Nuclear reflection asymmetry in cluster approach

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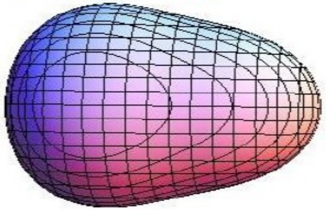
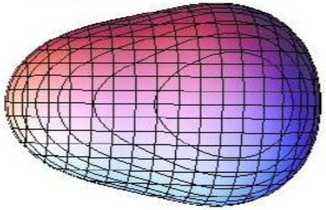
Reflection Asymmetric Deformation

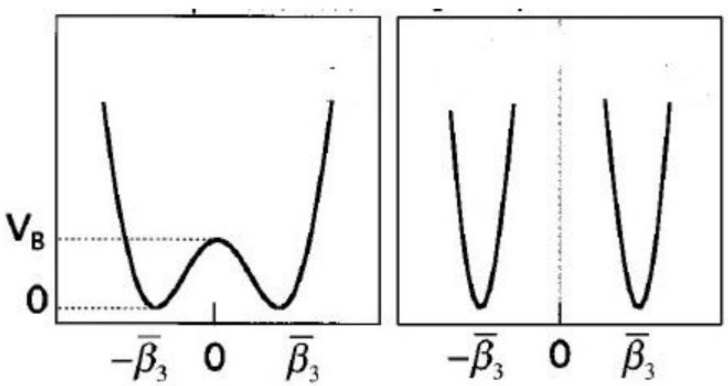
Intrinsic states $\Psi(\beta_{30})$ and $\Psi(-\beta_{30})$ are physically equivalent.

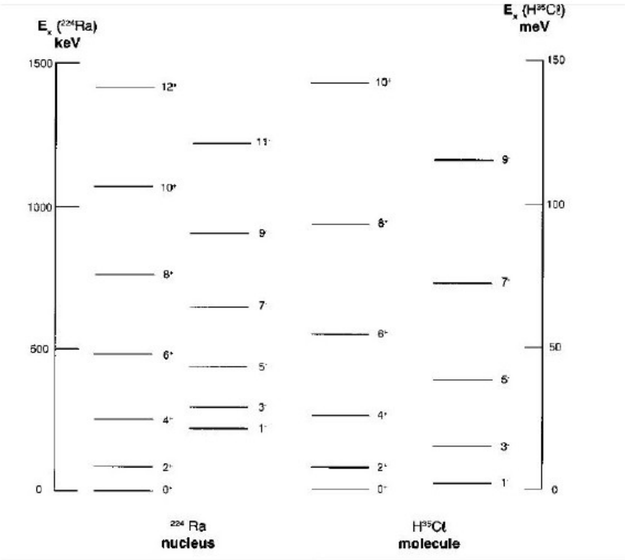
β_{20}=0.6, β_{30}=-0.5



β_{20}=0.6, β_{30}=0.5







$$\Psi_{p,IMK} = \sqrt{\frac{2I+1}{16\pi^2}}\left(\Phi_{n,K}(\xi)D^I_{MK} + p(-1)^{I+K}\Phi_{n,\overline{K}}(\xi)D^I_{M,-K}\right)$$

Wave function in ξ defined by the equation:

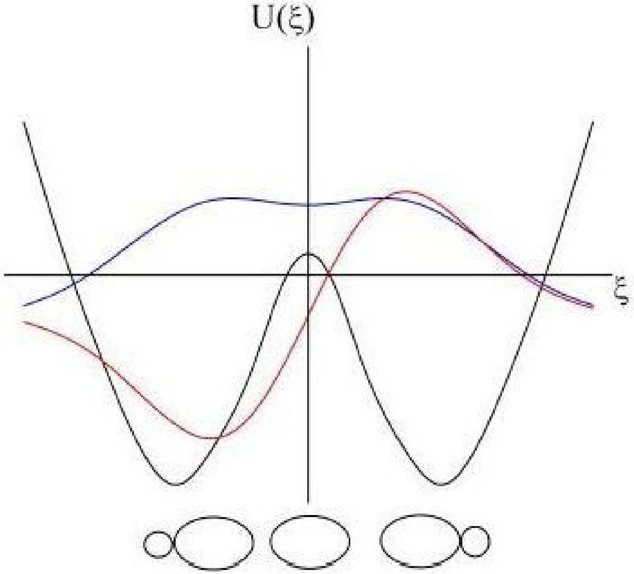
$$\left(-\frac{\hbar^2}{2B_\xi}\frac{d^2}{d\xi^2} + U(\xi) + \frac{\hbar^2}{2\Im(\xi)}I(I+1)\right)\Psi_{n,K}(\xi) = E_{n,K}\Psi_{n,K}(\xi),$$

where

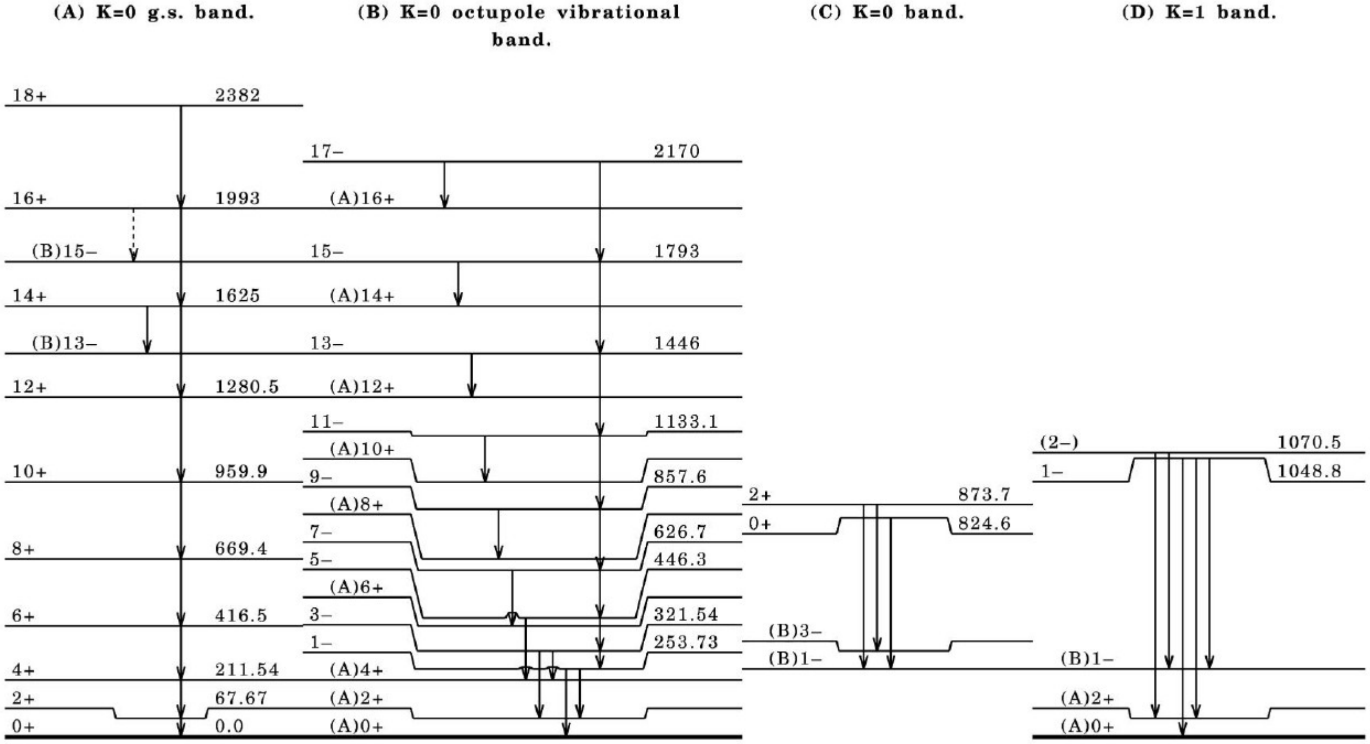
$$\Im(\xi) = 0.85(\Im_1^r + \Im_2^r + m_0\frac{A_1A_2}{A}R^2)$$

Exitation spectra:

$$I^p(\text{ for } K=0) = 0^+, 1^-, 2^+ ...$$
$$I^p(\text{ for } K\neq 0) = K^\pm, (K+1)^\pm ...$$

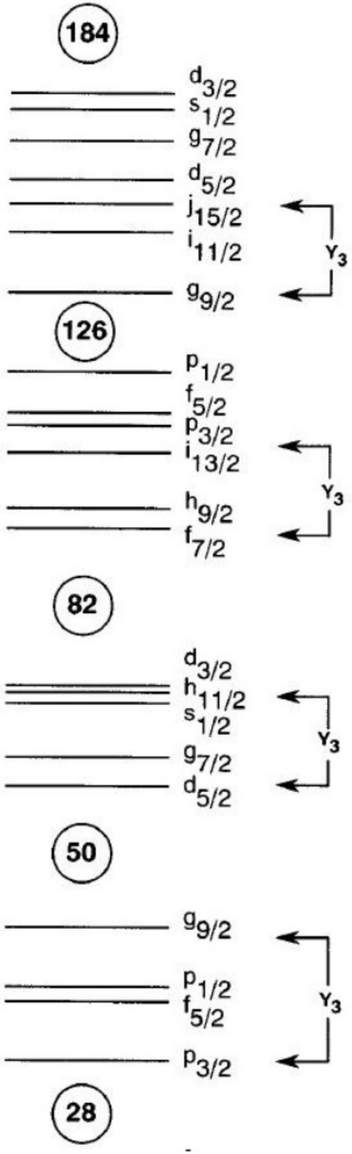


Experimental spectrum : ^{226}Ra



(from www.nndc.bnl.gov/ensdf)

ΔL= ΔJ=3 or clustering?



The strong reflection asymmetric correlations near the ground state can be microscopically associated with the appearance of **orbital pairs with $\Delta j = \Delta l = 3$ near the Fermi surface.**

Besides the actinides, the similar situation occurs in the nuclei with masses near **A ~56 and A ~ 134** that is in agreement with the experimental data.

The results of calculations within the shell-corrected liquid drop models and mean-field models show that nuclei in these mass Regions are either soft with respect to the octupole deformation or even octupole-deformed.

Nuclear spherical single-particle leveles.
Most important octupole couplings are indicated.

