

# PYGMY AND GIANT DIPOLE RESONANCES IN NEUTRON-RICH NUCLEI

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Seminar

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# Outline

## Introduction

### Part I: Main ingredients of the model

- Realization of QRPA
- Phonon-phonon coupling

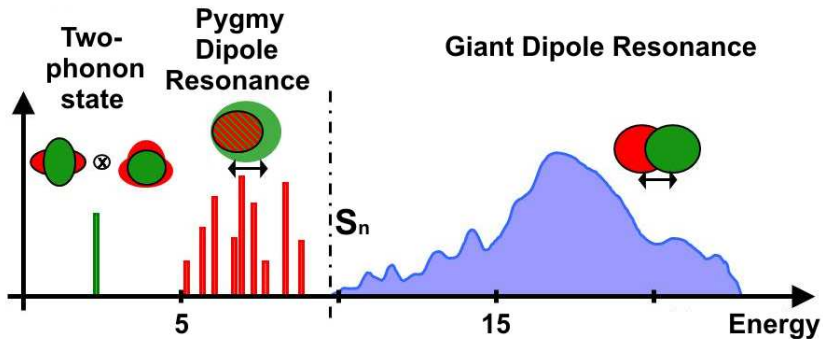
### Part II: Results and discussion

- Details of calculations
- Properties of the PDR and GDR
- Estimation of the symmetry energy

## Conclusions

# Introduction

$E1$  strength in (spherical) atomic nuclei



Courtesy: N. Pietralla

N. Arsenyev



# Relevance of the PDR

1. The PDR might play an important role in nuclear astrophysics. For example, the occurrence of the PDR could have a pronounced effect on neutron-capture rates in the r-process nucleosynthesis, and consequently on the calculated elemental abundance distribution.

*S. Goriely, Phys. Lett. B436, 10 (1998).*

2. The study of the pygmy  $E1$  strength is expected to provide information on the symmetry energy term of the nuclear equation of state. This information is very relevant for the modeling of neutron stars.

*C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. 86, 5647 (2001).*

3. New type of nuclear excitation: these resonances are the low-energy tail of the GDR, or if they represent a different type of excitation, or if they are generated by single-particle excitations related to the specific shell structure of nuclei with neutron excess.

*N. Paar, D. Vretenar, E. Khan, G. Colò, Rep. Prog. Phys. 70, 691 (2007).*

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## MAIN INGREDIENTS OF THE MODEL



# Realization of QRPA

The starting point of the method is [the HF-BCS calculations](#) of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

*J. P. Blaizot and D. Gogny, Nucl. Phys. A284, 429 (1977).*

The residual interaction in the particle-hole channel  $V_{res}^{ph}$  and in the particle-particle channel  $V_{res}^{pp}$  can be obtained as the second derivative of the energy density functional  $\mathcal{H}$  with respect to the particle density  $\rho$  and the pair density  $\tilde{\rho}$ , respectively.

$$V_{res}^{ph} \sim \frac{\delta^2 \mathcal{H}}{\delta \rho_1 \delta \rho_2} \quad V_{res}^{pp} \sim \frac{\delta^2 \mathcal{H}}{\delta \tilde{\rho}_1 \delta \tilde{\rho}_2}.$$

*G. T. Bertsch and S. F. Tsai, Phys. Rep. 18, 125 (1975).*

# Realization of QRPA

We employ the effective Skyrme interaction in the particle-hole channel

$$V(\vec{r}_1, \vec{r}_2) = t_0 \left(1 + x_0 \hat{P}_\sigma\right) \delta(\vec{r}_1 - \vec{r}_2) + \frac{t_1}{2} \left(1 + x_1 \hat{P}_\sigma\right) \left[ \delta(\vec{r}_1 - \vec{r}_2) \vec{k}^2 + \vec{k}'^2 \delta(\vec{r}_1 - \vec{r}_2) \right] \\ + t_2 \left(1 + x_2 \hat{P}_\sigma\right) \vec{k}' \cdot \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + \frac{t_3}{6} \left(1 + x_3 \hat{P}_\sigma\right) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \\ + iW_0 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[ \vec{k}' \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k} \right].$$

*T. H. R. Skyrme, Nucl. Phys. 9, 615 (1959).*

*D. Vautherin and D. M. Brink, Phys. Rev. C5, 626 (1972).*

The Hamiltonian includes the pairing correlations are generated by the density-dependent zero-range force in the particle-particle channel

$$V_{pair}(\vec{r}_1, \vec{r}_2) = V_0 \left( 1 - \eta \frac{\rho(r_1)}{\rho_{sat}} \right) \delta(\vec{r}_1 - \vec{r}_2),$$

where  $\rho_{sat}$  is the nuclear saturation density;  $\eta$  and  $V_0$  are model parameters. For example,  $\eta=0$  and  $\eta=1$  are the case of a volume interaction and a surface-peaked interaction, respectively.

