

# Nuclei at the Mass Limit: Discovery of Super Heavy Nuclei

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The seminar on Nuclear Physics of INPh (MSU)  
April 04, 2017, Moscow

**For more than 22 centuries**

**From Democritus (460 - 371 BC...)**

**to Dalton (1766 - 1844)**

**it was assumed that the objects around us are  
made up of tiny indivisible particles - **atoms** interacting  
with each other..**

- **All atoms of one element are similar and have the same weight,**
- **Atoms of different elements have different weights,**



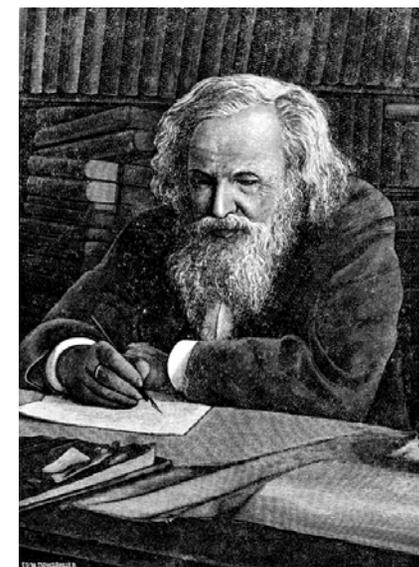
John Dalton 1808

- **The building bricks – molecules – are complex compositions of different atoms in certain proportions,**
- **The matter is neither formed nor is disappearing in chemical reactions**

**36 elements were known at that time**

# 63 elements were already known by that time

Ряды	Г р у п п ы э л е м е н т о в										
	0	I	II	III	IV	V	VI	VII		VIII	
0	Наводород										
1	Кислород	Водород H 1,008	—	—	—	—	—	—	—		
2	Гелий He 4,0	Литий Li 7,03	Бериллий Be 9,1	Бор B 11,0	Углерод C 12,0	Азот N 14,01	Кислород O 16,00	Фтор F 19,0			
3	Неон Ne 19,9	Натрий Na 23,05	Магний Mg 24,36	Алюминий Al 27,1	Кремний Si 28,2	Фосфор P 31,0	Сера S 32,06	Хлор Cl 35,45			
4	Аргон Ar 38	Калий K 39,15	Кальций Ca 40,1	Стронций Sr 44,1	Титан Ti 48,1	Ванадий V 51,2	Хром Cr 52,1	Марганец Mn 55,1	Железо Fe 55,9	Кобальт Co 59	Никель Ni 59
5		Медь Cu 63,6	Цинк Zn 65,4	Галлий Ga 70,0	Германий Ge 72,5	Мышьяк As 75	Селен Se 79,2	Бром Br 79,95			
6	Криптон Kr 81,8	Рубидий Rb 85,5	Стронций Sr 87,6	Иттрий Y 89,0	Цирконий Zr 90,6	Нобий Nb 94,0	Молибден Mo 96,0	—	Рутений Ru 101,7	Родий Rh 103,0	Палладий Pd 106,5
7		Серебро Ag 107,93	Кадмий Cd 112,4	Индий In 115,0	Олово Sn 119,0	Сурьма Sb 120,2	Теллур Te 127	Иод I 127			
8	Ксенон Xe 128	Цезий Cs 132,9	Барий Ba 137,4	Лантан La 138,9	Церий Ce 140,2	—	—	—	—	—	—
9		—	—	—	—	—	—	—	—	—	—
10	—	—	—	Иттербий Yb 173	—	Тантал Ta 183	Вольфрам W 184	—	Осмий Os 191	Иридий Ir 193	Платина Pt 194,8
11											
12	—	—	Радий Ra 225	—	Торий Th 232,5	—	Уран U 238,5				



**Dmitri Mendeleev  
1869**

## Genuine D.I. Mendeleev's Table

**Mendeleev's Table of elements (1869) showed  
for the first time**

**that the chemical properties**

**(or the nature of the interaction)**

**are regularly repeated with increasing the weight (or  
atomic number) of the elements.**

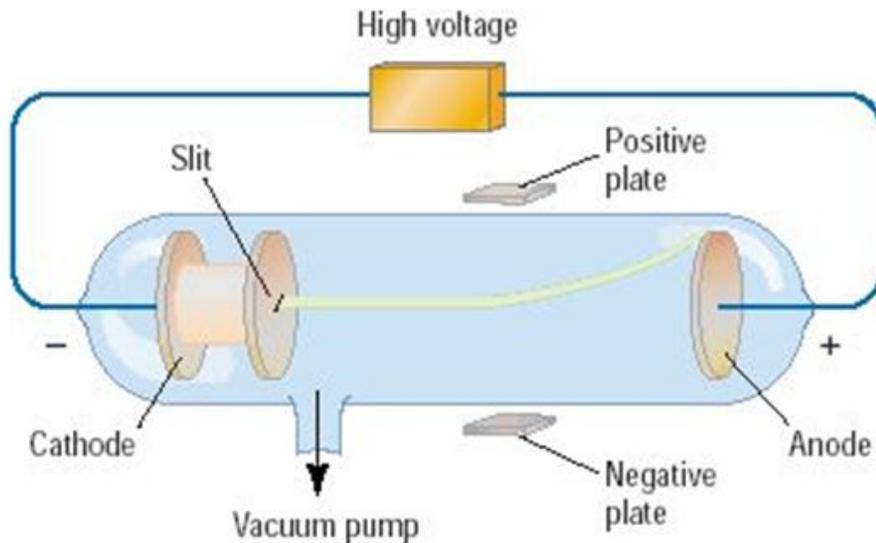
**It follows that:**

**the atoms are not "building bricks",**

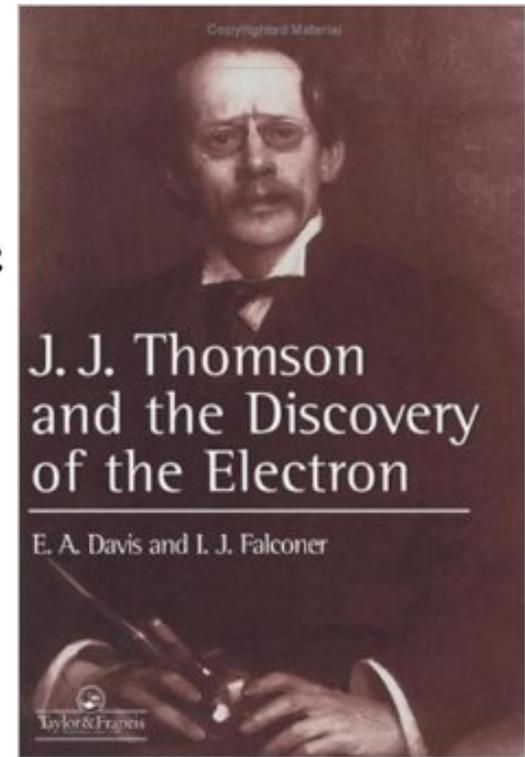
**They themselves are complex internal  
structures.**

# Discovery of the Electron

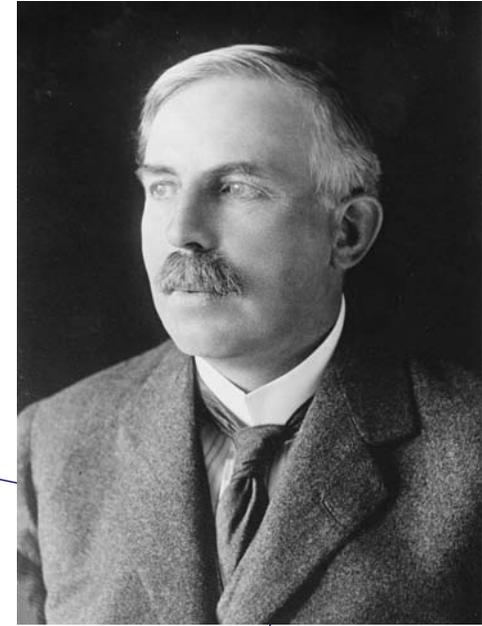
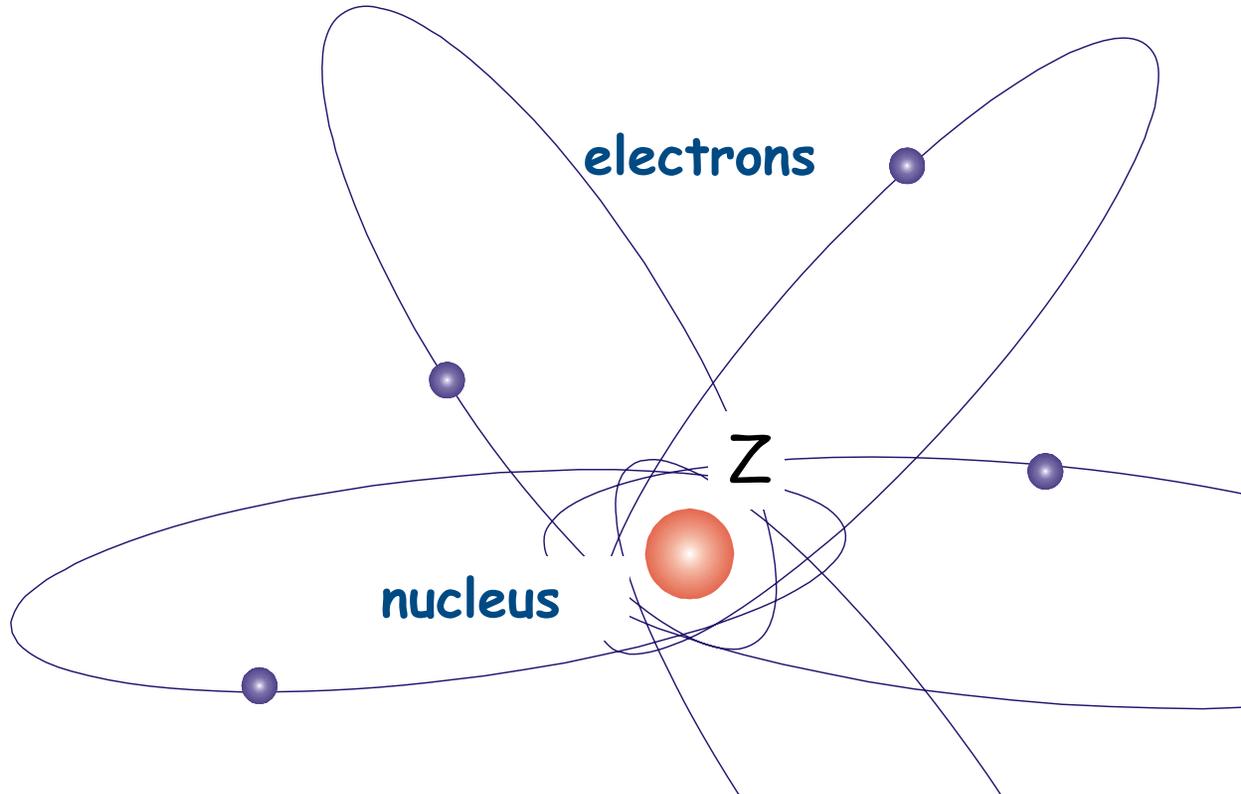
In 1897, J.J. Thomson used a cathode ray tube to deduce the presence of a negatively charged particle.



Cathode ray tubes pass electricity through a gas that is contained at a very low pressure.



86 elements were already known by that time



**Ernest Rutherford**  
March 7, 1911

**Planetary model of atom**

It became clear that existence of atom is limited by stability of its nucleus.

In attempts to describe the properties of nuclear matter, George Gamow made a daring assumption that atomic nucleus may be similar to a drop of positively charged liquid.



