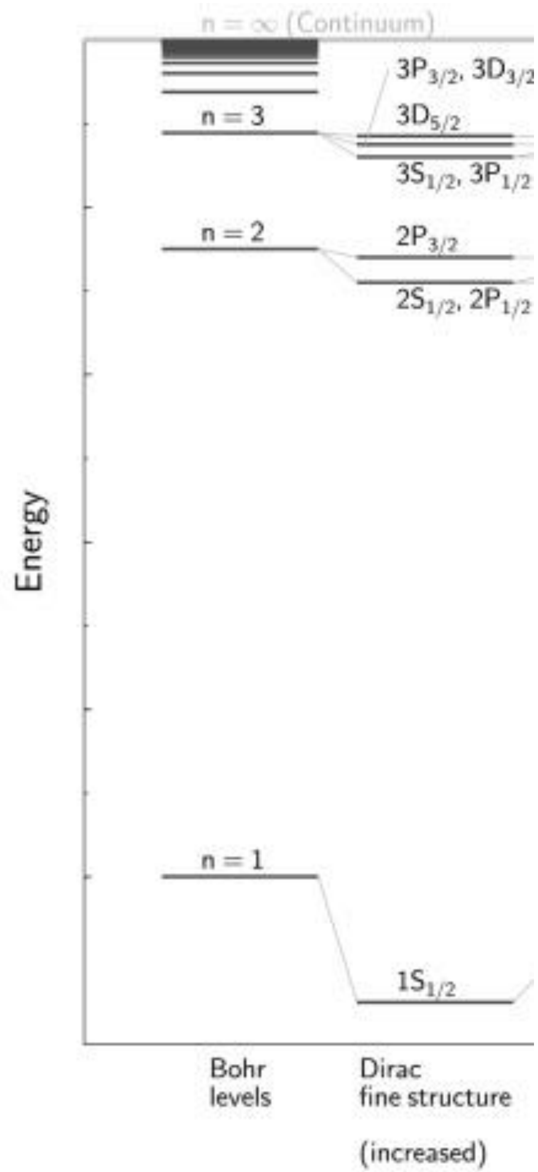
Lecture 4

Beyound the Dirac equation: QED and nuclear effects

Plan of the lecture

- Reminder from the last lecture: Bound-state solutions of Dirac equation
- Higher-order corrections to Dirac energies:
  - Radiative corrections (QED effects)
  - Hyperfine interaction

Dirac energy levels



- Energy values of Dirac particle bound to Coulomb potential are given by:

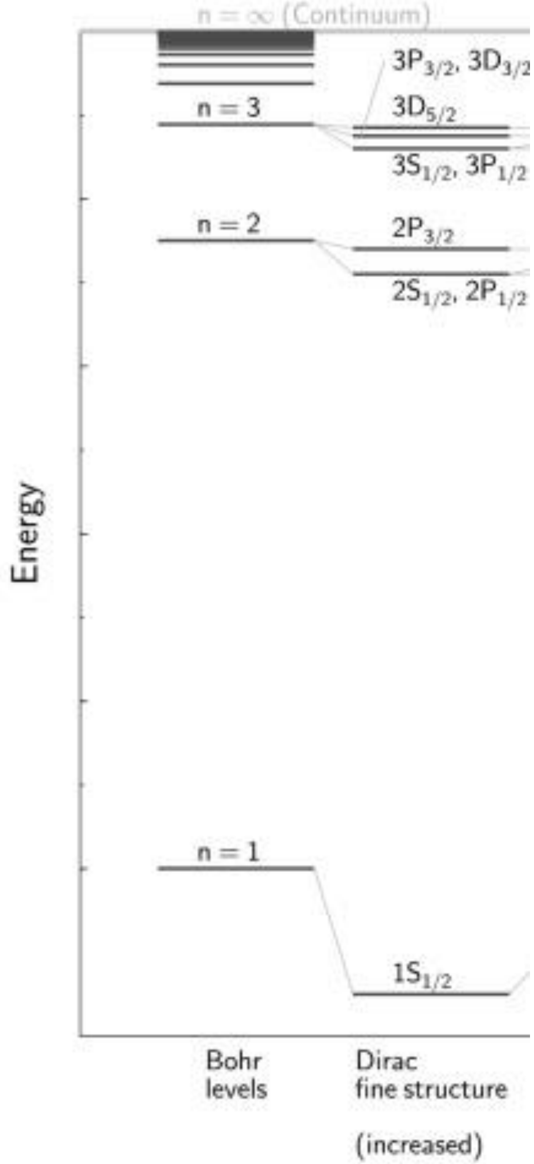
$$E_{nj} = mc^2 \Bigg/ \sqrt{1 + \left(\frac{Z\alpha}{n - |j+1/2| + \sqrt{(j+1/2)^2 - (Z\alpha)^2}} \right)^2}$$

$$\approx mc^2 \left(1 - \frac{1}{2}\frac{(\alpha Z)^2}{n^2} - \frac{1}{2}\frac{(\alpha Z)^4}{n^3}\left(\frac{1}{j+1/2} - \frac{3}{4n} \right) - \ldots \right)$$

Energy depends now on two quantum numbers: *n* and *j*.

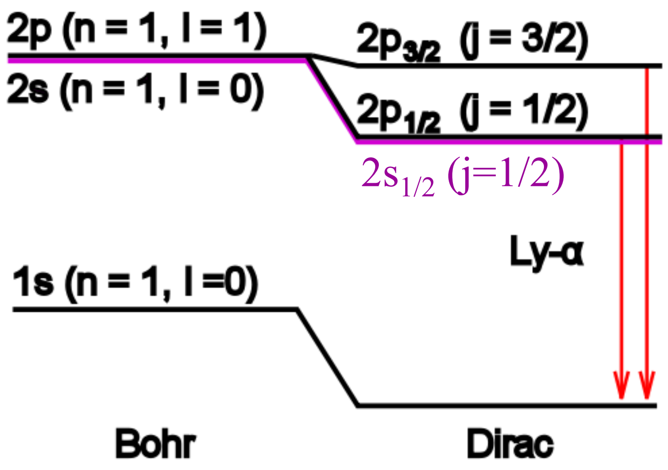
All state with the same *n* and *j* are degenerated!

Dirac energy levels



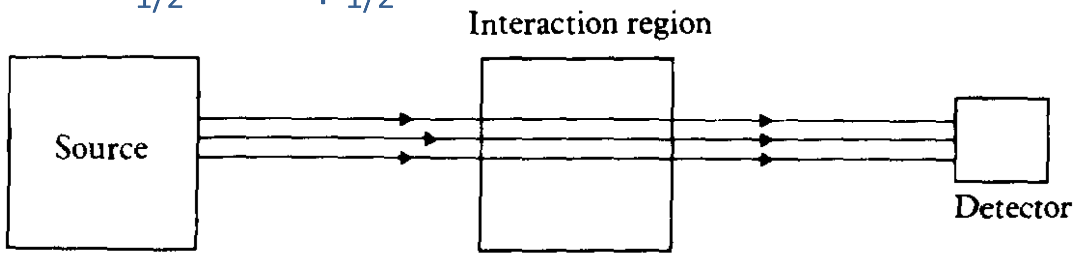
Are there any further corrections to Dirac energies?

$2s_{1/2} - 2p_{1/2}$ energy splitting



- From the middle of 30's several measurements have been reported which probably indicated that $2s_{1/2}$ and $2p_{1/2}$ levels do not coincide.
- The problem of these (first) experiments was their technique: optical spectroscopy of Ly-α lines.

Another approach has been used in brilliant experiment by Lamb and Retherford (1947) who used *microwave* techniques to stimulate a direct transition between $2s_{1/2}$ and $2p_{1/2}$ levels.



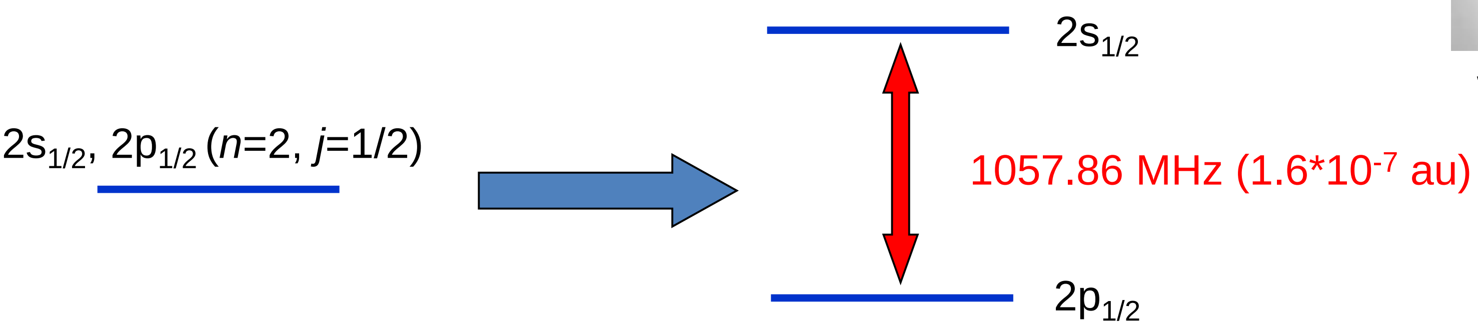
5.16 Schematic diagram of the Lamb–Retherford experiment. The source produces an atomic beam of hydrogen containing a small fraction of atoms in the $2s_{1/2}$ level. The beam is passed through a region of a radio-frequency electric field and a variable magnetic field and is detected by an apparatus which records only atoms in the $n = 2$ level.

