Transport in strongly coupled QFTs and gauge/gravity duality

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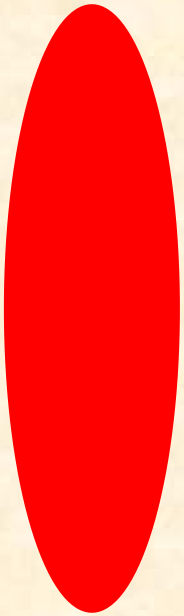
Heavy ion collision experiments at RHIC (2000-current) and LHC (2009-??) create hot and dense nuclear matter known as the “quark-gluon plasma”

(note: qualitative difference between p-p and Au-Au collisions)

Elliptic flow, jet quenching… - focus on transport in this talk

Evolution of the plasma “fireball” is described by relativistic fluid dynamics (relativistic Navier-Stokes equations): Landau; Bjorken

Need to know



thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

$$\alpha_s(T_{\text{RHIC}}) \sim O(1)$$

initial conditions (initial energy density profile)

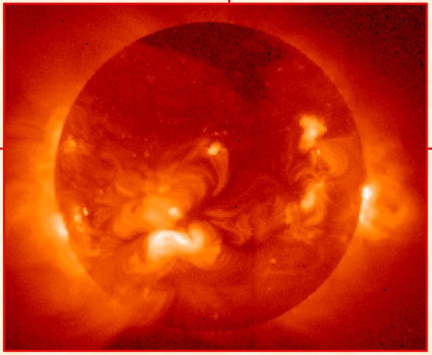
thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Quantum field theories at finite temperature/density

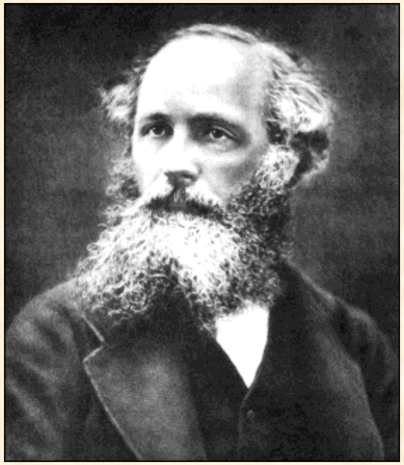
Thermodynamics

Kinetics



Equilibrium

Near-equilibrium



entropy
equation of state
....

transport coefficients
emission rates
....

perturbative — pQCD

non-perturbative — Lattice

perturbative — kinetic theory

non-perturbative — ????

Energy density vs temperature for various gauge theories

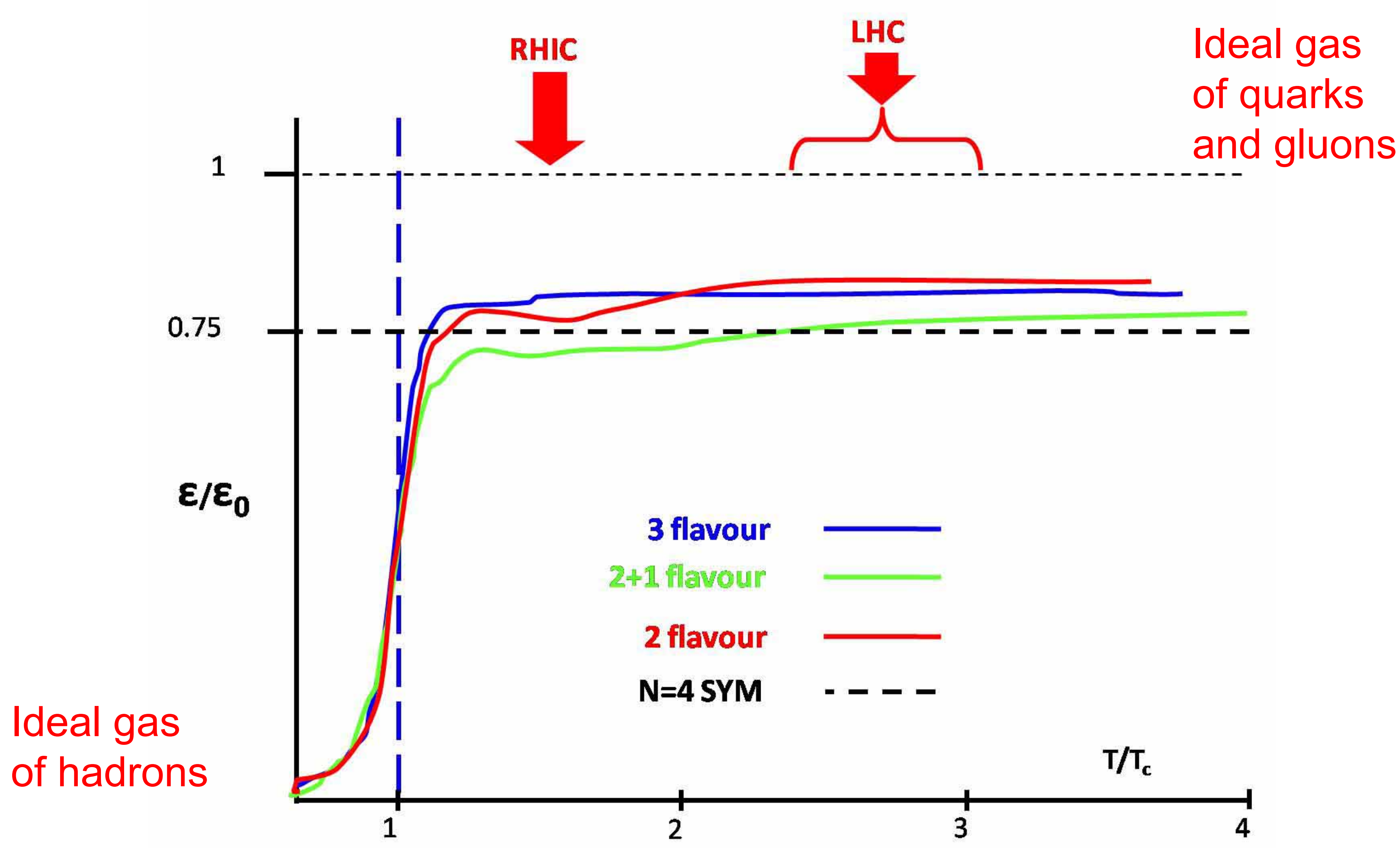


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

AdS/CFT correspondence

$\mathcal{N}=4$ supersymmetric $SU(N_c)$ YM theory in 4 dim $\Longleftrightarrow$ type IIB superstring theory on $AdS_5 \times S^5$backgrond

conjectured exact equivalence

Latest test: Janik'08

$$Z_{\mathrm{SYM}}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\mathrm{SYM}} = Z_{\mathrm{string}}[J]$$

Generating functional for correlation functions of gauge-invariant operators $\Longleftrightarrow$ String partition function

$$\langle \mathcal{O}\;\mathcal{O}\;\cdots\mathcal{O} \rangle$$

In particular

$$Z_{\mathrm{SYM}}[J] = Z_{\mathrm{string}}[J] \simeq e^{-S_{\mathrm{grav}}[J]}$$

$$\lambda \equiv g_{YM}^2\, N_c \gg 1$$

$$N_c \gg 1$$

Classical gravity action serves as a generating functional for the gauge theory correlators

$\mathcal{N} = 4$ supersymmetric YM theory

Gliozzi,Scherk,Olive'77
Brink,Schwarz,Scherk'77

- Field content:

$A_\mu \qquad \Phi_I \qquad \Psi_\alpha^A \qquad \text{all in the adjoint of } SU(N)$

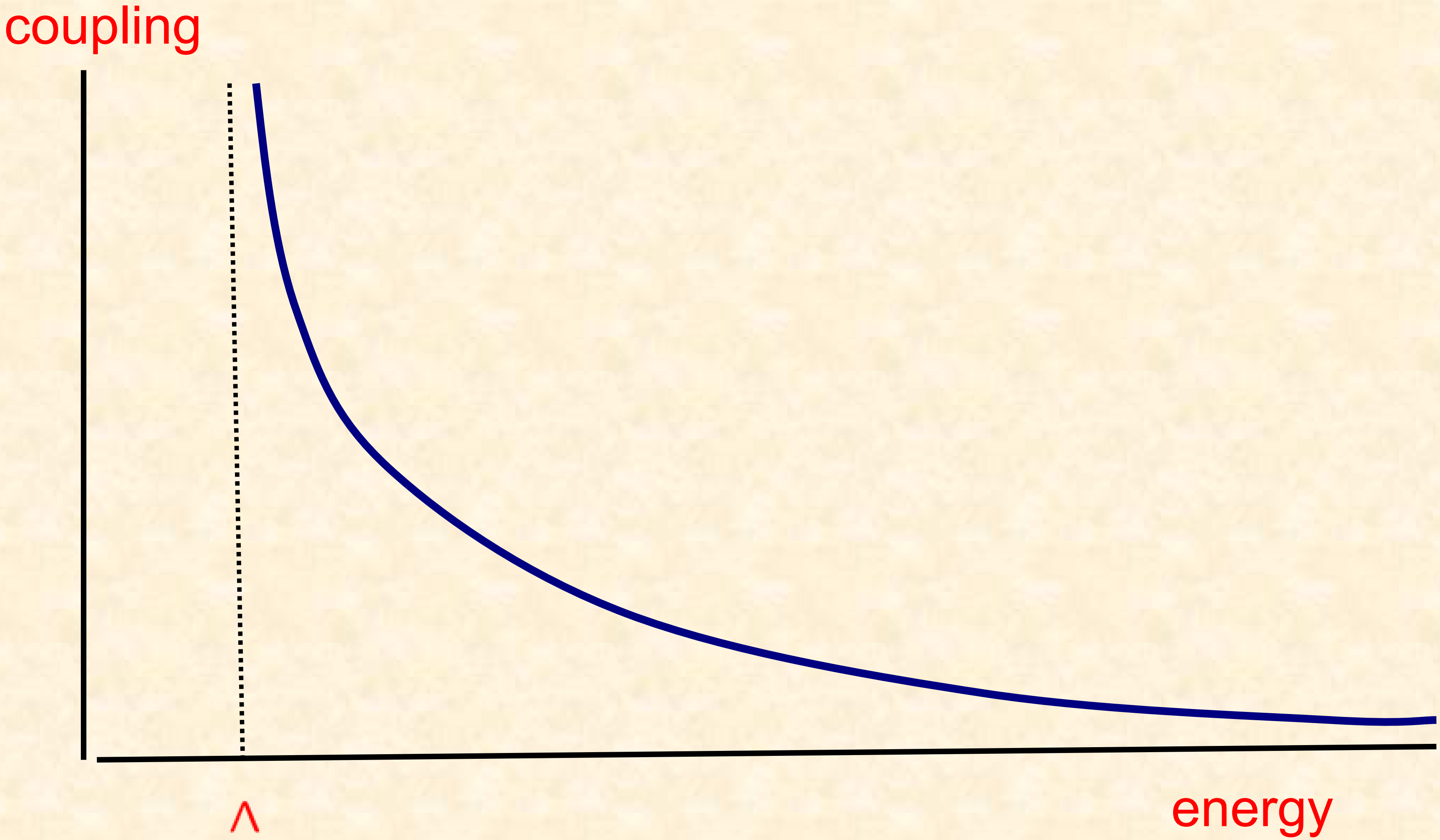
$I = 1 \ldots 6 \qquad A = 1 \ldots 4$

- Action:

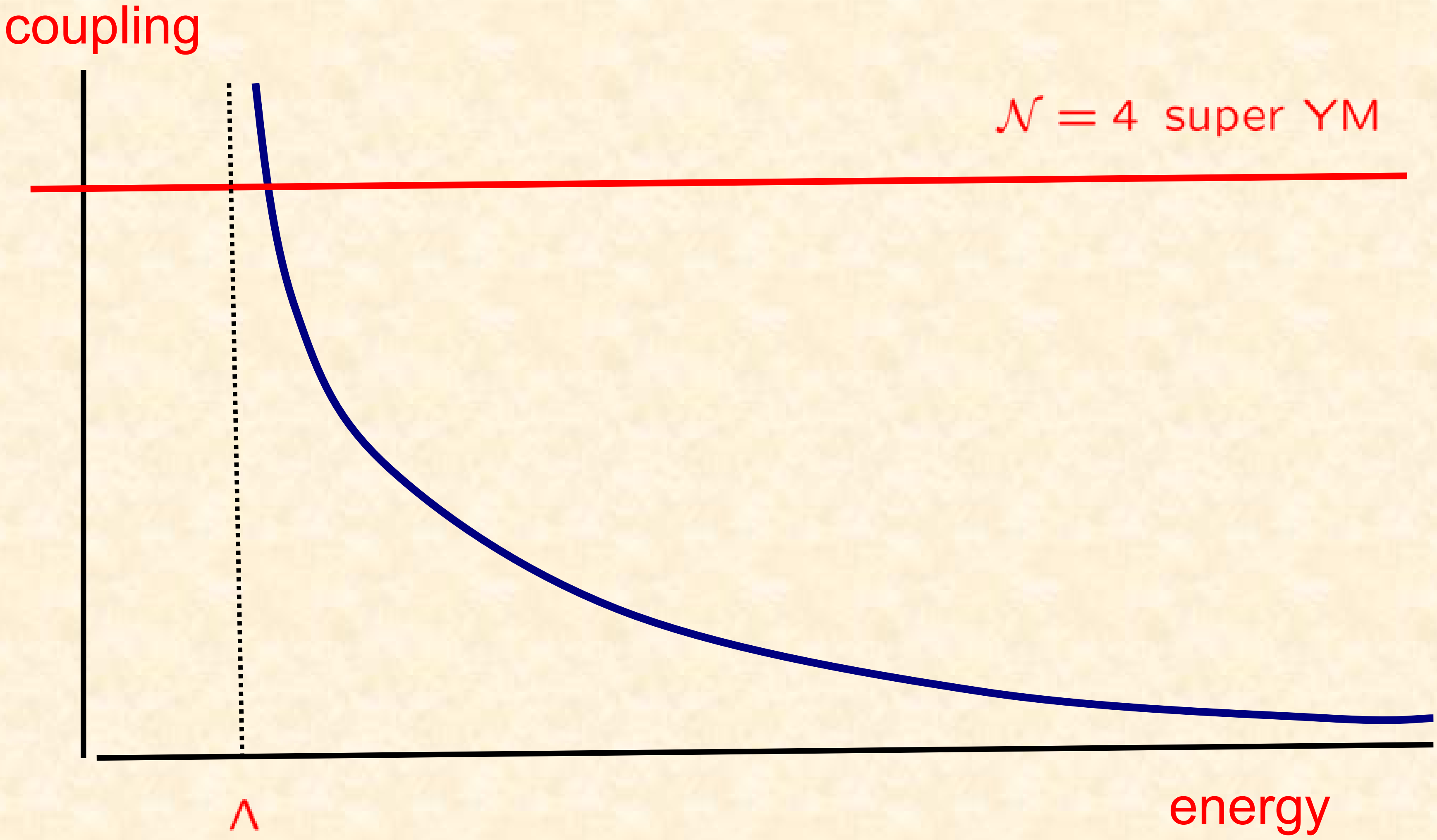
$$S = \frac{1}{g_{YM}^2} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_I)^2 - \frac{1}{2} [\Phi_I, \Phi_J]^2 + i \bar{\Psi} \Gamma^\mu D_\mu \Psi - \bar{\Psi} \Gamma^I [\Phi_I, \Psi] \right\}$$

