Nucleus2015, St.Petersburg, June 30, 2015

Annihilation of $\bar{p}$ and $\bar{n}$ with nucleons and nuclei

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1. Introduction

   Pomeranchuk prediction, $\sigma_{ann}(\bar{p}p)= \sigma_{ann}(\bar{p}n)$ at high energies

2. Confirmation of Pomeranchuk prediction

   & Theoretical analysis of $\sigma_{ann}(\bar{p}p)$ and $\sigma_{ann}(\bar{p}n)=\sigma_{ann}(\bar{n}p)$

3. Application of $\sigma_{ann}(\bar{p}$-nucleon) to $\sigma_{ann}(\bar{p}$-nucleus) in a

   Glauber model at high and low energies

4. Conclusions

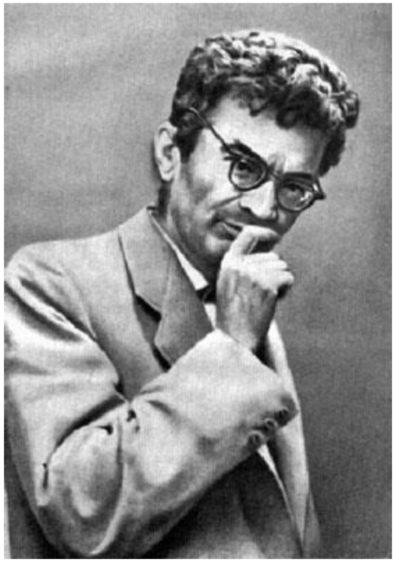
Collaborator: Teck-Ghee Lee

T.G.Lee and C.Y.Wong, Phys. Rev.C89,054601(2014)

Why study the annihilation of $\bar{p}$ and $\bar{n}$ with nucleons and nuclei

- The advent of FAIR at Darmstadt and Antiproton Decelerator at CERN will make $\bar{p}$ available for antimatter studies
- Annihilation of antimatter with matter is a part of the interaction between matter and antimatter
- The physics of the annihilation contains many interesting aspects of nuclear reaction theory

Pomeranchuk Prediction : $\sigma_{ann}(\bar{p}p) = \sigma_{ann}(\bar{p}n)$ at high energies



(1933-1966)

Because of the conservation of isospin

1. $\sigma_{elastic}(\bar{p}p) = \sigma_{elastic}(\bar{p}n)$
2. $\sigma_{tot}(\bar{p}p) = \sigma_{tot}(\bar{p}n)$

   [Now known as Pomeranchuk theorem]

3. Therefore,

   $\sigma_{ann}(\bar{p}p) = \sigma_{ann}(\bar{p}n) = \sigma_{ann}(\bar{n}p)$

   at high energies.

Pomeranchuk JETP 30,423 (1956); JETP 34,725(1958)

Pomeronchuk theorem is confirmed experimentally

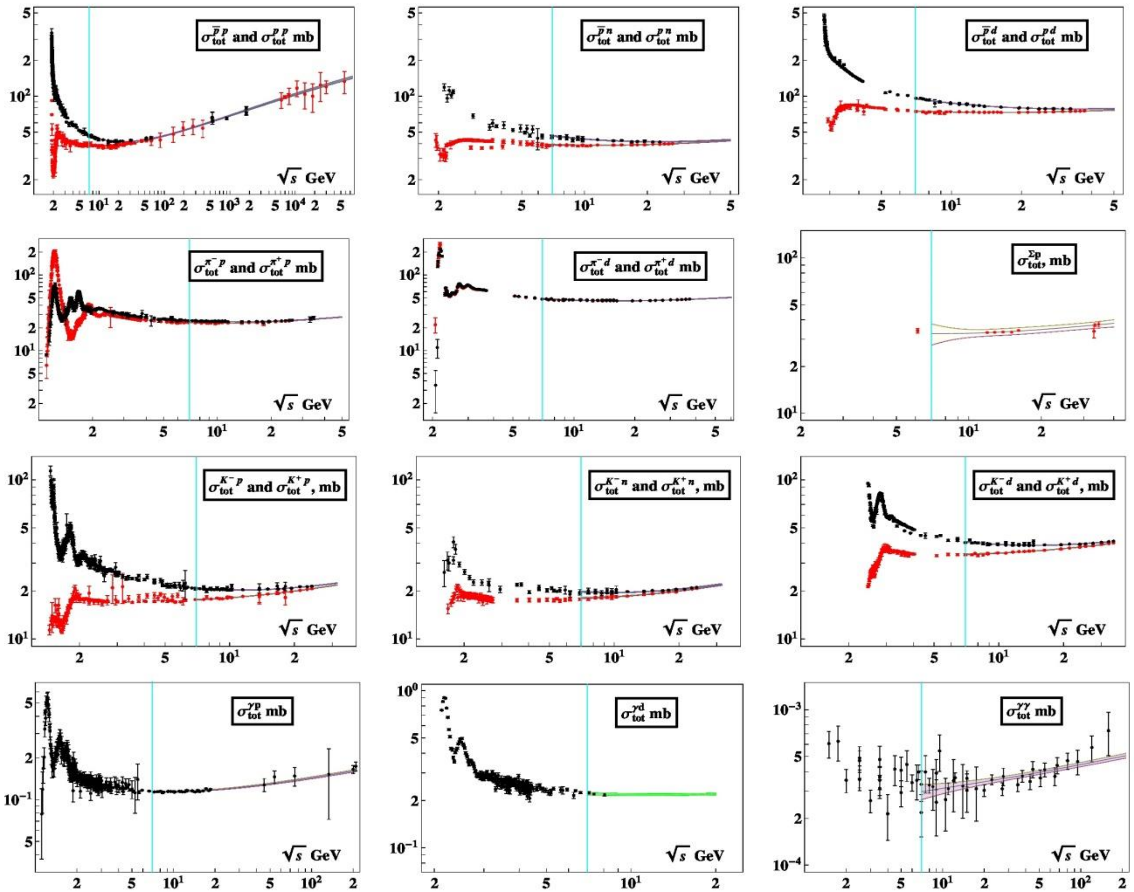


Figure 50.9: Summary of all total collision cross sections jointly fitted with available hadronic ρ parameter data. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html

