
Lecture 5

Further beyond the Dirac equation:
isotope shift, parity violation effects

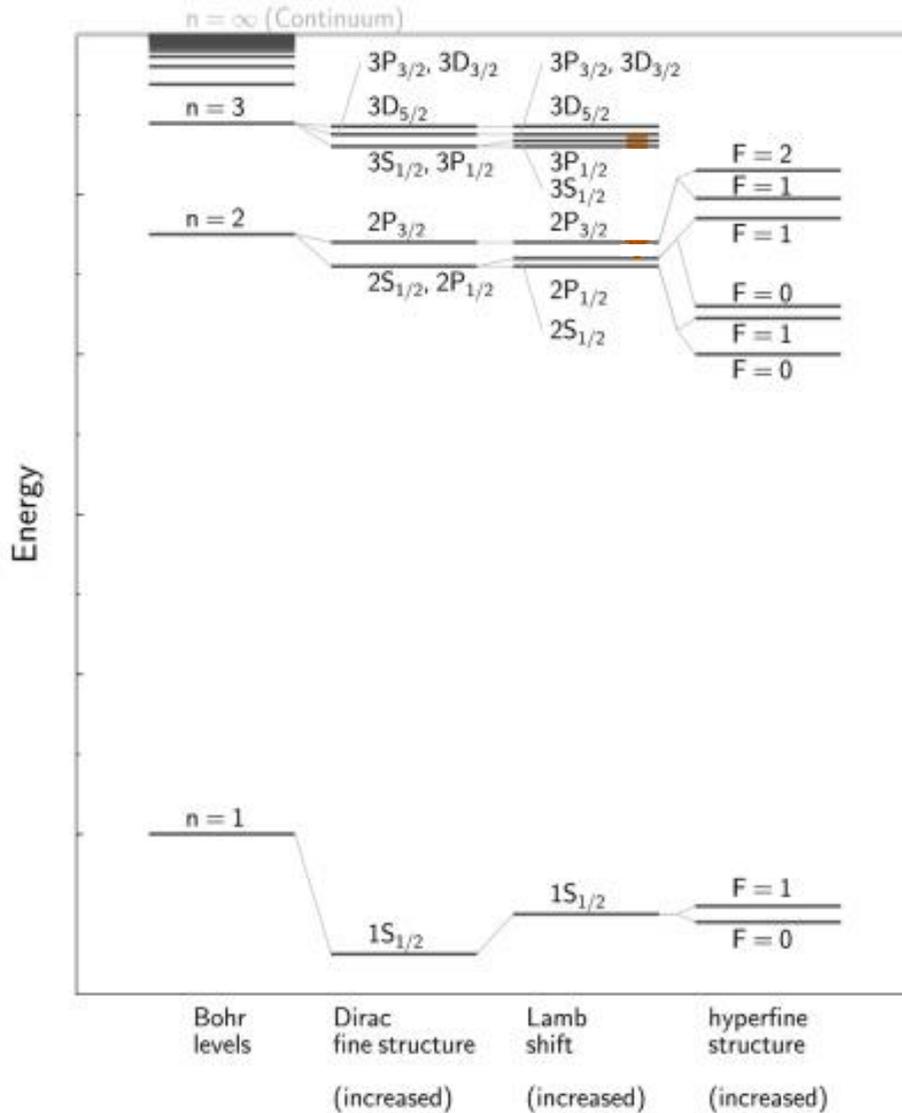
Plan of the lecture



- Isotope shift of energy levels: Mass and volume effects
 - Isotope shift in Ca^+ ions
 - Analysis of DR spectra of heavy ions

- Parity violation in heavy ions
 - Nuclear spin independent effect
 - Parity violation in nuclei: Anapole moment

Energy levels of hydrogen-like ions



- OK, we did a rather long way... Are there further corrections to the energy levels?

Dirac energy levels

- So far we have discussed Dirac equation for point-like, infinitely heavy nucleus:

$$\left(-i\hbar c \boldsymbol{\alpha} \cdot \nabla - \frac{Ze^2}{r} + m_e c^2 \alpha_0 \right) \psi(\mathbf{r}) = E \psi(\mathbf{r})$$

- In reality, nucleus has some finite mass \Rightarrow *mass shift*
- ... and finite size \Rightarrow *field shift*

- Mass shift has been discussed already in non-relativistic quantum mechanics.

- We have to introduce *reduced* mass: $m_e \rightarrow \frac{m_e M}{m_e + M}$

- This leads only to shift (not splitting!) of energy levels, moreover:

$$\frac{m_e}{M_p} \approx \frac{1}{1836} \quad \Rightarrow \quad \frac{m_e}{M_U} < 0.000003$$

Filed shift

- To estimate volume effect we will use the well known approximation and assume that nuclear charge is uniformly distributed within a sphere of radius:

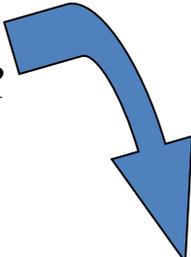
$$r = r_0 A^{1/3}$$

$r = 1.2 \times 10^{-15} m$ (indicated by a red arrow pointing to r_0)

mass number (indicated by a blue arrow pointing to A)

- In this model, electrostatic potential due to the nucleus deviates from pure Coulomb one and is given by:

$$V(r) = \begin{cases} \frac{Ze^2}{2R} \left(\frac{r^2}{R^2} - 3 \right) & r \leq R \\ -\frac{Ze^2}{r} & r > R \end{cases}$$

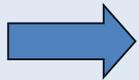

$$\left(-i\hbar c \boldsymbol{\alpha} \cdot \nabla + V(r) + m_e c^2 \alpha_0 \right) \psi(\mathbf{r}) = E \psi(\mathbf{r})$$

Filed shift

- Let us re-write Dirac equation as:

$$\left(-i\hbar c \boldsymbol{\alpha} \cdot \nabla - \frac{Ze^2}{r} + m_e c^2 \alpha_0 + \underbrace{V'(r)}_{\text{perturbation}} \right) \psi(\mathbf{r}) = E \psi(\mathbf{r})$$

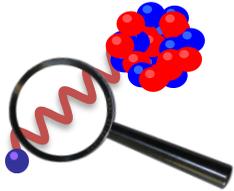
- Where “perturbation term” is:
$$V'(r) = \begin{cases} \frac{Ze^2}{2R} \left(\frac{r^2}{R^2} + \frac{2R}{r} - 3 \right) & r \leq R \\ 0 & r > R \end{cases}$$



First-order energy shift:

$$\Delta E = \langle \psi_{n\kappa m} | V' | \psi_{n\kappa m} \rangle \propto \frac{Z^2 R^4}{n^3}$$

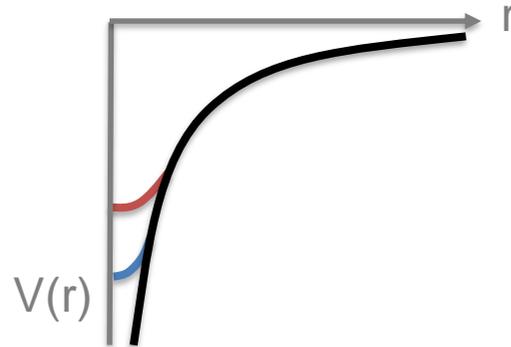
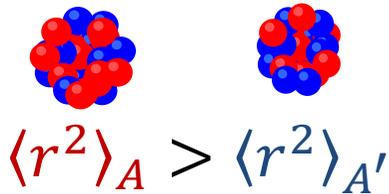
How to measure the isotope shift?



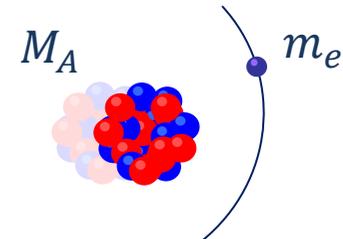
Is the electron-nucleus interaction just an interaction of two point-like charges, one of which (nucleus) is infinitely heavy?

$$V(r) = -\frac{Ze^2}{r}$$

Field shift
due to different charge radii



Mass shift
due to finite nuclear mass



The difference in the transition frequency for two isotopes with masses M_A and $M_{A'}$ and (mean square) nuclear radii:

$$\nu_A - \nu_{A'} = K \frac{M_A - M_{A'}}{M_A M_{A'}} + F \delta \langle r^2 \rangle_{AA'}$$

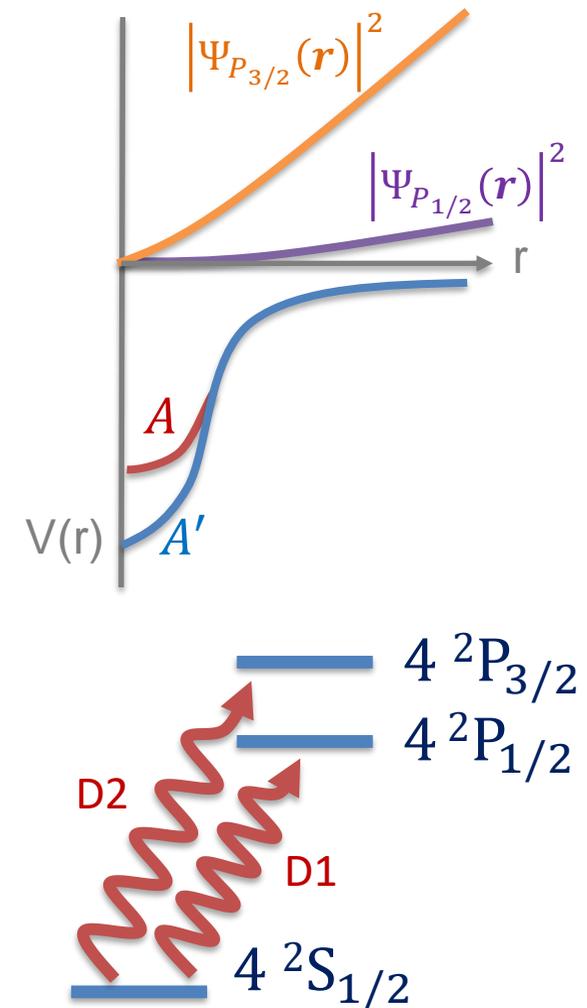
Isotope shift in Ca⁺ ion

Recently, high-precision measurements have been performed at the QUEST institute to compare isotope shifts for the $S_{1/2} \rightarrow P_{1/2}$ and $S_{1/2} \rightarrow P_{3/2}$ transitions in Ca⁺ ion.