P.A. Butler & W. Nazarewicz, Rev. Mod. Phys. 68, 349 (1996)

Motion in mass-asymmetry

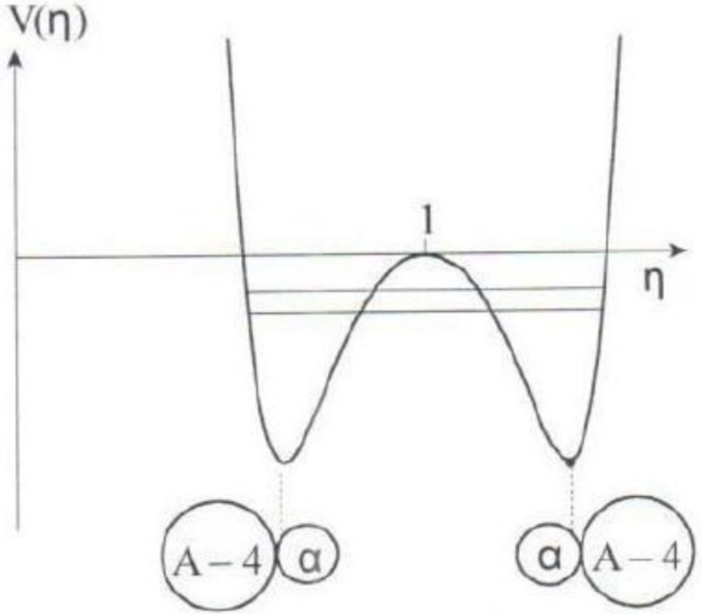
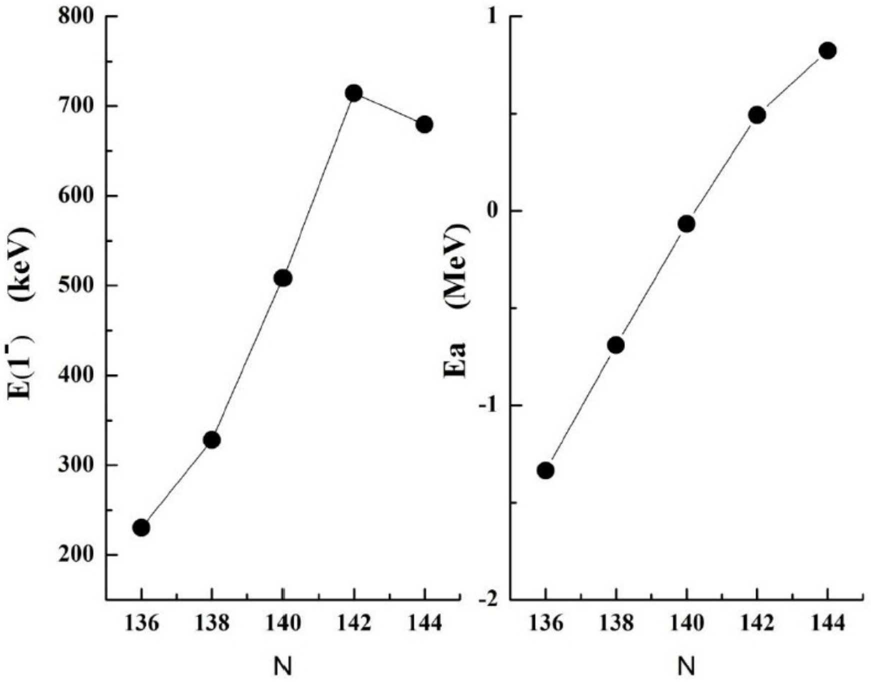


Fig. 2. Schematic picture of the potential in the mass asymmetry and of the two states with different parities (parallel lines, lower state is with positive parity, higher state is with negative parity).

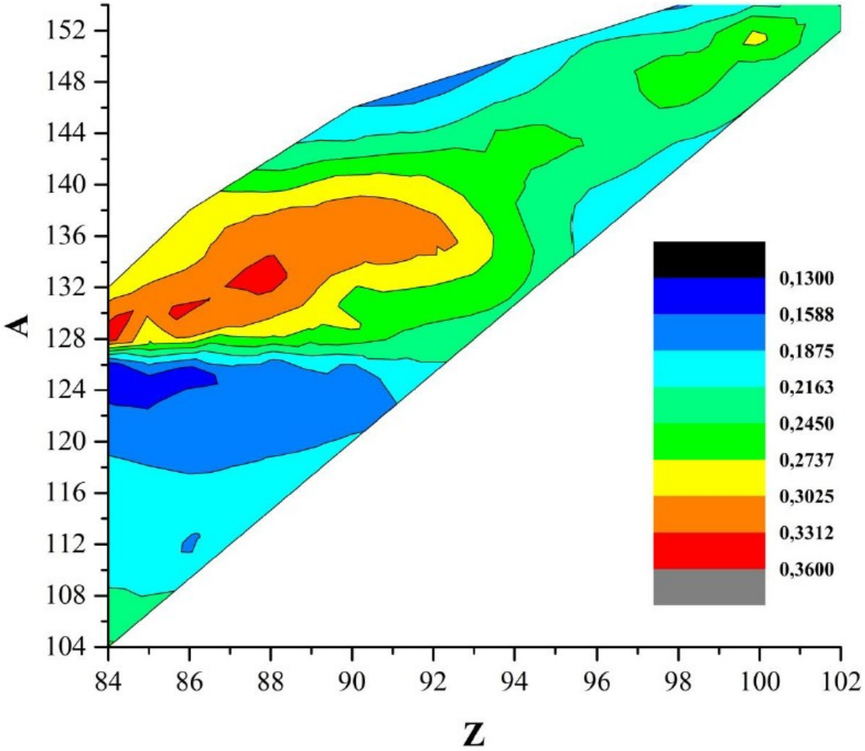
Energy of α–DNS for Th isotopes

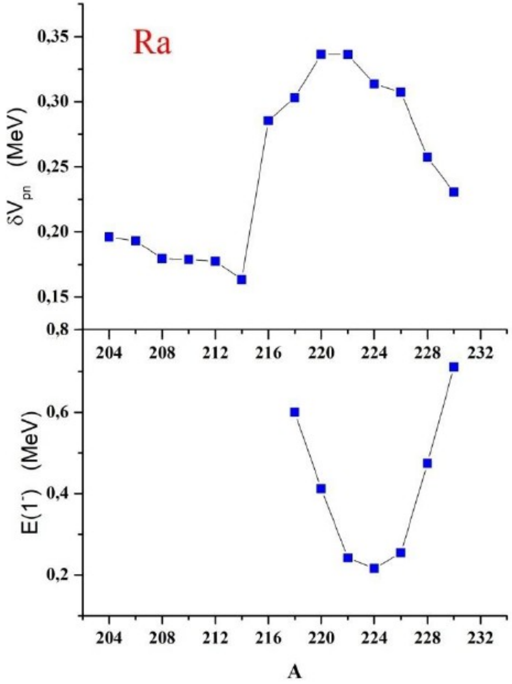


The dynamics of the reflection asymmetric collective motion can be treat as the motion in mass-asymmetry degrees of freedom.

$$\delta V_{pn} = -0.25\left[B(Z,N) - B(Z,N-2) - B(Z-2,N) + B(Z-2,N-2)\right]$$
$$= -0.25\left[S_{2n} + S_{2p} - S_{\alpha}\right]$$

R.F. Casten & R.B. Cakirli, Phys. Scri 91, 033004 (2016)





Alpha-decay half-lives

The half-life of the parent nucleus (Z,A) against alpha-decay

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma}.$$

Width of alpha-decay

$$\Gamma = \frac{\hbar \omega_0}{\pi} S_\alpha P_\alpha$$

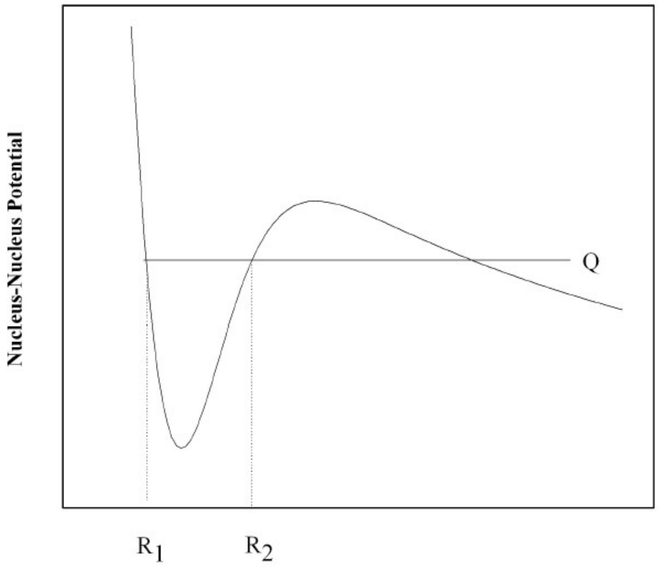
.

ω_0 – assault frequency;
S_α – alpha-particle preformation factor;
P_α – probability of penetration through the barrier in nucleus-nucleus potential.

WKB Approximation

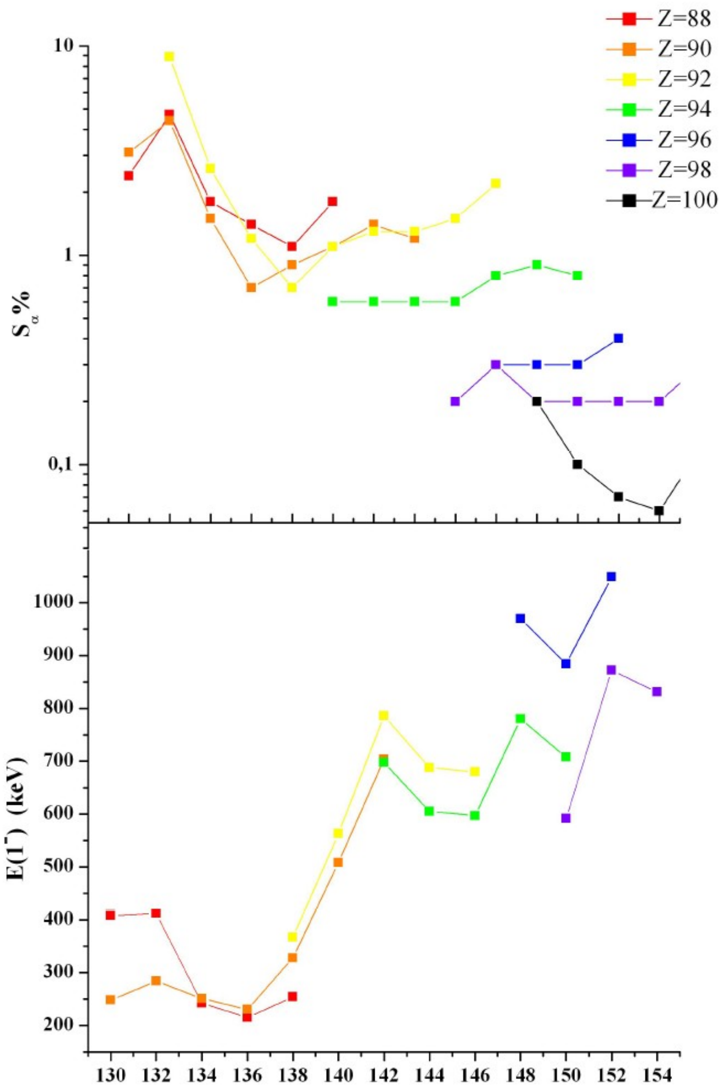
According to the WKB approximation the probability

$$P_\alpha = \exp\left[-\frac{2}{\hbar}\int_{R_1}^{R_2}\sqrt{2\mu(U(R,\eta_\alpha)-Q)}\right]$$