References

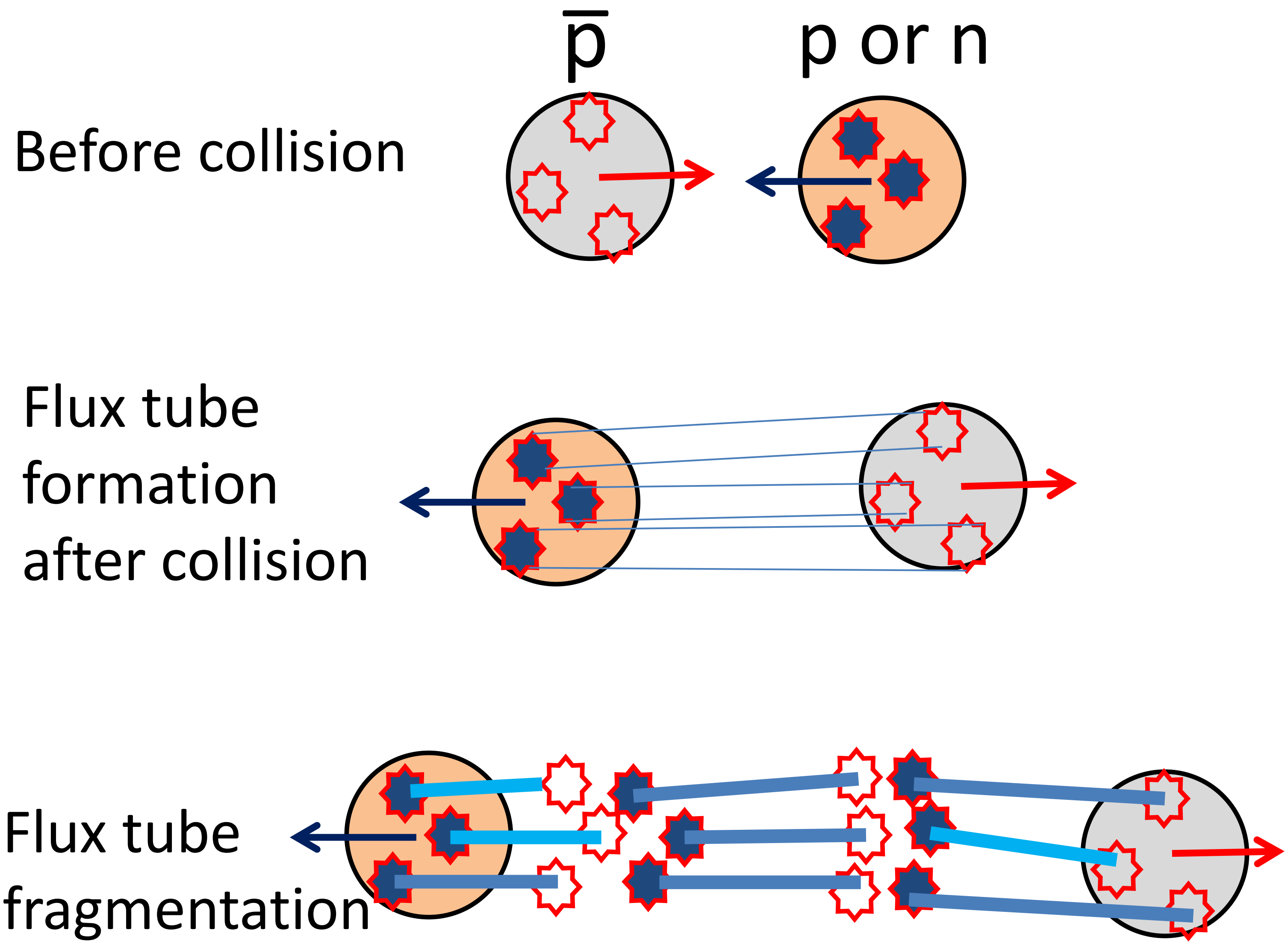
[1] G. Antchev *et al.*, Europhys. Lett. **101**, 21004 (2013).

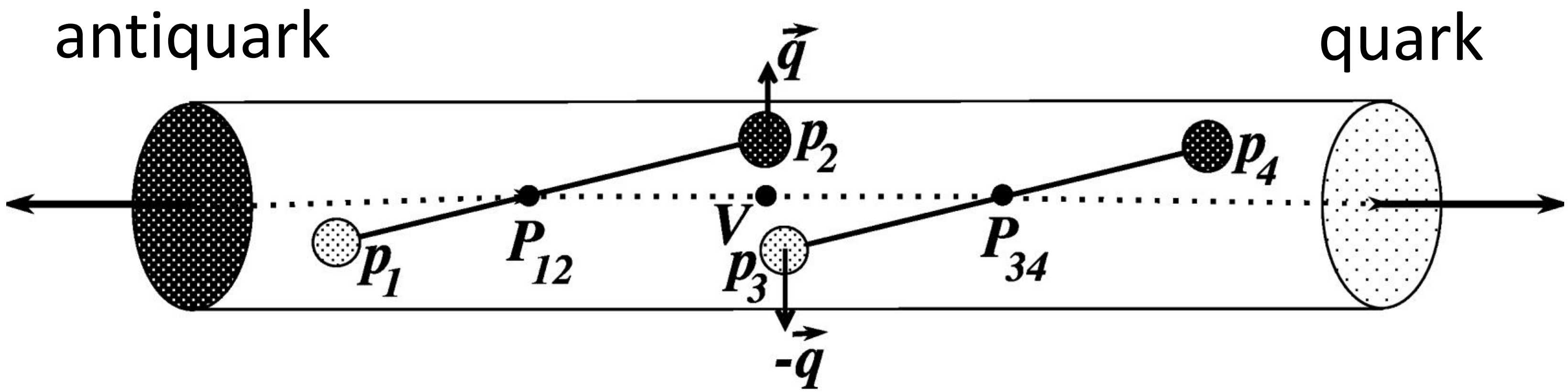
[2] P. Abreu *et al.*, Phys. Rev. Lett. **109**, 062002 (2012).

[3] J.R. Cudell *et al.*: Phys. Rev. **D65**, 074024 (2002); Phys. Rev. Lett. **89**, 201801 (2002).

[4] K. Igi and M. Ishida: Phys. Rev. **D66**, 034023 (2002); Phys. Lett. **B622**, 286 (2005).

