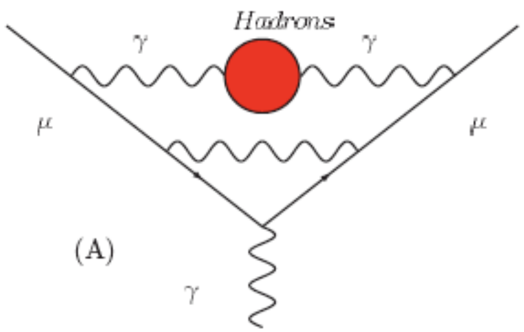
  vs 2003: $\boxed{a_\mu^{HLO} = 7110\,(58) \times 10^{-11}}$ Davier et al, EPJC31 (2003) 503

  Belle's data included + <u>IB corrections revisited & updated</u>

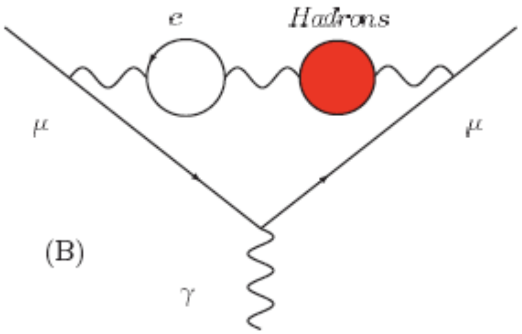
  (Marciano & Sirlin '88; Cirigliano, Ecker, Neufeld '01-'02, Flores-Baez et al. '06 &' 07)
- The discrepancy with e^+e^- data is smaller, but it's still there! Inconsistencies in e^+e^- or τ data? All possible isospin-breaking (IB) effects taken into account? Recent claims that e^+e^- & τ data are consistent after IB effects & vector meson mixings considered (Benayoun et al.'07 & '09).

The hadronic higher-order (HHO) contributions: VP

HHO: Vacuum Polarization

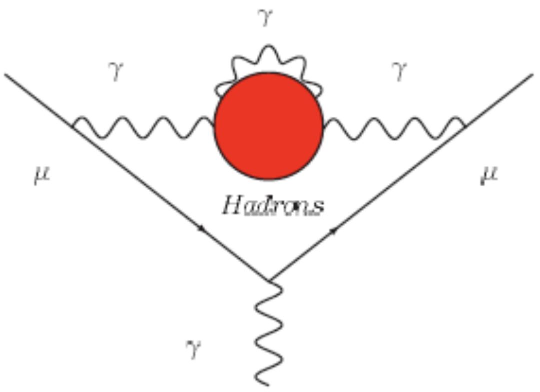


(A)



(B)

Already included in a_μ^{HLO}



$O(a^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

$$\boxed{a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}}$$

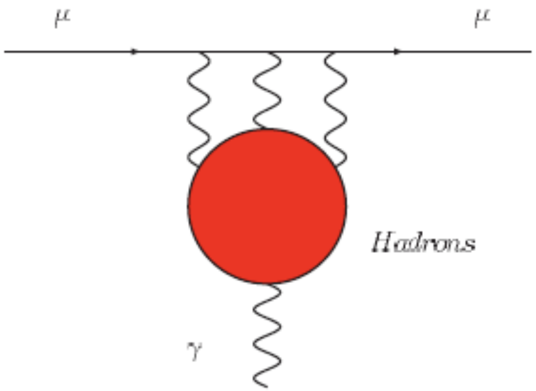
Krause '96, Alemany et al. '98, Hagiwara et al. '03 & '06

Shifts by ~ -3×10^{-11} if τ data are used instead of the e^+e^- ones

Davier & Marciano '04.

The hadronic higher-order (HHO) contributions: LBL

HHO: Light-by-light contribution



- Unlike the HLO term, for the hadronic l-b-l term one must rely on theoretical approaches.
- This term had a troubled life! Its recent determinations vary between:

$a_\mu^{HHO}(lbl)$ = + 80 (40) x 10^{-11} Knecht & Nyffeler '02
$a_\mu^{HHO}(lbl)$ = +136 (25) x 10^{-11} Melnikov & Vainshtein '03
$a_\mu^{HHO}(lbl)$ = +105 (26) x 10^{-11} Prades, de Rafael, Vainshtein '09
$a_\mu^{HHO}(lbl)$ = +116 (39) x 10^{-11} Jegerlehner & Nyffeler '09

(results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02)

- Upper bound: $a_\mu^{HHO}(lbl) < \sim 160 \times 10^{-11}$ Erler & Sanchez 2006, Pivovarov 2002
- Lattice? Very hard, but in progress: Rakow et al, Hayakawa et al., ...
- It's likely to become the ultimate limitation of the SM prediction.