*A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C77, 024322 (2008).*

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# Realization of QRPA

We introduce the phonon creation operators

$$Q_{\lambda\mu i}^+ = \frac{1}{2} \sum_{jj'} \left[ X_{jj'}^{\lambda i} A^+(jj'; \lambda\mu) - (-1)^{\lambda-\mu} Y_{jj'}^{\lambda i} A(jj'; \lambda - \mu) \right],$$
$$A^+(jj'; \lambda\mu) = \sum_{mm'} C_{jmj'm'}^{\lambda\mu} \alpha_{jm}^+ \alpha_{j'm'}^+.$$

The index  $\lambda$  denotes total angular momentum and  $\mu$  is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum  $|0\rangle$  and one-phonon excited states are  $Q_{\lambda\mu i}^+|0\rangle$  with the normalization condition

$$\langle 0|[Q_{\lambda\mu i}, Q_{\lambda\mu i'}^+]|0\rangle = \delta_{ii'}.$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}.$$

Solutions of this set of linear equations yield the one-phonon energies  $\omega$  and the amplitudes  $X, Y$  of the excited states.





# Phonon-phonon coupling (PPC)

To take into account the effects of the phonon-phonon coupling (PPC) in the simplest case one can write the wave functions of excited states as a linear combination of one- and two-phonon configurations

$$\Psi_{\nu}(JM) = \left[ \sum_i R_i(J\nu) Q_{JMi}^+ + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) [Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+]_{JM} \right] |0\rangle$$

with the normalization condition

$$\sum_i R_i^2(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} \left[ P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) \right]^2 = 1.$$

V. G. Soloviev, *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Inst. of Phys., Bristol 1992).

# Phonon-phonon coupling (PPC)

Using the variational principle in the form

$$\delta \left( \langle \Psi_\nu(JM) | \mathcal{H} | \Psi_\nu(JM) \rangle - E_\nu [\langle \Psi_\nu(JM) | \Psi_\nu(JM) \rangle - 1] \right) = 0,$$

one obtains a set of linear equations for the unknown amplitudes  $R_i(J\nu)$  and  $P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)$ :

$$(\omega_{Ji} - E_\nu) R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) = 0;$$

$$\sum_i U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) R_i(J\nu) + 2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - E_\nu) P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) = 0.$$

$U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji)$  is the matrix element coupling one- and two-phonon configurations:

$$U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) = \langle 0 | Q_{Ji} \mathcal{H} [Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+]_J | 0 \rangle.$$

These equations have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. A22, 397 (2004).

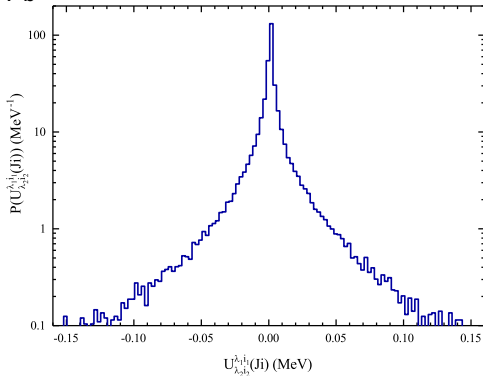
A. P. Severyukhin, N. N. Arsenyev, N. Pietralla, Phys. Rev. C86, 024311 (2012).

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# Phonon-phonon coupling (PPC)

Distribution of coupling matrix elements  $U_{\lambda_2 j_2}^{\lambda_1 j_1}(J_i)$  between the one- and two-phonon configurations in the PPC calculation of the GDR strength function for  $^{208}\text{Pb}$



A. P. Severyukhin, S. Åberg, N. N. Arsenyev R. G. Nazmitdinov, Phys. Rev. C98, 044319 (2018).

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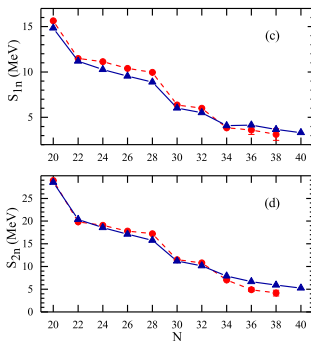
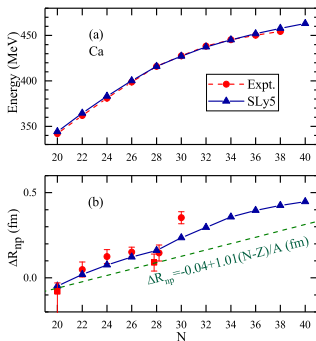
## RESULTS AND DISCUSSION



# Details of calculations

In the case of the Ca and Ni isotopes, we use the Skyrme interaction **SLy5**. The pairing strength  $V_0 = -270 \text{ MeV}\cdot\text{fm}^3$  is fitted to reproduce the experimental neutron pairing energies near  $^{50}\text{Ca}$  and  $^{68}\text{Ni}$ .