(super)conformal field theory = coupling doesn't run

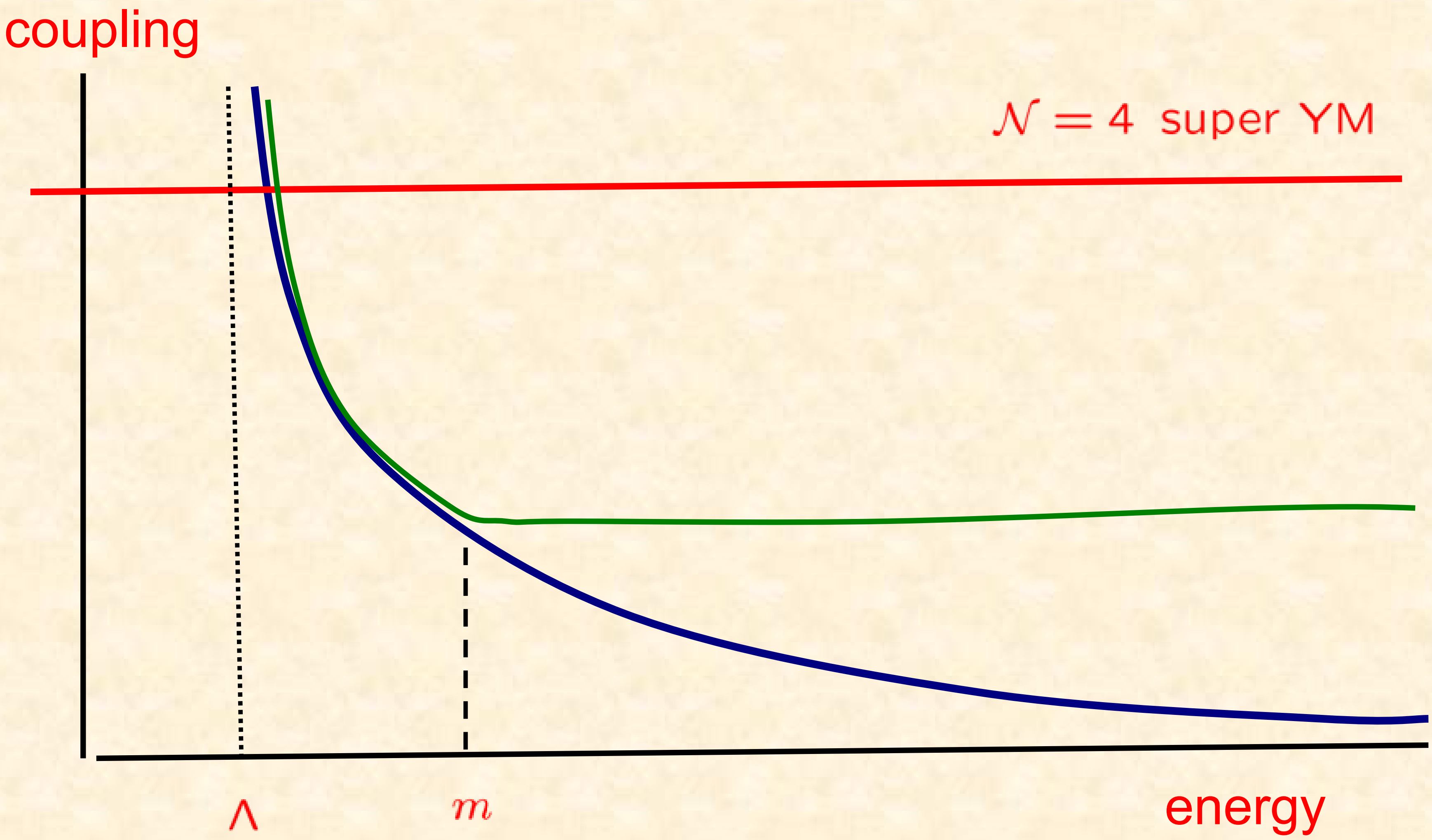
Dual to QCD? (Polchinski-Strassler)



Dual to QCD? (Polchinski-Strassler)



Dual to QCD? (Polchinski-Strassler)



At zero temperature, N=4 SYM is obviously a very bad approximation to QCD

However:

At finite temperature $T > T_c$ it is qualitatively similar to QCD

- ✓ supersymmetry broken
- ✓ non-Abelian plasma (with additional d.o.f.)
- ✓ area law for spatial Wilson loops
- ✓ Debye screening
- ✓ spontaneous breaking of Z_N symmetry at high temperature
- ✓ hydrodynamics

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law $\partial_t j^0 + \partial_i j^i = 0$

Constitutive relation

[Fick's law (1855)] $j_i = -D\, \partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$

Diffusion equation $\partial_t j^0 = D \nabla^2 j^0$

Dispersion relation $\omega = -i\, D\, q^2 + \cdots$

Expansion parameters: $\omega \ll T\,, \quad q \ll T$

First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

Second-order transport (kinetic) coefficients

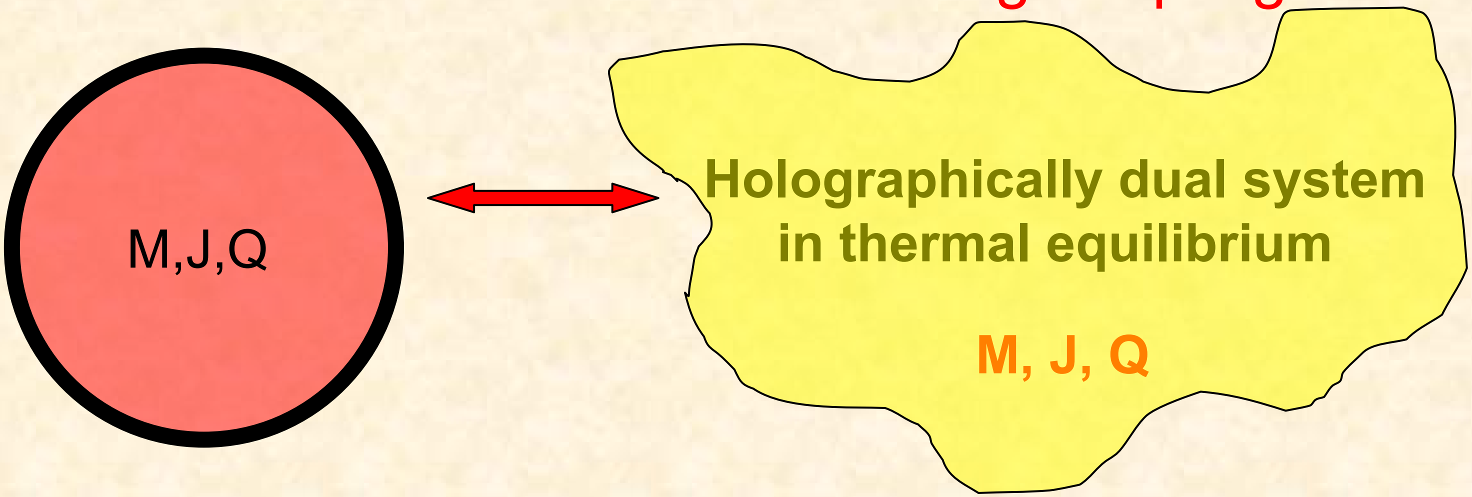
(for theories conformal at T=0)

Relaxation time	τ_Π
Second order trasport coefficient	λ_1
Second order trasport coefficient	λ_2
Second order trasport coefficient	λ_3
Second order trasport coefficient	κ

In non-conformal theories such as QCD, the total number of second-order transport coefficients is quite large

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box"\ h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$

$$\omega = -iDq^2 + \cdots$$

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual
the correlator can be computed using
the gauge theory/gravity duality

Example: stress-energy tensor correlator in $4d$ $\mathcal{N}=4$ SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Zero temperature, Euclid: $G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$

Finite temperature, Mink:

$$< T_{tt}(-\omega, -q), T_{tt}(\omega, q) >^{\text{ret}} = \frac{3 N_c^2 \pi^2 T^4 q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

(in the limit $\omega/T \ll 1\,, \quad q/T \ll 1$)

The pole
(or the lowest quasinormal freq.)

$$\omega = \pm \frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2 \sqrt{3} T^2} q^3 + \cdots$$

Compare with hydro:

$$\omega = \pm v_s q - \frac{i}{2sT} \left(\zeta + \frac{4}{3}\eta \right) q^2 + \cdots$$

In CFT: $v_s = \frac{1}{\sqrt{3}}\,, \quad \zeta = 0$

$$\Rightarrow \quad \eta = \pi N_c^2 T^3/8$$

Also, $s = \pi^2 N_c^2 T^3/2$ (Gubser, Klebanov, Peet, 1996)

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty\,, \quad g_{YM}^2 N_c \to \infty$

Shear viscosity $\quad \eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2} \right) \right]$

Bulk viscosity $\quad \zeta = 0$

for non-conformal theories see Buchel et al; G.D.Moore et al Gubser et al.

Charge diffusion constant $\quad D_R = \frac{1}{2\pi T} + \cdots$

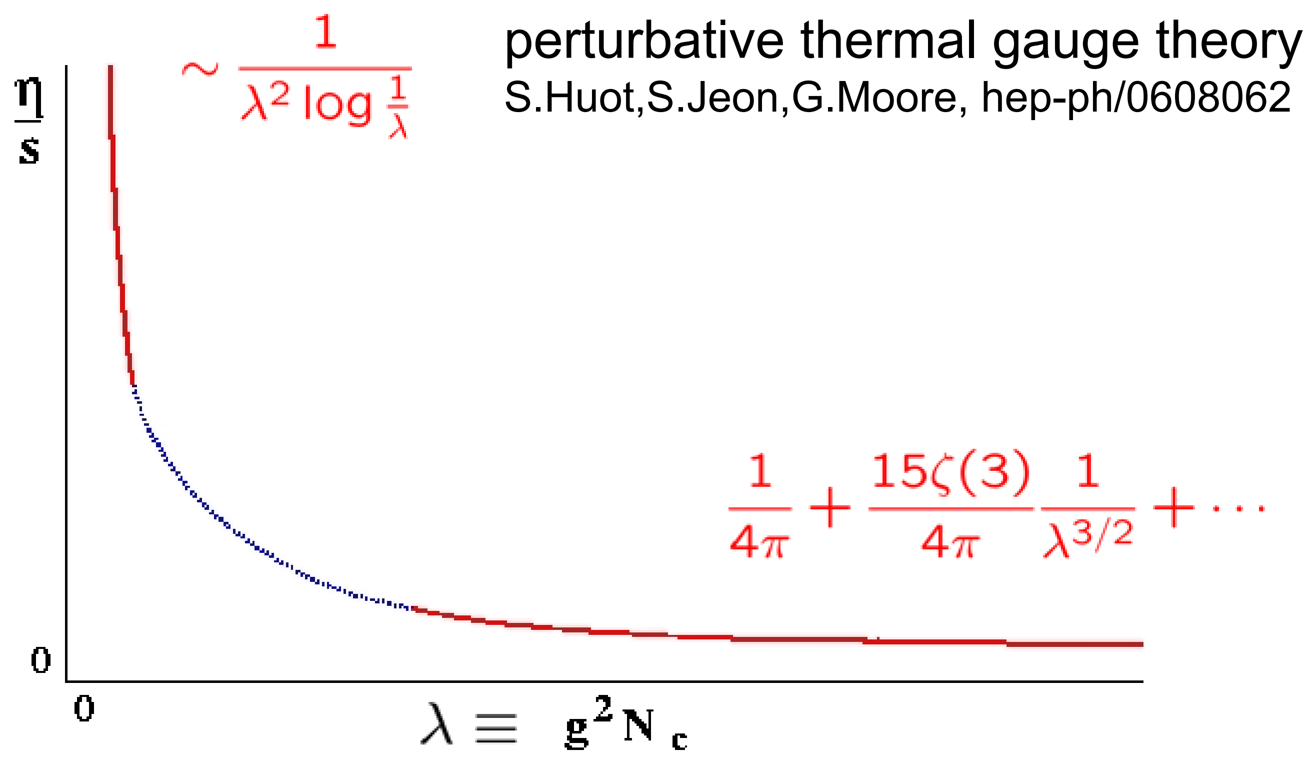
Supercharge diffusion constant $\quad D_s = \frac{2\sqrt{2}}{9\pi T}$

NEW! (G.Policastro, 2008)

Thermal conductivity $\quad \frac{\kappa_T \, \mu^2}{\eta \; T} = 8\pi^2 + \cdots$

Electrical conductivity $\quad \sigma = e^2 \frac{N_c^2 \, T}{16\,\pi} + \cdots$

Shear viscosity in $\mathcal{N} = 4$ SYM



Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Electrical conductivity in $\mathcal{N}=4$ SYM

Weak coupling: $\sigma = 1.28349 \, \frac{e^2 \, (N_c^2 - 1) \, T}{\lambda^2 \left[\ln \lambda^{-1/2} + O(1) \right]}$

$\lambda \ll 1$

Strong coupling: $\sigma = \frac{e^2 \, N_c^2 \, T}{16 \, \pi} + O\Big(\frac{1}{\lambda^{3/2}}\Big)$

$\lambda \gg 1$

* Charge susceptibility can be computed independently: $\Xi = \frac{N_c^2 T^2}{8}$

D.T.Son, A.S., hep-th/0601157

Einstein relation holds: $\frac{\sigma}{e^2 \Xi} = D_{U(1)} = \frac{1}{2\pi T}$

Universality of η/s

Theorem:

For a thermal gauge theory, the ratio of shear viscosity to entropy density is equal to $1/4\pi$
in the regime described by a dual gravity theory

(e.g. at $g^2_{YM} N_c = \infty \, , N_c = \infty$ in $\mathcal{N} = 4$ SYM)

Remarks:

- Extended to non-zero chemical potential:

  Benincasa, Buchel, Naryshkin, hep-th/0610145

- Extended to models with fundamental fermions in the limit $N_f/N_c \ll 1$

  Mateos, Myers, Thomson, hep-th/0610184

- *String/Gravity dual to QCD is currently unknown*

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w) dw^\mu dw^\nu$$

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

$$\sigma_{abs} = -\frac{16\pi G}{\omega} \operatorname{Im} G^R(\omega) = \frac{8\pi G}{\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Three roads to universality of η/s

