From Shell Model and Collective Nuclear Model to Nuclear Energy Density Functional and Chiral Effective Field Theory.

R.V.Jolos and E.A.Kolganova

Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna

May 14 , 2025

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1933. Elsasser „On the Pauli principle in nuclei“

„The Pauli principle together with the assumption of the central potential well, leads to shell structure in nuclei. The field due to N-1 nucleons acting on the Nth nucleon has probably spherical symmetry leading to the analogy with electron shells in atoms“

1934 – 1935 papers of Elsasser
Magic numbers: $Z = 50, 82$; $N = 126$.

Looking back, it is difficult to explain how the clear guidelines presented by Elsasser were ignored for almost 13 years.

1949. Nuclear shell model. Strong spin-orbit interaction

Spin-orbit coupling estimated by analogy to atomic physics have a correct sign within nuclei. However, its value is too small.

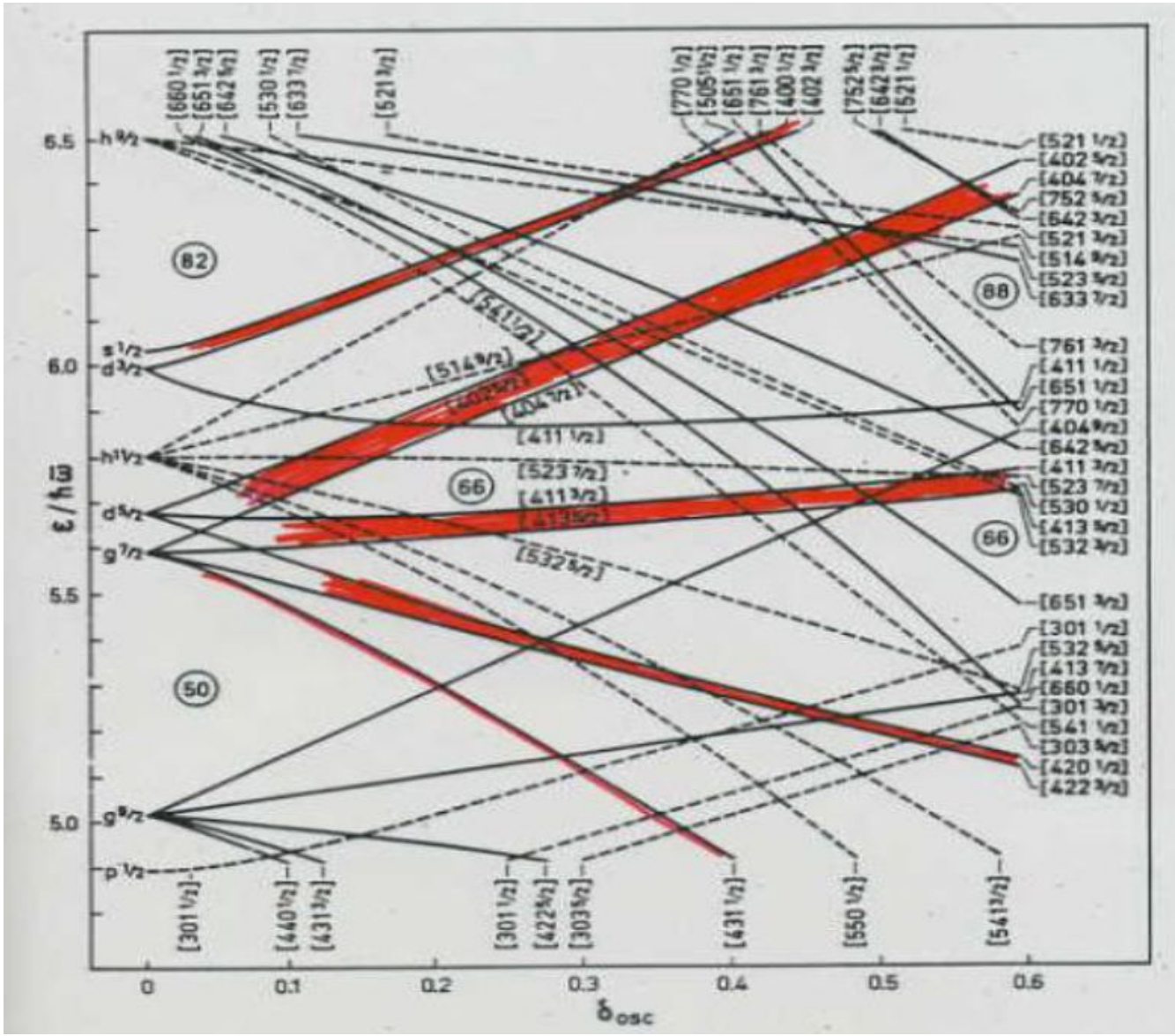
$$V_{so} = \frac{\hbar}{m^2c^2}\frac{1}{r}\frac{\partial V}{\partial r}\vec{s}\cdot\vec{l}$$

$$\equiv \lambda\frac{1}{r}\frac{\partial V}{\partial r}\vec{s}\cdot\vec{l}$$

with $\lambda = -0.044$ fm^2. In nucleus $\lambda = -0.5$ fm^2.

The reason for this discrepancy is that the average nuclear field also has a vector component (vector potential). The appearance of the vector component can be interpreted as the result of the coupling of nucleons with vector mesons. This potential is quite large in magnitude and is repulsive. Difference between scalar and vector potentials of about 50 MeV characterizes the binding of nucleons in the nucleus, and the sum, which is 700-800 MeV, determines the magnitude of the spinal-orbital interaction.

Pseudospin



Proton-neutron residual interaction

$^{128}_{50}Sn_{78}$ (E(2^+_1=1168 keV), $^{126}_{50}Sn_{76}$ (E(2^+_1=1141 keV),

$^{124}_{50}Sn_{74}$ (E(2^+_1=1132 keV),

$^{130}_{52}Te_{78}$ (E(2^+_1=839 keV),

$^{132}_{54}Xe_{78}$ (E(2^+_1=668 keV),

Part of the potential interaction near the minimum can be characterized by dimensionless size

$$\Lambda = \frac{\hbar^2}{ma^2}\frac{1}{V_0},$$

where a is the width and V_0 is the minimum depth. Typical values of Λ for liquids 0.15 to 0.20, for solids 0.01 - 0.10. For atomic nuclei $\Lambda = 0.4$. From this point of view, the nucleus resembles a liquid. At the same time, despite similarity with liquids associated with a short-acting character nuclear forces, nucleons are characterized by a large mean free path in the nucleus.

The average distance between nucleons in the nucleus is noticeable greater than the distance corresponding to the position of the minimum potential attraction, while in molecules the average distance between atoms coincides with the position of the minimum.

Microscopic approach in the Theory of Nuclear Structre. 1960 y.