R_1, R_2 – are the turning points of the WKB action intergral determined by the equation

$$U(R_{1,2},\eta_\alpha) = Q$$



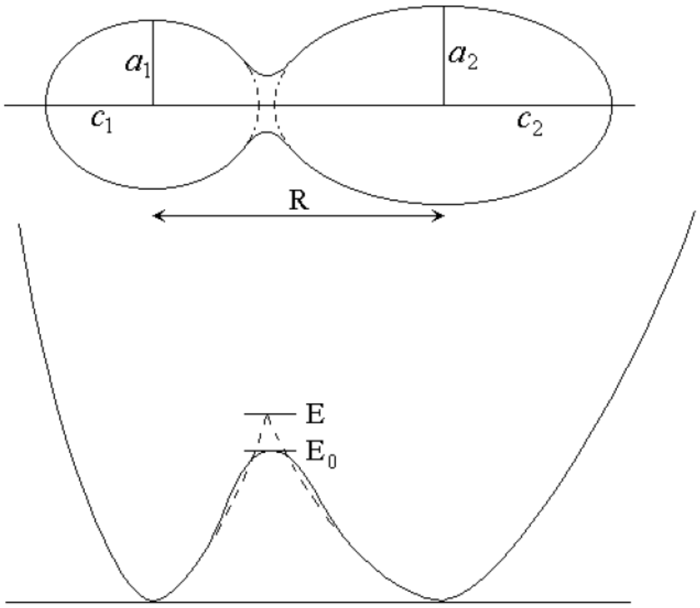
The value of α-particle preformation factor obtained from the experiment as:

$$S_\alpha^{exp} = T_{1/2}^{exp} / T_{1/2}^{\alpha}$$

$T_{1/2}^{\alpha}$ – *half-life of α-particle dinuclear system.*

Energies of $E(1^-)$–*states as a function of neutron number.*

Two-center shell model



J. Maruhn and W. Greiner, Z. Phys. 251 (1972) 431.

Instead of using the deformation parameters one can consider the degrees of freedom related to the two-center shell model or to the dinuclear system model.

G. G. Adamian et al., Int. J. Mod. Phys. E5 (1996) 191.

Degrees of Freedom of Dinuclear System Model

The dinuclear system (A,Z) consists of a configuration of two touching nuclei (clusters) (A_1,Z_1) and (A_2,Z_2) with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$, which keep their individuality.

DNS has totally 15 collective degrees of freedom which govern its dynamics.

- *Relative motion of the clusters* $\mathbf{R} = (R, \theta_R, \phi_R)$
- *Rotation of the clusters* $\Omega_1 = (\phi_1, \theta_1, \chi_1)$, $\Omega_2 = (\phi_2, \theta_2, \chi_2)$
- *Intrinsic excitations of the clusters* (β_1, γ_1), (β_2, γ_2)
- *Nucleon transfer between the clusters* ξ, ξ_Z

Mass asymmetry $\xi = \frac{2A_2}{A_1+A_2}$. Charge asymmetry $\xi_Z = \frac{2Z_2}{Z_1+Z_2}$

Nuclear wave function is thought as a superposition of DNS configurations with various mass asymmetries, including mononucleus (no clusters) configurations.

Potential Energy of the Dinuclear System

$$U(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) = B_1(\beta^{(1)}) + B_2(\beta^{(2)}) - B_{12} + V(R,\xi,\beta_{2\mu})$$

where, B_1, B_2 and B_{12} are the binding energies of the fragments and the compound nucleus, respectively.

The nucleus-nucleus potential

$$V(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) = V_{Coul}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) + V_{nucl}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)})$$

is the sum of the nuclear interaction potential $V_{nucl}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)})$

and of the Coulomb potential

$$V_{Coul}(R,\xi,\beta_{2\mu}) = \frac{e^2 Z_1 Z_2}{R} + \frac{3}{5}\frac{e^2 Z_1 Z_2}{R^3} R_{01}^2 \sum_{i,\mu} \beta_{2\mu}^{(i)*} Y_{2\mu}(\theta_i,\phi_i) + ...$$

Nuclear Interaction in Dinuclear System

$$V_{nucl}(R,\xi,\beta_{2\mu}) = \int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{R}-\mathbf{r}_2)F(\mathbf{r}_1-\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2$$

$$\rho_i(\mathbf{r}) = \frac{\rho_{00}}{1+\exp\left(\frac{s(\mathrm{r})}{a_{0i}}\right)}, \quad \rho_{00} = 0.17\ \mathrm{fm}^{-3}$$

$$F(\mathbf{r}_1-\mathbf{r}_2) = C_0\left(F_{in}\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}} + F_{ex}\left(1-\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}}\right)\right)\delta(\mathbf{r}_1-\mathbf{r}_2)$$

$$\rho_0(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r})$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{N_1-Z_1}{A_1}\,\frac{N_2-Z_2}{A_2}$$

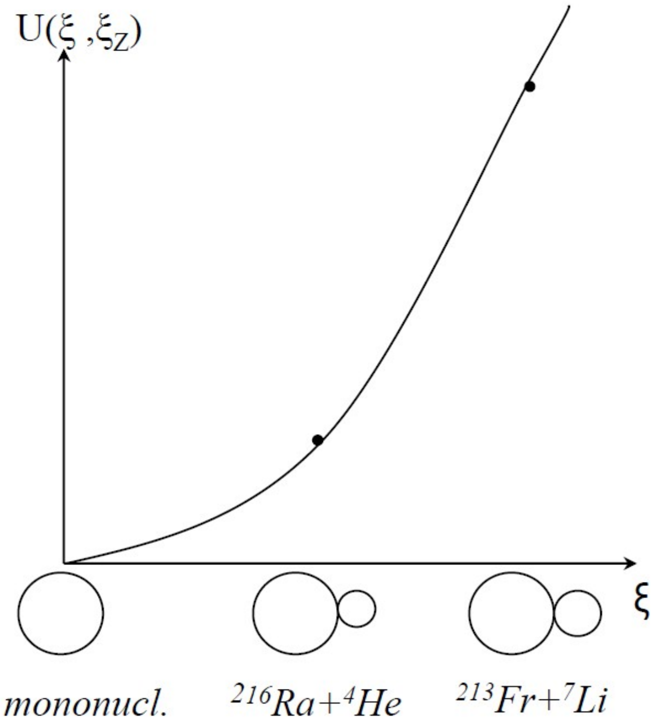
$$C_0 = 300\ \mathrm{MeV\ fm}^3,\ \ f_{in} = 0.09,\ \ f_{ex} = -2.59,\ \ f'_{in} = 0.42,\ \ f'_{ex} = 0.54$$

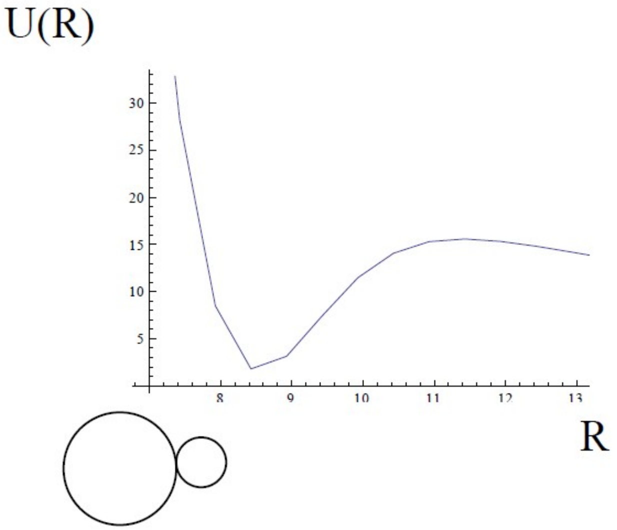
Potential Energy of the Dinuclear System

$$U(\xi,\xi_Z) = B_1(\xi,\xi_Z) + B_2(\xi,\xi_Z) - B + V(\mathbf{R}=\mathbf{R_m},\xi,\xi_\mathbf{Z})$$

$$V(\mathbf{R},\xi,\xi_\mathbf{Z}) = \mathbf{V_{coul}}(\mathbf{R},\xi,\xi_\mathbf{Z}) + \mathbf{V_N}(\mathbf{R},\xi,\xi_\mathbf{Z})$$

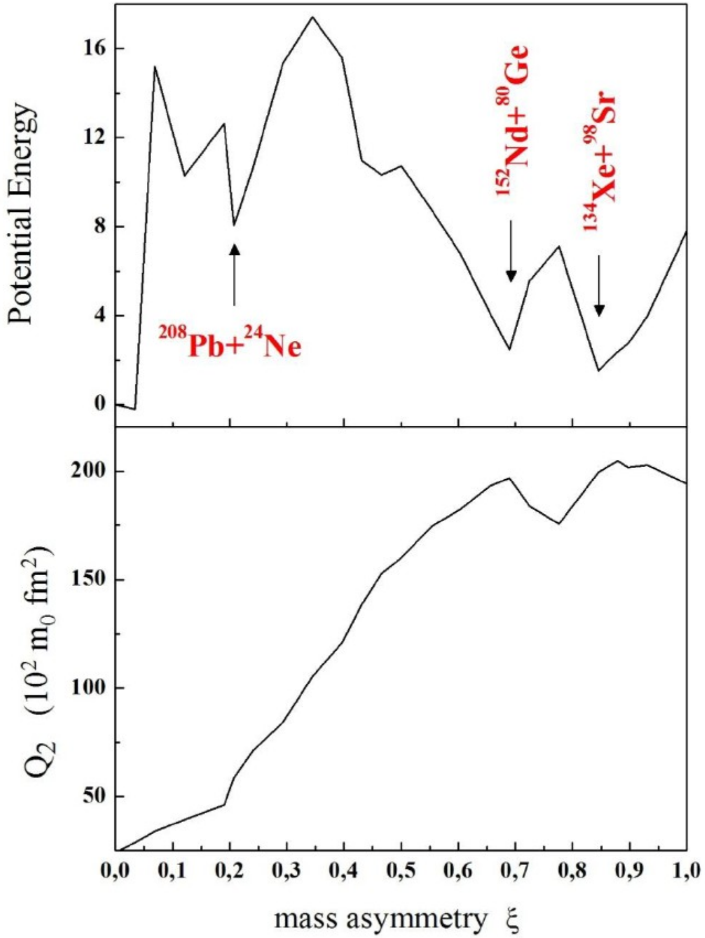
$$V_N(\mathbf{R},\xi,\xi_Z) = \int \rho_1(\mathbf{r_1})\rho_2(\mathbf{r_2})F(\mathbf{r_1}-\mathbf{r_2}+\mathbf{R})d\mathbf{r_1}d\mathbf{r_2}$$





