



***Features of nuclear many-body dynamics:
from pairing to clustering***

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Outline

- Configuration interaction approach
 - SU(3) symmetry based configurations
 - Computational examples, bosonic
 - SU(3) components in realistic and random systems
- Cluster-Nucleon Configuration Interaction Model.
 - Center of mass and translational invariance
 - Traditional cluster spectroscopic amplitudes
 - Norm kernel, orthogonality condition model, RGM
- Applications, nuclear clustering
 - Realistic examples with effective hamiltonian
 - Distribution of clustering strength
 - Pairing and clustering
 - ab-initio no-core shell model calculations

From shell model to configuration interactions

State, equivalent to operator (polymorphism)

$$|\Psi\rangle \equiv \hat{\Psi}^\dagger |0\rangle = \sum_{\{1,2,3,\dots,A\}} \langle 1, 2 \dots A | \Psi \rangle \hat{a}_1^\dagger \hat{a}_2^\dagger \dots \hat{a}_A^\dagger |0\rangle$$

Construct relevant many-body operators, configurations

- Proper symmetry quantum numbers (S,T,J)
- Use SU(3) classification + permutation group

$$|\Phi_{(n,0):L}^\eta\rangle \equiv \left(\hat{\Phi}_{(n,0):L}^\eta\right)^\dagger |0\rangle \equiv \left| \{n_i^{\alpha_i}\} [f]=[4](n,0) : L, S=0, T=0 \right\rangle$$

Methods

- Direct diagonalization of Casimir operators of SU(3), J^2 , T^2 ...
- Coupling and U(N) Clebsh-Gordan coefficients (via diagonalization)
- Casimir projection techniques. Generators of algebra.

Notations

$\{n_i^{\alpha_i}\}$ configuration

α_i number of particles

n_i oscillator shell

$$n = \sum_i \alpha_i n_i$$

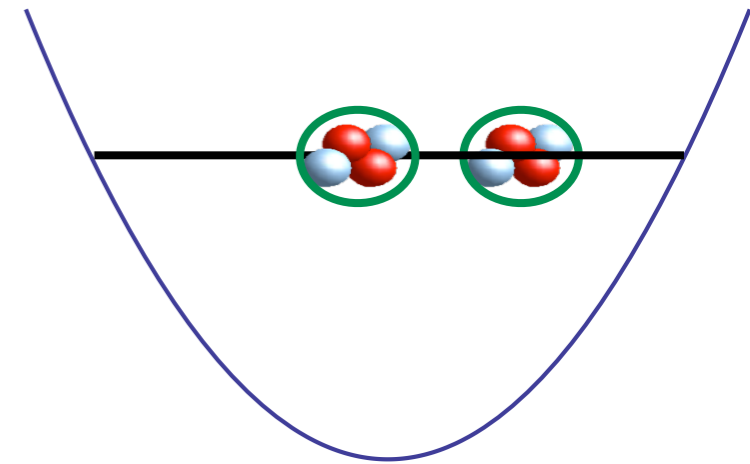
$$A = \sum \alpha_i$$

Computational test; Bosonic nature of 4-nucleon operators

If Φ^\dagger is thought of as being a boson then $\Phi\Phi^\dagger = 1 + N_b$

$$|\Psi_D\rangle = |\Phi\rangle \quad \langle\Phi_D|\hat{\Phi}\hat{\Phi}^\dagger|\Psi_D\rangle = \langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle = 2$$

$$L = S = T = 0$$



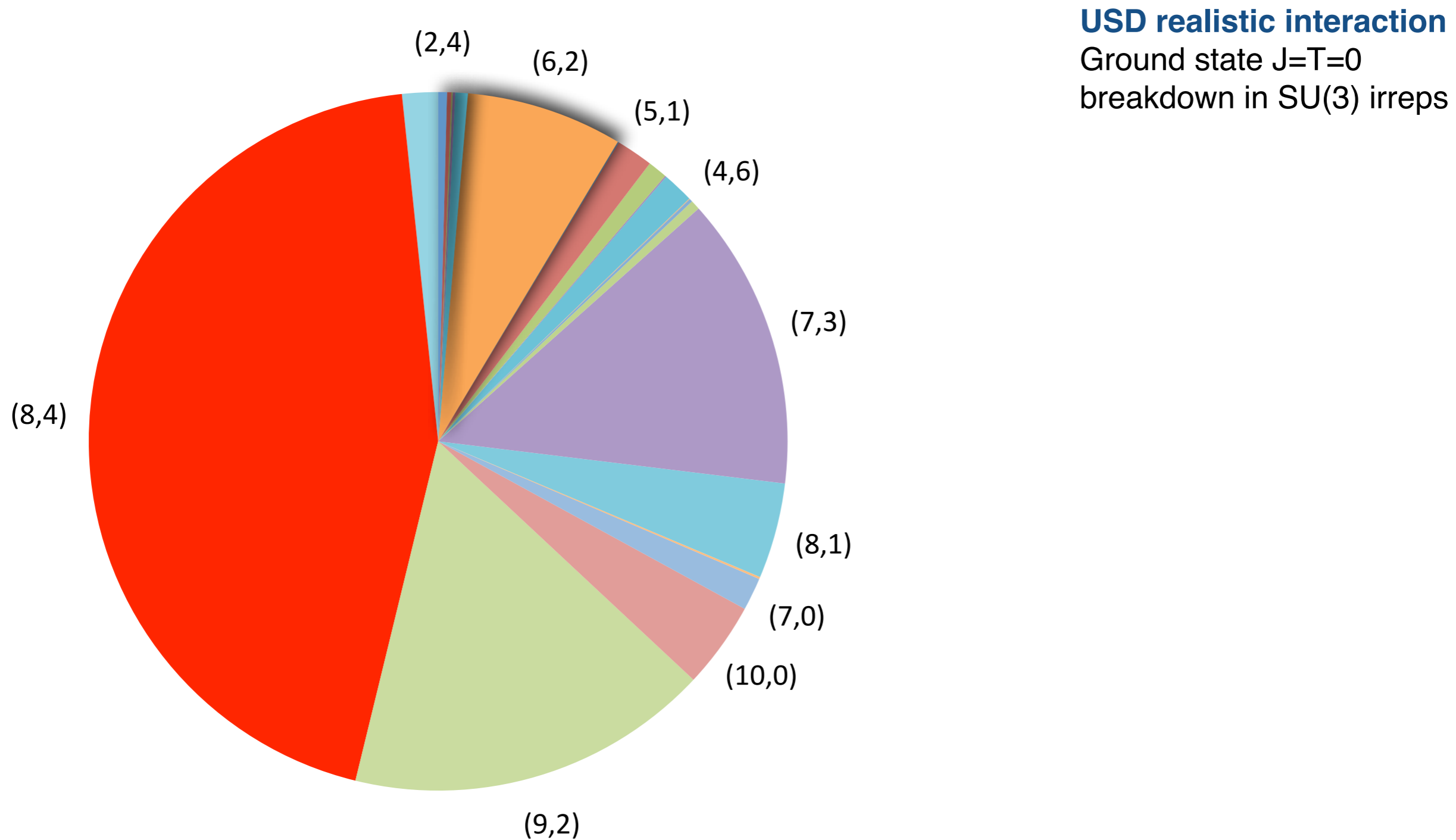
Coefficients of fractional parentage

$$\mathcal{F}_{nl} = \langle\Psi_P(R_P)|\hat{\mathcal{A}}\{\Phi_{(n,0):l}(R_\alpha)\Psi_D(R_D)\}\rangle \equiv \langle\Psi_P(R_P)|\Phi_{(n,0):l}^\dagger(R_\alpha)|\Psi_D(R_D)\rangle$$