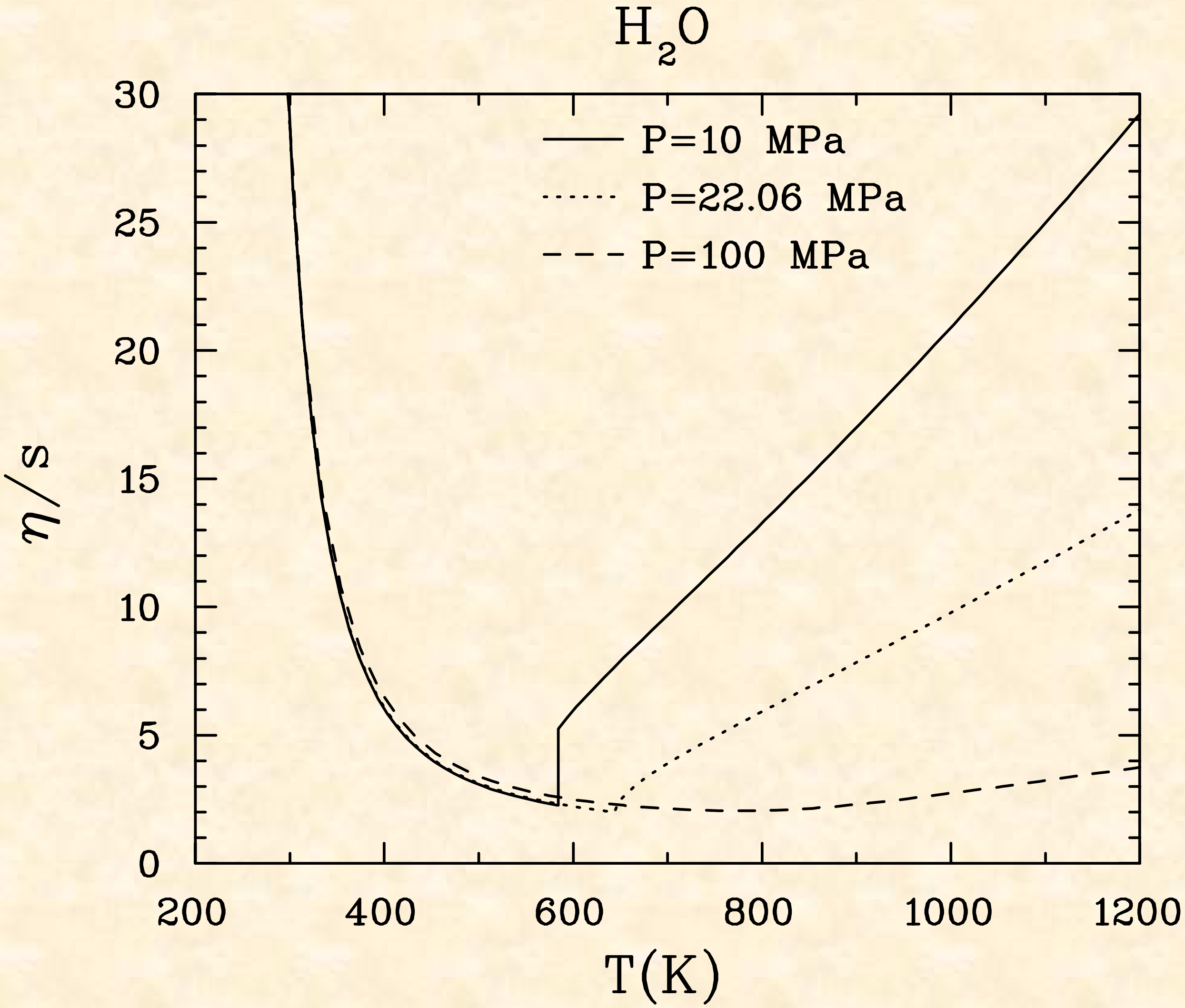
- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., 0806.3797 [hep-th], P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175



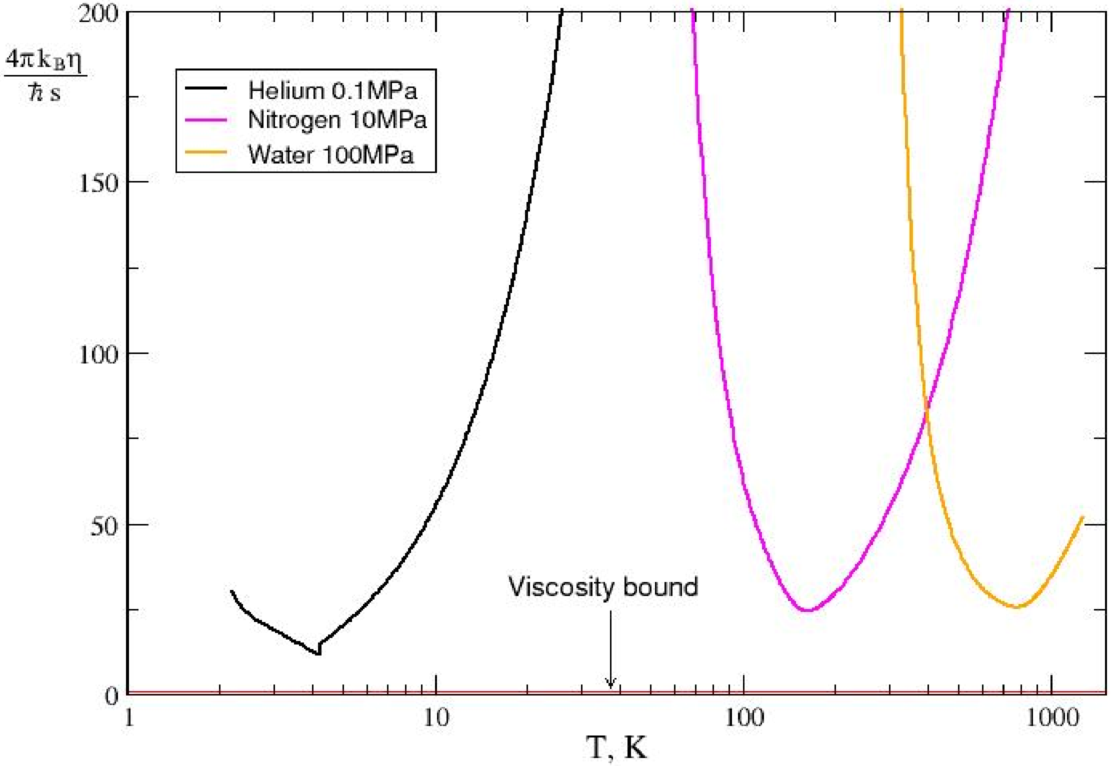
$(\eta/s)_{\text{min}} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

A viscosity bound conjecture

?

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

A hand-waving argument

$$\eta \sim \rho\, v\, l \sim \rho\, v^2 \tau \sim n\, m\, v^2 \tau \sim n\, \epsilon \tau$$

$$s \sim n$$

Thus $$\frac{\eta}{s} \sim \epsilon\tau \geq \hbar$$?

Gravity duals fix the coefficient: $$\frac{\eta}{s} \geq \hbar/4\pi$$

Shear viscosity - (volume) entropy density ratio from gauge-string duality

In ALL theories (in the limit where dual gravity valid) : $\frac{1}{4\pi} + \text{corrections}$

In particular, in N=4 SYM: $\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi}\frac{1}{\lambda^{3/2}} + \cdots$

Other higher-derivative gravity actions

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

Y.Kats and P.Petrov: 0712.0743 [hep-th]

M.Brigante, H.Liu, R.C.Myers, S.Shenker and S.Yaida: 0802.3318 [hep-th], 0712.0805 [hep-th].

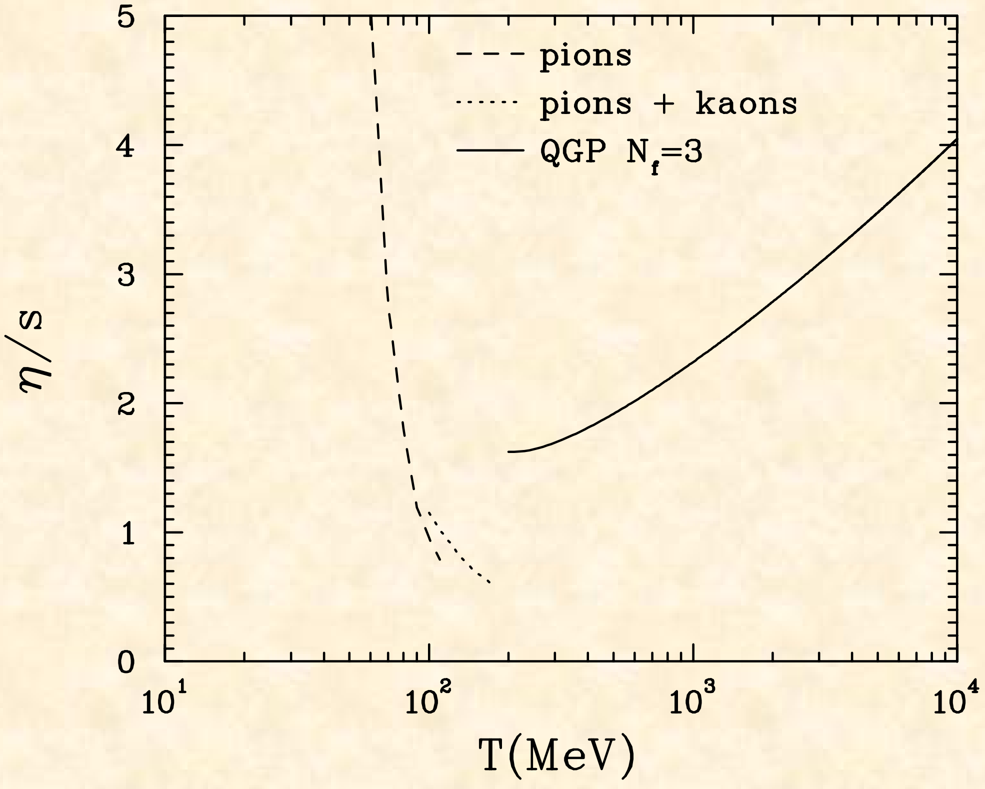
R.Myers,M.Paulos, A.Sinha: 0903.2834 [hep-th] (and ref. therein – many other papers)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 8c_1 + \ldots\right)$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right)$$ for superconformal Sp(N) gauge theory in d=4

Also: The species problem: T.Cohen, hep-th/0702136; A. Dolbado, F.Llanes-Estrada: hep-th/0703132

Shear viscosity - (volume) entropy density ratio in QCD

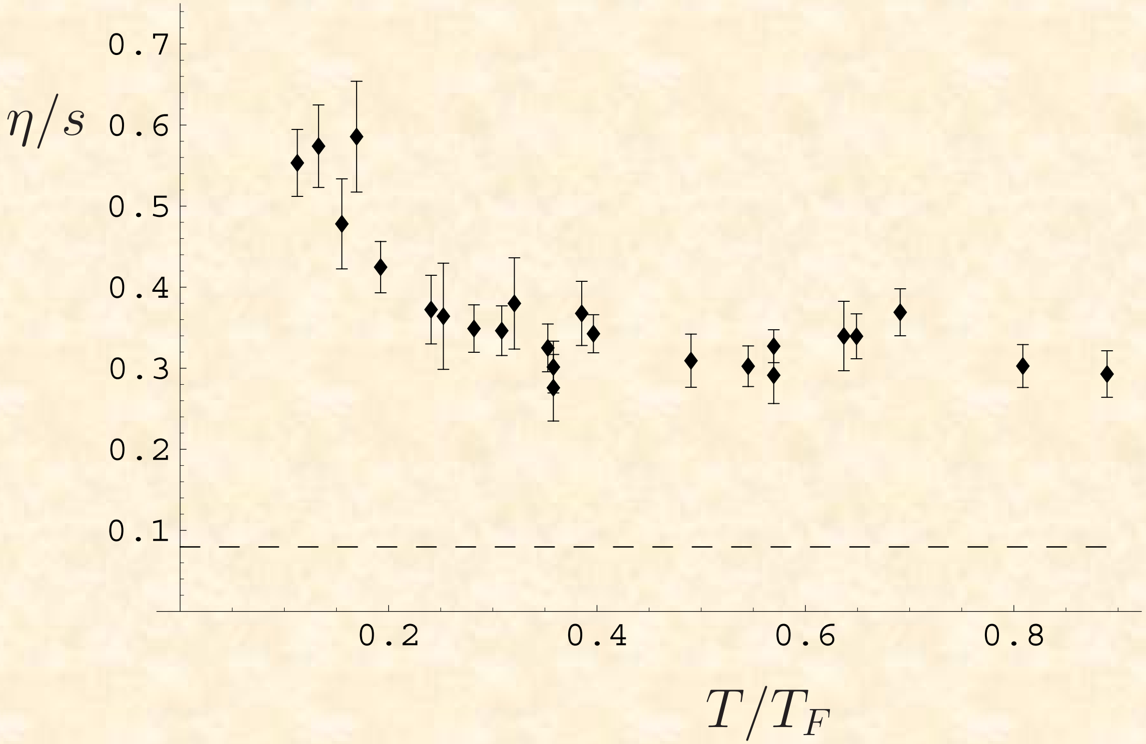


$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} F\left(\frac{\Lambda_{QCD}}{T}, N_c\right)$$

$$\frac{\eta}{s} \sim \frac{1}{\alpha_s^2 \log \alpha_s^{-1}}$$

The value of this ratio strongly affects the elliptic flow in hydro models of QGP