These transitions exhibit different behavior with respect to the field shift.

$$f = \frac{F_{D2}}{F_{D1}}$$

Theoretical model	f
Hydrogenic	1.0051
Dirac-Fock	1.0010
Dirac-Fock + Core Pol.	1.0009
CCSD	1.0029
CCSD(T)	1.0048
MBPT	1.0011
CI+MBPT	1.0014 (4)
Experimental value	1.0085 (12)

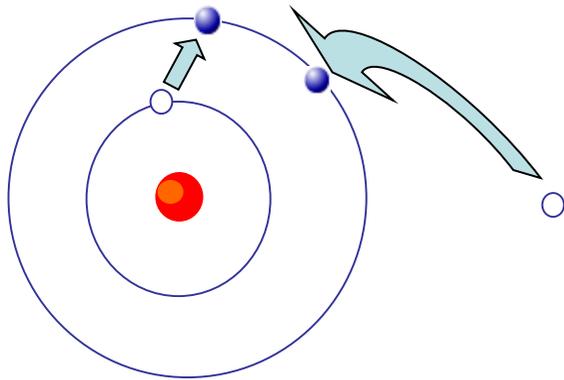


What is the reason for this big disagreement between experiment and theory? Many-electron or nuclear effects? QED?

Dielectronic recombinations

- One may consider process reversed to Auger decay: dielectronic recombination

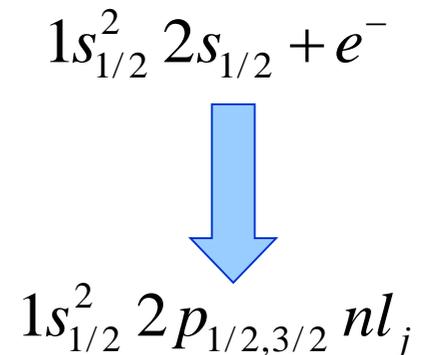
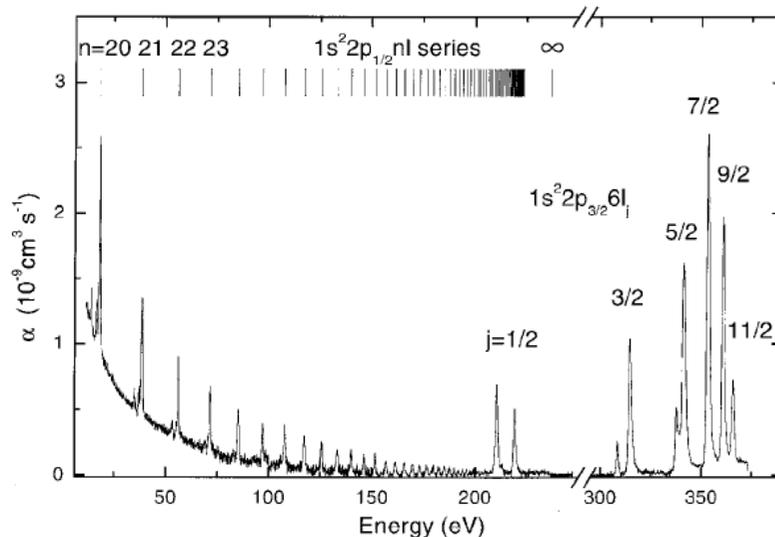
Very important for spectroscopic purposes: dielectronic recombination is resonant capture process!



$$T_{kin} + E_b = E^{**}$$

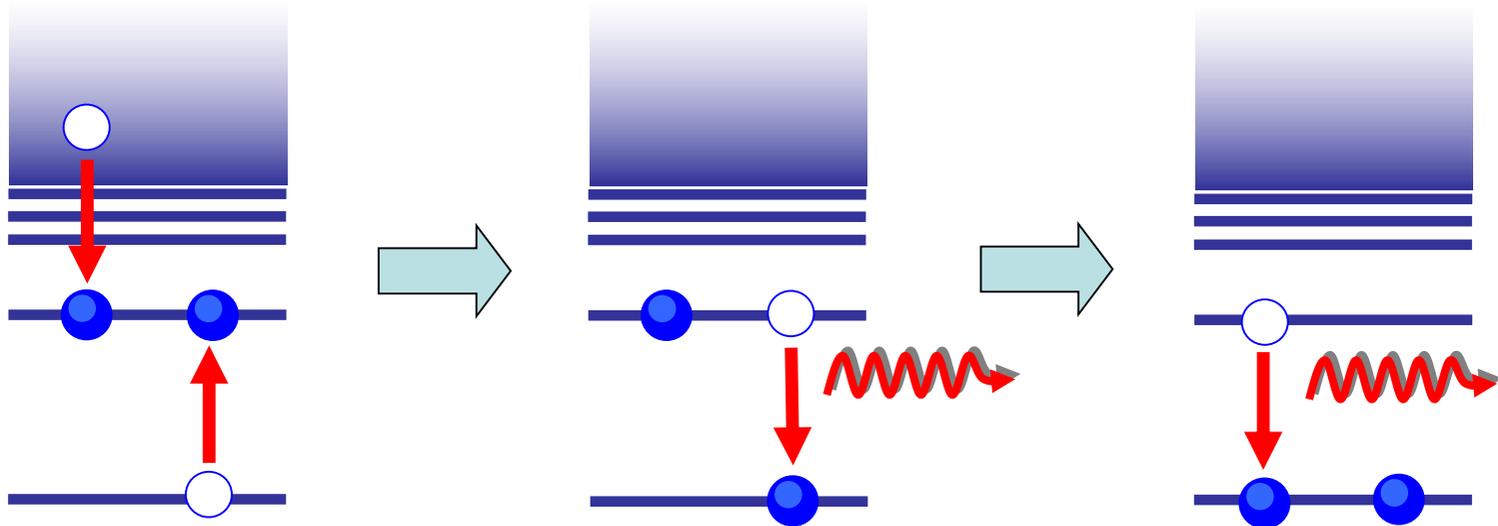
One can vary kinetic energy of electrons (or ion beam) and to study DR cross sections (rates).

- Example: DR of initially lithium-like bismuth ions Bi^{80+}



Dielectronic recombination

- Dielectronic recombination (DR) is a resonant process in which a free electron is captured into an ion under the simultaneous excitation of bound electron(s), and where the multiply excited ion is stabilized afterwards by photon emission.



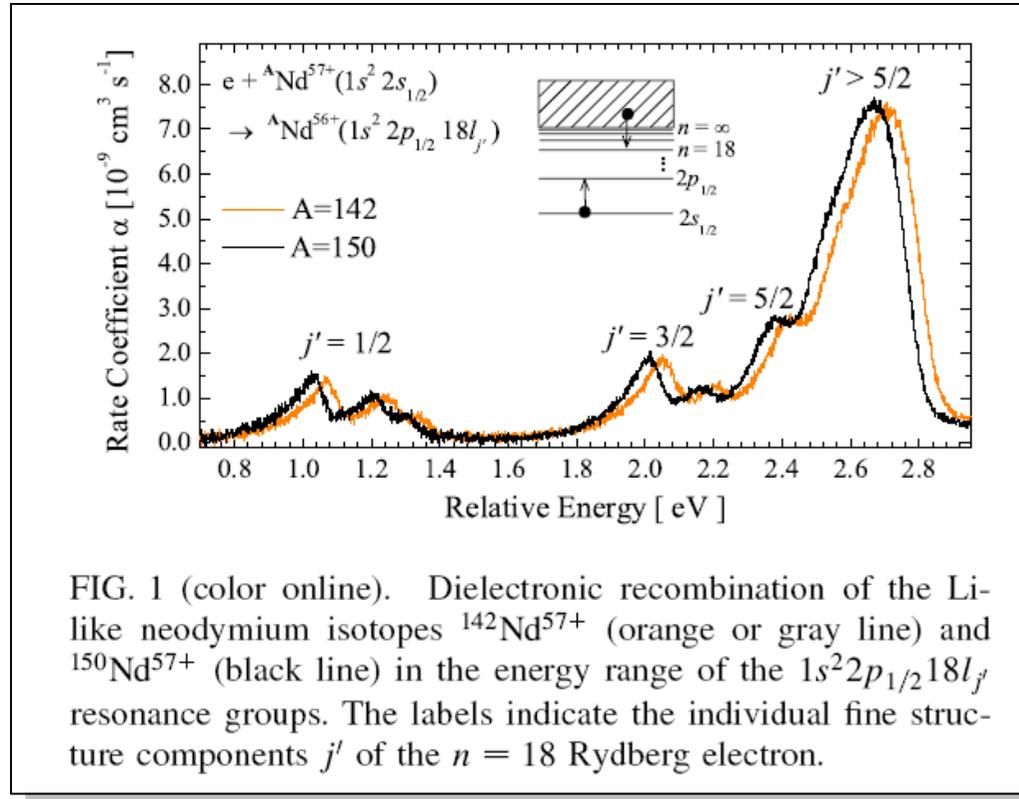
Isotope shift measurements with heavy ions

- DR spectroscopy may help us to resolve (hyper) fine structure of ionic levels.