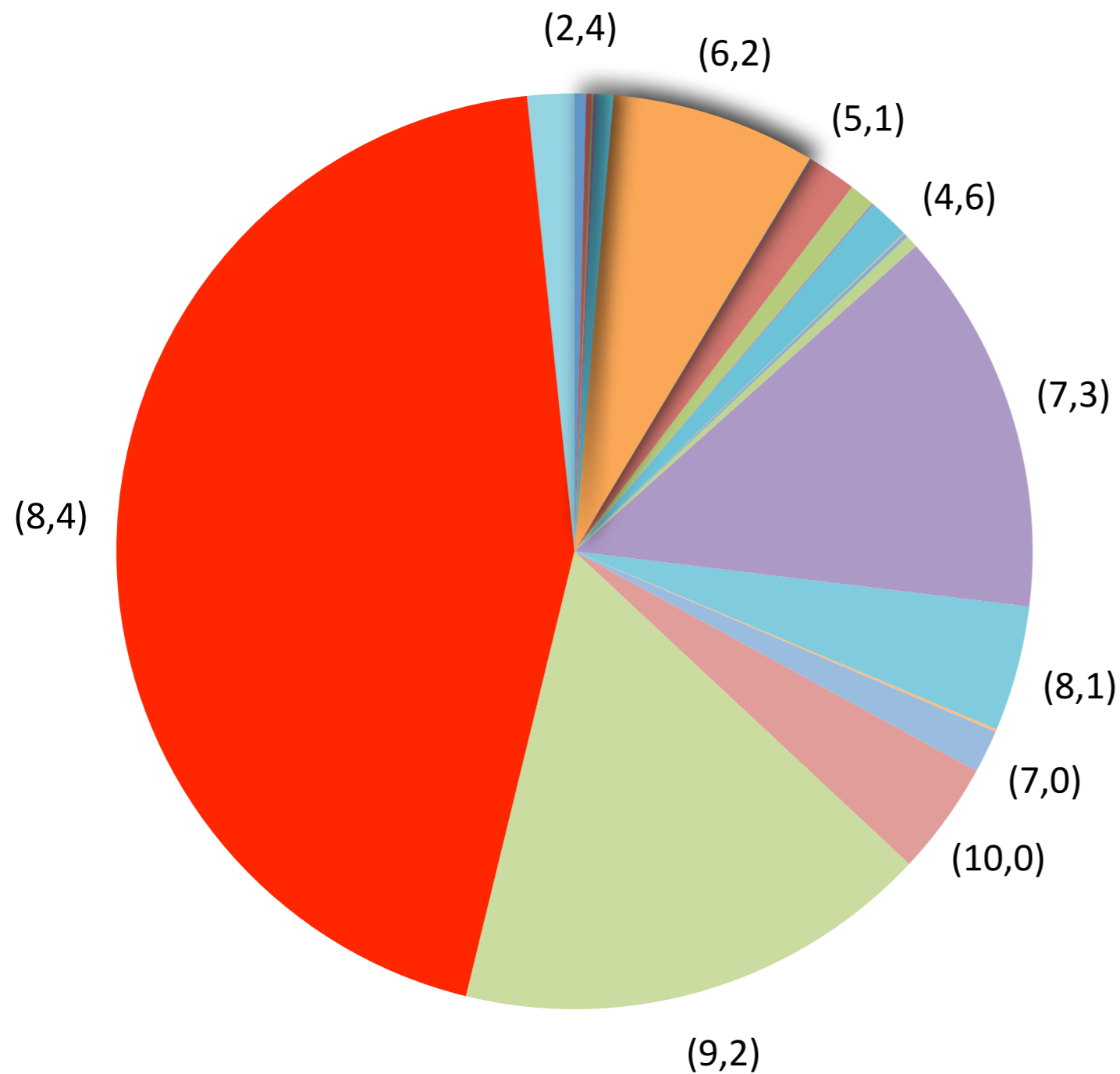
Φ	Ψ_P	$ \langle\Psi_P \hat{\Phi}^\dagger \Psi_D\rangle ^2$	$\langle\Psi_D \hat{\Phi}\hat{\Phi}^\dagger \hat{\Psi}_D\rangle$
$(p)^4 (4, 0)$	$(p)^8 (0, 4)$	1.42222*	1.42222
$(sd)^4 (8, 0)$	$(sd)^8 (8, 4)$	0.487903	1.20213
$(fp)^4 (12, 0)$	$(fp)^8 (16, 4)$	0.292411	1.41503
$(sdg)^4 (16, 0)$	$(sdg)^8 (24, 4)$	0.209525	1.5278