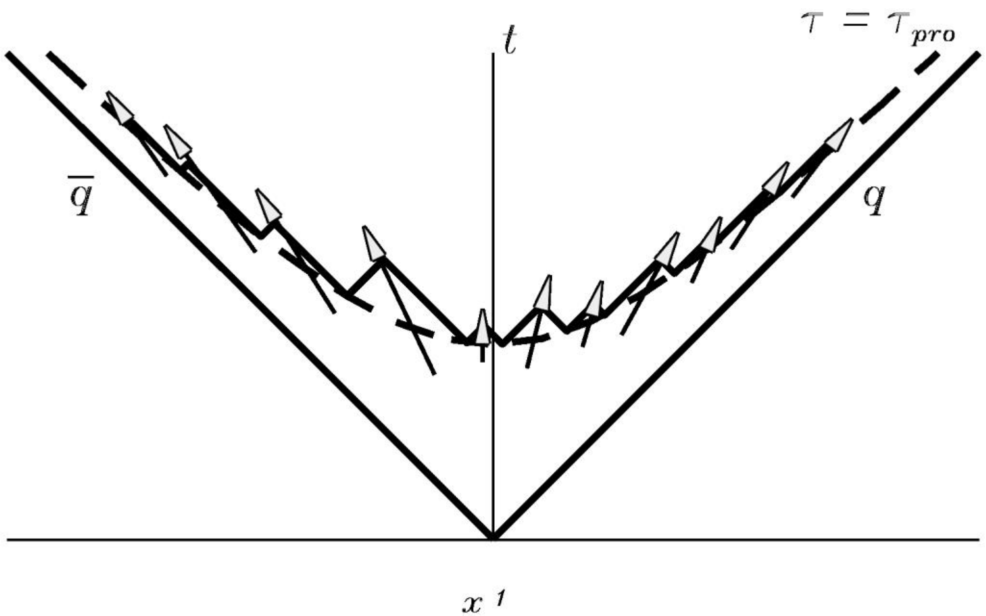
Flux-tube model of nucleon-antinucleon annihilation gives $\sigma_{ann}(\bar{p}p) = \sigma_{ann}(\bar{p}n)\ ) = \sigma_{ann}(\bar{n}p)$





Fragmentation of a color flux tube into pions

Space-time picture of a flux tube fragmentation



See e.g. CYWong, "*Introduction to High-Energy Heavy-Ion Collisions*"

Quark model description of the annihialtion process

1. Flux tube model of annihilation gives

   $$\sigma_{ann}(\bar{p}p) = \sigma_{ann}(\bar{p}n)$$

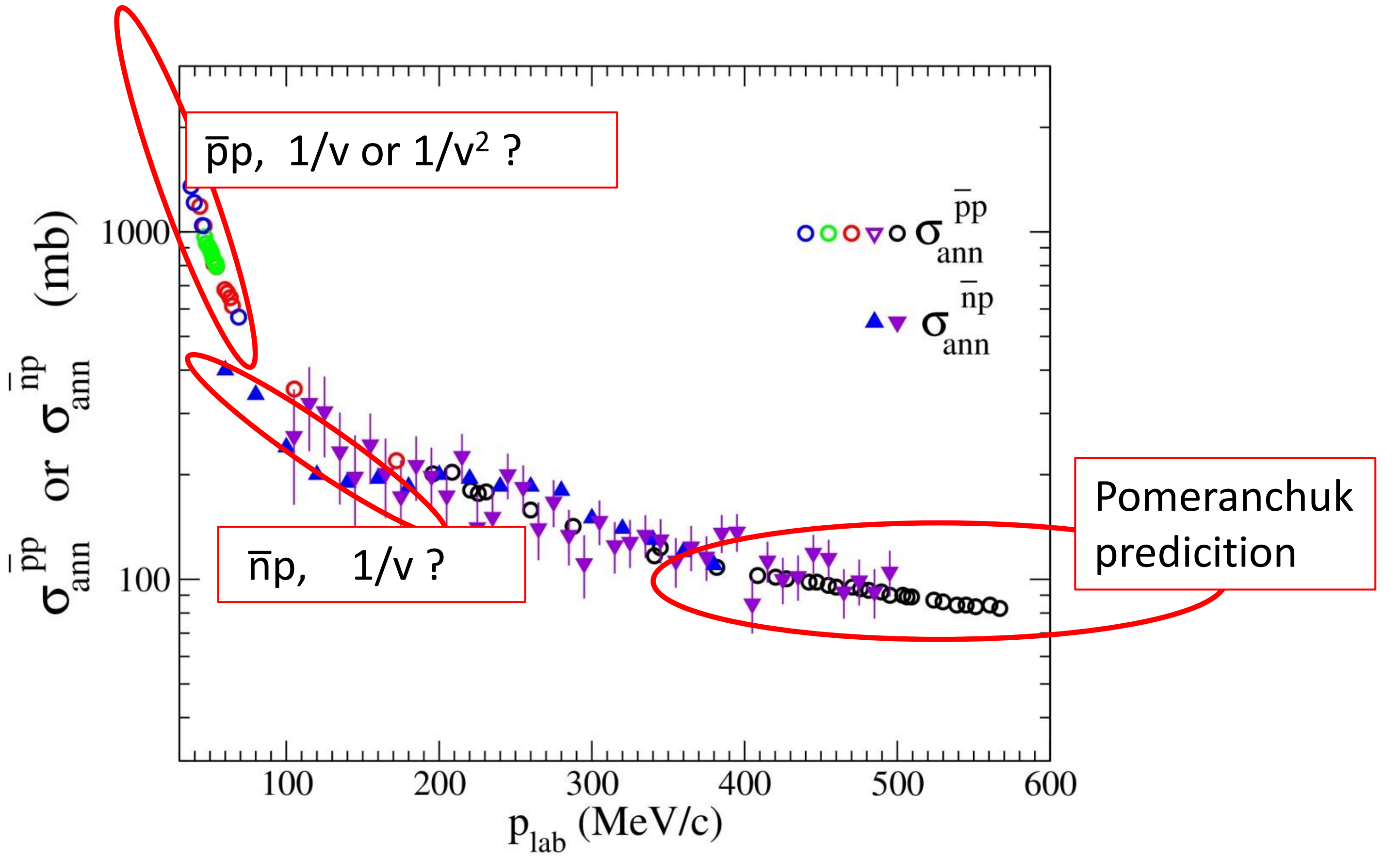
   in agreement with the Pomranchuk prediction

2. The quark model of annihilation by annihilation of a quark and an antiquark of the same flavor gives

   $$\sigma_{ann}(\bar{p}p) = (5/4)\,\sigma_{ann}(\bar{p}n)$$

   in disagreement with the Pomranchuk prediction

Experimental data agree with Pomeranchuk prediction:



1. $\sigma_{ann}(\bar{p}p) \sim \sigma_{ann}(\bar{n}p)$ at high energies
2. What is the behavior of $\sigma_{ann}(\bar{p}p)$ at small p_{lab}
3. What is the behavior of $\sigma_{ann}(\bar{n}p)$ at small p_{lab}

Theoretical analysis of $\sigma_{ann}(\bar{p}p)$ and $\sigma_{ann}(\bar{p}n)$

$$\sigma_{\rm ann}(\bar{p}X) = \frac{\pi}{k^2}\sum_{L=0}^{L_{\max}}(2L+1)T_L(k)G_L(k)$$