The muon g-2: Standard Model vs. Experiment

- Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

$a_\mu^{\mathrm{SM}} \times 10^{11}$		$\Delta a_\mu \times 10^{11}$	σ
[1]	116 591 788 (51)	292 (81)	3.6
[2]	116 591 773 (53)	307 (82)	3.7
[3]	116 591 782 (59)	298 (86)	3.4
[4]	116 591 777 (51)	303 (81)	3.7
[5]	116 591 929 (52)	151 (82)	1.8

with $a_\mu^{HHO}(lbl) = 105\ (26) \times 10^{-11}$. $\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$.

[1] Eidelman at ICHEP06 & Davier at TAU06.
[2] Hagiwara, Martin, Nomura, Teubner, PLB649 (2007) 173.
[3] F. Jegerlehner and A. Nyffeler, arXiv:0902.3360.
[4] Davier et al, arXiv:0906.5443v2 August 2009 (all e^+e^- data) .
[5] Davier et al, arXiv:0906.5443v2 August 2009 (τ data).

- The th error is now about the same as the exp. one!

The muon g-2 and the bounds on the Higgs mass

MP, W.J. Marciano & A. Sirlin

[PRD78, 013009 (2008)]

The Hadronic Contribution to $\alpha(M_Z^2)$ ...

The effective fine-structure constant at the scale M_Z^2 is given by:

$$\alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha(M_Z)} \qquad \text{with} \qquad \Delta\alpha = \Delta\alpha_{\rm lep} + \Delta\alpha^{(5)}_{\rm had} + \Delta\alpha_{\rm top}$$

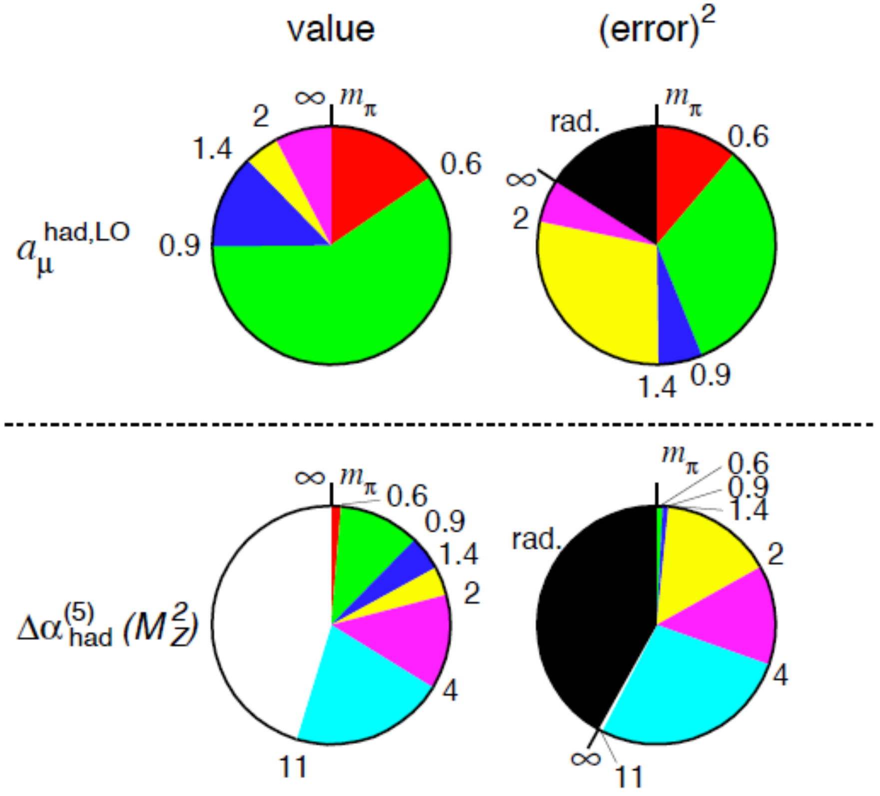
The light quarks part is determined by:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z) = \frac{M_Z^2}{4\alpha\pi^2}\, P\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{M_Z^2 - s}$$

Progress due to significant improvement of the data:

$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ =

0.02800 (70)	Eidelman, Jegerlehner'95
0.02775 (17)	Kuhn, Steinhauser 1998
0.02749 (12)	Troconiz, Yndurain 2005
0.02758 (35)	Burkhardt, Pietrzyk 2005
0.02768 (22)	Hagiwara et al. 2006
0.02761 (23)	F. Jegerlehner 2008