* For p-shell the result is known analytically 64/45

Classic Example: ^{24}Mg , 8 nucleons in sd-shell

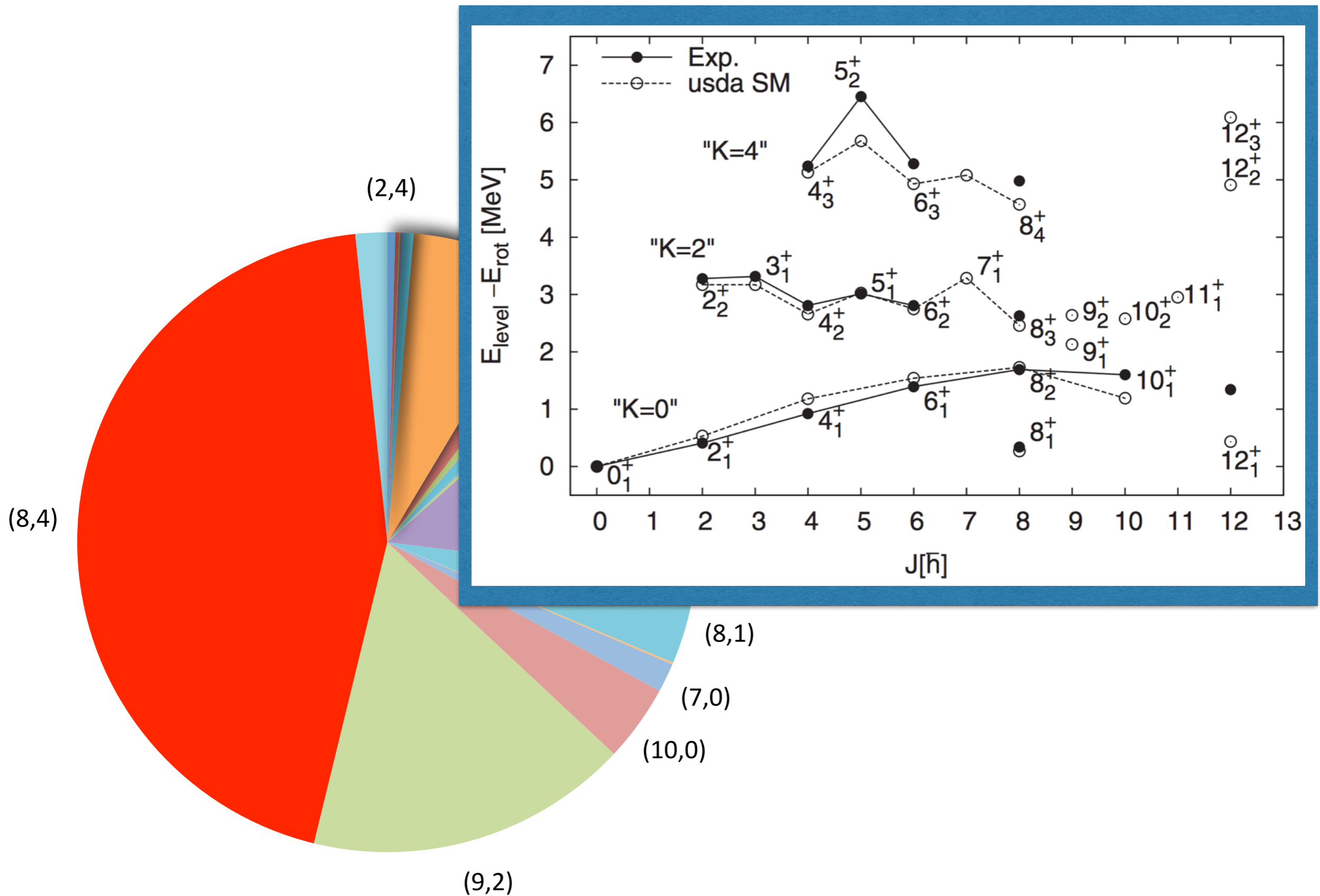


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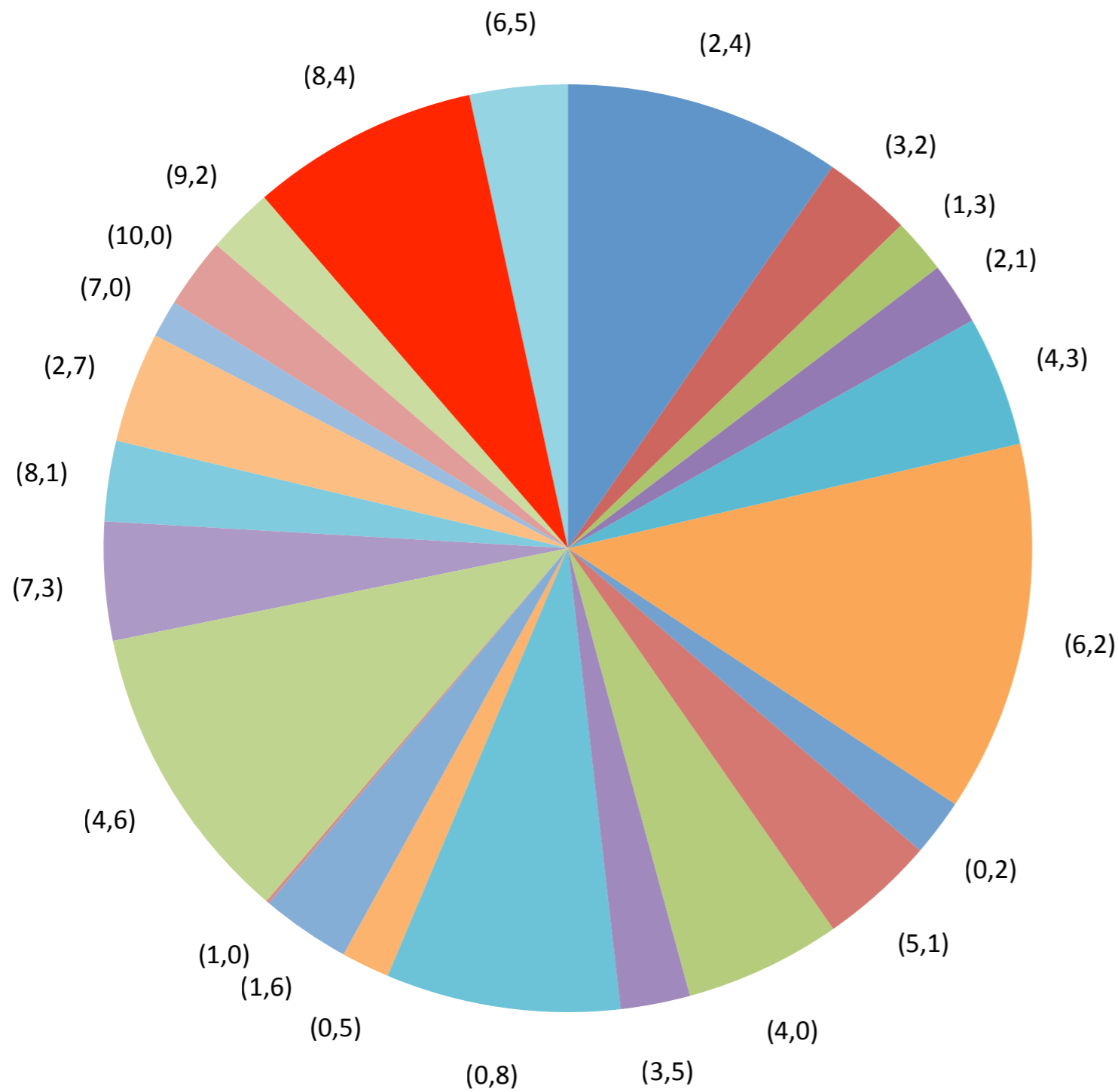


USD realistic interaction
Ground state $J=T=0$
breakdown in SU(3) irreps

Classic Example: ^{24}Mg , 8 nucleons in sd-shell



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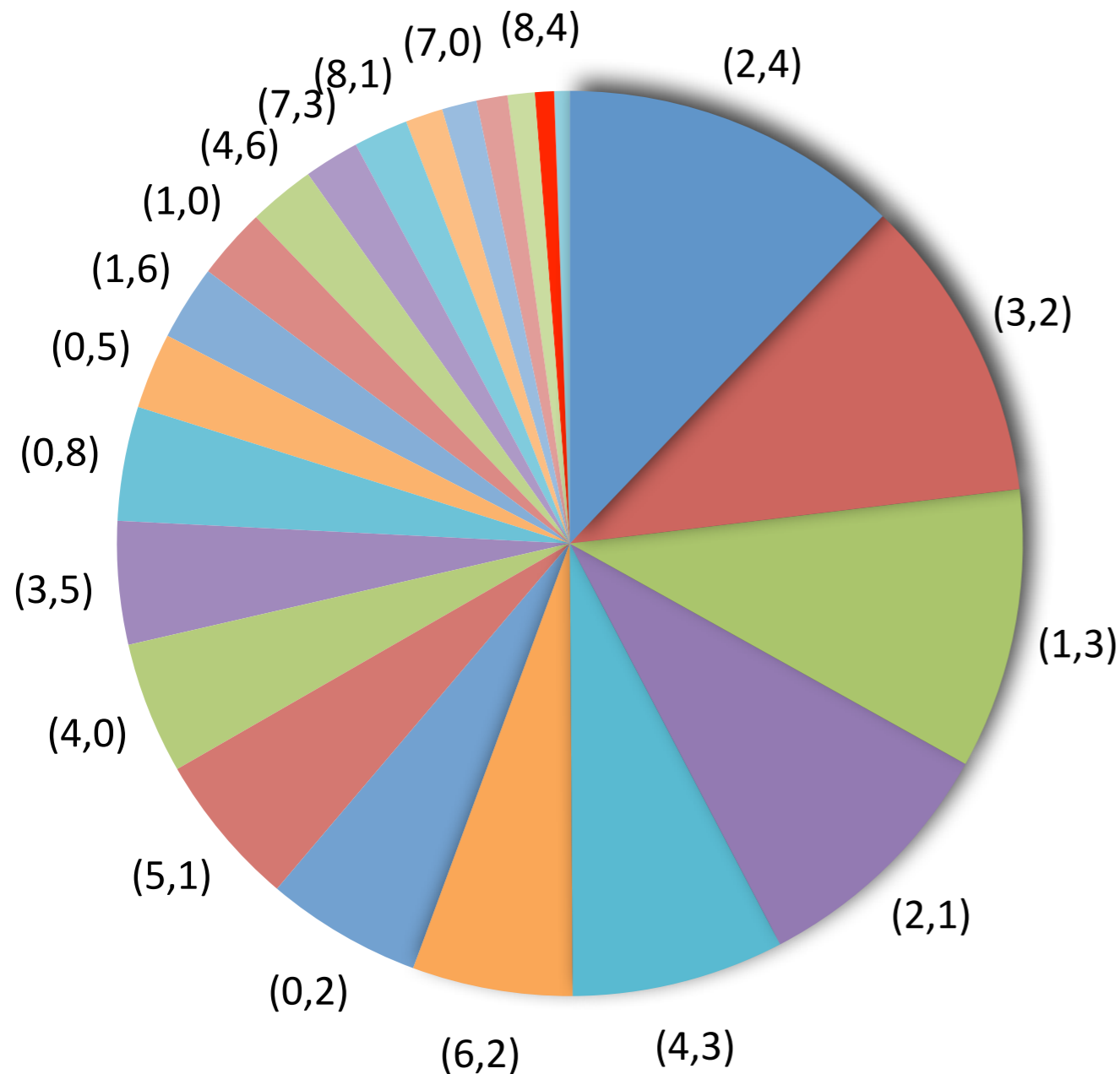


Pairing and spin-orbit
Ground state $J=T=0$
breakdown in SU(3) irreps

Two-body random interaction on an oscillator shell

Two-body random ensemble

- Statistical study of Hamiltonian matrices
- Two-body matrix elements selected at random, but distributions are basis invariant (GOE)
- Important for generic numerical techniques
- Emergence of realistic patterns: dominance of $J=0$ g.s.; rotational bands.



**For ^{24}Mg model space
 $J=T=0$ g.s. happens in 60% of cases**

Weight of SU(3) irreps in all ground states

Traditional Cluster Spectroscopic Characteristics

Radial amplitude function of cluster-daughter system relative to parent

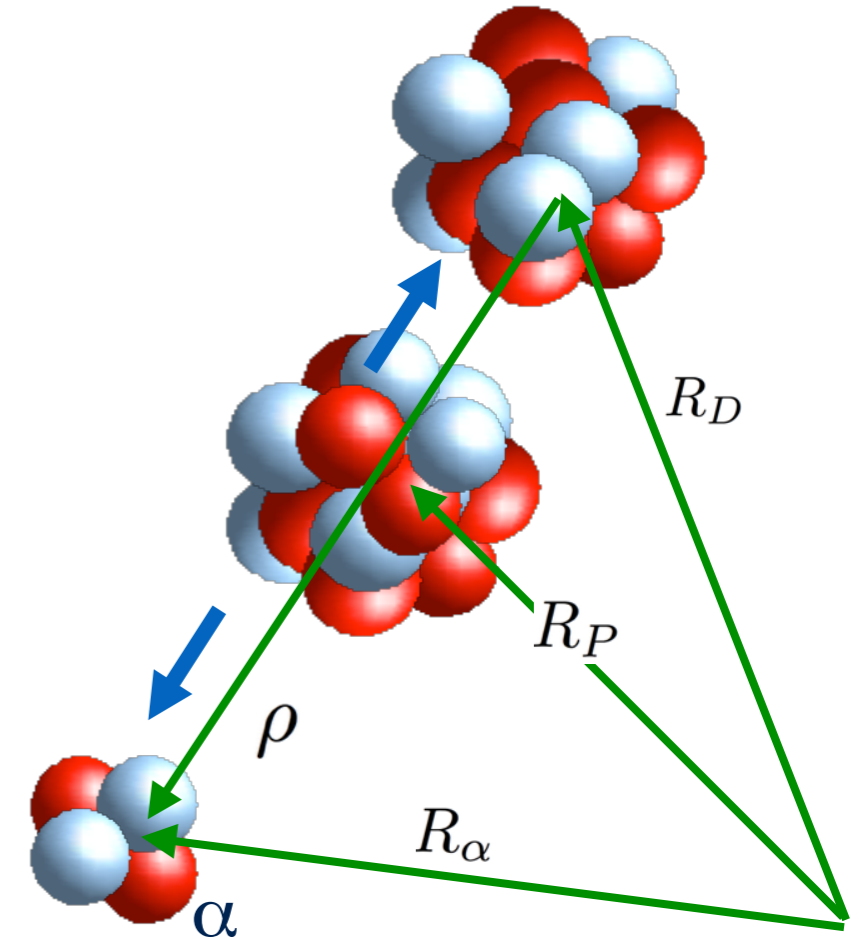
$$\varphi_l(\rho') = \left\langle \hat{A} \left\{ \frac{\delta(\rho - \rho')}{\rho^2} Y_{lm}(\Omega_\rho) \Psi'_\alpha \Psi'_D \right\} \middle| \Psi'_P \right\rangle$$