Viscosity-entropy ratio of a trapped Fermi gas



$\eta/s \sim 4.2$ in units of $\frac{\hbar}{4\pi k_B}$

T.Schafer, cond-mat/0701251

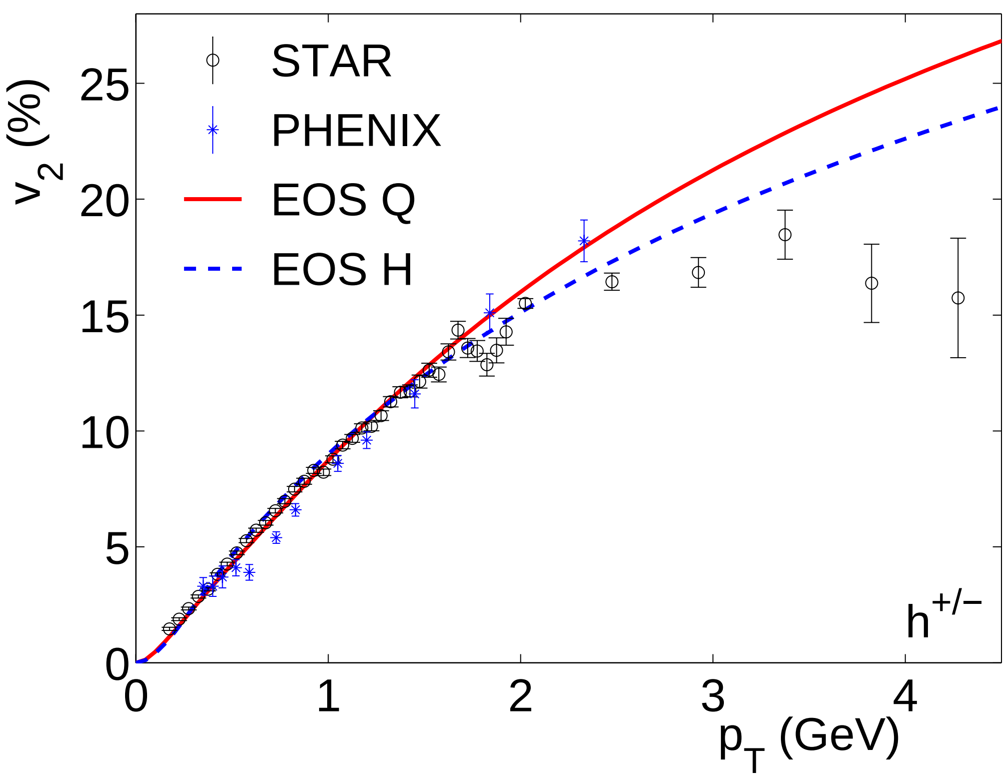
(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$0 < \eta/s \leq 0.5$

e.g. Baier, Romatschke, nucl-th/0610108

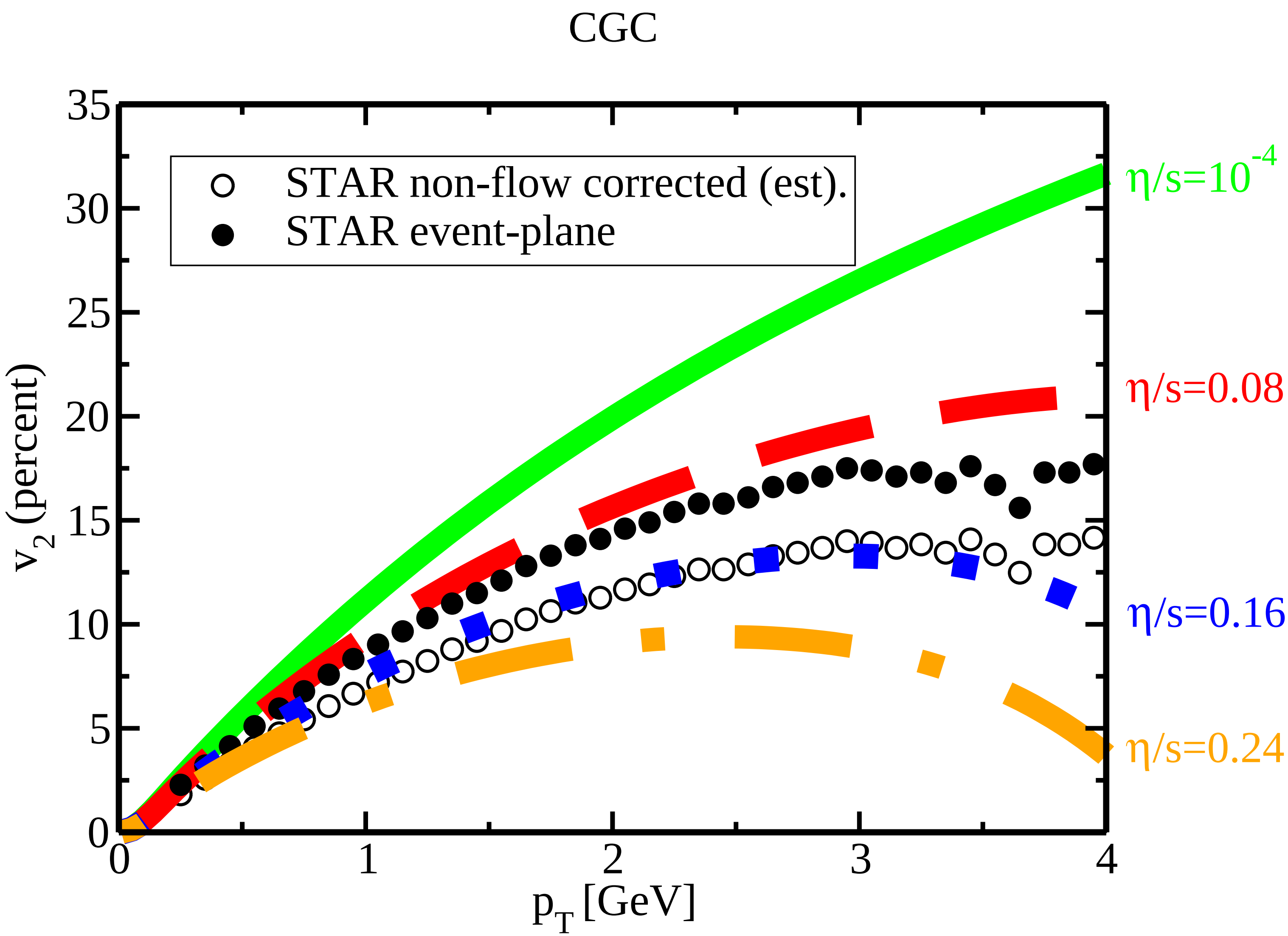
Perturbative QCD:

$\eta/s\,(T_{RHIC}) \approx 1.6 \sim 1.8$

Chernai, Kapusta, McLerran, nucl-th/0604032

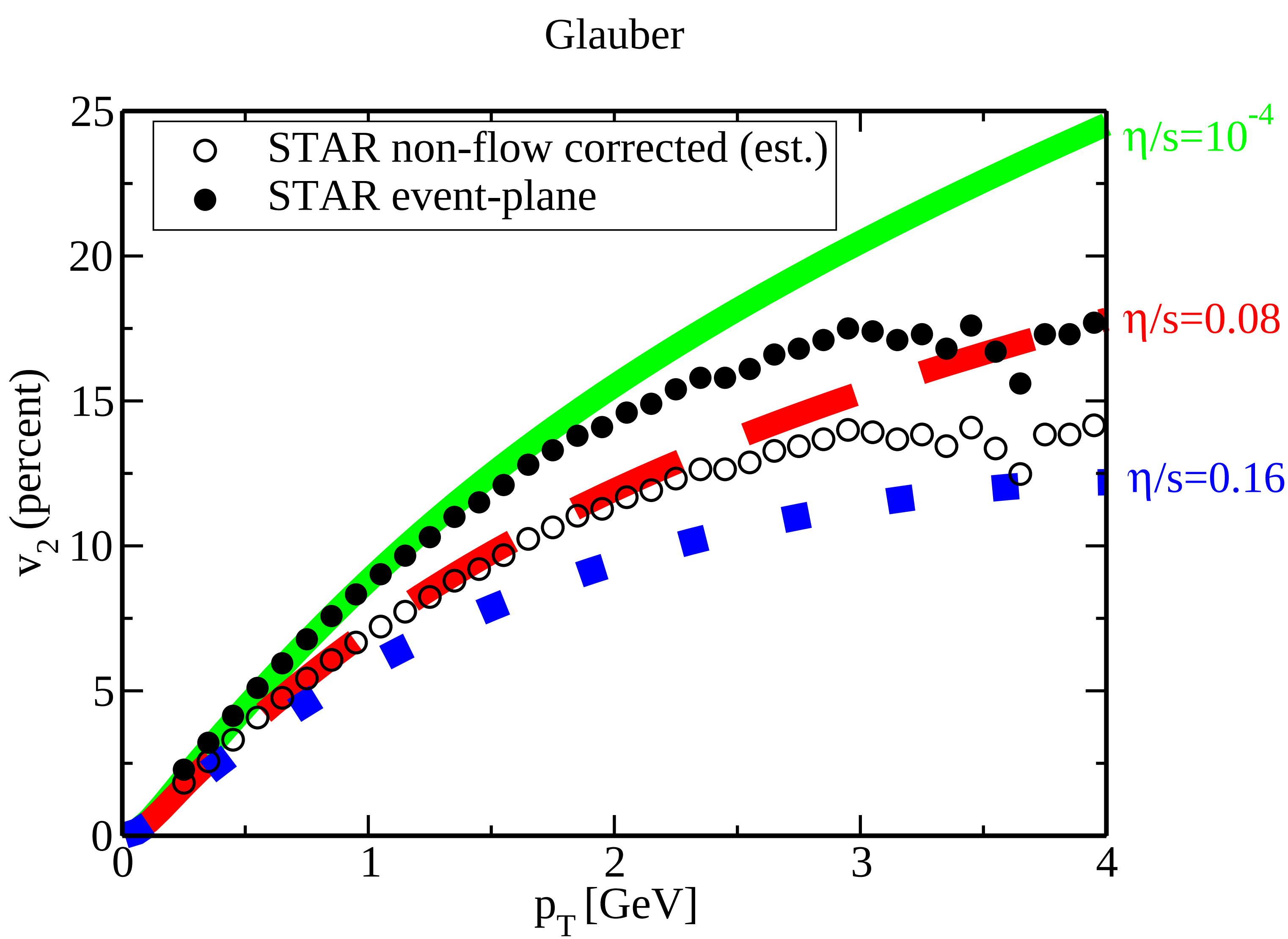
SYM: $\eta/s \approx 0.09 \sim 0.28$

Elliptic flow with color glass condensate initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Elliptic flow with Glauber initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Viscosity/entropy ratio in QCD: current status

Theories with gravity duals in the regime where the dual gravity description is valid

[Kovtun, Son & A.S] [Buchel] [Buchel & Liu, A.S]

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

(universal limit)

QCD: RHIC elliptic flow analysis suggests

$$0 < \frac{\eta}{s} < 0.5$$

QCD: (Indirect) LQCD simulations

H.Meyer, 0805.4567 [hep-th]

$$0.08 < \frac{\eta}{s} < 0.16$$

$$1.2 T_c < T < 1.7 T_c$$

Trapped strongly correlated cold alkali atoms

T.Schafer, 0808.0734 [nucl-th]

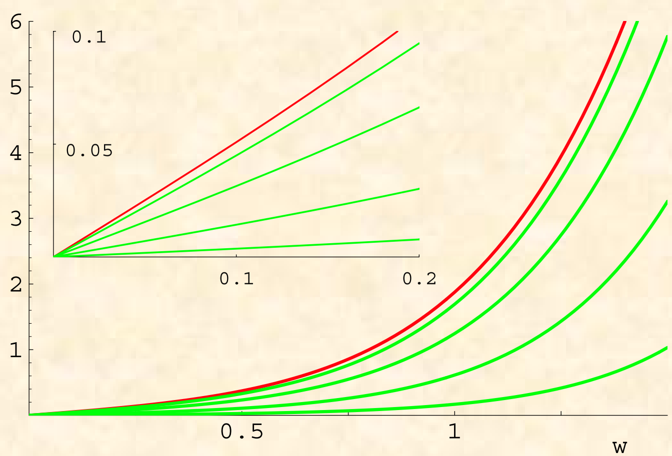
$$\left(\frac{\eta}{s}\right)_{\text{min}} \approx 0.5$$

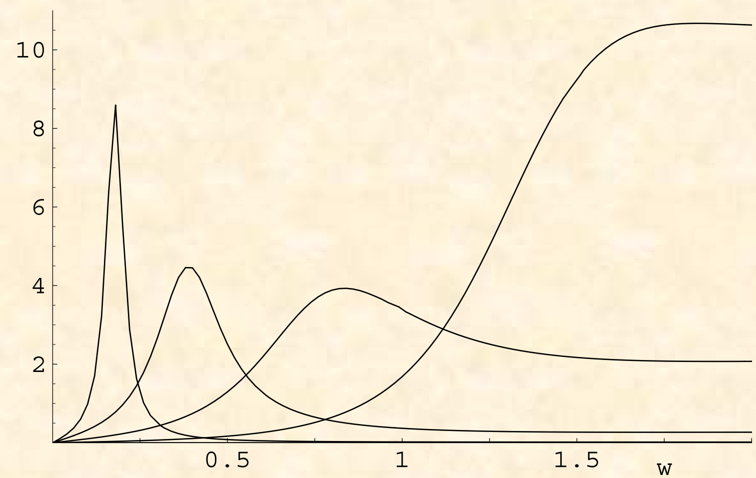
Liquid Helium-3

$$\left(\frac{\eta}{s}\right)_{\text{min}} \approx 0.7$$

Spectral sum rules for the QGP

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle [T_{\mu\nu}(x) T_{\alpha\beta}(0)] \rangle = -2\,\mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$



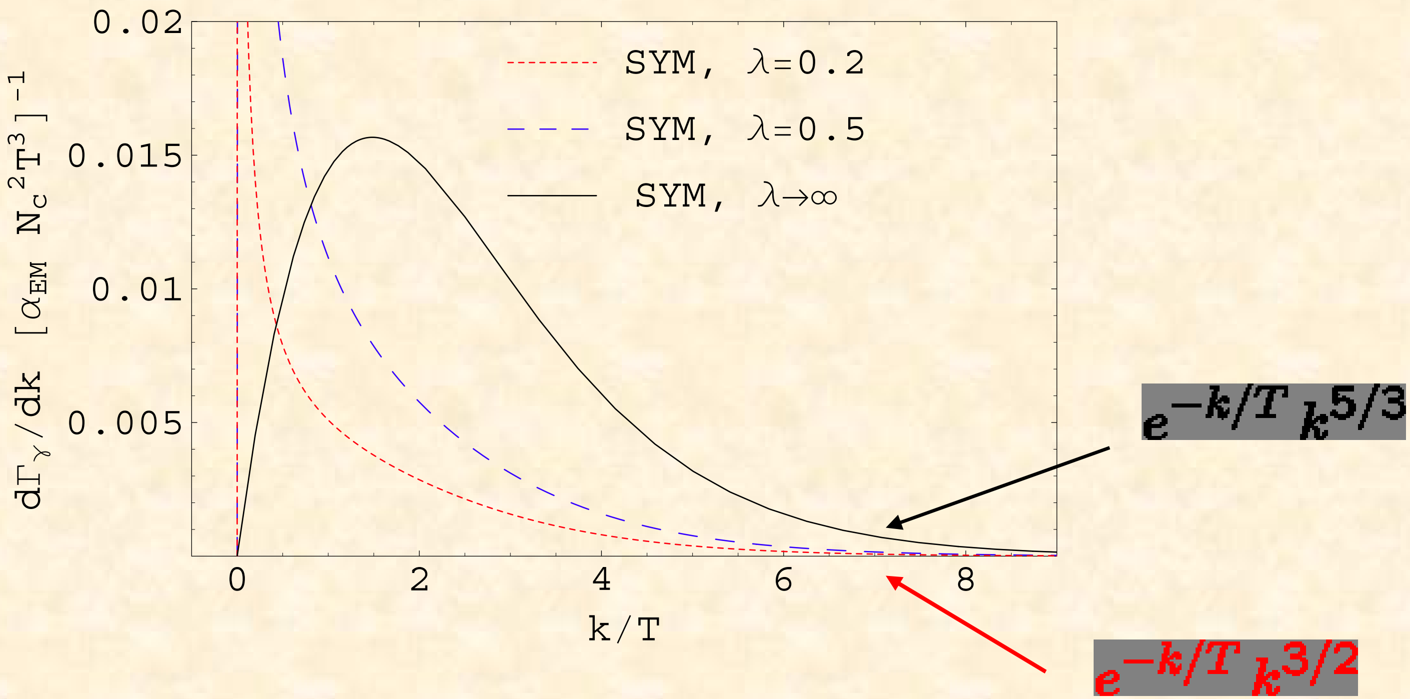


$$\frac{2}{5}\epsilon = \frac{1}{\pi}\int \frac{d\omega}{\omega}\left[\chi_{xy,xy}(\omega) - \chi^{T=0}_{xy,xy}(\omega)\right]$$