Hagiwara et al (HMNT) 2006

... and the EW Bounds on the SM Higgs mass

- The dependence of SM predictions on the Higgs mass, via loops, provides a powerful tool to set bounds on its value.
- Comparing the theoretical predictions of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$
  [convenient formulae in terms of M_H, M_{top}, $\Delta\alpha_{had}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$ by Degrassi, Gambino, MP, Sirlin '98; Degrassi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06]

with

M_W = 80.399 (25) GeV [LEP+Tevatron]

$\sin^2\theta_{\rm eff}^{\rm lept}$ = 0.23153 (16) [LEP+SLC]

and

$\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02768 (22) [HMNT '07]

M_{top} = 172.4 (1.2) GeV [CDF-D0, Aug '08]

$\alpha_s(M_Z)$ = 0.118 (2) [PDG '08]

we get

$M_H = 88^{+32}_{-24}$ GeV & $M_H < 145$ GeV 95%CL

- The value of $\Delta\alpha_{had}^{(5)}(M_Z)$ is a key input of these EW fits...

Back to Δa_μ: how do we explain it?

- Δa_μ can be explained in many ways: errors in HHO-LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or New Physics.
- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

a_μ^{HLO} :
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \; s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)}$:
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

$(\varepsilon>0)$, in the range:

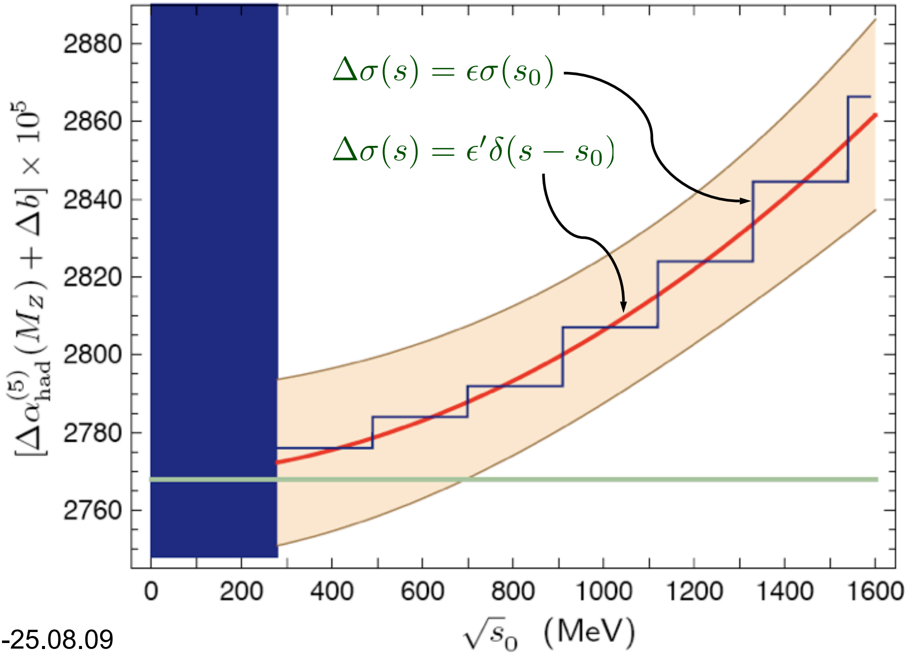
$$\sqrt{(s)} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Shifts of a_μ^{HLO} and $\Delta\alpha_{had}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ is adjusted to bridge the g-2 discrepancy, the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ increases by:

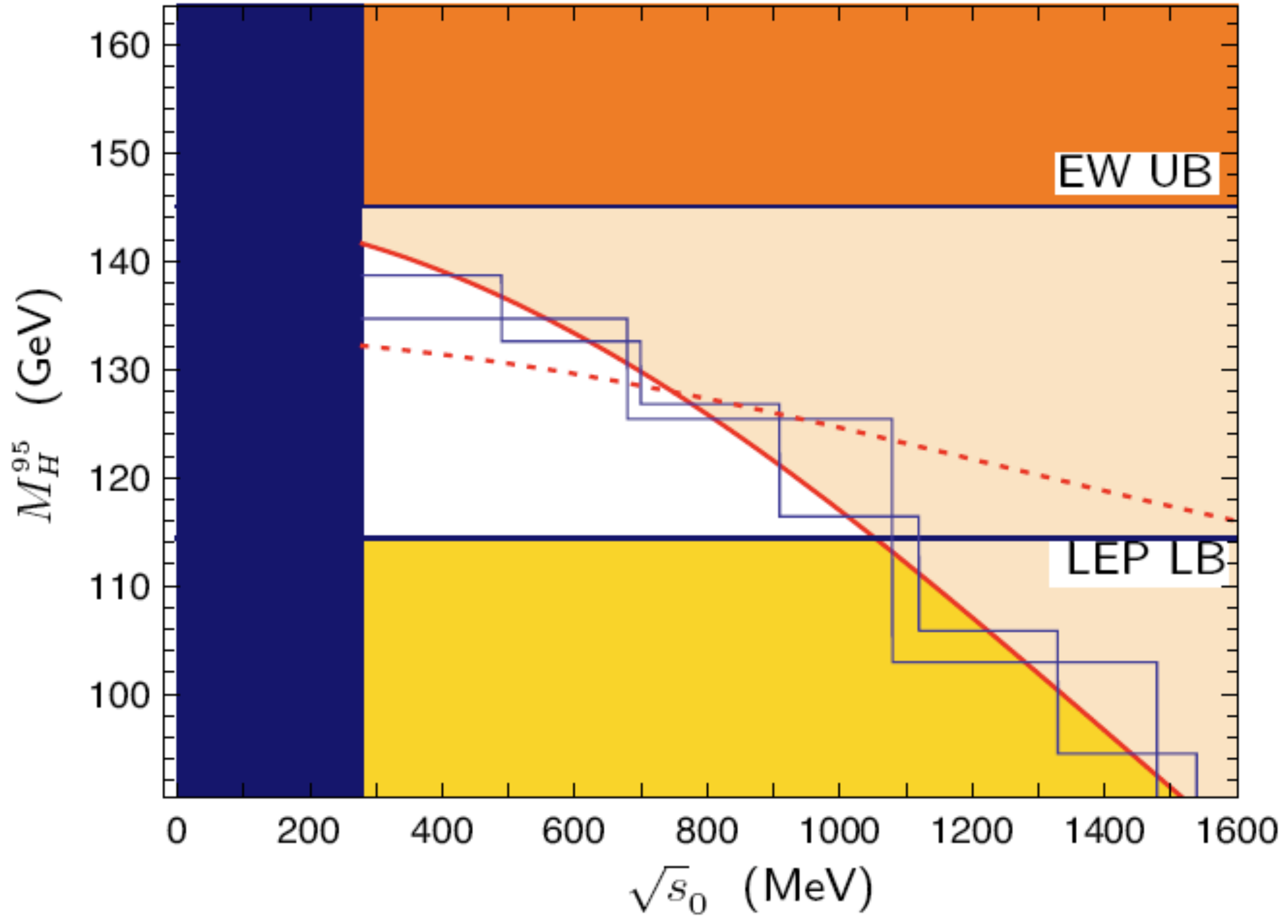
$$\Delta b(\sqrt{s_0}, \delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2)\,\sigma(t^2)\,t\,dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2)\,\sigma(t^2)\,t\,dt}$$

- Adding this shift to $\Delta\alpha_{had}^{(5)}(M_Z) = 0.02768(22)$ [HMNT07] , with $\Delta a_\mu = 302(88) \times 10^{-11}$ [HMNT07], we obtain:



The muon g-2: connection with the SM Higgs mass

- How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?