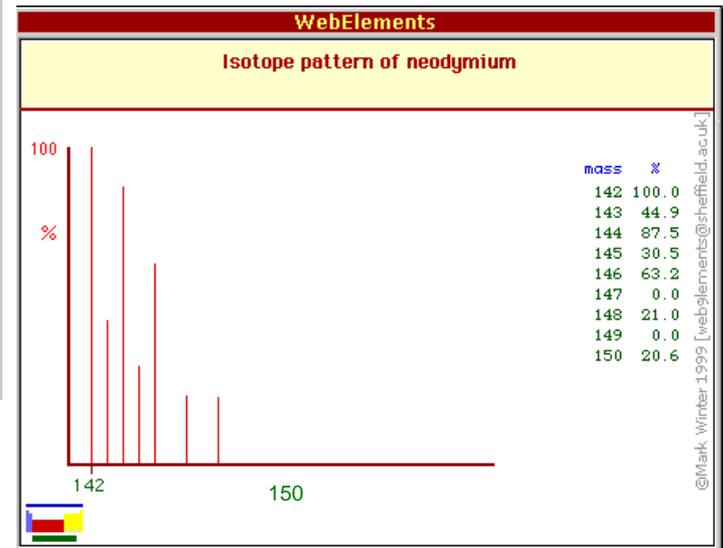
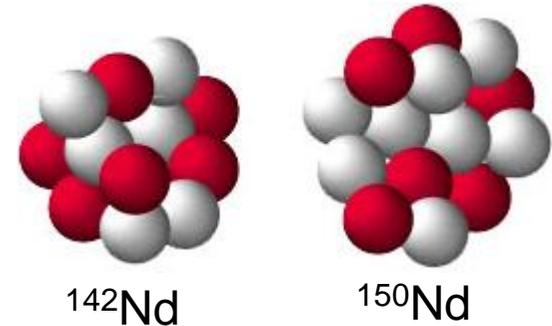
PRL 100, 073201 (2008)

PHYSICAL REVIEW LETTERS

week ending
22 FEBRUARY 2008



Example: isotope shift in DR spectra of neodymium ions.



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 - Parity violation in nuclei: Anapole moment

Unified electro-weak interaction

Standard Model of FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS

Leptons spin = 1/2

Flavor	Mass GeV/c ²	Electric charge
ν_e electron neutrino	<1·10 ⁻⁸	0
e electron	0.000511	-1
μ muon	<0.0002	0
M muon	0.106	-1
ν_τ tau neutrino	<0.02	0
T tau	1.7771	-1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.003	2/3
d down	0.006	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	175	2/3
b bottom	4.3	-1/3

Structure within the Atom

BOSONS

Unified Electroweak spin = 1

Name	Mass GeV/c ²	Electric charge
γ photon	0	0
W^-	80.4	-1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c ²	Electric charge
g gluon	0	0

Color Charge
Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and W and Z bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons
One cannot isolate quarks and gluons. They are confined in color-neutral particles called hadrons. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons. These are the particles seen to emerge. Two types of hadrons have been observed in nature: mesons $q\bar{q}$ and baryons qqq .

Residual Strong Interaction
The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property	Interaction	Gravitational	Weak	Electromagnetic	Strong
		Mass-Energy	Flavor	Electric Charge	Color Charge
Acts on:		All	Quarks, Leptons	Electrically charged	Color Charge, Gluons
Particles experiencing:		All	Quarks, Leptons	Electrically charged	Color Charge, Gluons
Particles mediating:		Graviton (not yet discovered)	W^+ W^- Z^0	γ	Gluons
Strength relative to electromagnetism for two u quarks at:		10 ⁻⁴¹	0.8	1	25
for two u quarks at:		10 ⁻⁴¹	10 ⁻⁴	1	60
for two protons in nucleus		10 ⁻³⁶	10 ⁻⁷	1	Not applicable to hadrons

Mesons $q\bar{q}$

Mesons are bosonic hadrons. There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c ²	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	-1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.380	0

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons. There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c ²	Spin
p	proton	uud	1	0.938	1/2
\bar{p}	anti-proton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	-1	1.672	3/2

Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or - charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0 , γ , and η_c , but not K^0 or Ω^0) are their own antiparticles.

Figures
These diagrams are an artist's conception of physical processes. They are not exact and have no meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

An electron and positron annihilate at high energy can annihilate to produce γ and Z^0 mesons via a virtual Z boson or a virtual photon.

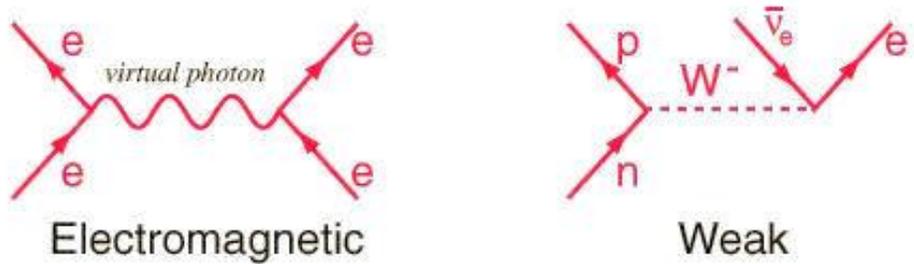
Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure
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One of the biggest successes of the Standard Model is the unification of the electromagnetic and the weak forces into the so-called electroweak force.

Note that electromagnetic interaction preserves spatial parity while weak interaction – not!



Unified electro-weak interaction

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Figures

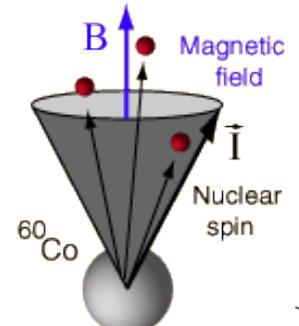
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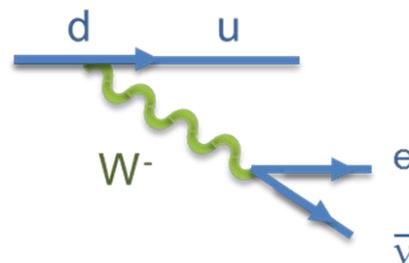
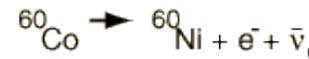
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Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

Beta emission is preferentially in the direction opposite the nuclear spin, in violation of conservation of parity.



Wu, 1957



Parity violation (PV) is first time observed in famous Wu experiment (1956) on the beta-decay of cobalt nuclei.

http://hyperphysics.phy-astr.gsu.edu/

Further extensions of the Standard Model ?

mass →	2.4 MeV/c ²	1.27 GeV/c ²	171.2 GeV/c ²	0	≈126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	γ photon	H Higgs boson
QUARKS					
	4.8 MeV/c ²	104 MeV/c ²	4.2 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	g gluon	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS				GAUGE BOSONS	
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

Is our knowledge about the Standard Model (and, in particular, about its electroweak segment) complete?

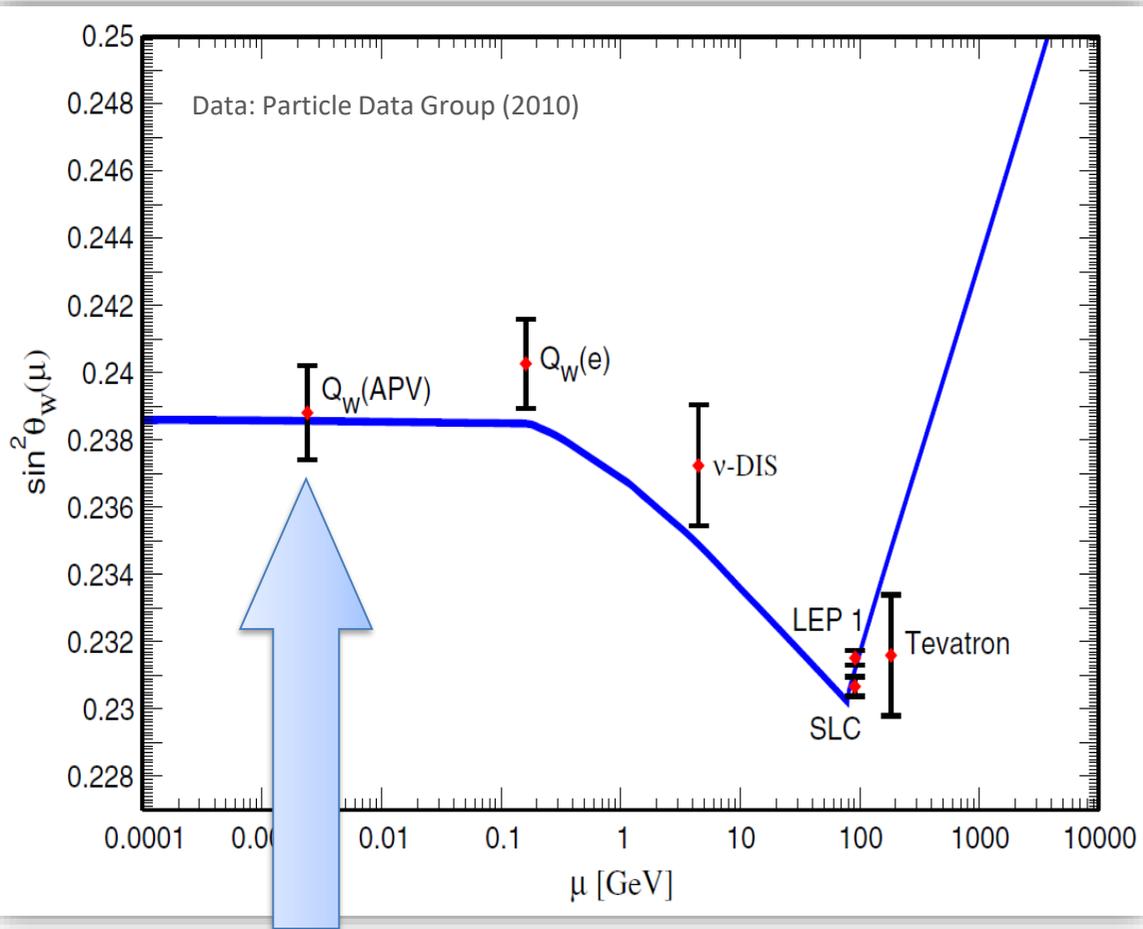
New **experimental results** from the LHC collaborations might indicate the existence of new particles beyond the SM:

- Di-photon bump at 750 GeV ?
- Charged Higgs boson in $H \rightarrow tb$ decay ?
- W' in high-mass di-boson resonances?

Various **theoretical extensions** of the standard model suggest the existence of additional W' and Z' bosons which can modify the parameters of the electroweak interaction.