In N=4 SYM at ANY coupling

P.Romatschke, D.Son, 0903.3946 [hep-ph]

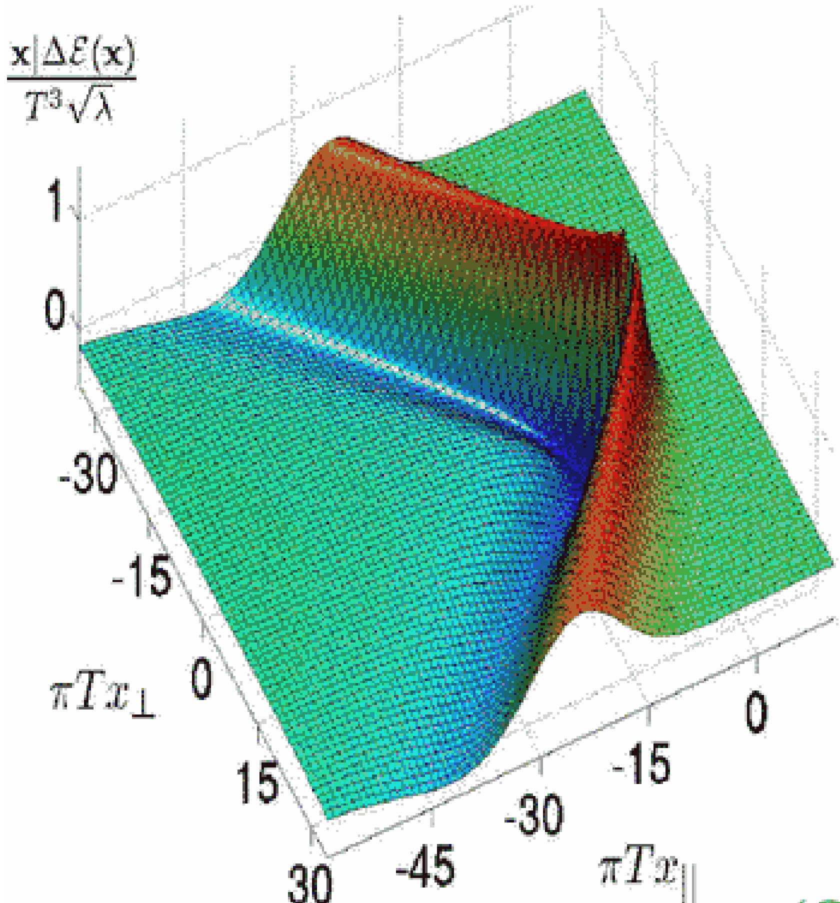
Photoproduction rate in SYM

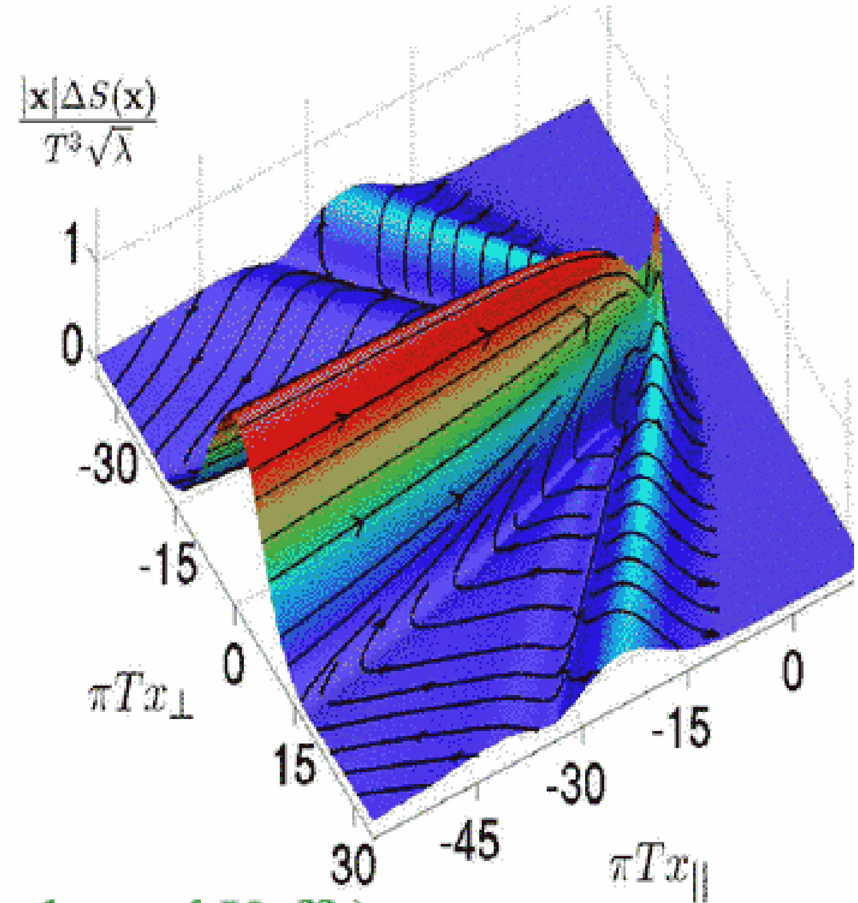


(Normalized) photon production rate in SYM for various values of 't Hooft coupling

$$\frac{d\Gamma_\gamma}{dk\,\alpha_{em}N_c^2T^3} = n_B(k)\left(\frac{k}{4\pi T}\right)^2 \left|{}_2F_1\left(1-\frac{(1+i)k}{4\pi T}, 1+\frac{(1-i)k}{4\pi T}; 1-\frac{ik}{2\pi T}; -1\right)\right|^{-2}$$

Energy and Momentum Density





(Chesler and Yaffe)

Other avenues of (related) research

Bulk viscosity for non-conformal theories (Buchel, Benincasa, Gubser, Moore…)

Non-relativistic gravity duals (Son, McGreevy,…)

Gravity duals of theories with SSB, AdS/CMT (Kovtun, Herzog, Hartnoll, Horowitz…)

Bulk from the boundary, time evolution of QGP (Janik,…)

Navier-Stokes equations and their generalization from gravity (Minwalla,…)

Quarks moving through plasma (Chesler, Yaffe, Gubser,…)

New directions

S. Hartnoll
"Lectures on holographic methods for condensed matter physics",
0903.3246 [hep-th]

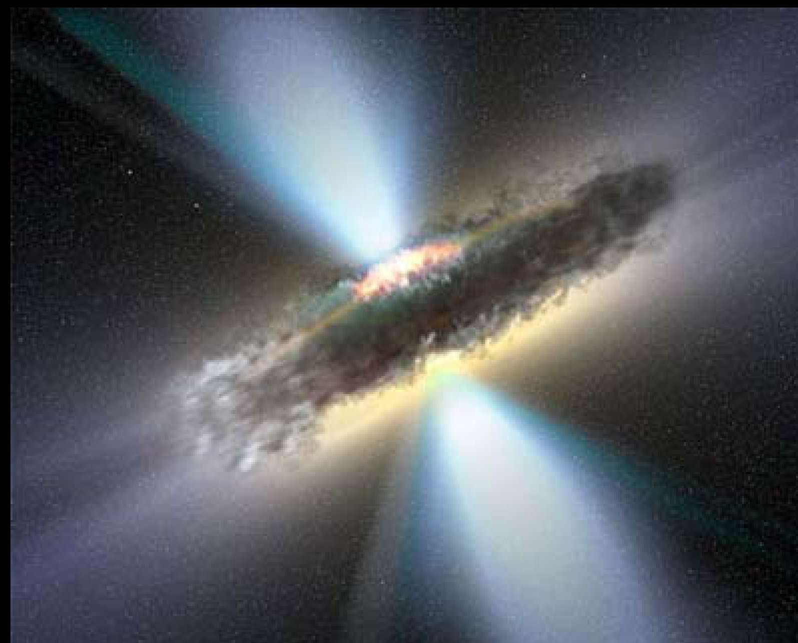
C. Herzog
"Lectures on holographic superfluidity and superconductivity",
0904.1975 [hep-th]

M. Rangamani
"Gravity and hydrodynamics: Lectures on the fluid-gravity correspondence",
0905.4352 [hep-th]

THANK YOU

Hydrodynamic properties of strongly interacting hot plasmas in 4 dimensions

can be related (for certain models!)



to fluctuations and dynamics of 5-dimensional black holes

AdS/CFT correspondence: the role of J

$$Z_{\rm SYM}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x} \rangle_{\rm SYM} \simeq e^{-S_{\rm grav}[J]}$$

For a given operator $\mathcal{O}$, identify the source field J, e.g. $T^{\mu\nu} \Longleftrightarrow h_{\mu\nu}$

$$e^{-S_{{\rm grav},\mathcal{M}}[\phi_{\rm BG}+\delta\phi]} = Z[J = \delta\phi|_{\partial\mathcal{M}}]$$

$\delta\phi$ satisfies linearized supergravity e.o.m. with b.c. $\delta\phi \to \delta\phi_0 \equiv J$

The recipe:

To compute correlators of $\mathcal{O}$, one needs to solve the bulk supergravity e.o.m. for $\delta\phi$ and compute the on-shell action as a functional of the b.c. $\delta\phi \equiv J$

Warning: e.o.m. for different bulk fields may be coupled: need self-consistent solution

Then, taking functional derivatives of $e^{-S_{\rm grav}[J]}$ gives $\langle \mathcal{O}\,\mathcal{O} \rangle$

Holography at finite temperature and density

$$\langle \mathcal{O} \rangle = \frac{\mathrm{tr}\rho\mathcal{O}}{\mathrm{tr}\rho}$$

$$\rho = e^{-\beta H + \mu Q}$$

$$H \rightarrow T^{00} \rightarrow T^{\mu\nu} \rightarrow h_{\mu\nu}$$

$$Q \rightarrow J^{0} \rightarrow J^{\mu} \rightarrow A_{\mu}$$

Nonzero expectation values of energy and charge density translate into nontrivial background values of the metric (above extremality)=horizon and electric potential = CHARGED BLACK HOLE (with flat horizon)

$$ds^2 = -F(u)\,dt^2 + G(u)\left(dx^2 + dy^2 + dz^2\right) + H(u)\,du^2$$

$T = T_H$ temperature of the dual gauge theory

$$A_0 = P(u)$$

$\mu = P(\text{boundary}) - P(\text{horizon})$ chemical potential of the dual theory

Gauge-string duality and QCD

Approach I: use the gauge-string (gauge-gravity) duality to study N=4 SYM and similar theories, get qualitative insights into relevant aspects of QCD, look for universal quantities

(exact solutions but limited set of theories)

Approach II: bottom-up (a.k.a. AdS/QCD) – start with QCD, build gravity dual approximation

(unlimited set of theories, approximate solutions, systematic procedure unclear)

(will not consider here but see e.g. Gürsoy, Kiritsis, Mazzanti, Nitti, 0903.2859 [hep-th])

Approach III: solve QCD

Approach IIIa: pQCD (weak coupling; problems with convergence for thermal quantities)

Approach IIIb: LQCD (usual lattice problems + problems with kinetics)

Over the last several years, holographic (gauge/gravity duality) methods were used to study strongly coupled gauge theories at finite temperature and density

These studies were motivated by the heavy-ion collision programs at RHIC and LHC (ALICE, ATLAS) and the necessity to understand hot and dense nuclear matter in the regime of intermediate coupling $\alpha_s(T_{\text{RHIC}}) \sim O(1)$

As a result, we now have a better understanding of thermodynamics and especially kinetics (transport) of strongly coupled gauge theories

Of course, these calculations are done for theoretical models such as N=4 SYM and its cousins (including non-conformal theories etc).

We don't know quantities such as $\frac{\eta}{s}\left(\frac{\Lambda_{\text{QCD}}}{T}\right)$ for QCD

New transport coefficients in $\mathcal{N} = 4$ SYM

Sound dispersion:

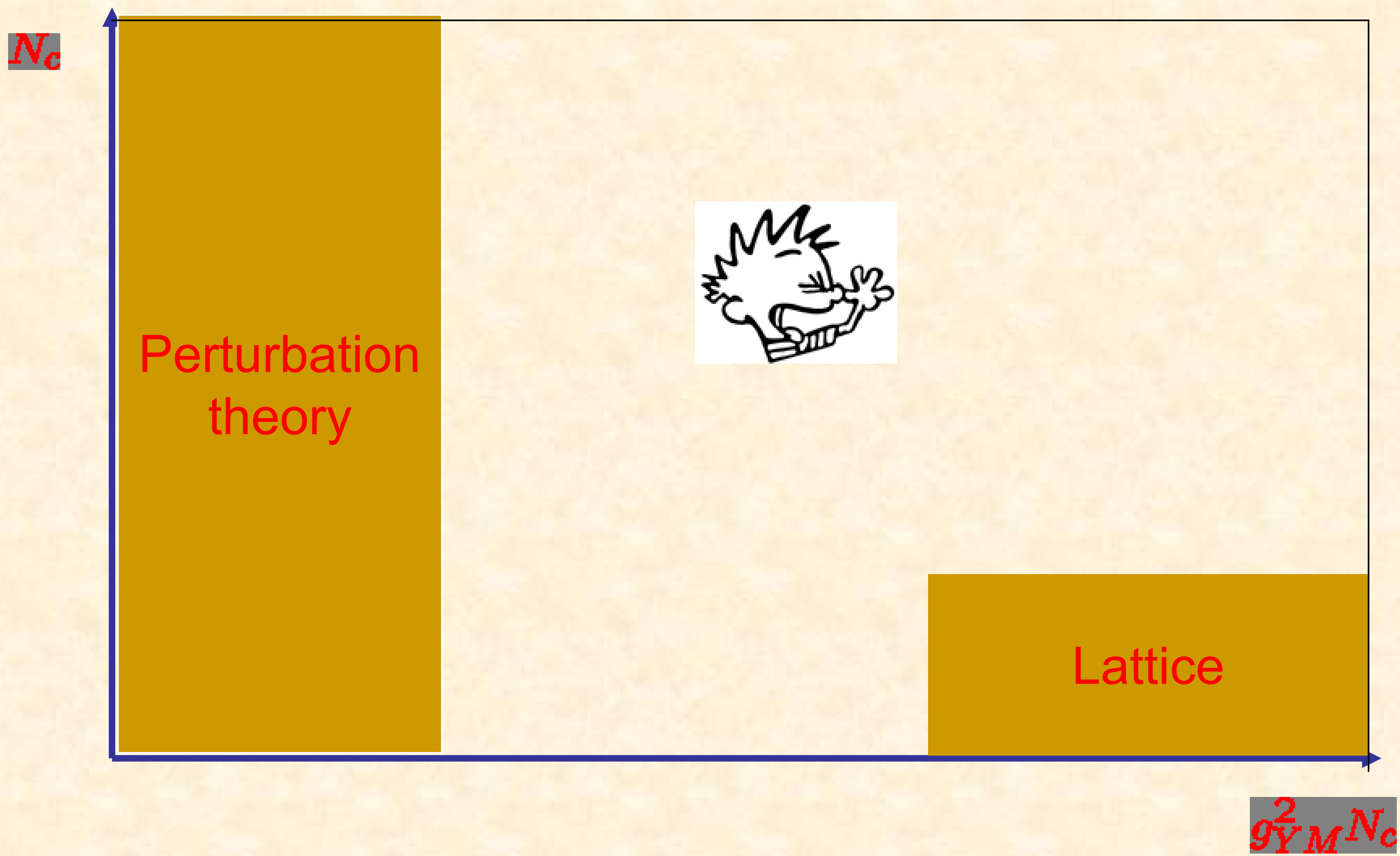
$$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 + \frac{3-2\ln 2}{24\pi^2\sqrt{3}T^2}q^3 + \cdots$$