*E. Chabanat et al., Nucl. Phys. A635, 231 (1998).*



*N. N. Arsenyev et al., Phys. Rev. C95, 054312 (2017).*

*M. Wang et al., Chin. Phys. C41, 030003 (2017).*

*M. Tanaka et al., PRL 124, 102501 (2020).*

*A. Trzcńska et al., PRL 87, 082501 (2001).*

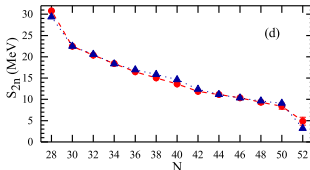
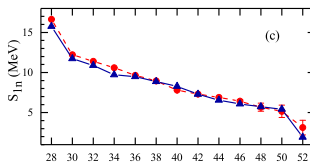
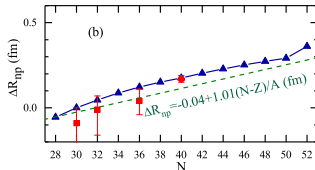
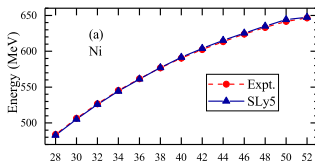
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# Details of calculations

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*E. Chabanat et al., Nucl. Phys. A635, 231 (1998).*



*N. N. Arsenyev et al., in preparation.*

*M. Wang et al., Chin. Phys. C41, 030003 (2017).*

*D. M. Rossi et al., PRL 111, 242503 (2013).*

*A. Trzcińska et al., PRL 87, 082501 (2001).*

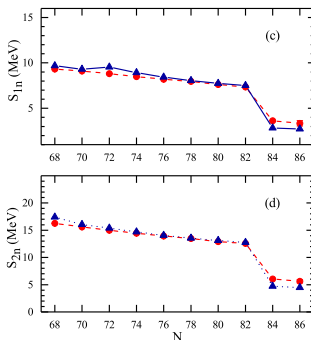
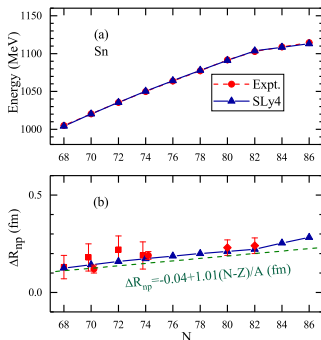
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# Details of calculations

In the case of the Sn isotopes, we use the Skyrme interactions **SLy4**. The pairing strength  $V_0 = -940 \text{ MeV}\cdot\text{fm}^3$  is fitted to reproduce the experimental neutron pairing energies near  $^{130}\text{Sn}$ .

*E. Chabanat et al., Nucl. Phys. A635, 231 (1998).*



*N. N. Arsenyev et al., in preparation.*

*M. Wang et al., Chin. Phys. C41, 030003 (2017).*

*A. Klimkiewicz et al., Phys. Rev. C76, 051603 (2007).*

*A. Trzcńska et al., PRL 87, 082501 (2001).*

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# Details of calculations

The dipole transition probabilities can be expressed as

$$B(E1; 0_{gs}^+ \rightarrow 1_1^-) = \left| e_{eff}^{(n)} \langle j | \hat{M}^{(n)} | 0 \rangle + e_{eff}^{(p)} \langle j | \hat{M}^{(p)} | 0 \rangle \right|^2,$$

where  $\hat{M}^{(p)} = \sum_i^Z r_i Y_{1\mu}(\hat{r}_i)$  and  $\hat{M}^{(n)} = \sum_i^N r_i Y_{1\mu}(\hat{r}_i)$ . The spurious  $1^-$  state is excluded from the excitation spectra by introduction of the effective neutron  $e_{eff}^{(n)} = -Z/A e$  and proton  $e_{eff}^{(p)} = N/A e$  charges.