George Gamow 1928

In fact, the density of nuclear liquid is  $10^{15}$  times more than that of water

**Charged Liquid Drop Model  
of the atomic nucleus.**

**G. Gamow, 1928.**

**With this beautiful macroscopic and, in this sense, classical model of nucleus that presents nuclei as drops of charged liquid of constant density:**

**G.A.Gamov -developed the theory of alpha-decay of the atomic nuclei (1928)**

**C.F. von Weizsäcker - wrote the famous formula for the binding energy of nuclei (1933)**

**N.Bohr and J.Wheeler - developed the theory of nuclear fission (1939)**

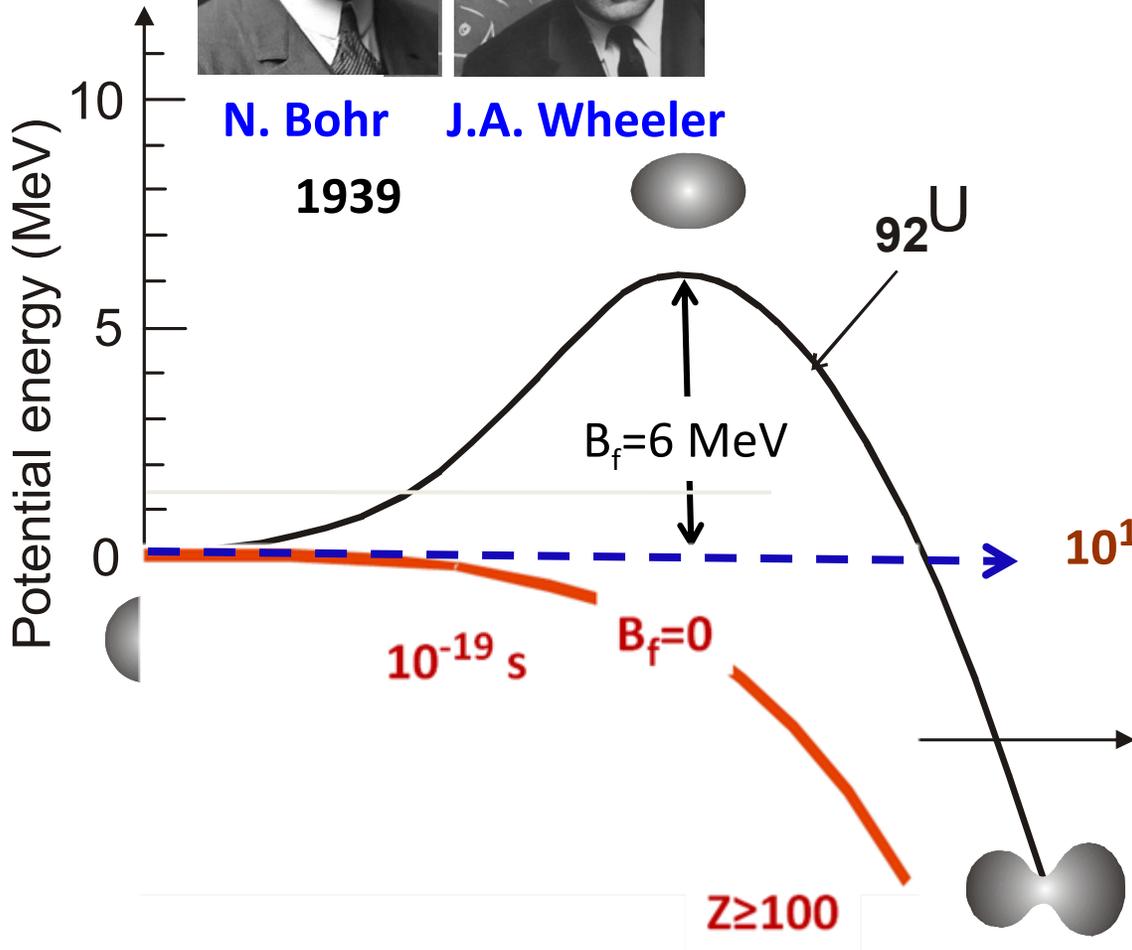
We will be discussing today the problem of synthesis and properties of extremely heavy nuclei to get answers to questions about:

- How big the nuclei can be?
- What of proton and neutron number they may have?
- Where is the mass limit of the nuclei?
- How many chemical elements can be and what are their properties?