– Pair correlations. U-V Bogoliubov transformation

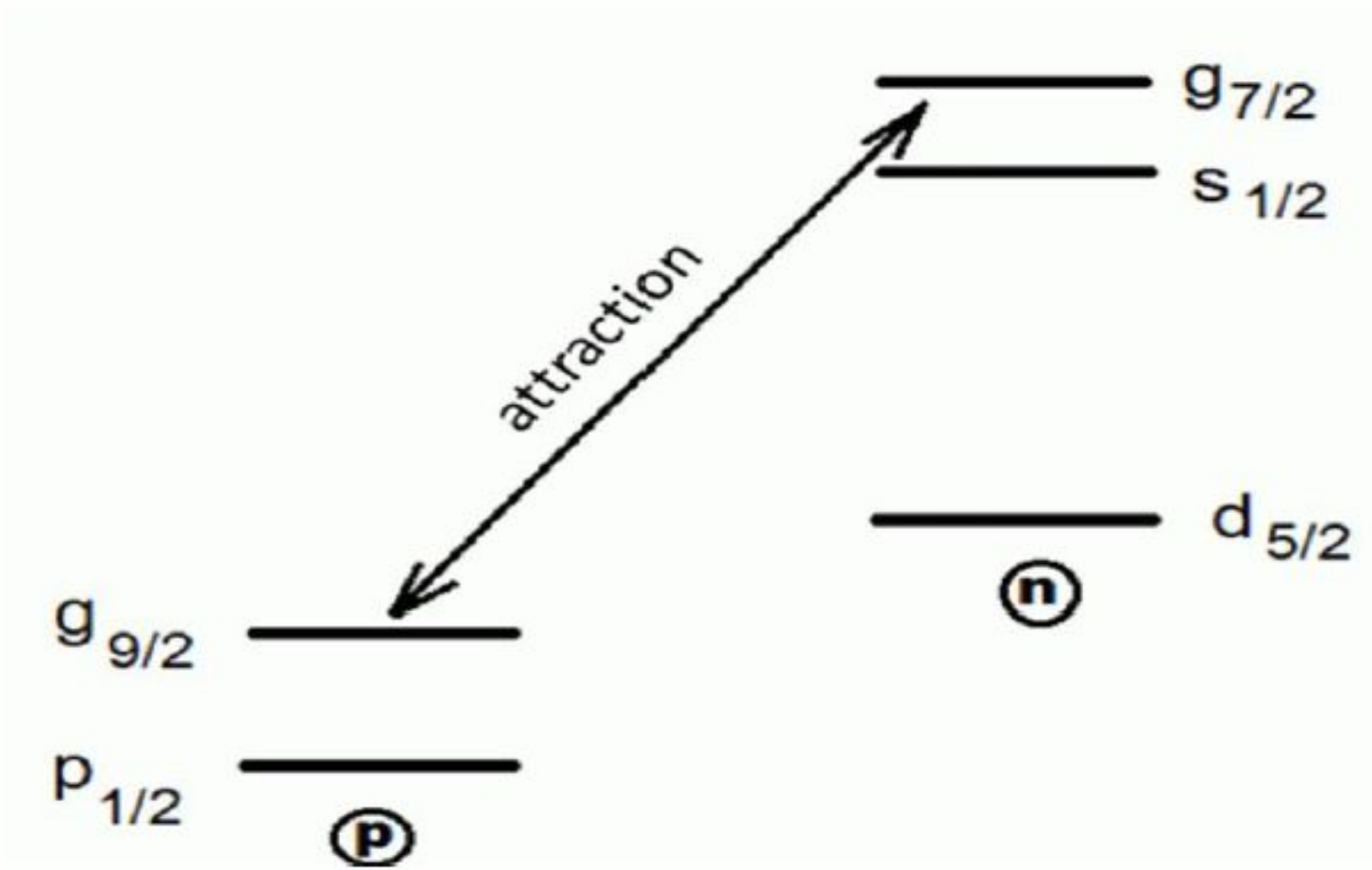
– Random Phase approximation

– Tom Kuo and Gerry Brown: the first attempt to perform a microscopic nuclear-structure calculation starting from the free nucleon-nucleon (NN) potential.
The KB work was grounded in the general belief, which came out between the end of 1950s and the beginning of 1960s, that a new generation of nuclear structure calculations for finite systems, based on first principles and free from phenomenological inputs, had to be started.

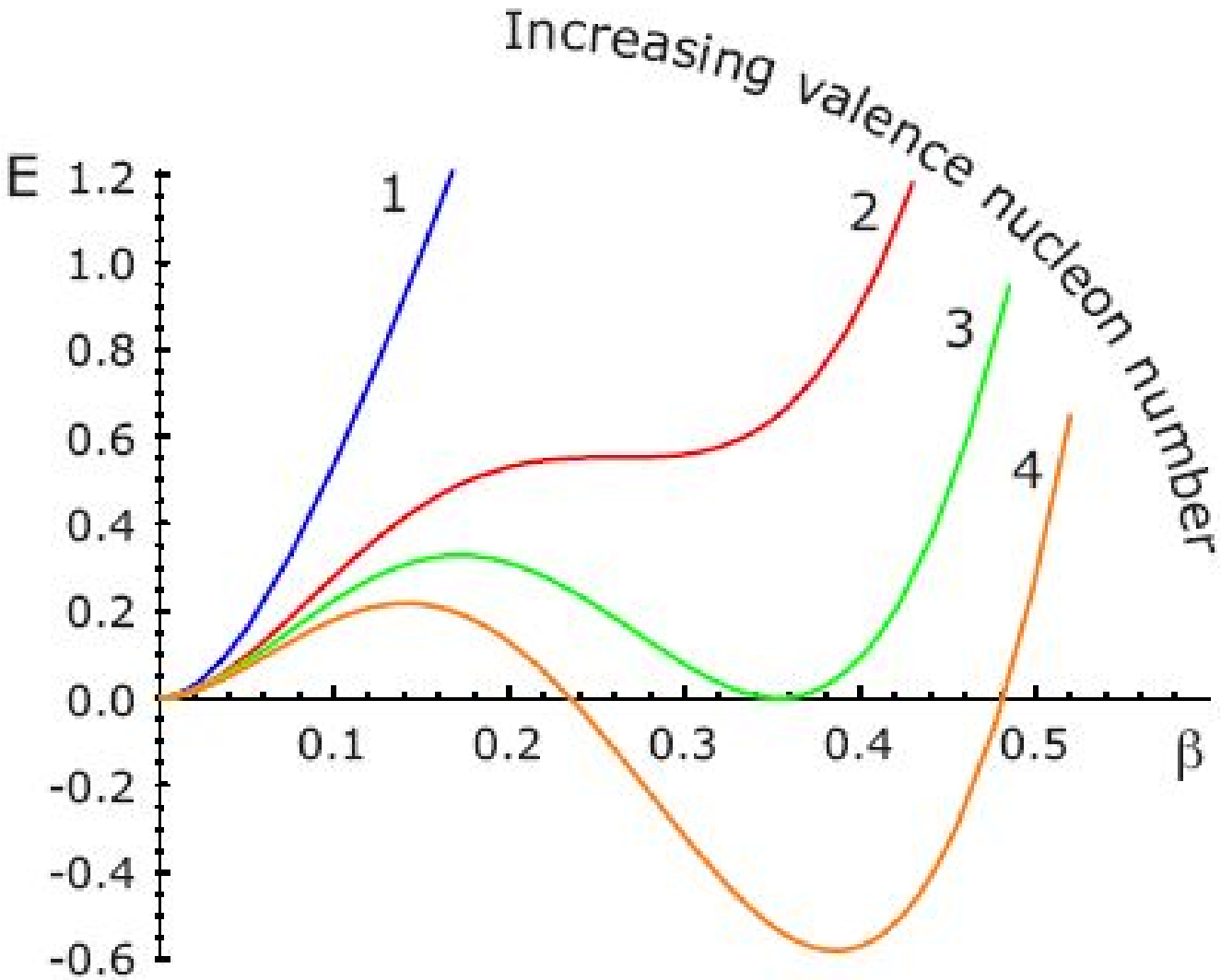
Federman-Pittel mexanism of nuclear deformation.