$2s_{1/2} - 2p_{1/2}$ energy splitting

- According to Dirac theory, levels $2s_{1/2}$ and $2p_{1/2}$ should be degenerated (since the have the same *j*).

- However, in 1947 Willis Lamb and Robert Retherford have a small difference in energy between these two levels!

Willis Eugene Lamb
1955 Nobel prize



- To compare: energy of $2s_{1/2}$ and $2p_{1/2}$ levels is -0.125 au.

Obviously: some effects which beyond the Dirac theory have to be taken into account!
By which???

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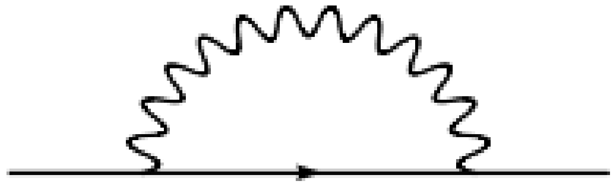
Idea of radiative corrections

- From the end of 1930th : interaction of electron with radiation filed. But which field?



- Ideas: electron may interact with its own field. The Coulomb potential is therefore perturbed by a small amount and the degeneracy of the two energy levels is removed.

- But: problems with divergence of results!

- In 1947 Hans Bethe has shown how to identify the divergent terms and to subtract them from the theoretical expression.

Hans Bethe

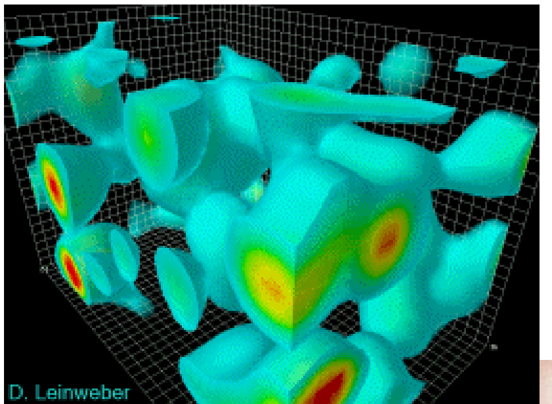
Development of
Quantum Electrodynamics (QED)
and Quantum Field Theory (QFT)

Idea of radiative corrections

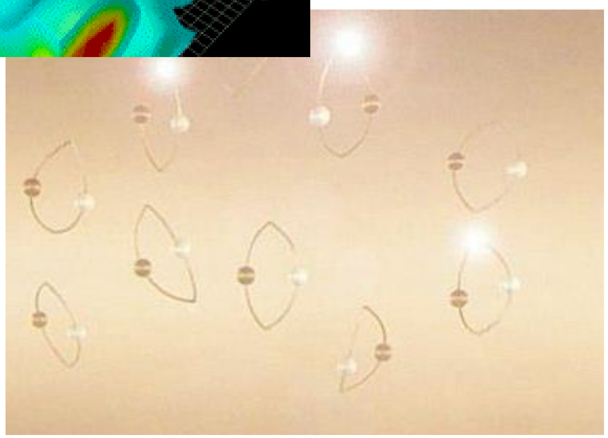
- What is vacuum?

- With “classical” vacuum it is clear: it is a volume of space that is essentially empty of “everything”.

“An Experiment on a Bird in the Air Pump”
by Joseph Wright of Derby



QCD vacuum fluctuations



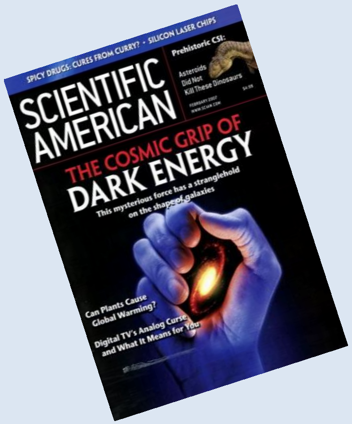
According to present-day understanding of what is called quantum vacuum, it is "by no means a simple empty space”.

The quantum vacuum is not truly empty but instead contains fleeting electromagnetic waves and particles that pop into and out of existence.

Virtual particles

- The uncertainty principle allows virtual particles (each corresponding to a quantum field) continually materialize out of the vacuum, propagate for a short time and then vanish.

“Philosophical aspects” of quantum mechanics and QFT!



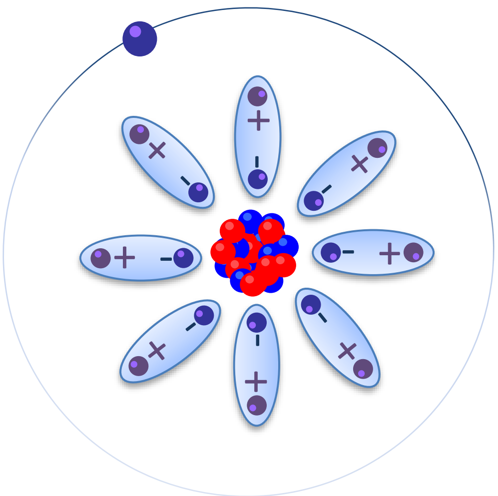
“Are virtual particles really constantly popping in and out of existence? Or are they merely a mathematical bookkeeping device for quantum mechanics?”

http://www.sciam.com/article.cfm?id=are-virtual-particles-rea&topicID=13

QED effects

Vacuum polarization

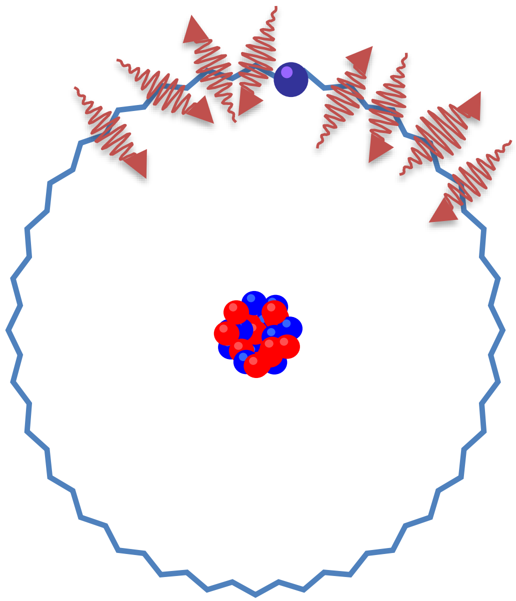
We can for a short time to "borrow" energy from vacuum and to create electron-positron pairs.



The electron in atom "sees" the nuclear charge as screened by these electron-positron pairs.

Self-energy

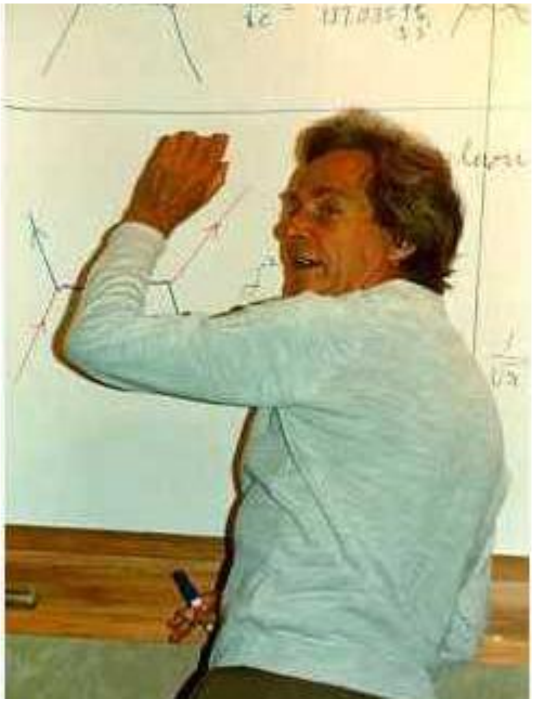
Electron can also emit and absorb a virtual photon. We can see this process as interaction of electron with ist own electromagnetic field.



How to describe QED effects?

Feynman technique

- Feynman diagrams is a nice tool to perform calculations of scattering processes.
- Each Feynman diagram an amplitude for some process!
- In order to "translate" the Feynman diagram to language of formulas, one has to use set of rules.

