Chiral symmetry breaking and Chiral Magnetic Effect in lattice gluodynamics with background magnetic field

ITEP Lattice

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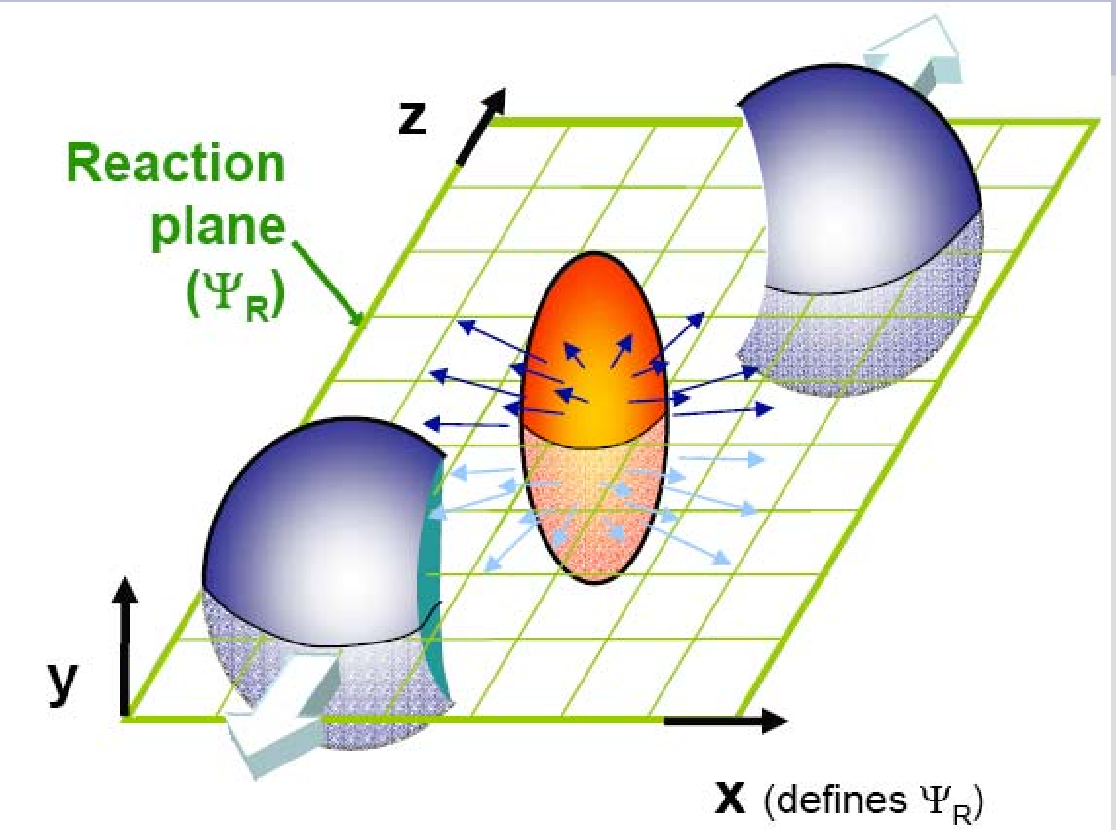
I use a lot of slides made by M.N. Chernodub and P.V. Buividovich and some made by D.E. Kharzeev

Plan

- Strong magnetic (not chromo-magnetic!) fields
- Lattice simulations with magnetic fields
- Chiral symmetry breaking: Dirac eigenmodes and their localization
- Magnetization of the vacuum: first lattice results
- Chiral Magnetic Effect? - a new effect at RHIC?
- Conclusions

Magnetic fields in non-central collisions

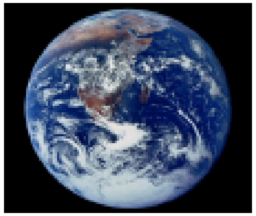


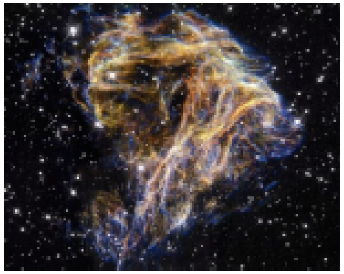
The medium is filled with electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

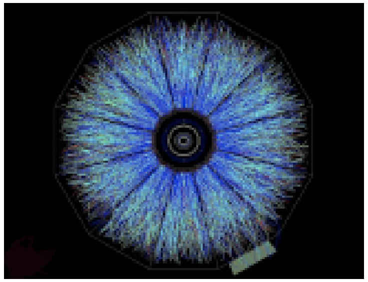
Comparison of magnetic fields





The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau = 0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

Another estimation of magnetic fields, now in «eV»

- Early Universe:
  $B \sim 10^{16}$ T or $\sqrt{eB} \sim 1$ GeV
- Compact dense stars, such as magnetars:
  $B \sim 10^{10}$ T or $\sqrt{eB} \sim 1$ MeV
- Heavy-ion collisions in very strong laser pulses (experiments as PHELIX at GSI and ELI:)
  $B \sim 10^{7}$ T or $\sqrt{eB} \sim 0.01$ MeV
- Noncentral heavy-ion collisions:
  $B \sim 10^{15}$ T or $\sqrt{eB} \sim 10$ MeV $\ldots 300$ MeV

Effects observed at very strong magnetic fields on the lattice

1. Enhancement of the chiral condensate (arXiv:0812.1740)

2. Magnetization of the QCD vacuum (arXiv:0906.0488)

3. Chiral Magnetic Effect (arXiv:0907.0494)

[signatures of “CP violation” at non-zero topological charge first discussed by Fukushima, Kharzeev, Warringa, McLerran ’07-’08 + data from RHIC: Conference “Quark Matter 2009, 30 March - 4 April” in Knoxville, USA]

Magnetic fields in our lattice simulations

- We perform simulations in quenched $SU(2)$ lattice gauge theory with overlap Dirac operator

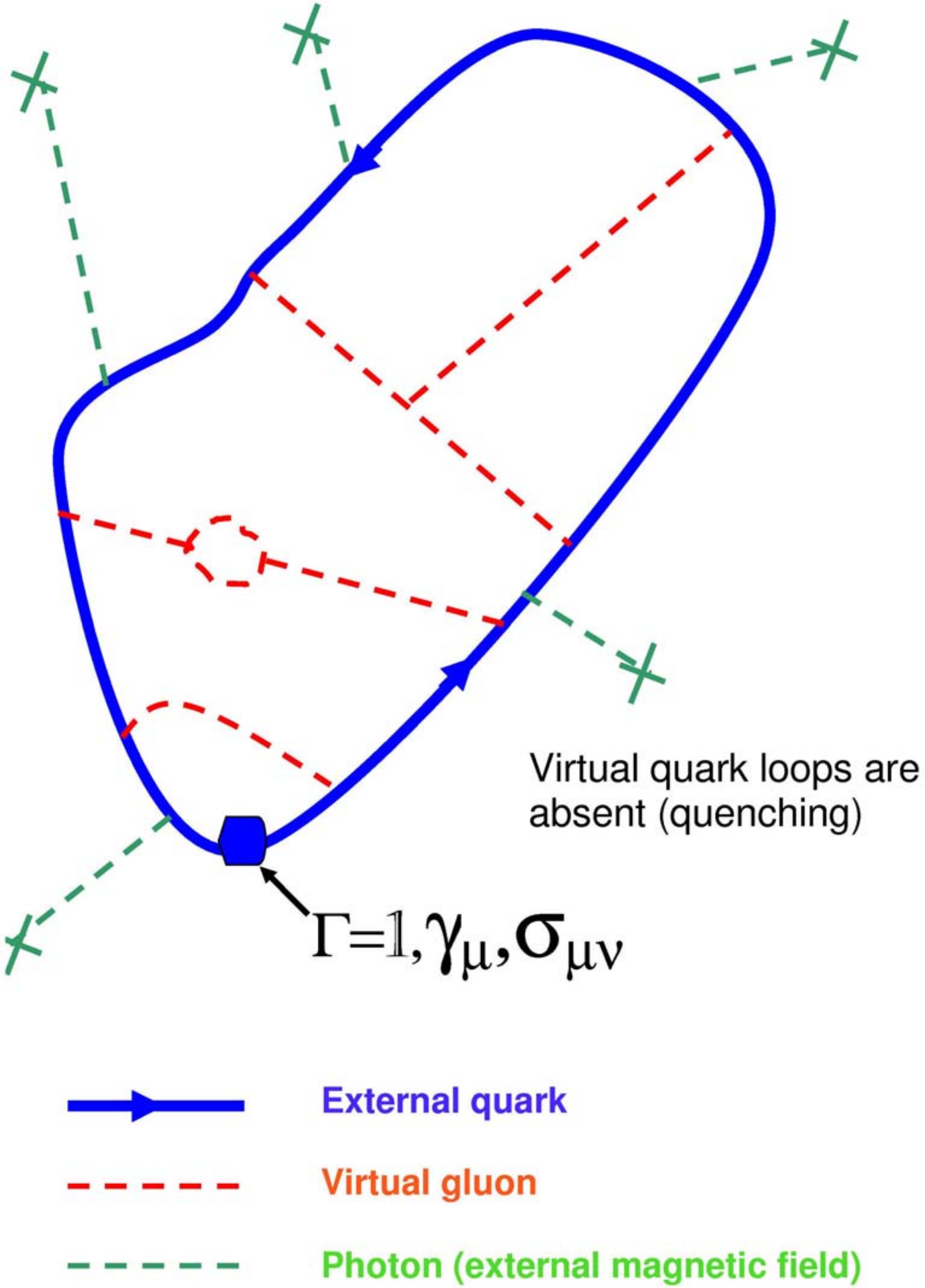
$$\mathcal{Z}_{\mathrm{QCD}}[Y] = \int \mathrm{D}A \int \mathrm{D}\Psi \int \mathrm{D}\bar{\Psi}\, e^{-S_{\mathrm{YM}}(A) - S_F(A, Y, \Psi, \bar{\Psi})}$$
$$= \int \mathrm{D}A \det[\mathcal{D}(A + Y) + m]\, e^{-S_{\mathrm{YM}}(A)}$$

$$S_F(A, Y, \Psi, \bar{\Psi}) = \int \mathrm{d}^4x\, \bar{\Psi}(x)[\mathcal{D}(A + Y) + m]\Psi(x)$$