Touching configuration
$R=R_m$

Driving Potential for ^{232}U



The potential energy of the DNS

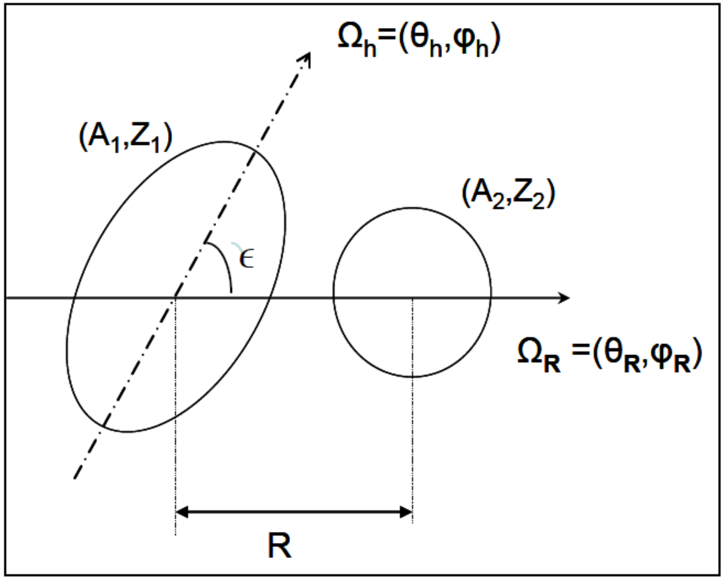
$$V(\xi) = E_1(\xi) + E_2(\xi) + V_N(R,\xi) + V_C(R,\xi)$$

Mass quadrupole moments of the DNS

$$Q_2(\xi,R) = 2m_0 \frac{A_1 A_2}{A_1 + A_2} R^2 + Q_2(A_1) + Q_2(A_2)$$

Degrees of freedom (for Simplified Hamiltonian)

We assume that collective motion of nucleus in mass-asymmetry degree of freedom leads to the admixture of the very asymmetric cluster configurations to the intrinsic nucleus wave function and creates deformations with even- and odd-multipolarities.



-Mass-asymmetry motion, $\xi=2A_2/A$

-Rotation of the molecular system, $\boldsymbol{\Omega_R=(\theta_R,\varphi_R)}$

-Rotation of the heavy cluster, $\boldsymbol{\Omega_h=(\theta_h,\varphi_h)}$ *(for deformed heavy fragment)*

or

-harmonic quadrupole oscillations of the heavy fragment, $\boldsymbol{\beta_{2\mu}}$ *(for spherical heavy fragment)*

Approximation of the Potential Energy

$$U(\xi,\epsilon) = V(\xi) + \frac{C_0\xi}{2}\sin^2(\epsilon)$$

$$V(\xi) = \sum_{n=0,2,4,6} a_n \xi^n$$

$$\sin^2\epsilon = \frac{2}{3}\left(1 - \frac{4\pi}{5}\left[Y_2(\Omega_h) \times Y_2(\Omega_{\mathbf{R}})\right]_{(0,0)}\right)$$

a_{2n} are fitted to reproduce
exp. binding energy of nucleus;
calc. energy of the α–DNS;
calc. energy of the Li–DNS;

Nucleus	β	$C\xi_\alpha$
^{230}U	0.173	3.57
^{232}U	0.182	3.96
^{234}U	0.198	4.76
^{236}U	0.207	5.23
^{238}U	0.215	5.57

Hamiltonian of the Model
(Deformed heavy fragment)

Hamiltonian

$$\boxed{\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{V}_{int} \\ \hat{H}_0 &= -\frac{\hbar^2}{2B}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + \frac{\hbar^2}{2\Im_h}\hat{l}_h^2 + \frac{\hbar^2}{2\mu R_m^2}\hat{l}_{\mathbf{R}}^2 + V_0(\xi) \\ \hat{V}_{int} &= \frac{C_1\xi}{2}\sum_{\mu} Y_{2\mu}^*(\Omega_h)Y_{2\mu}(\Omega_{\mathbf{R}})\end{aligned}}$$

eq2

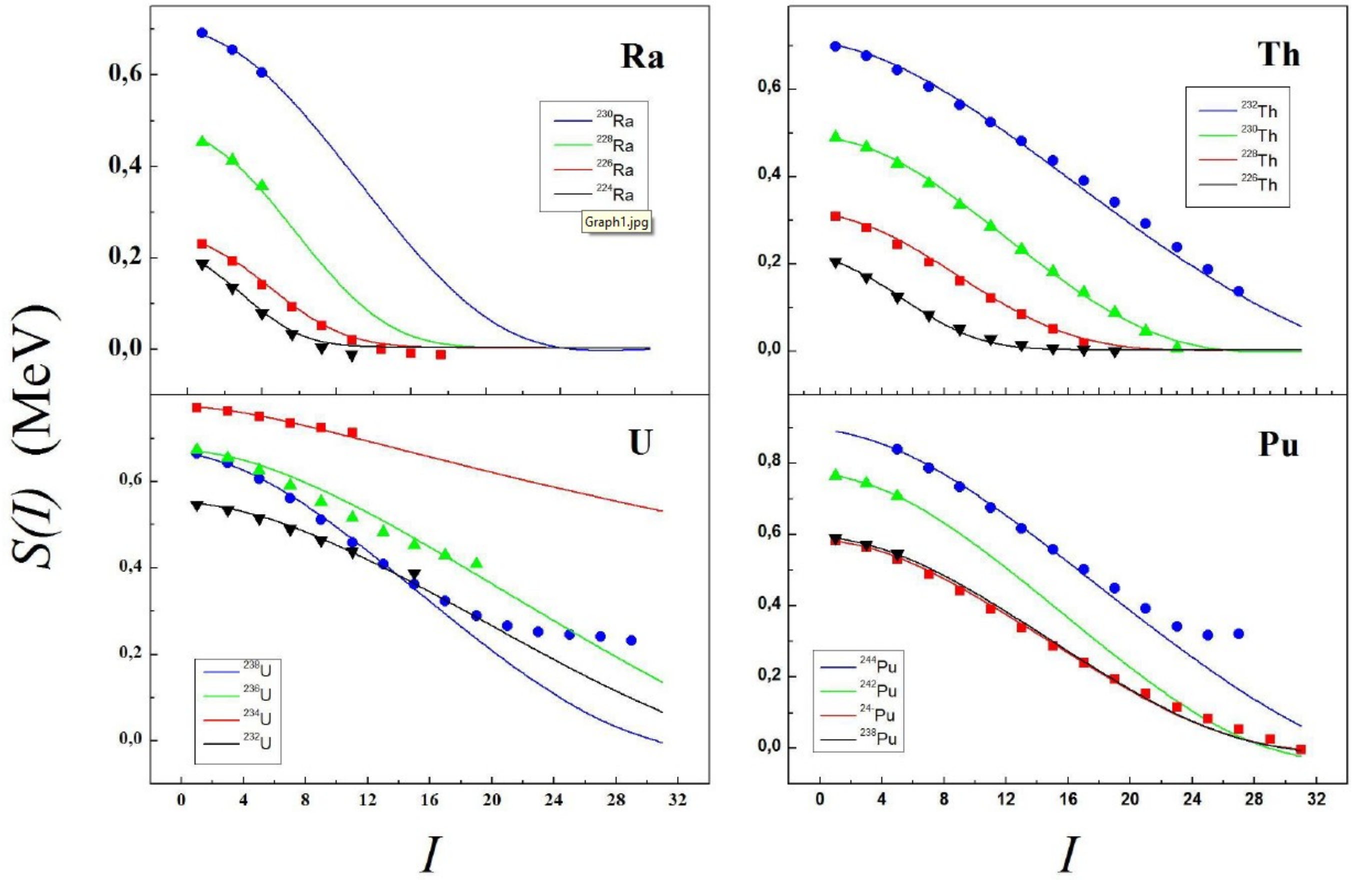
Wave Functions

$$\Psi_{LM\pi} = \sum_{l_h l_R n} a_{nl_hl_R}^{(LM)}(\xi)\left[Y_{l_h}(\Omega_h) \times Y_{l_R}(\Omega_R)\right]_{(L,M)}$$

Parity of the states is determined by the angular momentum of the relative motion

$$\pi = (-1)^{l_R}$$

Parity splitting in alternating parity bands



$$S(I^-) = E(I^-) - \frac{(I+1)E^+_{(I-1)} + IE^+_{(I+1)}}{2I+1}$$

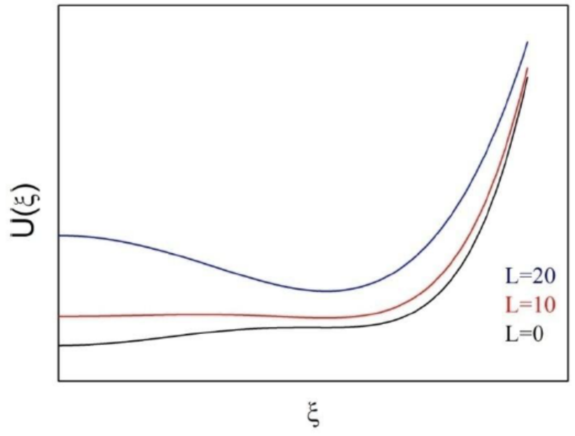
EPJ WC 107, 03009, (2016)

Angular Momentum Dependence of the Parity Splitting

Assumption for wave function

$$\Psi_{LM}(\xi,\Omega_h,\Omega_R)=\Phi(\xi,L)g_{LM}(\xi_0,\Omega_h,\Omega_R)$$

where ξ_0 is the average mass asymmetry.



Hamiltonian in mass asymmetry

$$H(\xi,L)=-\frac{\hbar^2}{2B}\frac{1}{\xi^{3/2}}\frac{\partial}{\partial\xi}\xi^{3/2}\frac{\partial}{\partial\xi}+U_0(\xi)+\frac{\hbar^2 L(L+1)}{2J(\xi)}$$