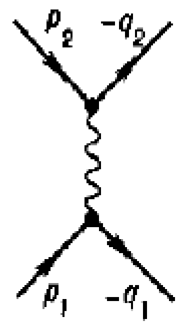


vertex		
virtual photon		$\frac{1}{(2\pi)^4 i}\frac{\varepsilon_{\mu,\nu}}{-k^2}$
virtual electron (positron)		$\frac{1}{(2\pi)^4 i}\frac{m+\hat{p}}{m^2-p^2}\hat{p}=\gamma^\mu\rho_\mu$
photon		$\frac{(e^\alpha(k))_\mu}{(2\pi)^{(3/2)}\sqrt{2k_0}}$
outgoing electron		$(2\pi)^{-3/2}\vec{v}_\sigma(\rho)$
incoming electron		$(2\pi)^{-3/2}\vec{v}_\rho(\rho)$

Feynman technique

vertex		$(2\pi)^4\, ie\gamma^\mu\, \delta^{(4)}(p + k - p')$
virtual photon		$\frac{1}{(2\pi)^4 i}\frac{\varepsilon_{\mu,\nu}}{-k^2}$
virtual electron (positron)		$\frac{1}{(2\pi)^4 i}\frac{m+\hat{p}}{m^2-p^2}\hat{p} = \gamma^\mu \rho_\mu$
photon		$\frac{(e^\alpha(k))_\mu}{(2\pi)^{(3/2)}\sqrt{2k_0}}$
outgoing electron		$(2\pi)^{-3/2}\vec{v_\sigma}(\rho)$
incoming electron		$(2\pi)^{-3/2}\vec{v_\rho}(\rho)$

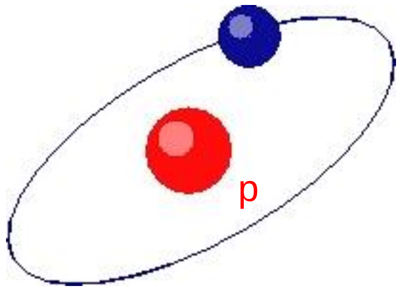
- Amplitude for one of the simplest processes (electron-electron or Moller scattering)

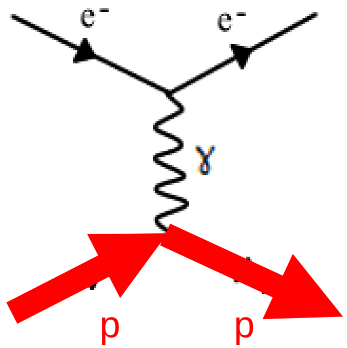


$$M(p_1,p_2,-q_1,-q_2) = \frac{e^2}{i(2\pi)^2}\delta(p_1+p_2+q_1+q_2)\frac{g_{\mu,\nu}}{(p_1+q_1)^2}\vec{v_\sigma}(-q_1)\gamma^\mu v_\rho(p_1)\vec{v_\kappa}(-q_2)\gamma^\nu v_\lambda(p_2)$$

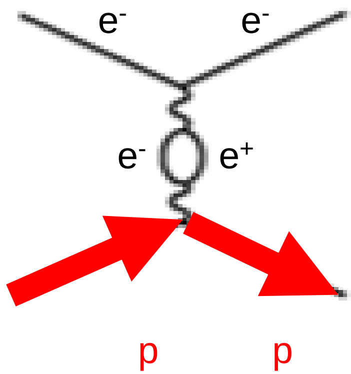
Back to the Lamb shift

- Interaction of electron with atomic/ionic nucleus in zero approximation:

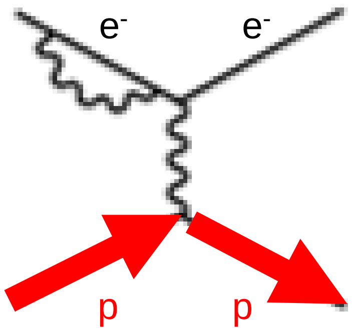




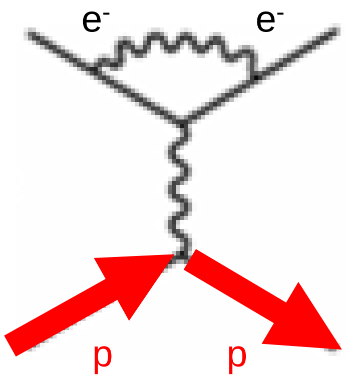
But we can now consider the next-order corrections:



vacuum polarization

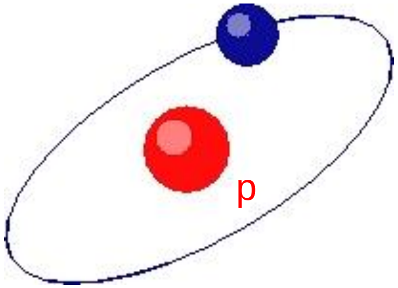


self energy

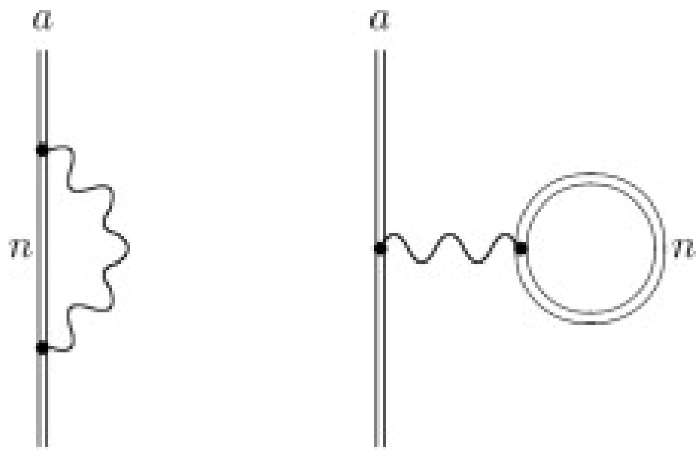


vertex correction
(anomalous magnetic moment)

Back to the Lamb shift

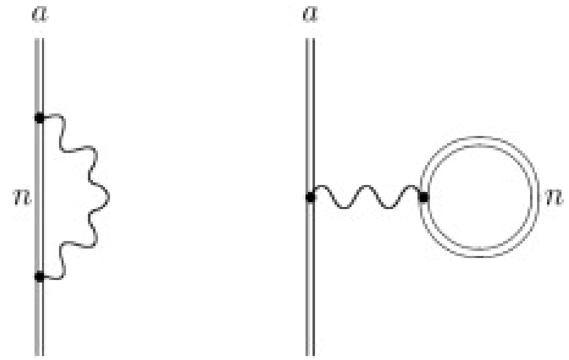


- We need to take into account all virtual-photon exchange diagrams!
- We use double line to present electron in the Coulomb field.
- The diagrams for QED corrections then look as:



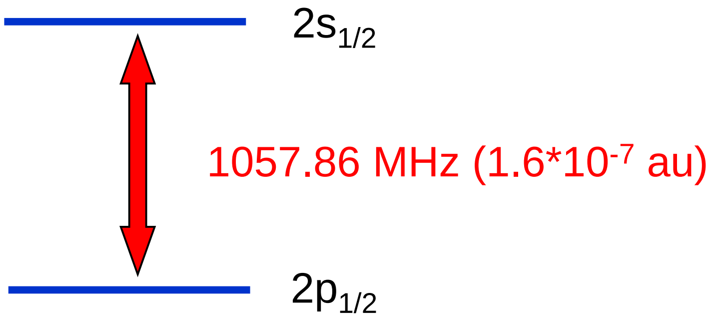
Lamb shift in neutral hydrogen

- Remembering that every Feynman diagram can be attributed to some amplitude, one may evaluate the numerical value of the Lamb shift.



- For the neutral hydrogen we find:

Effect	Energy contribution
Vacuum polarization	-27 MHz
Electron mass renormalization	+1017 MHz
Anomalous magnetic moment	+68 MHz
Total	+1058 MHz



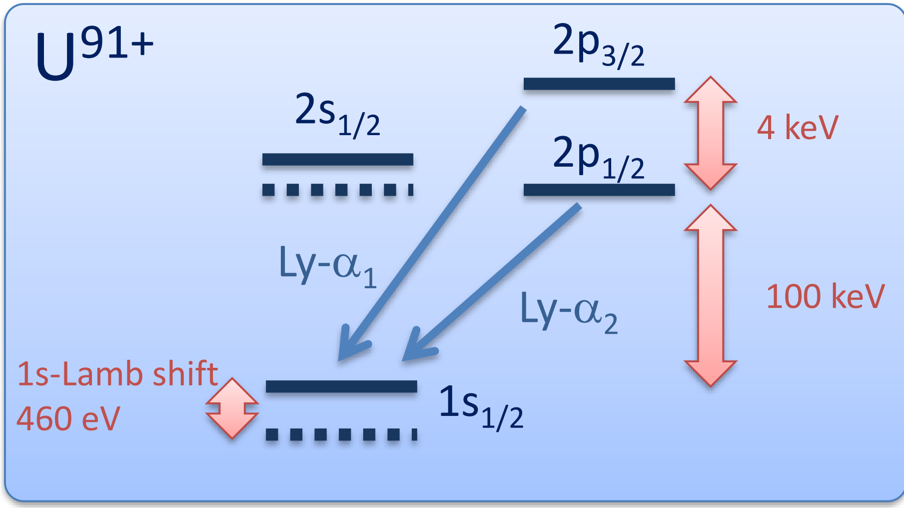
- Another important result: Lamb shift increases with increasing of nuclear charge Z as $\Delta E \sim (\alpha Z)^4/n^3$

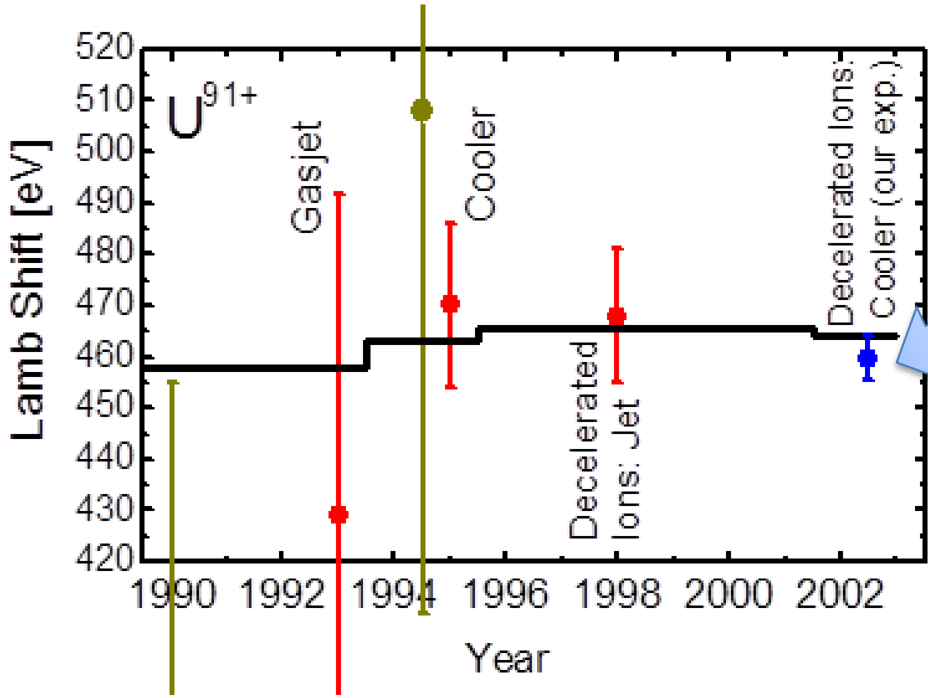
Experiments with high-Z, hydrogen-like ions are performed!