$A \in SU(2)$, $Y \in U(1)$

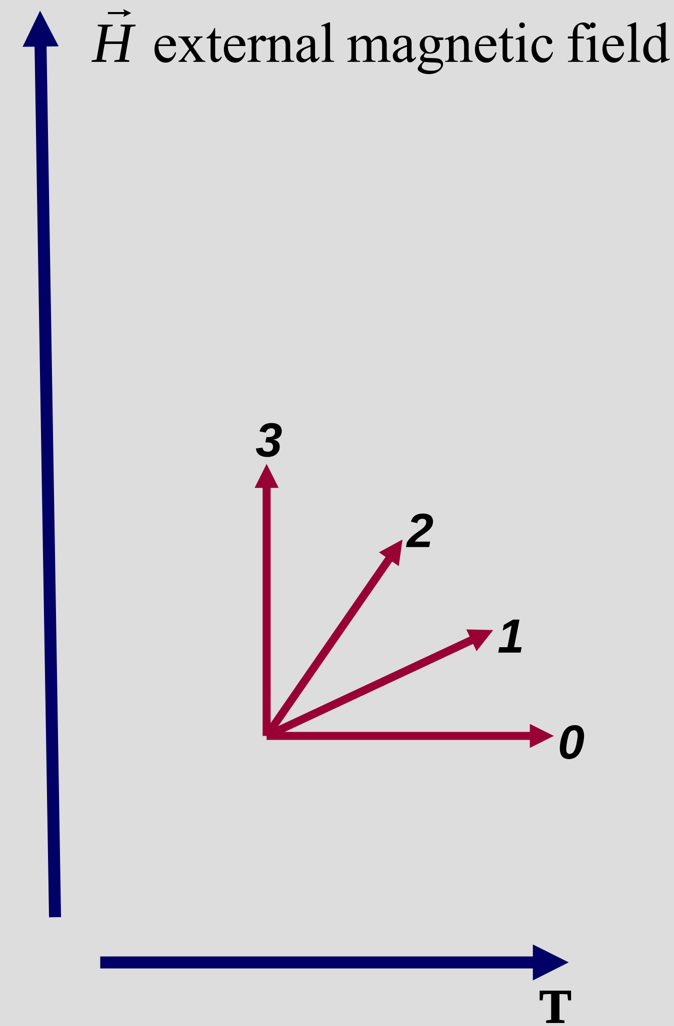
- Quenching: $\det[\mathcal{D}(A + Y) + m] \to 1$
- Magnetic field $B = dY$ is introduced into the Dirac operator $\mathcal{D}$

1) We use the overlap Dirac operator on the lattice
2) We perform simulations in the quenched limit



We calculate $< \overline{\psi}\, \Gamma \psi >; \ \Gamma = 1, \gamma_\mu, \sigma_{\mu\nu}$

in the external magnetic field and in the presence of the vacuum gluon fields



Quenched vacuum, overlap Dirac operator, external magnetic field

$$eB = \frac{2\pi qk}{L^2}; eB \geq 250\ Mev$$

Free massless fermions in (2+1) dimensions

$$< \overline{\psi}\psi > = -\frac{eB}{2\pi}$$

V.P.Gusynin, V.A.Miransky, I.A.Shovkovy, hep-ph/9405262I

Physics: strong magnetic fields reduce the system to D = (0 + 1) !!!

Physics: magnetic field is the only dimensionful parameter!!!

Free massless fermions in (3+1) dimensions

$$< \bar{\psi}\psi > = -\frac{eB}{4\pi^2} m \ln \frac{\Lambda_{UV}^2}{m^2}$$

V.P.Gusynin, V.A.Miransky, I.A.Shovkovy, hep-ph/9412257

Physics: strong magnetic fields reduce the system to D = (1 + 1) !!!

Physics: the poles of fermionic propagator are at Landau levels

Chiral condensate in QCD

$$\Sigma = -<\overline{\psi}\psi>$$

- Chiral perturbation theory gives [Shushpanov, Smilga 1997]

$$\Sigma = \Sigma_0 \left(1 + \frac{eB\ln 2}{16\pi^2 F_\pi^2}\right)$$

- Physics: Pion loop in external field, all possible electromagnetic insertions summed as in Euler-Heisenberg lagrangian
- AdS/CFT prediction: $\Sigma = \Sigma_0 \left(1 + cB^2\right)$

[A. Gorsky, A. Zayakin 2008]

How to calculate the chiral condensate?

1. Calculate the spectrum of the Dirac operator:

$$\not{D}(A,Y)\psi_k(A,Y) = i\lambda_k(A,Y)\psi_k(A,Y)$$

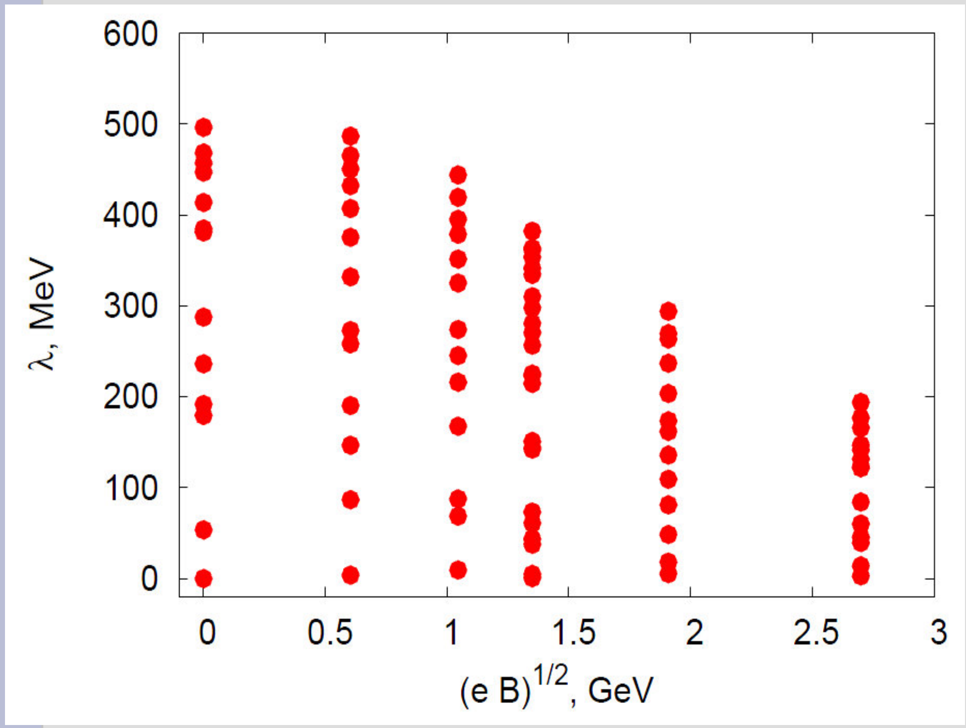
2. Find density of the Dirac eigenmodes:

$$\rho(\lambda) = \sum_k \delta\left(\lambda - \lambda_k\right)$$

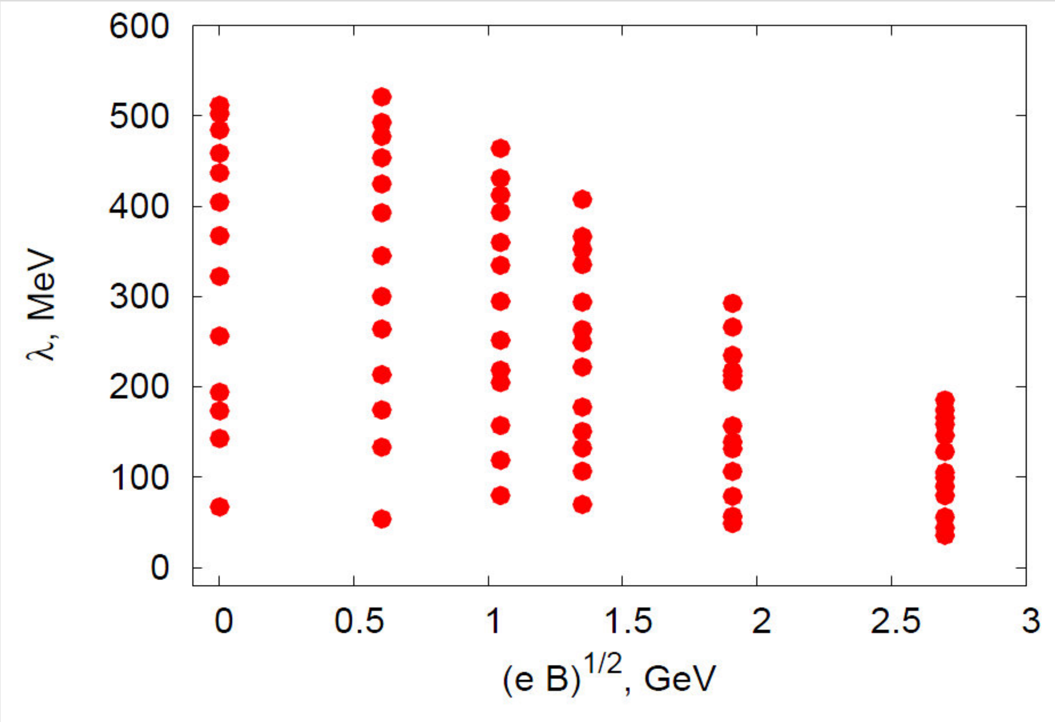
3. Use the Banks-Casher relation:

$$\langle 0|\, \bar{q}q \,|0\rangle = -\lim_{\lambda\to 0} \frac{\pi\rho(\lambda)}{V}$$

