
Lecture 6

Electron bridge processes, Critical electromagnetic fields

Plan of the lecture



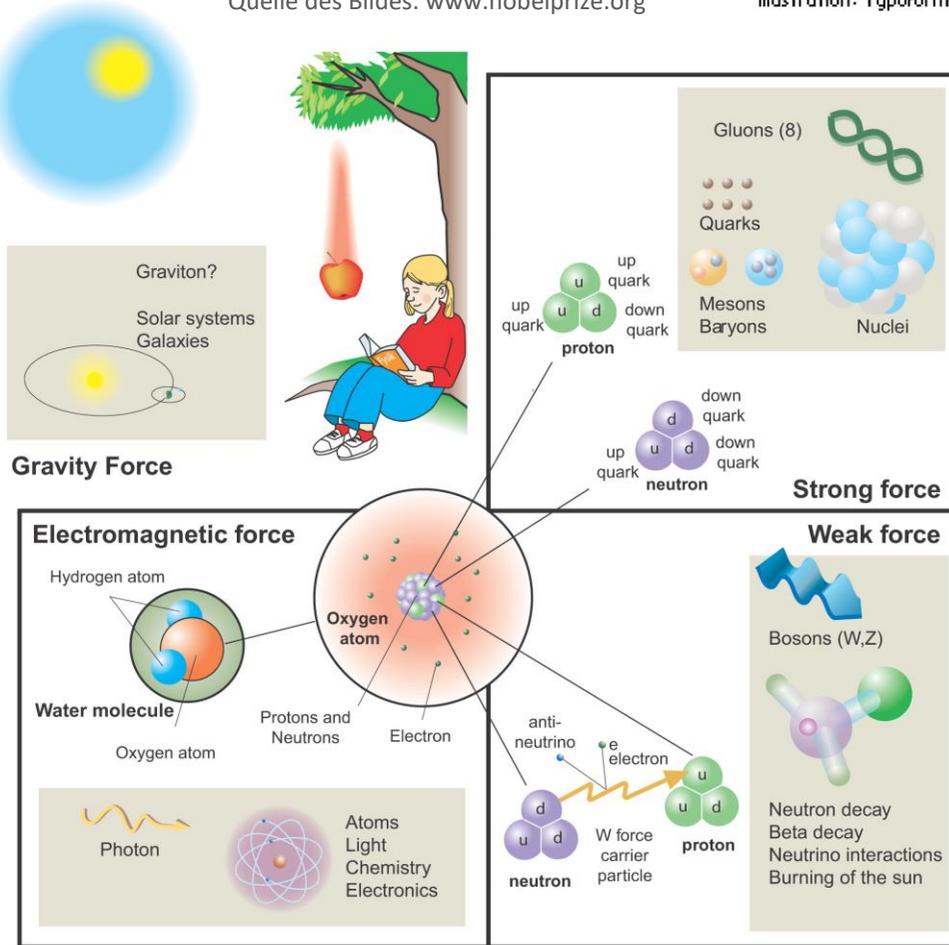
- Why do we need high-precision atomic/nuclear clocks?
- Search for the ^{229}Th isomeric state
 - Electron bridge processes
 - Nuclear excitation by two-photon transition

- Physics of critical electromagnetic fields
 - Ion-ion collisions
 - Formation of heavy quasi-molecules

Fundamental physical constants

Quelle des Bildes: www.nobelprize.org

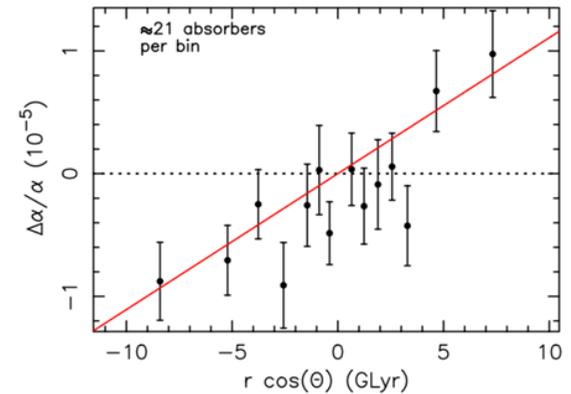
Illustration: Typoform



The standard model (today's "theory of everything") contains many physical constants that describe properties of both particles and interactions.

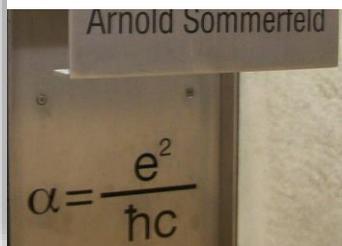
These constants are believed to be both universal in nature and having constant value in time. But is it true?

The structure describes the electromagnetic



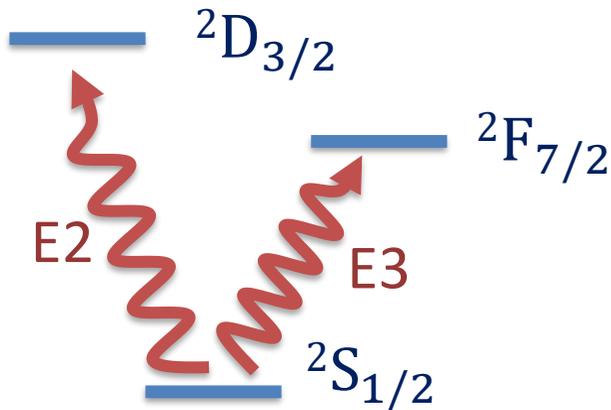
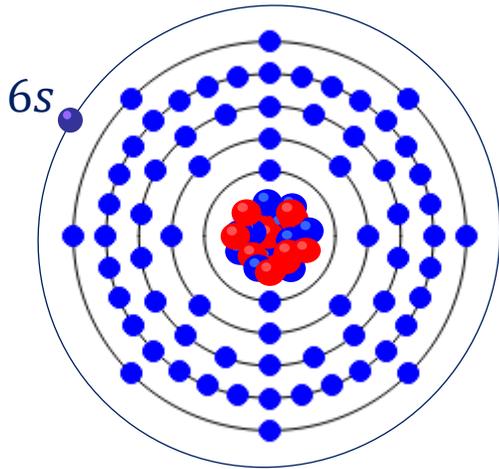
There were recent indications that the fine-structure constant varies with position and time.

J. K. Webb *et al.*, Phys. Rev. Lett. **107**, 191101 (2011)



Variation of fine-structure constant

Yt⁺ ion: symmetric core (68 electrons)
plus 1 electron in 6s state



Atomic levels can be shifted in different ways under the variation of the fine structure constant (as well as other constants like electron and proton masses):

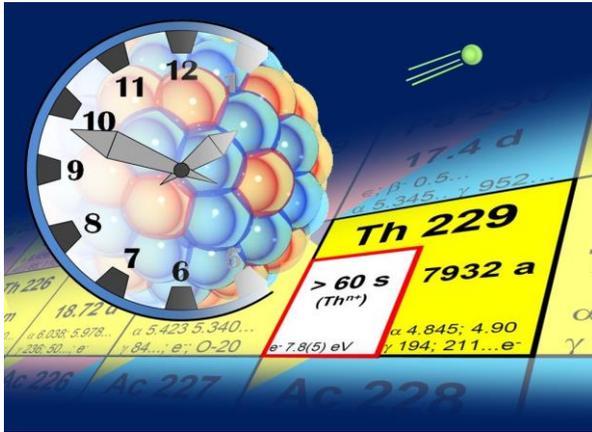
$$\Delta E = f(\Delta\alpha, \dots)$$

If the high accuracy of atomic clocks is achieved, this can be used to search for time variation of α .

Recent experiment with Cs and Yt⁺ atomic clocks have improved the limit on the time variation of α :

$$\frac{1}{\alpha} \frac{d\alpha}{dt} = -0.20(20) \times 10^{-16} / \text{yr}$$

Nuclear clocks and “thorium puzzle”



Observation of nuclear transition in ^{229}Th is important for the development of novel nuclear clocks.

A number of various schemes are proposed to observe this transition. Very promising are atomic processes in which excitation of electronic shell is transferred to a nucleus.