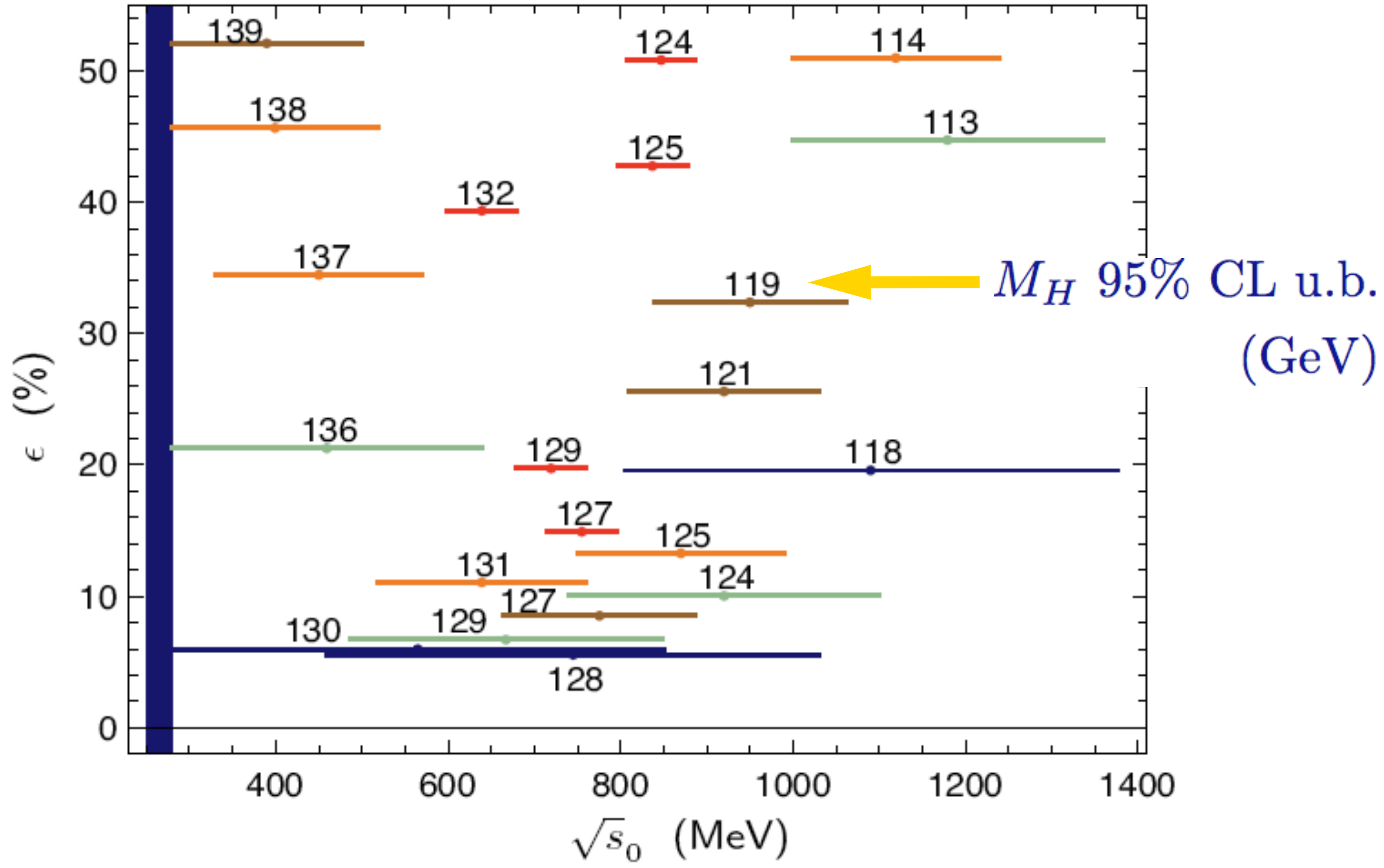


The muon g-2: connection with the SM Higgs mass (2)

- The LEP direct-search lower bound is M_H^{LB} = 114.4 GeV (95%CL).
- The hypothetical shifts $\Delta\sigma = \varepsilon\sigma(s)$ that bridge the muon g-2 discrepancy conflict with the LEP lower limit when $\sqrt{s_0} >$ ~1.2GeV (for bin widths δ up to several hundreds of MeV).
- While the use of tau data in the calculation of a_μ^{HLO} reduces the muon g-2 discrepancy, it increases the value of $\Delta\alpha_{had}^{(5)}(M_Z)$, lowering the M_H upper bound (tension with the M_H lower bound).
- Recent claim: e^+e^- & tau data consistent below ~1 GeV (after isospin viol. effects & vector meson mixings). We could thus assume that Δa_μ is fixed by hypothetical errors above ~1GeV (where disagreement persists). If so, M_H^{UB} falls below M_H^{LB} !!
- Scenarios where Δa_μ is accommodated without affecting M_H^{UB} are possible, but considerably more unlikely.

How realistic are these shifts Δσ(s) ?

- How realistic are these shifts Δσ(s) when compared with the quoted exp. uncertainties? Study the ratio ε=Δσ(s)/σ(s):



How realistic are these shifts $\Delta\sigma(s)$? (2)

- The minimum ε is ~ +4%. It occurs if σ is multiplied by $(1+\varepsilon)$ in the whole integration region (!), leading to M_H^{UB} ~ 70 GeV (!!)
- As the quoted exp. uncertainty of $\sigma(s)$ below 1 GeV is ~ a few per cent (or less), the possibility to explain the muon g-2 with these shifts $\Delta\sigma(s)$ appears to be unlikely.
- If, however, we allow variations of $\sigma(s)$ up to ~6% (7%), M_H^{UB} is reduced to less than ~130 GeV (131 GeV). E.g., the ~6% shift in the interval [0.6, 1.2] GeV, required to fix Δa_μ, lowers M_H^{UB} to 126GeV. Tension with the M_H >~120GeV "vacuum stability" bound.
- Reminder: the above M_H upper bounds, like the LEP-EWWG ones, depend on the value of $\sin^2\theta_{\rm eff}^{\rm lept}$. They also depend on M_t & its unc. δM_t. We prepared simple formulae to translate easily M_H upper bounds discussed above into new values corresponding to M_t & δM_t inputs different from those employed here.

Conclusions

- g: Beautiful examples of interplay between theory and experiment: g_e probed at <ppt → α and extraordinary test of QED's validity; g_μ probed at <ppb → test of the full SM and great opportunity to unveil (or just constrain) "New Physics" effects!