**A. Bohr and B. Mottelson, Nuclear Structure Vol. II (Benjamin, New York 1975).**

To construct the wave functions of the  $1^-$  states, in the present study we take into account all two-phonon terms that are constructed from the phonons with multipolarities  $\lambda \leq 5$ . All dipole excitations with energies below 35 MeV and 15 most collective phonons of the other multipolarities are included in the wave function. We have checked that extending the configuration space plays a minor role in our calculations.

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## Details of calculations

The integral characteristics of  $E1$  strength function are the centroid energy  $\bar{E}$  and the spreading width  $\Gamma$

$$\bar{E} = \frac{m_1}{m_0} \quad \text{and} \quad \Gamma = 2.35 \sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2},$$

where  $m_k = \sum_i B(E1; 0_{gs}^+ \rightarrow 1_i^-) \cdot E_{1_i^-}^k$  are the energy-weighted moments.

*G. F. Bertsch, P. F. Bortignon, R. A. Broglia, Rev. Mod. Phys. 55, 287 (1983).*

The  $E1$  transition strength function  $b(E1; \omega)$  which is determined as follows:

$$b(E1; \omega) = \sum_i B(E1; 0_{gs}^+ \rightarrow 1_i^-) \rho(\omega - E_{1_i^-}),$$

where is the Lorentz weight (the Lorentz averaging parameter is  $\Delta=1$  MeV)

$$\rho(\omega - E_{1_i^-}) = \frac{\Delta}{2\pi} \frac{1}{\left(\omega - E_{1_i^-}\right)^2 + \left(\frac{\Delta}{2}\right)^2}.$$

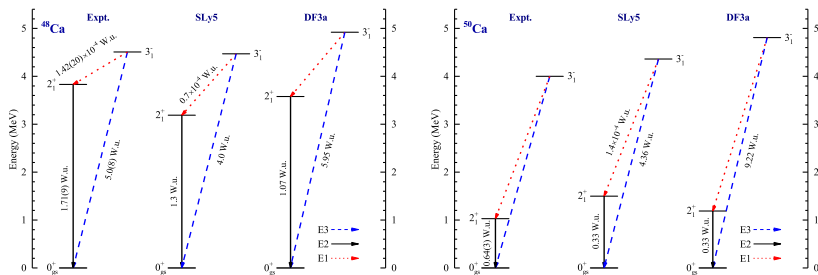
	$\lambda_1^\pi$	Energy, MeV		$B(E\lambda; 0_{gs}^+ \rightarrow \lambda_1^\pi), e^2b^\lambda$	
		Expt.	Theory	Expt.	Theory
$^{46}\text{Ca}$	$2_1^+$	1.346	2.05	$0.0127 \pm 0.0023$	0.0070
	$3_1^-$	3.614	4.57	$0.006 \pm 0.003$	0.0049
	$4_1^+$	2.575	2.30		0.00035
	$5_1^-$	4.184	4.67		0.00027
$^{48}\text{Ca}$	$2_1^+$	3.832	3.19	$0.00968 \pm 0.00105$	0.0065
	$3_1^-$	4.507	4.47	$0.0083 \pm 0.0020$	0.0038
	$4_1^+$	4.503	3.51		0.00035
	$5_1^-$	5.729	4.52		0.00026
$^{50}\text{Ca}$	$2_1^+$	1.027	1.50	$0.00375 \pm 0.00010$	0.0018
	$3_1^-$	3.997	4.36		0.0045
	$4_1^+$	4.515	3.75		0.00051
	$5_1^-$	5.110	4.45		0.00029

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).*

<http://www.nndc.bnl.gov/ensdf/> [16 May (2017)]

# $^{48}\text{Ca}$ vs $^{50}\text{Ca}$

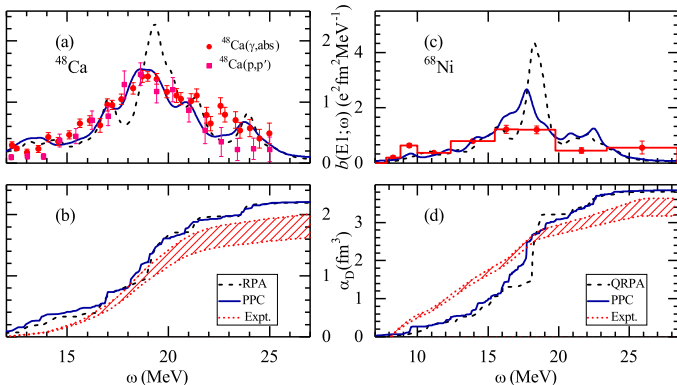
## SLy5



***N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. WC 194, 04002 (2018).***

***E. E. Saperstein et al., JETP Letter 104, 218 (2016).***

**<http://www.nndc.bnl.gov/ensdf/> [14 January (2019)]**



$$\alpha_D = \frac{\hbar c}{2\pi^2} \int \frac{\sigma_\gamma(\omega)}{\omega^2} d\omega$$