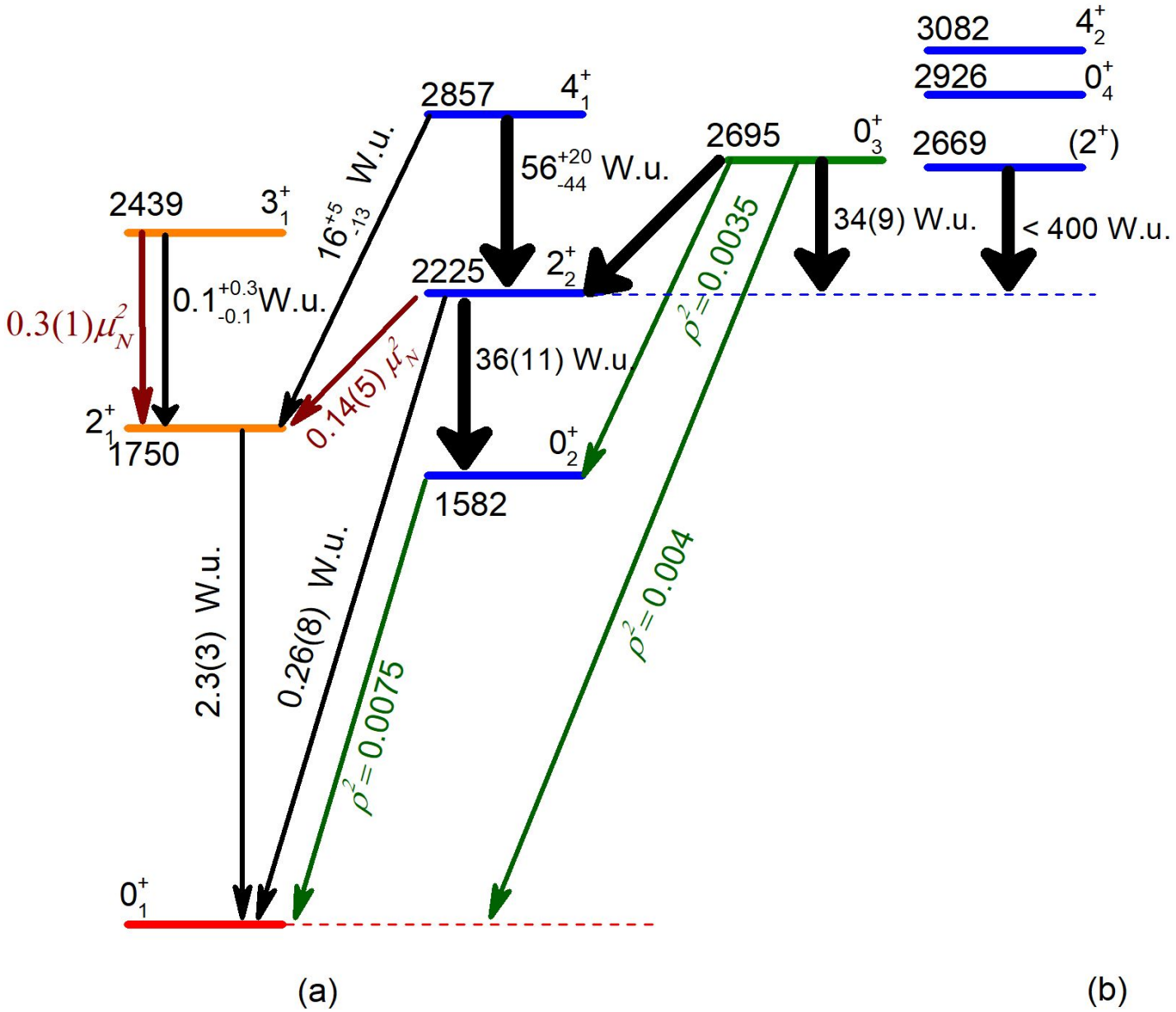
– The key role in the Federman-Pittel mechanism of the phase transition from spherical to deformed nuclear shape is played by the monopole proton-neutron interaction of nucleons filling the single particle orbits with a strong spacial overlap. This interaction is characterized by its specific dependence on the orientation of spin and isospin.

Federman-Pittel mechanism of deformation.

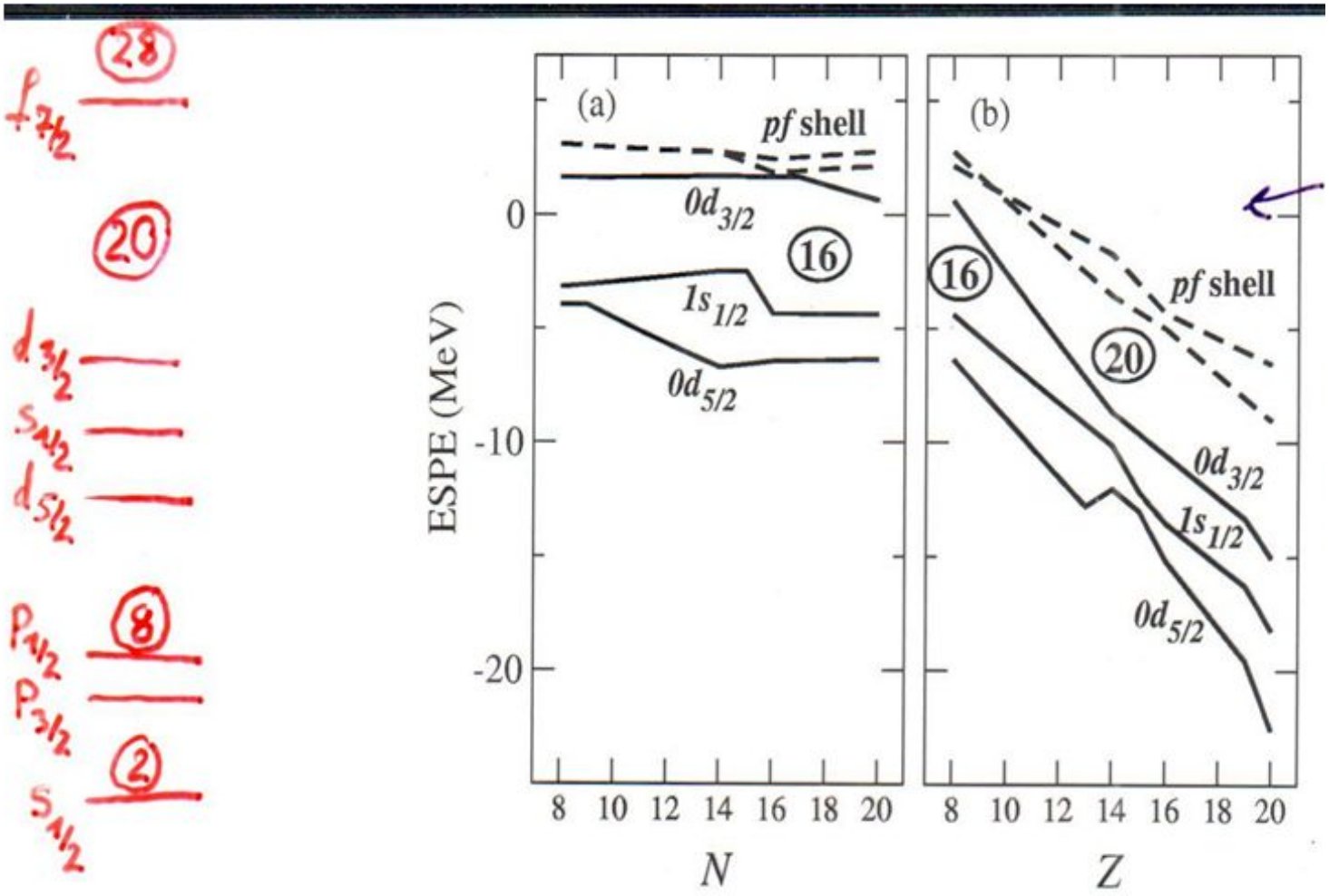


Federman-Pittel mechanism of deformation.




3082
4_2^+
2926
0_4^+
2857
4_1^+
2695
0_3^+
2669
(2^+)
2439
3_1^+
2225
2_2^+
2_1^+
1750
1582
0_2^+
0_1^+
16^{+5}_{-13} W.u.
56^{+20}_{-44} W.u.
34(9) W.u.
< 400 W.u.
$0.1^{+0.3}_{-0.1}$W.u.
$0.3(1)\mu_N^2$
$0.14(5)\,\mu_N^2$
36(11) W.u.
2.3(3) W.u.
0.26(8) W.u.
ρ^2=0.0035
ρ^2=0.0075
ρ^2=0.004
(a)
(b)

Изменение магических чисел.