NATURE | ARTICLE



日本語要約

Direct detection of the ^{229}Th nuclear clock transition

Lars von der Wense, Benedict Seiferle, Mustapha Laatiaoui, Jürgen B. Neumayr, Hans-Jörg Maier, Hans-Friedrich Wirth, Christoph Mokry, Jörg Runke, Klaus Eberhardt, Christoph E. Düllmann, Norbert G. Trautmann & Peter G. Thirolf

[Affiliations](#) | [Contributions](#) | [Corresponding author](#)

Nature **533**, 47–51 (05 May 2016) | doi:10.1038/nature17669

Received 16 December 2015 | Accepted 16 March 2016 | Published online 04 May 2016

Citation

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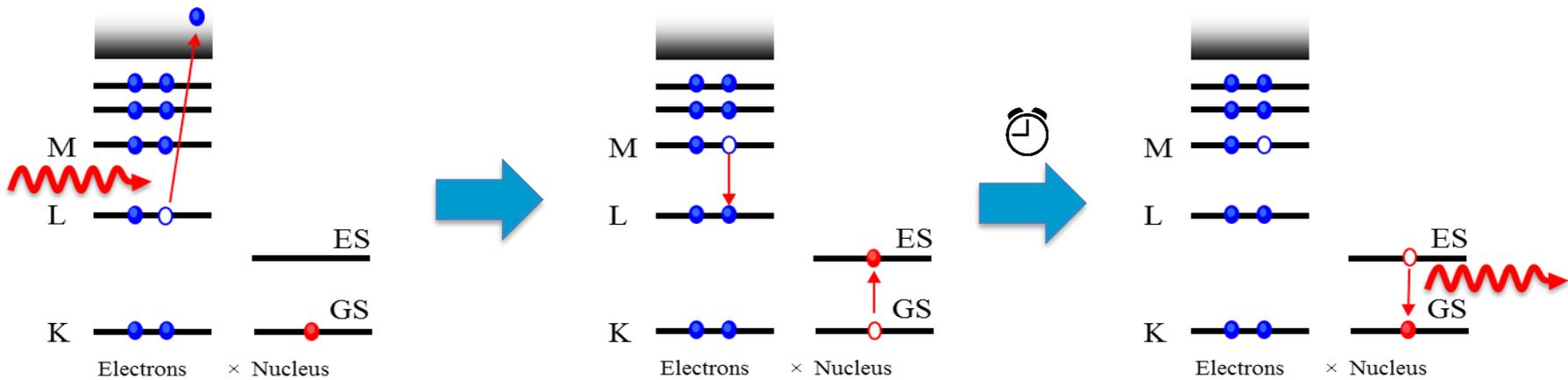
Article metrics

Abstract

[Abstract](#) • [References](#) • [Author information](#) • [Extended data figures and tables](#)

Today's most precise time and frequency measurements are performed with optical atomic clocks. However, it has been proposed that they could potentially be outperformed by a nuclear clock, which employs a nuclear transition instead of an atomic shell transition. There is only one known nuclear state that could serve as a nuclear clock using currently available technology, namely, the isomeric first excited state of ^{229}Th (denoted $^{229\text{m}}\text{Th}$). Here we report the direct detection of this nuclear state, which is further confirmation of the existence of the isomer and lays the foundation for precise studies of its decay parameters. On the basis of this direct detection, the isomeric energy is constrained to between 6.3 and 18.3 electronvolts, and the

Nuclear excitation by electron transitions



Nucleus is in its ground state (GS). Atomic shell vacancy is produced.

Bound-electron transition leads to the nuclear excitation.

Radiative nuclear decay by gamma-ray emission.

NEET – nuclear excitation by electron transition. Bound-electron transitions being **energetically very close** to nuclear excitations can directly induce them.

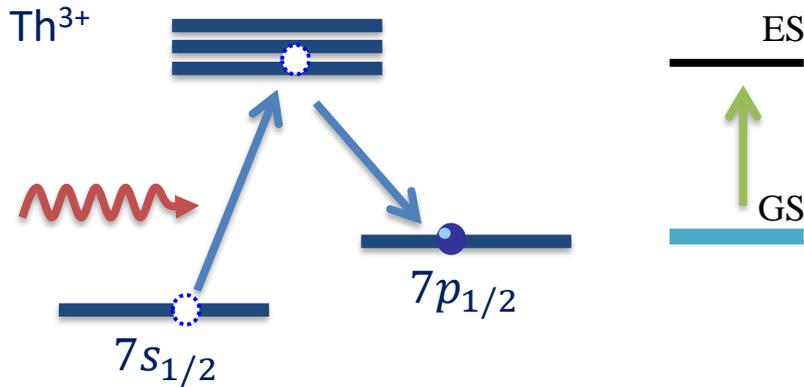
Fine adjustment of atomic and nuclear transition energies is crucial!

Nuclear excitation by electron transitions

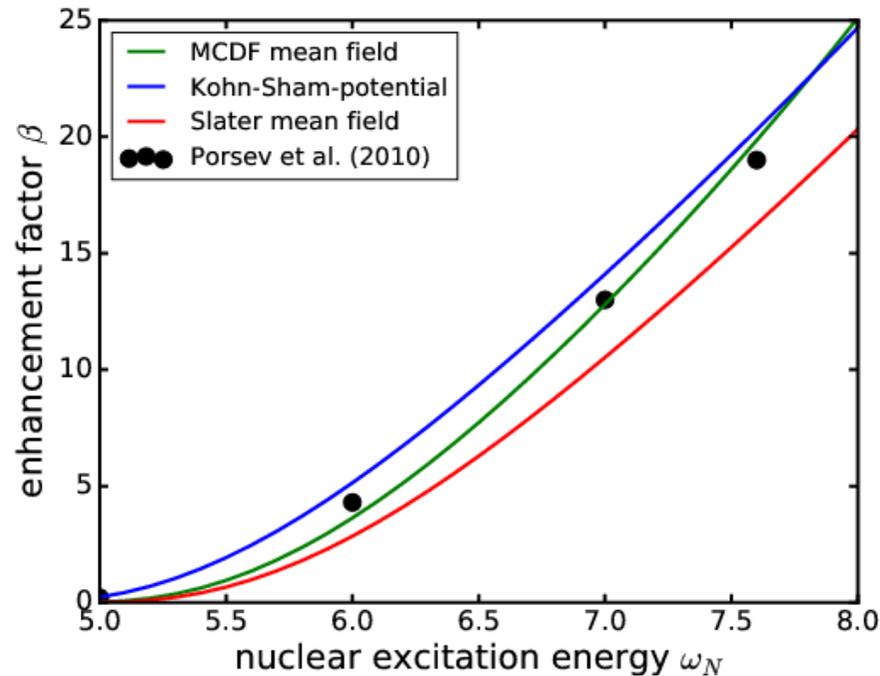
The key parameter is the NEET probability: $P_{NEET} = W_{NEET}/W_{tot}$

Nucleus	Experiment	References	Theory	References
^{237}Np	$2.1(0.6) \times 10^{-4}$	Saito et al., 1980	1.5×10^{-7} 3.1×10^{-12} 1.9×10^{-9} 2.6×10^{-4} 2.2×10^{-12}	Pisk et al., 1989 Tkalya, 1992 Ho et al., 1993 Ljubicic, 1998 Harston, 2001
^{197}Au	$5.1(3.6) \times 10^{-5}$ $5.0(0.6) \times 10^{-8}$	Shinohara et al., 1995 Kishimoto et al., 2000	3.5×10^{-5} 1.3×10^{-7} 2.4×10^{-7} 2.2×10^{-5} 3.6×10^{-8}	Pisk et al., 1989 Tkalya, 1992 Ho et al., 1993 Ljubicic, 1998 Harston, 2001
^{189}Os	$5.7(1.7) \times 10^{-9}$ $2.0(1.4) \times 10^{-8}$ $<9 \times 10^{-10}$	Shinohara et al., 1987 Lakosi et al., 1995 Ahmad et al., 2000	2.5×10^{-7} 1.1×10^{-10} 2.1×10^{-9} 2.3×10^{-7} 1.1×10^{-10}	Pisk et al., 1989 Tkalya, 1992 Ho et al., 1993 Ljubicic, 1998 Harston, 2001

Nuclear excitation by electron transition (NEET)