- The discrepancy Δa_μ is more than 3σ if e^+e^- data are used. With tau data, the deviation is ~2σ. BaBar 2π? More e^+e^- data & analyses eagerly awaited! QED & EW ready for new g-2 exp! LBL??

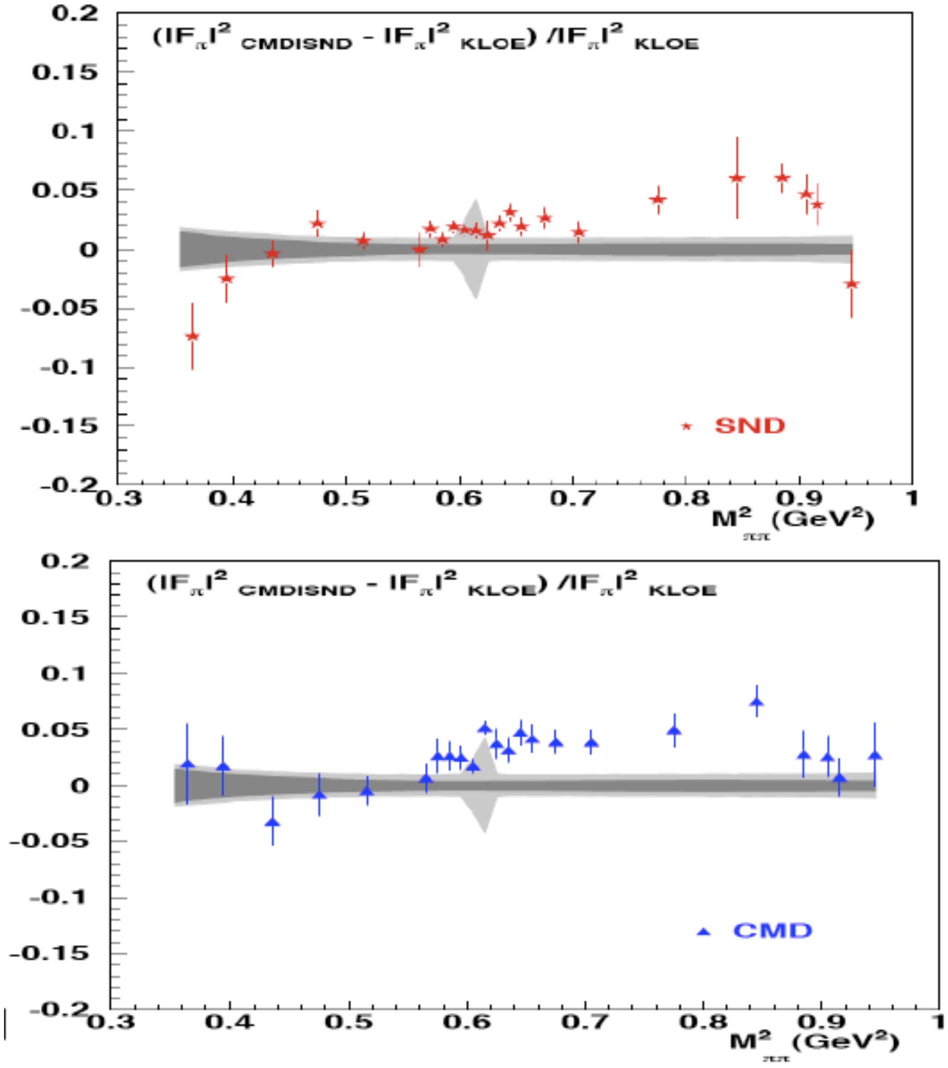
- Δa_μ can be due to New Physics, or to problems in a_μ^{SM} (or a_μ^{EXP}). Can it be due to hypothetical mistakes in the hadronic $\sigma(s)$? An increase $\Delta\sigma(s)$ could bridge Δa_μ, leading however to a decrease on the EW upper bound on the SM Higgs mass M_H ...

- By means of a detailed analysis we conclude that solving Δa_μ via an increase of $\sigma(s)$ is unlikely in view of current exp. error estimates. However, if this turns out to be the solution, then the M_H upper bound drops to about 130 GeV which, in conjunction with the LEP 114 GeV direct lower limit, leaves a rather narrow window for M_H.