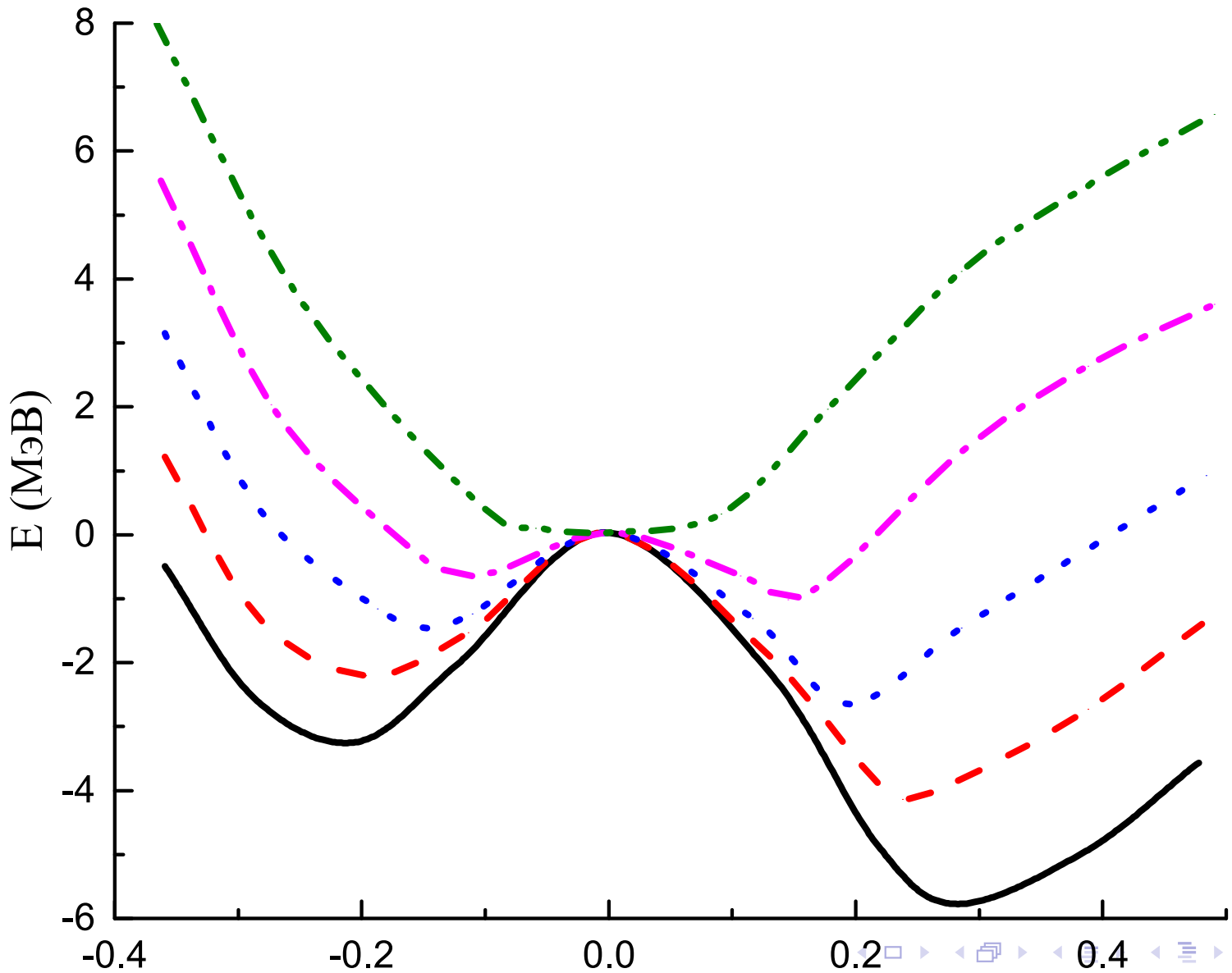


Jan-Teller effect.

– Deformation occurs as a result of the polarization of the nucleus core by valence nucleons. The interaction between valence nucleons and oscillations of the nucleus core leads to the nuclear Jahn-Teller effect, i.e. to instability of the spherical symmetry of the mean field under deformation that reduce the symmetry.

– The degenerate state of the nucleus becomes unstable due to the interaction of valence nucleons with the core. This leads to decrease in the symmetry of the system.

Jan-Teller effect.



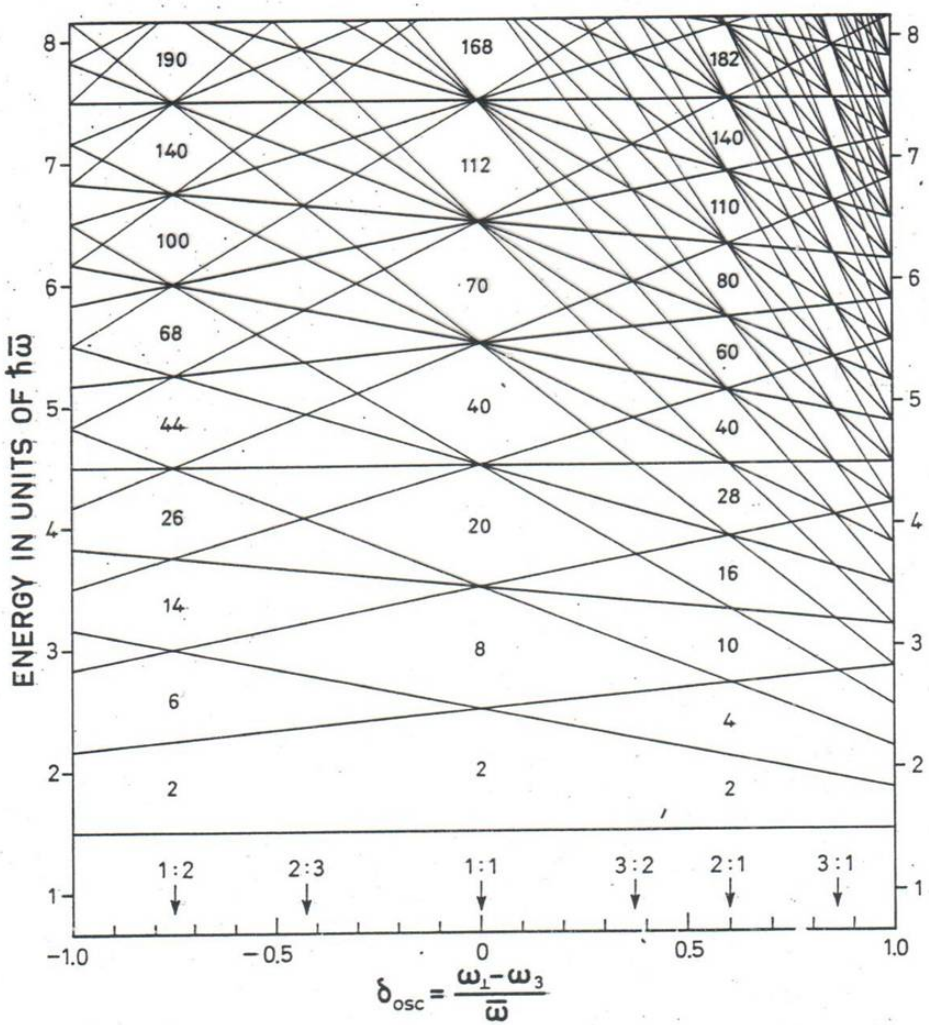


Figure 6-48 Single-particle spectrum for axially symmetric harmonic oscillator potentials. The eigenvalues are measured in units of $\bar{\omega} = (2\omega_\perp + \omega_3)/3$, and the deformation parameter δ_{osc} is that defined by Eq. (5-11). The arrows mark the deformations corresponding to the indicated rational ratios of frequencies $\omega_\perp : \omega_3$.

Consider a system of N interacting particles described by the Hamiltonian

$$\hat{H} \equiv \hat{T} + \hat{v} + \hat{W}$$

Let Ψ be the N-body w.f. and $n(\vec{r})$ the corresponding particle density. Theorem:
–The nongenerate g.s.w.f. is a unique functional of the g.s. density $n_0(\vec{r})$

$$\Psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N) = \Psi_0[n_0(\vec{r})].$$

As a consequence the g.s. expectation value of any observable is a functional of $n_0(\vec{r})$

$$E_0 \equiv E[n_0(\vec{r})] = \langle \Psi[n_0] | \hat{H} | \Psi[n_0] \rangle$$

There exists functional $F[n]$ such that the energy functional can be written as

$$E[n] = F[n] + \int d^3 r v(\vec{r}) n(\vec{r})$$

The functional $F[n]$ is universal in the sense that for given NN-interaction it does not depend on $v(\vec{r})$
The formal definition of the H-K functional is

$$F[n] = \langle \Psi[n] | \hat{T} | \Psi[n] \rangle + \langle \Psi[n] | \hat{W} | \Psi[n] \rangle$$

The H-K theorem gives no practical guide to the construction of the universal density functional.

The H-K theorem resembles in some respects «Method of contraction description» in statistical physics:

$$t \gg \tau_0 \quad = \quad \frac{r_c}{\bar{v}}$$

$$f_s(x_1, x_2, ..., x_N; t) \quad \rightarrow \quad f_s(x_1, x_2, ..., x_N; f_1(x', t))$$

Functional $f_s(x_1, x_2, ..., x_N; f_1(x', t))$ is universal and do not depend on initial conditions.
The fact is that multi-particle distribution functions, unlike single-particle ones, change rapidly over time on the order of the collision time. Therefore, they will have time to adapt to each instantaneous value of a single-particle function, which noticeably changes only for a time significantly exceeding the collision time.

$$\left(-\frac{\hbar^2}{2M}\nabla^2 + U_{eff}\right)\varphi_i = \epsilon_i\varphi$$

$$U_{eff} = \frac{\delta F[\rho(r)]}{\delta\rho}$$

$$\rho(r) = \sum_{i=1}^{N} |\varphi_i(r)|^2$$

Comparison of the Kohn-Sham scheme and the Hartree-Fock method

– The key of the Kohn-Sham scheme is to map the interacting many bode system to independent particle system. By solving selfconsistently the independent particle problem, one obtains the particle density and the ground state energy of the interacting problem.