Evolution of the eigenmodes with increase of the magnetic field



Q=1

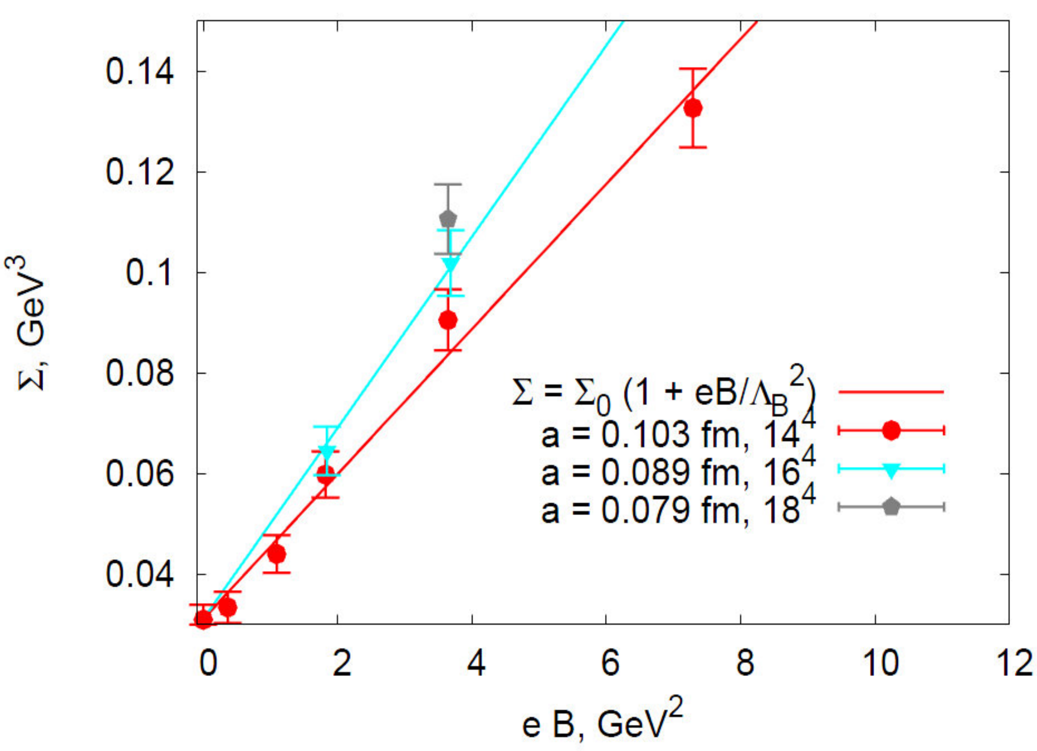


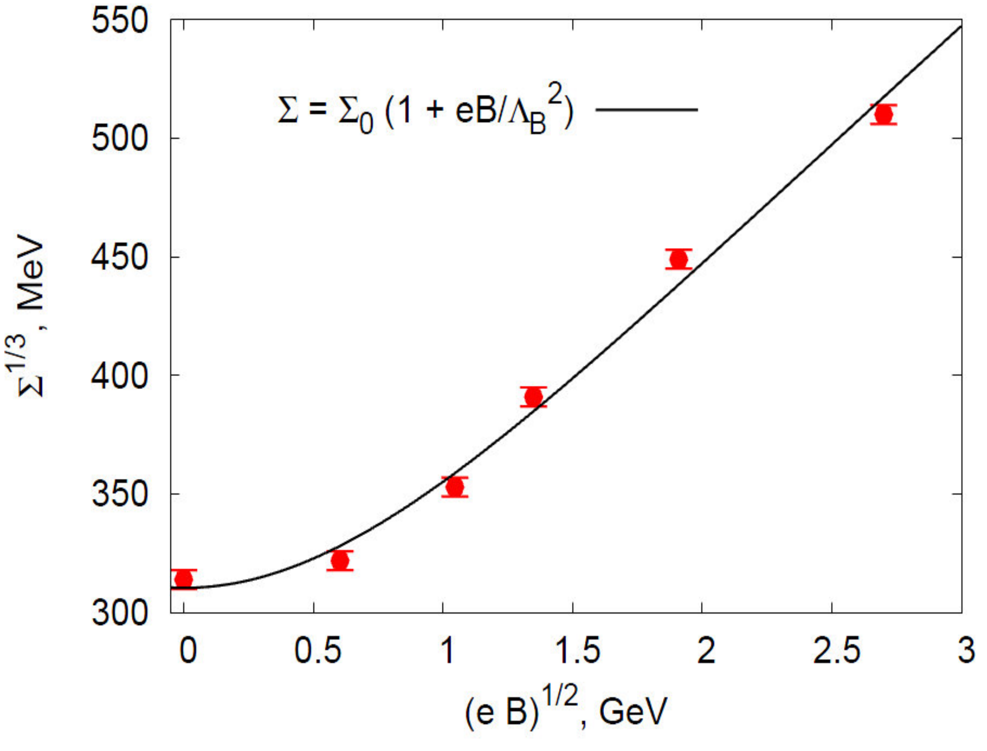
Q=0

Features:

1) The stronger the field the denser near-zero eigemodes
2) The exact zero eigenmode is insensitive to the field!

Chiral condensate vs. field strength





We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!

Lattice results vs. chiral perturbation theory

$$\Sigma = \Sigma_0 \left(1 + \frac{eB}{\Lambda_B^2}\right)$$

- Our value for Λ_B:

$$\Lambda_B^{\rm fit} = (1.41 \pm 0.14 \pm 0.20)\,\mathrm{GeV}$$

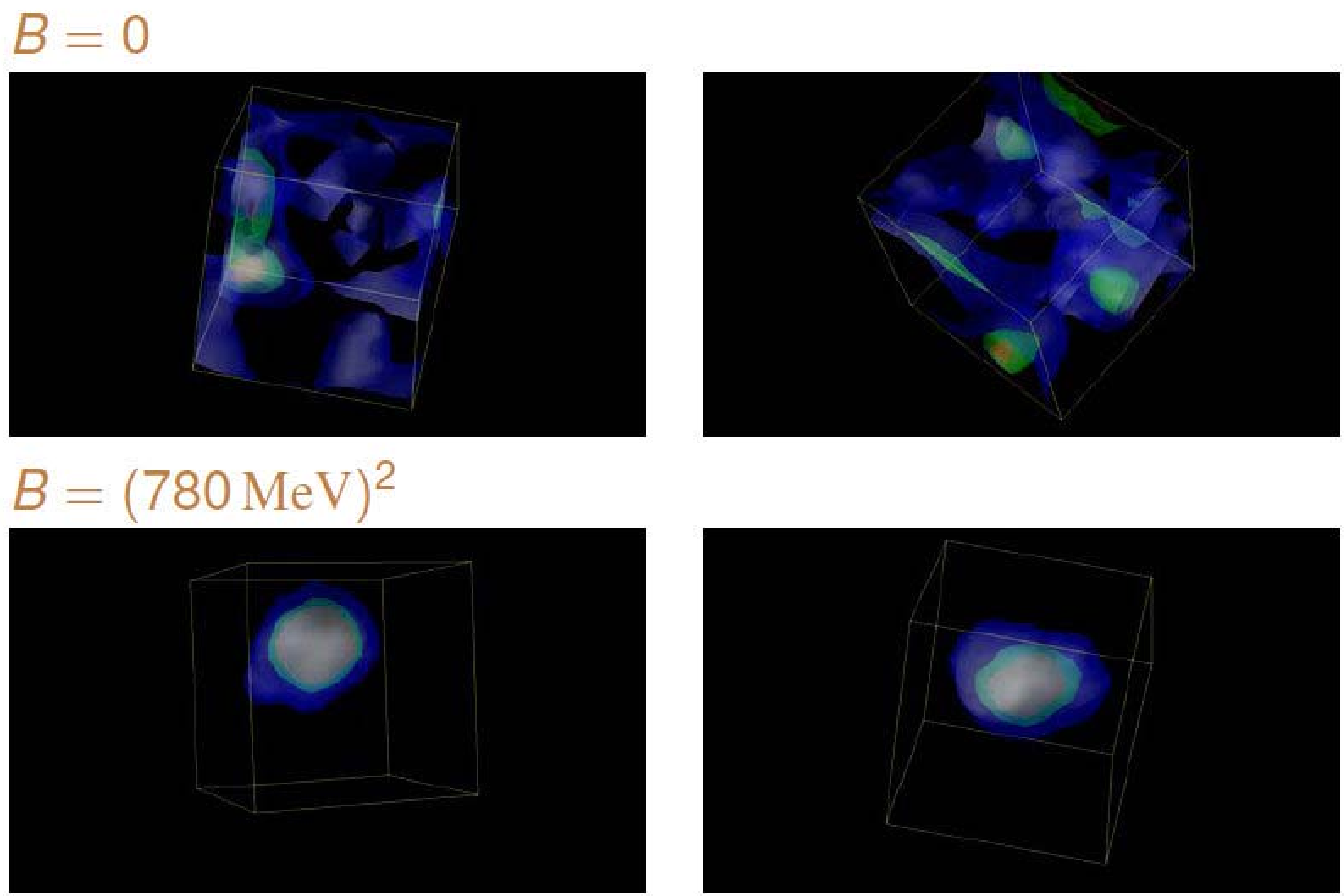
- χPT result:

$$\Lambda_B^{\chi PT} = 1.96\,GeV \quad (F_\pi = 130\,MeV - real\ world)$$
$$\Lambda_B^{\chi PT} = 1.36\,GeV \quad (F_\pi = 90\,MeV - quenched)$$

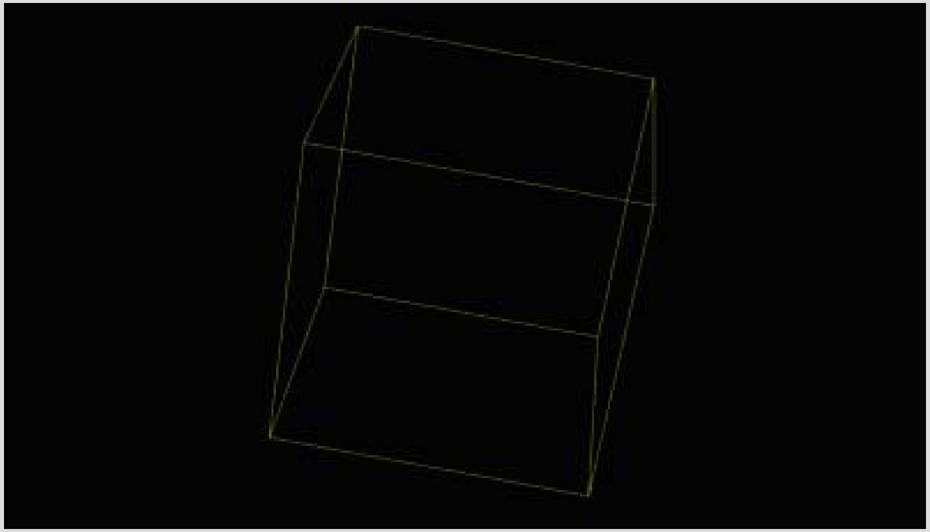
- Chiral condensate at $B = 0$: $\Sigma_0^{\rm fit} = [(310 \pm 6)\ \mathrm{MeV}]^3$