$$\xi=0;$$

$$U(\xi,L)=U(\xi,L=0)+\frac{\hbar^2}{2}\frac{L(L+1)}{J_h}$$

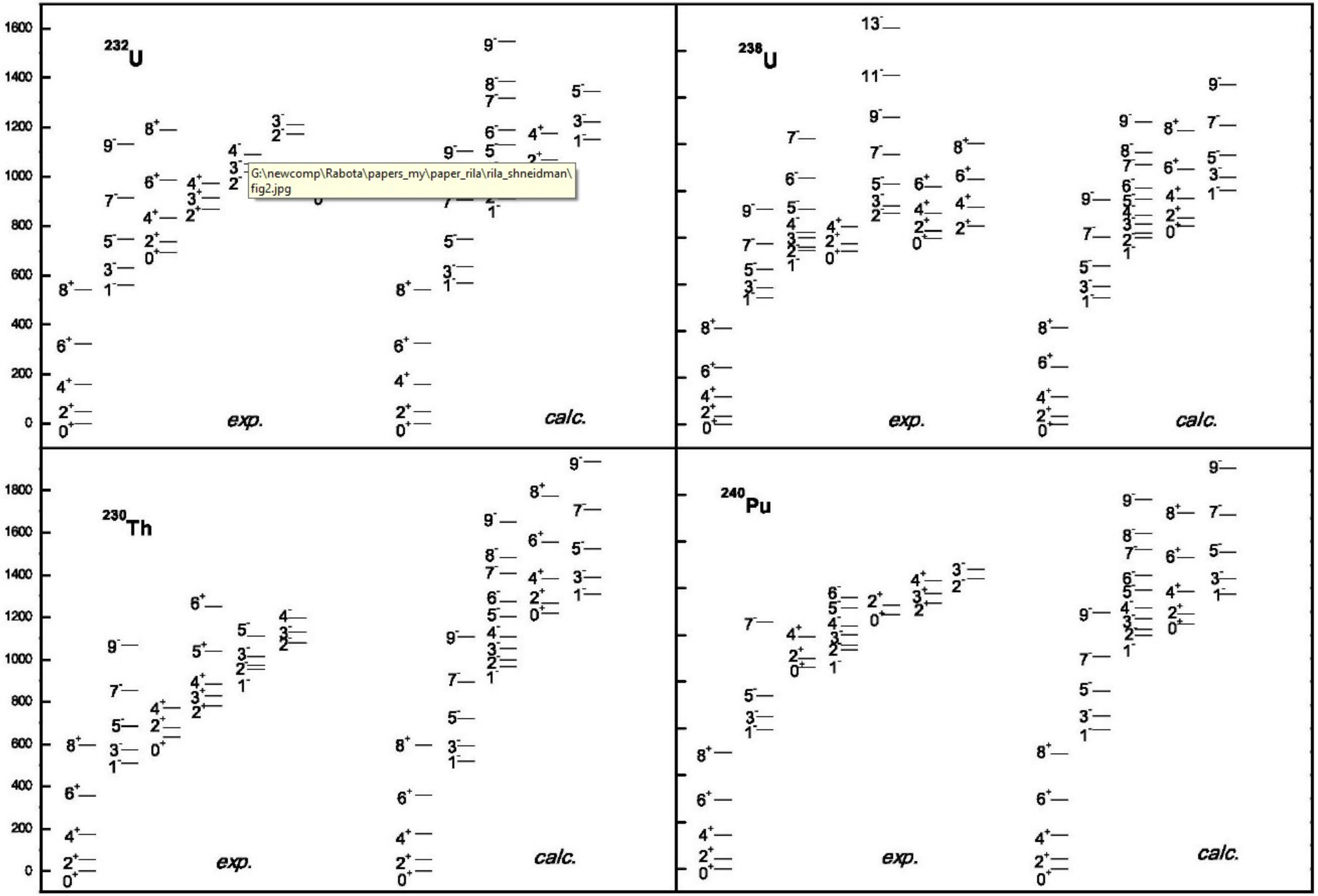
$$\xi=1;$$

$$U(\xi,L)=U(\xi,L=0)+\frac{\hbar^2}{2}\frac{L(L+1)}{J_{tot}}$$

$$\boxed{J_{tot}>J_h}$$

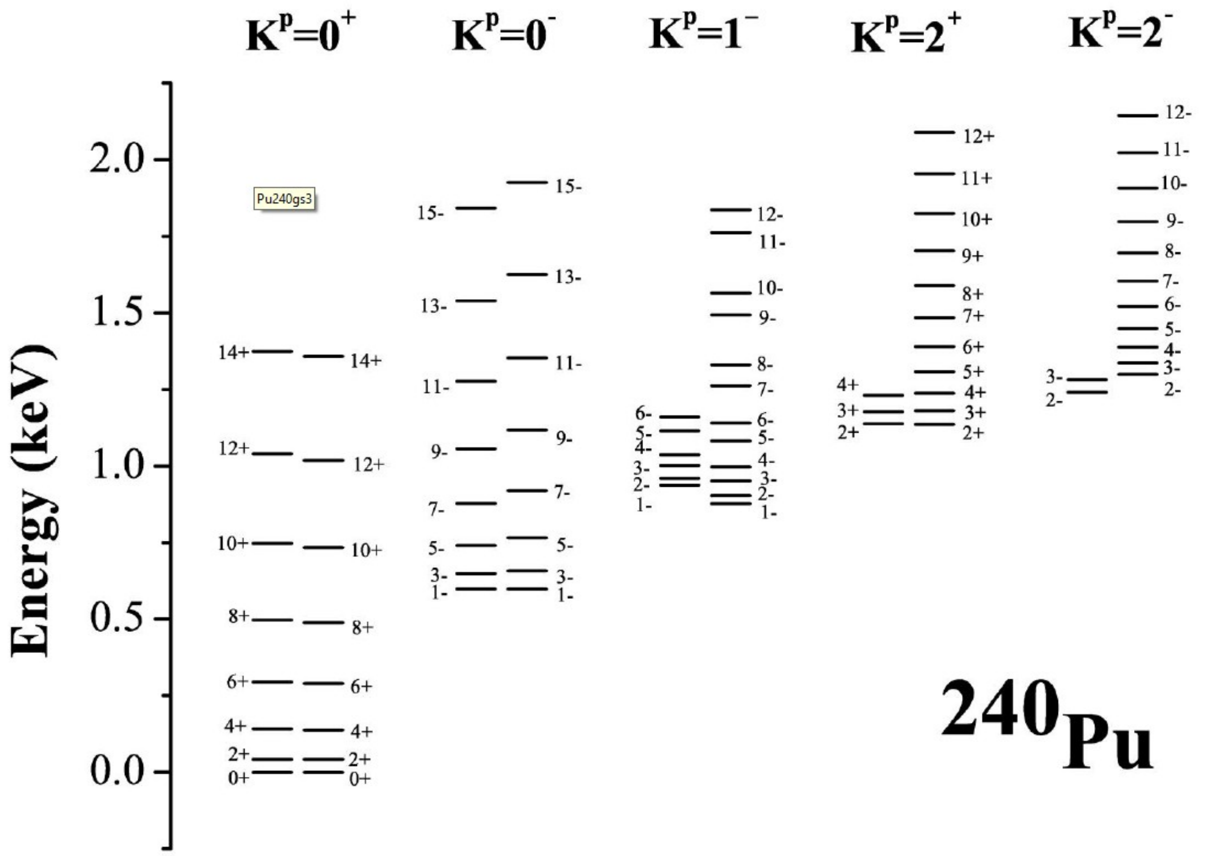
As a result average mass asymmetry ξ_0 increases with angular momentum.

Results of Calculation for Heavy Actinides



Ground-State Well (^{240}Pu)

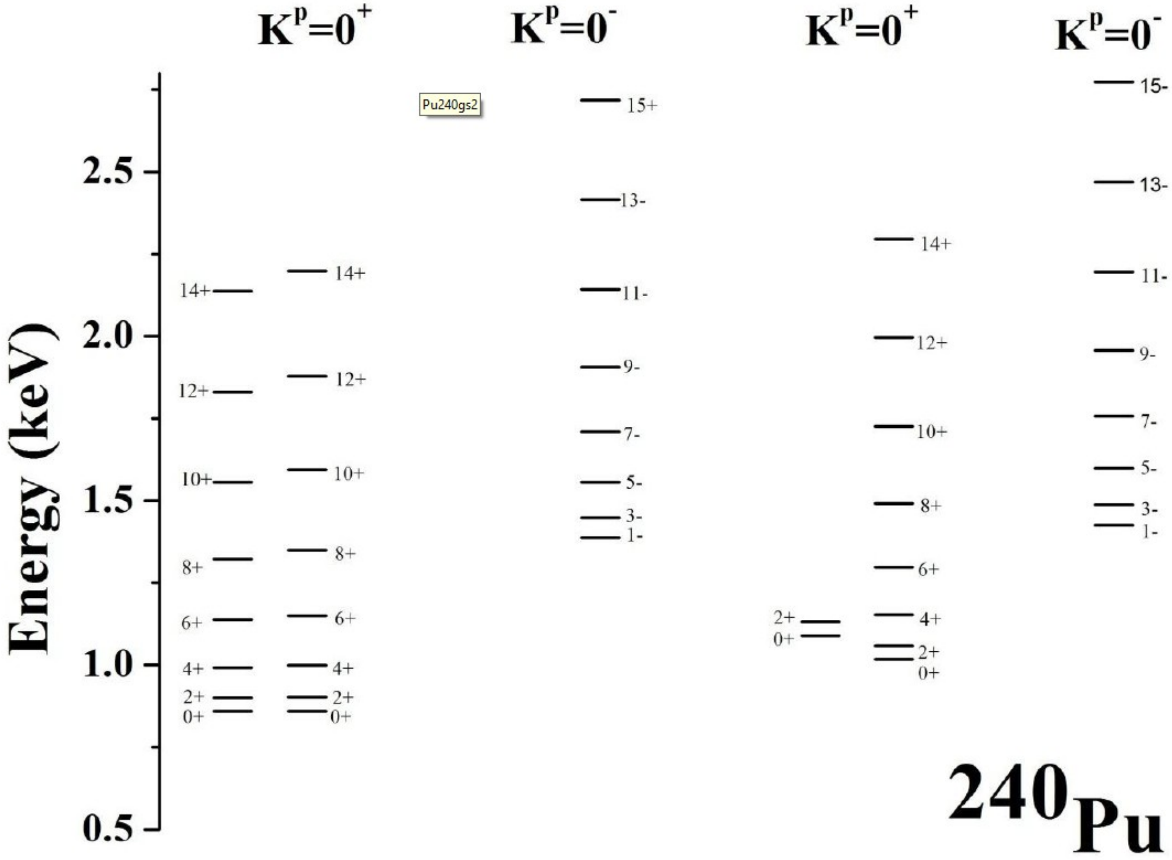
(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)



PRC92, 034302 (2015)

Ground-State Well (^{240}Pu) –continued

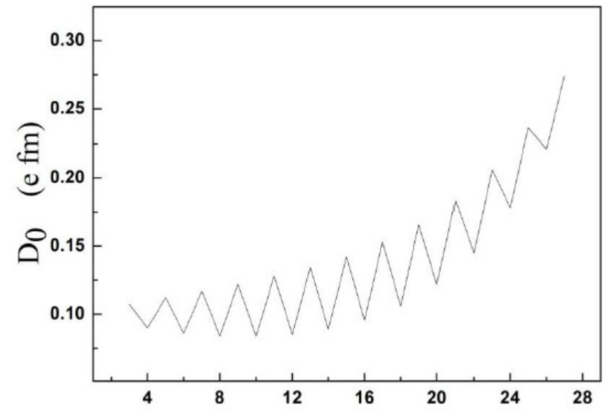
(Exp. data are taken from: http://www.nndc.bnl.gov/ensdf/)