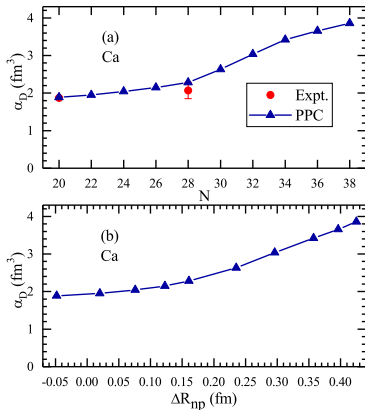
*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Part. Nucl. 50, 528 (2019).*

*J. Birkhan et al., PRL 118, 252501 (2017).*

*D. M. Rossi et al., PRL 111, 242503 (2013).*

# The dipole polarizability $\alpha_D$

SLy5



	$\alpha_D(^{48}\text{Ca})$
SLy5:	2.28
EDF-averaged:	2.306(89)
<i>ab initio</i> :	2.19 ÷ 2.60
Expt.:	2.07(22)

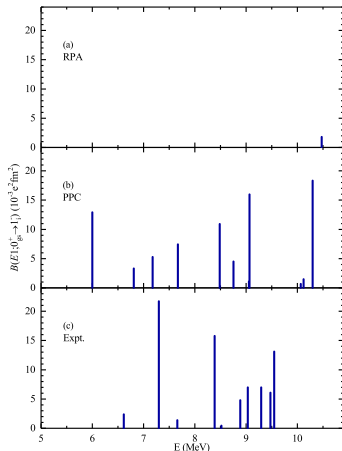
*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).*

*J. Piekarewicz et al., Phys. Rev. C85, 041302(R) (2012).*

*G. Hagen et al., Nature Phys. 12, 186 (2016).*

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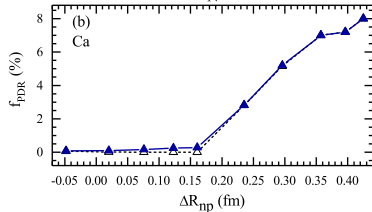
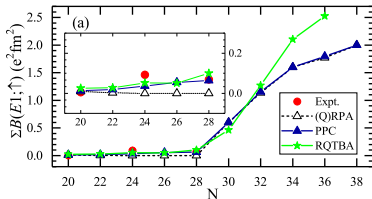
The dominant contribution in the wave function of the  $1^-$  states comes from the two-phonon configurations ( $> 60\%$ ). These states originate from the fragmentation of the RPA states above 10 MeV.

	$\sum B(E1; \uparrow)$	$\sum B(E1; \uparrow) \cdot E$
SLy5:		
RPA	0.00	0.00
PPC	0.06	0.50
QTBA:	0.071	0.509
RQTBA:	0.10	0.95
Expt.:	0.0687(75)	0.570(62)

***N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).***

***T. Hartmann et al. PRL 93, 192501 (2004).***

***I. A. Egorova and E. Litvinova, Phys. Rev. C94, 034322 (2016).***



$$f_{\text{PDR}} = \frac{\sum_k^{\leq 10 \text{ MeV}} B(E1; 0_{gs}^+ \rightarrow 1_k^-) \cdot E_{1_k^-}}{14.8NZ/A \text{ e}^2\text{fm}^2\text{MeV}}$$

The strong increase of the summed  $E1$  strength below 10 MeV [ $\sum B(E1)$ ], with increasing neutron number from  $^{48}\text{Ca}$  till  $^{58}\text{Ca}$ .

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C95, 054312 (2017).*

*I. A. Egorova and E. Litvinova, Phys. Rev. C94, 034322 (2016).*

Nucleus	$\sum B(E1; \uparrow) \text{ (e}^2\text{fm}^2\text{)}$			$f_{PDR} \text{ (\%)}$		
	(Q)RPA	PPC	Expt.	(Q)RPA	PPC	Expt.
$^{48}\text{Ca}$	0.00	0.06	0.08(1)	0.00	0.28	0.33(4)
$^{50}\text{Ca}$	0.57	0.58	–	2.81	2.82	–
$^{68}\text{Ni}$	0.97	1.10	–	4.20	4.61	5.0(15)
$^{70}\text{Ni}$	1.06	1.15	3.26(54)	4.54	4.62	6.3(11)

$$f_{PDR} = \frac{\sum_{k \leq 10(12)}^{\text{MeV}} B(E1; 0_{gs}^+ \rightarrow 1_k^-) \cdot E_{1_k^-}}{14.8NZ/A \text{ e}^2\text{fm}^2\text{MeV}}$$