Localization of Dirac Eigenmodes

Typical densities of the chiral eigenmodes vs. the strength of the external magnetic field



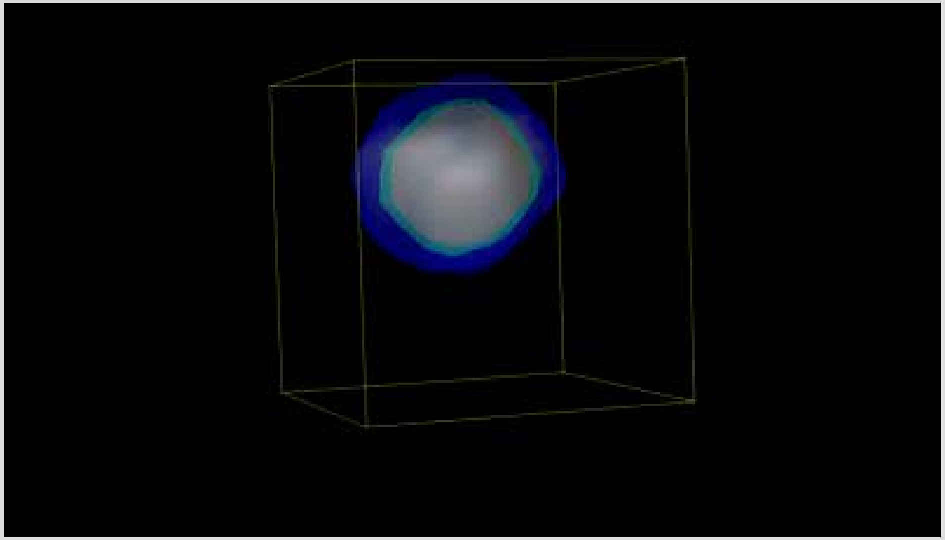
Localization of Dirac Eigenmodes



Zero fields, B=0

no clear visible localization

[but in fact the localization exists on 2D surfaces]



Non-zero fields, B ~ a few GeV^2

Localization appears at Lowest Landau Level

The magnetic susceptibility of the Yang-Mills vacuum

- External magnetic field induces nonzero magnetic moment of charged particles
- Parameterized by magnetic susceptibility χ_M

$$\langle 0|\, \bar{q}\, [\gamma_\mu, \gamma_\nu]\, q\, |0\rangle = \langle 0|\, \bar{q} q\, |0\rangle \chi_M (F)\, F_{\mu\nu}$$

- Generalization of the Banks-Casher formula:

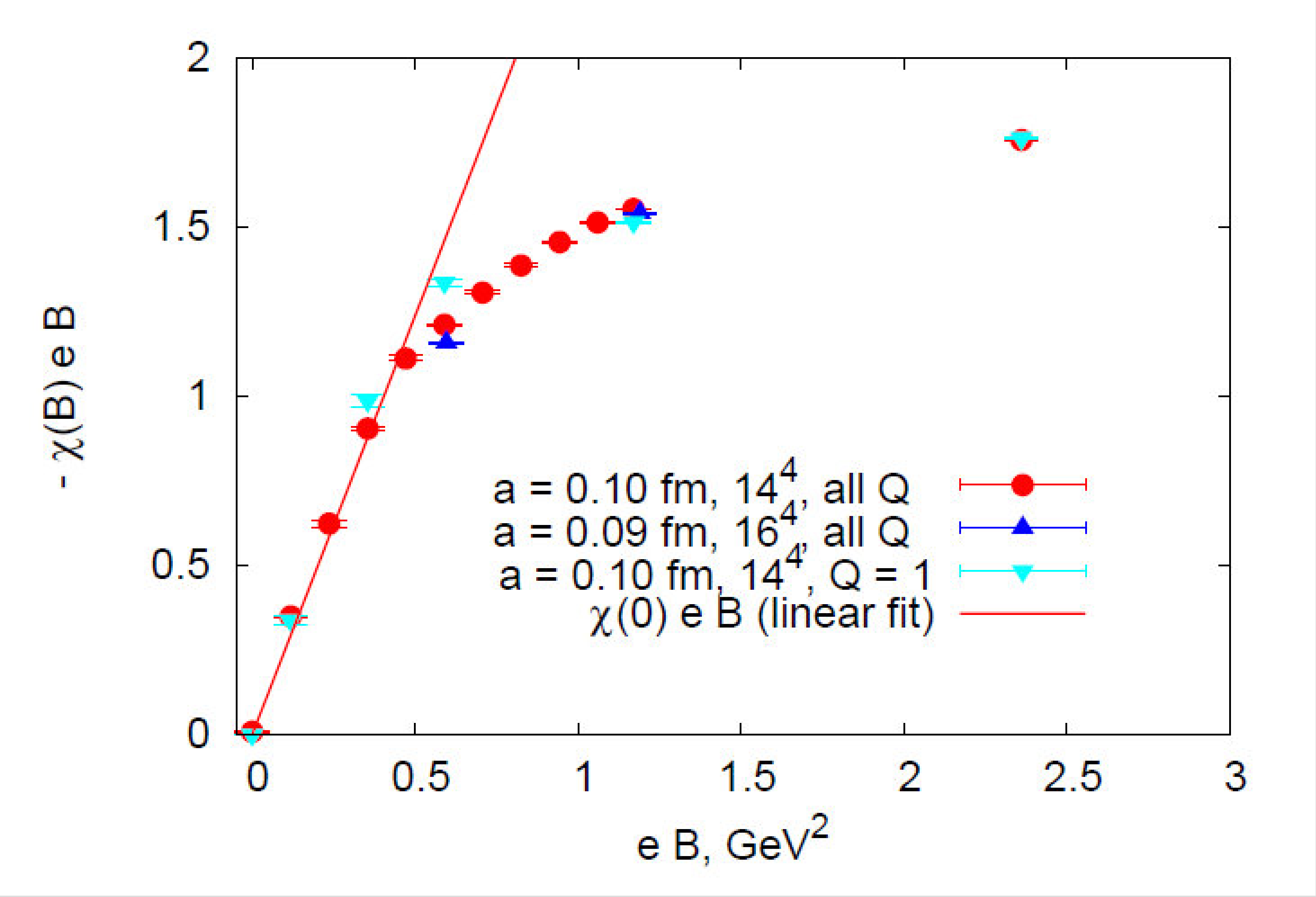
$$\langle 0|\, \bar{q} \mathcal{O} q\, |0\rangle = - \lim_{\lambda \to 0} \frac{\pi \rho (\lambda)}{V} \int d^4 x\, \psi^\dagger_\lambda \mathcal{O} \psi_\lambda$$

- This yields:

$$\chi_M (F)\, F_{\mu\nu} = \lim_{\lambda \to 0} \int d^4 x\, \psi^\dagger_\lambda\, [\gamma_\mu, \gamma_\nu]\, \psi_\lambda$$

- Expressed via magnetic moments for low eigenmodes

Magnetization of the vacuum as a function of the magnetic field



Magnetic susceptibility - results

- Theoretically,

$$\chi = -cN_c / (8\pi^2 f_\pi^2)$$

- In OPE $c = 2$ [Vainshtein'02]
- AdS/QCD with hard wall gives $c = 2.15$ [Gorsky, Krikun'09]
- Our first-principle (quenched) result gives:

$$c = 1.6 \pm 0.1 \quad (F_\pi = 130\, MeV - real\ world)$$
$$c = 0.80 \pm 0.05 \quad (F_\pi = 90\, MeV - quenched)$$

Magnetic susceptibility - results

$$<\bar{\psi}\psi> \chi = -46(3) Mev \leftrightarrow \text{our result}$$

$$<\bar{\psi}\psi> \chi \approx -50 Mev \leftrightarrow \text{QCD sum rules}$$

(I. I. Balitsky, 1985, P. Ball, 2003.)

Chiral Magnetic Effect

[Fukushima, Kharzeev, Warringa, McLerran ’07-’08]