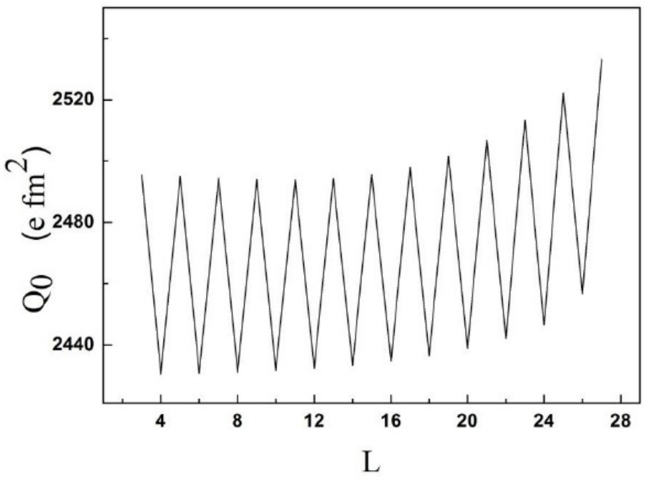


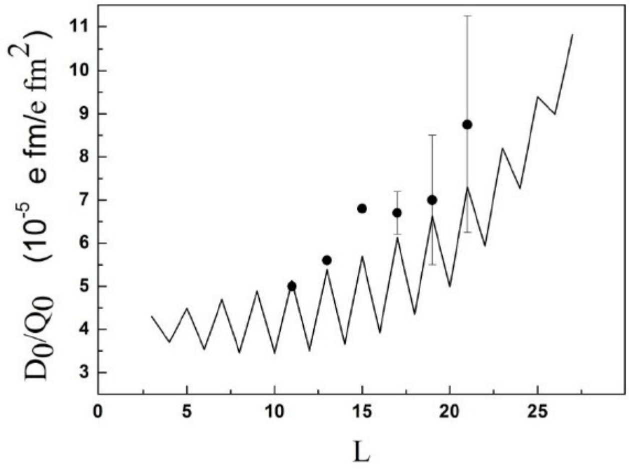
PRC92, 034302 (2015)

Electromagnetic Transition in ^{240}Pu

(I. Wiedenhöver et al., Phys. Rev. Lett. 83, Number 11, (1999))

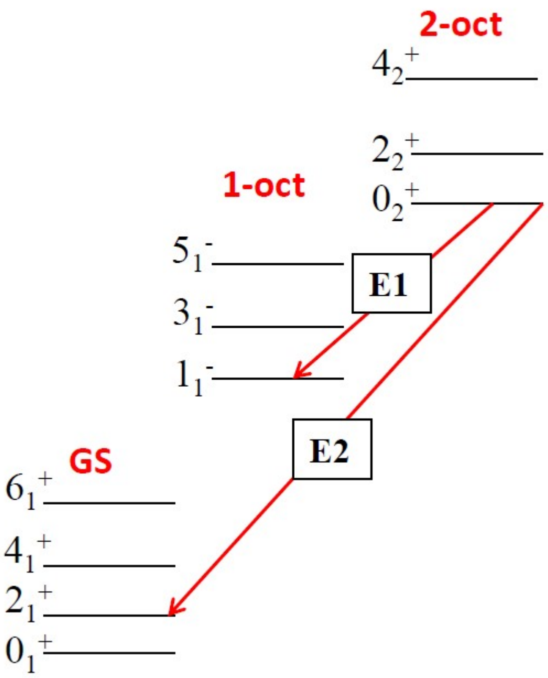






Ratio of transition dipole and quadrupole moments extracted from the *E1* and *E2* branchings $E1(I^- \longrightarrow (I-1)^+)/E2(I^- \longrightarrow (I-2)^-)$ as a function of the initial spin *I*.

Electromagnetic Transition in ^{240}Pu

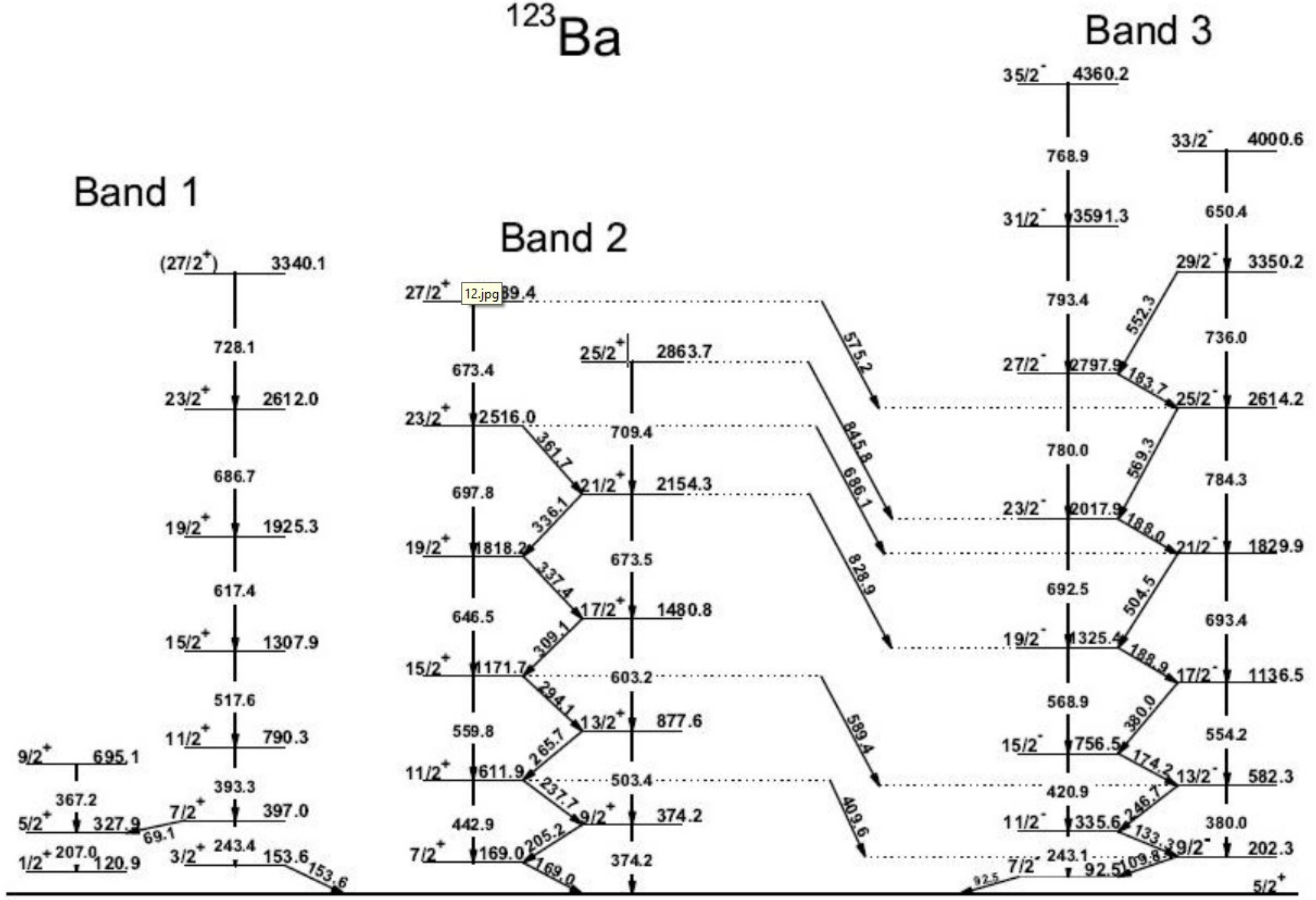


Experimental B(E1)/B(E2) ratios (R_{exp}) are compared to the calculation of our model for the low-spin members of the Kπ = 0+ 2 rotational band in ^{240}Pu.

I_i^π	$I_{f,E1}^\pi$	$I_{f,E2}^\pi$	R_{exp} $(10^{-6}\ \mathrm{fm}^{-2})$	R_{DNS} $(10^{-6}\ \mathrm{fm}^{-2})$
0_2^+	1_1^-	2_1^+	13.7(3)	19.17
2_2^+	1_1^-	0_1^+	99(15)	99.95
2_2^+	1_1^-	2_1^+	26(2)	39.15
2_2^+	1_1^-	4_1^+	5.9(3)	8.57
2_2^+	3_1^-	0_1^+	149(22)	165.60
2_2^+	3_1^-	2_1^+	39(2)	64.9
2_2^+	3_1^-	4_1^+	8.9(5)	14.2
4_2^+	3_1^-	6_1^+	4.4(11)	6.9
4_2^+	5_1^-	6_1^+	4.7(13)	10.59

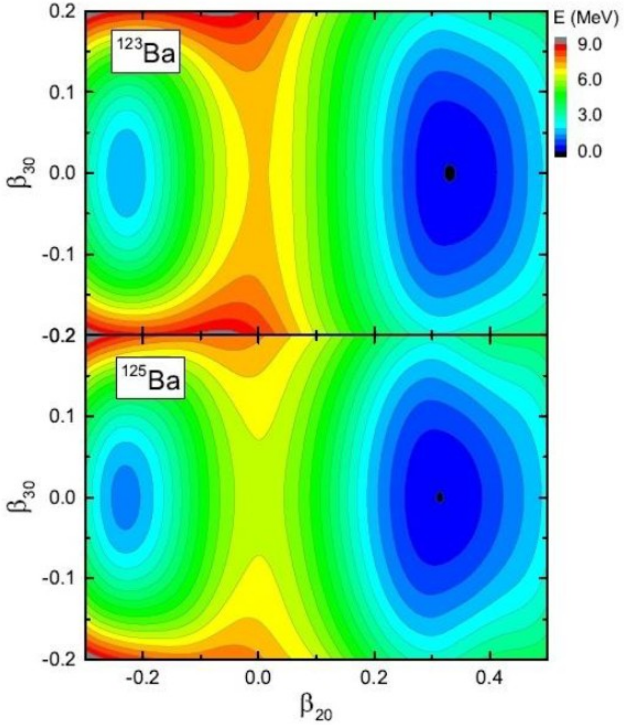
(Exp. Data are from *M. Spieker et al., PRC88, 041303(R), (2013)*)

Reflection-asymmetric correlations in ^{123}Ba



PRC94, 021301(R) (2016)

PES for ^{123,125}Ba



Calculations have been performed in the frame of MDC-RMF model.

Although the minimum of the nuclear potential energy corresponds to the reflection-symmetric shape, PES for ^{123,135}Ba are very soft with respect to the reflection-asymmetric deformation.