Transmission coefficient $T_L(k) = \dfrac{4s_L KR}{\Delta_L^2 + (s_L + KR)^2}$

through a square well $V = -V_0$ and radius R,

$$K = \sqrt{2\mu(E+V_0)},\ s_L = R\left[\frac{g_L(df_L/dr) - f_L(dg_L/dr)}{g_L^2 + f_L^2}\right]_{r=R},\ \Delta_L = R\left[\frac{g_L(df_L/dr) + f_L(dg_L/dr)}{g_L^2 + f_L^2}\right]_{r=R}$$

$$f_L(r) = \left(\frac{\pi kr}{2}\right)^{1/2} J_{L+1/2}(kr), \qquad g_L(r) = \left(\frac{\pi kr}{2}\right)^{1/2} N_{L+1/2}(kr)$$

Blatt and Weiskopf (1952)

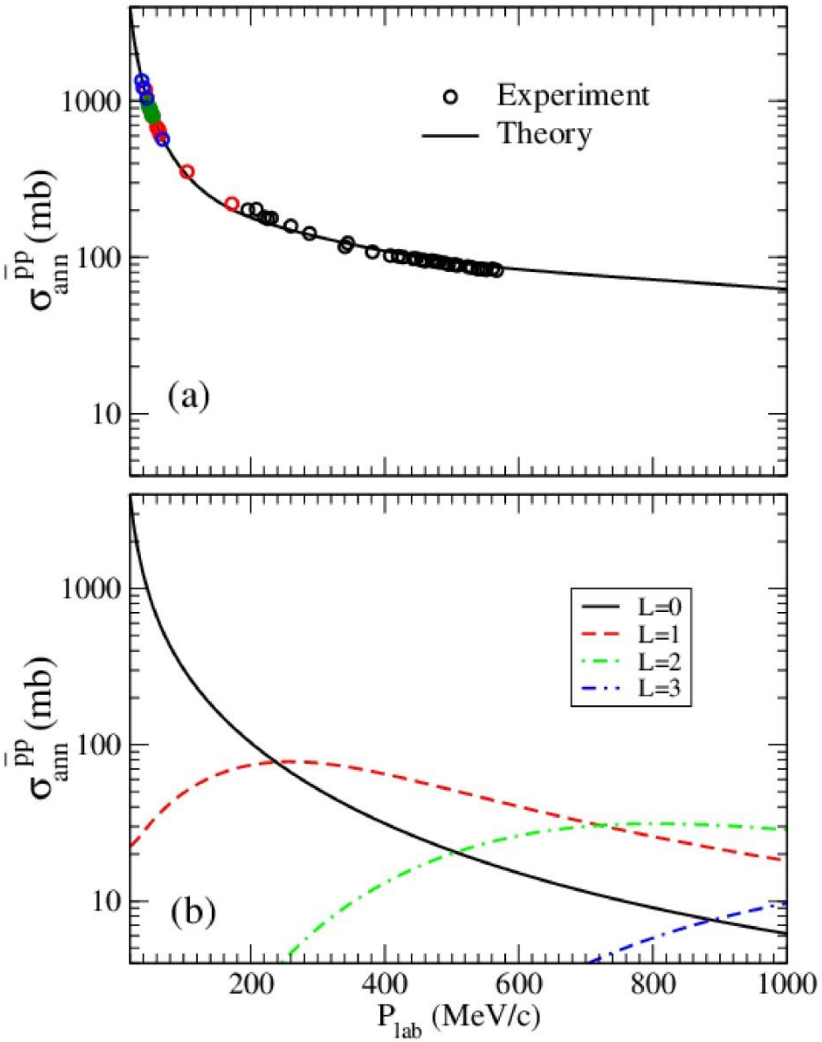
Coulomb Gamow factor, for $\bar{p}p$ initial state interaction in $\sigma_{\rm ann}(\bar{p}p)$,

$$G_L(k) = \frac{(L^2+\xi^2)[(L-1)^2+\xi^2)...(1+\xi^2)}{[L!]^2}\frac{2\pi\xi}{\exp\{2\pi\xi\}-1} \text{ where } \xi = -\frac{\alpha}{v}$$

CY Wong, Phys.Rev.D60,114025(1999)

$G_L(k) = 1$ for $\bar{p}n$, there is no initial state interaction in $\sigma_{\rm ann}(\bar{n}p)$.

Comparison of $\sigma_{ann}(\bar{p}p)$ with theory



Square well
$V=-V_0=-85$ MeV
$R=0.8$ fm

Behavior of $\sigma_{ann}(\bar{p}p)$ and $\sigma_{ann}(\bar{p}n)$ at small p_{lab}

At low energies, the s - wave contribution dominates,

$$\sigma_{\text{ann}}(\bar{p}p) = \frac{\pi}{k^2} T_0 G_0$$

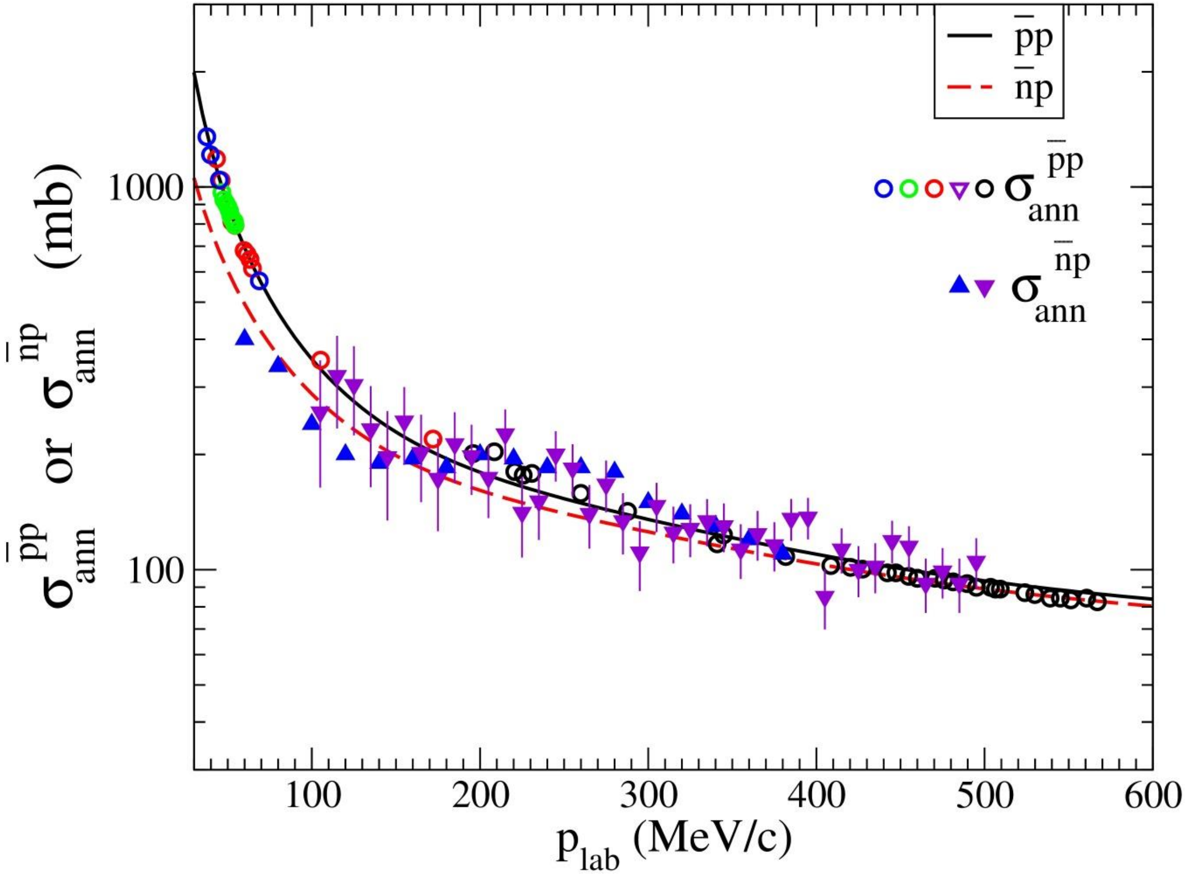
$$= \frac{\pi}{k^2}\left(\frac{4kK}{(k+K)^2}\right)\left(\frac{2\pi\xi}{\exp(2\pi\xi)-1}\right)$$