– The Hartree-Fock equations are obtained by minimizing the energy; solving the Hartree-Fock equations one obtains the „best“ energy under the single-determinant constraint. Hartree-Fock method is approximation intrinsically. The Kohn-Sham equations are obtained by mapping; solving the Kohn-Sham equations one obtaines the best particle density. Kohn-Sham method is an exact method, in principle.

– In practical calculations $F[\rho(r)]$ must be approximated since the existing theorem makes no statement about its structure..

Skyrme functional contains the following components of the densities and currents:

$$\rho_0(\vec{r}) = \sum_{\sigma,\tau} \rho(\vec{r},\sigma,\tau;\vec{r},\sigma,\tau)$$

$$\rho_1(\vec{r}) = \sum_{\sigma,\tau} \rho(\vec{r},\sigma,\tau;\vec{r},\sigma,\tau)\tau$$

$$\vec{s}_0(\vec{r}) = \sum_{\sigma,\sigma',\tau} \rho(\vec{r},\sigma,\tau;\vec{r},\sigma',\tau)\vec{\sigma}_{\sigma'\sigma}$$

$$\vec{s}_1(\vec{r}) = \sum_{\sigma,\sigma',\tau} \rho(\vec{r},\sigma,\tau;\vec{r},\sigma',\tau)\vec{\sigma}_{\sigma'\sigma}\tau$$

$$
\begin{aligned}
\vec{j}_T(\vec{r}) &= \frac{\imath}{2}(\nabla' - \nabla)\rho_T(\vec{r},\vec{r}')|_{\vec{r}=\vec{r}'} - \text{current} \\
\vec{\mathcal{J}}_T(\vec{r}) &= \frac{\imath}{2}(\nabla' - \nabla) \times S_T(\vec{r},\vec{r}')|_{\vec{r}=\vec{r}'} - \text{spin} - \text{current} \quad \text{tensor} \\
\tau_T(\vec{r}) &= \nabla \cdot \nabla' \rho_T(\vec{r},\vec{r}')|_{\vec{r}=\vec{r}'} - \text{kinetic} \quad \text{density} \\
\vec{T}_T(\vec{r}) &= \nabla \cdot \nabla' S_T(\vec{r},\vec{r}')|_{\vec{r}=\vec{r}'} - \text{kinetic} \quad \text{spin} \quad \text{density}
\end{aligned}
$$

The Skyrme functional contains systematically all possible bilinear terms in the local densities and currents up to 2nd order in the derivatives, which are invariant with respect to parity, time-reversal, rotational, translational and isospin transformations.

$$E_{sk} = \sum_{T=0,1} \Big\{ C_T^{\rho}\rho_T^2 + C_T^{\Delta\rho}\rho_T\Delta\rho_{\rho_T} + C_T^{\tau}\rho_T\tau_T + C_T^{J}\vec{\mathcal{J}}_T^2$$
$$+C_T^{\Delta J}\rho_T\nabla\cdot\vec{\mathcal{J}} + C_T^{S}\vec{S}_T^2 + C_T^{\Delta S}\vec{s}_T\cdot\Delta\vec{S}_T + C_T^{ST}\vec{S}_T\cdot\vec{T}_T$$
$$+C_T^{\nabla S}(\nabla\cdot S_T)^2 + C_T^{j}\vec{j}_T^2 + C_T^{\nabla j}\vec{S}_T\cdot\nabla\times\vec{j}_T \Big\}$$

In applications the functional is parametrized directly by fitting the coefficients to the gs data without references to any NN-interaction.

$$
\begin{aligned}
\mathcal{E} &= \mathcal{E}_{kin} + \mathcal{E}_v + \mathcal{E}_s + \mathcal{E}_{Coul} + \mathcal{E}_{sl} + \mathcal{E}_{anom}, \\
\mathcal{E}_v &= \frac{2}{3}\varepsilon_F^0 \rho_0 \left[a_+^v \frac{1 - h_{1+}^v x_+}{1 + h_{2+}^v x_+} x_+^2 + a_+^v \frac{1 - h_{1-}^v x_-}{1 + h_{2-}^v x_-} x_-^2 \right], \\
\mathcal{E}_s &= \frac{2}{3}\varepsilon_F^0 \rho_0 \frac{a_+^s r_0^2 (\nabla x_+)^2}{1 + h_+^s x_+ + h_\nabla^s r_0^2 (\nabla x_+)^2}, \\
x_\pm &= \frac{(\rho_n \pm \rho_p)}{2\rho_0}.
\end{aligned}
$$

Such expressions allow an extrapolation to very high densities.

In 1935 Weizsäcker proposed that an energy density approach could be effective for calculating nuclear binding energy:

$$E = \int F d^3r$$

$$F = \frac{\hbar^2}{32\pi^2 M}\frac{(\nabla\rho)^2}{\rho} + \frac{4\pi\hbar^2}{5M}\left(\frac{3\rho}{8\pi}\right)^{5/3} - f(\rho) \qquad (1)$$

Bethe and Bacher in 1936 further developed Weizsäcker's idea and introduced the nuclear mass formula:

$$E(N,Z) = a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_I \frac{(N-Z)^2}{A} \qquad (2)$$

a_v	a_s	a_I	a_c	χ_E
-15.46	16.71	22.84	0.698	3.30 MeV

$$\chi^2_E = \sum \frac{|E^{exp}_{NZ} - E(N,Z)|^2}{N_E}$$
$$N_E = 2375 \qquad (3)$$

$$
\begin{aligned}
\mathcal{E}[\rho_n,\rho_p] &= \mathcal{E}_{kin}[\rho_n,\rho_p]+\mathcal{E}_C[\rho_n,\rho_p]+\mathcal{E}_{int}[\rho_n,\rho_p] \\
\mathcal{E}_{kin}[\rho_n,\rho_p] &= \sum_{\tau=n,p}\frac{\hbar^2}{2m_\tau}\left[\frac{1}{9}|\nabla\rho_\tau^{1/2}|^2+\frac{3}{5}(3\pi^2)^{2/3}\rho_\tau^{5/3}\right]+\ldots \\
\mathcal{E}_{int}[\rho_n,\rho_p] &= (\eta-\frac{1}{9})\sum_{\tau=n,p}\frac{\hbar^2}{2m_\tau}|\nabla\rho_\tau^{1/2}|^2+\sum_{j=0}^{2}\mathcal{E}_j(\rho)\beta^{2j} \\
\mathcal{E}_j(\rho) &= a_j\rho^{5/3}+b_j\rho^2+c_j\rho^{7/3} \\
\rho &= \rho_n+\rho_p, \quad \beta=\frac{(\rho_n-\rho_p)}{(\rho_n+\rho_p)}.
\end{aligned}
$$

The equation that determine equilibrium density of a nucleus is obtained by minimizing

$$\begin{aligned} E(N,Z) &= \int d^3r \mathcal{E}[\rho_n,\rho_p]. \\ &- \eta\frac{\hbar^2}{2m_\tau}\nabla^2\rho_\tau^{1/2} + U_\tau\rho_\tau^{1/2} = \mu_\tau\rho_\tau^{1/2} \\ U_\tau &= \frac{\partial\mathcal{E}[\rho_n,\rho_p]}{\partial\rho_\tau} \end{aligned}$$

η	$\tilde{b}_0$	$\tilde{c}$	$\tilde{a}_1$	$\tilde{b}_1$	χ_E
0.471	-3.15166	2.12378	1.048	-0.610	2.59 MeV

$\tilde{a}_j = a_j\rho_0^{2/3}/\varepsilon_F$, $\tilde{b}_j = b_j\rho_0/\varepsilon_F$, $\tilde{c}_j = c_j\rho_0^{4/3}/\varepsilon_F$

ρ-number density.
ϕ-local momentum potential: $\vec{p} = m\vec{v} = \nabla\phi$.
Lagrangian:

$$\mathcal{L} = -\rho(\dot{\phi} + \frac{1}{2m}(\nabla\phi)^2)^2 - \mathcal{E}(\rho) - \eta\frac{\hbar^2}{2m}(\nabla^2\rho^{1/2})^2$$

Euler-Lagrange equations:

$$\dot{\rho} + \nabla(\rho v) = 0$$

$$\dot{\phi} + \frac{mv^2}{2} + \mathcal{E}'(\rho) - \eta\frac{\hbar^2}{2m}\frac{\nabla^2\rho^{1/2}}{\rho^{1/2}} = 0$$

$$\rho = \rho_0 + \delta\rho$$

This hydrodynamic theory can be reformulated as

$$\begin{aligned}
\Psi &= \sqrt{\rho}\exp(\frac{\imath}{\tilde{\hbar}}\phi), \quad (\tilde{\hbar}=\eta^{1/2}\hbar) \\
\mathcal{L}(\Psi,\dot{\Psi}) &= \Psi^{+}\left(\imath\tilde{\hbar}\partial_t+\frac{\tilde{\hbar}^2\nabla^2}{2m}\right)\Psi-\mathcal{E}(\rho) \\
\imath\tilde{\hbar}\dot{\Psi} &= -\frac{\tilde{\hbar}^2\nabla^2}{2m}\Psi+\mathcal{E}'(\rho)\Psi
\end{aligned}$$

The paper of M.M.Johnson and E.Teller (1955) was, in fact, the first attempt to construct REDF. The following statements were made:

1. Nuclear interaction is strong. It means that at high energies many mesons can be created, i.e. in NN-collisions several mesons can be in virtual states.