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, in preparation.*

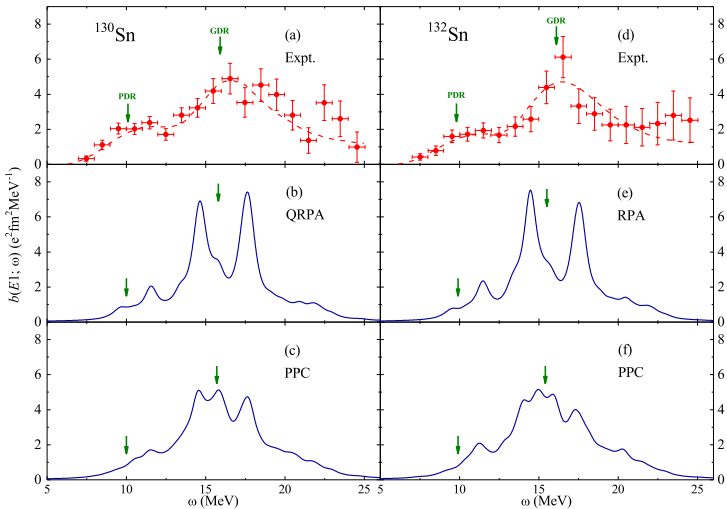
*J. Birkhan et al., PRL 118, 252501 (2017).*

*D. M. Rossi et al., PRL 111, 242503 (2013).*



# The dipole strength distributions of $^{130,132}\text{Sn}$

SLy4



***N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. WC 38, 17002 (2012).***

***A. Klimkiewicz et al., Nucl. Phys. A788, 145c (2007).***

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Integral characteristics of the GDR in  $^{124-134}\text{Sn}$

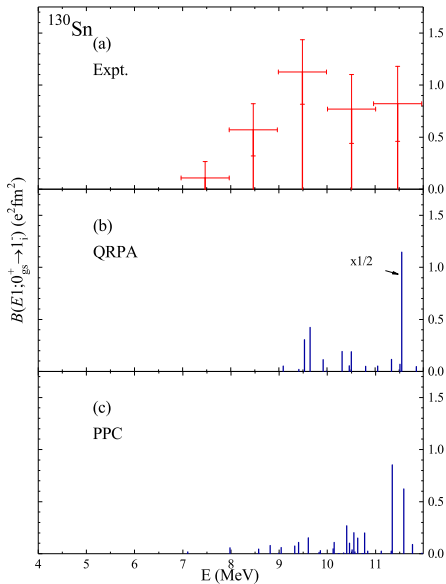
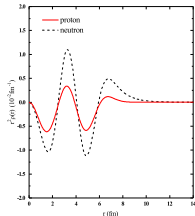
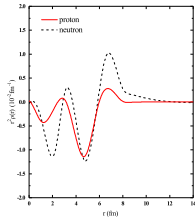
	$\bar{E}$ (MeV)			$\Gamma$ (MeV)		
	Expt.	Theory		Expt.	Theory	
		(Q)RPA	PPC		(Q)RPA	PPC
$^{124}\text{Sn}$	15.3	16.4	16.3	4.8	4.4	4.7
$^{126}\text{Sn}$		16.2	16.2		4.4	4.7
$^{128}\text{Sn}$		16.1	16.0		4.7	4.7
$^{130}\text{Sn}$	15.9(5)	15.8	15.7	4.8(17)	4.8	4.8
$^{132}\text{Sn}$	16.1(7)	15.5	15.4	4.7(21)	4.9	5.0

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC 38, 17002 (2012).*

*P. Adrich et al., PRL 95, 132501 (2005).*

# Low-energy $E1$ strength distributions of $^{130}\text{Sn}$

SLy4



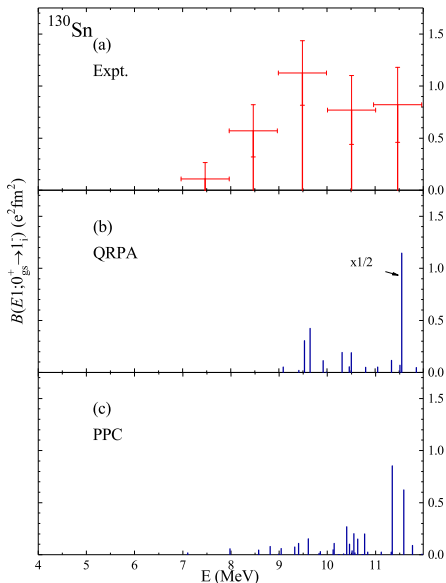
**N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. WC 38, 17002 (2012).**