$$= \pi \underbrace{\left(\frac{4kK}{k^2(k+K)^2}\right)}_{\frac{1}{v}} \underbrace{\left(\frac{2\pi\xi}{\exp(2\pi\xi)-1}\right)}_{\frac{1}{v}} \propto \frac{1}{v^2}$$

Landau & Liftshitz, *Quantum Mechanics*, p. 349

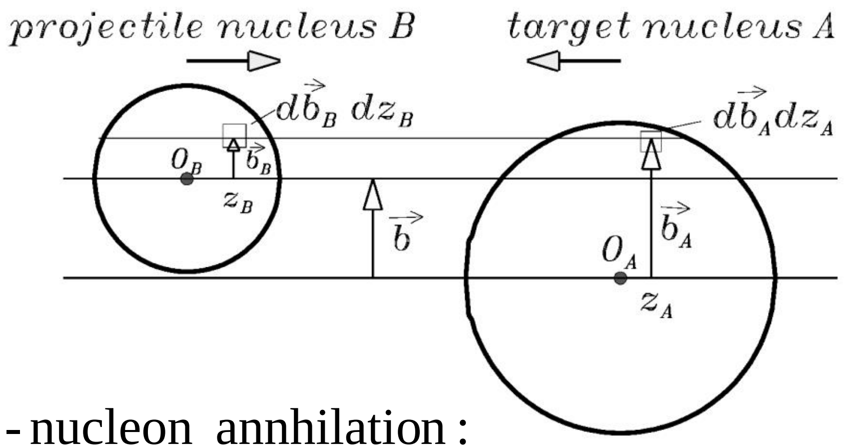
$$\sigma_{\text{ann}}(\bar{n}p) = \frac{\pi}{k^2} T_0 G_0$$

$$= \frac{\pi}{k^2}\left(\frac{4kK}{(k+K)^2}\right) \propto \frac{1}{v}$$



Behavior of $\sigma_{ann}(\bar{p}p)$ and $\sigma_{ann}(\bar{n}p)$ can be understood in terms of T_L and G_L

Glauber model of nucleus-nucleus collision



Probability element of a $\bar{p}$ - nucleon annhilation :

$$dP = \underbrace{\rho_A(\vec{b}_A z_A)dz_A d\vec{b}_A}_{\text{probability of finding a nucleon in } A} \quad \underbrace{\rho_B(\vec{b}_B z_B)dz_B d\vec{b}_B}_{\text{probability of finding a } \bar{p} \text{ in } B} \quad \underbrace{t(\vec{b} - \vec{b}_A + \vec{b}_B)\sigma_{\text{ann}}(\bar{p}p)}_{\text{probability of a } \bar{p}-\text{nucleon annihilation}}$$

$T(b)\sigma_{\text{ann}}(\bar{p}p)$, the probability of a $\bar{p}$ - nucleon annhilation is

$$T(b)\sigma_{\text{ann}}(\bar{p}p) = \int dP = \int \rho_A(\vec{b}_A z_A)dz_A d\vec{b}_A \;\; \rho_B(\vec{b}_B z_B)dz_B d\vec{b}_B \; t(\vec{b} - \vec{b}_A + \vec{b}_B)\sigma_{\text{ann}}(\bar{p}p)$$

$$T(b) = \int \underbrace{\rho_A(\vec{b}_A z_A)dz_A}_{T_A(\vec{b}_A)} d\vec{b}_A \;\; \underbrace{\rho_B(\vec{b}_B z_B)dz_B}_{T_B(\vec{b}_B)} d\vec{b}_B \;\; t(\vec{b} - \vec{b}_A + \vec{b}_B)$$

If $T_A(\vec{b}_A) = \exp(-b_A^2 / 2\beta_A^2)/2\pi\beta_A^2$, $\;t(\vec{b}) = \exp(-b^2/2\beta_{\bar{p}p}^2)/2\pi\beta_{\bar{p}p}^2$,

then $\quad T(\vec{b}) = \exp(-b^2/2\beta^2)/2\pi\beta^2 \; ; \quad \beta^2 = \beta_A^2 + \beta_B^2 + \beta_{\bar{p}p}^2$

See e.g. CYWong, Chapter 12, "*Introduction to High-Energy Heavy-Ion Collisions*"

Glauber model of p̄-nucleus annihilation cross sections at high energies

At high energies, $\sigma_{\text{ann}}(\bar{p}p) = \sigma_{\text{ann}}(\bar{p}n)$,

$\sigma_{\text{ann}}(\bar{p}A) = \int d^2b \left\{1 - [1 - T(b)\sigma_{\text{ann}}(\bar{p}p)]^A\right\}$