2. In heavy nuclei the average number of mesons is much larger than one. Due to Bose statistics they can be in one and the same state. The w.f. of this quantum state corresponds to the nuclear potential.

3. This meson must be scalar. This meson is not necessary is an elementary particle.

The next step has been done by H.-P.Dürr. He indicated that Dirac's equation symmetries do possible an introduction of both scalar attractive and vector repulsive potentials. In the stationary limit:

$$H = \vec{\alpha} \cdot \vec{p} + \beta M - \beta S + V_0, \tag{4}$$

where V_0 is a time like component of the vector potential.

REDF based on the meson exchange theory.

Only as few mesons as possible are included.
π: J=0, T=1 and $P = -$
σ: J=0, T=0, P=+
ω: J=1, T=0, $P = -$
ρ: J=1, T=1, $P = -$
Without pions

$$L_{int} = -g_\sigma \bar{\psi}\sigma\psi - g_\omega \bar{\psi}\gamma_\mu \omega^\mu \psi - g_\rho \bar{\psi}\gamma_\mu \vec{\tau}\vec{\rho}^\mu \psi - e\bar{\psi}\gamma_\mu A^\mu \psi$$

The following Dirac equation is derived using this Lagrangian

$$\left(\gamma_\mu(\imath\partial^\mu + V^\mu) + M + S\right)\psi = 0$$

where $S(x) = g_\sigma \sigma(x)$, $V^\mu(x) = g_\omega \omega^\mu(x) + g_\rho \vec{\tau}\vec{\rho}^\mu(x) + eA^\mu(x)$.
Since meson masses are large Laplace operator can be neglected in the stationary equations for meson fields, in qualitative consideration and σ, ω^0 and ρ_3^0 becomes proportional to the corresponding nuclear densities.

For the total energy we obtain

$$E = \int d^3r H(r) = \sum_{i=1}^{A} \int d^3r \psi_i^+(r)(-\imath\vec{\alpha}\cdot\nabla + \beta M)\psi_i(r)$$
$$+\frac{1}{2}\int d^3r \left(m_\sigma^2\sigma^2 - m_\omega^2(\omega^0)^2 - m_\rho^2(\rho_3^0)^2\right)$$
$$+ \int d^3r \left(g_\sigma\rho_s\sigma + g_\omega\rho_v\omega^0 + g_\rho\rho_3\rho_3^0 + e\rho_c A^0\right).$$

Relativistic energy density functional based on QCD sum rules and ideas of Chiral EFT. N.Finelli, N.Kaiser, D.Vretenar, W.Weise.

The construction of EDF is based on the following conjectures:

1. The nuclear gs is characterized by strong scalar and vector fields which have their origin in the in-medium changes of the scalar quark condensate and of the quark density.

2. The long- and intermediate-range interactions arise from chiral (pionic) fluctuations superimposed on the condensate backgroun fields.

Relativistic energy density functional based on QCD sum rules and ideas of Chiral EFT.

The energy functional is assumed to be decomposed into three terms:

$$F[\rho] = T_{kin}[\rho] + E_H[\rho] + E_{xc}[\rho]$$

E_H is the Hartree energy. We assume that large scalar and vector mean fields, that have their origin in the in-medium changes of the chiral condensate and the quark energy, determine E_H. Chiral (pionic) fluctuations including one- and two-pion exchange are incorporated in the E_{xc}.

Relativistic energy density functional based on QCD sum rules and chiral EFT.

The relativistic Lagrangian includes: isoscalar-scalar (S), isoscalar-vector (V), isovector-scalar (TS) and isovector-vector (TV) effective four-fermion interaction vertices with density dependent coupling strengths.

Relativistic energy density functional based on QCD sum rules and ideas of Chiral EFT.

The gs energy of the even-even nucleus with A-nucleons is presented as:

$$E_0 = \sum_{k=1}^{A} \varepsilon_k - \frac{1}{2}\int d^3r \Big\{ [G_s^{(0)} + G_s^{(\pi)}(\rho)]\rho_s^2 + G_{TS}^{(\pi)}(\rho)\rho_{S3}^2$$
$$+[G_V^{(0)} + G_V^{(\pi)}(\rho)]\rho^2 + G_{TV}^{(\pi)}(\rho)\rho_3^2 + \frac{\partial G_s^{(\pi)}(\rho)}{\partial\rho}\rho_s^2\rho$$
$$+\frac{\partial G_{TS}^{(\pi)}(\rho)}{\partial\rho}\rho_{s3}^2\rho + \frac{\partial G_V^{(\pi)}(\rho)}{\partial\rho}\rho^3 + \frac{\partial G_{TV}^{(\pi)}(\rho)}{\partial\rho}\rho_3^2\rho$$
$$+D_S^{(\pi)}\rho_s\nabla^2\rho_s + e\rho_{ch}A^{(0)} \Big\}$$

where ε_k denotes single particle Kohn-Sham energies.

Relativistic energy density functional based on QCD sum rules and ideas of Chiral EFT.

The coupling constants are decomposed as

$$\begin{aligned} G_i(\hat{\rho}) &= G_i^{(0)} + G_i^{(\pi)}(\hat{\rho}), \text{where} \quad i = S, V \\ G_i(\hat{\rho}) &= G_i^{(\pi)}(\hat{\rho}), \text{where} \quad i = TS, TV \end{aligned}$$

The density independent part arise from isoscalar-scalar and -vector background fields, whereas $G_i^{(\pi)}(\hat{\rho})$ are generated by one- and two-pion exchange dynamics.

QCD suum rules.
M.A.Shifman, A.I,Vainstein, and V.I.Zakharov
B.L.Ioffe
E.G.Drukarev and E.M.Levin

The following estimates follows from the QCD sum rules:

$$G_S^{(0)} = -\frac{\sigma_N M_N}{m_\pi^2 f_\pi^2}$$

$$G_V^{(0)} = -\frac{4(m_u + m_d) M_N}{m_\pi^2 f_\pi^2}$$

$$\frac{\sigma_N}{m_u + m_d}\rho_s = < \bar{q}q >_p - < \bar{q}q >_0$$

Relativistic energy density functional based on QCD sum rules and ideas of Chiral EFT.

The resulting expressions for the density dependent couplings are:

$$
\begin{aligned}
G_s^{(\pi)} &= c_{s1} + c_{s2}\rho^{1/3} + c_{s3}\rho^{2/3} + c_{s4}\rho, \\
G_v^{(\pi)} &= \bar{c}_{v1} + \bar{c}_{v2}\rho^{1/3} + \bar{c}_{v3}\rho^{2/3} + \bar{c}_{v4}\rho, \\
G_{TS}^{(\pi)} &= c_{ts1} + c_{ts2}\rho^{1/3} + c_{ts3}\rho^{2/3} + c_{ts4}\rho, \\
G_{TV}^{(\pi)} &= c_{tv1} + c_{tv2}\rho^{1/3} + c_{tv3}\rho^{2/3} + c_{tv4}\rho,
\end{aligned}
$$

Relativistic energy density functional based on QCD sum rules and chiral EFT.

The quantitative accuracy of the calculated binding energies and radii is such that deviations from empirical data are usually less than 0.5% throuhgout the nuclear chart. It has to be noted, however, that these results still do not reach the accuracy of the best phenomenological mass tables (modern Skyrme based phenomenological mass formulas).

Relativistic energy density functional based on QCD sum rules and ideas of Chiral EFT.