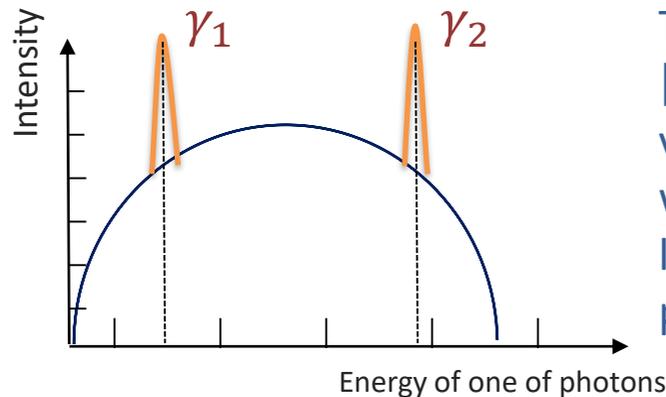
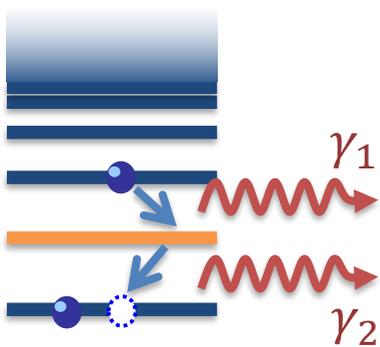
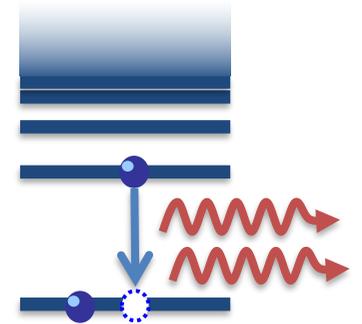
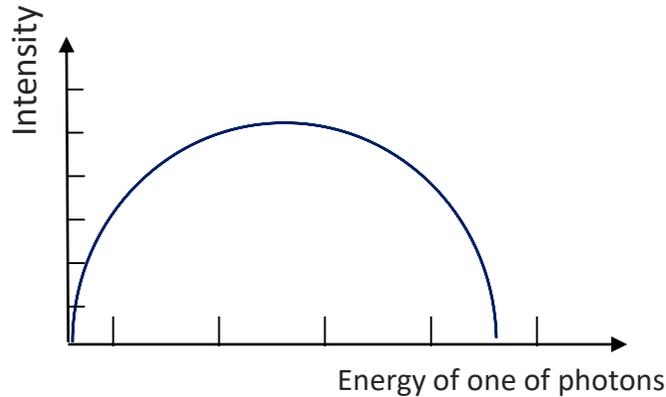
Special attention is currently paid to two-step excitation-and-decay processes. Detailed calculations have been performed, for example, for Th³⁺ ions.



Two-photon transitions with cascades

In contrast to single-photon transitions, the two-photon decay has a continuous spectrum. Photon energies just need to satisfy the energy conservation: $E_f - E_i = \hbar\omega_1 + \hbar\omega_2$

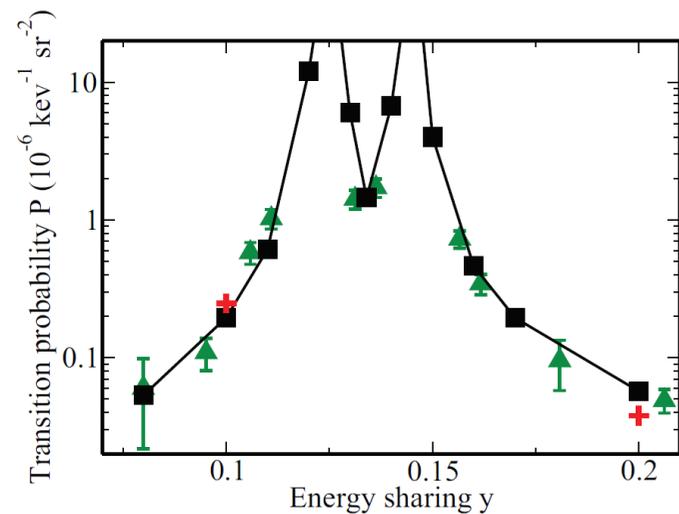
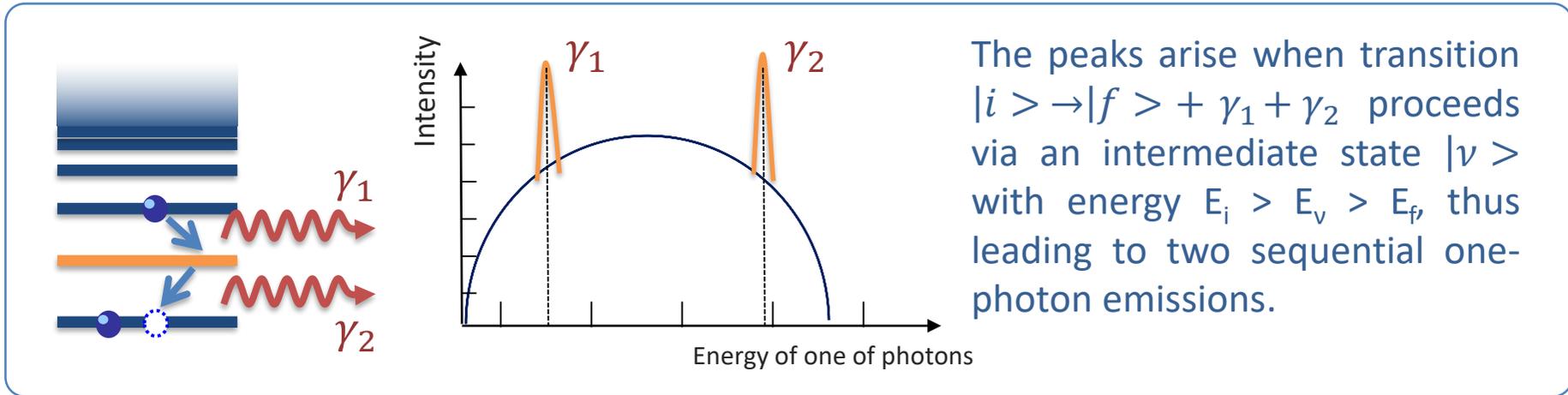
If there are no real states between initial and final states, the energy spectrum is smooth.



The peaks arise when transition $|i\rangle \rightarrow |f\rangle + \gamma_1 + \gamma_2$ proceeds via an intermediate state $|v\rangle$ with energy $E_i > E_v > E_f$, thus leading to two sequential one-photon emissions.

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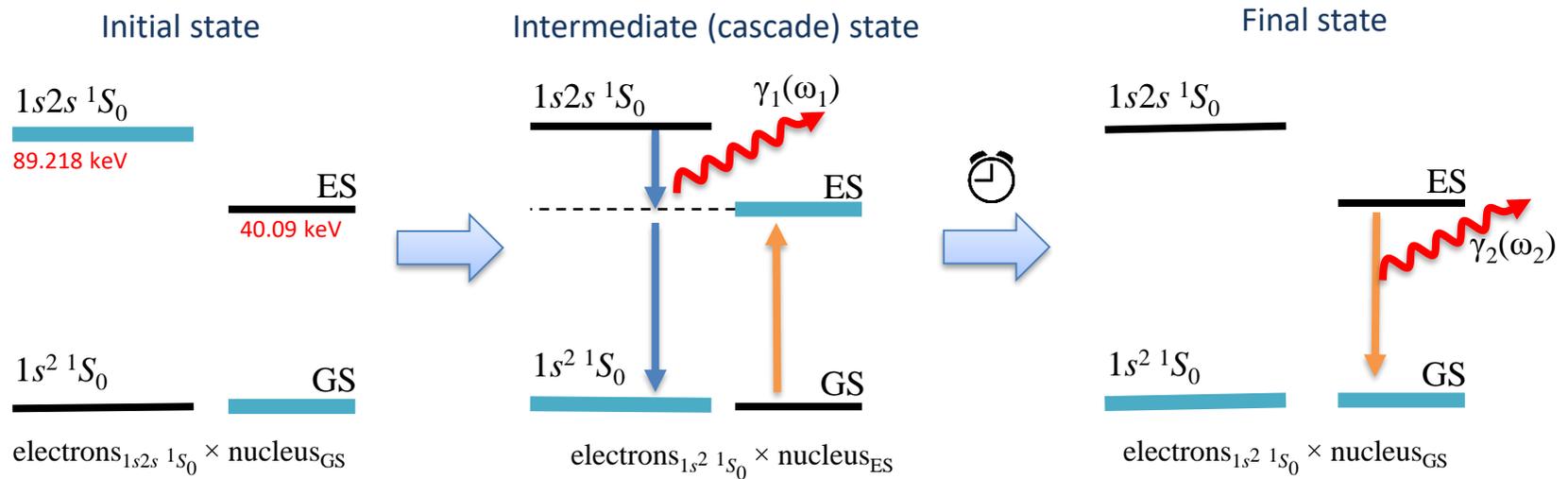


“Resonance” two-photon spectroscopy provides important information about atomic structure.