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# Low-energy $E1$ strength distributions of $^{130}\text{Sn}$

SLy4



PDR width:

1.0 MeV – QRPA

1.8 MeV – PPC

< 3.4 MeV – an upper limit

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. WC 38, 17002 (2012).*

N. Arsenyev



Integral characteristics of the PDR in  $^{124-134}\text{Sn}$

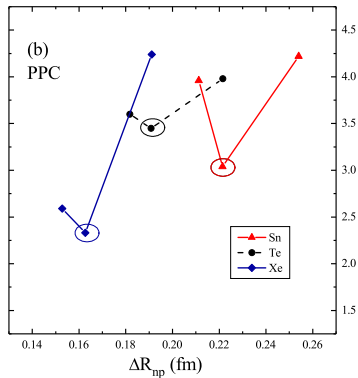
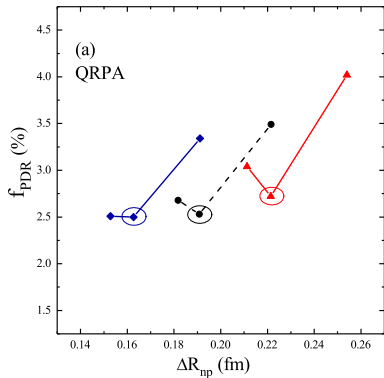
	$\bar{E}$ (MeV)			$\sum B(E1; \uparrow)$ ( $e^2\text{fm}^2$ )		
	Expt.	Theory		Expt.	Theory	
		(Q)RPA	PPC		(Q)RPA	PPC
$^{124}\text{Sn}$	6.97	9.7	9.1	0.379(45)	0.86	0.59
$^{126}\text{Sn}$		10.1	10.0		1.82	1.86
$^{128}\text{Sn}$		10.0	10.0		1.63	1.78
$^{130}\text{Sn}$	10.1(7)	10.0	10.0	2.4(7)	1.40	1.80
$^{132}\text{Sn}$	9.8(7)	9.9	9.9	1.3(8)	1.27	1.42
$^{134}\text{Sn}$		9.1	9.1		2.05	2.14

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. WC 38, 17002 (2012).*

*P. Adrich et al., PRL 95, 132501 (2005).*

*B. Özel et al., Nucl. Phys. A788, 385c (2007).*

# PDR fractions $f_{\text{PDR}}$ as functions of the neutron skin $\Delta R_{\text{np}}$ SLy4



$$f_{\text{PDR}} = \frac{\sum_k^{\leq 11 \text{ MeV}} B(E1; 0_{gs}^+ \rightarrow 1_k^-) \cdot E_{1_k^-}}{14.8NZ/A \text{ e}^2 \text{ fm}^2 \text{ MeV}}$$

*N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Part. Nucl. 48, 876 (2017).*

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# Estimation of the symmetry energy

One of the key properties of nuclear matter is the symmetry energy ( $J$ ), which is particularly important in modeling nuclear matter and finite nuclei because it probes the isospin part of the Skyrme interaction. It is defined as

$$J(\rho) = \frac{\hbar^2}{6m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} + \frac{1}{8} t_0 (2x_0 + 1) \rho + \frac{1}{24} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} [3t_1 x_1 - t_2 (4 + 5x_2)] + \frac{1}{48} t_3 (2x_3 + 1) \rho^{\alpha+1}.$$

Saturation properties of all Skyrme parametrizations used in this work.

	SLy4	SLy5	SIII	SGII	SkM*	SkP	SkI2	SkI3	SK255	LNS
$J$	32.00	32.01	28.16	26.83	30.03	30.00	33.37	34.83	37.40	33.43
$\rho_{\text{sat}}$	0.160	0.161	0.145	0.158	0.160	0.163	0.158	0.158	0.157	0.175

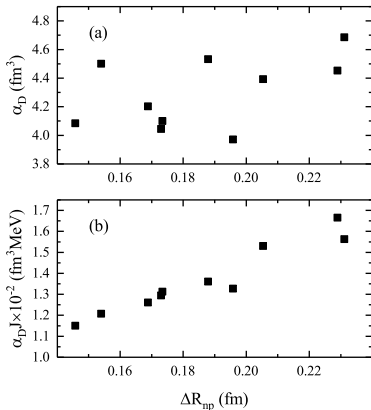
M. Dutra et al., Phys. Rev. C85, 035201 (2012).