The neutral weak interaction currents which lead to the parity-violating effects in atomic systems attract special attention!

Weinberg angle



Besides high-energy LHC experiments, precision electroweak measurements in **atomic physics** attract currently much attention since they allow to explore low-energy regime!

Weinberg angle is one of the key parameters of the electro-weak theory.

It describes how the photon and neutral boson Z are “produced” from original vector bosons:

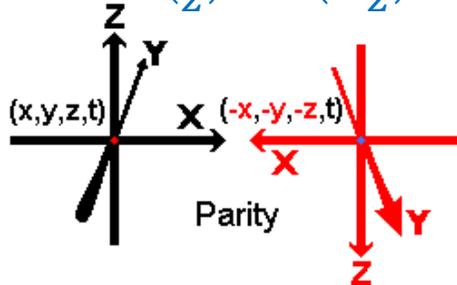
$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

This formula, again, shows that the weak and electromagnetic interactions are representations of the same force.

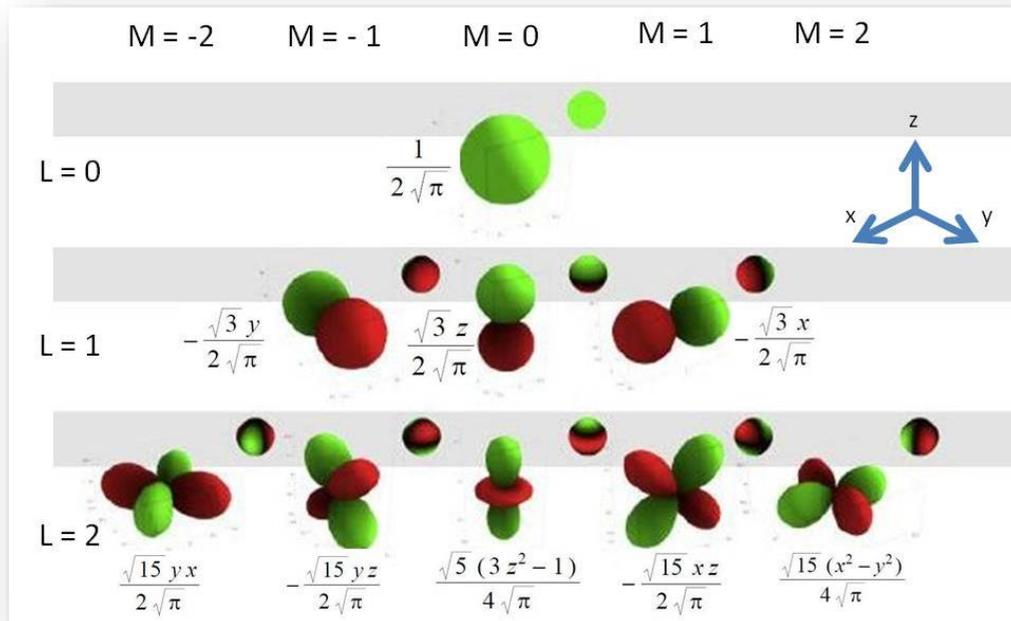
Parity of atomic states (just a reminder!)

The parity operator commutes with the (non-relativistic) Hamiltonian of the hydrogen-like atom.

Therefore eigenfunctions of the Hamiltonian, i.e. atomic wavefunctions, have particular symmetry properties:

$$\hat{P}\mathbf{r} = \hat{P} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$


$$\psi_{nlm}(\mathbf{r}) = R_{nl}(r) Y_{lm}(\theta, \varphi), \quad \hat{P}\psi_{nlm}(\mathbf{r}) = \psi_{nlm}(-\mathbf{r}) = (-1)^l \psi_{nlm}(\mathbf{r})$$



S-, D-, G- atomic states have positive parity, while P-, F-, H- have negative parity.

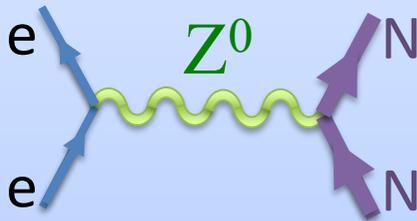
This is all true if interaction inside an atom is purely electromagnetic....

Parity violating interactions

The effective Hamiltonian of the PV electron-nucleus interaction can be cast in the form:

$$H_{PV} = \frac{G_F}{\sqrt{2}} \left(-\frac{Q_W}{2} \gamma_5 + \frac{\kappa}{I} \boldsymbol{\alpha} \cdot \mathbf{I} \right) \rho(\mathbf{r})$$

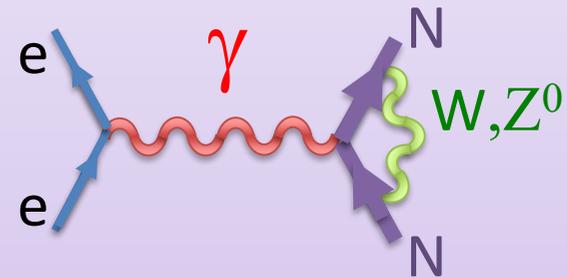
Dominant part is the nuclear-spin-independent (NSI) interaction that arises due to exchange of neutral Z^0 boson between nucleus and electrons.



+ Weak charge Q_w characterizes NSI part:

$$Q_W = Z(1 - 4 \sin^2 \theta_W) - N$$

The nuclear-spin-dependent (NSD) part comes mainly from the electromagnetic interaction with weakly interacting nucleons.



+ NSD is characterized by the coupling constant κ :

$$|\kappa| \approx \frac{|Q_W|}{100}$$

How we can observe the parity-violating interactions?

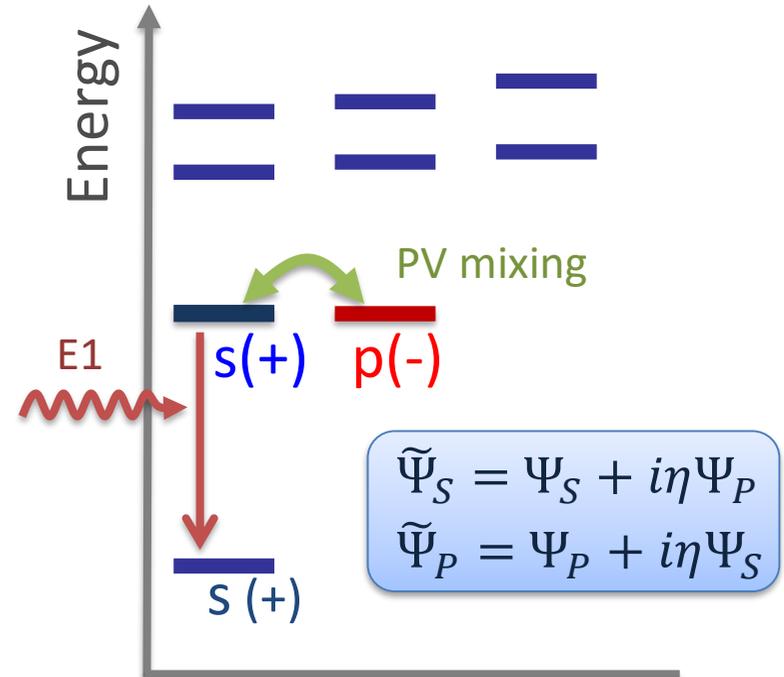
Atomic parity violation

The parity-violating (NSI and NSD) interactions lead to the mixing of atomic levels with opposite parity.

Mixing coefficient is given by:

$$\eta = \frac{\left\langle \Psi_S \left| \frac{G_F}{\sqrt{2}} \left(-\frac{Q_W}{2} \gamma_5 + \frac{\kappa}{I} \boldsymbol{\alpha} \cdot \mathbf{I} \right) \rho(\mathbf{r}) \right| \Psi_P \right\rangle}{E_S - E_P - i \Gamma/2}$$

Energy splitting should be small!



Total angular momenta “selection rules”:

- Nuclear spin independent interaction mixes only states with the same J 's
- Nuclear spin dependent interaction can mix levels with different J 's but same F 's



- If the parity is broken due to the weak interaction this transition becomes possible!

PV experiments with neutral atoms

PV experiments with neutral atoms have provided us with valuable information on the weak interaction.

But! Analysis of these experiments is rather difficult task because of “many-electron nature” of systems.

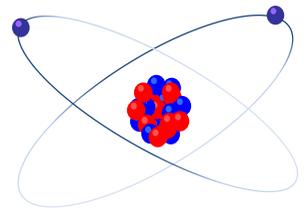
- Alternatively, we may explore APV effects as appear in few-electron ions!
- Few-electron ions may be perfect candidates for PV studies:
 - Relatively simple atomic systems
 - Large electron-nucleus overlap
 - Effect scales as Z^5 (in contrast to Z^3 in neutral systems)
 - Levels with opposite parities might be almost degenerated

PV experiments with few-electron ions

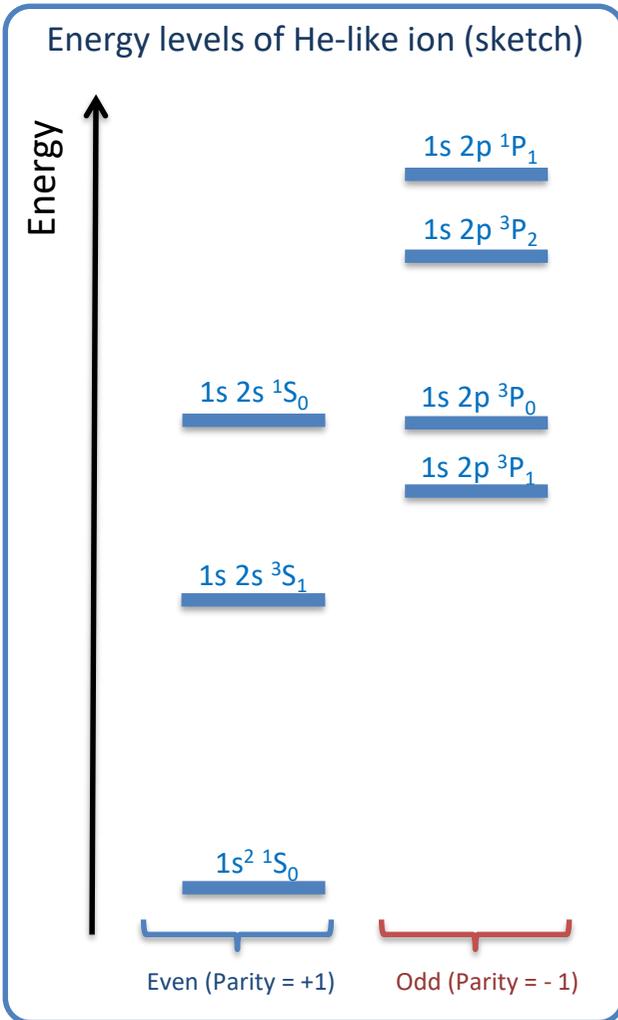
A large number of parity-violation studies have been proposed in the recent years for ions in different charge states: from singly-ionized to hydrogen-like.