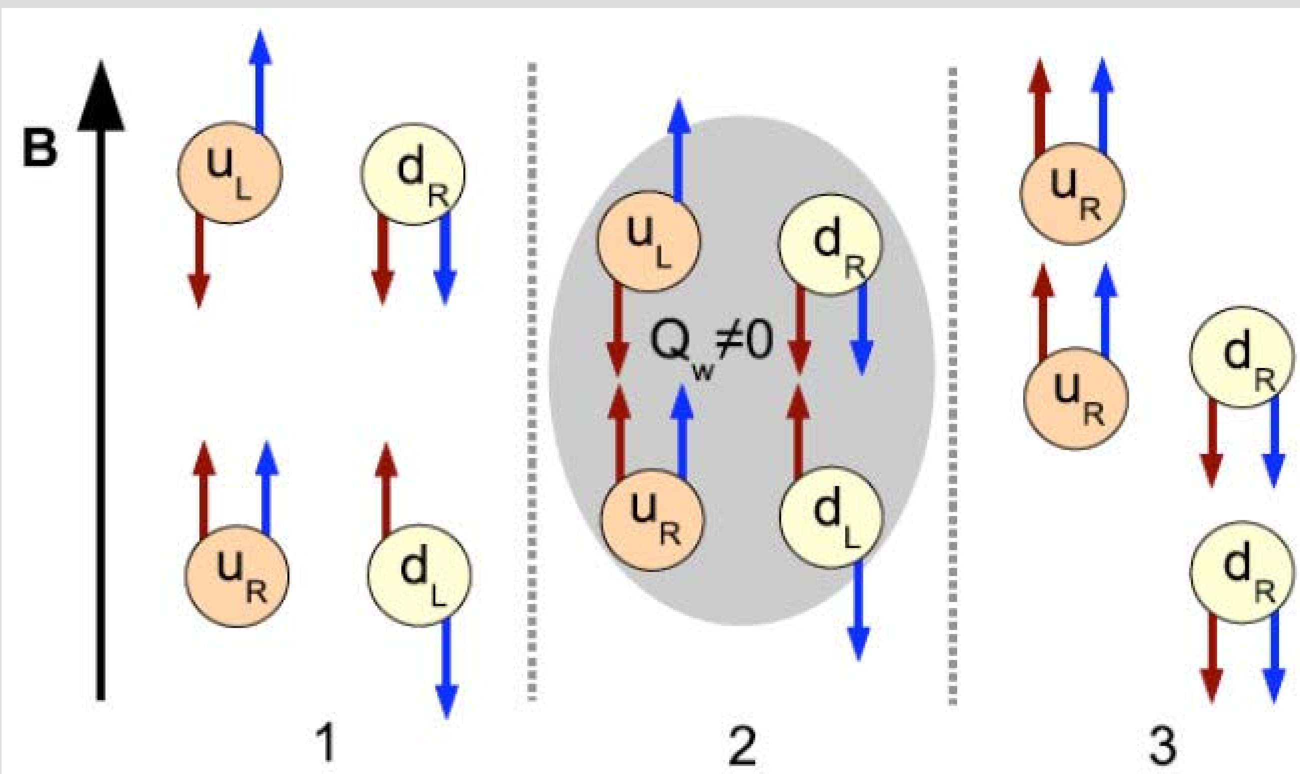
Electric current appears at regions
1. with non-zero topological charge density
2. exposed to external magnetic field

***Experimentally observed at RHIC* :**
charge asymmetry of produced particles at heavy ion collisions

Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

Theoretically: electric dipole moment at regions
1. with non-zero topological charge density
2. exposed to external magnetic field.

Experimentally: leads to charge asymmetry of produced particles at heavy ion collisions (currently at RHIC)



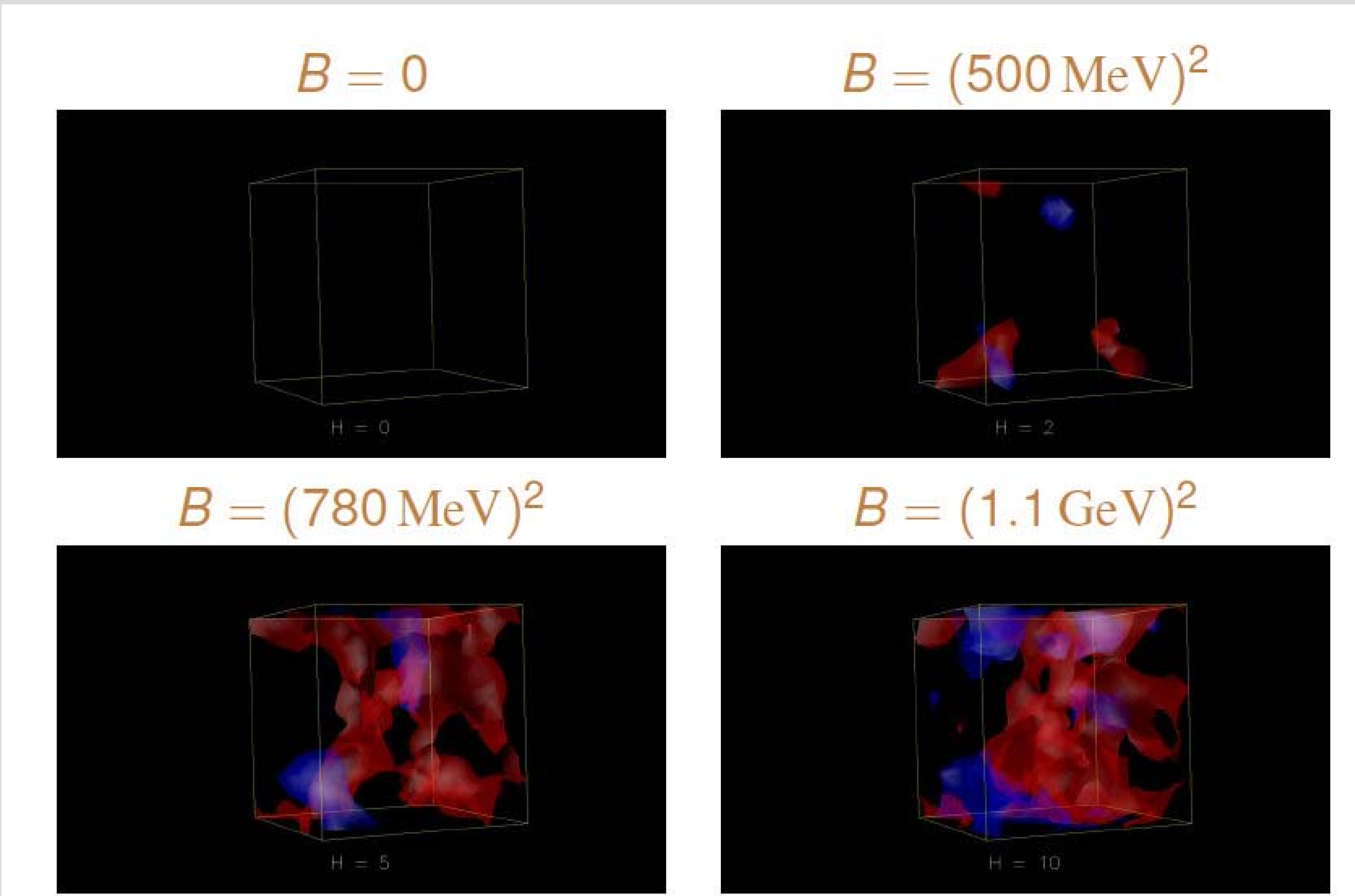
Red: momentum
Blue: spin

u-quark: q=+2/3
d-quark: q= - 1/3

Effect of topology:
$u_L \rightarrow u_R$
$d_L \rightarrow d_R$

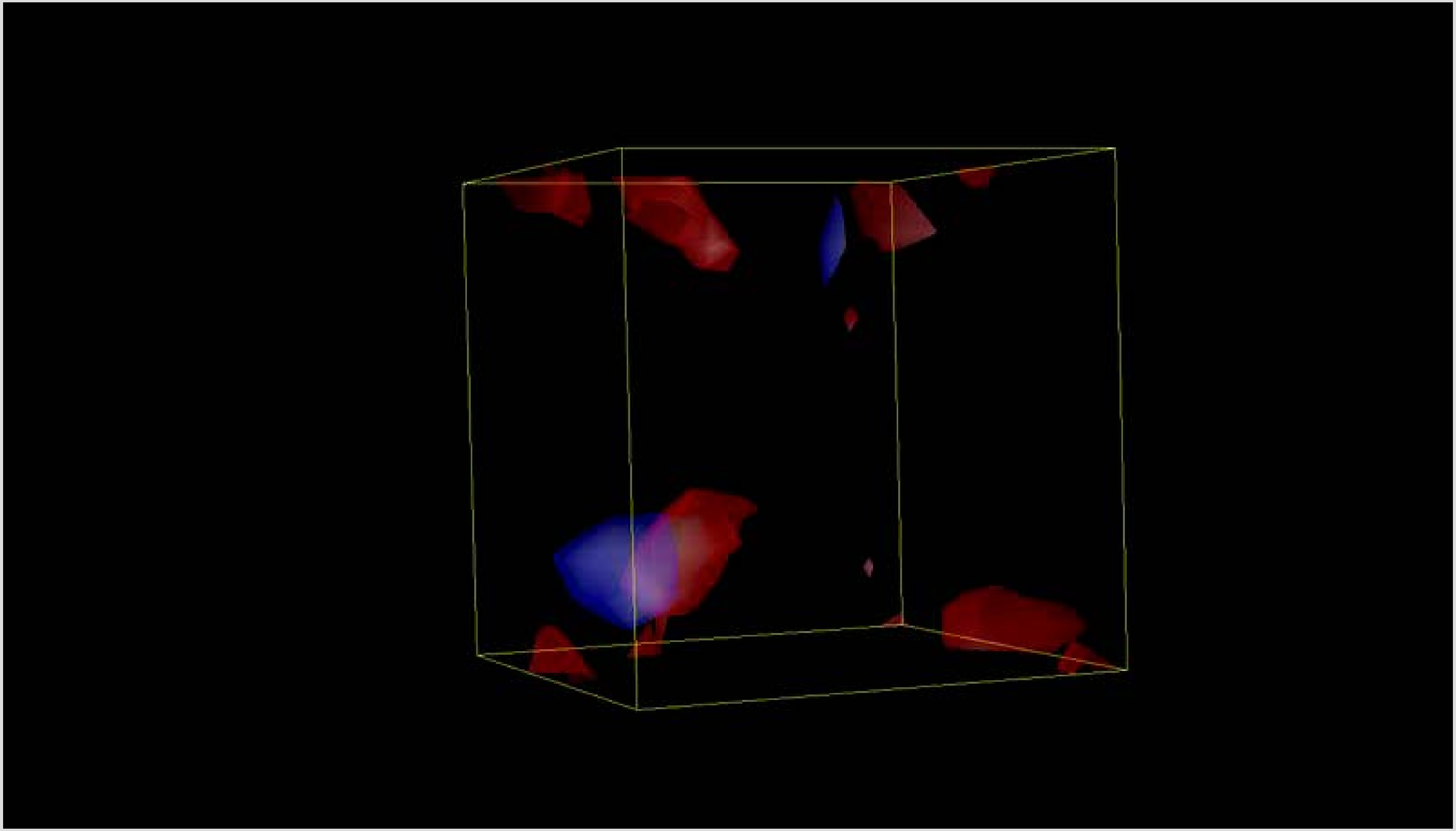
Chiral Magnetic Effect on the lattice, qualitative picture