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# Correlations: $\alpha_D$ vs $\Delta R_{np}$ and $J$

$^{68}\text{Ni}$



In statistics, the Pearson correlation coefficient ( $r_{XY}$ ) is a measure of linear correlation between two sets of data. The Pearson coefficient calculate as

$$r_{XY} = \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{\sigma_X \sigma_Y},$$

where  $\bar{X}$ ,  $\bar{Y}$  represent the mean values and  $\sigma_X$ ,  $\sigma_Y$  the root mean square of the sample.

*N. E. Solonovich, N. N. Arsenyev, A. P. Severyukhin, in preparation.*

*N. Draper and H. Smith, Applied Regression Analysis (Wiley, New York, 1998).*

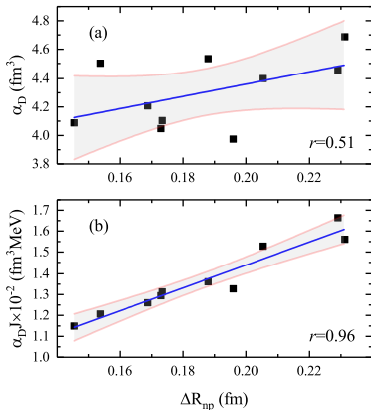
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*N. E. Solonovich, N. N. Arsenyev, A. P. Severyukhin, in preparation.*

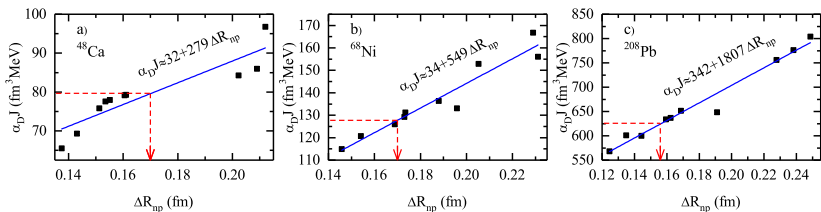
*N. Draper and H. Smith, Applied Regression Analysis (Wiley, New York, 1998).*

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# Estimation of the symmetry energy $J$

We carried out a theoretical analysis of the recently measured  $\alpha_D$  and  $\Delta R_{np}$  in  $^{48}\text{Ca}$ ,  $^{68}\text{Ni}$ , and  $^{208}\text{Pb}$  to extract information about the symmetry energy, by using a strong correlation between  $\alpha_D J$  and  $\Delta R_{np}$ . Combining the experimental data and the RPA theory constraints yields the interval of  $J = 31 - 38$  MeV.



*N. E. Solonovich, N. N. Arsenyev, A. P. Severyukhin, submitted Phys. Part. Nucl. Letters.*

*A. Tamii et al., PRL 107, 062502 (2011).*

*D. M. Rossi et al., PRL 111, 242503 (2013).*

*J. Birkhan et al., Phys. Rev. Lett. 118, 252501 (2017).*

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# Conclusions

Starting from the Skyrme mean-field calculations, the properties of the electric dipole strength in neutron-rich Ca, Ni and Sn isotopes are studied by taking into account the coupling between one- and two-phonons terms in the wave functions of excited states. It is shown that the PPC have small influence on the dipole polarizability.

The strong enhancement of the PDR strengths are studied by taking into account with the PPC configurations. We find the strong increase of the summed  $E1$  strength below 10 MeV [ $\sum B(E1; \uparrow)$ ], with increasing neutron number from  $^{48}\text{Ca}$  until  $^{50}\text{Ca}$ . The comparison of the  $^{68}\text{Ni}$  and  $^{70}\text{Ni}$  summed strength values, in particular if integrated in the region below 12 MeV, shows an insignificant increase with neutron number. We have also found the impact of the neutron shell closure on the PDR strength.

We carried out a theoretical analysis of the recently measured  $\alpha_D$  and  $\Delta R_{np}$  in  $^{48}\text{Ca}$ ,  $^{68}\text{Ni}$ , and  $^{208}\text{Pb}$  to extract information about the symmetry energy, by using a strong correlation between  $\alpha_D$  and  $\Delta R_{np}$ . Combining the experimental data and the RPA theory (using a broad set of Skyrme functionals) constraints yields the interval of  $J=31-38$  MeV.

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(JINR Dubna)



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(INP Orsay)

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THE END

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