# Nuclear fission



**N. Bohr** **J.A. Wheeler**  
**1939**

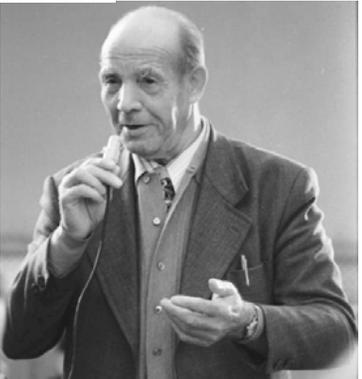


**G.N. Flerov**



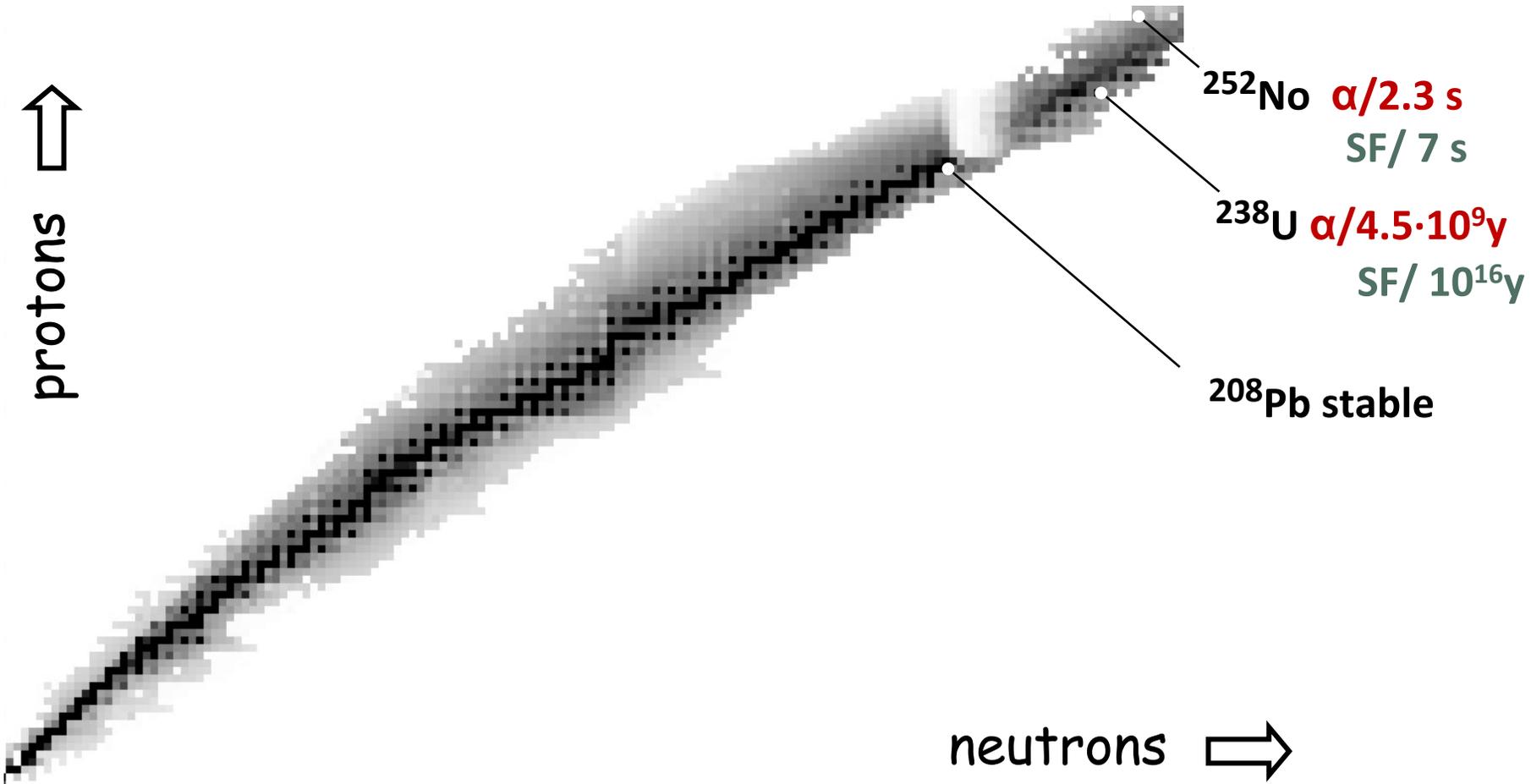
**1940**

**K.A. Petrzhak**

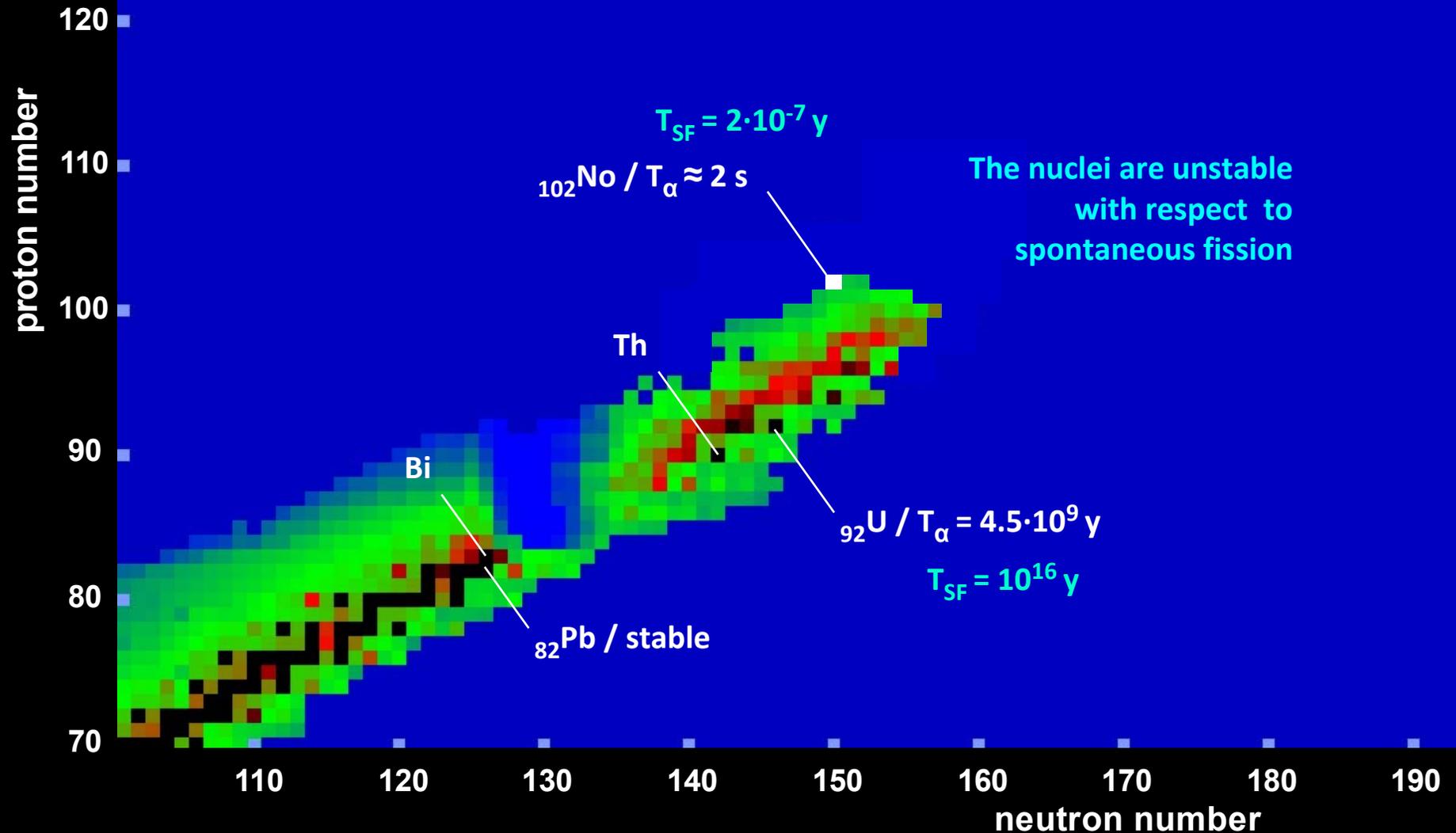


# Chart of the Nuclides

SF

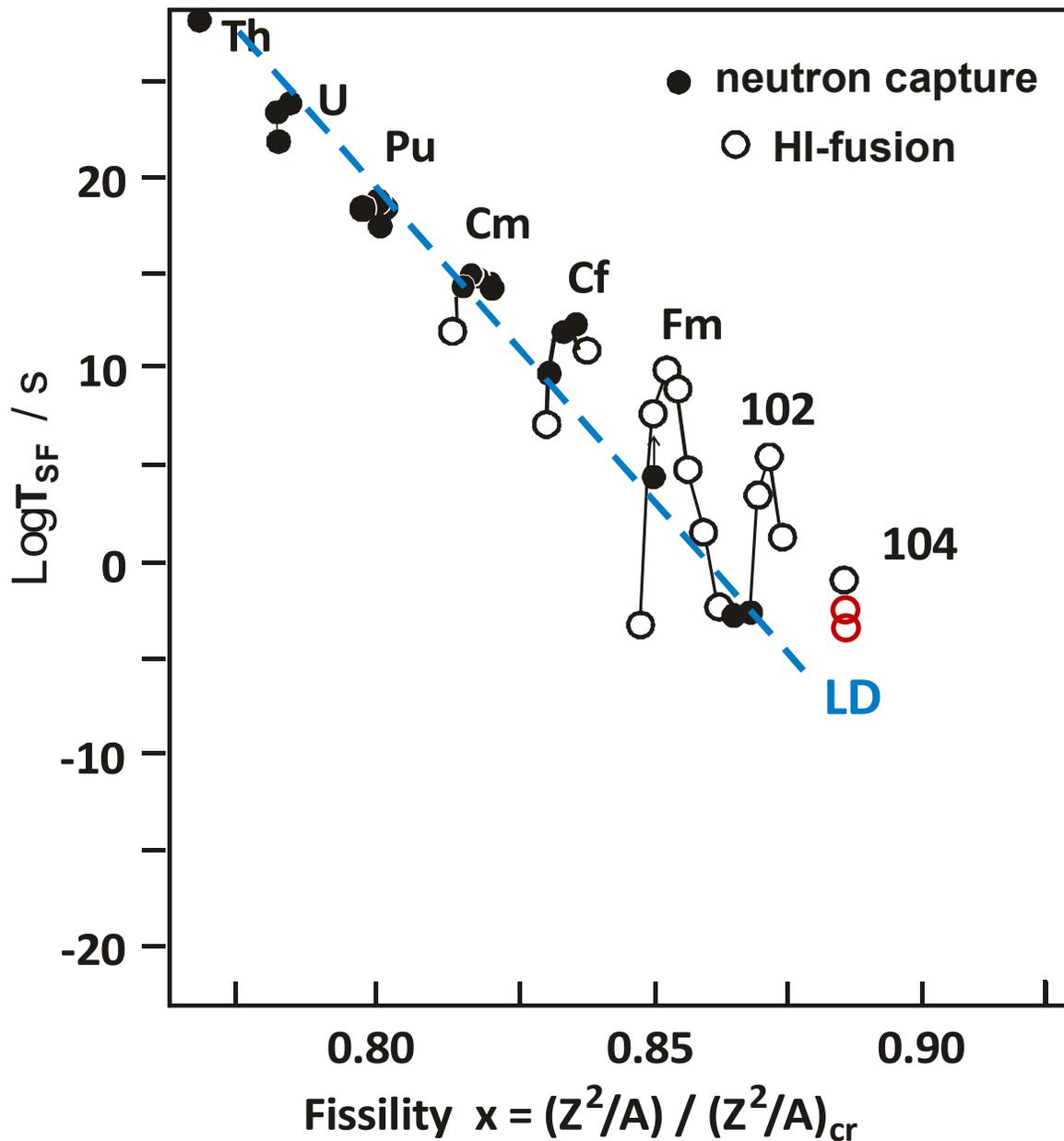


about 50 years ago...



1962 JINR, Dubna

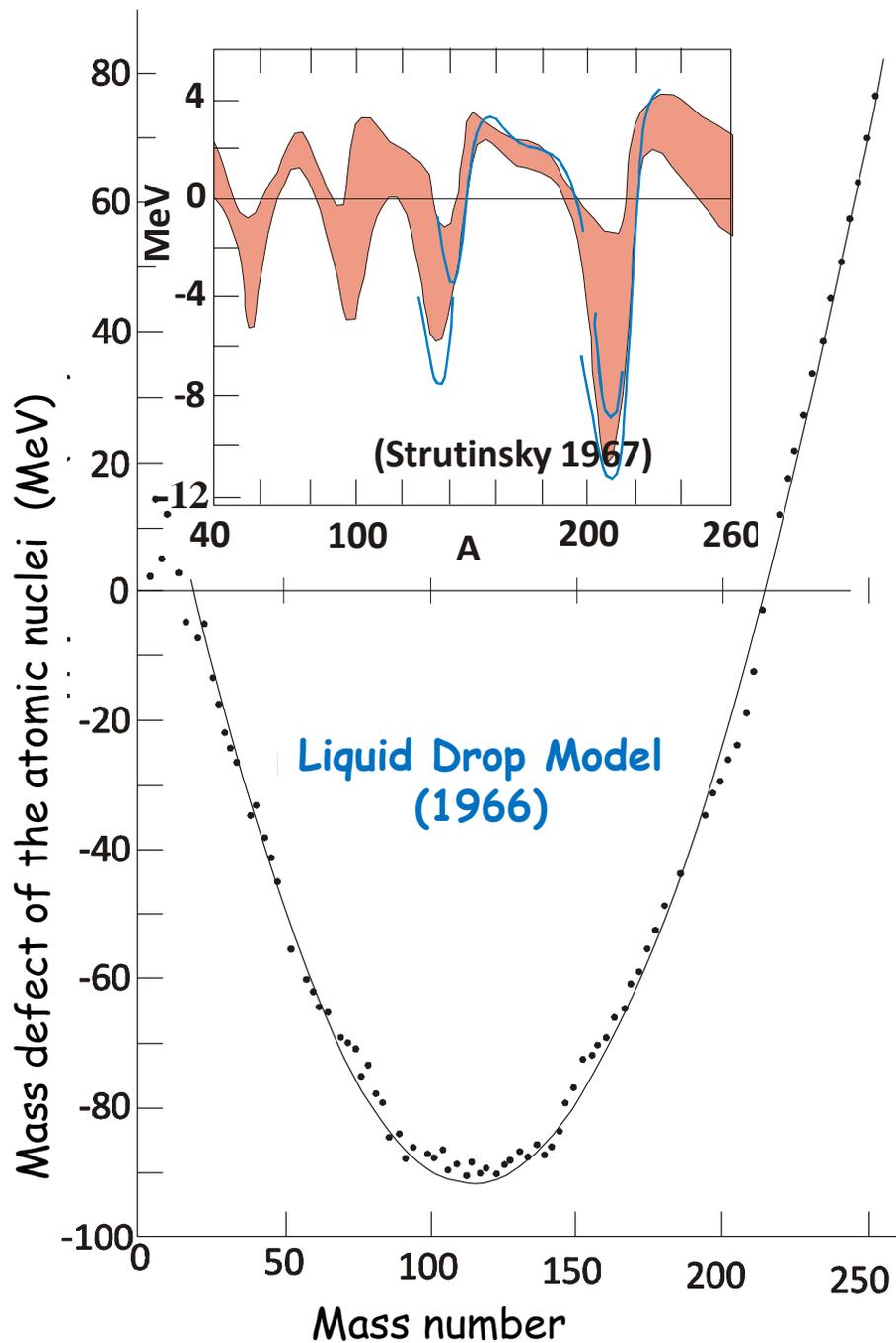
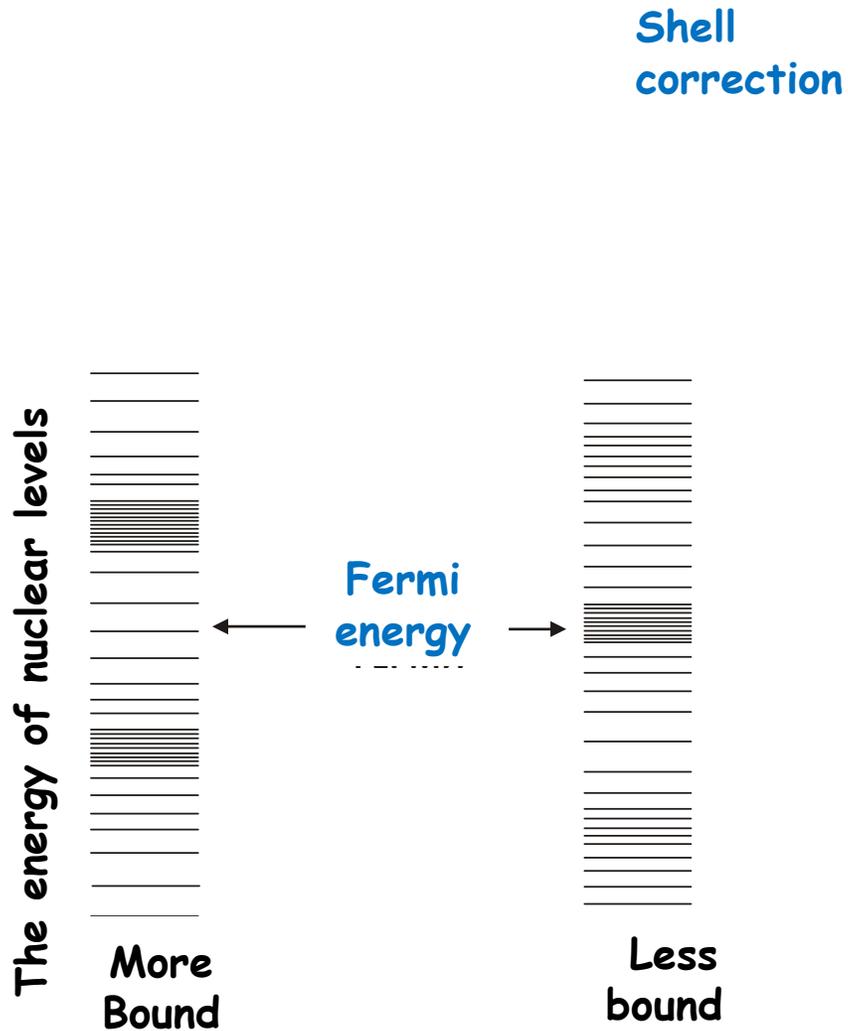
Search for Element 104  
in  $^{242}\text{Pu} + ^{22}\text{Ne}$  reaction