Density of the electric charge vs. magnetic field



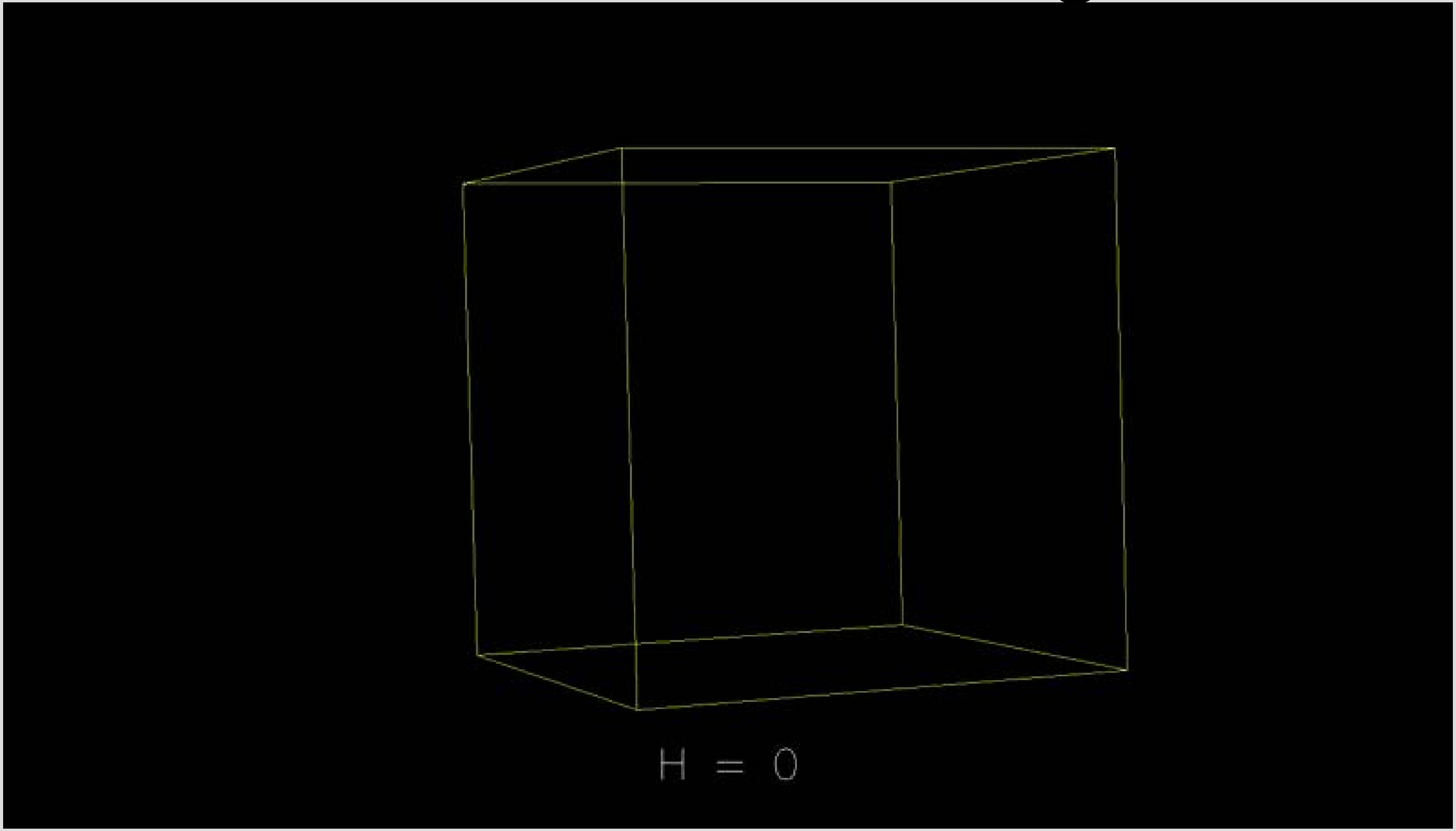
Chiral Magnetic Effect on the lattice, qualitative picture

Non-zero field, subsequent time slices

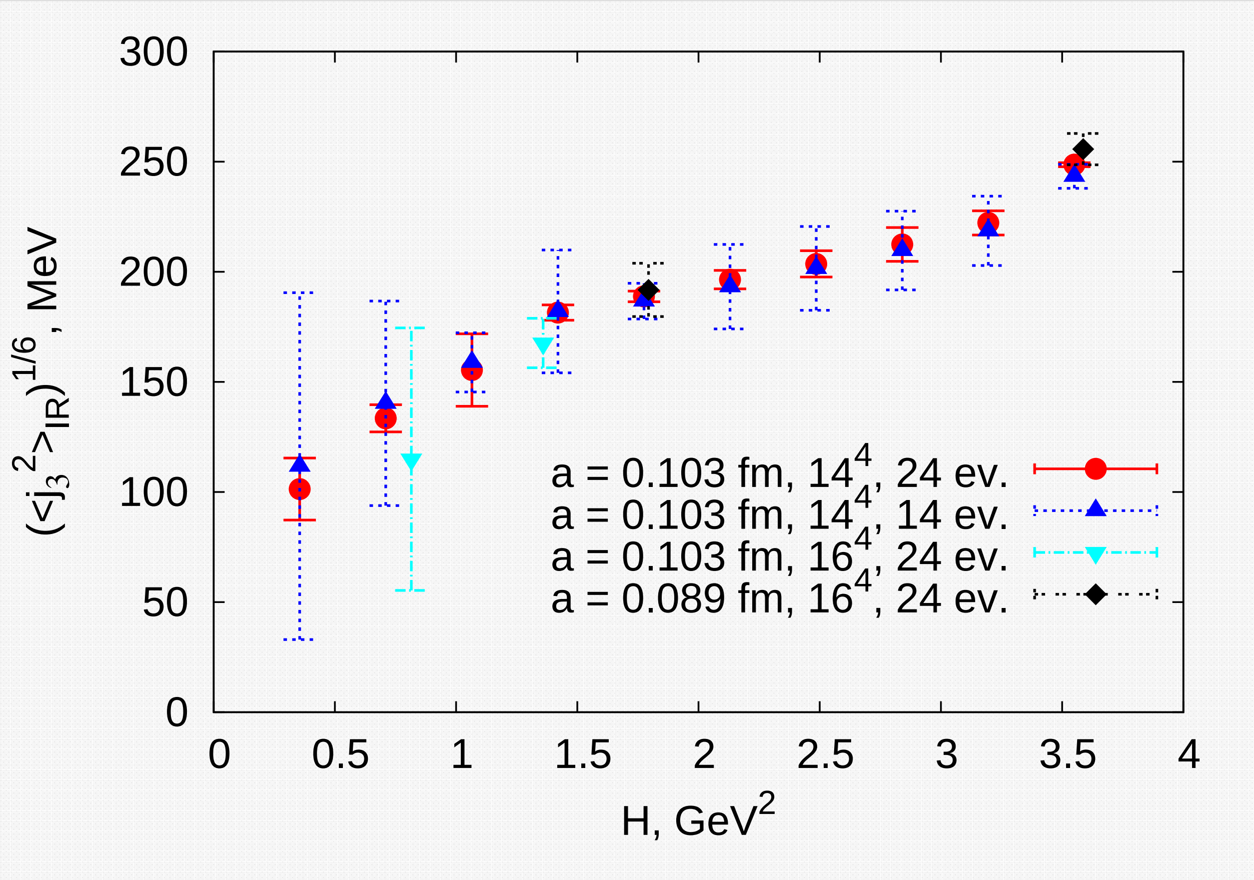


Chiral Magnetic Effect on the lattice, qualitative picture

Effect of field increasing



Chiral Magnetic Effect on the lattice, numerical results



Regularized electric current:

$$< j_3^2 >_{IR} = < j_3^2(H,T) > - < j_3^2(0,0) >, \quad j_3 = \overline{\psi}\gamma_3\psi$$

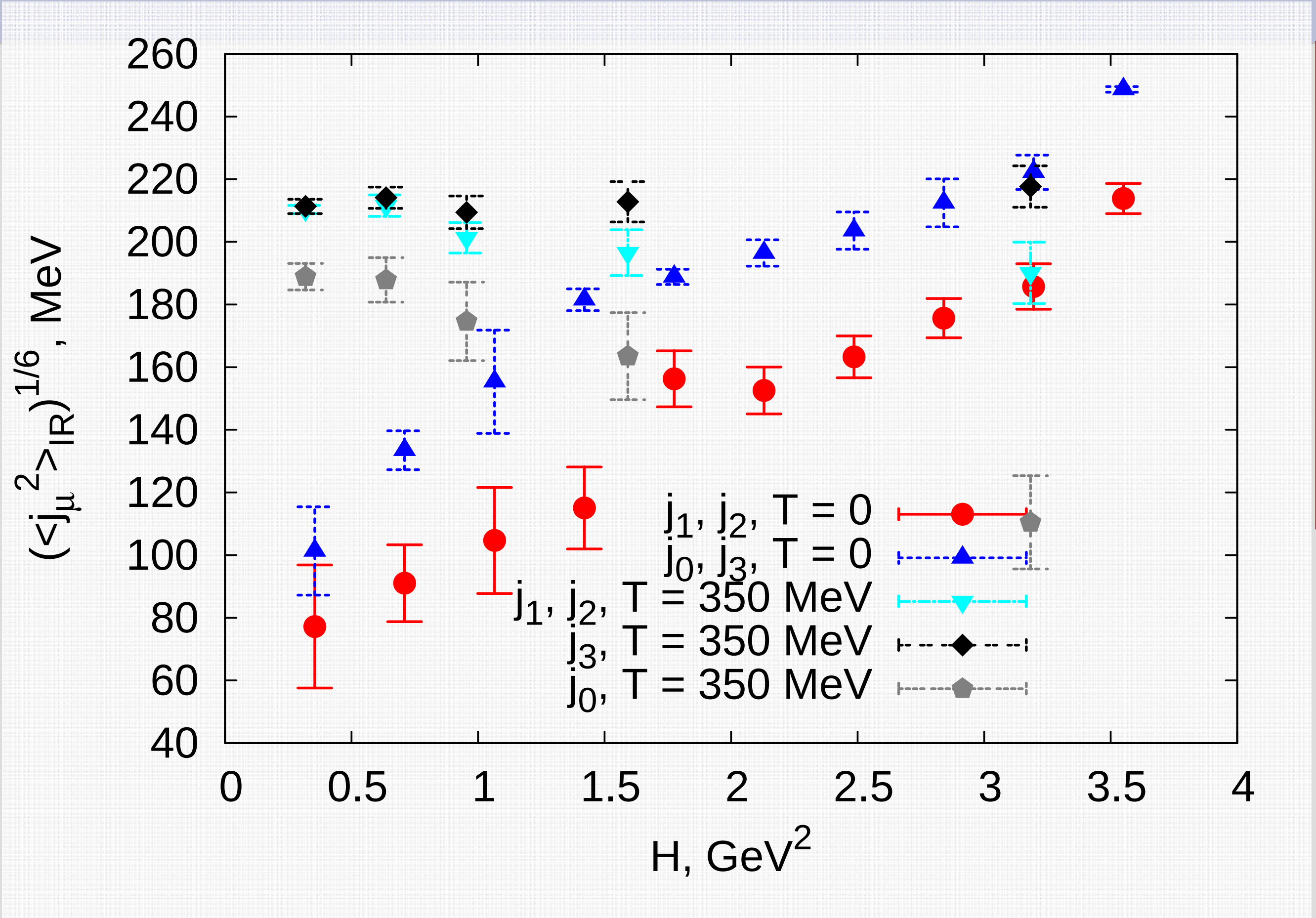
Chiral Magnetic Effect on the lattice, numerical results