1s-Lamb shift in heavy ions

- Spin and QED effects on the electron structure become of great importance and can be observed.

- During the last two decades a number of experiments have been performed to explore ground-state Lamb shift.





1s-Lamb Shift (U^{91+})

Experiment: 459.8 eV ± 4.6 eV
Theory: 463.95 eV

A. Gumberidze et al. PRL **94** (2005) 223001

- New series of measurements performed in the framework of FOCAL experiment are currently under analysis.

1s-Lamb shift in heavy ions

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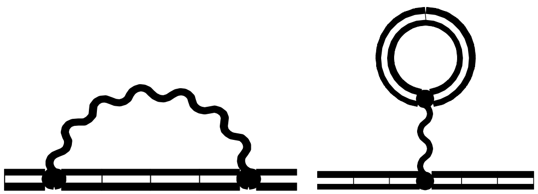
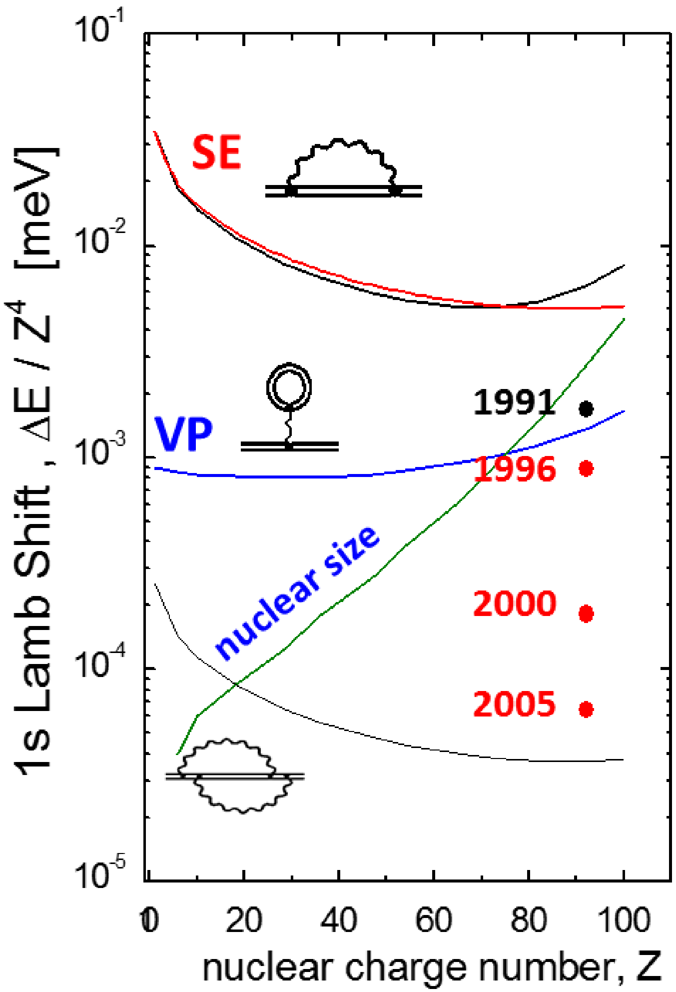
- During the last two decades a number of experiments have been performed to explore ground-state Lamb shift.



U^{91+}	Self-energy	Vacuum polarization	Nuclear size
	355.0 eV	-88.6 eV	198.7 eV



- The present goal is ±1 eV accuracy!

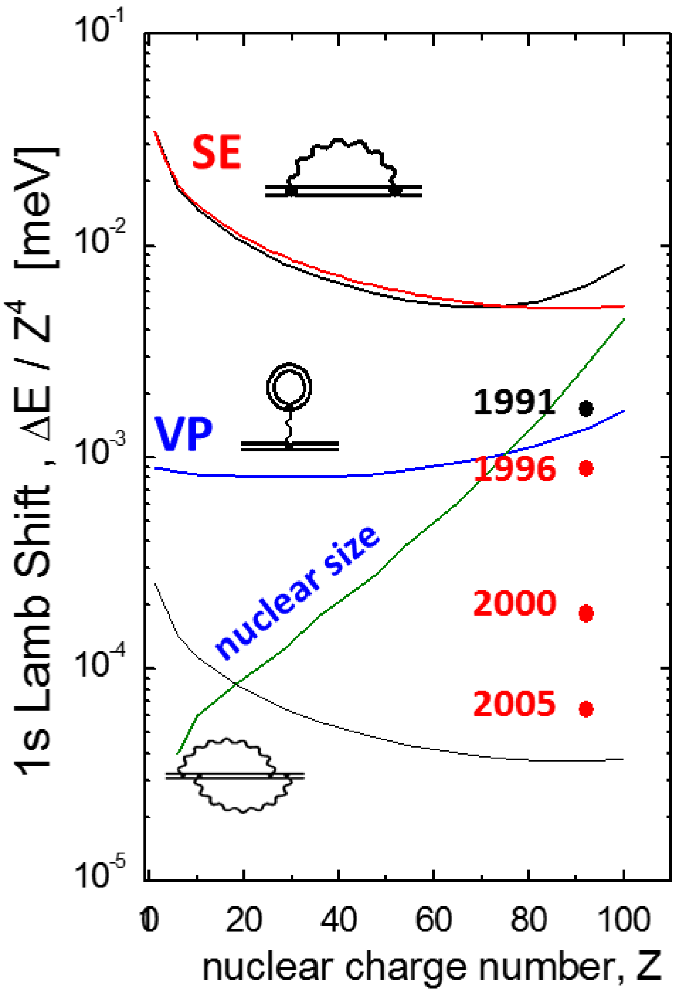
1s-Lamb shift in heavy ions

- The Lamb shift can be parameterized as:

$$\underbrace{\Delta E = \alpha/\pi\ (\alpha Z)^4\ F(\alpha Z)\ m_e c^2}$$

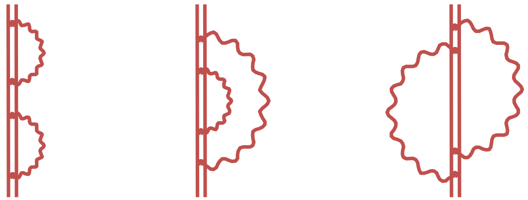
Low Z-Regime: $\alpha Z << 1$
$F(\alpha Z)$: series expansion in αZ

High Z-Regime: $\alpha Z \approx 1$
$F(\alpha Z)$: series expansion in αZ not appropriate

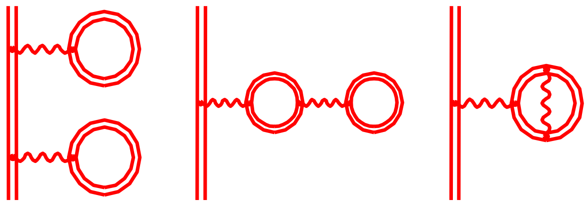


QED is the most accurate field theory. But: Theory of (non-perturbative) QED in high fields still under construction.