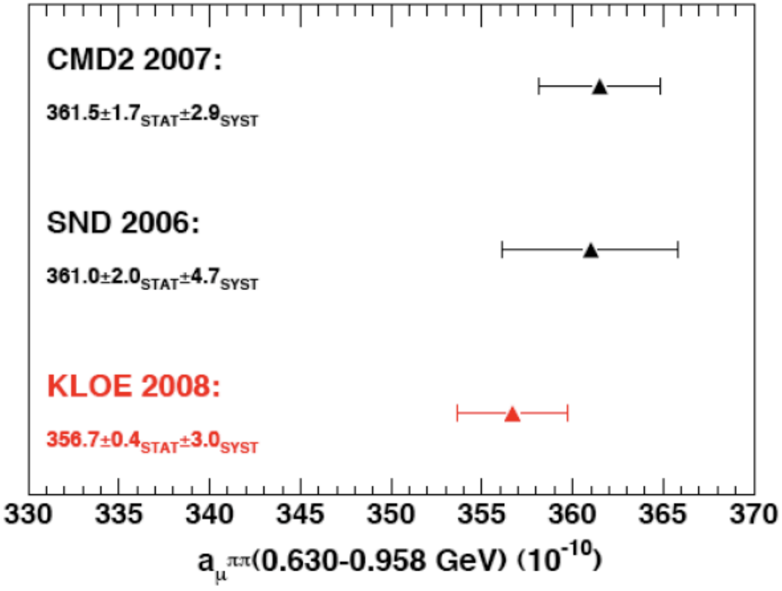
The End

Back-up Slides

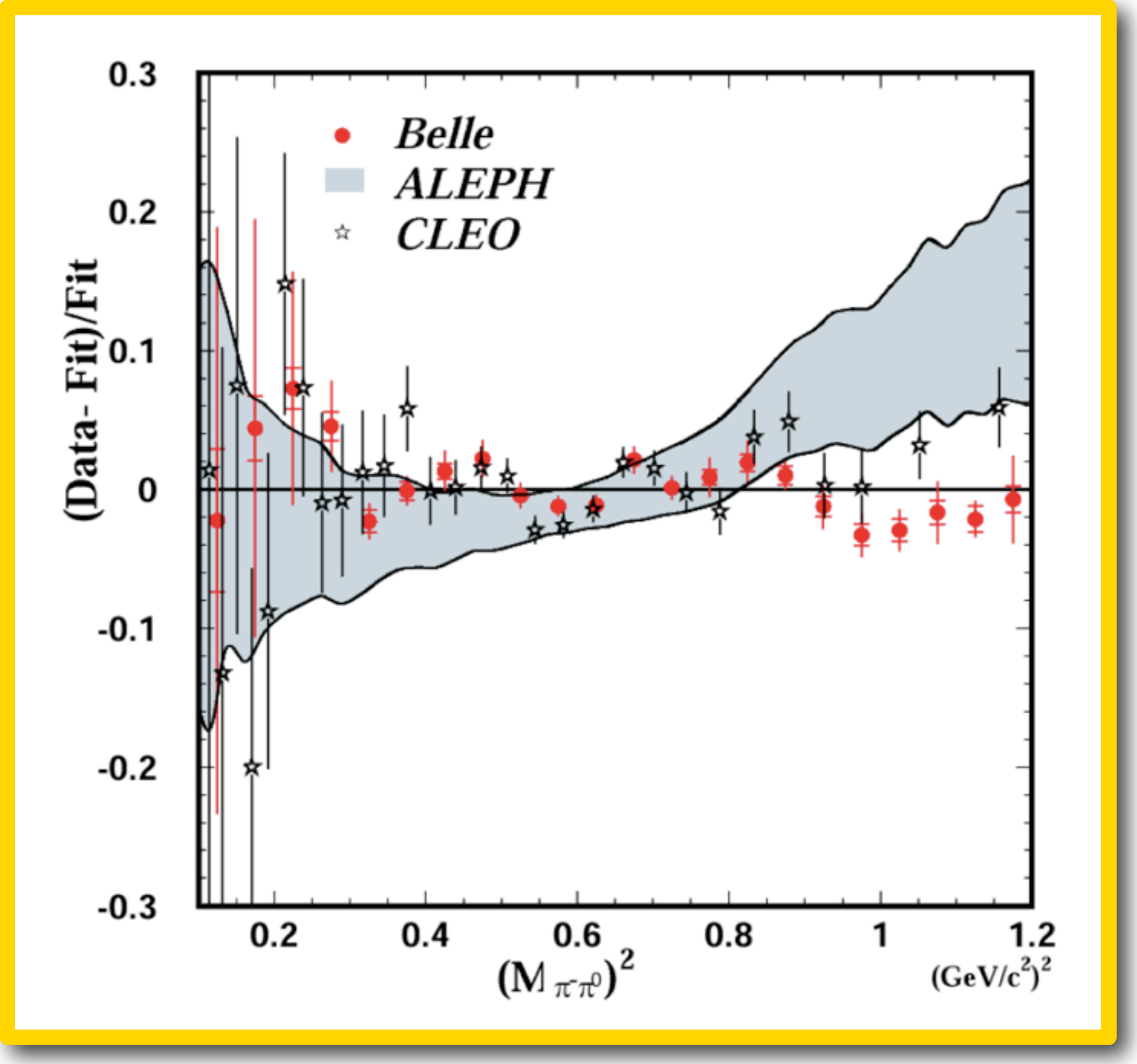
CMD2 & SND vs KLOE



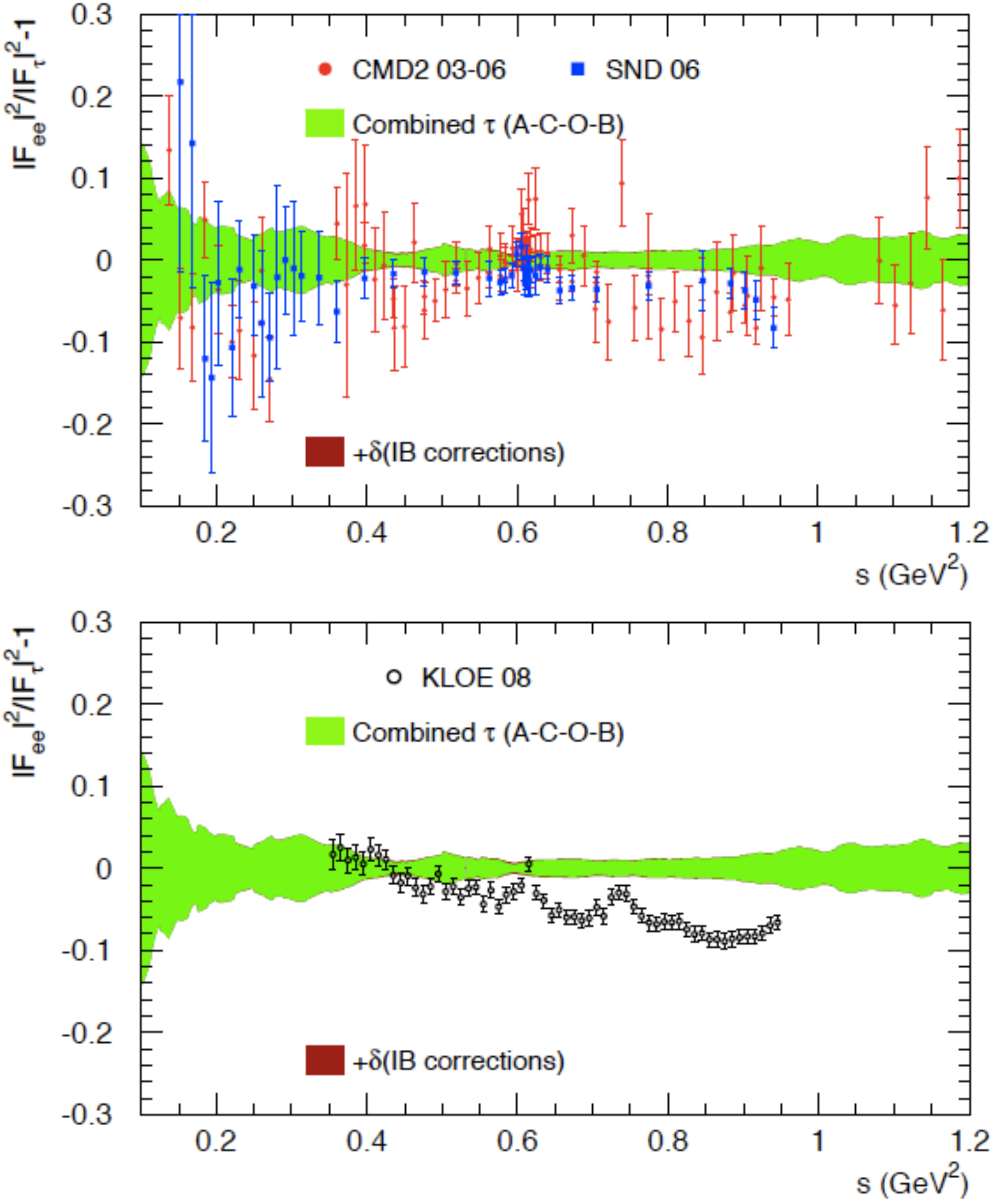
band: KLOE error
data points: CMD2/SND experiments



G. Venanzoni, Tau08, Novosibirsk, September 2008



Fujikawa, Hayashii, Eidelman [for the Belle Collab.], arXiv:0805.3773, May '08



Davier et al,
arXiv:0906.5443v2