$3d \rightarrow 1s$ transition in a gold atom as a function of the energy sharing parameter $y = \hbar\omega_1 / (\hbar\omega_1 + \hbar\omega_2)$.

Nuclear excitation by two-photon transition

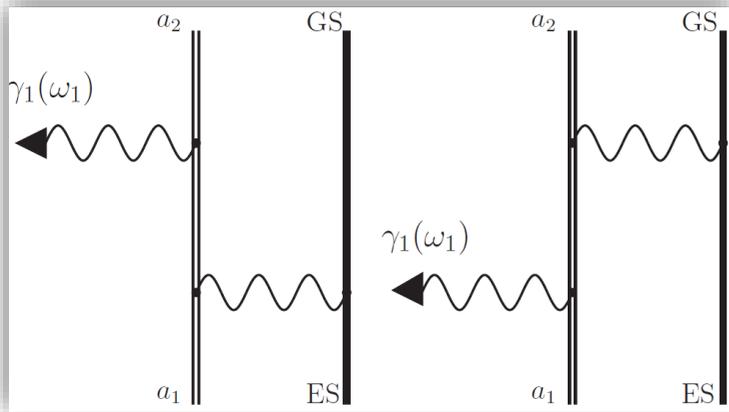
We have considered the two-photon $1s\ 2s\ 1S_0 \rightarrow 1s^2\ 1S_0$ (2E1) decay in helium-like $^{225}\text{Ac}^{87+}$ ion.



Considering electrons and nucleus as a “united” system the cascade decay with nuclear excitation in the intermediate state is taken place:

$$|1s\ 2s\ ^1S_0\rangle \times \text{GS} \rightarrow |1s^2\ ^1S_0\rangle \times \text{ES} + \gamma_1 \rightarrow |1s^2\ ^1S_0\rangle \times \text{GS} + \gamma_2$$

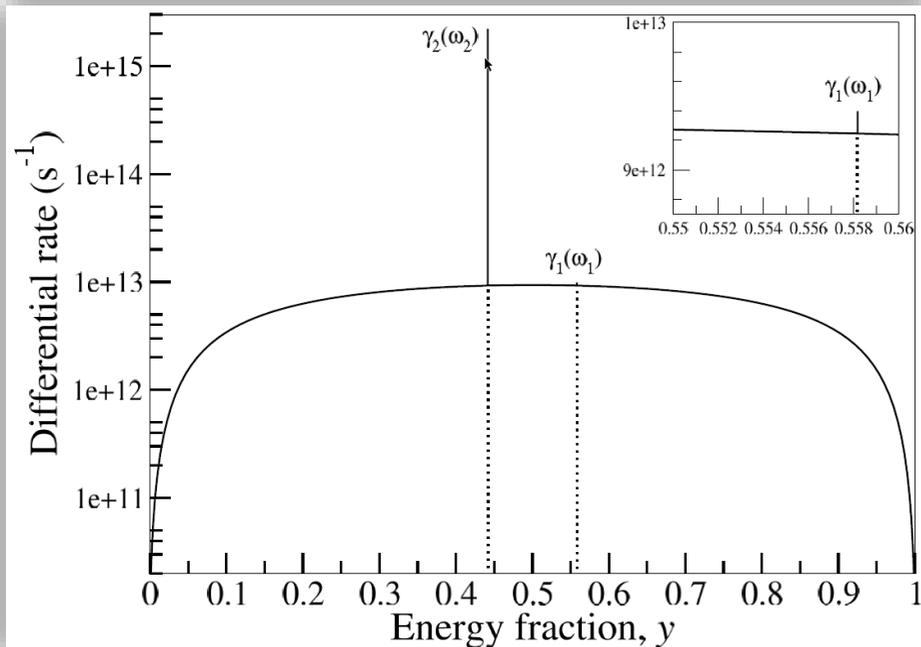
Nuclear excitation by two-photon transition



Based on the relativistic QED approach, we have performed theoretical analysis of the NETP process.

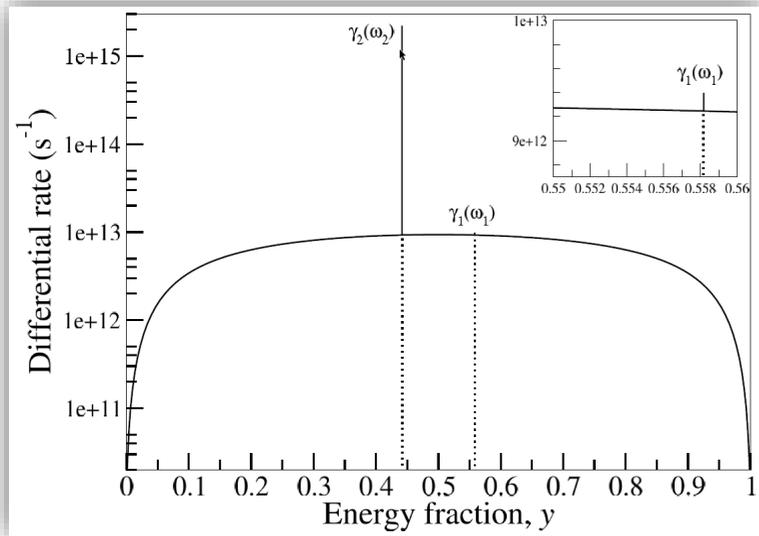
This requires the evaluation of the second order amplitude:

$$S_{\text{NETP}}^{(3)} = \frac{1}{\Delta E - \omega_{\text{ES}} - \omega_1 - i(\Gamma_{I_s 2s^1 S_0} + \Gamma_{\text{ES}})/2} \times \frac{e^2}{4\pi} \int d^3 r_1 d^3 r_2 d^3 R \bar{\psi}_{a_1}(\mathbf{r}_1) \times \left\{ \gamma_0 \frac{1}{|\mathbf{r}_1 - \mathbf{R}|} S(\varepsilon_{a_2} - \omega_1, \mathbf{r}_1, \mathbf{r}_2) \gamma^\mu A_\mu^*(\omega_1, \mathbf{r}_2) + \gamma^\mu A_\mu^*(\omega_1, \mathbf{r}_1) S(\varepsilon_{a_1} + \omega_1, \mathbf{r}_1, \mathbf{r}_2) \gamma_0 \frac{1}{|\mathbf{r}_2 - \mathbf{R}|} \right\} \times \psi_{a_2}(\mathbf{r}_2) \Psi_{\text{ES}}^\dagger(\mathbf{R}) \hat{\rho}_{\text{fluc}}(\mathbf{R}) \Psi_{\text{GS}}(\mathbf{R})$$

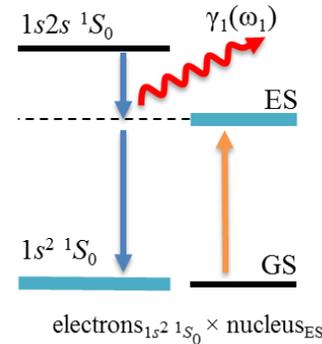


Based on our calculations, we found the NETP probability: $P_{\text{NETP}} = 3.5 \times 10^{-9}$!