For light nuclei, the thickness function has a Gaussian shape

$$T(b) = \frac{1}{2\pi\beta^2}\exp\left\{-\frac{b^2}{2\beta^2}\right\}$$

$$\beta^2 = \beta_{\bar{p}}^2 + \beta_A^2 + \sigma_{\text{ann}}(\bar{p}p)/2\pi$$

Then the corss section can be obtained analytically,

$$\sigma_{\text{ann}}(\bar{p}A) = 2\pi\beta^2 \sum_{n=1}^{A} \frac{1-(1-f)^n}{n},$$

$$f = \frac{\sigma_{\text{ann}}(\bar{p}p)}{2\pi\beta^2}$$

For $f \ll 1$ and small values of A,

$$\sigma_{\text{ann}}(\bar{p}A) \approx A\sigma_{\text{ann}}(\bar{p}p)$$

Glauber model of p̄-nucleus annihilation cross sections at high energies (contd)

For heavy nuclei, the thickness function has a sharp cutoff distribution

$$T(b) = \frac{3\sqrt{R^2 - b^2}}{2\pi R^3}\,\theta(R-b)$$

$$R = R_{\bar{p}} + R_A + R_{\bar{p}p}$$

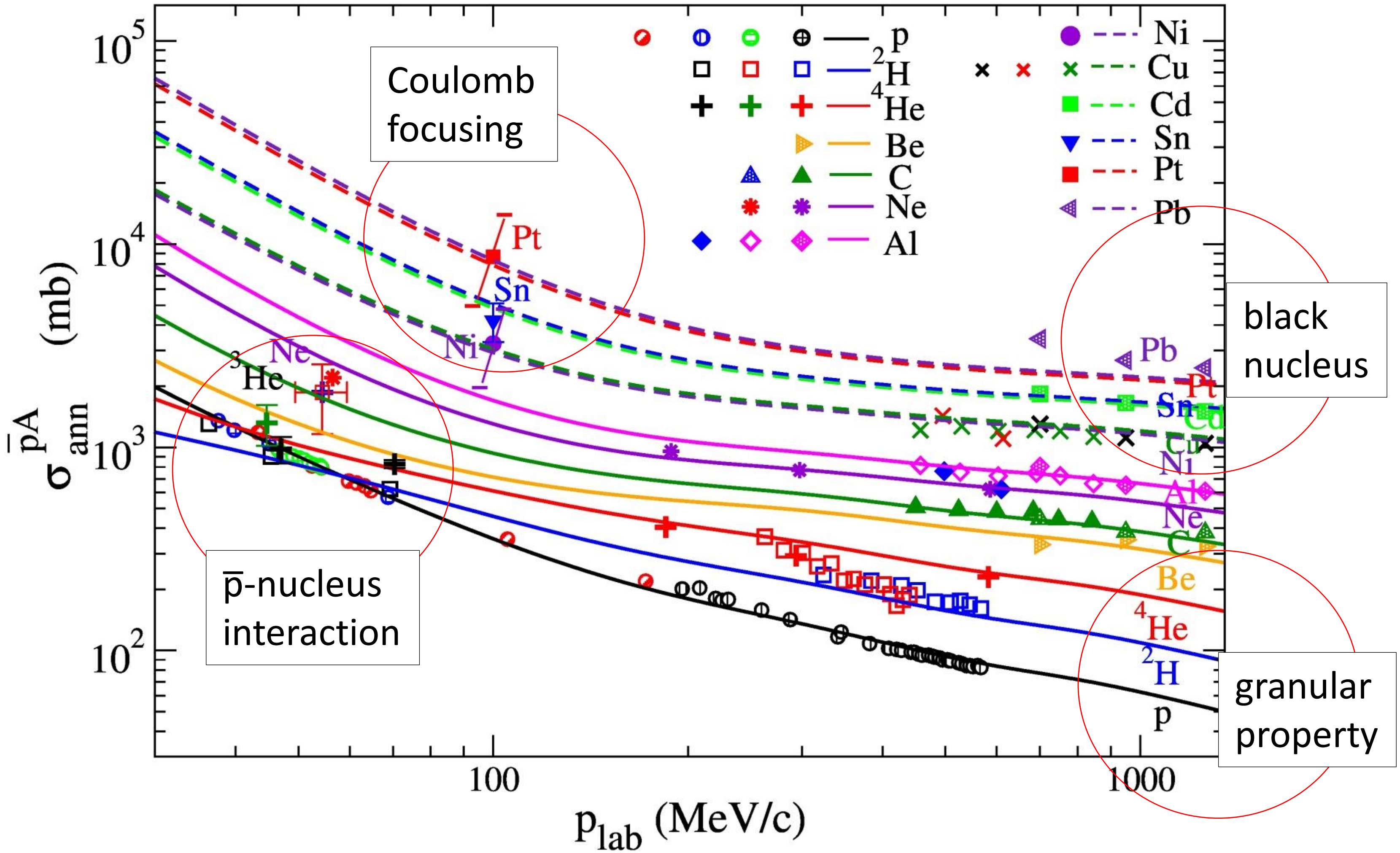
Then the corss section can be obtained analytically,

$$\sigma_{\text{ann}}(\bar{p}A) = \pi R^2 \left(1 + \frac{2}{F^2}\left[\frac{1-(1-F)^{A+2}}{A+2} - \frac{1-(1-F)^{A+1}}{A+1} \right] \right),$$

$$F = \frac{\sigma_{\text{ann}}(\bar{p}p)}{2\pi R^2/3}$$

For finite value of F and large values of A, then $(1-F)^{A+1} \to 0$,

$$\sigma_{\text{ann}}(\bar{p}A) \approx \pi R^2 \text{ for large } A \text{ at high energies.}$$



Coulomb focusing
black nucleus
p̄-nucleus interaction
granular property
p
^{2}H
^{4}He
Be
C
Ne
Al
Ni
Cu
Cd
Sn
Pt
Pb
^{3}He
$\sigma_{ann}^{\bar{p}A}$ (mb)
p_{lab} (MeV/c)
10^5
10^4
10^3
10^2
100
1000

For lower energies, we need to modify the Glauber model

Three effects:

(1) At low energies, $\sigma_{\text{ann}}(\bar{p}p) \neq \sigma_{\text{ann}}(\bar{p}n)$,

$$\sigma_{\text{ann}}(\bar{p}A) = \int d^2b \left\{1 - [1 - T_N(b)\sigma_{\text{ann}}(\bar{p}n)]^N [1 - T_Z(b)\sigma_{\text{ann}}(\bar{p}p)]^Z \right\}$$