Expand radial motion in HO wave functions

$$\varphi_l(\rho) = \sum_n \langle \phi_{nl} | \varphi_l \rangle \phi_{nl}(\rho)$$

Expansion amplitudes are translationally invariant overlaps

$$\langle \phi_{nl} | \varphi_l \rangle = \langle \hat{A} \{ \phi_{nlm}(\rho) \Psi'_\alpha \Psi'_D \} | \Psi'_P \rangle$$



Summary of notations used

$\phi_{nl}(\rho)$ radial HO wf.

$\phi_{nlm}(\mathbf{r})$ full single-particle oscillator wf.

$\Phi_{(n,0):L}^\eta$ many-body symmetry-based configuration

$\Psi_{P,D..}$ arbitrary many-body (m-scheme) state or operator

$\Psi'_{P,D..}$ arbitrary translationally invariant many-body state

$\varphi_l(\rho)$ projected radial wf, traditional amplitude

$\psi_l(\rho)$ projected radial function, new amplitude

Translational invariance

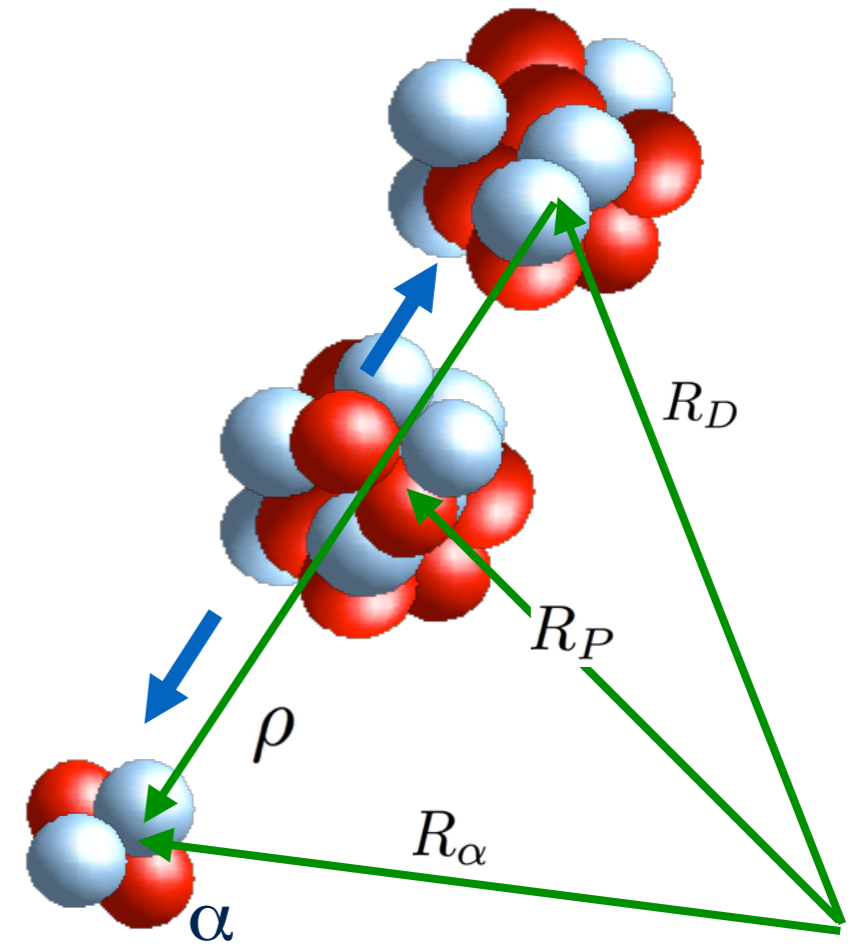
Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D) \Psi'_D \quad \Psi_P = \phi_{000}(\mathbf{R}_P) \Psi'_P$$

Assume alpha structure to be $(0s)^4$

$$|\Psi_\alpha\rangle \equiv \left| (0s)^4 [f] = [4] (0, 0) : L=0, S=0, T=0 \right\rangle$$

$$\Psi_\alpha = \phi_{000}(\mathbf{R}_\alpha) \Psi'_\alpha$$



Factorizing center of mass in overlap integral

$$\langle \Psi_P | \hat{A} \{ \phi_{nlm}(\mathbf{R}_\alpha) \Psi'_\alpha \Psi_D \} \rangle = \langle \Psi'_P | \hat{A} \{ \phi_{nlm}(\boldsymbol{\rho}) \Psi'_\alpha \Psi'_D \} \rangle \times \langle \phi_{000}(\mathbf{R}_P) \phi_{nlm}(\boldsymbol{\rho}) | \phi_{nlm}(\mathbf{R}_\alpha) \phi_{000}(\mathbf{R}_D) \rangle$$

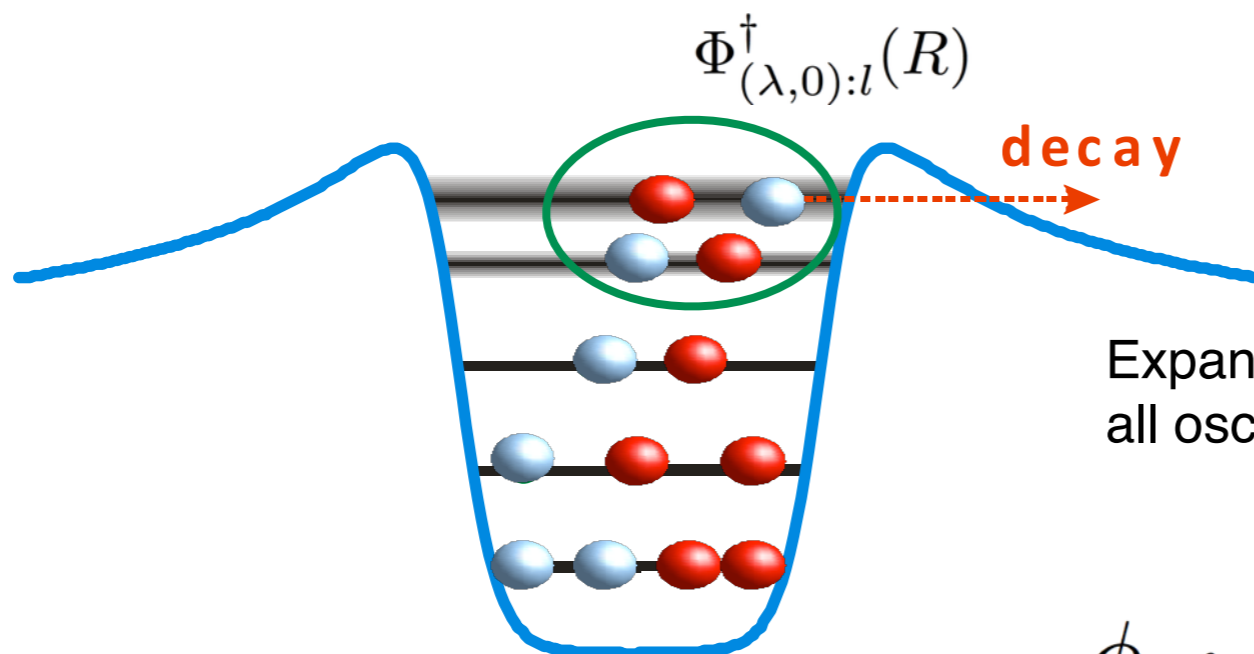
Recoil factor (inverse of Talmi-Moshinsky coefficient)

$$\mathbf{R}_P = \frac{m_D \mathbf{R}_D + m_\alpha \mathbf{R}_\alpha}{m_D + m_\alpha}, \quad \boldsymbol{\rho} = \mathbf{R}_D - \mathbf{R}_\alpha$$

$$\langle \phi_{000}(\mathbf{R}_P) \phi_{nlm}(\boldsymbol{\rho}) | \phi_{nlm}(\mathbf{R}_\alpha) \phi_{000}(\mathbf{R}_D) \rangle \equiv \langle 00, nl : l | \{nl\}_{m_\alpha}, \{00\}_{m_D} : l \rangle$$

$$\mathcal{R}_{nl} \equiv \left(\langle 00, nl : l | \{nl\}_{m_\alpha}, \{00\}_{m_D} : l \rangle \right)^{-1} = (-1)^n \left(\frac{m_D + m_\alpha}{m_D} \right)^{n/2}$$

Cluster coefficients for SU(3) components



Expand SU(3) 4-nucleon structure in intrinsic+ relative
all oscillator quanta of excitation are in relative motion.

$$\phi_{nlm}(\mathbf{R}_\alpha) \Psi'_\alpha = \sum_{\eta} X_{nl}^{\eta} \Phi_{(n,0):lm}^{\eta}$$

$$X_{nl}^{\eta} \equiv \langle \Phi_{(n,0):lm}^{\eta} | \phi_{nlm}(\mathbf{R}_\alpha) \Psi'_\alpha \rangle = \sqrt{\frac{1}{4^n} \frac{n!}{\prod_i (n_i!)^{\alpha_i}} \frac{4!}{\prod_i \alpha_i!}}$$

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).