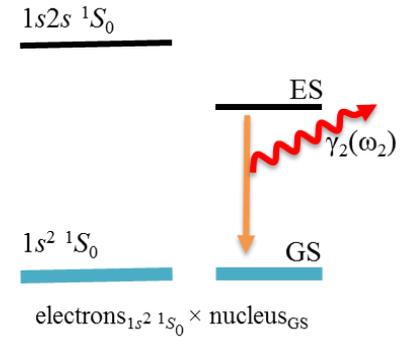
Detection of the NETP process



Intermediate (cascade) state

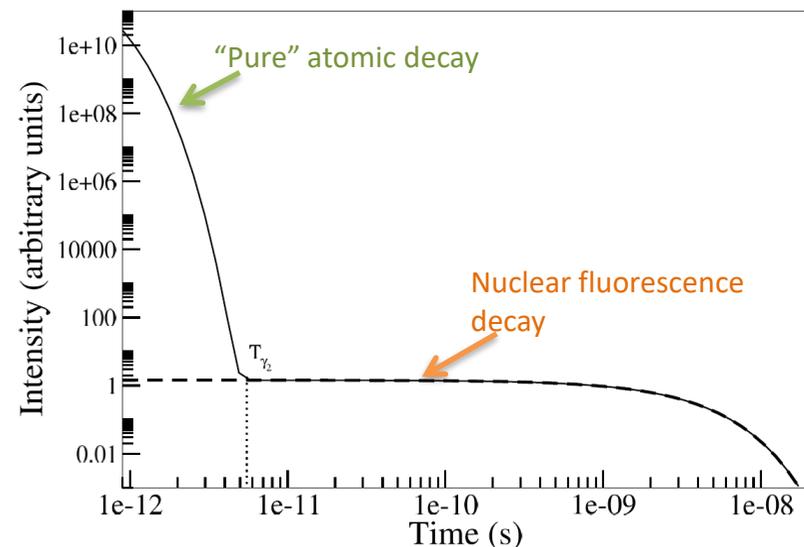


Final state



Electron decay photon γ_1 is emitted in a same time scale as the photons from “pure” two-photon decay and its observation requires detectors with high resolution.

Nuclear fluorescence photon γ_2 is emitted with a time delay and can be clearly identified. We need to perform time-delayed spectroscopy!



Plan of the lecture

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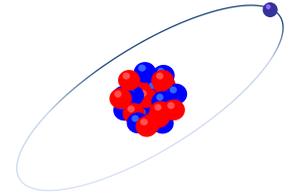


- Physics of critical electromagnetic fields
 - Ion-ion collisions
 - Formation of heavy quasi-molecules

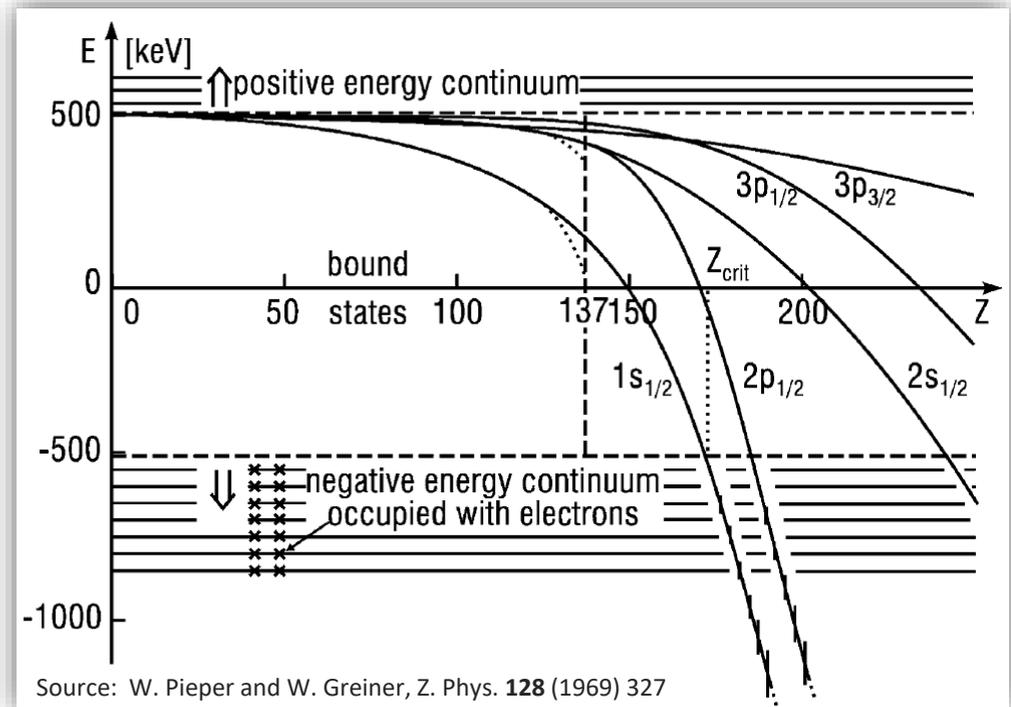
Critical electromagnetic fields

- Dirac energy of a *single* hydrogen-like ion (for the point-like nucleus):

$$E_{nj} = mc^2 / \sqrt{1 + \left(\frac{Z\alpha}{n - |j + 1/2| + \sqrt{(j + 1/2)^2 - (Z\alpha)^2}} \right)^2}$$

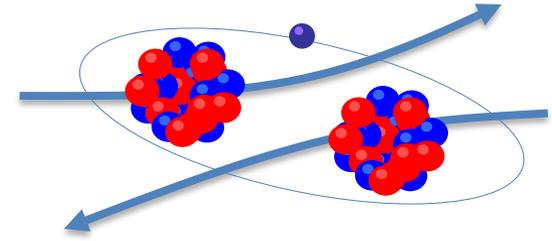


- What happens if we increase the nuclear charge Z ?
- If nuclear charge of the ion is greater than Z_{crit} the ionic levels can “dive” into Dirac’s negative continuum.
- Physical vacuum becomes unstable: creation of pairs may take place!



Heavy quasi-molecules

- Alternatively, we may form strong electromagnetic fields in (rather) slow collisions of two heavy ions.
- Since velocity of an electron is much higher comparing to collision velocity may think of formation of (quasi) molecule!

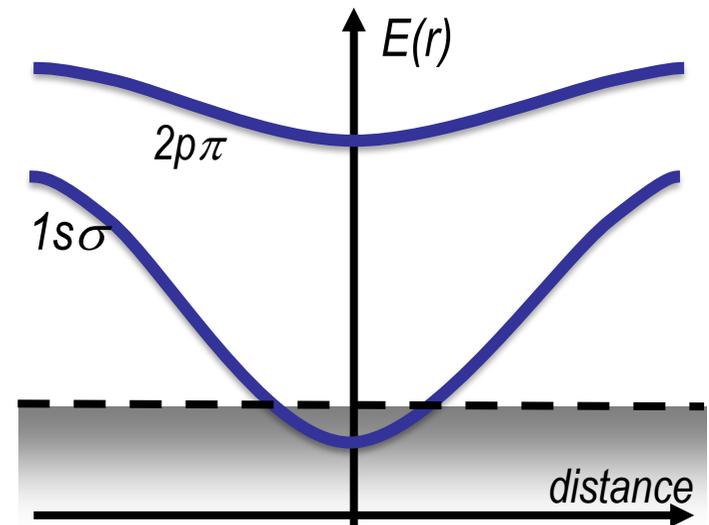


- ⊕ In heavy ions velocity of electron being in ground state of U^{91+} is:

$$v_e = (\alpha Z) c \approx 0.7c$$

- ⊕ At the same time velocity of colliding ions is about:

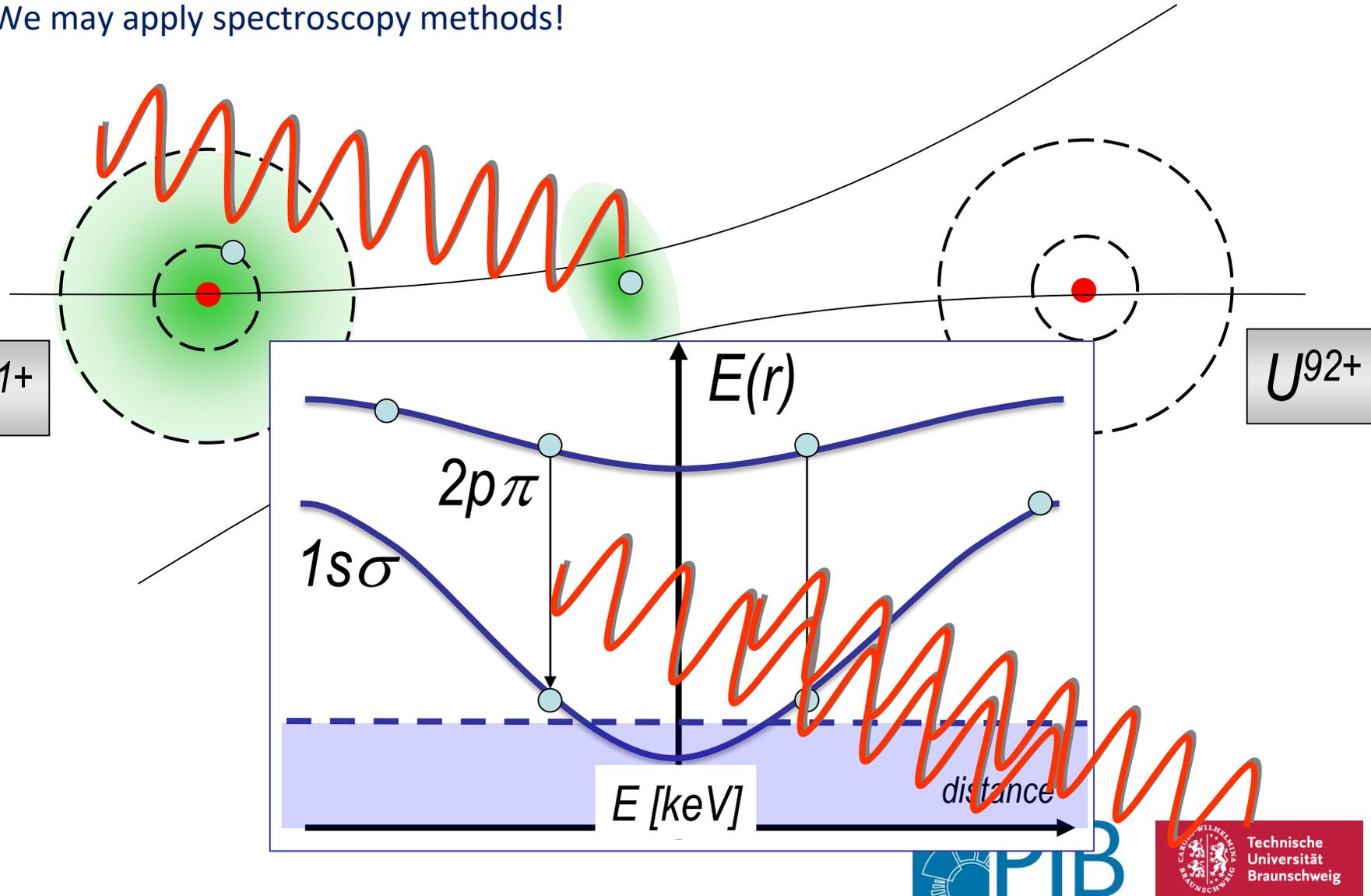
$$v_{ion} \approx 0.07c$$



- We need to find ways to control formation of quasi-molecules!

Spectroscopy of quasi-molecules

- How can we “control” formation and dynamics of quasi-molecular systems?
- We may apply spectroscopy methods!



Spectroscopy of quasi-molecules

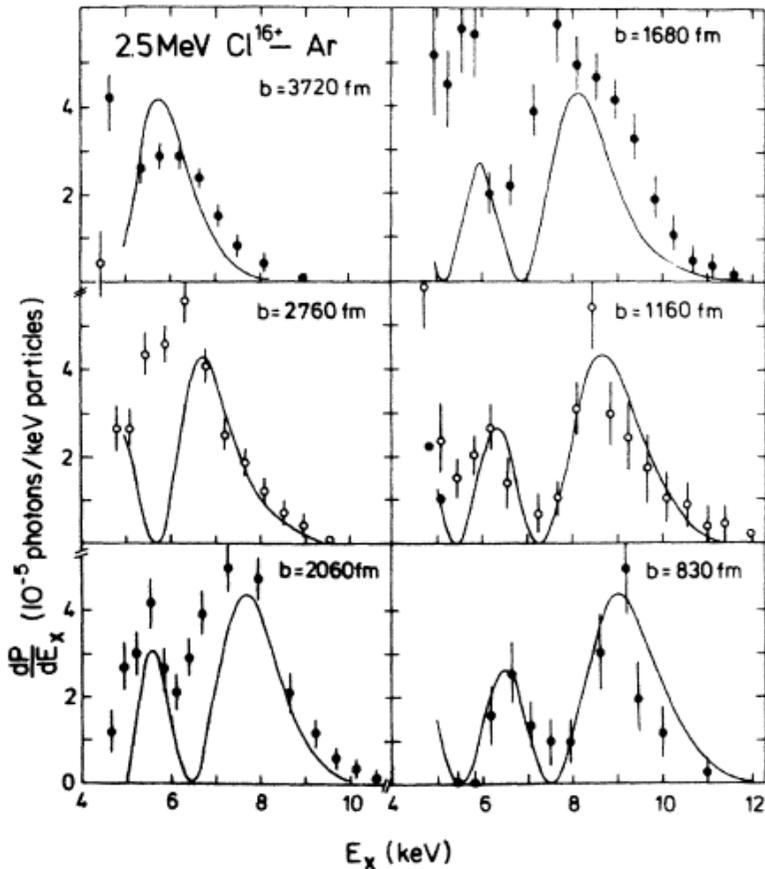
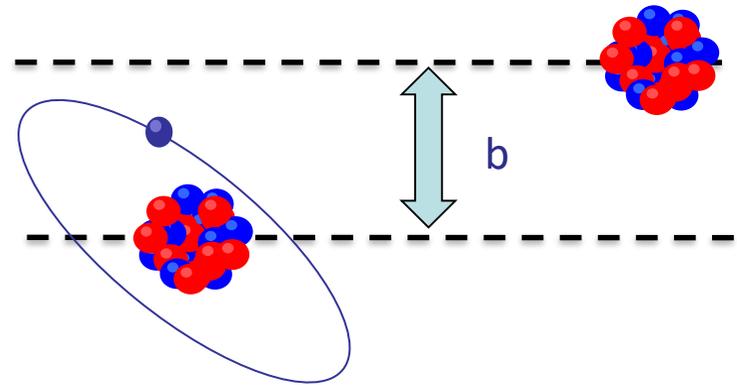


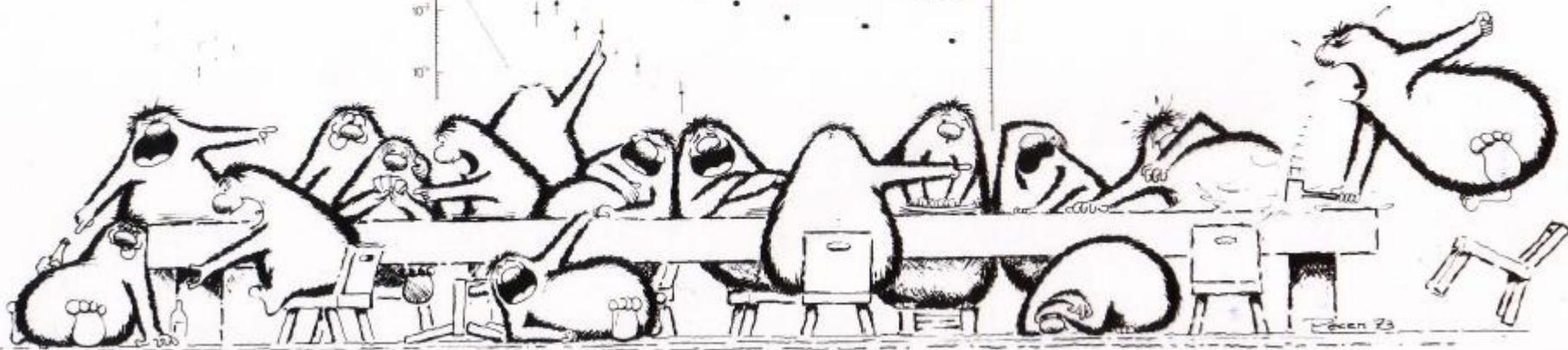
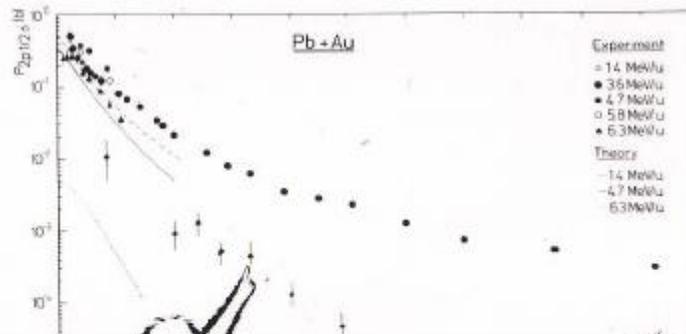
FIG. 8. X-ray-emission probabilities as a function of x-ray energy at different impact parameters for the beam energy of 2.5 MeV.