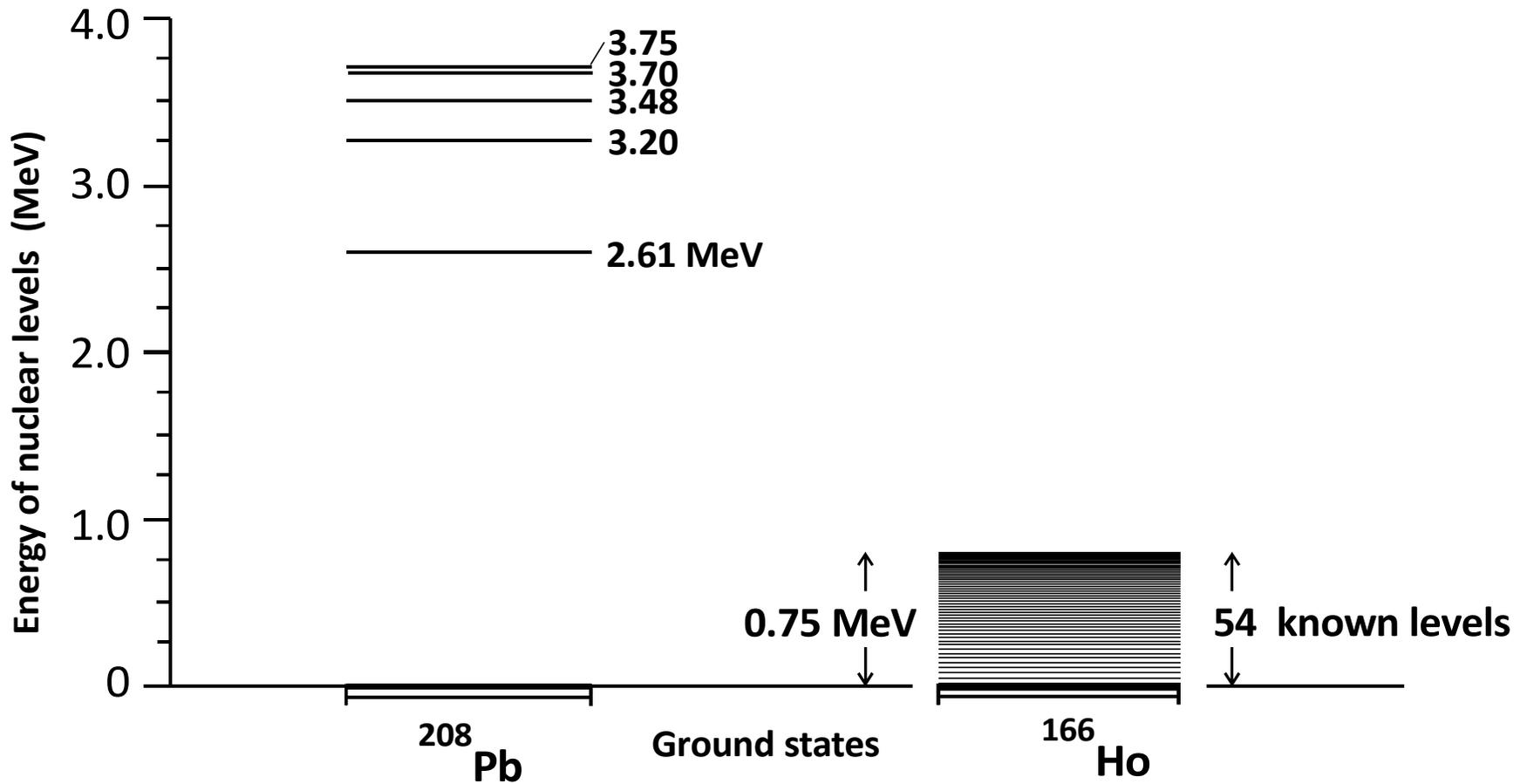




# Nuclear structure and stability of the heaviest nuclei

# Nuclear Shells





In this interpretation, the manifestation of the nuclear structure is similar to the effect of the electron shells closure in the atoms,—

${}^2\text{He}$ ,  ${}^{10}\text{Ne}$ ,  ${}^{18}\text{Ar}$ ,  ${}^{54}\text{Xe}$ ,  ${}^{86}\text{Rn}$  known as “noble gases” located in the last column of the Table of Elements.

Maybe that's why the appearance of this effect in the nucleus has been called **nuclear shells closure ....**

However the analogy is very conditional because of a very dense nuclear matter can not have the structure of the atom

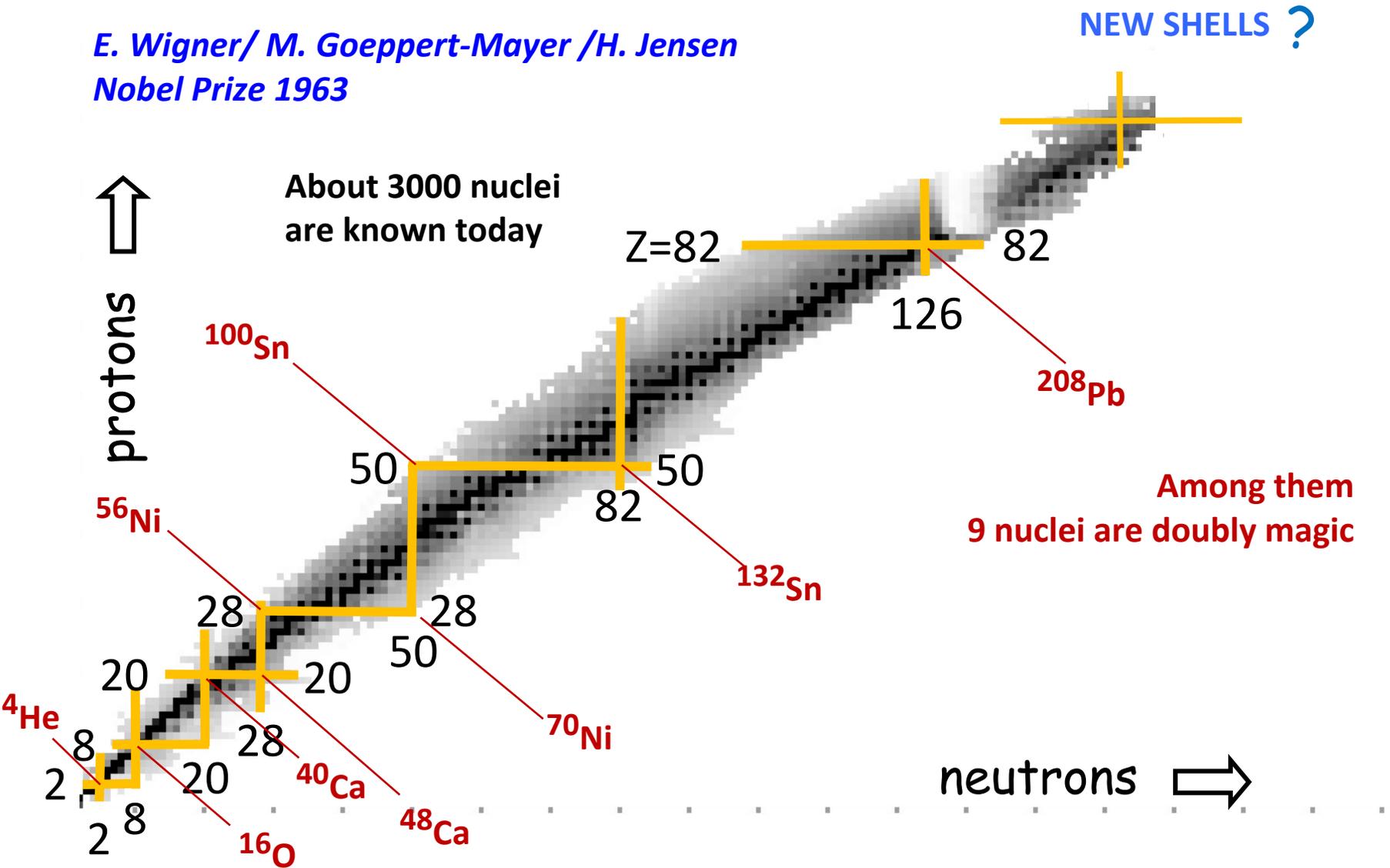
(remember the model of atom of Rutherford - Bohr)

7

132.9	137.3	138.9	178.5	180.9	183.9	186.2	190.2	190.2	195.1	197.0	200.5	204.4	207.2	209.0	(210)	(210)	(222)
87	88	89	104	105	106	107	108	109	110	111	112		114		116		118
<u>Fr</u>	<u>Ra</u>	Ac~	<u>Rf</u>	<u>Db</u>	<u>Sg</u>	<u>Bh</u>	<u>Hs</u>	<u>Mt</u>	---	---	---		---	---	---		---
(223)	(226)	(227)	(257)	(260)	(263)	(262)	(265)	(266)	0	0	0		0	0	0		0