Kubo:

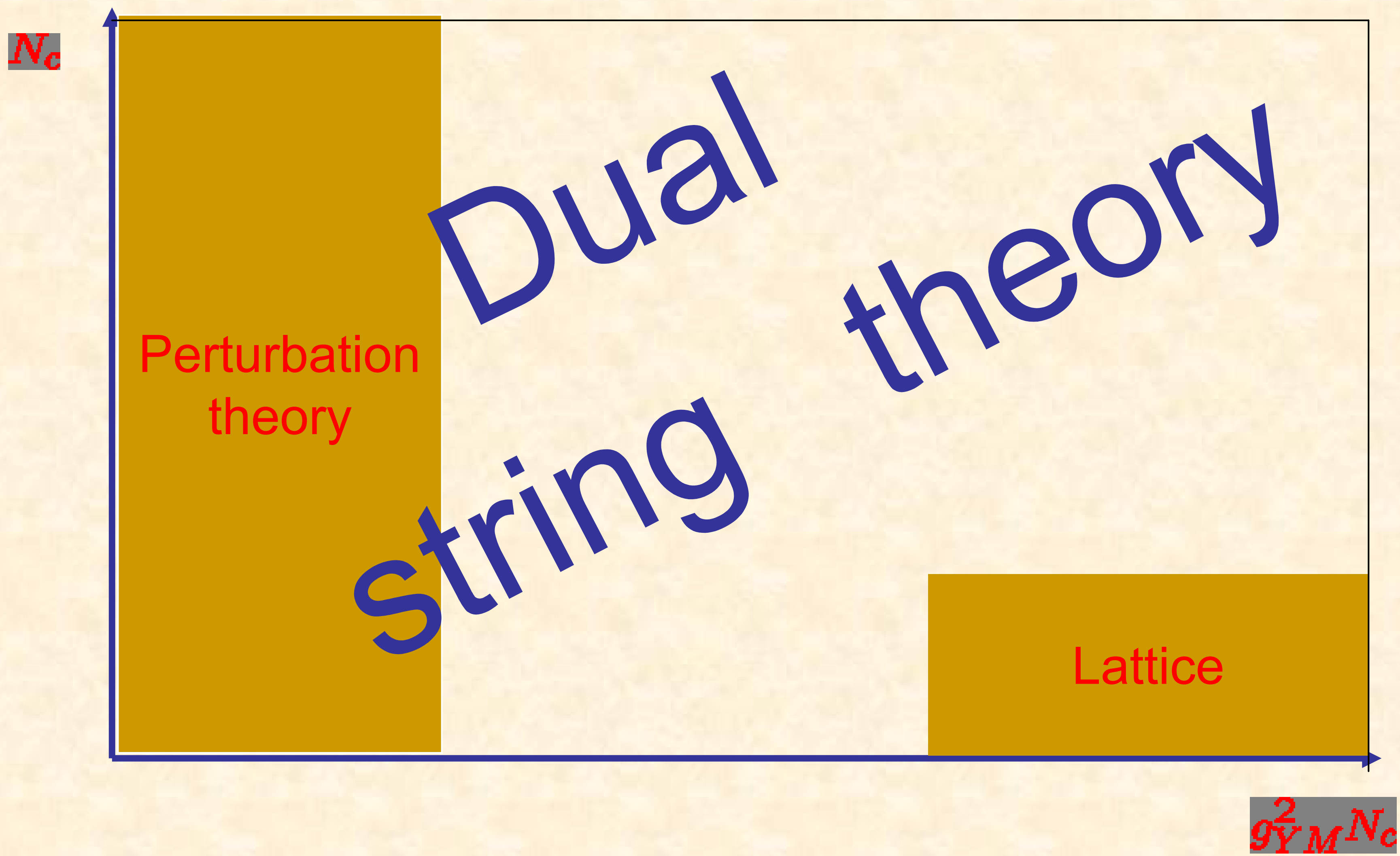
$$G_R^{xy,xy}(\omega,q) = -\frac{\pi^2 N_c^2 T^4}{4}\left[i\mathfrak{w} - \mathfrak{w}^2 + \mathfrak{k}^2 + \mathfrak{w}^2\ln 2 - \frac{1}{2}\right] + O(\mathfrak{w}^3, \mathfrak{w}\mathfrak{k}^2)$$

$$\mathfrak{w} = \omega/2\pi T\,, \quad \mathfrak{k} = q/2\pi T$$

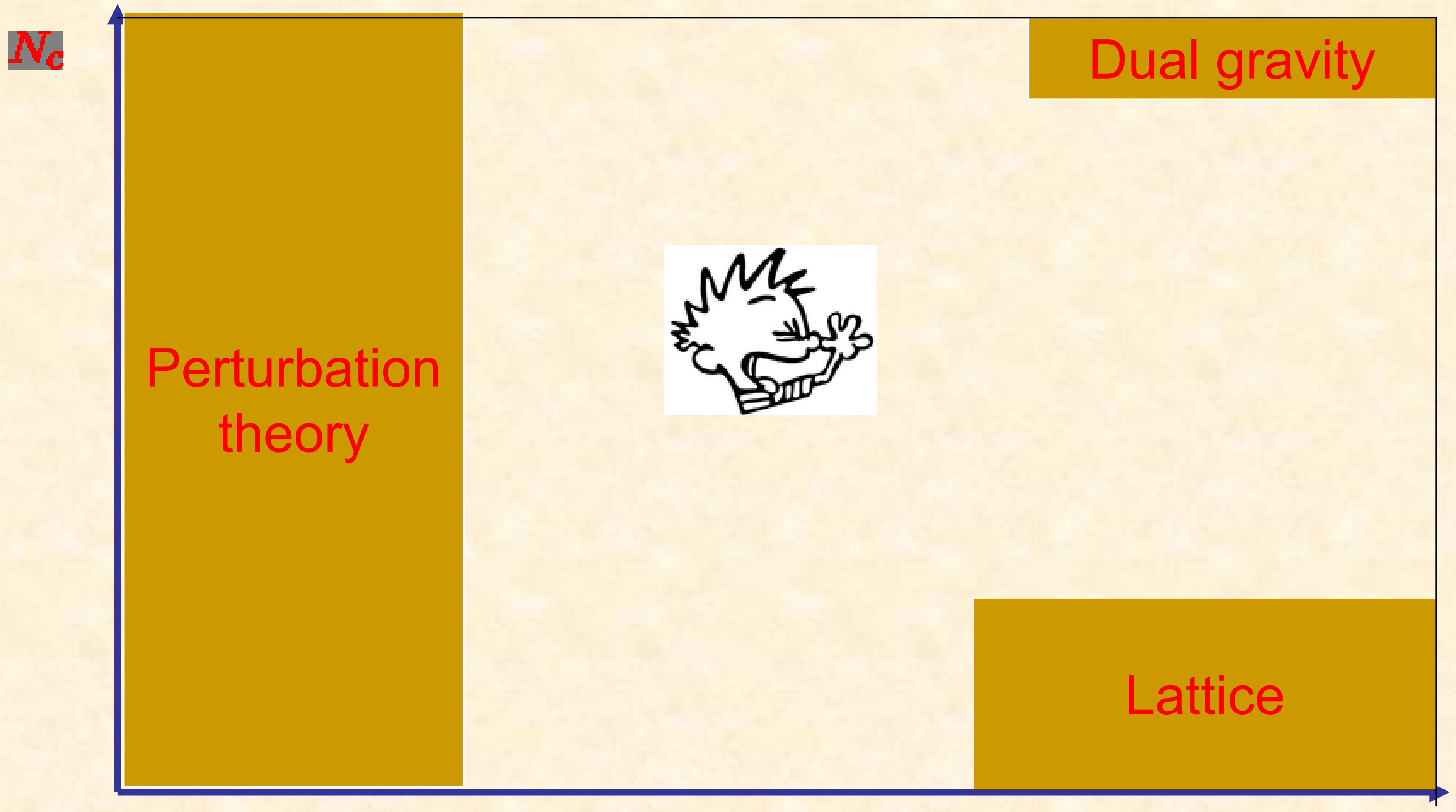
Our understanding of gauge theories is limited…



Conjecture:
specific gauge theory in 4 dim =
specific string theory in 10 dim



In practice: gravity (low energy limit of string theory) in 10 dim = 4-dim gauge theory in a region of a parameter space



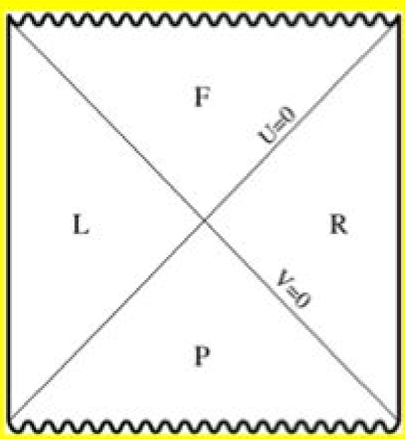
Can add fundamental fermions with $N_f \ll N_c$

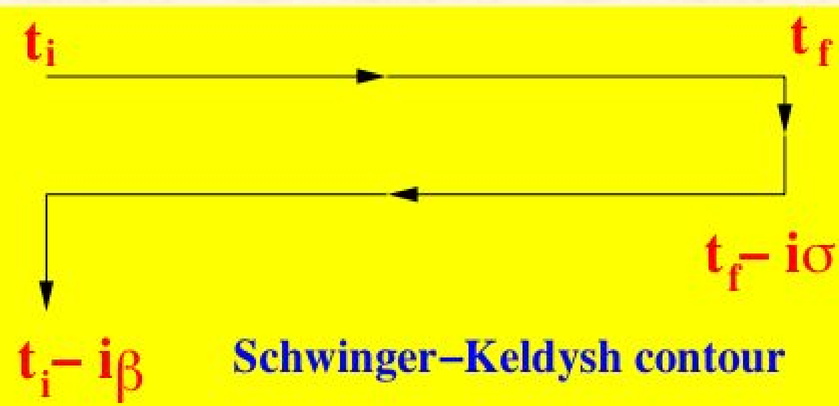
Computing real-time correlation functions from gravity

To extract transport coefficients and spectral functions from dual gravity, we need a recipe for computing Minkowski space correlators in AdS/CFT

The recipe of [D.T.Son & A.S., 2001] and [C.Herzog & D.T.Son, 2002] relates real-time correlators in field theory to Penrose diagram of black hole in dual gravity

Quasinormal spectrum of dual gravity = poles of the retarded correlators in 4d theory [D.T.Son & A.S., 2001]





Schwinger–Keldysh contour

Example: R-current correlator in $4d\ \mathcal{N}=4$ SYM

in the limit $N_c \to \infty\,, \quad g_{YM}^2 N_c \to \infty$

Zero temperature:

$$< J_i(x)\, J_i(y) > \sim \frac{N_c^2}{|x-y|^6}$$

$$G_E(k) = \frac{N_c^2 k_E^2}{32\pi^2} \ln k_E^2$$

$$G^{\text{ret}}(k) = \frac{N_c^2 k^2}{32\pi^2}\left(\ln|k^2| - i\,\pi\theta\,(-k^2)\,\text{sgn}\,\omega\right) \qquad k^2 = -\omega^2 + q^2$$

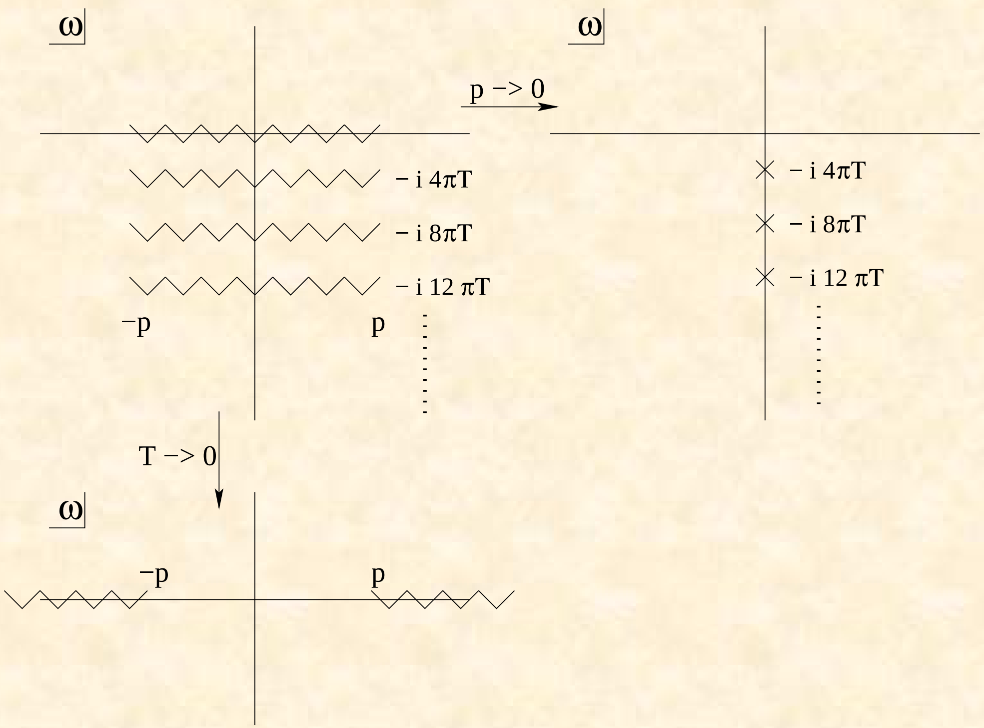
Finite temperature: $G^{\text{ret}}(\omega, q)$

Poles of G^{ret} = quasinormal spectrum of dual gravity background

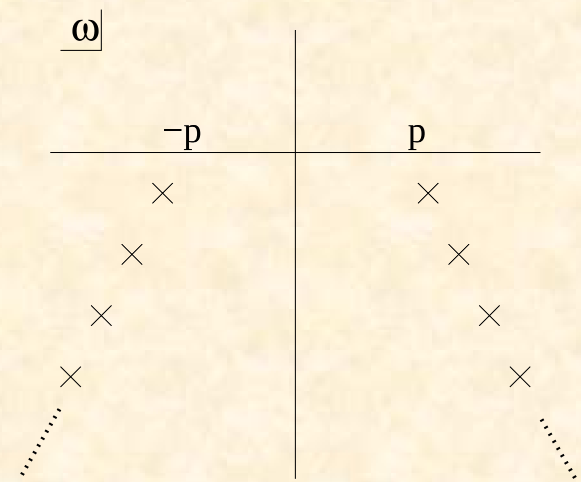
(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