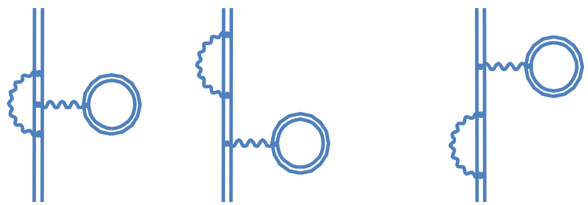
QED correction in second order α



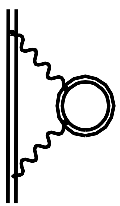
SE – SE



VP – VP

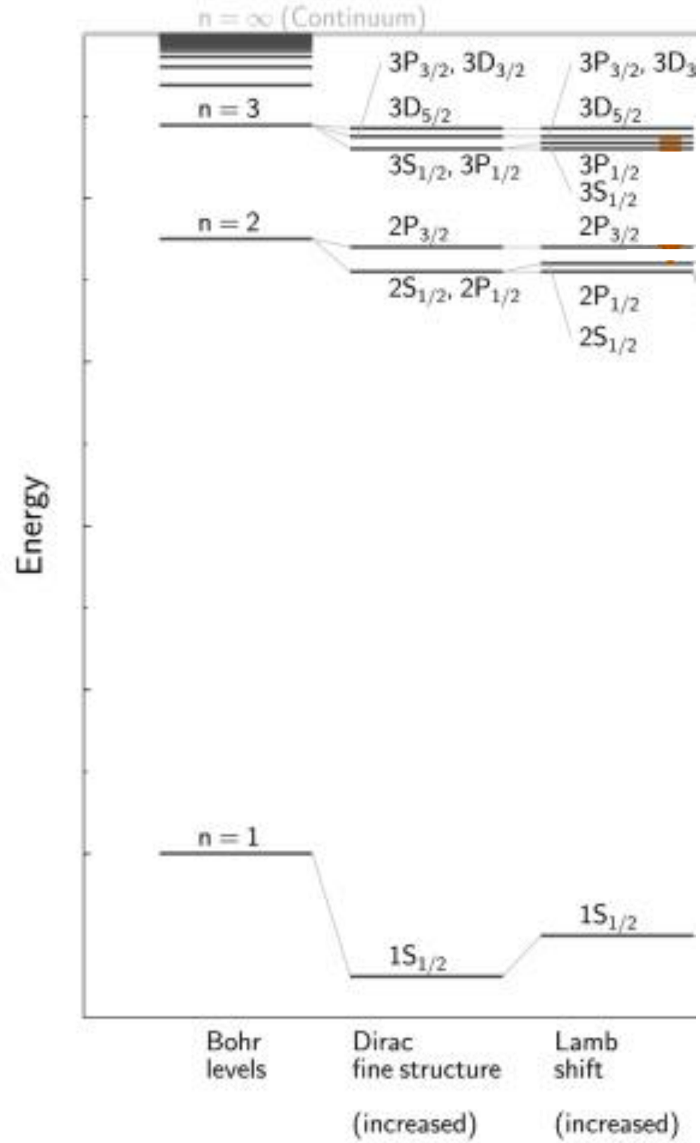


VP – VP



S(VP)E

Energy levels of hydrogen-like ions



Do we expect some more effects which can influence the bound-state structure of hydrogen-like ions?

Actually, yes!

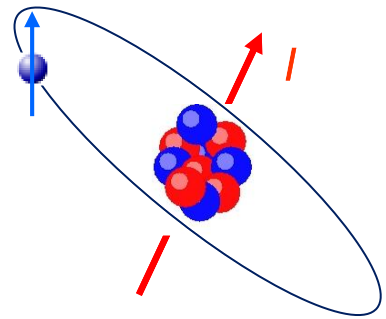
Plan of the lecture

- Reminder from the last lecture: Bound-state solutions of Dirac equation
- Higher-order corrections to Dirac energies:
  - Radiative corrections (QED effects)
  - Hyperfine interaction

Nuclear spin and magnetic moment

- Until now we have assumed in our analysis that nucleus has *zero* nuclear spin.

- However, there are many isotopes having non-zero (integer or half-integer) nuclear spin I.



Nucleus	*Spin* I	*Landé factor* g_I	*Magnetic moment* $\mathcal{M}_N$ (*in nuclear magnetons*)
proton p	1/2	5.5883	2.79278
neutron n	1/2	−3.8263	−1.91315
deuteron $^2_1$D	1	0.85742	0.85742
$^3_2$He	1/2	−4.255	−2.1276
$^4_2$He	0	—	0
$^{12}_6$C	0	—	0
$^{16}_8$O	0	—	0
$^{39}_{19}$K	3/2	0.2609	0.3914
$^{67}_{30}$Zn	5/2	0.35028	0.8757
$^{85}_{37}$Rb	5/2	0.54108	1.3527
$^{129}_{54}$Xe	1/2	−1.5536	−0.7768
$^{133}_{55}$Cs	7/2	0.7369	2.579
$^{199}_{80}$Hg	1/2	1.0054	0.5027
$^{201}_{80}$Hg	3/2	−0.37113	−0.5567

- Associated with each nuclear spin there is a magnetic moment:

$$\boldsymbol{\mu}_I = g_I\, \mu_N \boldsymbol{I}/\hbar$$

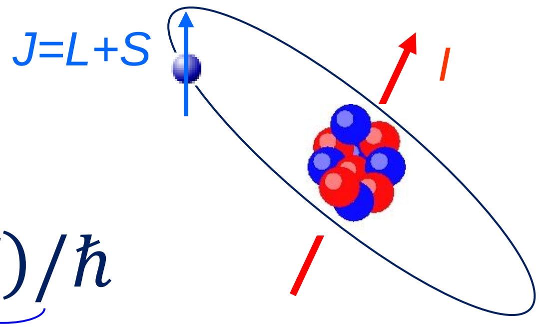
nuclear magneton

nuclear g factor

How non-zero nuclear spin may affect energy levels of ions?

Hyperfine interaction

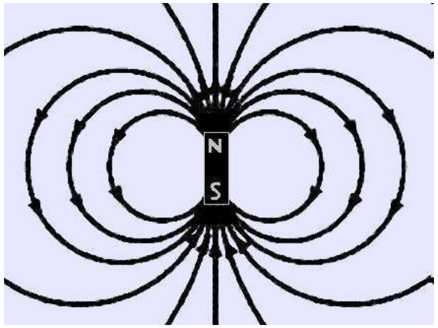
- The magnetic field due to the magnetic dipole moment of the nucleus will interact with the electron dipole momentum:



$$\underbrace{\boldsymbol{\mu}_I = g_I\, \mu_N \boldsymbol{I}/\hbar}_{\text{nuclear magnetic dipole moment}} \qquad \boldsymbol{\mu} = \underbrace{\mu_0(\boldsymbol{L} + g\boldsymbol{S})/\hbar}_{\text{electron moment}}$$

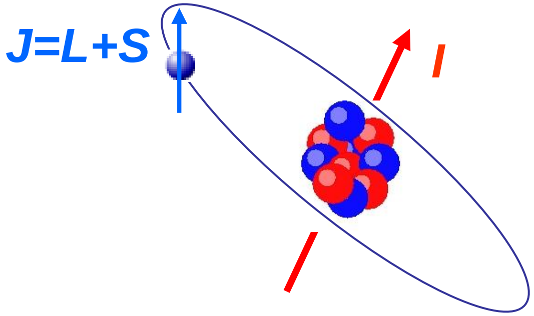
- i.e. it will interact both with electron orbital momentum and spin.

$$\hat{H}_1 \propto \mu_N \mu_0\, \boldsymbol{L}\cdot\boldsymbol{I} \qquad \hat{H}_2 \propto \mu_0\, \boldsymbol{S}\cdot\boldsymbol{B}$$



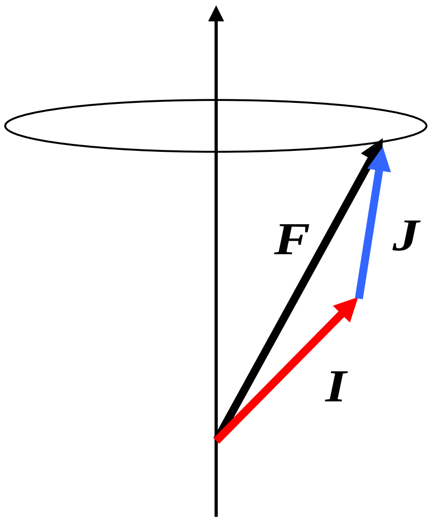
Again, we have to re-consider set of quantum numbers to describe our quantum states!
(Please, remind yourself the case of spin-orbit interaction).

Total angular momentum of an ion



- We shall introduce total angular momentum F of the system “electron+ion”

$$\vec{F} = \vec{J} + \vec{I}$$



- Again, any other angular momentum it satisfies:

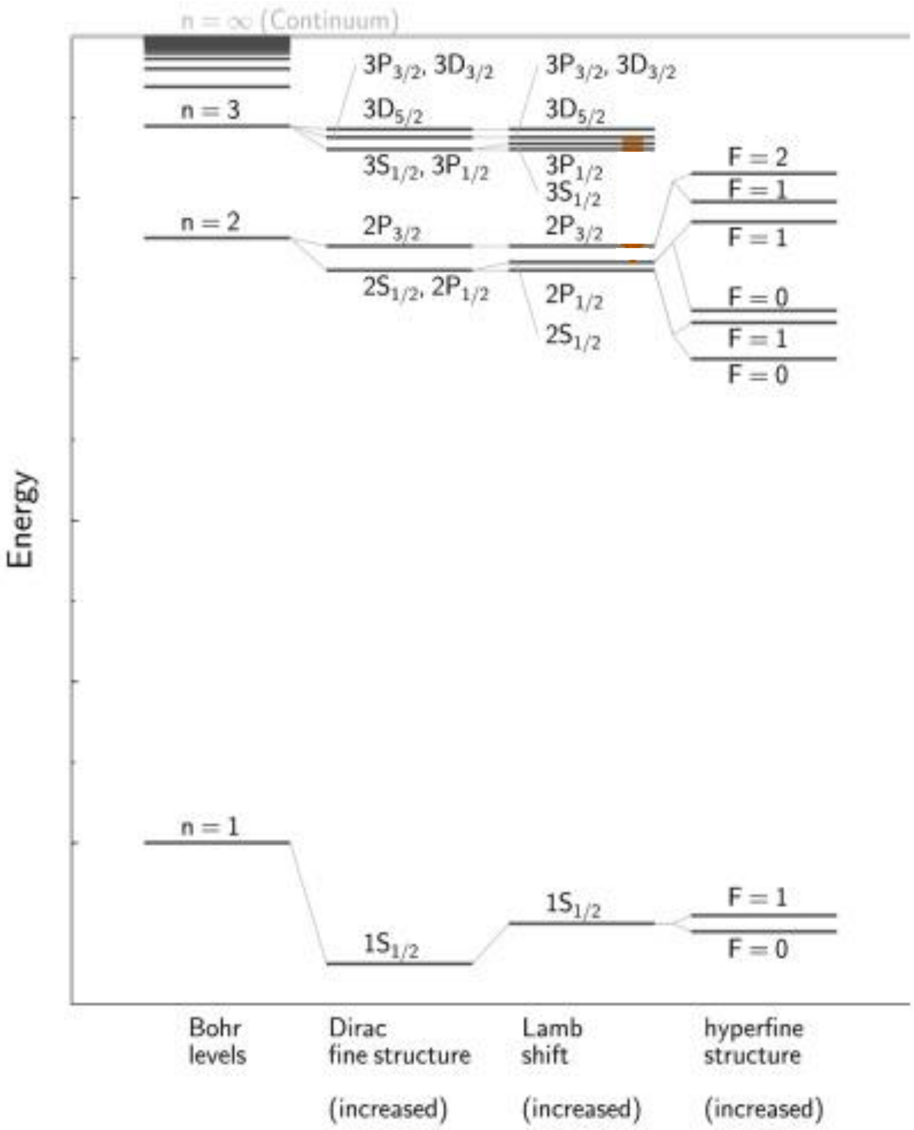
$$\hat{F}^2 \Psi_{FM_F} = F(F+1)\hbar^2 \Psi_{FM_F}$$