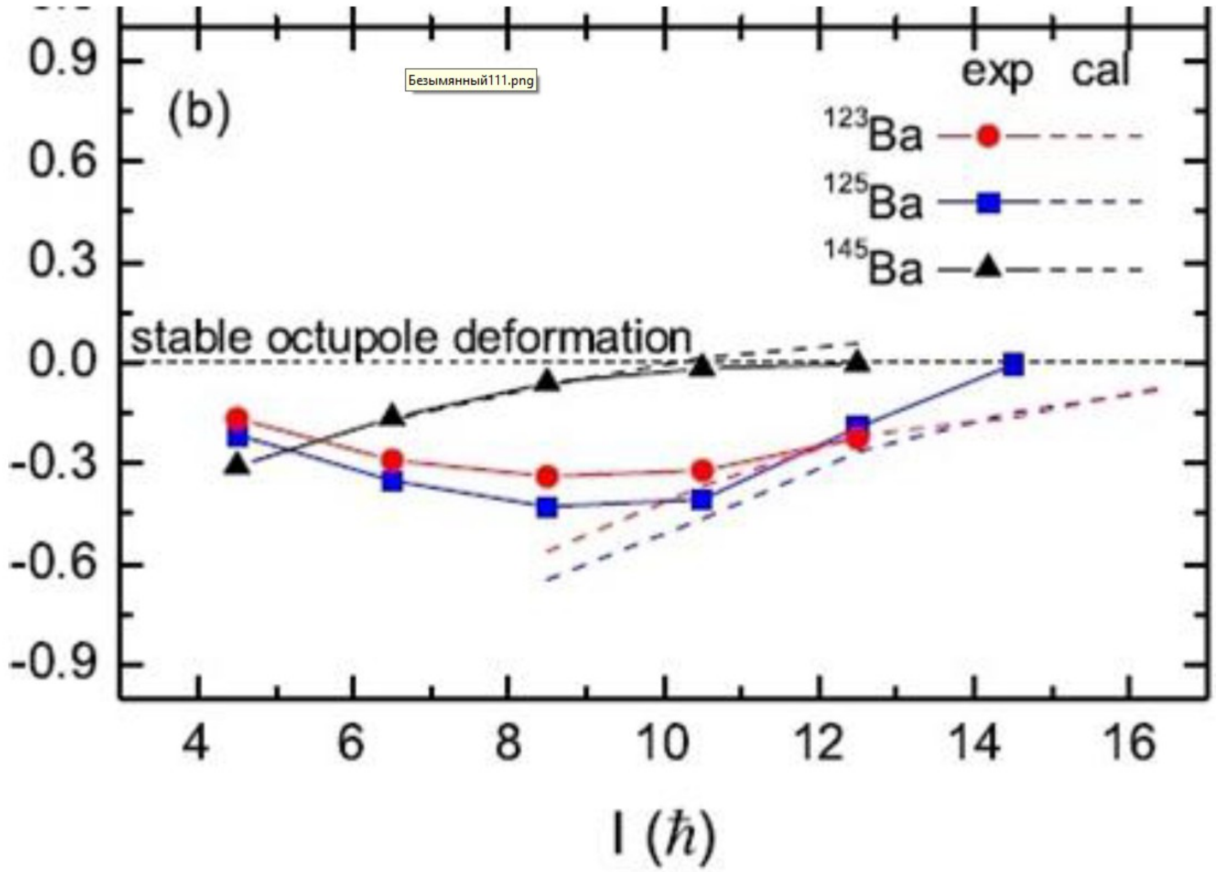
Using the DNS model one can estimate the critical value of angular momentum at which the stable reflection-asymmetric is developed.

$$I_{crit} \approx 13\hbar \quad \text{- for } {}^{123}\text{Ba},$$

$$I_{crit} \approx 12\hbar \quad \text{- for } {}^{125}\text{Ba}.$$

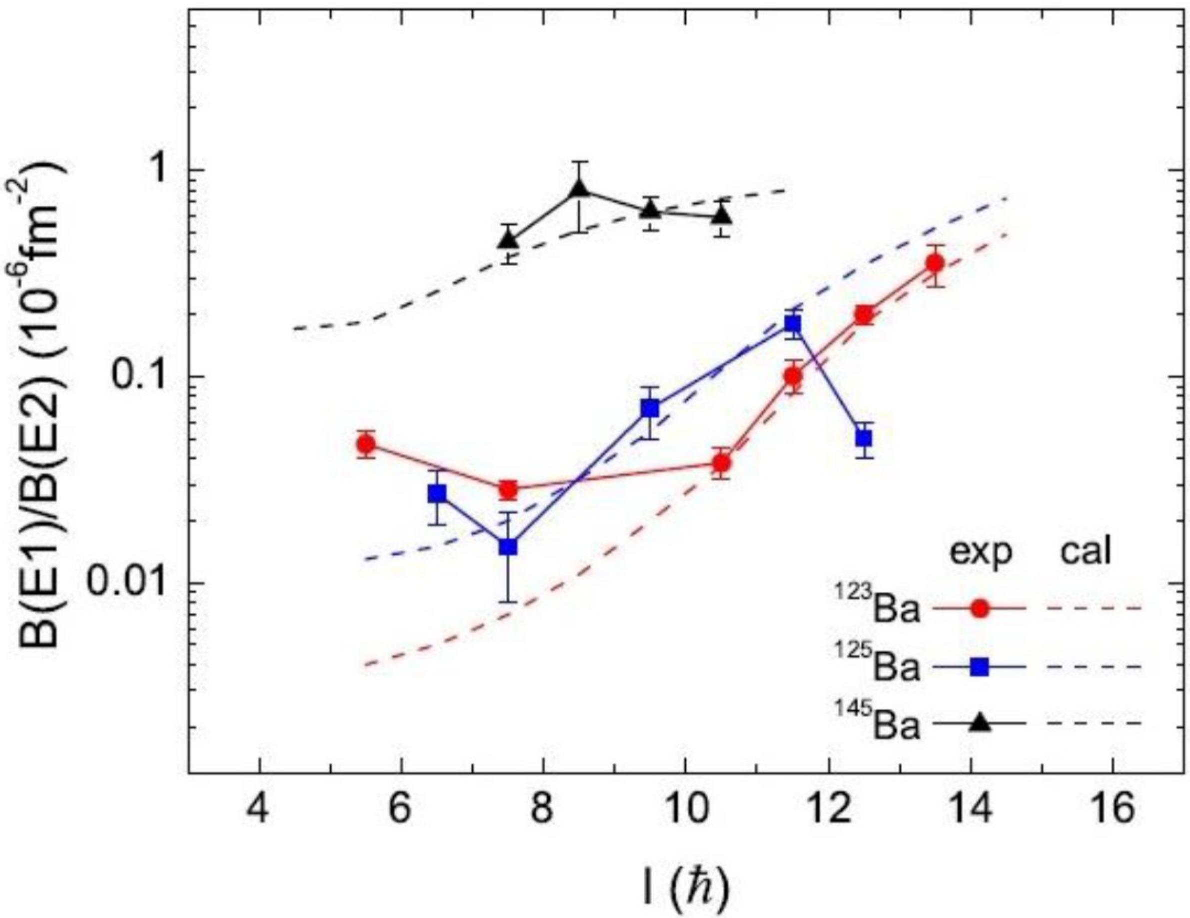
PRC94, 021301(R) (2016)

Parity splitting of ^{123,125,145}Ba



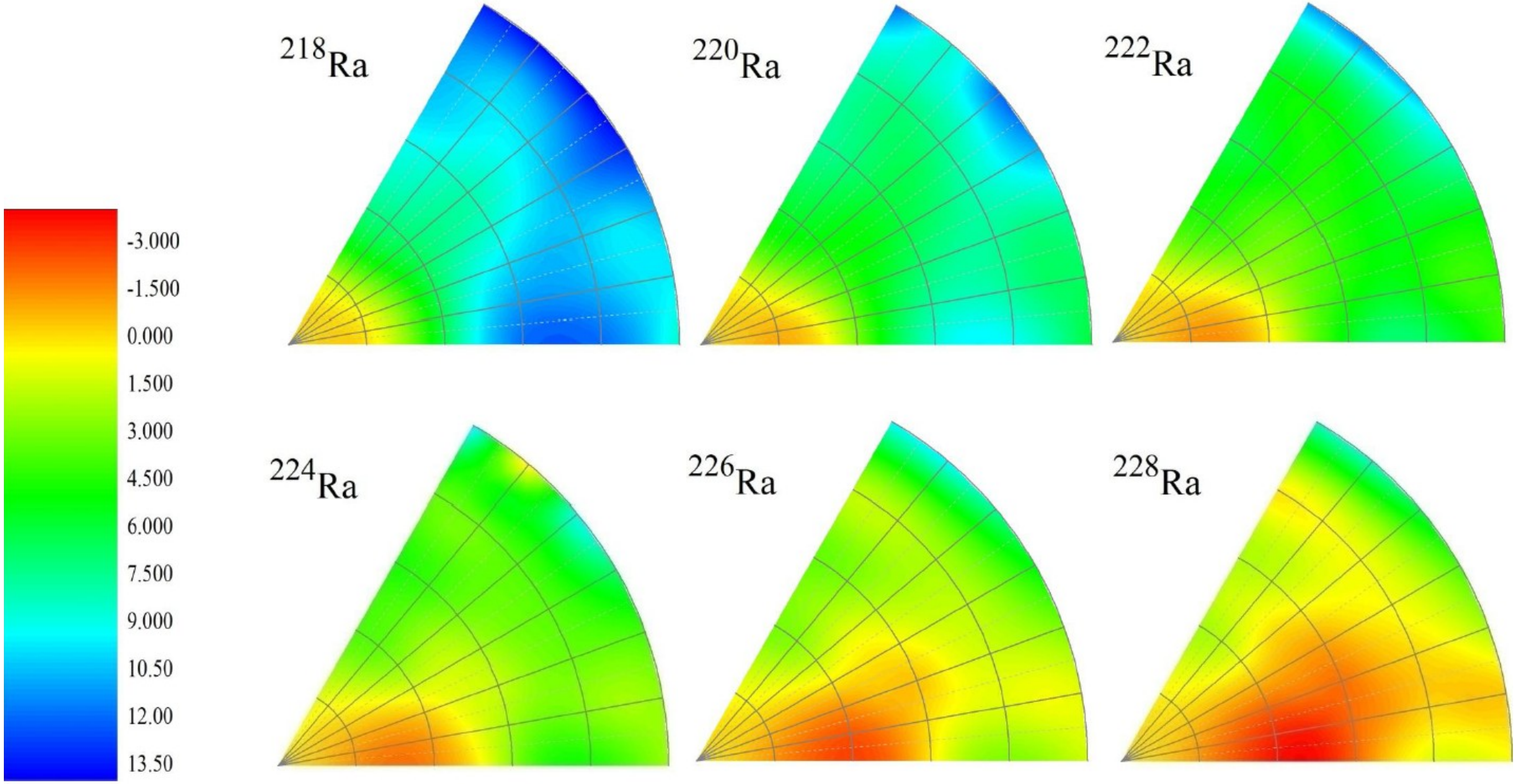
PRC94, 021301(R) (2016)

B(E1)/B(E2)-values for ^{123,125,145}Ba



PRC94, 021301(R) (2016)

Potential Energy Surface of various Ra isotopes



Calculation has been performed in MDC RMF model with PC-PK1 parameterization
(P.W. Zhao *et al.*, PRC 82, 054319 (2010)