(2) Coulomb initial-state interaction between $\bar{p}$ and Z proton charges

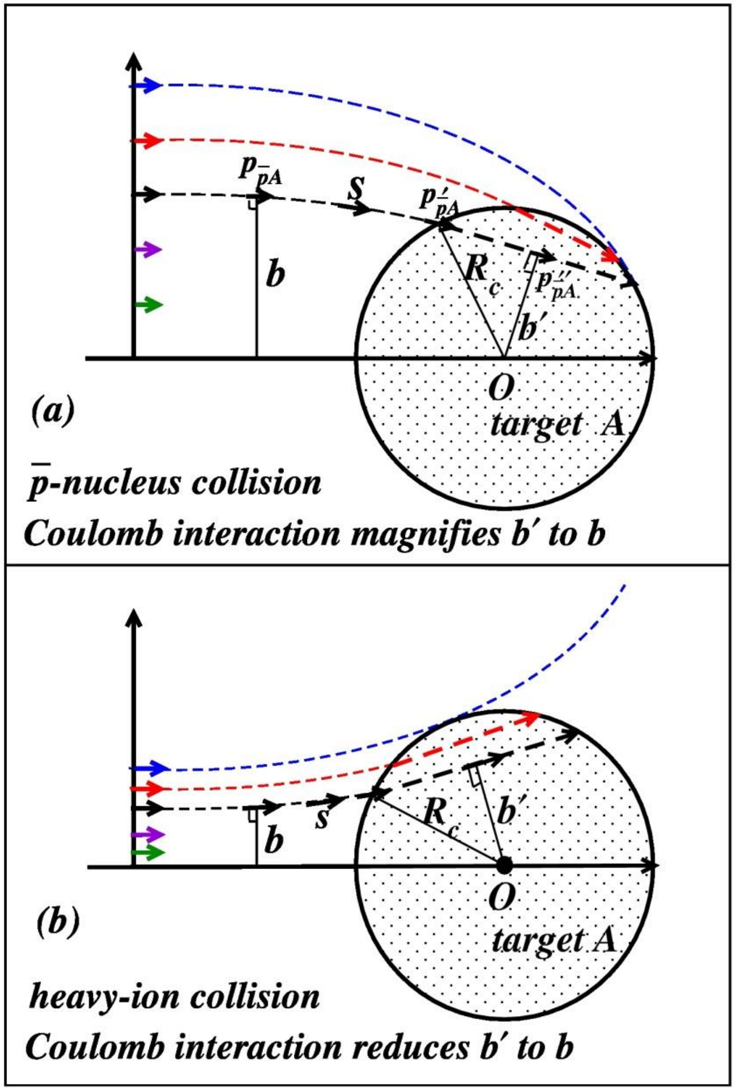
$$\sigma_{\text{ann}}(\bar{p}A) \to \Sigma_{\text{ann}}(\bar{p}A) = (1 - V_c(R)/E)\sigma_{\text{ann}}(\bar{p}A),$$

$$V_c(R) = -1 \times Ze^2/R$$

(3) Mean field nuclear interaction between $\bar{p}$ and A nucleons

$\Rightarrow$ momentum of $\bar{p}$ inside a nucleus is greater than in free space

Coulomb and nuclear interaction effects



$$E = \frac{p_{\bar{p}A}^2}{2\mu} = \frac{p_{\bar{p}A}'^2}{2\mu} + V_C(R_C)$$

$$p_{\bar{p}A}' = p_{\bar{p}A}\sqrt{1 - \frac{V_C(R_C)}{E}}$$

Conservation of angular momentum implies

$$p_{\bar{p}A}'\, b' = p_{\bar{p}A} b,$$

The initial-state Coulomb interaction modifies the cross section to

$$\Sigma_{\rm ann}^{\bar{p}A}(p_{\bar{p}A}) = \pi\, b^2 = \pi\, b'^2 \frac{p_{\bar{p}A}'^2}{p_{\bar{p}A}^2}$$

$$= \sigma_{\rm ann}^{\bar{p}A}(p_{\bar{p}A}')\left\{1 - \frac{V_C(R_C)}{E}\right\}$$

There are Coulomb and nuclear interaction inside the nucleus

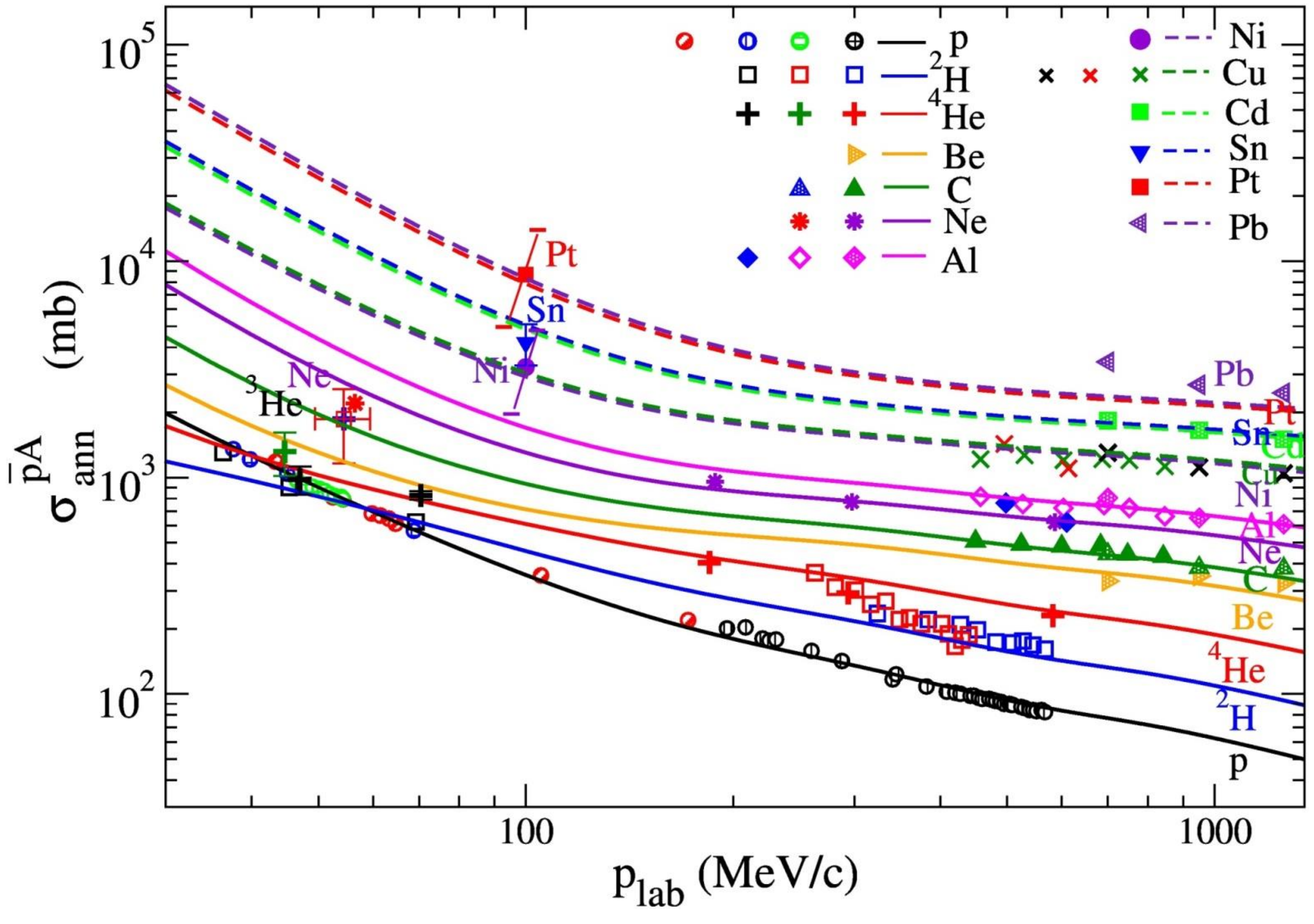
$$E = \frac{p_{\bar{p}A}^2}{2\mu} = \frac{p_{\bar{p}A}''^2}{2\mu} + V_C(r) + V_N(r)$$