The nuclear energy density functional developed above contains at most 7 significant parameters, each clearly related to specific properties of nuclei. The values of the parameters are adjusted to the properties of nuclear matter, binding energies, charge radii and differences between proton and neutron radii of spherical nuclei.
The resulting optimal parameter set is remarkably close to the anticipated QCD sum rules and Chiral perturbation theory values, with exception of two constants associated with 3-body correlations.

Chiral Effective Field Theory and Magic Numbers of Nucleons.

To explain the observed shell structure strong spin-orbit coupling term, comparable in magnitude to the strong nuclear force, was introduced into the single particle Hamiltonian.
Meanwhile, single particle states with quantum numbers (n,l,j=l+1/2) and (n-1,l+2,j=l+3/2)
were found to be nearly degenerate, leading to introduction of a pseudo orbital angular momentum $\tilde{l}$=l+1 and the concept of pseudospin symmetry.

In this regard, the problem arise to explane the large spin-orbit potential starting from nuclear force obtained in the framework of the ChEFT and employing nuclear $ab \quad initio$ methods. For this aim SRG and IMSRG can be used to decouple the Hamiltonian matrix elements connecting high- and low-momentum states.

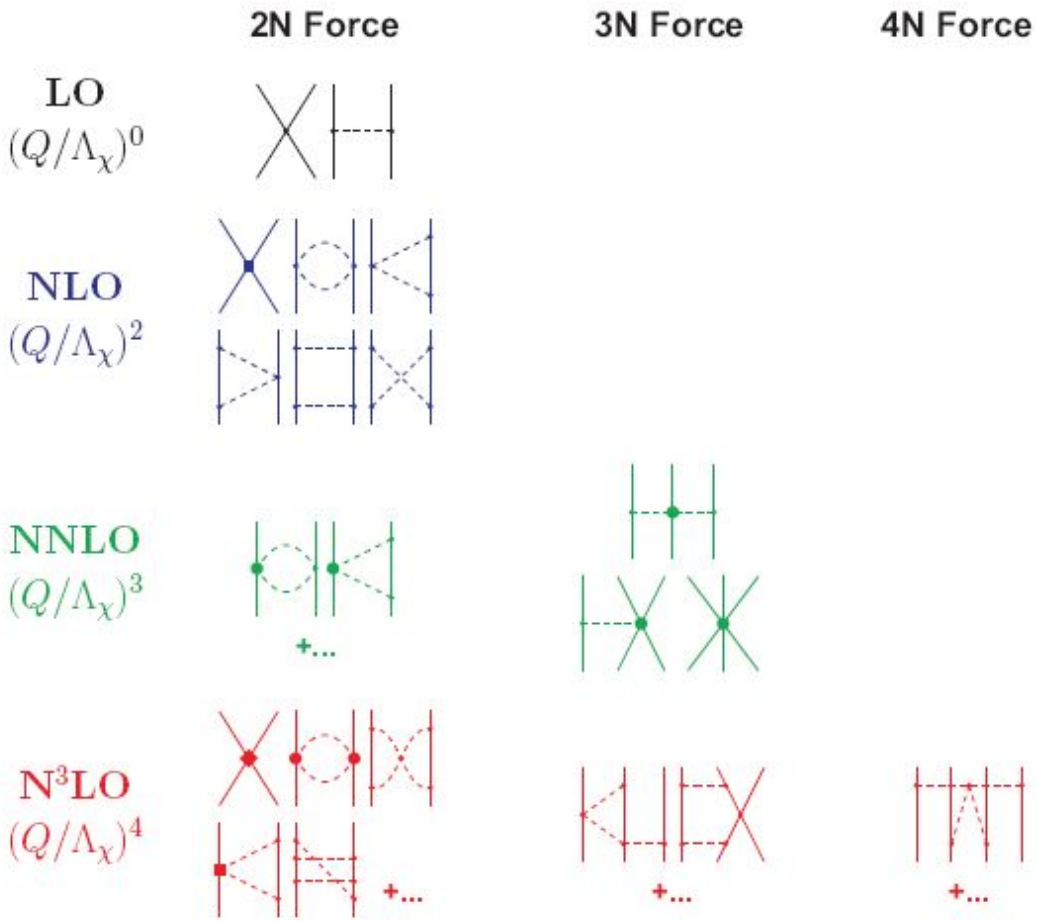


FIGURE 1. Hierarchy of nuclear forces in ChPT. Solid lines represent nucleons and dashed lines pions. Small dots, large solid dots, solid squares, and solid diamonds denote vertices of index $\Delta = 0, 1, 2$, and 4, respectively. Further explanations are given in the text.

1. The chiral dimension of the given Feynman diagram is $\nu = 4L - 2I_\pi - I_N + \sum_i d_i V_i$. L is a number of loops; $I_{N(I_\pi)}$ a number of nucleon(pion) propagators; d_i is a number of derivetives/pion masses for vertex of type i; V_i is a number of vertices of type i.

2. The goal to modify power counting ν is to provide predictions independent of cutoff.

3. m_N must be treated as larger than breakdown scale ($m_N \sim \frac{\Lambda^2}{Q}$).

4. OPE potential is singular in some partial waves.

5. Singular potentials can be modified to possess a well-defined solution by adding local contact term.

6. The strength of nuclear interaction decreases as orbital momentum increases. Thus, only a few attractive singular partial waves need to be provided with an additional contact term.

Low-momentum technology for two-nucleon interaction

Example of technique used to derive low-momentum NN interaction

$$T(k',k;k^2) = V_{NN}(k',k) + \frac{2}{\pi}\int^{\Lambda_\infty} \frac{V_{NN}(k',p)T(p,k;k^2)}{k^2-p^2}p^2dp$$
$$= V^{\Lambda}_{low\,k}(k',k) + \frac{2}{\pi}\int^{\Lambda} \frac{V^{\Lambda}_{low\,k}(k',p)T(p,k;k^2)}{k^2-p^2}p^2dp$$

for all $k, k' < \Lambda$. Observables for these momenta, such as phase shifts and deutron binding energy, are preserved.

Imposing $dT(k', k; k^2)/d\Lambda$=0 gives RG equation

$$\frac{dV^{\Lambda}_{low\,k}(k', k)}{d\Lambda} = \frac{2}{\pi}\frac{V^{\Lambda}_{low\,k}(k', \Lambda)T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}.$$

Different initial V_{NN} interactions collapse to the same nearly universal $V_{low\,k}$ at sufficiently small cutoff ($\Lambda \sim 2.0 fm^{-1}$).

S.K.Bogner et al. Phys.Lett. B 576 (2003) 265.

Similarity renormalization group (SRG)

An alternative path to decouple high-momentum from low-momentum physics is SRG, which is based on a continuous sequence of unitary transformations that suppress off-diagonal matrix elements driving the Hamiltonian towards a band diagonal form. The SRG potentials are automatically energy independent.
The evolution of the Hamiltonian with the parameter „s“ is a series of unitary transformations:

$$H_s = U_s H U_s^+ \equiv T_{rel} + V_s$$

$$H = T_{rel} + V$$

$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = \frac{dU_s}{ds} U_s = -\eta_s^+,$$

$$\eta_s = [G_s, H_s].$$

and

$$\frac{dH_s}{ds} = [[G_s, H_s], H_s].$$

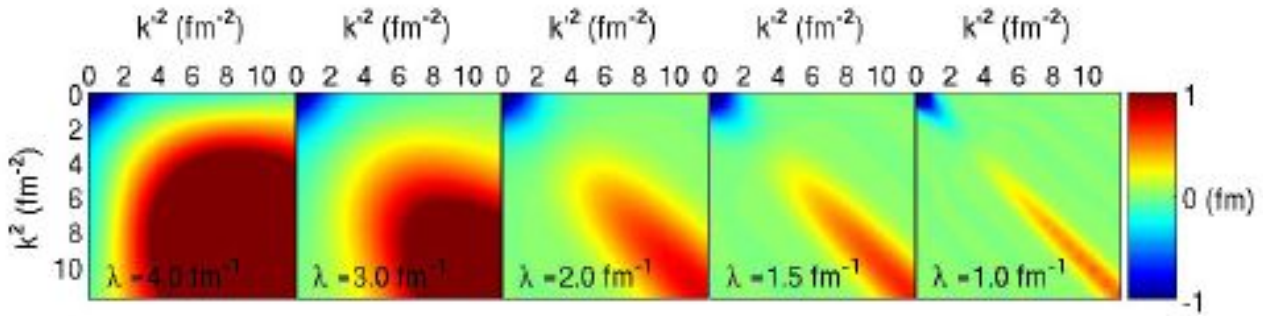
Applications to nuclear forces have used $G_s = T_{rel}$. Then it is possible to derive the following equation