$$\hat{F}_z \Psi_{FM_F} = \hbar M_F \Psi_{FM_F}$$

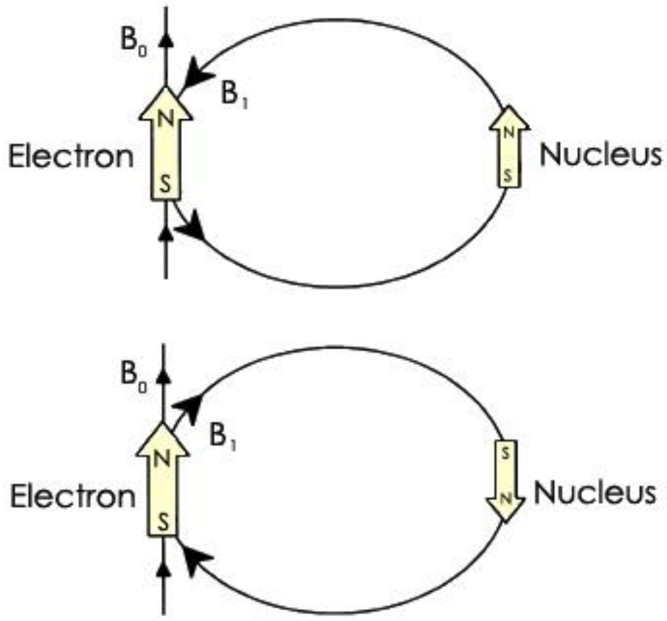
- Levels with the same n and j appear to be split one more time!

$$\Delta E_{HF} \approx \mu_I \left(F(F+1) - I(I+1) - J(J+1) \right)$$

Energy levels of hydrogen-like ions



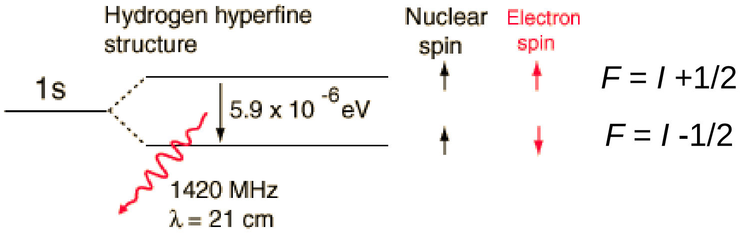
- Note: even levels with L=0 (s states) appear to be split because of "spin-spin" interaction.

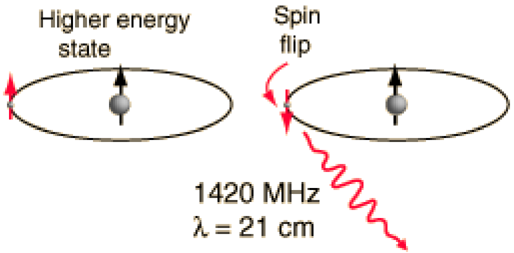


Hyperfine splitting in astrophysics

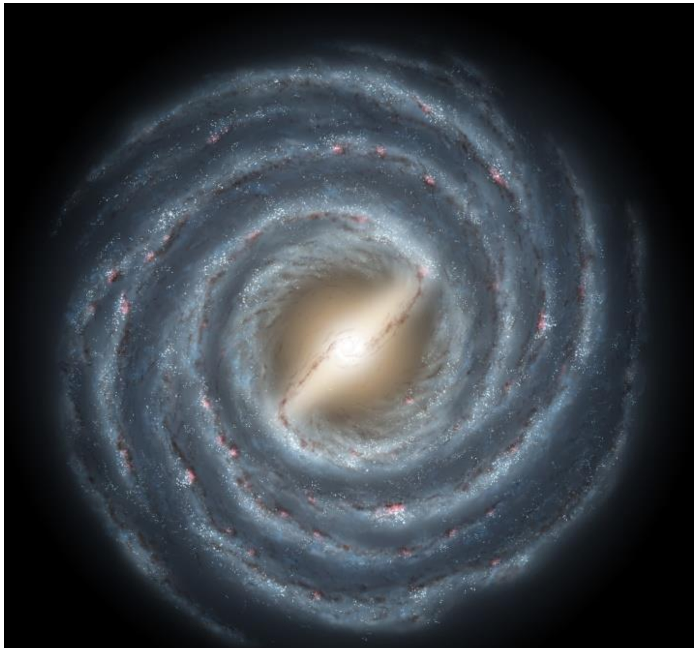
- For the case of ground $1s_{1/2}$ state (j=1/2) of hydrogen, HF interaction results in splitting of energy level into two levels.

- One can observe transition between two HF levels: famous 21 cm line in astrophysics!