- Interference pattern in the molecular orbital radiation can provide important information about collision conditions (i.e. impact parameter).



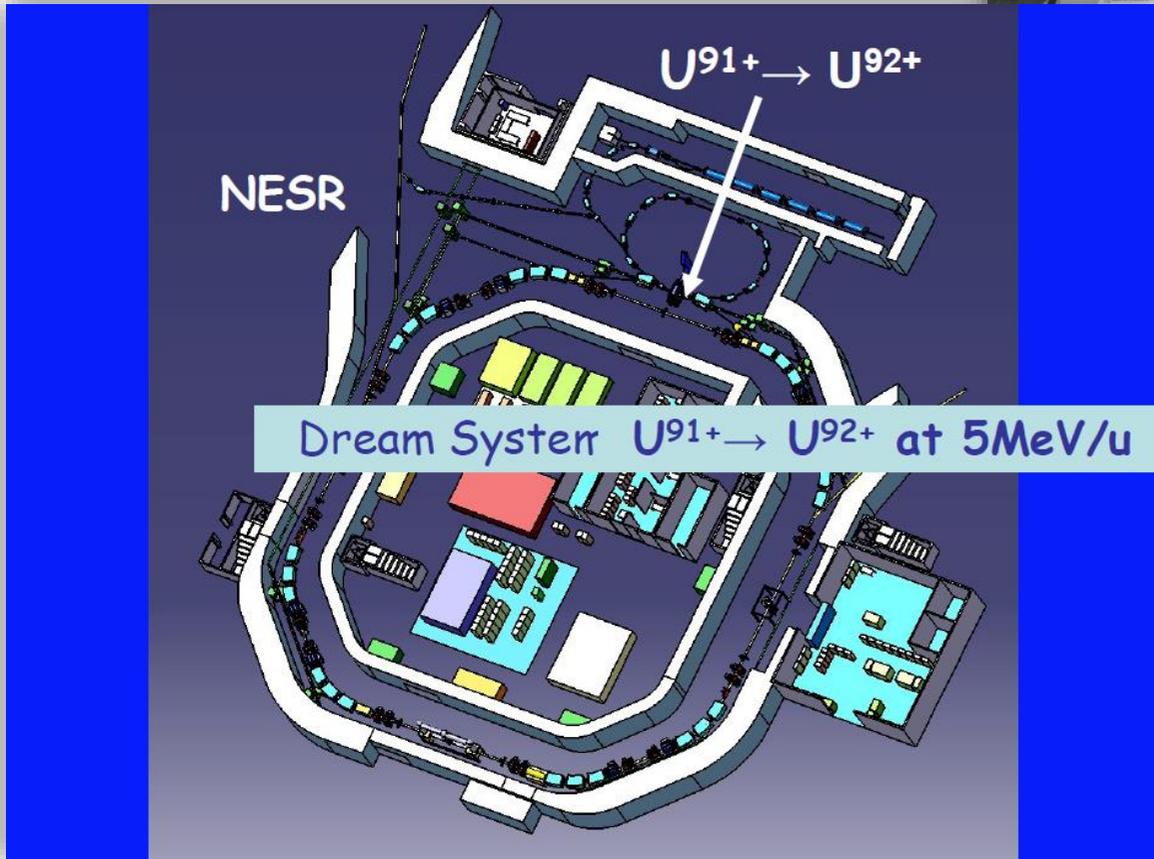
- OK, we can “control” collision dynamics! Have we finally seen antiprotons?

Spectroscopy of quasi-molecules



New experiments at FAIR

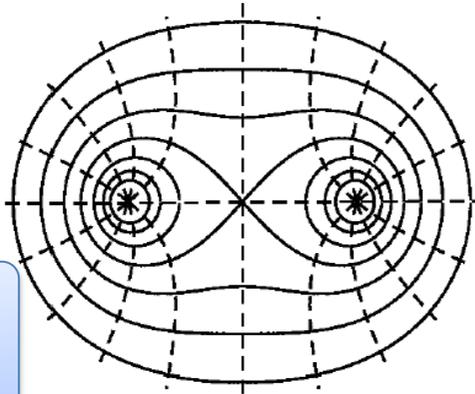
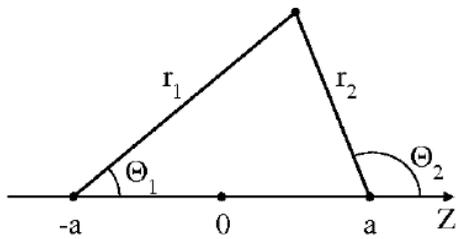
- New generation of “positron experiments” is likely to be performed at the FAIR facility in Darmstadt.



- There is need for novel theoretical techniques!

Two-center Dirac problem

- ▶ To deal with the two-center Dirac problem, we laid out a new approach based on application of B-spline (finite) basis sets constructed in Cassini coordinates.



$$w = \frac{\sqrt{r_1 r_2}}{a}, \quad \delta = \frac{\theta_1 + \theta_2}{2},$$

$$\phi = \phi$$

- ⊕ Hamiltonian in Cassini coordinates:

$$\hat{H}_{2C} = -i \frac{D^{1/4}}{aw} \left(\alpha_3 \frac{\partial}{\partial w} + \alpha_1 \frac{1}{w} \frac{\partial}{\partial \delta} \right) - i \alpha_2 \frac{1}{\rho} \frac{\partial}{\partial \phi} + V(w, \delta; a) + \beta m$$

- ▶ We seek the eigenfunctions (4-spinors) in the form

$$\Psi(w, \delta, \phi) = \psi_\mu(w, \delta) \exp(i\mu\phi)$$

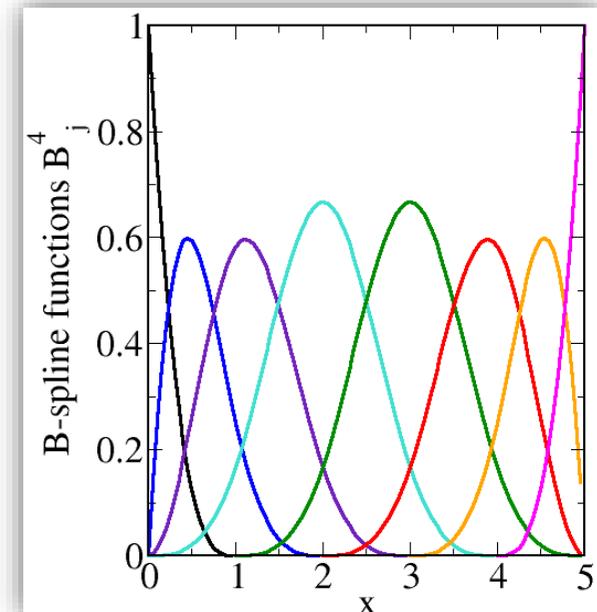
- ▶ With components $\psi_{\mu,i}(w, \delta) = \sum_{n=1}^N C_{\mu,i}(n) B_n(w, \delta)$

Expansion coefficients

(obtained from generalized eigenvalue problem)