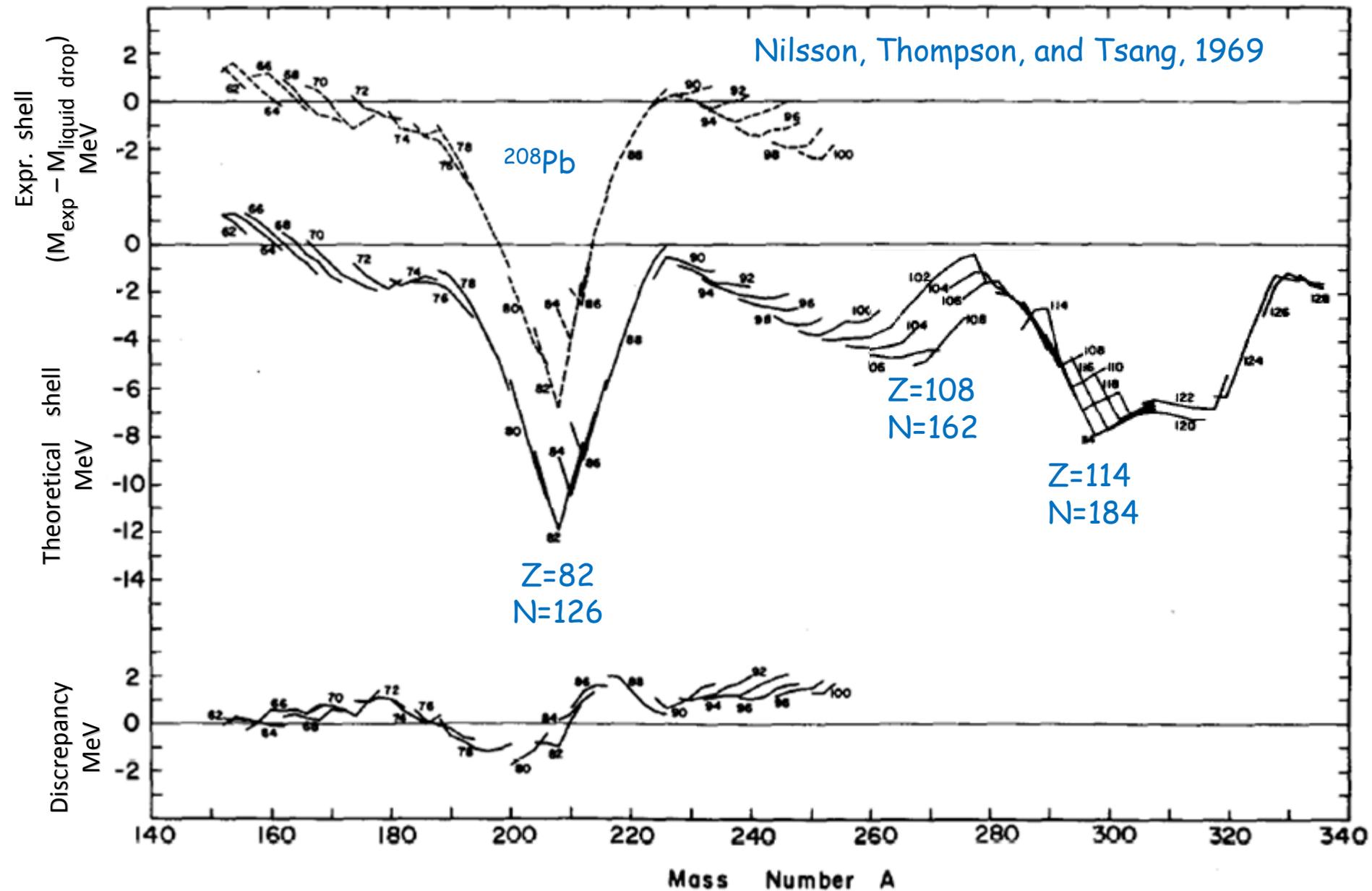
# Shell and magic numbers

*E. Wigner / M. Goeppert-Mayer / H. Jensen*  
Nobel Prize 1963

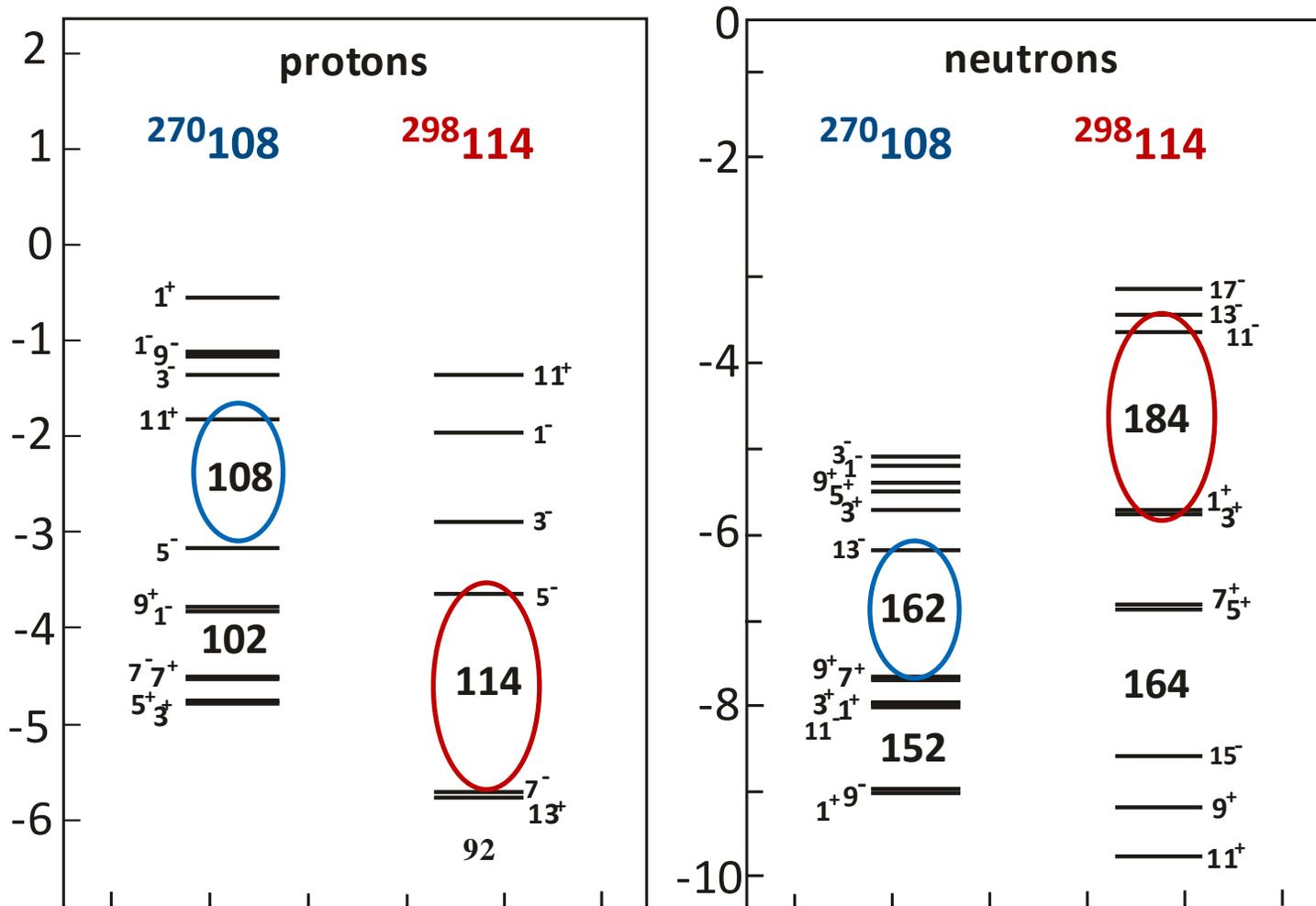


# Shell effect in the nuclear ground states (mass defect)

Nilsson, Thompson, and Tsang, 1969

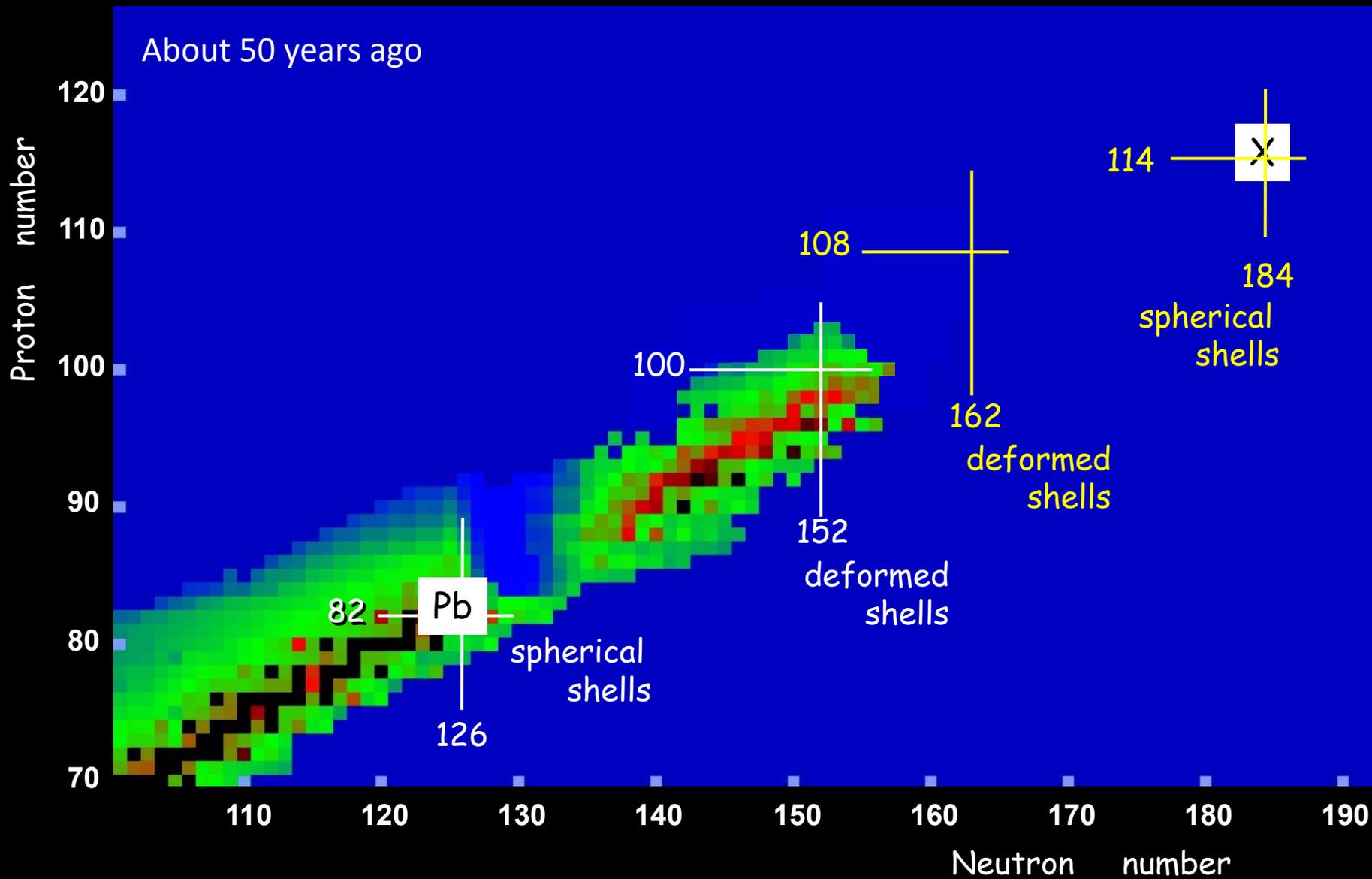


# Proton and neutron single particle energy spectra calculated for deformed $^{270}_{108}$ and $^{298}_{114}$ doubly magic nuclei