M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).

O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Traditional “old” spectroscopic factors

$$S_\ell^{(\text{old})} = \langle \varphi_\ell | \varphi_\ell \rangle = \int \rho^2 d\rho |\varphi_\ell(\rho)|^2 = \sum_n |\langle \phi_{nl} | \varphi_\ell \rangle|^2$$

$$\langle \phi_{nl} | \varphi_\ell \rangle = \mathcal{R}_{nl} \sum_\eta X_{nl}^\eta \mathcal{F}_{nl}^\eta$$

Recoil Factor Cluster Coefficient Fractional Parentage Coefficient

Issues with the traditional SF

- Non-orthogonal set of channels (over-complete set of configurations)
- Pauli exclusion principle
- Matching procedure, asymptotic normalization, connection to observables
- No agreement with experiment on absolute scale

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).

M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).

O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

W. Chung, J. van Hienen, B. H. Wildenthal, and C. L. Bennett, Phys. Lett. B 79, 381 (1978)

Orthogonality condition model, new SF

$$\psi_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{-1/2} \varphi_\ell(\rho)$$

Norm kernel and projection on relative motion

$$\langle \mathcal{P}_{\ell m}(\rho) | \Psi'_{\text{ch}} \rangle = \hat{\mathcal{N}}_\ell f_\ell = \int \mathcal{N}_\ell(\rho', \rho) f_\ell(\rho) \rho^2 d\rho.$$

$$\mathcal{N}_\ell(\rho', \rho'') = \langle \mathcal{P}_{\ell m}(\rho') | \mathcal{P}_{\ell m}(\rho'') \rangle = \left\langle \hat{\mathcal{A}} \left\{ \frac{\delta(\rho - \rho')}{\rho^2} Y_{\ell m}(\Omega_\rho) \Psi'_\alpha \Psi'_D \right\} \middle| \hat{\mathcal{A}} \left\{ \frac{\delta(\rho - \rho'')}{\rho^2} Y_{\ell m}(\Omega_\rho) \Psi'_\alpha \Psi'_D \right\} \right\rangle$$

$$S_\ell^{(\text{new})} \equiv \langle \psi_\ell | \psi_\ell \rangle = \int \rho^2 d\rho |\psi_\ell(\rho)|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)

S. G. Kadenskaya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).

R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.

T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).

H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286

Evaluation of norm kernel and SF in oscillator basis

$$\langle \phi_{n'l} | \hat{\mathcal{N}}_l | \phi_{nl} \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n'l m}(\boldsymbol{\rho}) \Psi'_\alpha \Psi'_D \} | \hat{\mathcal{A}} \{ \phi_{nl m}(\boldsymbol{\rho}) \Psi'_\alpha \Psi'_D \} \rangle$$

$$\langle \phi_{n'l} | \hat{\mathcal{N}}_l | \phi_{nl} \rangle = \mathcal{R}_{n'l} \mathcal{R}_{nl} \sum_{\eta \eta'} X_{n'l}^{\eta'} X_{nl}^\eta \langle 0 | \left\{ \hat{\Psi}_{(n',0):l}^{\eta'} \hat{\Psi}_D \right\} \left| \left\{ \hat{\Psi}_{(n,0):l}^\eta \hat{\Psi}_D \right\}^\dagger \right| 0 \rangle$$

4-body operator

We diagonalize norm kernel in oscillator basis $\hat{\mathcal{N}}_l |kl\rangle = N_{kl} |kl\rangle$

New Amplitudes $\langle \phi_{nl} | \psi_l \rangle = \sum_{k n'} \frac{1}{\sqrt{N_{kl}}} \langle \phi_{nl} | kl \rangle \langle kl | \phi_{n'l} \rangle \langle \phi_{n'l} | \varphi_l \rangle$

New SF

$$S_l^{(\text{new})} = \sum_k \frac{1}{N_{kl}} \left| \sum_n \langle kl | \phi_{nl} \rangle \langle \phi_{nl} | \varphi_l \rangle \right|^2$$

Statistical normalization property of new SF

$$\langle \phi_{n'l} | \hat{\mathcal{N}}_l | \phi_{nl} \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n'l m}(\boldsymbol{\rho}) \Psi'_\alpha \Psi'_D \} | \hat{\mathcal{A}} \{ \phi_{nl m}(\boldsymbol{\rho}) \Psi'_\alpha \Psi'_D \} \rangle$$

For an arbitrary set of parent states $\sum_P |\Psi_P\rangle \langle \Psi_P| \equiv \hat{1}$

Thus, norm comes from projector on old amplitudes $\hat{\mathcal{N}}_l = \sum_P |\varphi_l^{(P)}\rangle \langle \varphi_l^{(P)}|$

Sum of all new SF from all parent states to a given final state equals to the number of channels

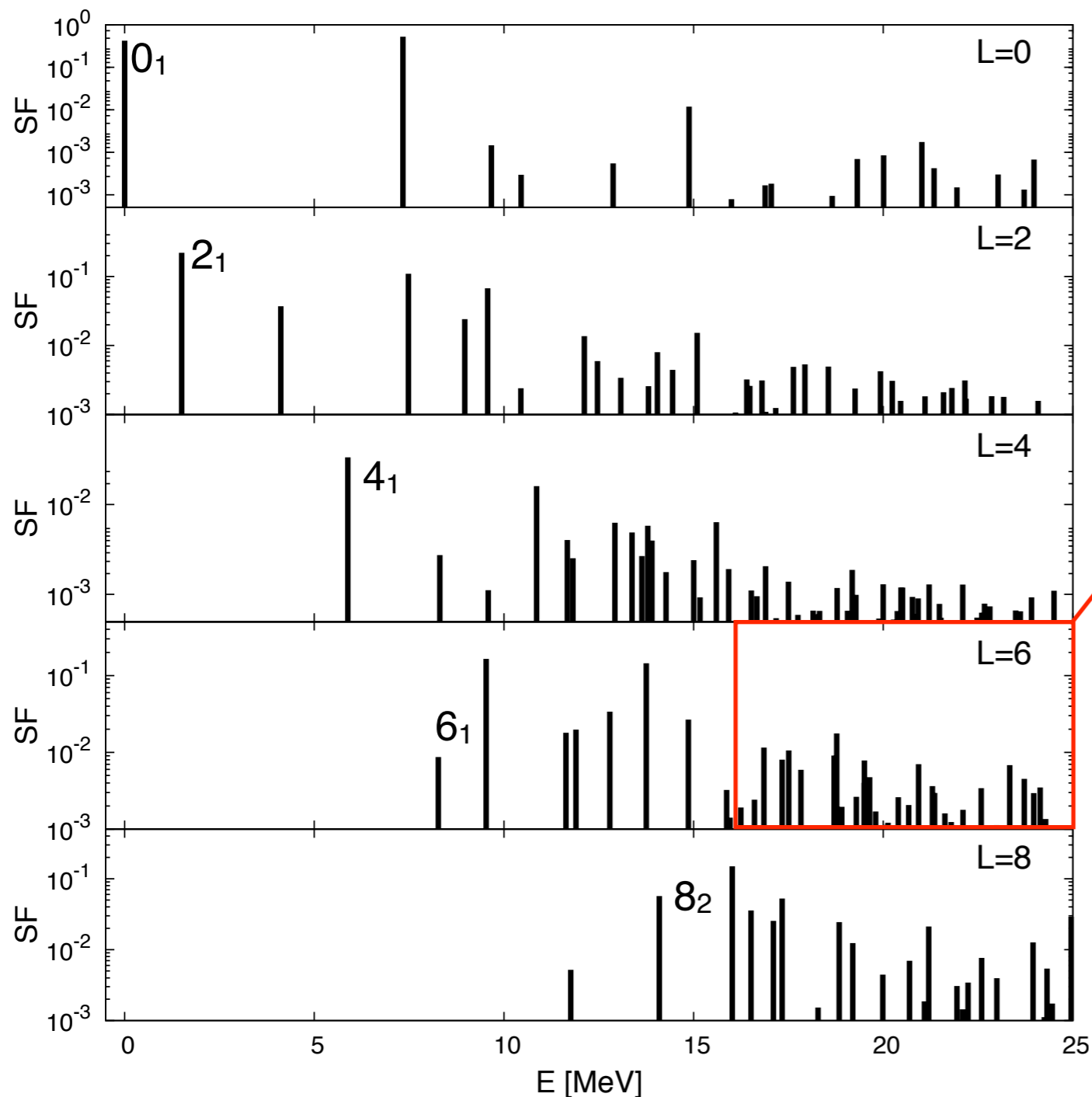
$$\sum_P S_l^{(\text{new})}(P) = \sum_P \langle \varphi_l^{(P)} | \hat{\mathcal{N}}_l^{-1} | \varphi_l^{(P)} \rangle = \sum_n 1$$