$T=0$

$F_{12}\neq 0$

$< j_1^2 > = < j_2^2 >$

$< j_3^2 > = < j_0^2 >$



$T>0$

$F_{12}\neq 0$

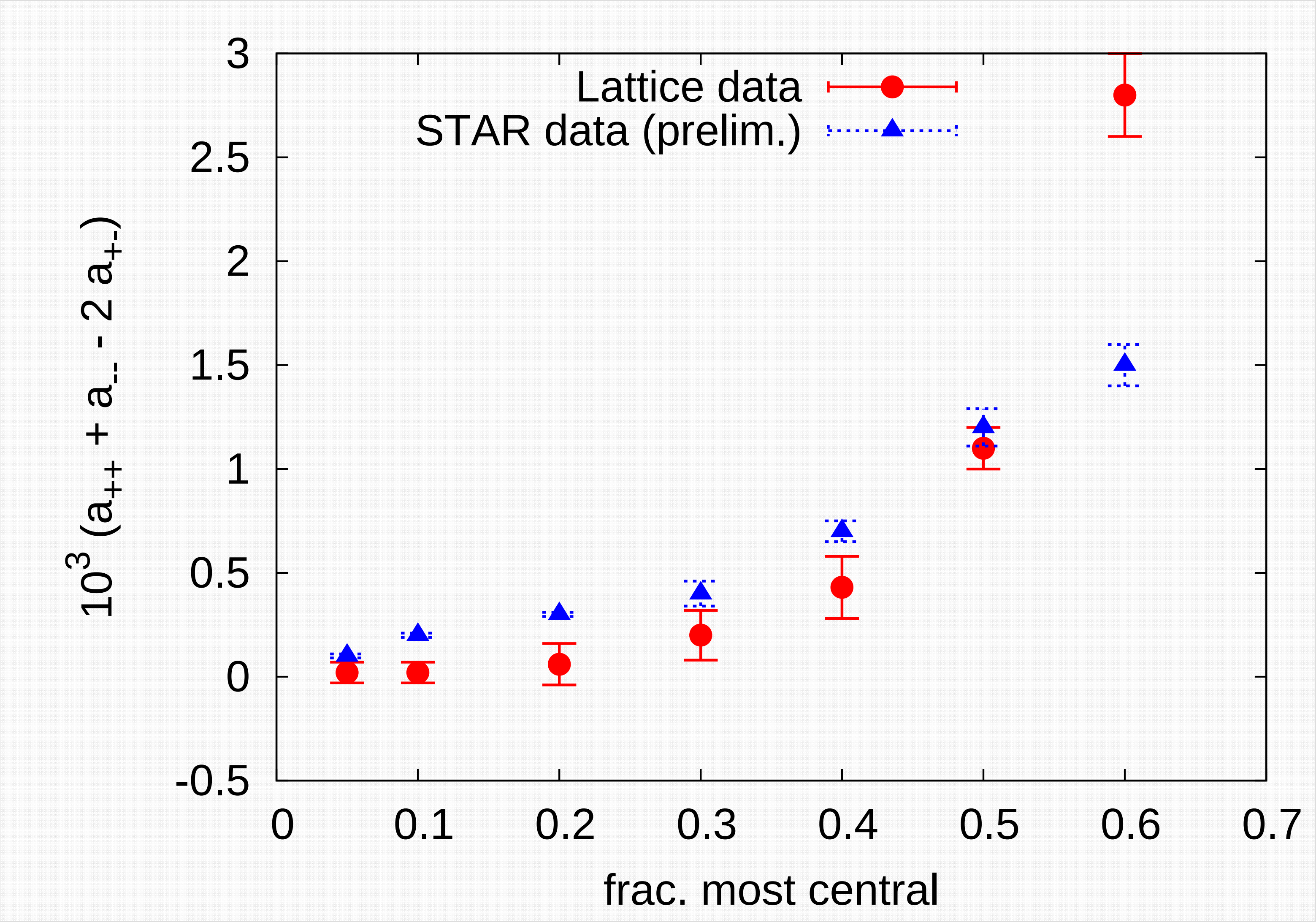
$< j_1^2 > = < j_2^2 >$

$< j_3^2 > \neq < j_0^2 >$

Regularized electric current:

$$< j_i^2 >_{IR} = < j_i^2 (H,T) > - < j_i^2 (0,0) >, \quad j_i = \bar{\psi}\gamma_i\psi$$

Chiral Magnetic Effect,
EXPERIMENT VS LATTICE DATA (Au+Au)



Chiral Magnetic Effect,
EXPERIMENT VS LATTICE DATA

$$a_{ab} = \frac{1}{N_e}\sum_{e=1}^{N_e}\frac{1}{N_a N_b}\sum_{i=1}^{N_a}\sum_{j=1}^{N_b}\cos\left(\phi_{ia}+\phi_{jb}\right)$$

experiment

$$\frac{\langle (\Delta Q)^2 \rangle}{N_q^2} = a_{++} + a_{--} - 2a_{+-}$$

$R \approx 5 \ fm$

$\rho \approx 0.2 \ fm$

$\tau \approx 1 \ fm$

our fit

$$= \frac{4\pi\tau^2\rho^2 R^2}{3N_q^2}\left(\langle j_{\parallel}^2 \rangle + 2\langle j_{\perp}^2 \rangle\right)$$