The momentum of $\bar{p}$ in the nuclear interior is

$$p_{\bar{p}A}'' \approx p_{\bar{p}A}\sqrt{1 - \frac{\langle V_C(r)\rangle + \langle V_N(r)\rangle}{E}}$$

The final cross section is $\Sigma_{\rm ann}^{\bar{p}A}(p_{\bar{p}A}) \approx \sigma_{\rm ann}^{\bar{p}A}(p_{\bar{p}A}'')\left\{1 - \frac{V_C(R_C)}{E}\right\}$

Inclusion of these three effects give a good description of $\sigma_{ann}(\bar{p}A)$



Curves are theoretical Results.

Uniform density T(b)

Gaussian density T(b)

Conclusions

1. Pomeranchuk prediction on $\sigma_{ann}(\bar{p}n)=\sigma_{ann}(\bar{p}n)=\sigma_{ann}(\bar{n}p)$ is supported by experimental data
2. Flux tube model of annihilation is in agreement with data and the Pomeranchuk prediction
3. The application of Glauber model at high energies gives a good description of $\bar{p}$-nucleus annihilation data at high energies
4. The extension of the Glauber model to include Coulomb and nuclear interactions lead to a qualitative description of the world data of $\bar{p}$-nucleus interaction from low to high energies

ANNIHILATION OF ANTINUCLEONS WITH NUCLEONS AND NUCLEI

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The annihilation of $\bar{p}$ and $\bar{n}$ with nucleons and nuclei are important properties in the interaction of matter with antimatter. Pomeranchuk suggested that $\sigma_{ann}(\bar{n}-p)$ is equal to $\sigma_{ann}(\bar{p}-p)$ at high energies [1]. On the other hand, from the naive quark model of counting the number of quark-antiquark pairs of the same flavor, $\sigma_{ann}(\bar{n}-p)$ would be (4/5) $\sigma_{ann}(\bar{p}-p)$. We have re-examined the nucleon-antinucleon annihilation cross sections, taking into account the nuclear and $p-\bar{p}$ Coulomb interactions, and found that the Pomeranchuk's suggested equality at high energies appears to be a reasonable concept, as shown in Fig. 1. On the basis of the elementary $\sigma_{ann}(\bar{n}-p)$ and $\sigma_{ann}(\bar{p}-p)$ cross sections as input, we extended the Glauber model for high-energy collisions [2] to both high and low energies, after taking into account effects of the nuclear interaction, the Coulomb interaction, and the change of the antinucleon momentum inside a nucleus [3]. The extended Glauber model captures the main features of the experimental antinucleon-nucleus annihilation cross sections [3]. At high energies, they exhibit the granular property for the lightest nuclei and the black-disk limit for the heavy nuclei. At low energies, they display the effect of antinucleon momentum increase due to the nuclear interaction for light nuclei, and the effect of focusing due to the attractive Coulomb interaction for antiproton annihilation for heavy nuclei, as shown in Fig. 2 for $\bar{p}$-nucleus annihilation cross sections.

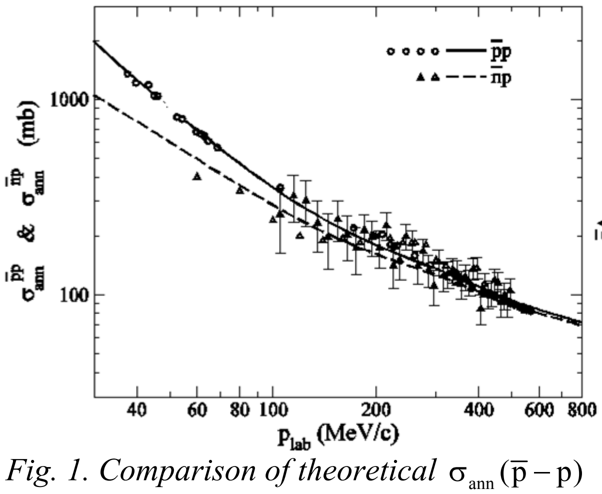

Fig. 1. Comparison of theoretical $\sigma_{ann}(\bar{p}-p)$ and $\sigma_{ann}(\bar{n}-p)$ curves with data.

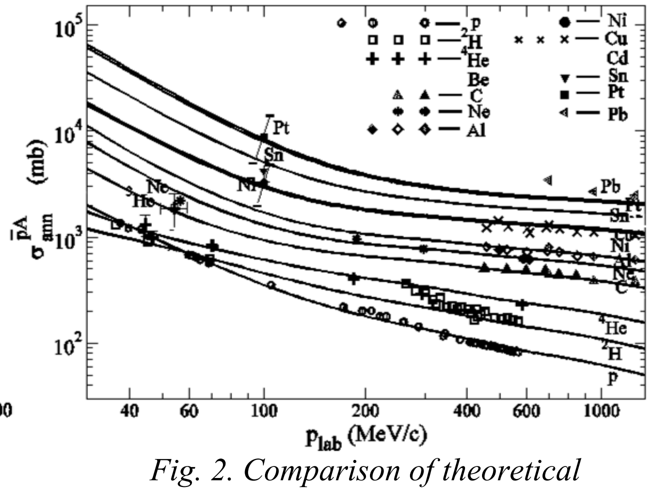

Fig. 2. Comparison of theoretical $\sigma_{ann}(\bar{p}-A)$ curves with data.

1. I.Ya.Pomeranchuk // JETP. 1956. V.423.
2. C.Y.Wong // Phys. Rev. D. 1984. V.961.
3. T.G.Lee, C.Y.Wong // Phys. Rev. C. 2014. V.89. 054601.