For example for one channel:

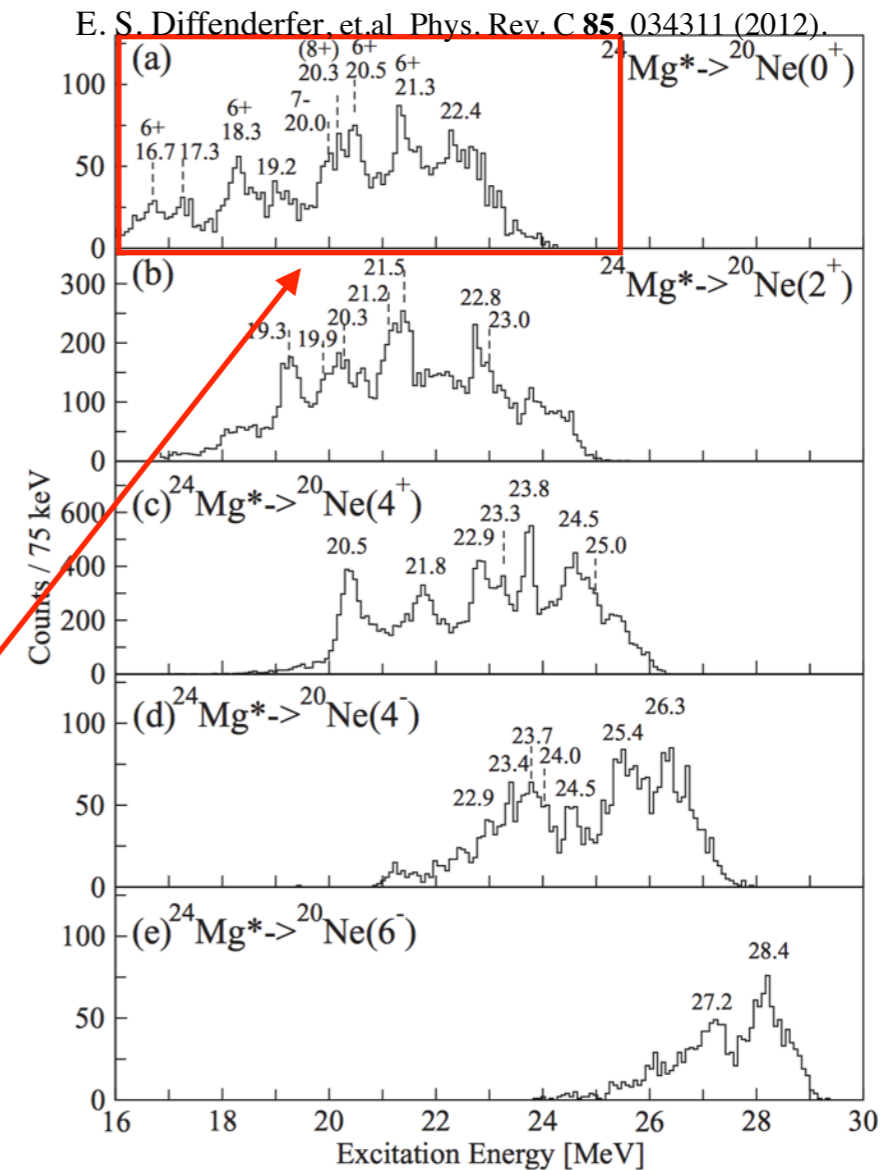
$$S_l^{(\text{new})}(P) = \frac{\mathcal{S}_l^{(\text{old})}(P)}{\sum_{P'} \mathcal{S}_l^{(\text{old})}(P')} = \frac{(\mathcal{F}_{nl}^{(P)})^2}{\sum_{P'} (\mathcal{F}_{nl}^{(P')})^2}$$

Alpha cluster spectroscopic factors in ^{24}Mg

Theoretical calculations in SD shell

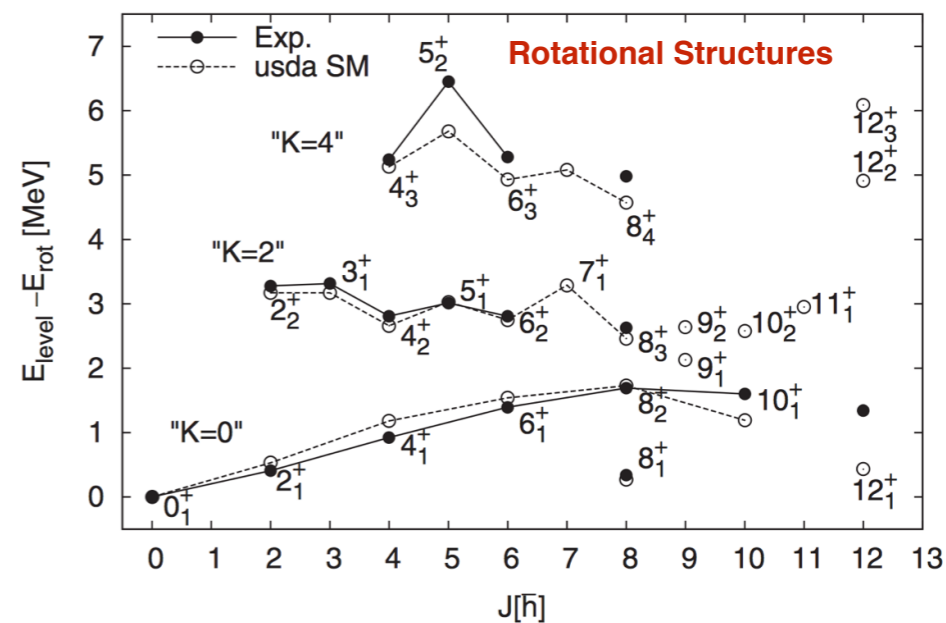


Experimental results



The sd valence space is considered with USDB interaction the operator is

$$|\Phi_{(8,0):L}\rangle = |(sd)^4[4](8,0), : LS = T = 0\rangle$$



Alpha clustering in p-sd shell

p-sd effective hamiltonian

SU(3) configurations

$$|\Phi_{(n,0):\ell}\rangle = |(p)^q(sd)^{4-q} [4](n, 0) : \ell, S=0, T=0\rangle$$

For positive parity

$$|\Phi_{(8,0):L}\rangle = |(sd)^4 [4] (8, 0), : LS = T = 0\rangle$$

$$|\Phi_{(6,0):L}\rangle = |p^2(sd)^4 [4] (6, 0), : LS = T = 0\rangle$$

$$|\Phi_{(4,0):L}\rangle = |p^4 [4] (4, 0), : LS = T = 0\rangle$$

Translational invariance violation in ^{16}O

We use Lawson approach $\beta = 30 \text{ MeV}/\hbar\omega$

$$\langle \bar{\Psi}_P | \beta (H_{\text{c.m.}} - \frac{3}{2} \hbar\omega) | \Psi_P \rangle \approx 0.3 \text{ MeV}$$