Analytic structure of the correlators

$g^2N = 0$



$g^2N = \infty$



Strong coupling: A.S., hep-th/0207133

Weak coupling: S. Hartnoll and P. Kumar, hep-th/0508092

Computing transport coefficients from dual gravity

Assuming validity of the gauge/gravity duality,
all transport coefficients are completely determined
by the lowest frequencies
in quasinormal spectra of the dual gravitational background

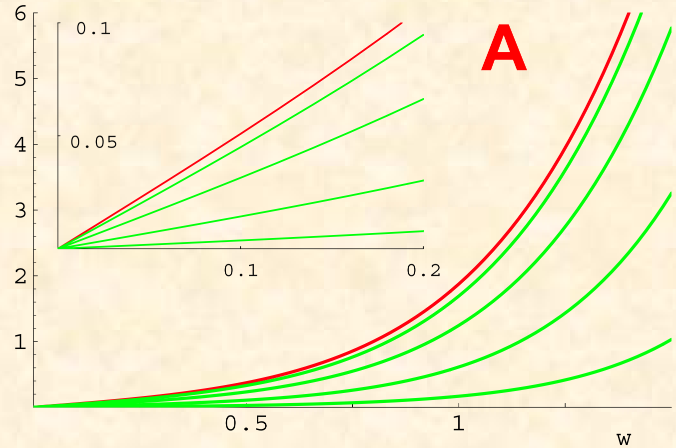
(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

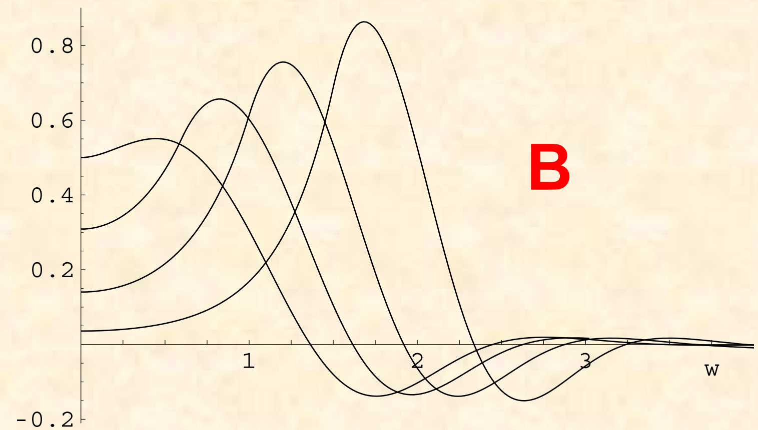
This determines kinetics in the regime of a thermal theory
where the dual gravity description is applicable

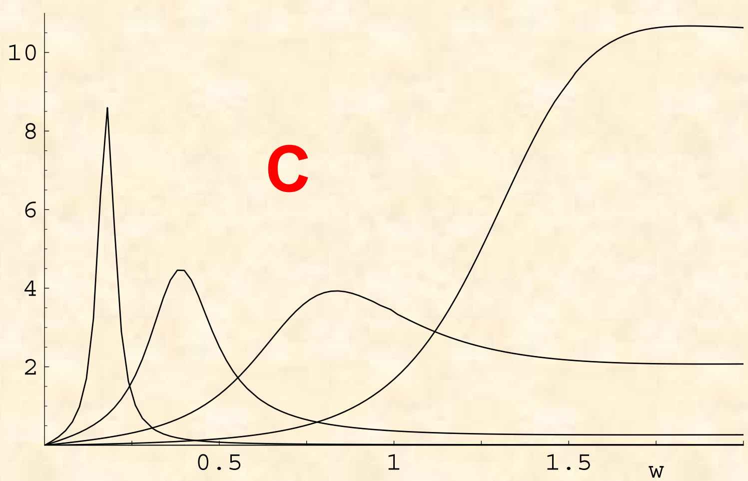
Transport coefficients and quasiparticle spectra can also be
obtained from thermal spectral functions $\chi = -2\,\mathrm{Im}\, G^R(\omega, q)$

Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle [T_{\mu\nu}(x) T_{\alpha\beta}(0)] \rangle = -2\, \mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$







A: scalar channel

B: scalar channel - thermal part

C: sound channel

Is the bound dead?

- Y.Kats and P.Petrov, 0712.0743 [hep-th]
  "Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory"

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right)$$

$\mathcal{N} = 2$ superconformal Sp(N) gauge theory in d=4

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \cdots\right)$$

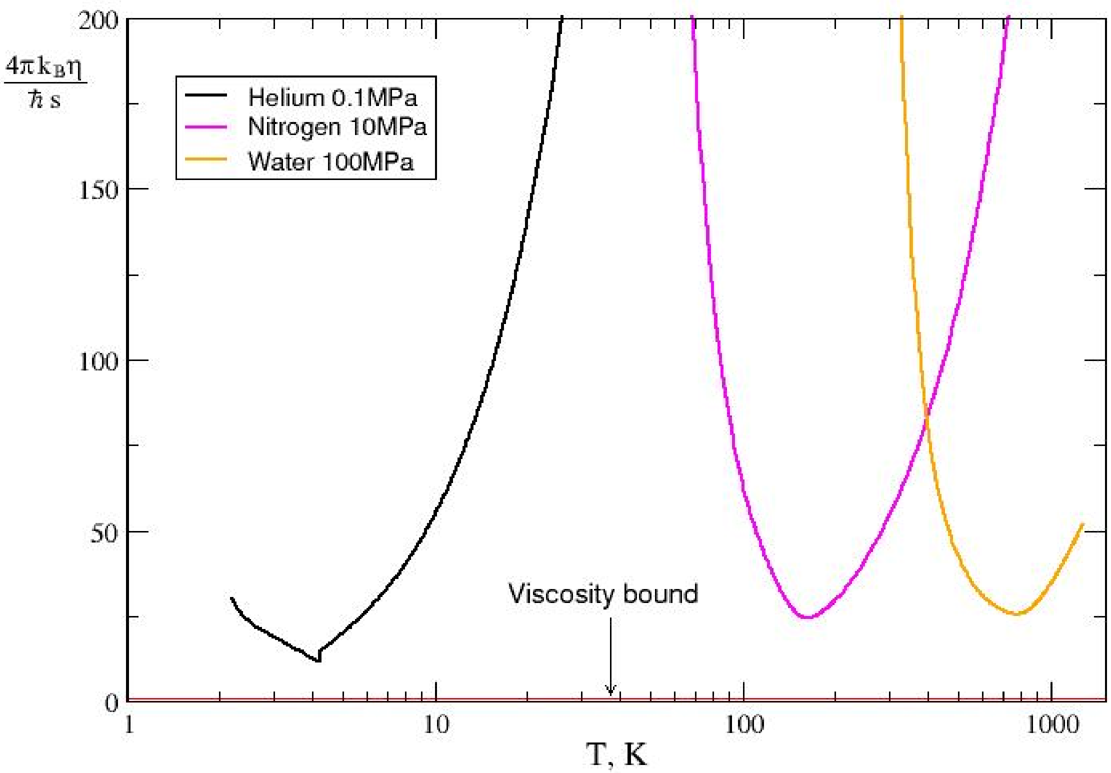
$$\frac{\eta}{s} = \frac{\hbar}{4\pi} F(a,c) \quad \text{for CFT ?}$$

- M.~Brigante, H.~Liu, R.~C.~Myers, S.~Shenker and S.~Yaida,
  ``The Viscosity Bound and Causality Violation,'' 0802.3318 [hep-th],
  ``Viscosity Bound Violation in Higher Derivative Gravity,'' 0712.0805 [hep-th].

- The "species problem"
  T.Cohen, hep-th/0702136, A.Dobado, F.Llanes-Estrada, hep-th/0703132

A viscosity bound conjecture

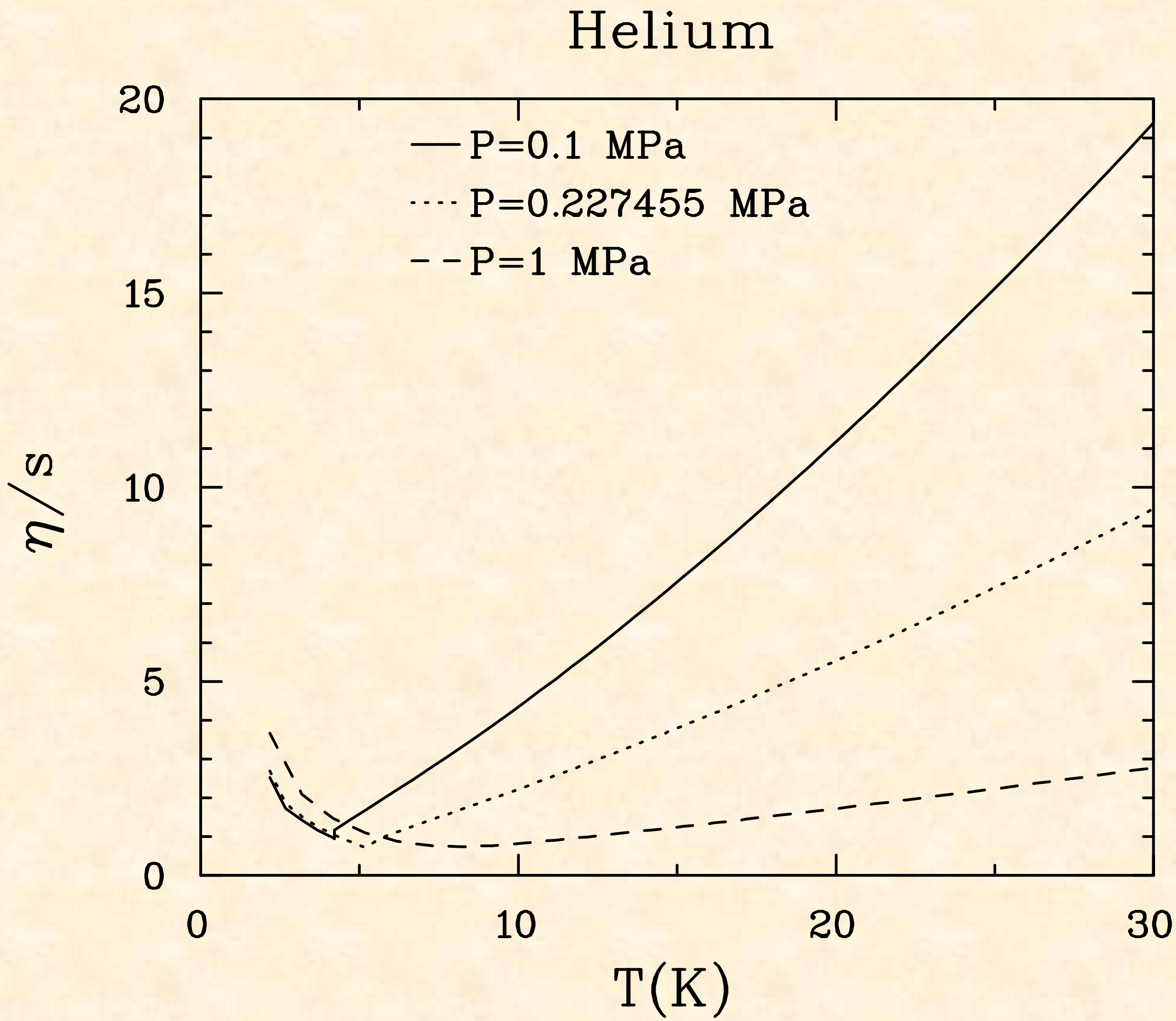
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

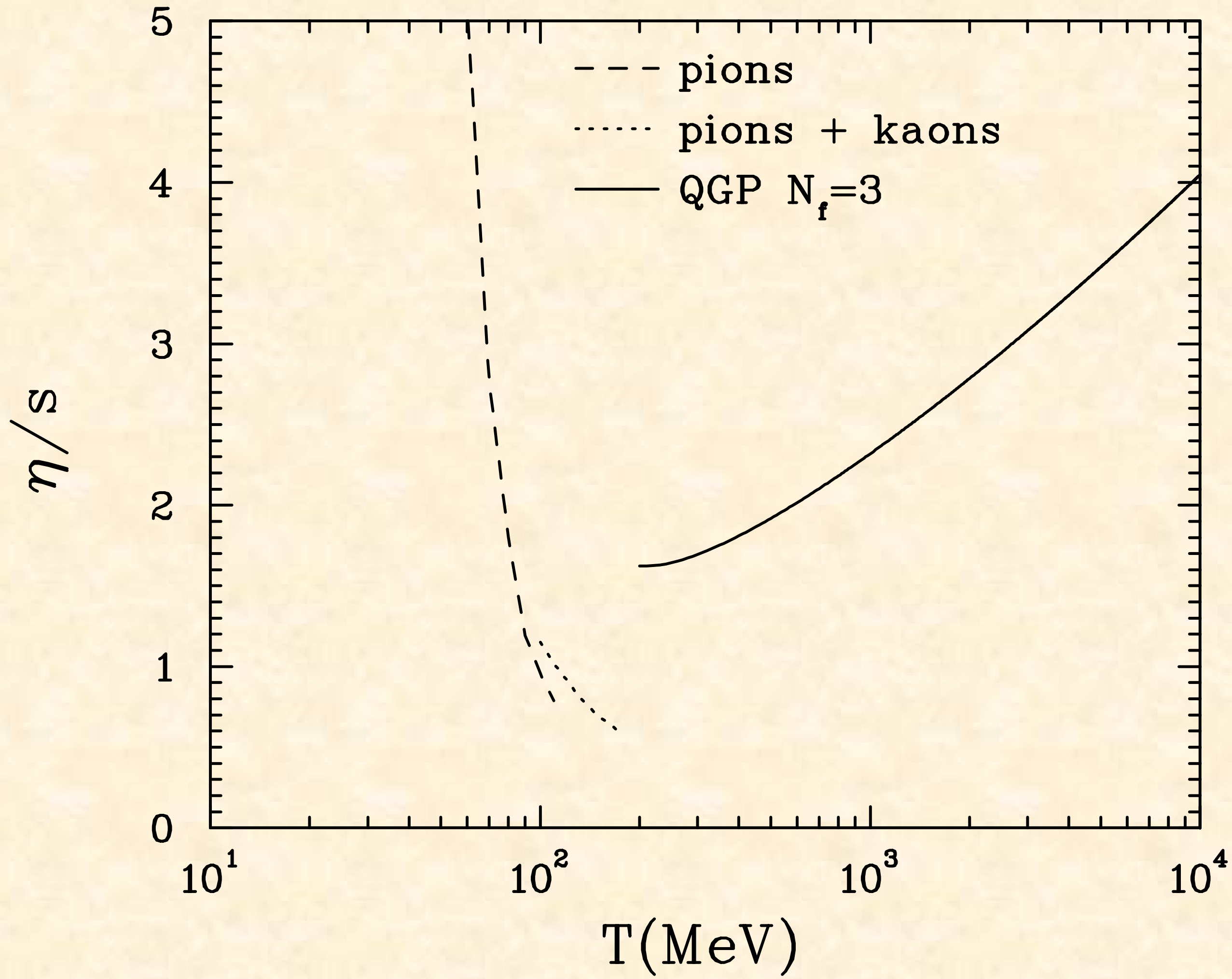
P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