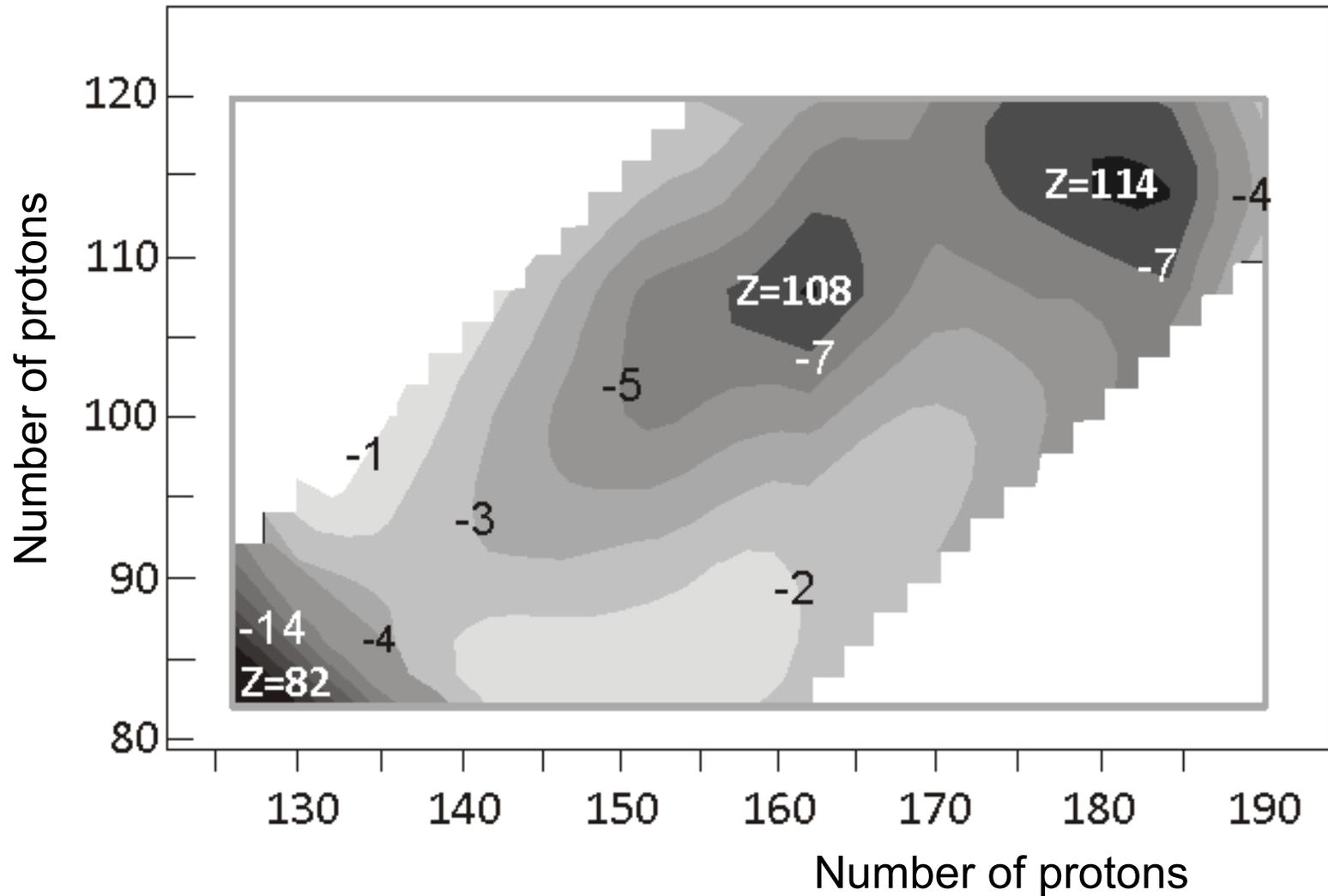


# Chart of nuclides

## Nuclear shells (macro-microscopic calculations)

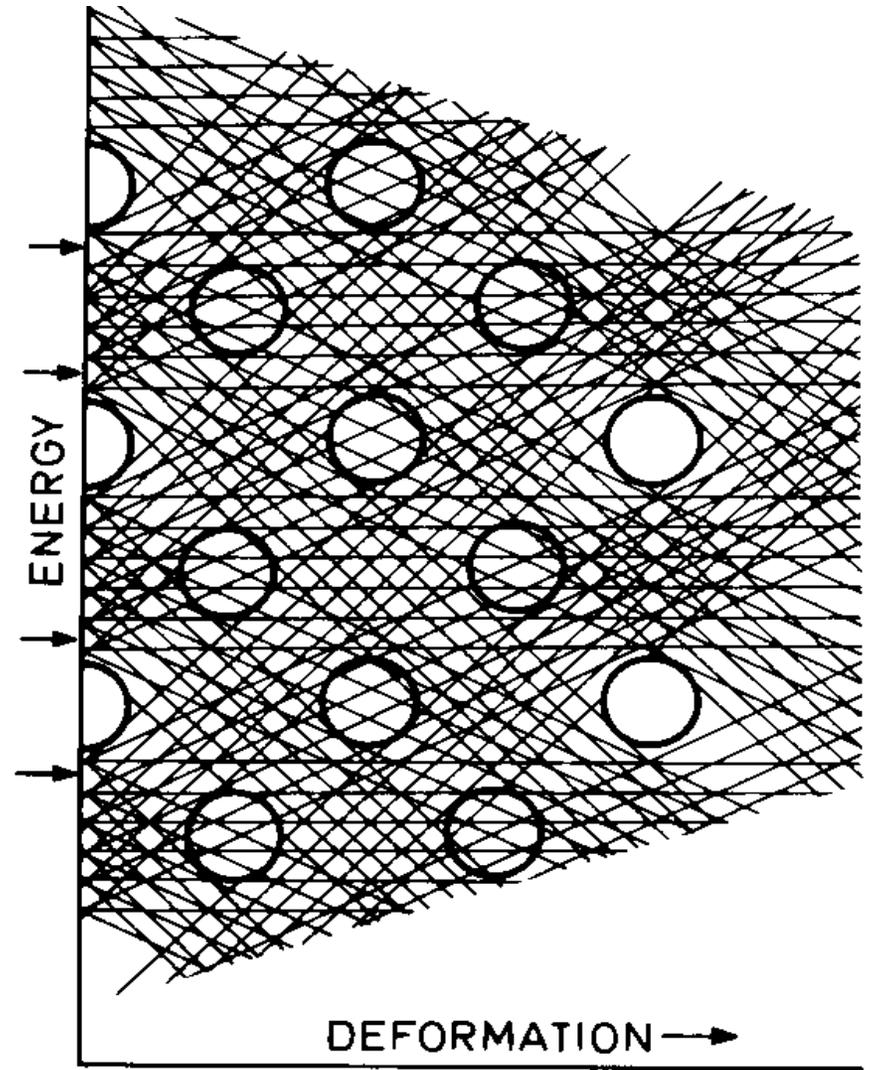
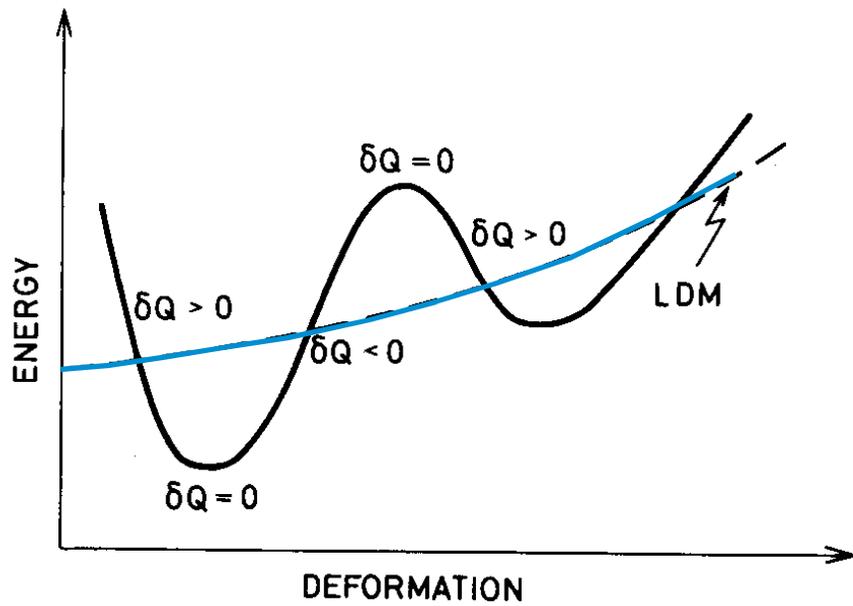


Map of the shell corrections (in MeV)  
to the liquid drop potential energy

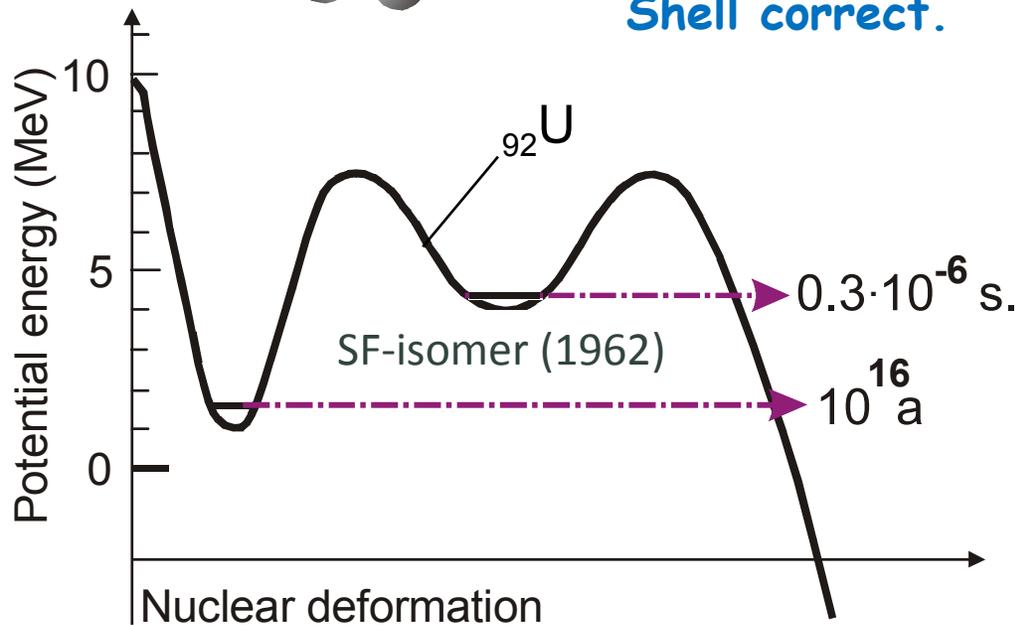
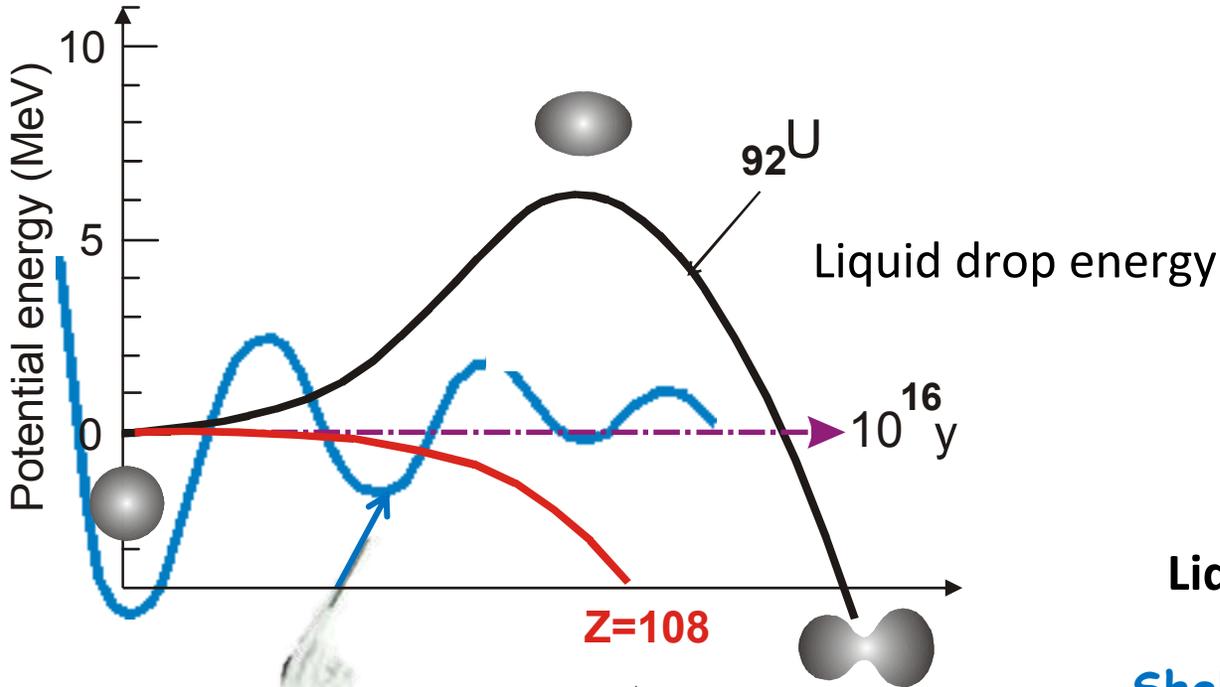