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)V_s(k,k')$$
$$+\frac{2}{\pi}\int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2)V_s(k,q)V_s(q,k')$$

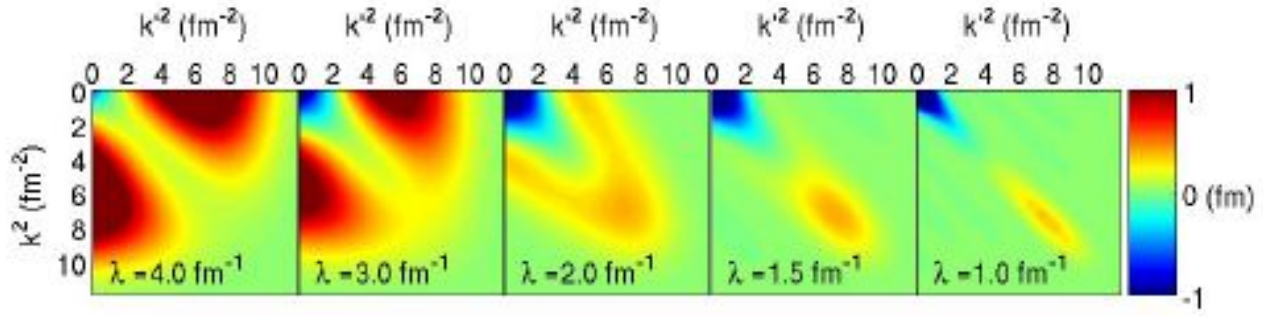
For matrix element far from diagonal

$$V_s(k,k') \approx V_{s=0}(k,k')e^{-s(k^2-k'^2)^2},$$

$\lambda \equiv s^{-1/4}\,(fm^{-1})$


k'² (fm⁻²)
k² (fm⁻²)
λ =4.0 fm⁻¹
λ =3.0 fm⁻¹
λ =2.0 fm⁻¹
λ =1.5 fm⁻¹
λ =1.0 fm⁻¹
1
0 (fm)
-1

(a)


k'² (fm⁻²)
k² (fm⁻²)
λ =4.0 fm⁻¹
λ =3.0 fm⁻¹
λ =2.0 fm⁻¹
λ =1.5 fm⁻¹
λ =1.0 fm⁻¹
1
0 (fm)
-1

(b)

The key to the simplification in the A-body case is in the use of the normal ordering with respect to a finite-density reference state. Starting from second quantized Hamiltonian with two- and three-body interactions

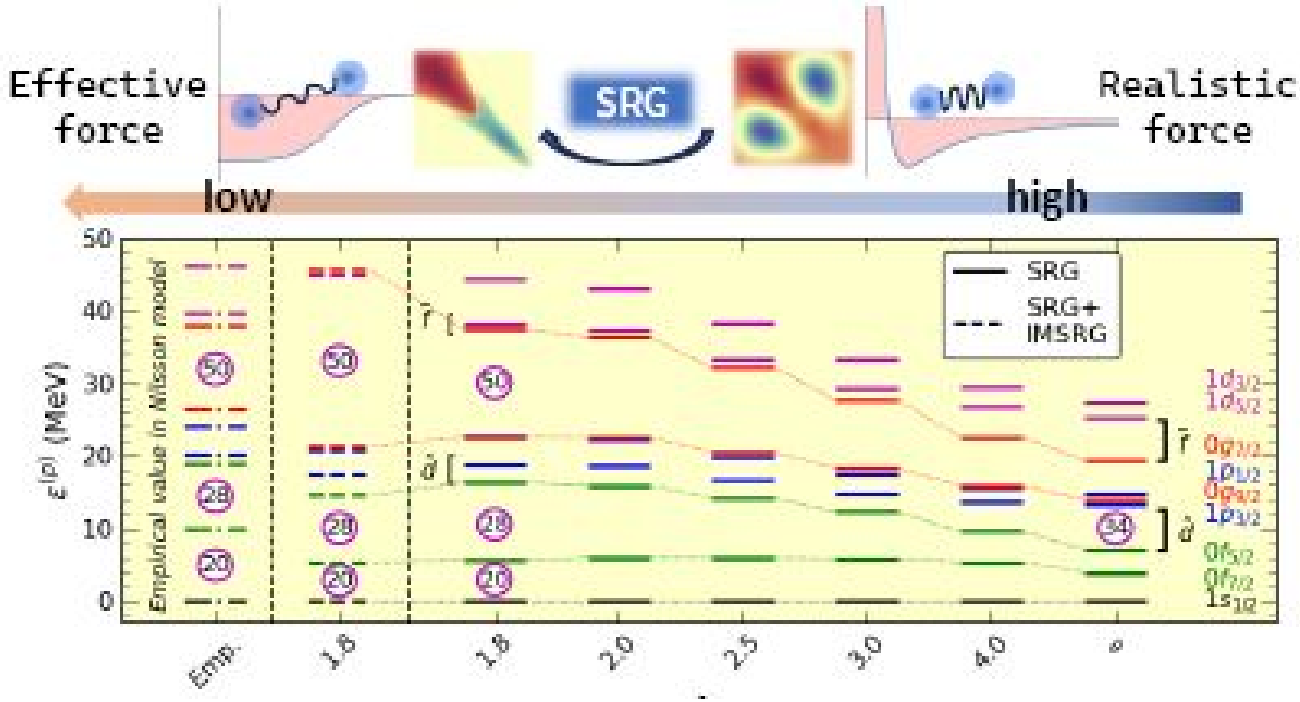
$$H = \sum_{12} T_{12} a_1^+ a_2 + \frac{1}{(2!)^2} \sum_{1234} < 12|V|34 > a_1^+ a_2^+ a_4 a_3 + \frac{1}{(3!)^2} \sum_{123456} < 123|V^{(3)}|456 > a_1^+ a_2^+ a_3^+ a_6 a_5 a_4 \quad (5)$$

and putting all operators in normal ordering with respect to finite-density Fermi vacuum $|\Phi >$ we obtain

$$H = E_0 + \sum_{12} f_{12}\{a_1^+ a_2\}$$
$$+\frac{1}{(2!)^2}\sum_{1234} < 12|\Gamma|34 > \{a_1^+ a_2^+ a_4 a_3\}$$
$$+\frac{1}{(3!)^2}\sum_{123456} < 123|\Gamma^{(3)}|456 > \{a_1^+ a_2^+ a_3^+ a_6 a_5 a_4\} \qquad (6)$$

The in-medium SRG evolution equations are obtained by evaluating $\frac{dH}{ds} = [\eta, H]$ with $H = E_0 + f + \Gamma$ and $\eta = [f, \Gamma]$.

SRG equations are iterated with increments δs. At each additional increment δs, the interaction from the previous step are inserted back into the right side of the SRG equations.
IT is convenient to switch to the variable $\lambda = s^{-1/4}$. In the limit $\lambda \to 0$, Hartree-Fock becomes exact for the evolved Hamiltonian; the zero-body term, E_0, approaches the interacting ground state energy, f approaches fully-dressed single-particle energies and the remaining matrix elements Γ approach a generalization of the quasiparticle interaction in Landau's theory of Fermi liquids.


Effective force
SRG
Realistic force
low
high
SRG
SRG+ IMSRG
Empirical value in Nilsson model
$\varepsilon^{(p)}$ (MeV)
50
40
30
20
10
0
Emp.
1.6
1.8
2.0
2.5
3.0
4.0
∞
$1d_{3/2}$
$1d_{5/2}$
$0g_{7/2}$
$1p_{1/2}$
$0g_{9/2}$
$1p_{3/2}$
$0f_{5/2}$
$0f_{7/2}$
$1s_{1/2}$

Conclusion

1. The parameters of the NEDF are fitted without reference to any NN-interaction.

2. NEDF is given as expansion in degrees of density and currents (excluding Fayans functional).

3. The changes of the quark condensate and quark density in the presence of the barionic matter are sources of strong scalar (attractive) and vector (repulsive) fields experienced by nucleons in the nucleus. These fields produce Hartree mean field nucleon potential, and are at the origin of the large energy spacings between spin-orbit partner states in nuclei.

Comments

Until now, it has not been possible to construct an interaction that would satisfy three basic conditions:

- Was realistic, i.e. described NN-phases.

- Correctly described the binding energies at the observed nuclear radius.

- Provided a good description of spectroscopy.