$(\eta/s)_{min} \sim 8.8$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

QCD



Chernai, Kapusta, McLerran, nucl-th/0604032

Shear viscosity at non-zero chemical potential

$\mathcal{N} = 4$ SYM

$q_i \in U(1)^3 \subset SO(6)_R$ $\Longleftrightarrow$ Reissner-Nordstrom-AdS black hole

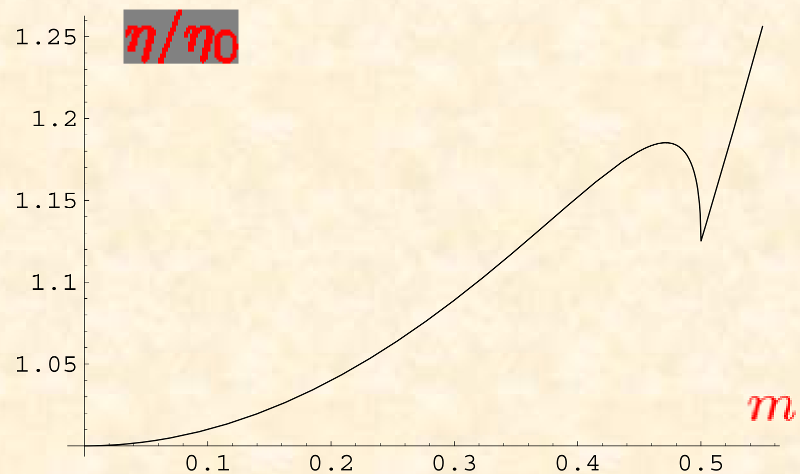
$Z = \mathrm{tr}\, e^{-\beta H + \mu_i q_i}$ with three R charges

(see e.g. Yaffe, Yamada, hep-th/0602074) (Behrnd, Cvetic, Sabra, 1998)

$$\eta = \pi N^2 T^3 \frac{m^2(1-\sqrt{1-4m^2}-m^2)^2}{(1-\sqrt{1-4m^2})^3}$$

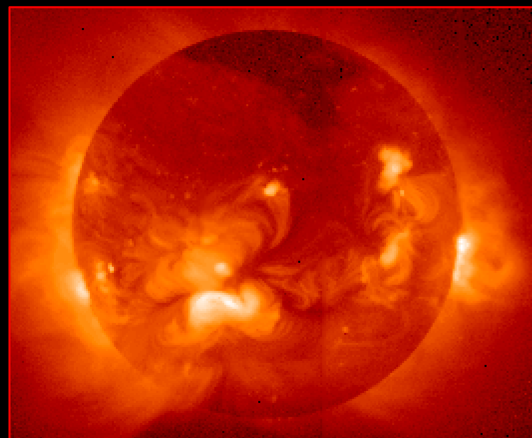
$m \equiv \mu/2\pi T$

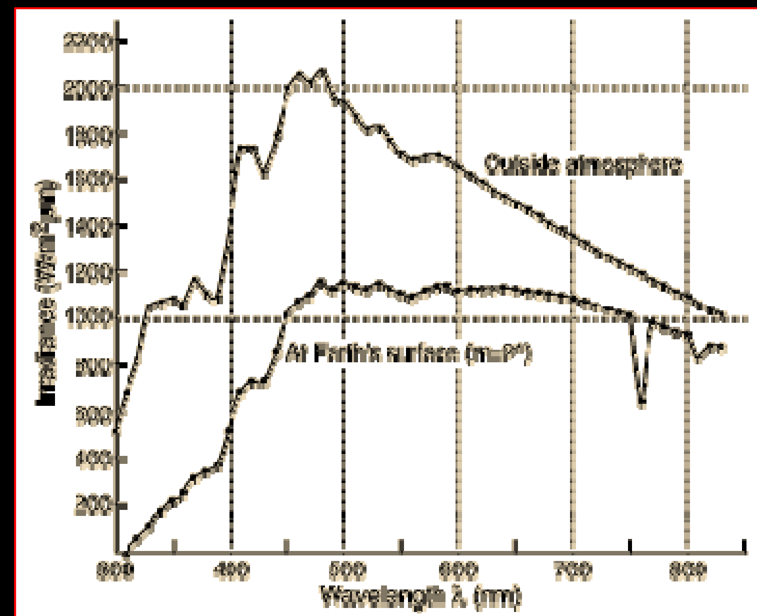
η/η_0

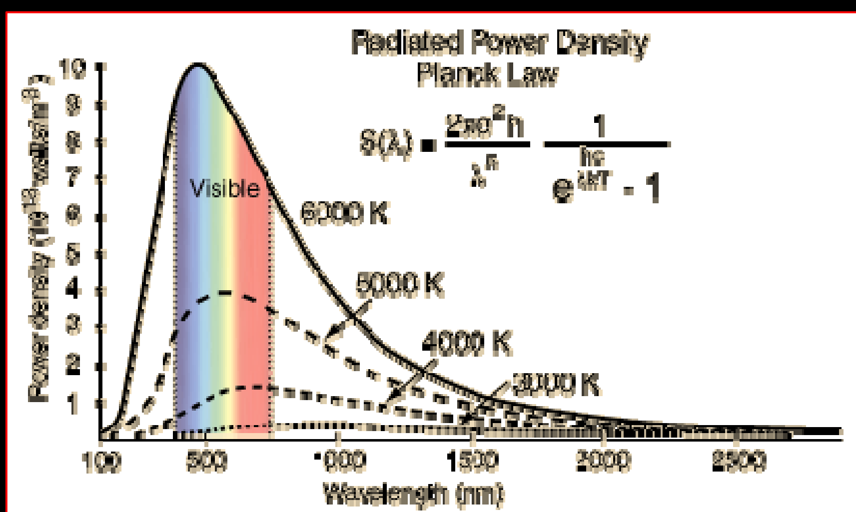


Photon and dilepton emission from supersymmetric Yang-Mills plasma

S. Caron-Huot, P. Kovtun, G. Moore, A.S., L.G. Yaffe, hep-th/0607237







Photon emission from SYM plasma

Photons interacting with matter: $e\, J_\mu^{\rm EM}\, A^\mu$

To leading order in e

$$d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3}\frac{e^2}{2|k|}\,\eta^{\mu\nu} C^<_{\mu\nu}(k^0 = |k|)$$

$$C^<_{\mu\nu} = \int d^4X e^{-iKX}\langle J_\mu^{\rm EM}(0) J_\nu^{\rm EM}(X)\rangle$$

Mimic $J_\mu^{\rm EM}$ by gauging global R-symmetry $U(1) \subset SU(4)$

$$\mathcal{L} = \mathcal{L}_{N=4\,\rm SYM} + e\, J_\mu^3\, A^\mu - \frac{1}{4} F_{\mu\nu}^2$$

Need only to compute correlators of the R-currents J_μ^3


Now consider strongly interacting systems at finite density
and LOW temperature

Probing quantum liquids with holography

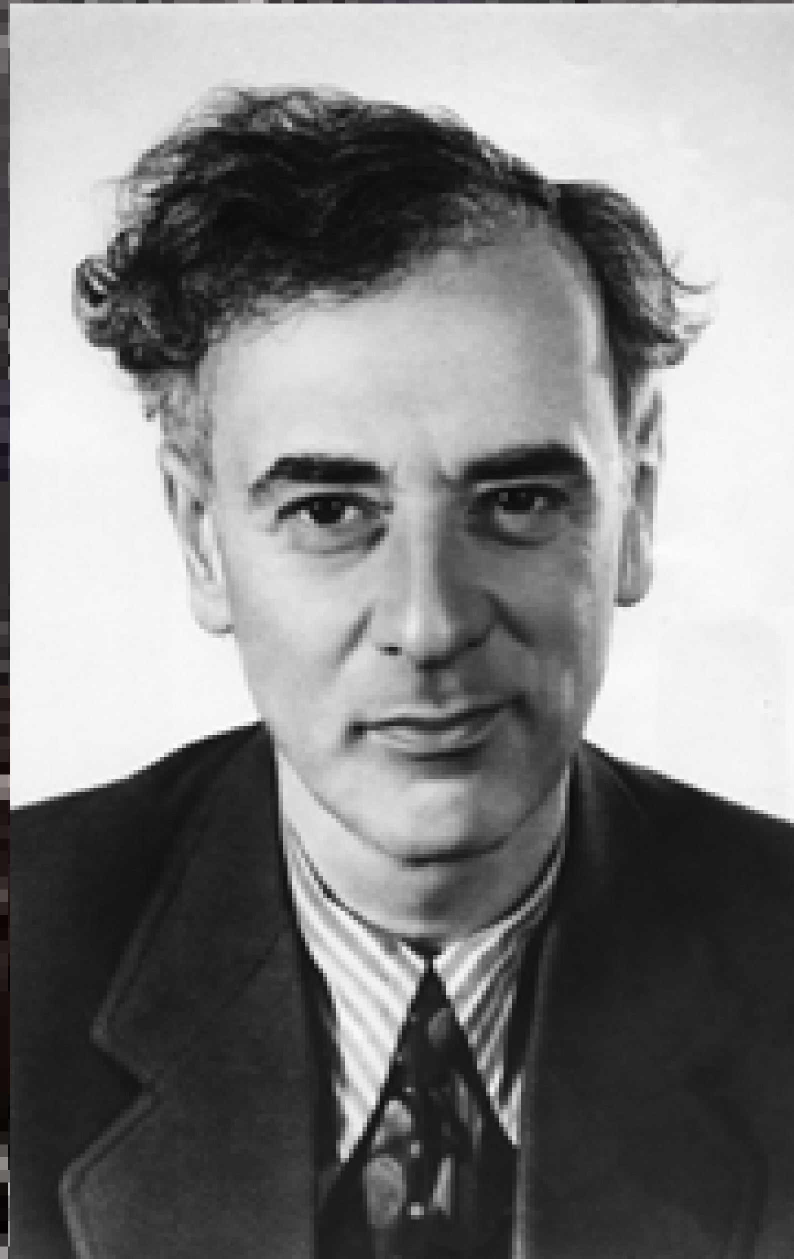
Quantum liquid in p+1 dim	Low-energy elementary excitations	Specific heat at low T
Quantum Bose liquid	phonons	$\sim T^p$
Quantum Fermi liquid	fermionic quasiparticles + bosonic branch	$\sim T$

Departures from normal Fermi liquid occur in

- 3+1 and 2+1 –dimensional systems with strongly correlated electrons

- In 1+1 –dimensional systems for any strength of interaction (Luttinger liquid)

One can apply holography to study strongly coupled Fermi systems at low T



L.D.Landau (1908-1968)

The simplest candidate with a known holographic description is

$SU(N_c)$ $\mathcal{N}=4$ SYM coupled to N_f $\mathcal{N}=2$ fundamental hypermultiplets

at finite temperature T and nonzero chemical potential associated with the "baryon number" density of the charge $U(1)_B \subset U(N_f)$

There are two dimensionless parameters: $\frac{n_q^{1/3}}{T}$ $\frac{M}{T}$

n_q is the baryon number density

M is the hypermultiplet mass

The holographic dual description in the limit $N_c \gg 1, \; g_{YM}^2 N_c \gg 1, \; N_f$ finite is given by the D3/D7 system, with D3 branes replaced by the AdS-Schwarzschild geometry and D7 branes embedded in it as probes.

Karch & Katz, hep-th/0205236

AdS-Schwarzschild black hole (brane) background

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \left(1-\frac{r_H^4}{r^4}\right)^{-1}\frac{R^2}{r^2}dr^2$$

D7 probe branes

$$S_{DBI} = -N_f T_{D7}\int d^8\xi\sqrt{-\det(g_{ab}+2\pi\alpha' F_{ab})}$$

The worldvolume U(1) field A_μ couples to the flavor current J^μ at the boundary

Nontrivial background value of A_0 corresponds to nontrivial expectation value of J^0

We would like to compute

- the specific heat at low $(Tn_q^{-1/3} \ll 1)$ temperature
- the charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$

★ The specific heat (in p+1 dimensions):

$$c_V = \mathcal{N}_q\, p \left(\frac{4\pi}{p+1}\right)^{2p+1} \frac{T^{2p}}{n_q}\left[1 + O(T n_q^{-\frac{1}{p}})\right]$$

(note the difference with Fermi $c_V \sim T$ and Bose $c_V \sim T^p$ systems)

★ The (retarded) charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$
has a pole corresponding to a propagating mode (zero sound)
- even at zero temperature

$$\omega = \pm\frac{q}{\sqrt{p}} - \frac{i\,\Gamma(\frac{1}{2})\,q^2}{n_q^{\frac{1}{p}}\Gamma(\frac{1}{2}-\frac{1}{2p})\Gamma(\frac{1}{2p})} + O(q^3)$$

(note that this is NOT a superfluid phonon whose attenuation scales as q^{p+1})

New type of quantum liquid?

Epilogue

- On the level of theoretical models, there exists a connection between near-equilibrium regime of certain strongly coupled thermal field theories and fluctuations of black holes
- This connection allows us to compute transport coefficients for these theories
- At the moment, this method is the only theoretical tool available to study the near-equilibrium regime of strongly coupled thermal field theories
- The result for the shear viscosity turns out to be universal for all such theories in the limit of infinitely strong coupling
- Influences other fields (heavy ion physics, condmat)

A hand-waving argument

$$\eta \sim \rho v l \sim \rho v^2 \tau \sim n m v^2 \tau \sim n \epsilon \tau$$

$$s \sim n$$

Thus $\dfrac{\eta}{s} \sim \epsilon \tau \geq \hbar$

Gravity duals fix the coefficient: $\dfrac{\eta}{s} \geq \hbar / 4\pi$

Outlook

- Gravity dual description of thermalization ?

- Gravity duals of theories with fundamental fermions:

  - phase transitions
  - heavy quark bound states in plasma
  - transport properties

- Finite ‘t Hooft coupling corrections to photon emission spectrum

- Understanding 1/N corrections

- Phonino

Equations such as $R_{\mu\nu} - 2\nabla_\mu \nabla_\nu \Phi + \frac{1}{4} H_\mu^{\lambda\rho} H_{\nu\lambda\rho} + O(\alpha') = 0$

describe the low energy $E \ll 1/l_s$ limit of string theory

As long as the dilaton is small, and thus the string interactions are suppressed, this limit corresponds to classical 10-dim Einstein gravity coupled to certain matter fields such as Maxwell field, p-forms, dilaton, fermions

Validity conditions for the classical (super)gravity approximation

- curvature invariants should be small: $\mathcal{R} \sim R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} \ll 1/l_s^4$

- quantum loop effects (string interactions = dilaton) should be small: $g_s \ll 1$

In AdS/CFT duality, these two conditions translate into

$$\lambda \equiv g_{YM}^2 N_c \gg 1$$

and

$$N_c \gg 1$$

for $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills theory in 4 dim

The challenge of RHIC (continued)

Rapid thermalization ??

Large elliptic flow ★

Jet quenching ✦

Photon/dilepton emission rates ★

The bulk and the boundary in AdS/CFT correspondence

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dz^2}{z^2}$$

UV/IR: the AdS metric is invariant under $z \to \Lambda z \quad x \to \Lambda x$

z plays a role of inverse energy scale in 4D theory