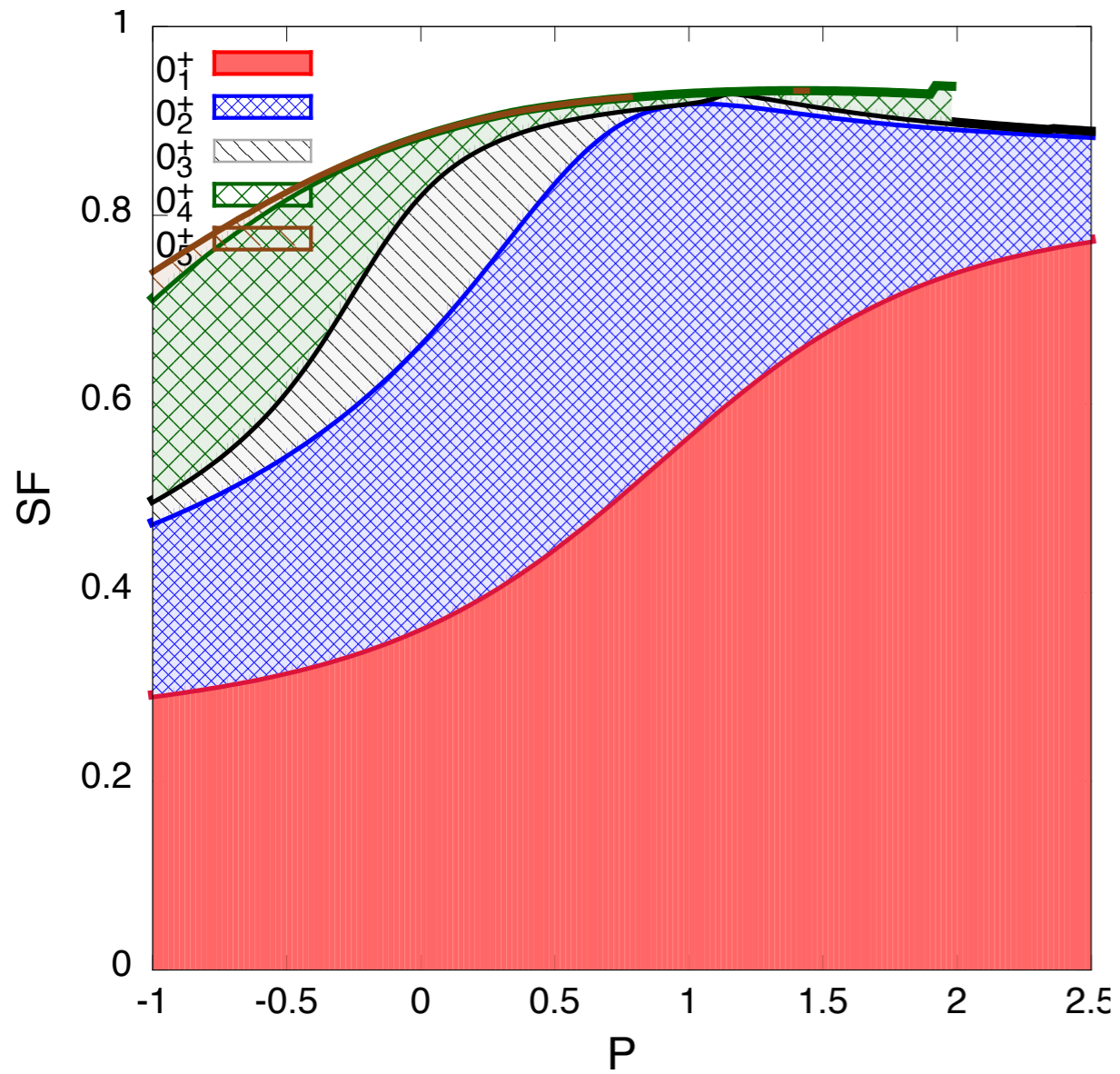
E. K. Warburton and B. A. Brown, Phys. Rev. C 46 (1992) 923

Y. Utsuno and S. Chiba, Phys. Rev. C 83 021301(R) (2011)

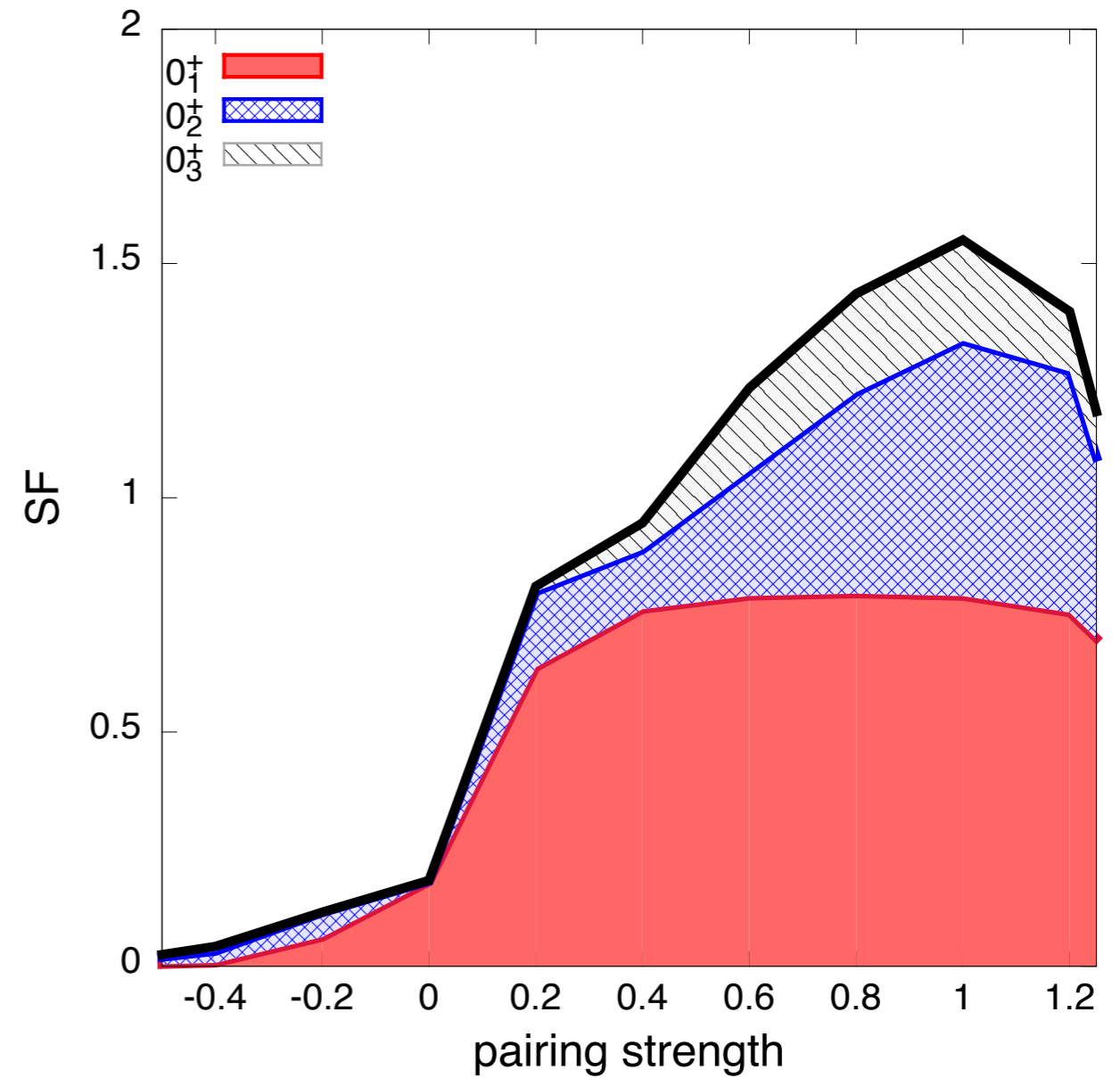
Interplay of clustering and nuclear superconductivity

What happens if all pairing matrix elements are rescaled?

^{24}Mg



^{16}O



Detailed shell model analysis of ^{10}Be ($^{10}\text{Be} \rightarrow ^6\text{He} + \alpha$) and experimental data

J_s^π	S_l	E_x^{th}	Γ_α^{th}	E_x^{exp}	Γ_α^{exp}	$\theta_\alpha^2(r_1)$	$\theta_\alpha^2(r_2)$
0_1^+	0.686	0.000		0			
2_1^+	0.563	3.330		3.368			
0_2^+	0.095	4.244		6.197			
2_2^+	0.049	5.741		5.958			
2_3^+	0.052	6.123		(a)			
1_1^-	0.027	6.290		5.96			
3_1^-	0.098	6.926		7.371		0.42 ^(b,c)	
2_4^+	0.116	7.650	$3 \cdot 10^{-4}$	7.542	$5 \cdot 10^{-4}$	1.1 ^(b,c)	0.19
0_3^+	0.023	8.068	17				
4_1^+	0.049	8.933	4.7				
1_2^-	0.045	9.755	180	10.57			
3_2^-	0.046	9.897	61				
2_5^+	0.027	10.819	50	9.56	141 ^(e)		0.074
2_6^+	0.023	11.295	43				
0_5^+	0.153	11.403	800				
4_2^+	0.370	11.426	180	10.15	185 ^(c)	1.5 ^(c)	0.38
5_1^-	0.148	11.440	150	11.93	200		0.20
1_5^-	0.013	12.650	76				
6_1^+	0.013	13.134	24				
5_2^-	0.128	13.545	250	13.54 ^(c,f)	99	1.0 ^(c)	0.051
2_{10}^+	0.040	13.789	240				
4_3^+	0.011	13.992	20	11.76	121		0.066
4_4^+	0.022	14.233	40				
0_6^+	0.018	14.252	120				
3_7^-	0.014	14.468	77				
5_3^-	0.059	14.992	180				
4_5^+	0.161	15.071	800	15.3(6 ⁻) ^(d)	800 ^(e)		0.16

(a) The existence of this state is suggested by the existence of 8.070 MeV isobaric analog state in ^{10}B , see analogous discussion in Ref. [20];

(b) Widths deduced from the isobaric analog channel $^{10}\text{B} \rightarrow ^6\text{Li}(0^+) + \alpha$ [21, 22];

(c) results from Ref. [22];

(d) results from Ref. [23].