Hamiltonian of the DNS model

The kinetic energy operator of the DNS then becomes

$$\hat{T} = -\frac{\hbar^2}{2B(\xi_0)}\frac{1}{\mu(\xi)}\frac{\partial}{\partial\xi}\mu(\xi)\frac{\partial}{\partial\xi}$$

$$+\frac{\hbar^2}{2\mu(\xi)R^2}\hat{l}_0^2+\frac{\hbar^2}{2}\sum_{n=1}^{2}\sum_{k=1}^{3}\frac{\hat{l}_{(n)k}^2}{I_k^{(n)}(\beta_n,\gamma_n)} \qquad (\equiv \hat{T}_{rot})$$

$$-\frac{\hbar^2}{2}\sum_{n=1}^{2}\frac{1}{D_n(\xi_0)}\left(\frac{1}{\beta_n^4}\frac{\partial}{\partial\beta_n}\beta_n^4\frac{\partial}{\partial\beta_n}+\frac{1}{\beta_n^2}\frac{1}{\sin 3\gamma_n}\frac{\partial}{\partial\gamma_n}\sin 3\gamma_n\frac{\partial}{\partial\gamma_n}\right)$$

$$(\equiv \hat{T}_{\mathrm{int}\,r})$$

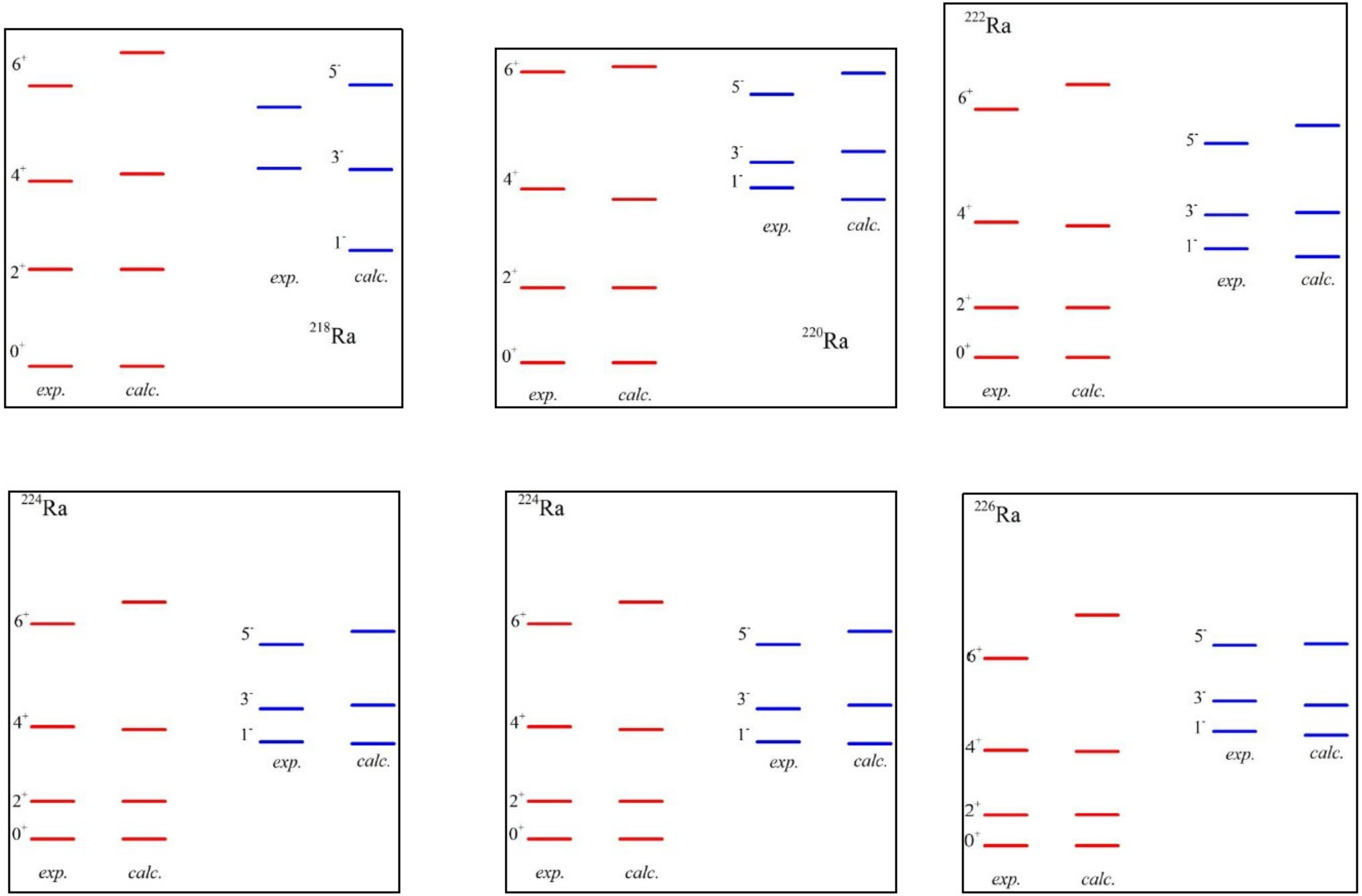
The potential energy of the DNS is

$$V(\xi)=E_1(\xi,\beta_1,\gamma_1)+E_2(\xi,\beta_2,\gamma_2)+V_N(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})+V_C(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})$$

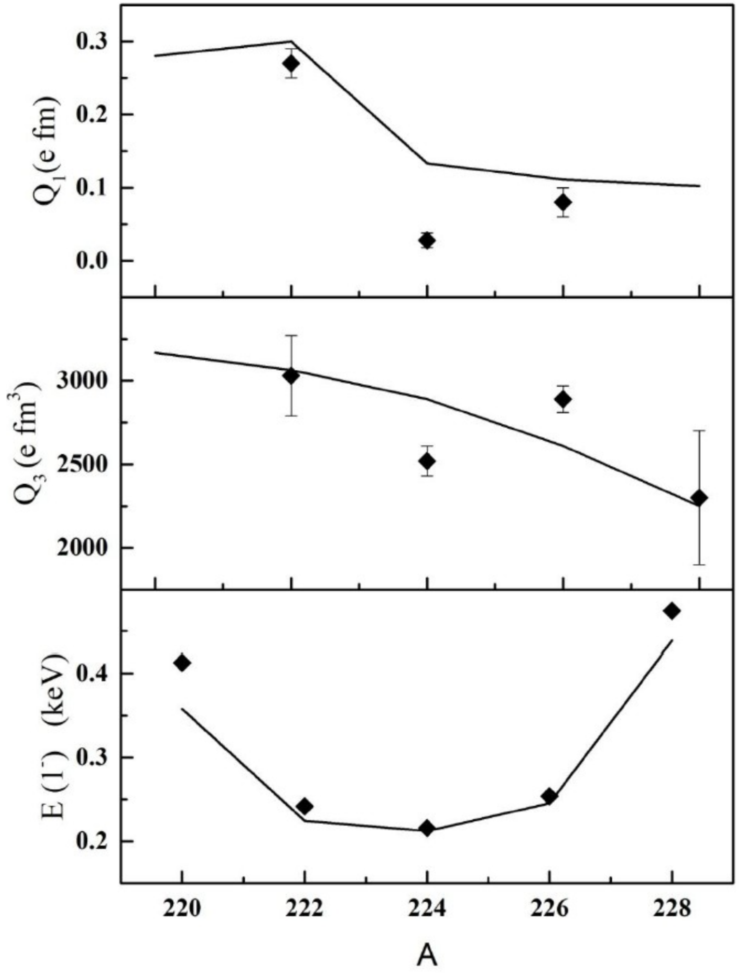
If only heavy fragment is deformed:

$$V(\xi)=V_0(\xi)+V_{\mathrm{int}}(\xi)\left(\beta_2\cdot Y_2(\Omega_R)\right)$$

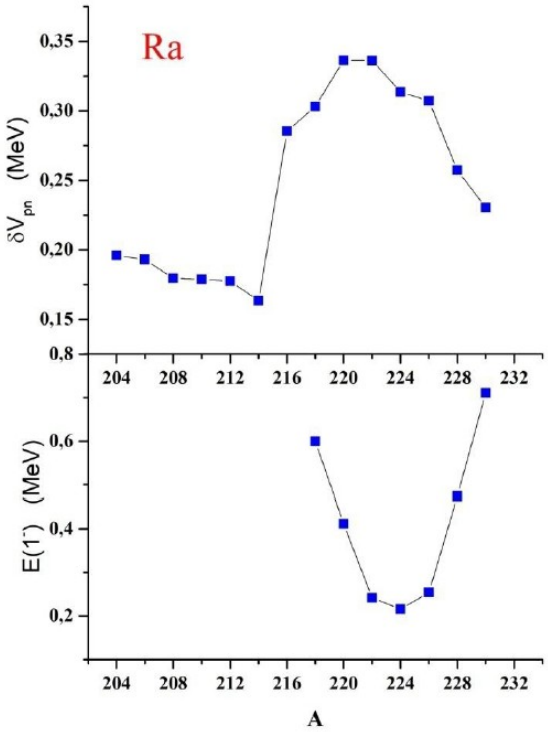
Lowest collective excitations in Ra isotopes



Maximum of *B(E3)* : ^{222}Ra
Minimum of *E(1$^-$)* : ^{224}Ra



L.P. Gaffney et al., Nature 497, 199 (2013) ***(^{224}Ra)***
H.J. Wollersheim et al., NPA 556, 261 (1993) ***(^{226}Ra)***
L.P. Gaffney et al.,in preparation ***(^{222,228}Ra)***



$$B(E3) \sim \xi^2$$

$$\Delta E(1^-) \sim V_B^{-1} \sim \frac{1}{\xi \ \beta_2}$$

Alpha-clustering structure in the ground-state band of ^{212}Po

(Phys. Lett. B 821, 136624 (2021))

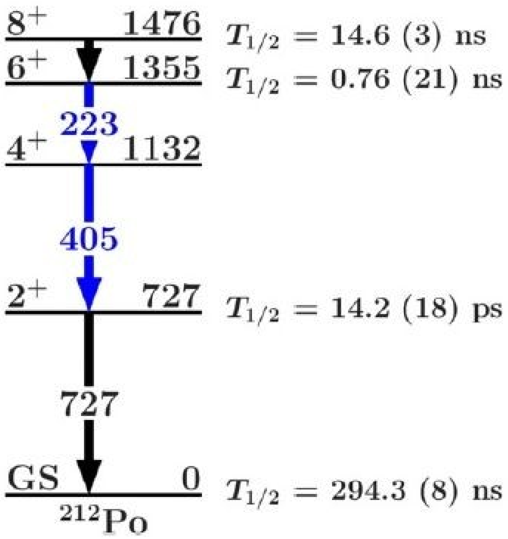


Fig. 1. Partial level scheme of the low-lying yrast states of ^{212}Po.

B(E2) values and α-branching ratios from the low-lying yrast states. The theoretical values are calculated in the α-clustering model from the present work. The experimental B(E2) values are taken from this analysis and from Ref. [10].

State I	B(E2; I → I-2) exp. (W.u.)	B(E2; I → I-2) theo. (W.u.)	α-branching ratios Ref. [10] (%)	α-branching ratios Ref. [13] (%)	α-branching ratios theo. (%)
2_1^+	2.6(3)	3.01	0.033	0.033	0.18
4_1^+	9.4(13)	5.39	~ 0.5	27	3.92
6_1^+	8.7(15)	6.23	~ 3	71	15.20
8_1^+	4.6(1)	5.90	~ 3	42	7.71