We will discuss today two (rather simple) cases of the parity violation in helium-like ions:

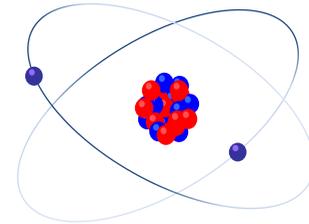
- Two-photon $^1S_0 \rightarrow ^3P_0$ transition (NSI mixing)
- Hyperfine-induced $^1S_0 \rightarrow ^3S_1$ transition (NSP mixing)



Helium-like ions (just a reminder!)



- Helium-like ions are among the most promising candidates for the PV studies in high-Z domain.



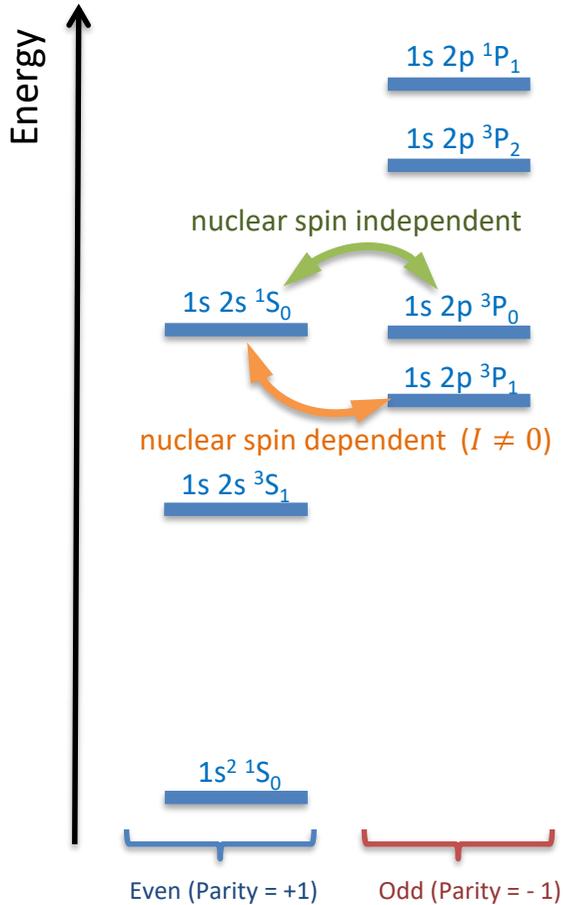
- To identify the states of this two-electron system we will use spectroscopic notations in the “jj-coupling” scheme as:

$$n_1 l_1 j_1 \quad n_2 l_2 j_2 \quad 2S+1 L_J$$

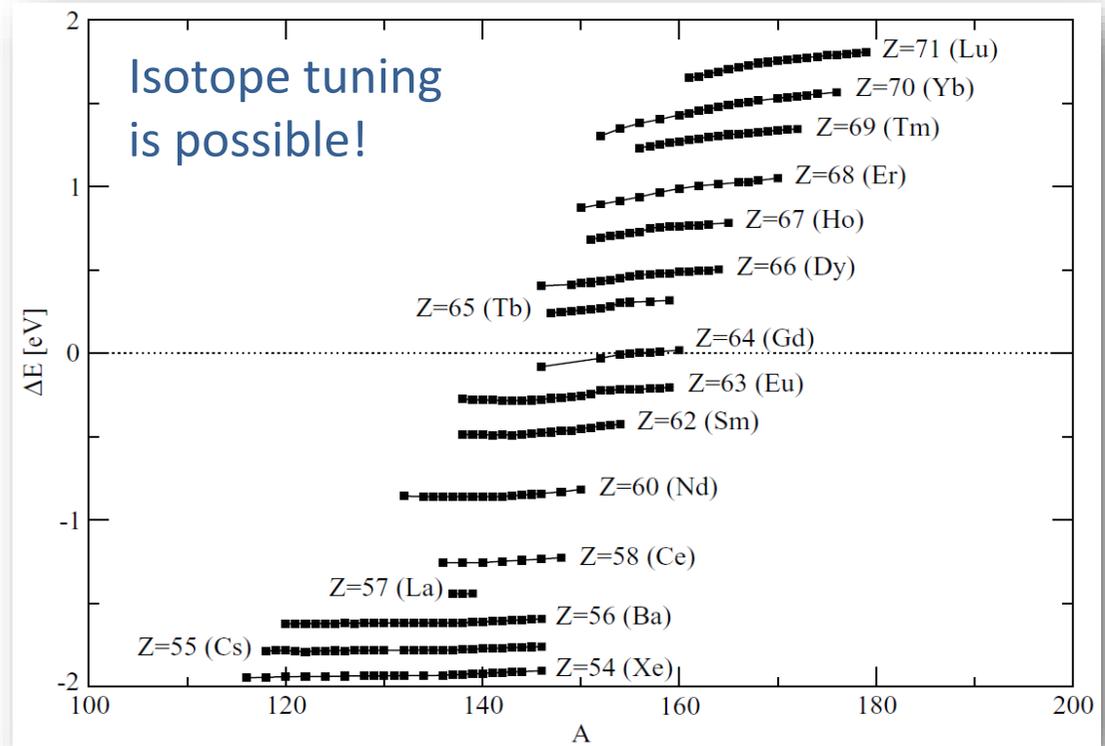
- For example, ground state: $1s^2 \ 1S_0$.
- Can we find in helium-like ions almost degenerate levels with opposite parity?

Parity-violation in helium-like ions

Energy levels of He-like ion (sketch)



Helium-like ions provide a unique tool for studying parity violation phenomena in atomic systems.



Don't forget: the closer levels to each other – the stronger parity violating mixing between them!

Many-electron calculations: Basics

- High-precision analysis of helium energy levels is a rather complicated task. We discuss here (very roughly and over-simplified!) just few important steps.

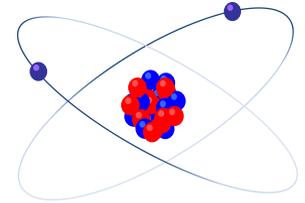


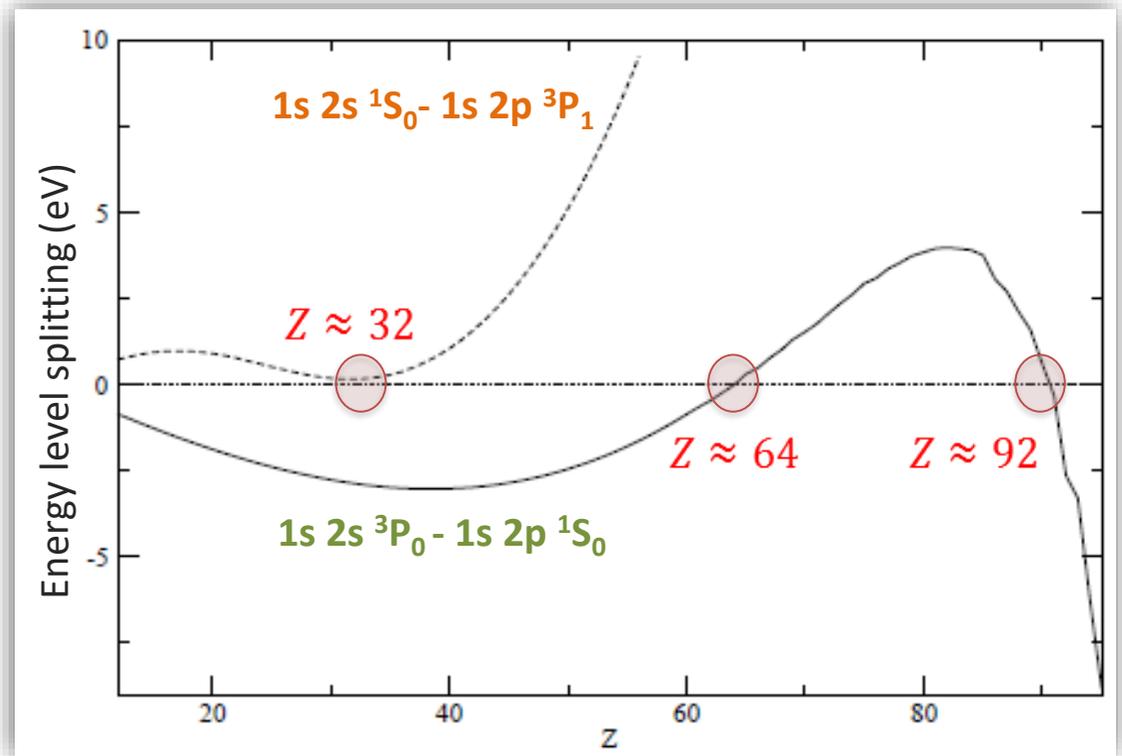
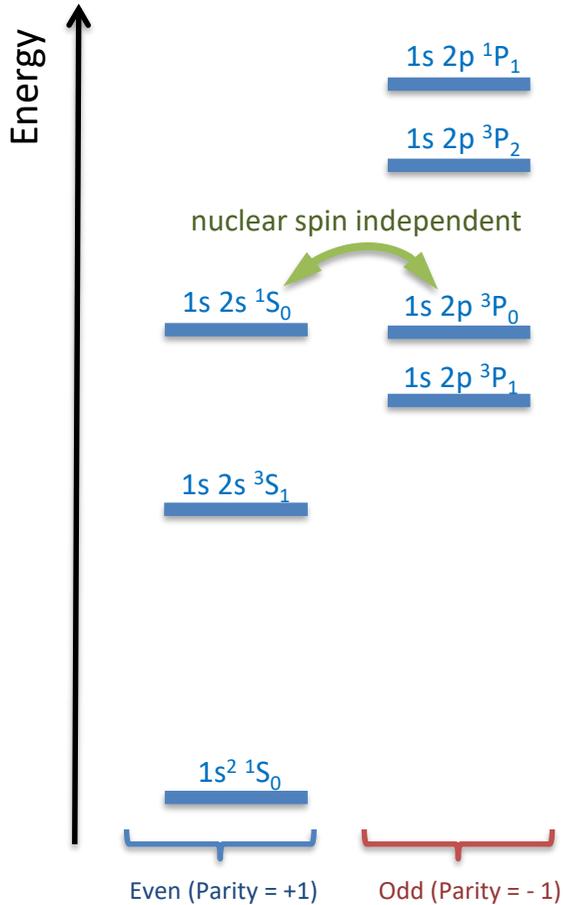
TABLE I. Contributions to the energies of the $1s2s\ ^1S_0$ and $1s2p\ ^3P_0$ states in He-like ^{238}U ions, relative to the ionization threshold. All energies are in eV.