# Shell effect with increasing of nuclear deformation

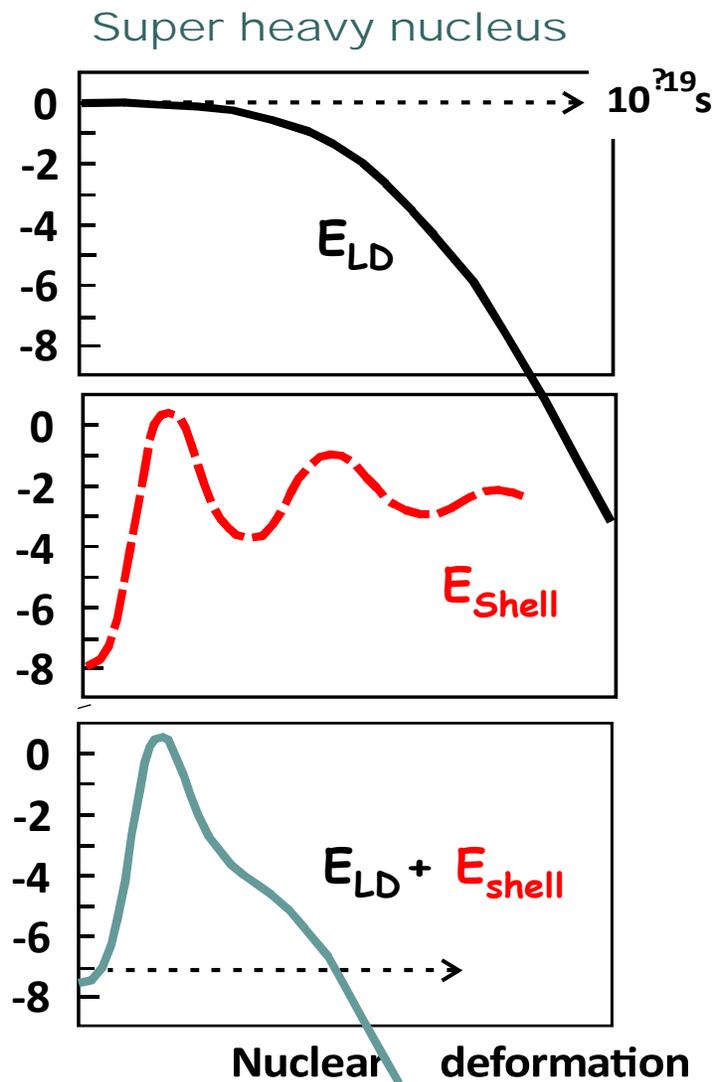
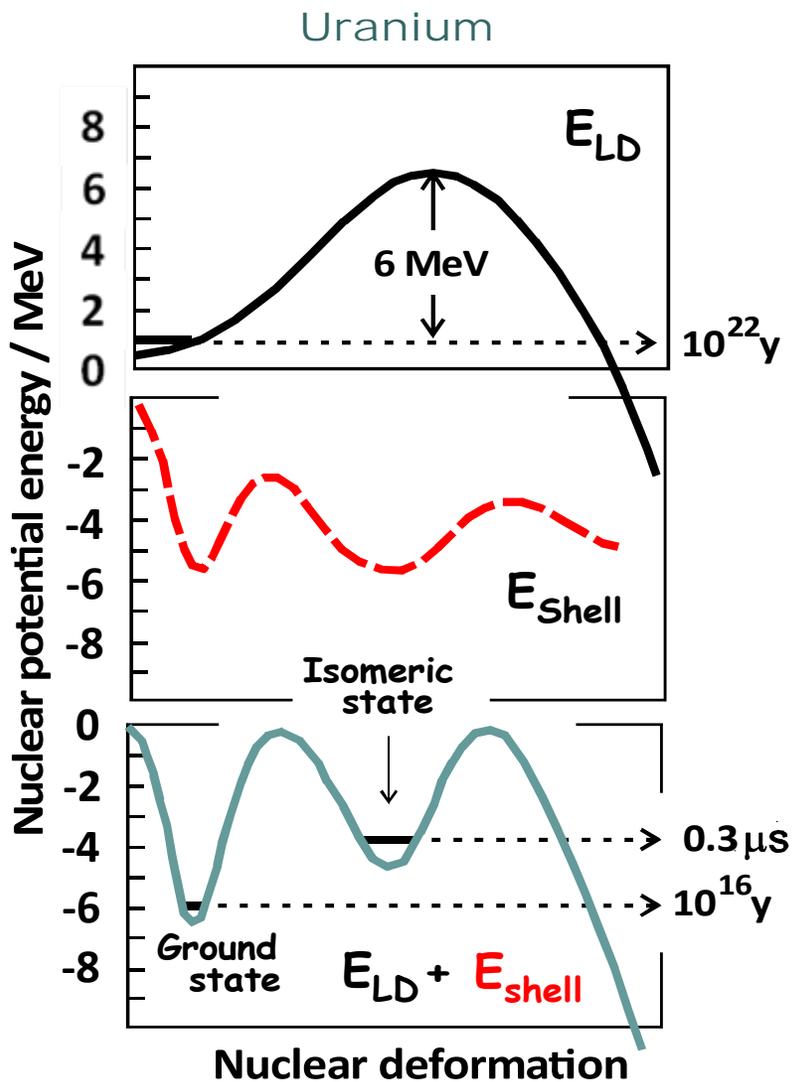
Nuclear levels depending on deformation