From: http://http://hyperphysics.phy-astr.gsu.edu

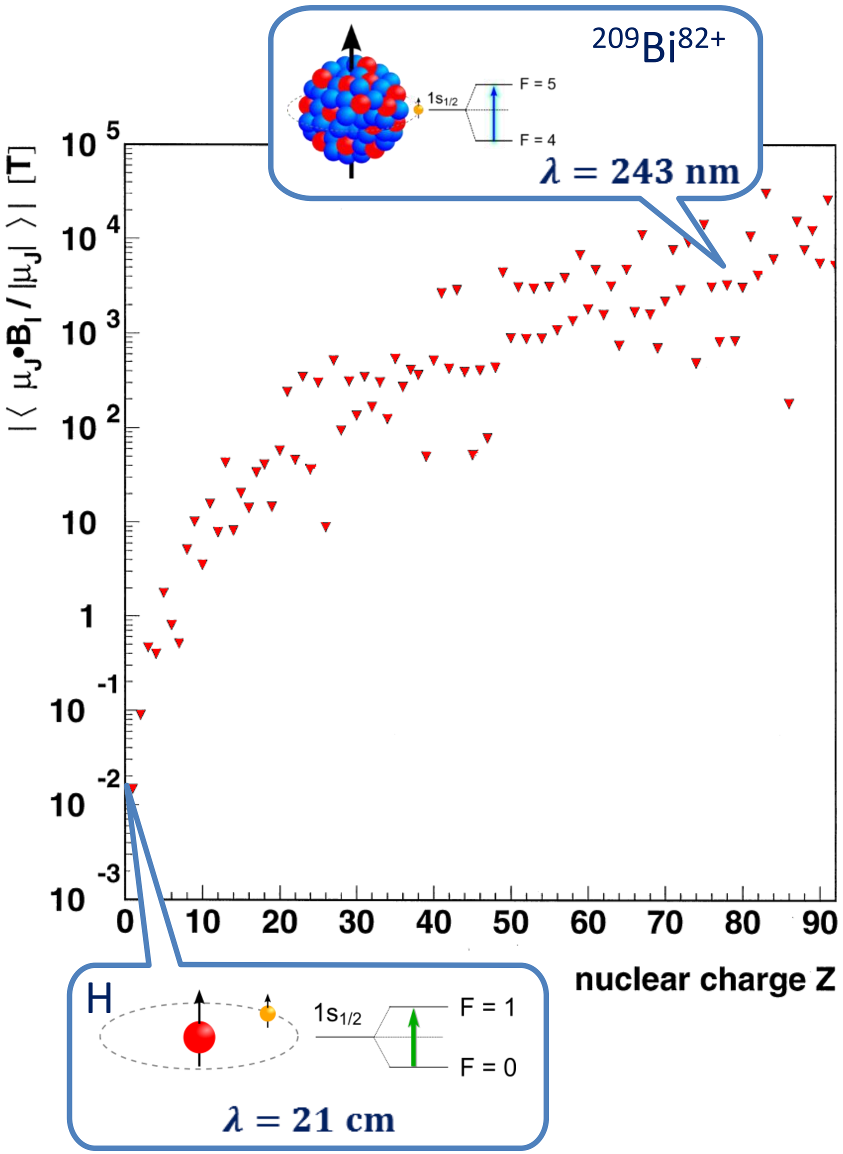


From http://antwrp.gsfc.nasa.gov/apod/ap050825.html

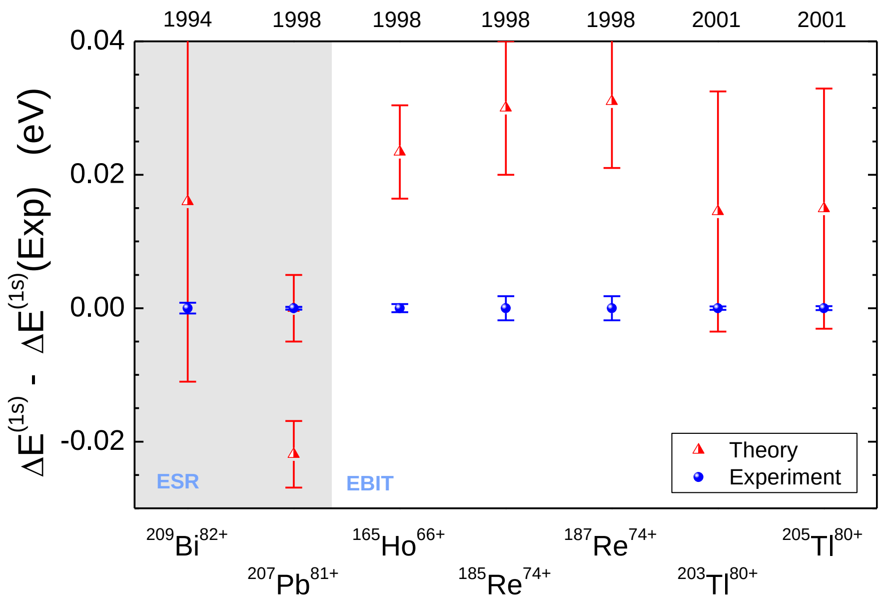
21 cm radiation is used, for example, to measure radial velocities of spiral arms of Milky Way.

Analysis of the properties of galaxies.

Hyperfine structure of heavy ions

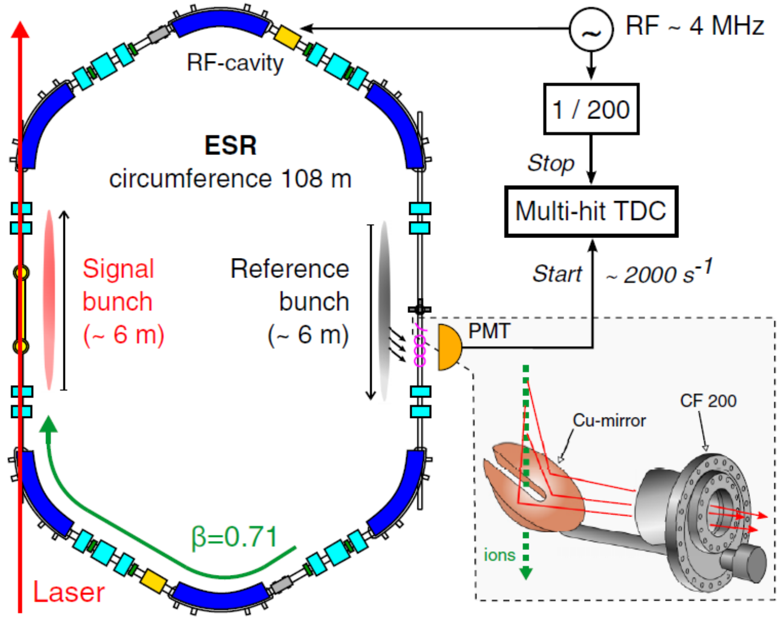


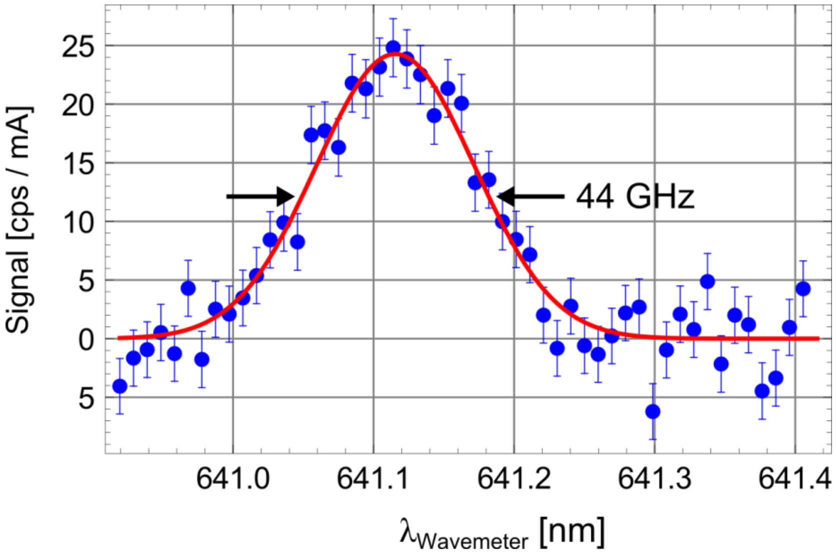
- The magnetic flux density at the nuclear surface is about $10^9 - 10^{10}$ T
- Hyperfine structure probes such extremely strong magnetic fields very close to the nuclear surface.
- A number of experiments were performed for 1s state of H-like ions.



Hyperfine structure of heavy ions

- A series of experiments have been performed to observe the hyperfine splitting of 2s state of Li-like $^{209}Bi^{80+}$.
- Theoretical evaluation of this splitting is very difficult task owing to the insufficiently known magnetic moment distribution inside the nucleus (Bohr-Weisskopf (BW) effect).





$$\lambda_{\text{Lab}} = \mathbf{641.112(24)}\ \text{nm}$$

M. Lochmann *et al*, Phys. Rev. A **90** (2014) 030501(R)

Atomic clocks based on hyperfine transitions

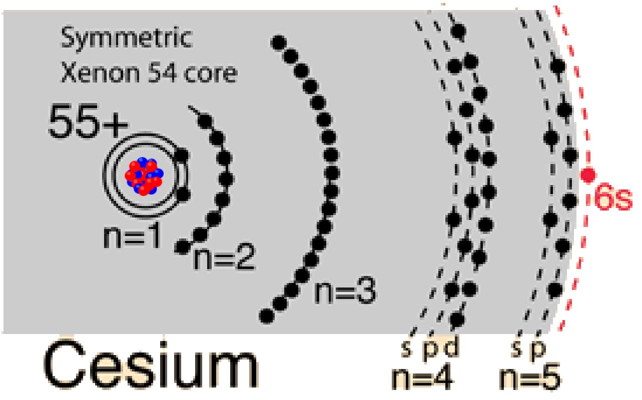
Very accurate clocks can be constructed by locking an electronic oscillator to the frequency of an atomic transition.

A second is defined as the duration of 9,192,631,770 cycles of microwave light absorbed or emitted by the hyper-fine transition of cesium-133 atoms undisturbed by external fields:

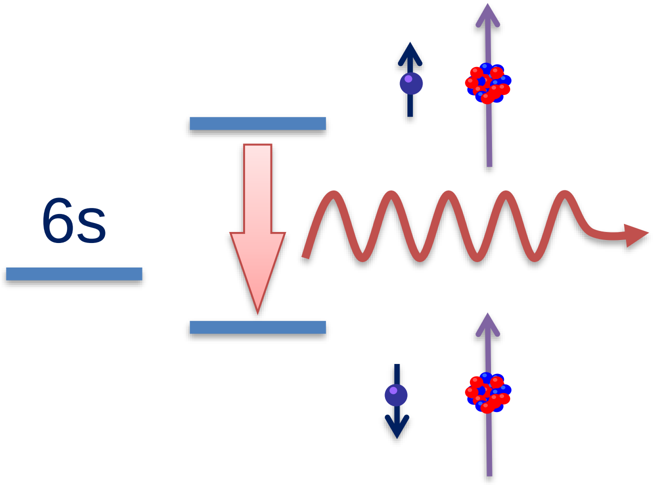
1 second = 9,192, 631,770 cycles transition

The relative standard uncertainty of the Cs clocks is about 10^{-16}.

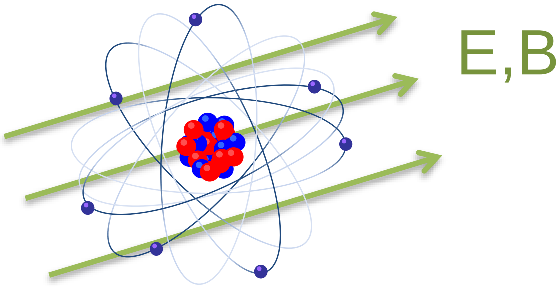
Can the accuracy of atomic clocks be further improved? And if “yes” – do we actually need it?



Quelle des Bildes: hyperphysics.phy-astr.gsu.edu

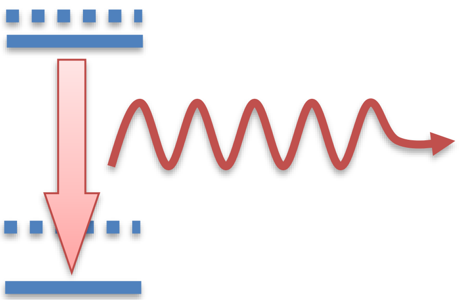


Atomic clocks: Improving accuracy



A second is defined by the transition in an atom undisturbed by external fields. But can we really "decouple" our atom from environment?