	$1s2s\ ^1S_0$	$1s2p\ ^3P_0$
Dirac energy (pointlike nucleus)	-34 215.481	-34 215.4811
Nuclear size effects ^a	37.738	4.4133
Total zeroth-order energy	-34 177.743(27)	-34 211.0678(56)
First-order correlation [30]	850.135	923.198
Second-order correlation	-6.5368	-5.6726
Higher-order correlation	-0.0005	-0.0174
Total electron correlations	843.598(1)	917.508(4)
One-electron QED correction [26]	49.547(75)	6.846(12)
Two-electron QED correction [17]	-3.8259(4)	-4.4740(3)
Higher-order QED correction [14]	-0.009(51)	0.002(73)
Nuclear polarization and recoil correction [27]	0.0890	0.0491
Total energy	-33 288.344(94)	-33 291.137(74)

^aA Fermi distribution for the nuclear charge with rms radius $\langle r^2 \rangle^{1/2} = 5.8569(33)$ fm [28] was adopted.

Parity-violation in helium-like ions

Energy levels of He-like ion (sketch)

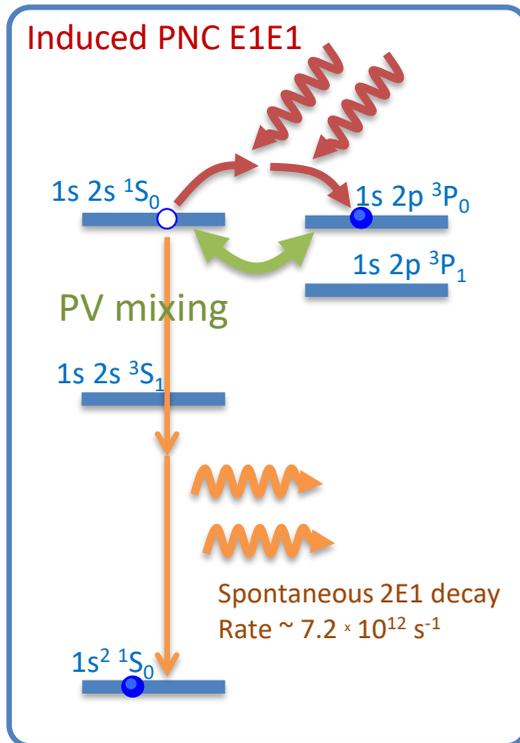


For elements around Gadolinium ($Z=64$) and Uranium ($Z = 92$), $1s2s \ ^3P_0$ and $1s2p \ ^1S_0$ levels are almost degenerate and, hence, strongly mixed:

$$\tilde{\Psi}_{1S_0} = \Psi_{1S_0} + i\eta\Psi_{3P_0}, \quad \eta = 10^{-6}$$

How to measure this mixing?

Induced two-photon transitions



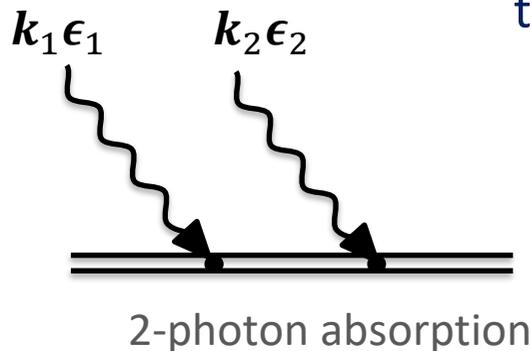
Simple idea is to induce direct transition between two states of interest. But... both states have zero total angular momentum.

One has to induce two-photon transition:



Simultaneous absorption (or emission) of two photons is one of the most known second-order QED processes.

Analysis of two-photon transitions can be traced back to the second-order perturbation theory:



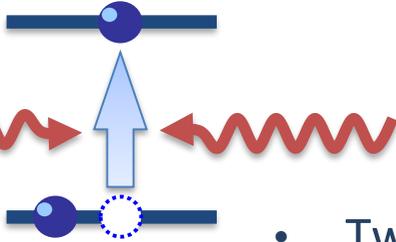
$$M_{2\gamma} = \sum_{\nu} \frac{\langle \tilde{\Psi}_P | \alpha A_1 | \Psi_{\nu} \rangle \langle \Psi_{\nu} | \alpha A_2 | \tilde{\Psi}_S \rangle}{E_{\nu} - E_S + \hbar\omega}$$

Two-photon transitions: Theory



- Analysis of two-photon transitions in many-electron systems is requires summation over the entire spectrum of the ion:

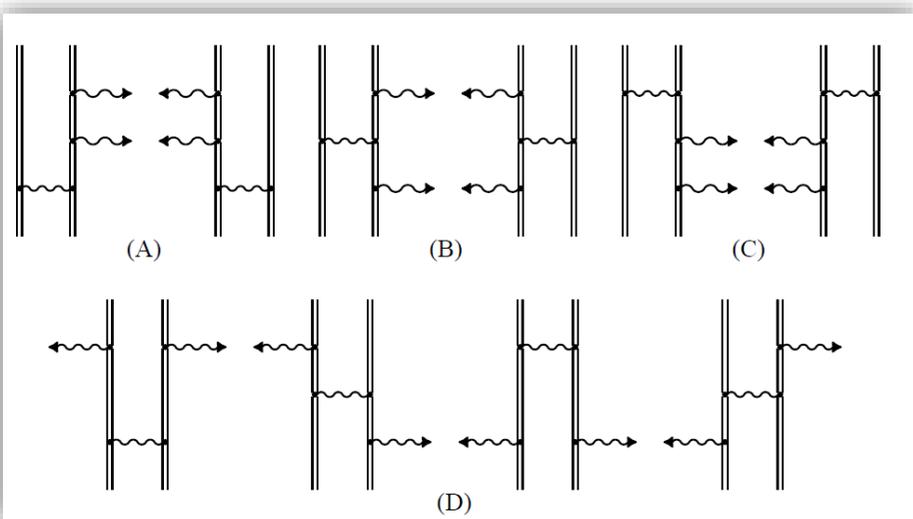
$$M_{2\gamma} = \sum_{\nu} \frac{\langle \tilde{\Psi}_f | \alpha A_1 | \Psi_{\nu} \rangle \langle \Psi_{\nu} | \alpha A_2 | \tilde{\Psi}_i \rangle}{E_{\nu} - E_S + \hbar\omega}$$



- Two independent theoretical approaches have been developed:

Analytical Green's function approach

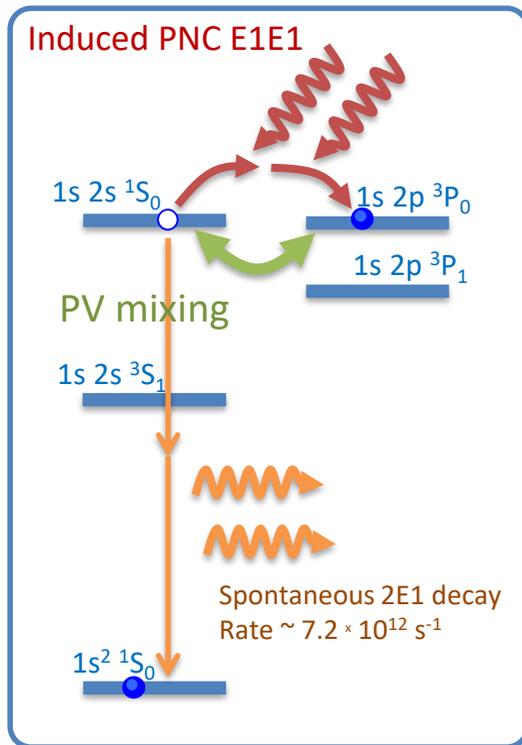
Final basis set approach



$$\Psi_n J^{\pi} M = \sum_r C_r(n) \underbrace{|\beta_r J^{\pi} M\rangle}$$

$$\begin{aligned} & \langle \mathbf{r}_1, \mathbf{r}_2 | \beta_r J^{\pi} M \rangle \\ &= \sum_{m_1 m_2} C_{m_1 m_2 M} \begin{vmatrix} \varphi_{j_1 m_1}(\mathbf{r}_1) & \varphi_{j_2 m_2}(\mathbf{r}_1) \\ \varphi_{j_1 m_1}(\mathbf{r}_2) & \varphi_{j_2 m_2}(\mathbf{r}_2) \end{vmatrix} \end{aligned}$$

Induced two-photon transitions



Induced two-photon transition rate:

$$\Gamma_{2\gamma} \propto I^2 \left| \sum_{\nu} \frac{\langle \tilde{\Psi}_P | \alpha A_1 | \Psi_{\nu} \rangle \langle \Psi_{\nu} | \alpha A_2 | \tilde{\Psi}_S \rangle}{E_{\nu} - E_S + \hbar\omega} \right|^2$$

Based on our many-body relativistic calculations we found that for helium-like uranium ion:

$$\begin{aligned} \Gamma_{2\gamma}(2E1) &= I^2 \eta^2 (\epsilon_1 \cdot \epsilon_2)^2 \times 4.35 \cdot 10^{-20} \\ &= I^2 (\epsilon_1 \cdot \epsilon_2)^2 \times 1.26 \cdot 10^{-33} \text{ s}^{-1} \end{aligned}$$

Polarization vectors should be parallel!