(e) Total width Γ^{tot} .

(f) In Ref. [22] the state was assigned spin-parity 6^+ .

$$r_1 = 4.77 \quad r_2 = 6.0 \text{ fm}$$

[21] A. N. Kuchera et al.: Phys. Rev. C 84 (2011) 054615.

[22] A. N. Kuchera <http://diginole.lib.fsu.edu/etd/8585/>

[23] D. R. Tilley et al.: Nucl. Phys. A 745 (2004) 155.

No-core shell model studies with JISP16 Hamiltonian, clustering in $^{10}\text{Be} \rightarrow ^6\text{He} + \alpha$

	psd	$N_{\text{max}}=0$	$N_{\text{max}}=2$	$N_{\text{max}}=4$	$N_{\text{max}}=4$	Exp
SF	0.686	0.713	0.622	0.609	0.687	0.55 [2]
operators	3	1	7	7	20	
Radius [fm]		3.4	4.0	4.0	4.5	4.7-6

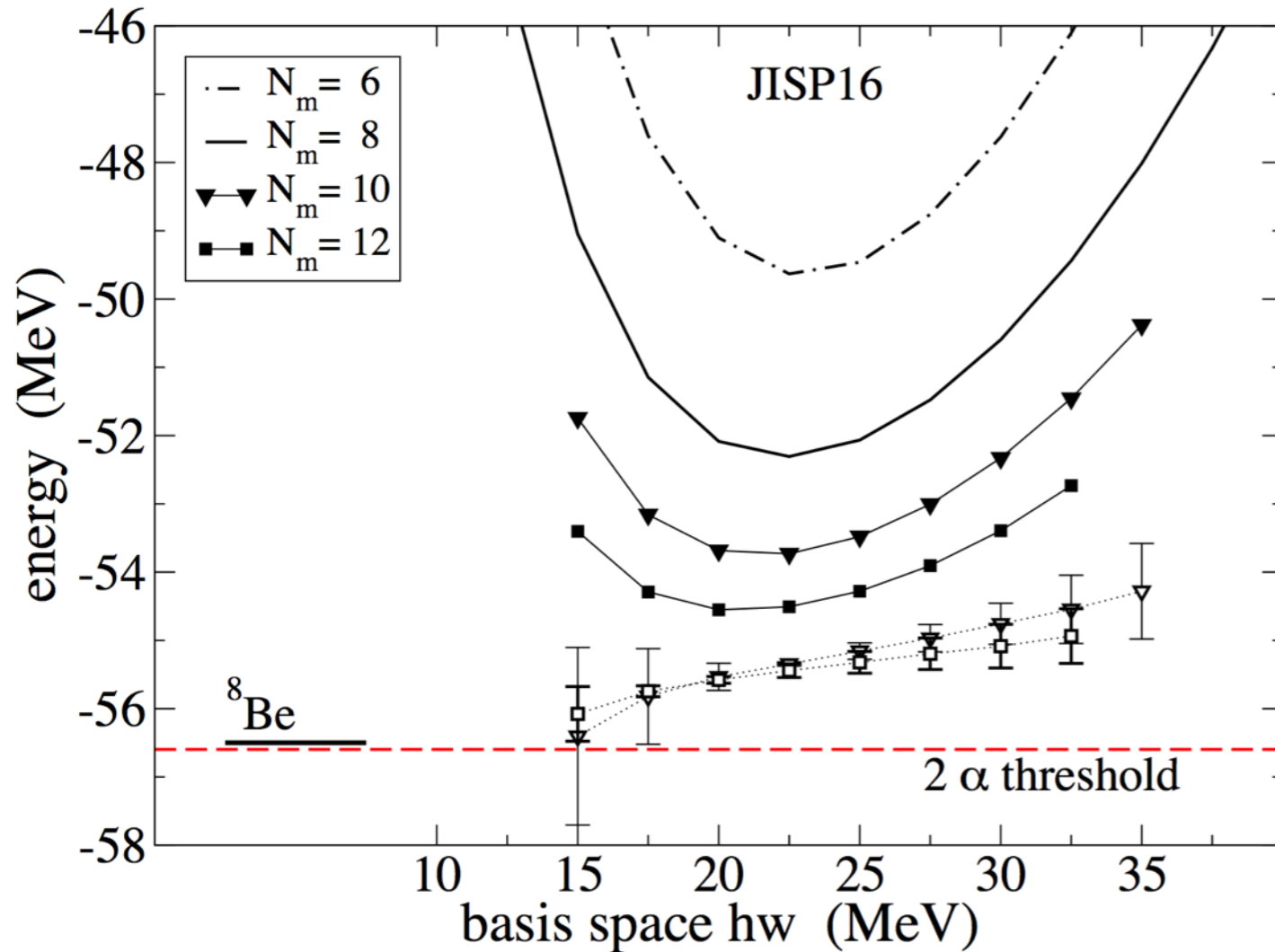
$$r \approx \sqrt{\frac{\hbar}{m\omega} \left(n_{\text{max}} + \frac{3}{2} \right)} \quad \hbar\omega = 20 \text{ MeV}$$

In order to get to $r=6$ fm, $n_{\text{max}}=16$ is needed, relative to core this is $N_{\text{max}}=10, 14$

[1] P. Maris and J. Vary, Int. J. Mod. Phys. E 22, 1330016 (2013); J. Vary private communication

[2] W. Oelert, Few-body Problems, Volume 3; Volume 1985 By E. Hadjimichael, W. Oelert, see page 252

No-core shell model studies with JISP16 Hamiltonian, clustering in ${}^8\text{Be} \rightarrow \alpha + \alpha$



	psd SM	$N_{\max}=0$	$N_{\max}=2$	$N_{\max}=4$
SF	0.993	0.897	0.801	0.781
operators	3	1	7	20
Radius [fm]		3.4	4.0	4.5

Summary, conclusions, and outlook

Results

- New look at cluster structure: from shell model to large scale configuration interaction method.
- Unified picture that includes transitional one-body observables, EM properties, and clustering.
- Demonstrated first results: good agreement with experiment and previous studies.
- Clustering and nuclear Hamiltonian.

Future studies

- Further studies and comparisons with data.
- no-core approach; di-neutron, clusters
- Extensions and modifications
 - Different oscillator frequencies, non-trivial ($0s^4$) state of alpha.
 - Full RGM/GCM approach
 - Non-orthogonal second quantization basis
- Overlapping resonances and physics of configuration interactions in continuum of reaction states.

Acknowledgements:

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Publications:

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M. L. Avila, G. V. Rogachev, V. Z. Goldberg, E. D. Johnson, K. W. Kemper, Y. M. Tchuvil'sky, and A. Volya, Phys. Rev. C 90, 024327 (2014).

FEATURES OF THE NUCLEAR MANY-BODY DYNAMICS: FROM PAIRING TO CLUSTERING

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Atomic nuclei are remarkable mesoscopic systems where single nucleon excitations coexist and interact with numerous collective features such as pairing correlations, clustering, shape dynamics, and superradiance [1–3]. Theories such as BCS, RPA, and algebraic models have been successful in addressing nuclear collectivities especially in the macroscopic limit. However, limited overlap between these theories makes it difficult to grasp the whole complex picture involving interplay between collective and non-collective components. Recently, the nuclear shell model has transformed into a powerful configuration interaction tool allowing for these questions to be studied microscopically.

In this work we advance the reach of the nuclear shell model approach towards clustering and other collective many-body phenomena. We explore the alpha spectroscopic factors of low-lying states, study the distribution of clustering strength, and discuss the structure of effective 4-body operators describing the in-medium alpha dynamics in multi-shell configuration spaces. We address interplay of clustering, pairing, collective particle-hole excitations, and decay processes, exploring both model and realistic examples.

1. A.Volya, V.Zelevinsky // *Phys. At. Nucl.* 2014. V.77. P.969.
2. A.Volya, Y.M.Tchuvil'sky // *J. Phys. Conf. Ser.* 2014. V.569. 012054.
3. A.Volya // *Phys. Rev. Lett.* 2008. V.100. 162501.