External fields lead to the shift of the energy levels (and, hence, of frequencies). For electric field, for example:

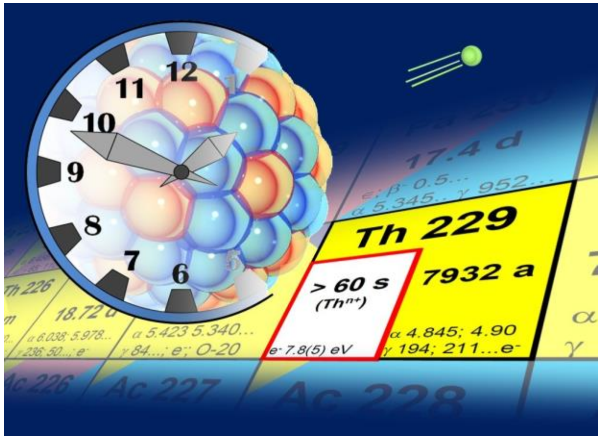


$$\Delta E \cong -\frac{1}{2}\,\alpha_0\,\mathcal{E} + \ldots$$

α_0 - is the polarizability which describes the response of an atom to external field

Alternative systems are searched which are less "coupled" to external fields. This will allow to make more accurate measurements.

Nuclear clocks: Even higher accuracy



Observation of nuclear transition in ^{229}Th is important for the development of novel nuclear clocks.

A number of various schemes are proposed to observe this transition. Very promising are atomic processes in which excitation of electronic shell is transferred to a nucleus.

NATURE | ARTICLE

日本語要約

Direct detection of the ^{229}Th nuclear clock transition

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Affiliations | Contributions | Corresponding author

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Abstract

Abstract • References • Author information • Extended data figures and tables

Today's most precise time and frequency measurements are performed with optical atomic clocks. However, it has been proposed that they could potentially be outperformed by a nuclear clock, which employs a nuclear transition instead of an atomic shell transition. There is only one known nuclear state that could serve as a nuclear clock using currently available technology, namely, the isomeric first excited state of ^{229}Th (denoted ^{229m}Th). Here we report the direct detection of this nuclear state, which is further confirmation of the existence of the isomer and lays the foundation for precise studies of its decay parameters. On the basis of this direct detection, the isomeric energy is constrained to between 6.3 and 18.3 electronvolts, and the

But why do we need so accurate measurements?

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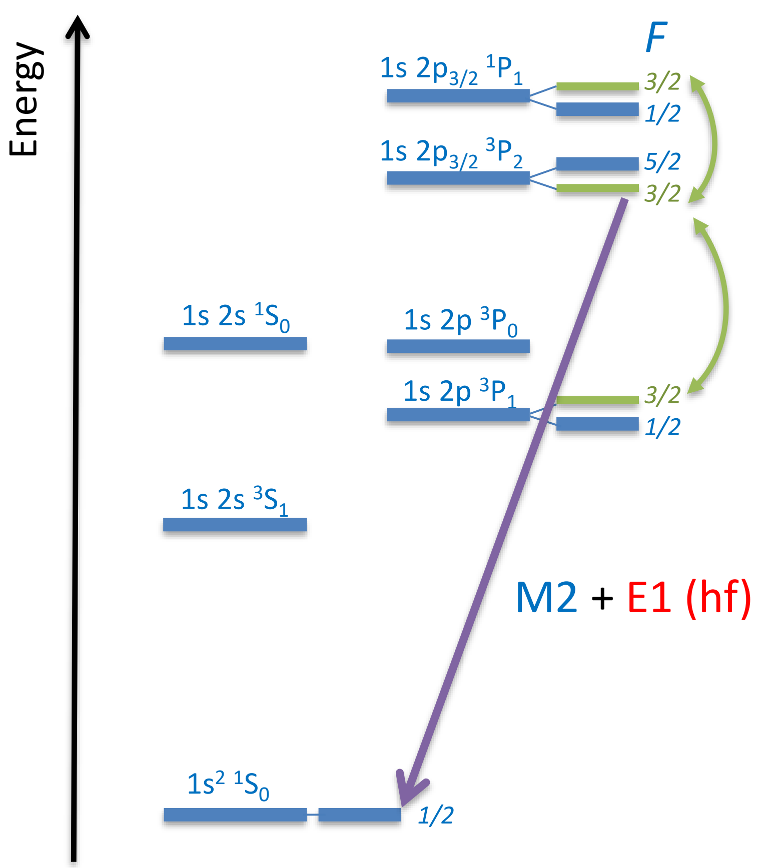
Hyperfine mixing of ionic levels



<u>Example</u>: Helium-like ion with the nuclear spin I=1/2

- Hyperfine interaction may affect not only the energy spectrum of highly-charged ions but also the properties of their characteristic emission.

- For example, hyperfine interaction leads to the mixing of $^3P_2$ and $^{1,3}P_1$ states of He-like ions:

$$\left|{}^3P_2\ F\right\rangle \rightarrow C_{^3P_2}\left|{}^3P_2\ F\right\rangle + C_{^3P_1}\left|{}^3P_1\ F\right\rangle + C_{^1P_1}\left|{}^1P_1\ F\right\rangle$$

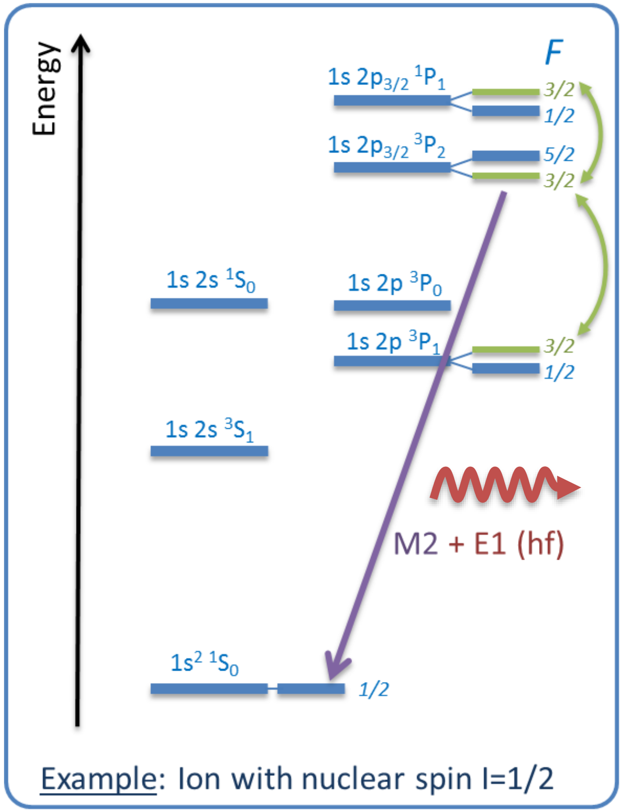
- As a result, the $^3P_2$ state can decay not only via the magnetic quadrupole (M2) but also the HF-induced electric dipole (E1) transition.

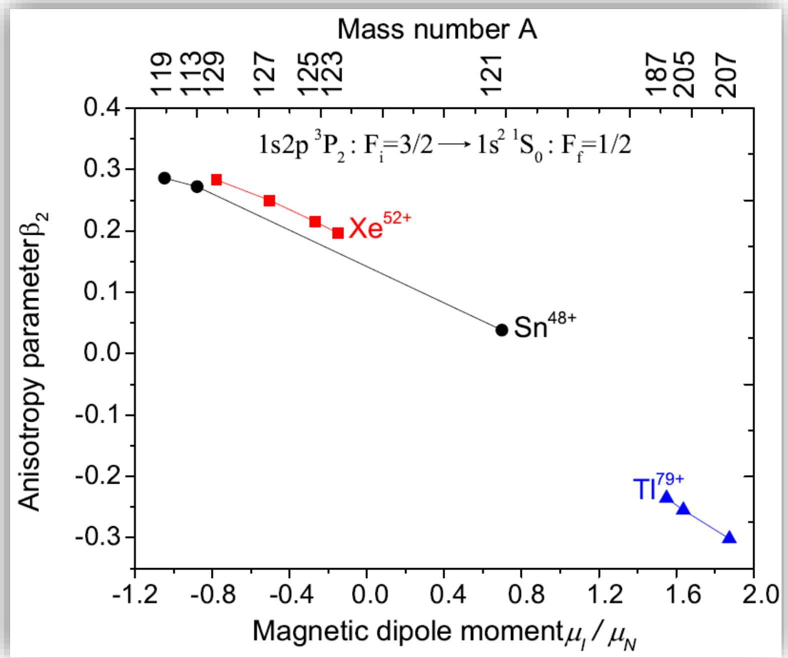
Kα transitions in helium-like ions

- The angular distribution of the hyperfine- as well as fine-structure resolved transitions in helium-like ions:

$$W(\theta) \sim 1 + \beta_2^{eff} P_2(\cos\theta)$$

- Owing to hyperfine-induced mixing between leading M2 and hf-E1 transitions, the Kα emission pattern appears to be very sensitive to the magnetic dipole moment.





A. Surzhykov, Y. Litvinov, Th. Stöhlker and S. Fritzsche, Phys. Rev. A **84** (2013) 052507
Z. W. Wu, A. Surzhykov and S. Fritzsche, Phys. Rev. A **90** (2014) 063422

Plan of the lecture

- Reminder from the last lecture: Bound-state solutions of Dirac equation
- Higher-order corrections to Dirac energies:
  - Radiative corrections (QED effects)
  - Hyperfine interaction