⊕ To “compete” with the spontaneous decay channel we have to induce two-photon E1E1 transition by polarized light with the intensity:

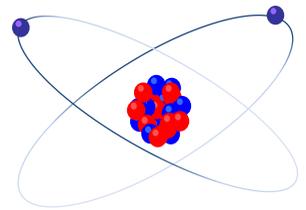
$$I \sim 10^{21} - 10^{22} \text{ W/cm}^2$$

PV experiments with few-electron ions

A large number of parity-violation studies have been proposed in the recent years for ions in different charge states: from singly-ionized to hydrogen-like.

We will discuss today two (rather simple) cases of the parity violation in helium-like ions:

- Two-photon $^1S_0 \rightarrow ^3P_0$ transition (NSI mixing)
- Hyperfine-induced $^1S_0 \rightarrow ^3S_1$ transition (NSP mixing)

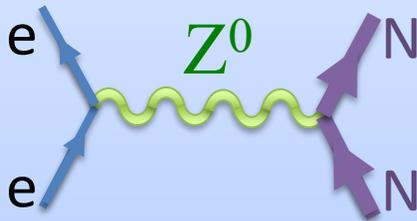


Parity violating interactions

The effective Hamiltonian of the PV electron-nucleus interaction can be cast in the form:

$$H_{PV} = \frac{G_F}{\sqrt{2}} \left(-\frac{Q_W}{2} \gamma_5 + \frac{\kappa}{I} \boldsymbol{\alpha} \cdot \mathbf{I} \right) \rho(\mathbf{r})$$

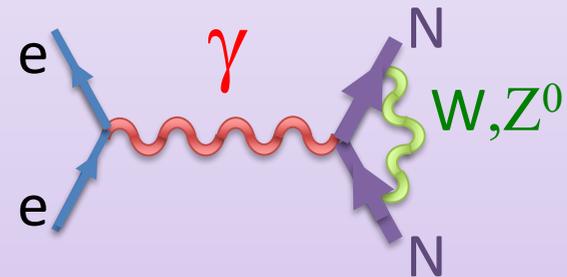
Dominant part is the nuclear-spin-independent (NSI) interaction that arises due to exchange of neutral Z^0 boson between nucleus and electrons.



+ Weak charge Q_w characterizes NSI part:

$$Q_W = Z(1 - 4 \sin^2 \theta_W) - N$$

The nuclear-spin-dependent (NSD) part comes mainly from the electromagnetic interaction with weakly interacting nucleons.



+ NSD is characterized by the coupling constant κ :

$$|\kappa| \approx \frac{|Q_W|}{100}$$

How we can observe the parity-violating interactions?

Nuclear spin-dependent PV effects

- ▶ Nuclear spin-dependent PV Hamiltonian:

$$H_{PV,NSD} = \frac{G_F \kappa}{\sqrt{2} I} \boldsymbol{\alpha} \cdot \mathbf{I} \rho(\mathbf{r})$$

- ▶ Within the shell model, considering a single valence nucleon with orbital momentum l around a spherical core of constant density:

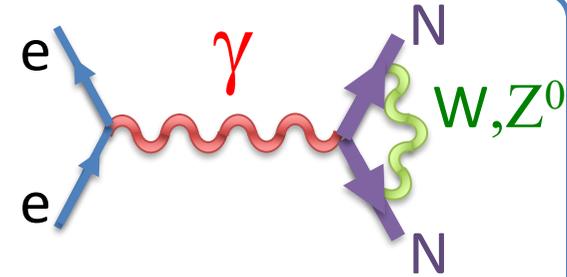
$$\kappa = \kappa_2 + \kappa_Q + \kappa_a$$

- ▶ The most important terms are anapole moment and $(V_e A_N)$ terms:

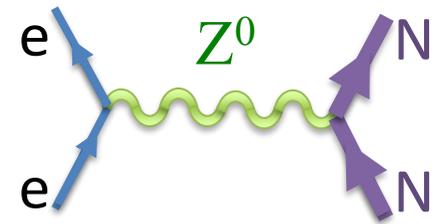
$$\kappa_a = \frac{K}{I+1} \cdot \frac{9}{10} g_\nu \mu_\nu \frac{\alpha}{m_p r_0} A^{2/3}$$

$$\kappa_2 = \frac{1/2 - K}{I+1} \cdot C_{2\nu}$$

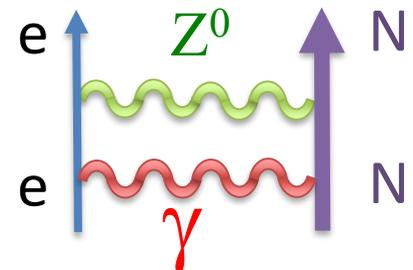
$$K = (I + 1/2)(-1)^{I+1/2-l}, \nu = P, N$$



Anapole moment



$(V_e A_N)$ weak neutral coupling



Hyperfine correction

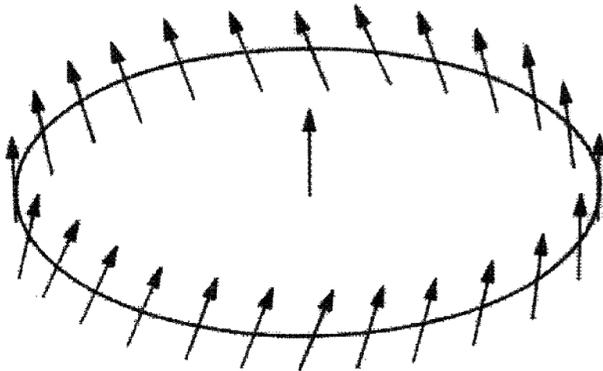
Nuclear anapole moment

- ▶ The effective one-body P-odd weak interaction between an unpaired nucleon and the nuclear core can then be obtained as:

$$\hat{W} = \frac{G}{2\sqrt{2}m_p} g[\boldsymbol{\sigma} \cdot \mathbf{p}\rho(r) + \rho(r)\boldsymbol{\sigma} \cdot \mathbf{p}]$$

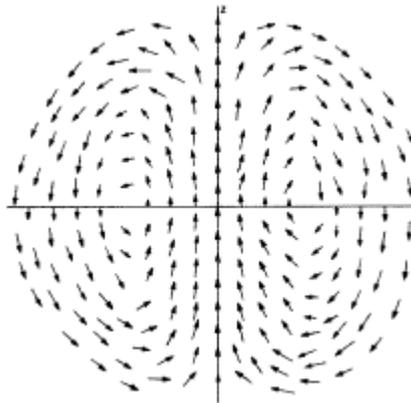
where ρ is the density of core nucleons.

- ▶ Approximate analytical solution of the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{W}$ is: $\psi = e^{i\theta\boldsymbol{\sigma} \cdot \mathbf{r}}\psi_0$

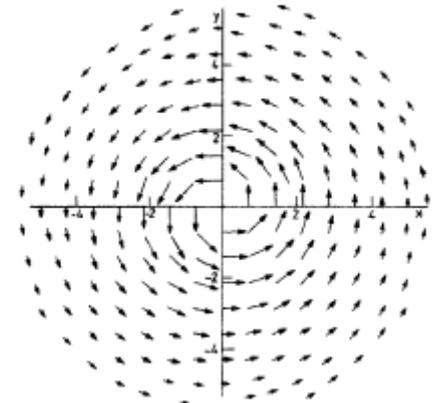


The spin helix occurs due to the parity violating nucleon–nucleus interaction. The degree of spin rotation is proportional to the distance from the origin and the strength of the weak interaction.

$$\mathbf{j} = -\frac{ie}{2m_p} q[\psi^\dagger \nabla \psi - (\nabla \psi^\dagger) \psi] + \frac{e\mu}{2m_p} \nabla \times (\psi^\dagger \boldsymbol{\sigma} \psi)$$



Current distribution (x-z plane)



Magnetic field (x-y plane)

Nuclear anapole moment

It was shown that within the nuclear shell model:

$$\kappa_a = \frac{K}{I+1} \cdot \frac{9}{10} g_\nu \mu_\nu \frac{\alpha}{m_p r_0} A^{2/3}$$

Parameter describing hadronic PV interaction between valence nucleon and nuclear core

Magnetic moment of a valence nucleon

The PV NN interaction is typically parameterized using the meson-exchange picture of Desplanques, Donoghue, and Holstein (DDH) which describes the constants g_ν in terms of the following combination of meson-nucleon parity non-conserving interaction constants:

$$g_p = 2.0 \times 10^5 W_\rho \left[176 \frac{W_\pi}{W_\rho} f_\pi - 19.5 h_\rho^0 - 4.7 h_\rho^1 + 1.3 h_\rho^2 - 11.3(h_\omega^0 + h_\omega^1) \right]$$

$$g_n = 2.0 \times 10^5 W_\rho \left[-118 \frac{W_\pi}{W_\rho} f_\pi - 18.9 h_\rho^0 + 8.4 h_\rho^1 - 1.3 h_\rho^2 - 12.8(h_\omega^0 - h_\omega^1) \right]$$