D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008),

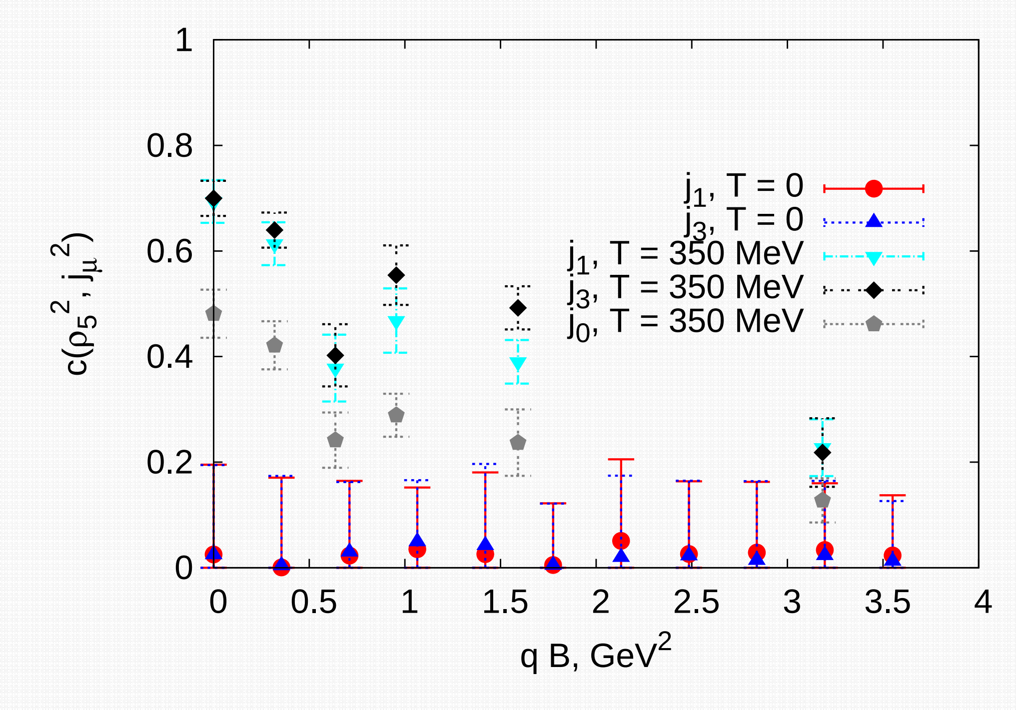
our lattice data at T=350 *Mev*

Chiral Magnetic Effect on the lattice, RESULTS and QUESTIONS

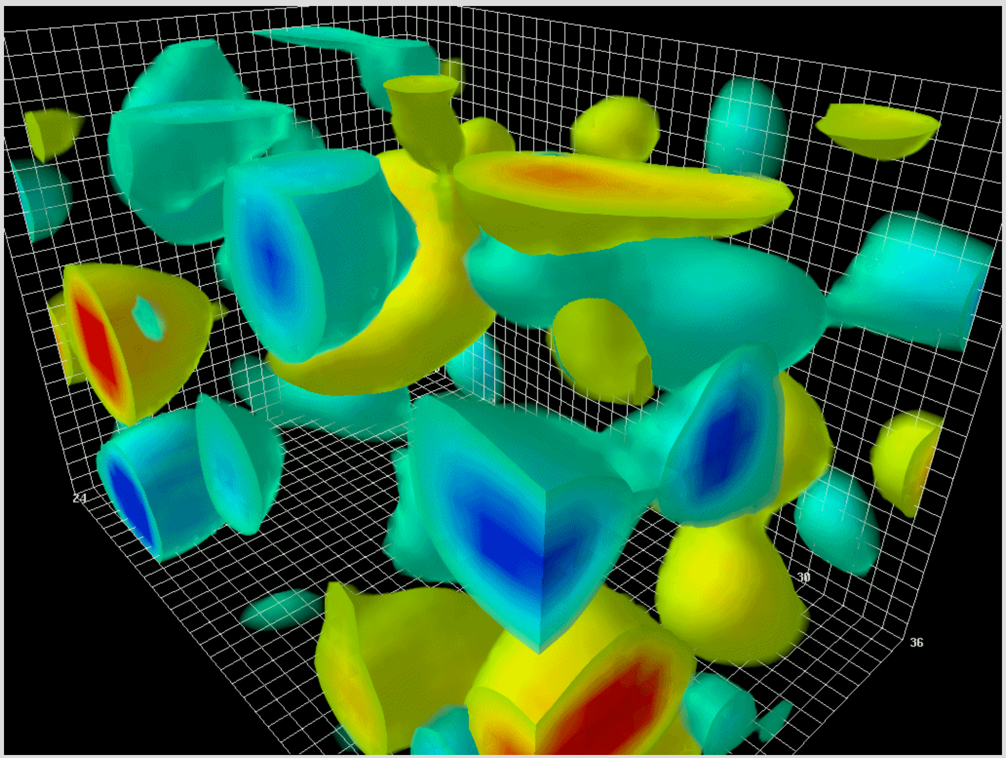
We see rather weak correlation between topological charge density and fluctuations of the density of the electric currents (What is the origin of CME effect?)

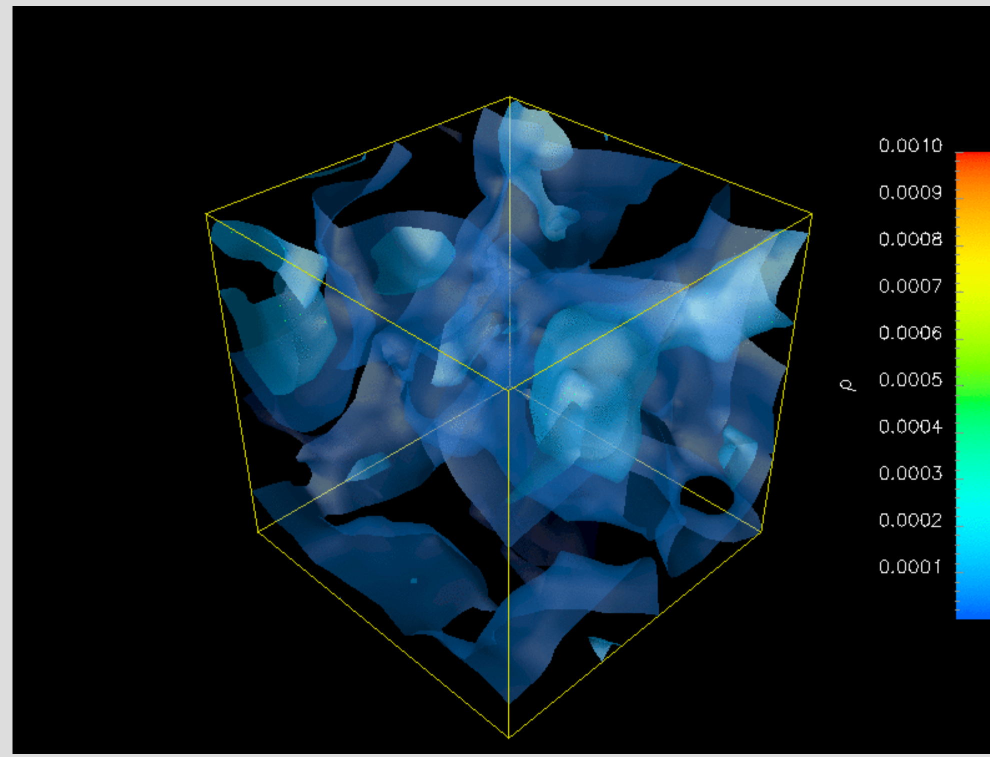
[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008), URL http://arxiv.org/abs/0808.3382.
[2] D. Kharzeev, R. D. Pisarski, and M. H. G.Tytgat, Phys. Rev. Lett. 81, 512 (1998), URL http://arxiv.org/abs/hep-ph/9804221.
[3] D. Kharzeev, Phys. Lett. B 633, 260 (2006), URL http://arxiv.org/abs/hep-ph/0406125.
[4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008), URL http://arxiv.org/abs/0711.0950.

$$c\left(\rho_5^2, j_\mu^2\right) = \frac{\langle \rho_5^2 j_\mu^2 \rangle - \langle \rho_5^2 \rangle \langle j_\mu^2 \rangle}{\sqrt{\langle \rho_5^4 \rangle}\sqrt{\langle j_\mu^4 \rangle}}$$



Chiral Magnetic Effect on the lattice, RESULTS and QUESTIONS





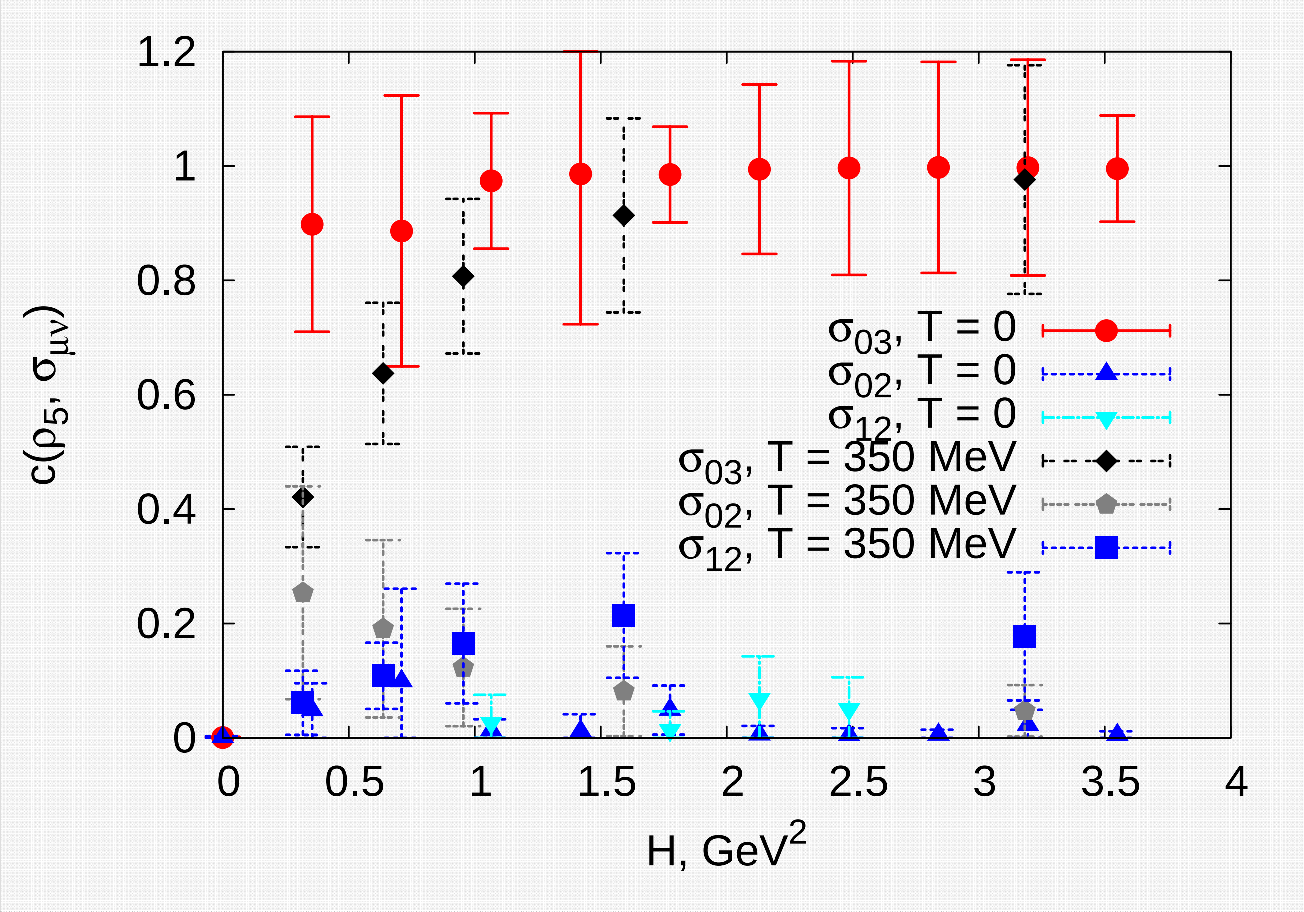
D. Leinweber
Topological charge density after vacuum cooling

S.Morozov, F.Gubarev, M.Polikarpov, V. Zakharov
Density of eigenfunctions of the Dirac equation in not cooled vacuum

Chiral Magnetic Effect on the lattice, RESULTS and QUESTIONS

- **Large correlation between square of the electric dipole moment**

$\sigma_{0i} = i\overline{\psi}[\gamma_0, \gamma_i]\psi$ **and chirality** $\rho_5 = \overline{\psi}\gamma_5\psi$



Conclusions

1. **We observe that the chiral condensate is proportional to the strength of the magnetic field, the coefficient of the proportionality agrees with Chiral Perturbation Theory. Microscopic mechanism for the chiral enhancement is the localization of fermion modes in the vacuum (arXiv:0812.1740)**

2. **The calculated vacuum magnetization is in a qualitative agreement with model calculations (arXiv:0906.0488)**

3. **We obsereve signatures of the Chiral Magnetic Effect, but the physics may differ from the model of Kharzeev, McLerran and Warringa (arXiv:0907.0494)**

4. **We observe very large correlation between electric dipole moment and chirality (not yet published)**