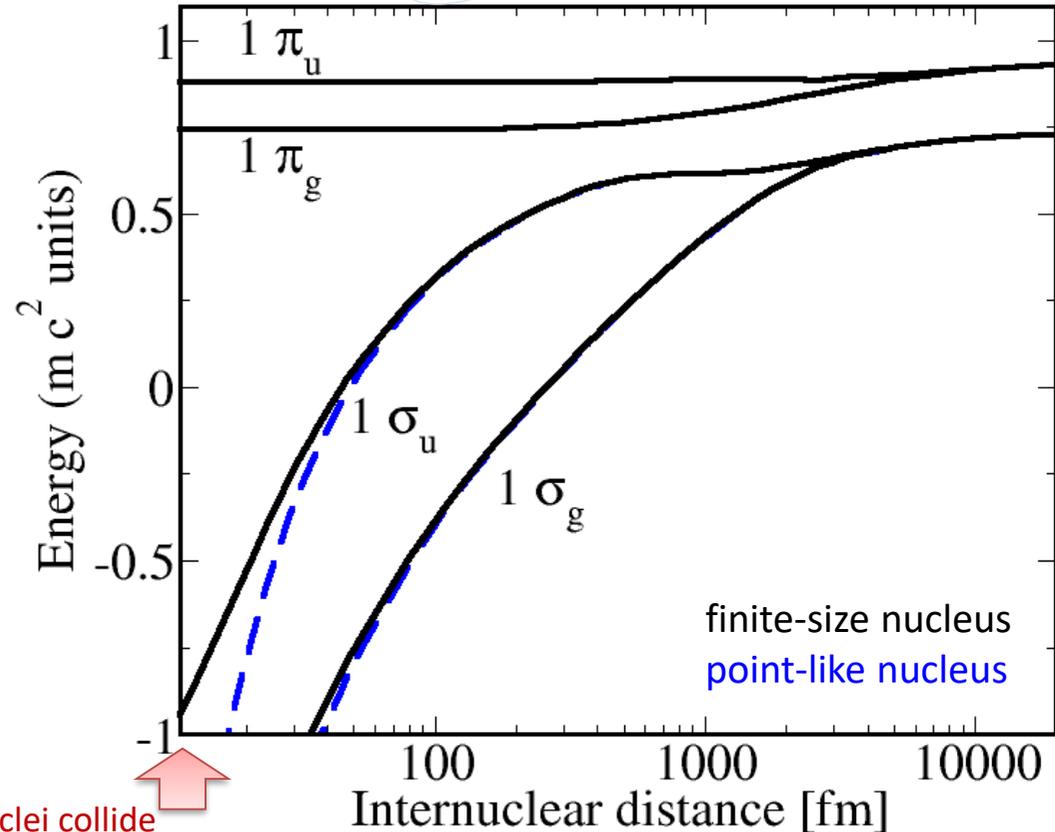
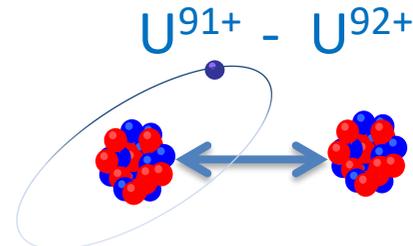
Basis functions

(constructed from B-splines)



Energy levels of quasi-molecules

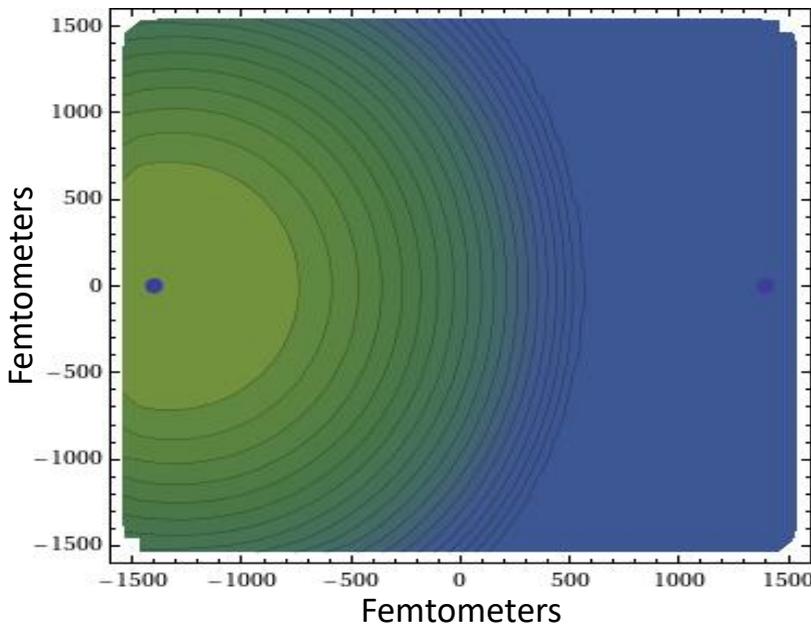
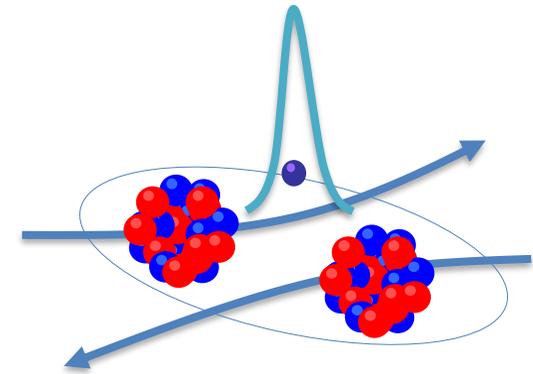
- Calculations have been carried out to study energy spectra of heavy quasi-molecules.
- Special attention has been paid to the nuclear-size effects.
- For small distances the finite-size effect plays an important role: it prevents first excited state from diving into the Dirac sea.



Electron dynamics accompanying slow ion collisions

- Of special interest is the study of fundamental processes accompanying slow ion collisions.
- Such an analysis requires solution of the time-dependent (two-center) Dirac equation.

$$\Phi(\mathbf{r}, t) = \sum_{\kappa} a_{\kappa}(\mathbf{r}) \Psi_{\kappa}(\mathbf{r}, \mathbf{R}(t)) e^{-i\epsilon_{\kappa} t}$$



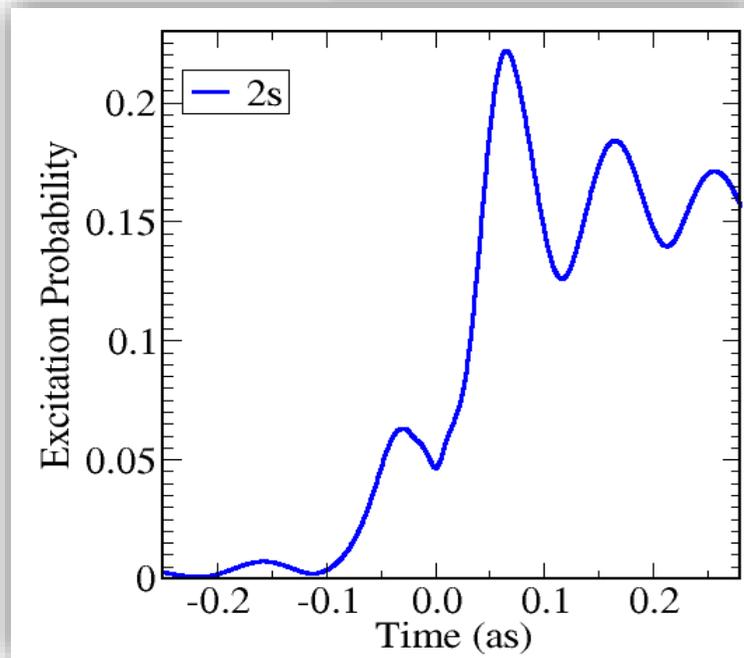
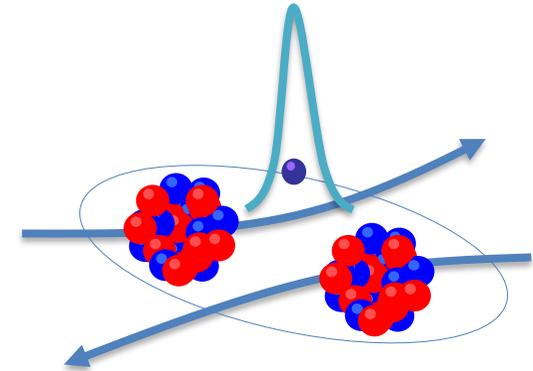
- ▶ Developed approach allows fast and efficient treatment of charge-transfer, excitation, ionization and even pair production processes!

Preliminary calculations: $\text{Pb}^{81+} + \text{Pb}^{82+}$ collision at energy of 3 MeV/u and zero impact parameter.

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Thank you for your attention!