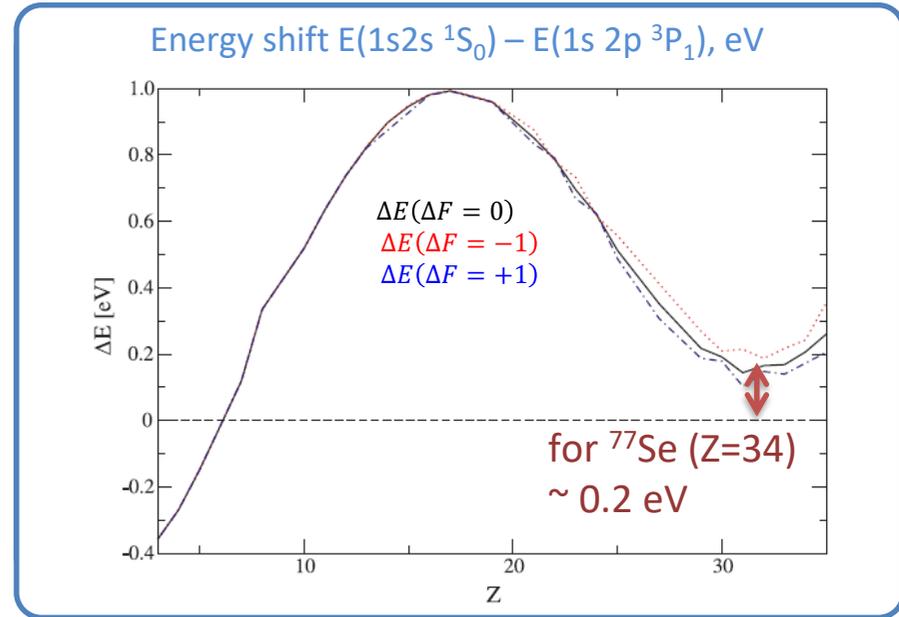
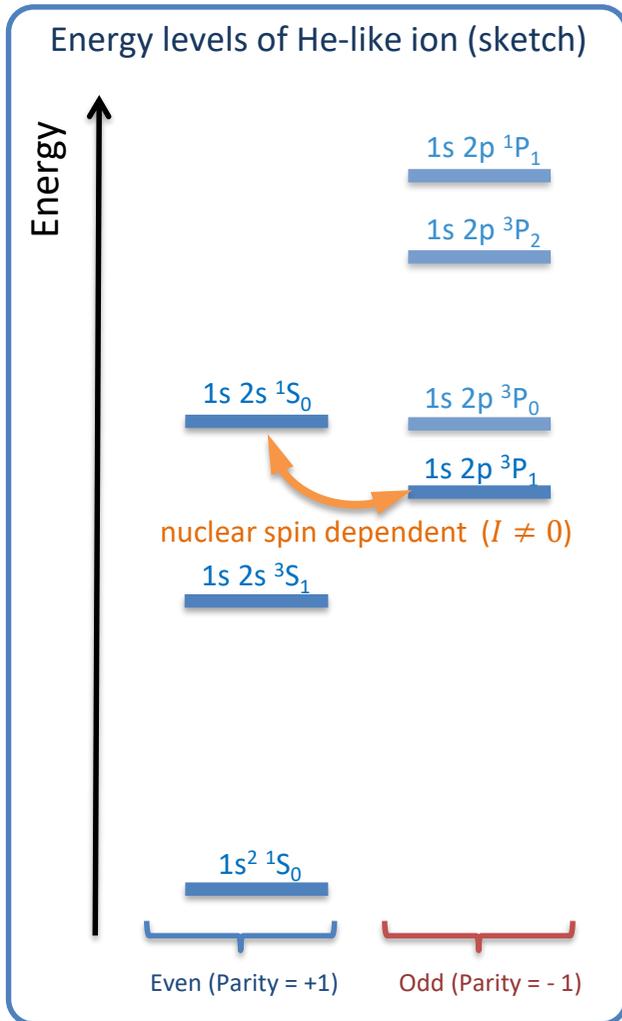
Cs experiment:

$$f_\pi = [7 \pm 2] \times 10^{-7}$$

DDH "b" coupling and effective coupling constants (g) at DDH values for the meson-nucleon

f_π	h_ρ^0	h_ρ^1	h_ρ^2	h_ω^0	h_ω^1	g_{pn}	g_{np}	$g_{pp} = g_{nn}$	g_p	g_n
4.6	-11.4	-0.19	-9.5	-1.9	-1.1	6.5	-2.2	1.5	4.5	0.2

Parity-violation in helium-like ions



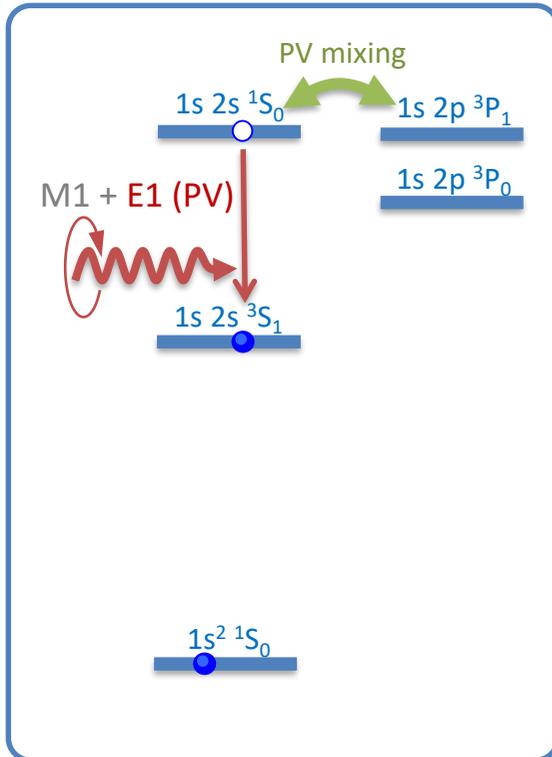
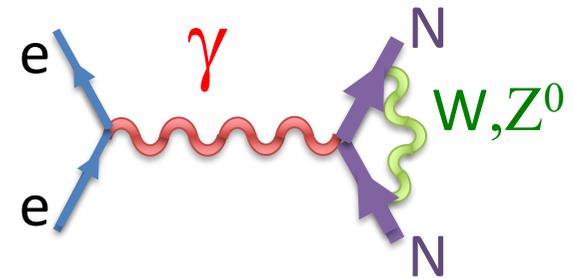
Owing to the (spin-dependent) part of PV Hamiltonian, the 2^1S_0 atomic state in He-like ions with nuclear spin $I \neq 0$ can be described as:

$$|1s\ 2s\ ^1S_0, F = I\rangle + i\eta|1s\ 2p\ ^3P_1, F = I\rangle$$

Mixing coefficient (for ^{77}Se) is $\eta \propto 10^{-9}$.

Towards analysis of nuclear PV effects

Novel schemes for studying the nuclear-spin-dependent part of the atomic parity violation have been also proposed.



Probability of the induced $1s2s\ ^1S_0 \rightarrow 1s\ 2s\ ^3S_1$ transition in He-like ions:

$$\Gamma_\lambda(M1 + E1) = (2I + 1) \cdot \Gamma_{M1} \cdot (1 + \lambda A)$$

parity-preserving rate (M1)

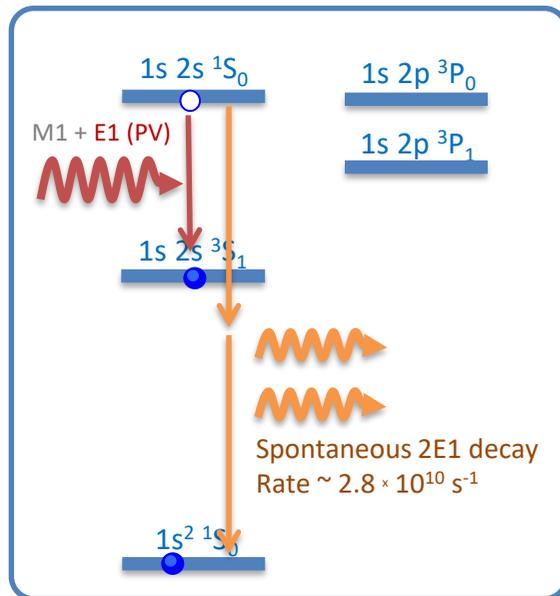
photon's helicity

Asymmetry coefficient $A = (\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-)$ is directly related to the nuclear-spin dependent mixing parameter: $A \propto \eta/(2I + 1)$

For medium-Z ions the asymmetry reaches $A \propto 10^{-7}$!

Nuclear PV effects: Estimates

- Based on the predictions of the Standard Model we have estimated the size of the expected asymmetry for several isotopes.
- Stable isotope of ^{77}Se seems to be most convenient for the study.



Ion	$t_{1/2}$	I	μ	$10^{-7} A$
$^{70}_{33}\text{As}$	53 min	4	2.106 100	0.801 74
$^{71}_{33}\text{As}$	65.3 h	5/2	1.673 500	1.251 77
$^{72}_{33}\text{As}$	26 h	2	-2.156 600	1.278 89
$^{74}_{33}\text{As}$	17.78 d	2	-1.597 000	1.335 09
$^{75}_{33}\text{As}$	stable	3/2	1.439 475	2.086 91
$^{76}_{33}\text{As}$	26.3 h	2	-0.906 000	1.403 86
$^{73}_{34}\text{Se}$	7.1 h	9/2	0.8700 000	0.623 39
$^{75}_{34}\text{Se}$	118.5 d	5/2	0.6700 000	1.084 38
$^{77}_{34}\text{Se}$	stable	1/2	0.535 0422	4.663 09
$^{79}_{34}\text{Se}$	65000 yr	7/2	-1.018 0000	0.787 13

- To induce M1+E1 (PV) transition in He-like Se we need to apply circularly polarized light of:
 - energy 43.85 eV (extreme ultraviolet)
 - intensity $I \propto 10^{12} - 10^{13}\ \text{W/cm}^2$ (to “compete” with the spontaneous E1E1 decay)

Plan of the lecture

- Isotope shift of energy levels: Mass and volume effects
 - Isotope shift in Ca^+ ions
 - Analysis of DR spectra of heavy ions

- Parity violation in heavy ions
 - Nuclear spin independent effect
 - Parity violation in nuclei: Anapole moment