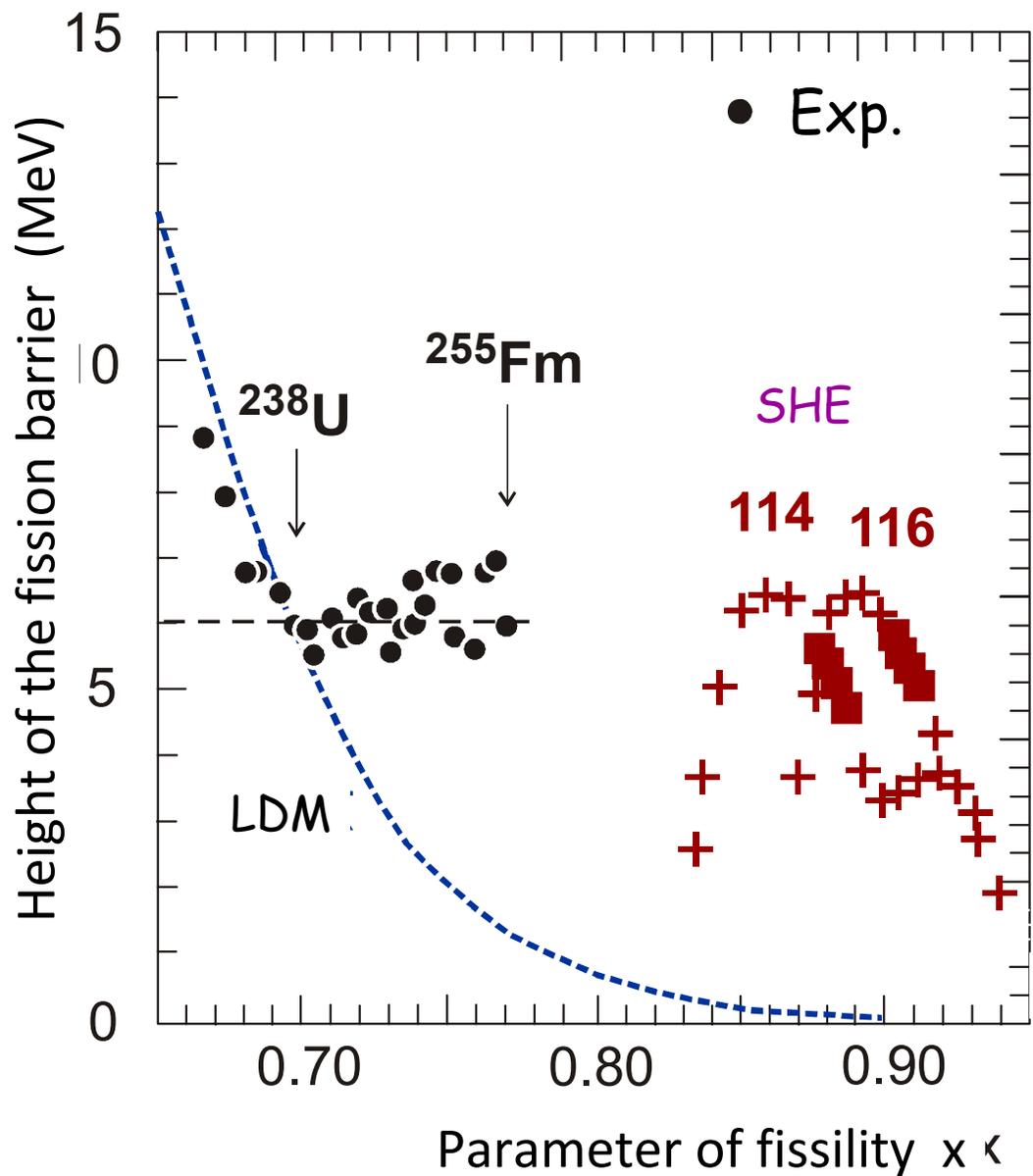
Shell effect



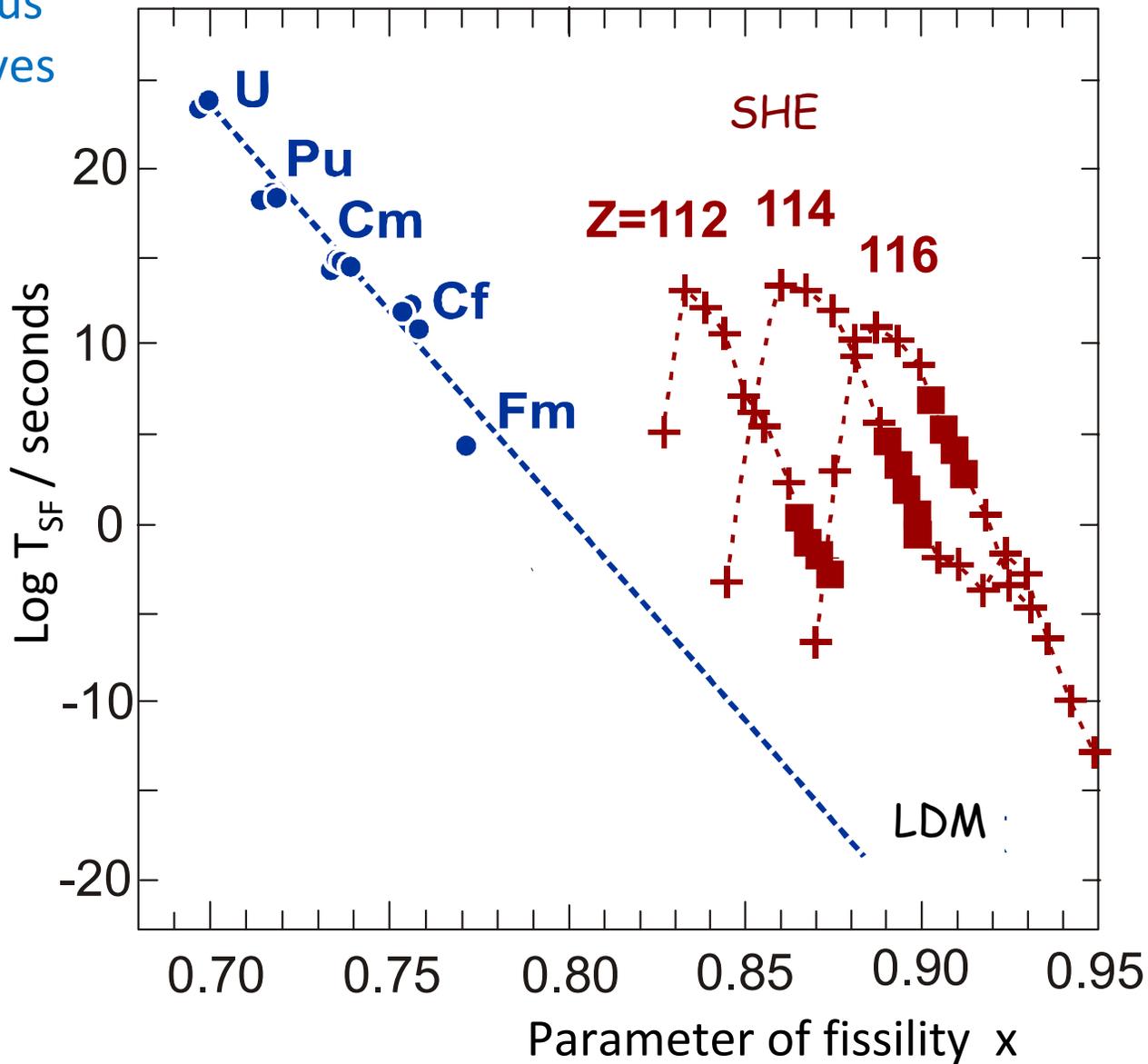
V.M. Strutinsky 1967

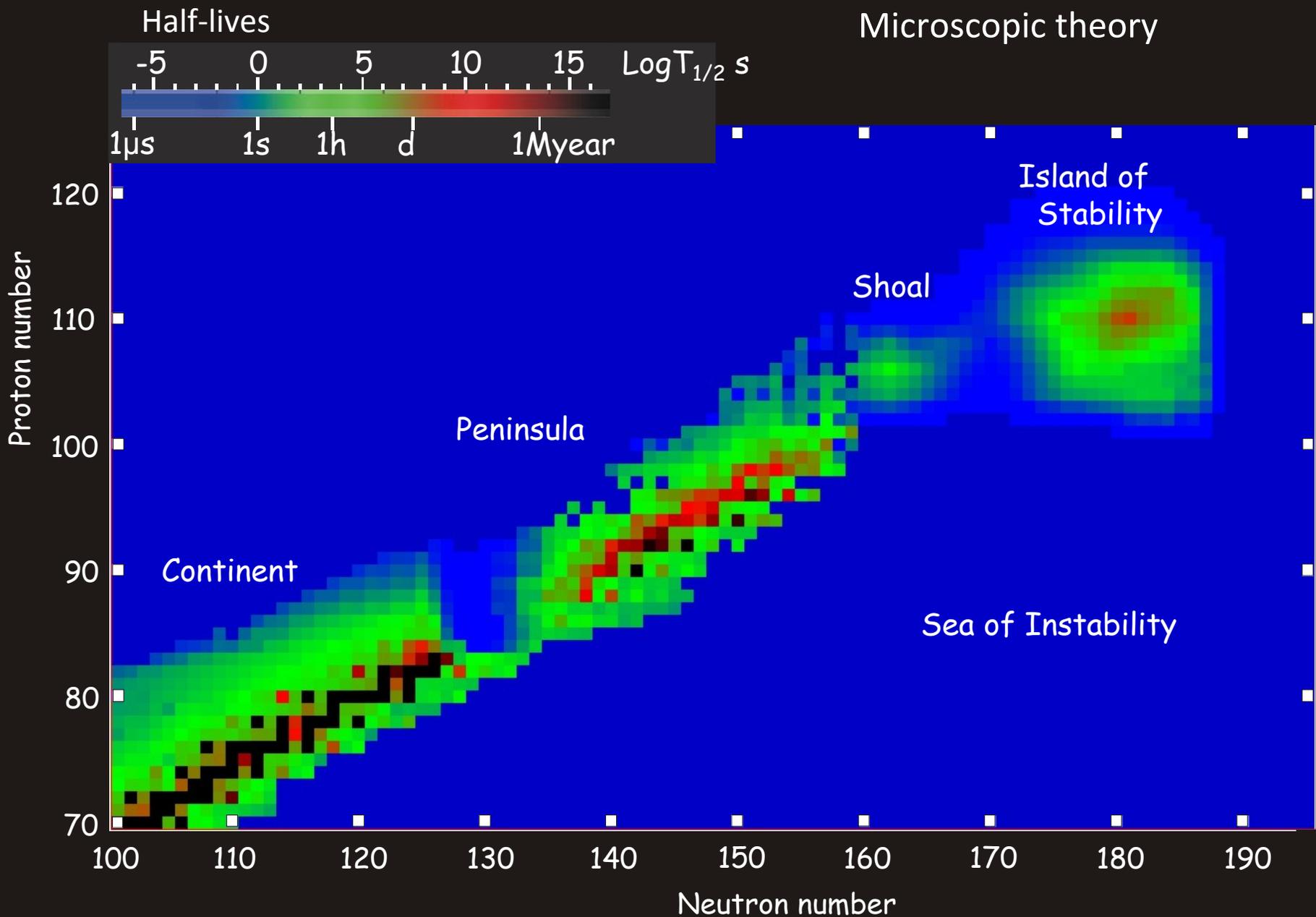


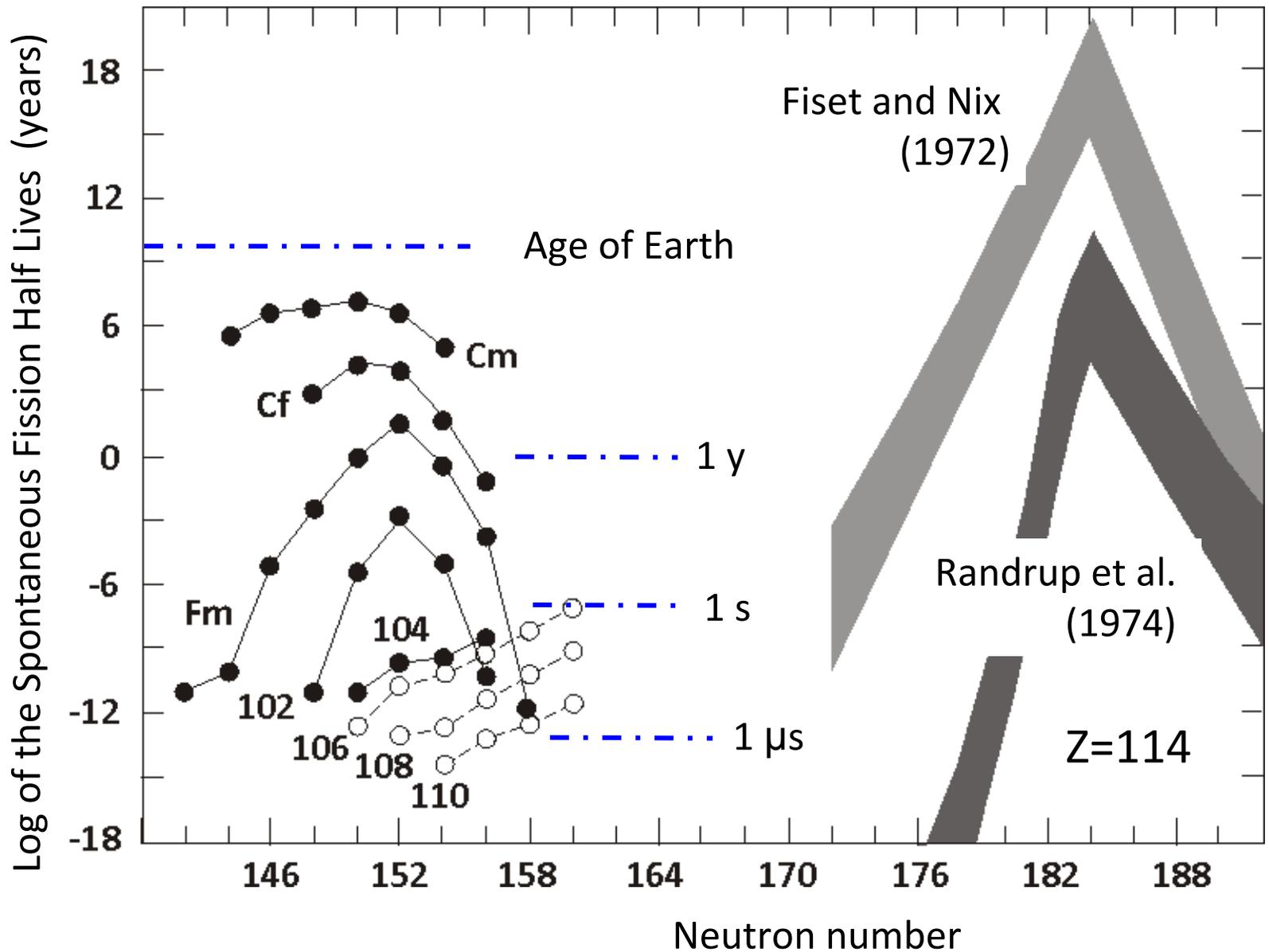
# Fission barriers



Partial Spontaneous  
Fission Half-Lives







# 15-year long assault on the "Islands of Stability"

1970-1985

**Los Alamos (USA)**

**Berkeley (USA)**

**Dubna (JINR)**

**Oak Ridge (USA)**

**Mainz (Germany)**

**Darmstadt (Germany)**

**Orsay (France)**

**Würenlingen (Switzerland)**

**Tokyo (Japan) some later**

The task of every laboratory was:

To find the method of producing/detecting of superheavy elements

**Search in nature: earth/lunar objects, cosmic rays,  
artificial synthesis of superheavy elements**

Use and develop a setup for synthesizing SHE

**High-flux reactor, even nuclear explosion, powerful  
accelerator of heavy ions,**

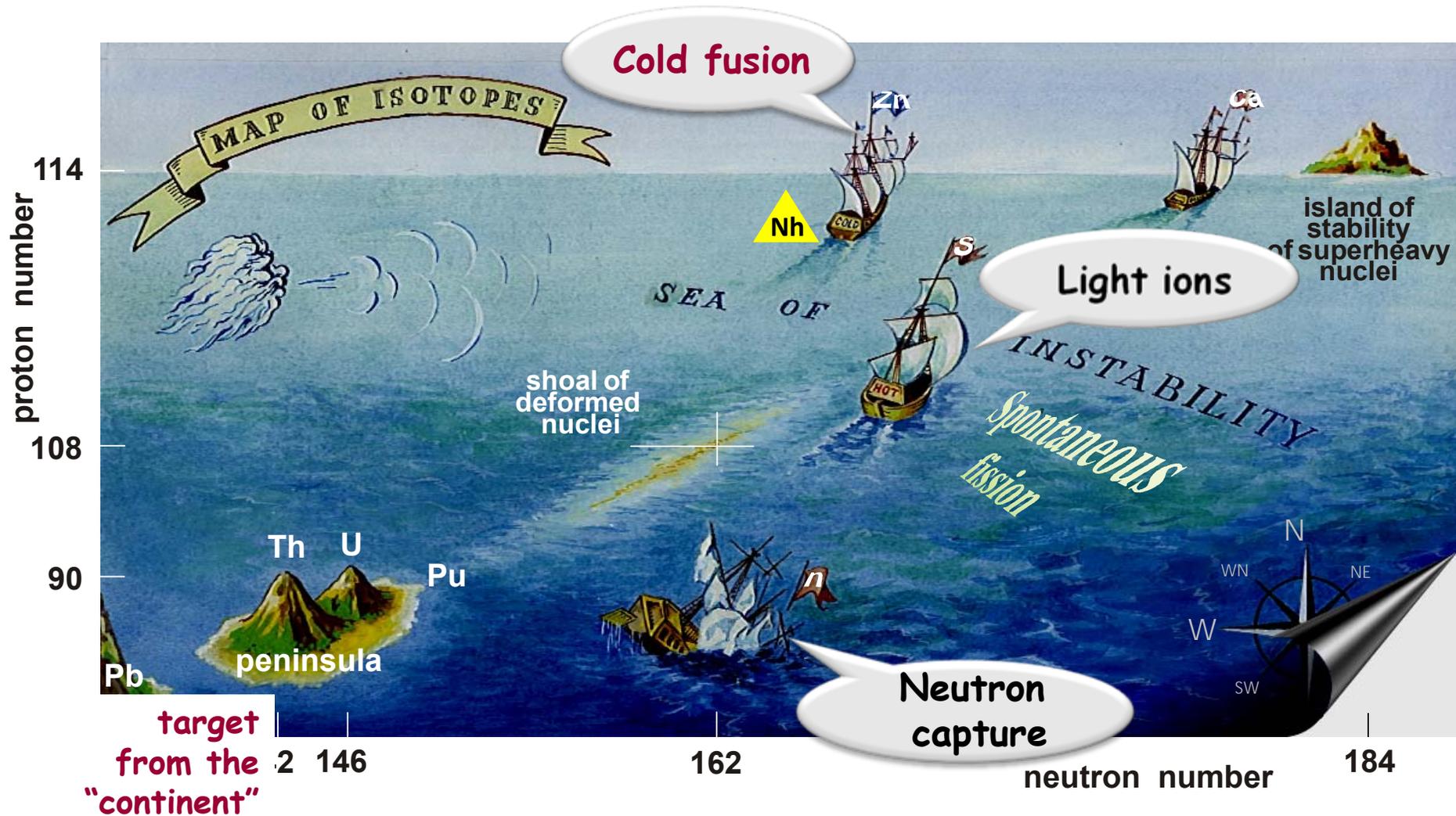
To develop setups and detectors for separating and registering the  
rarest events of formation and decay of superheavy nuclei

**separator/detector, spectrometer, chemical methods, etc.**

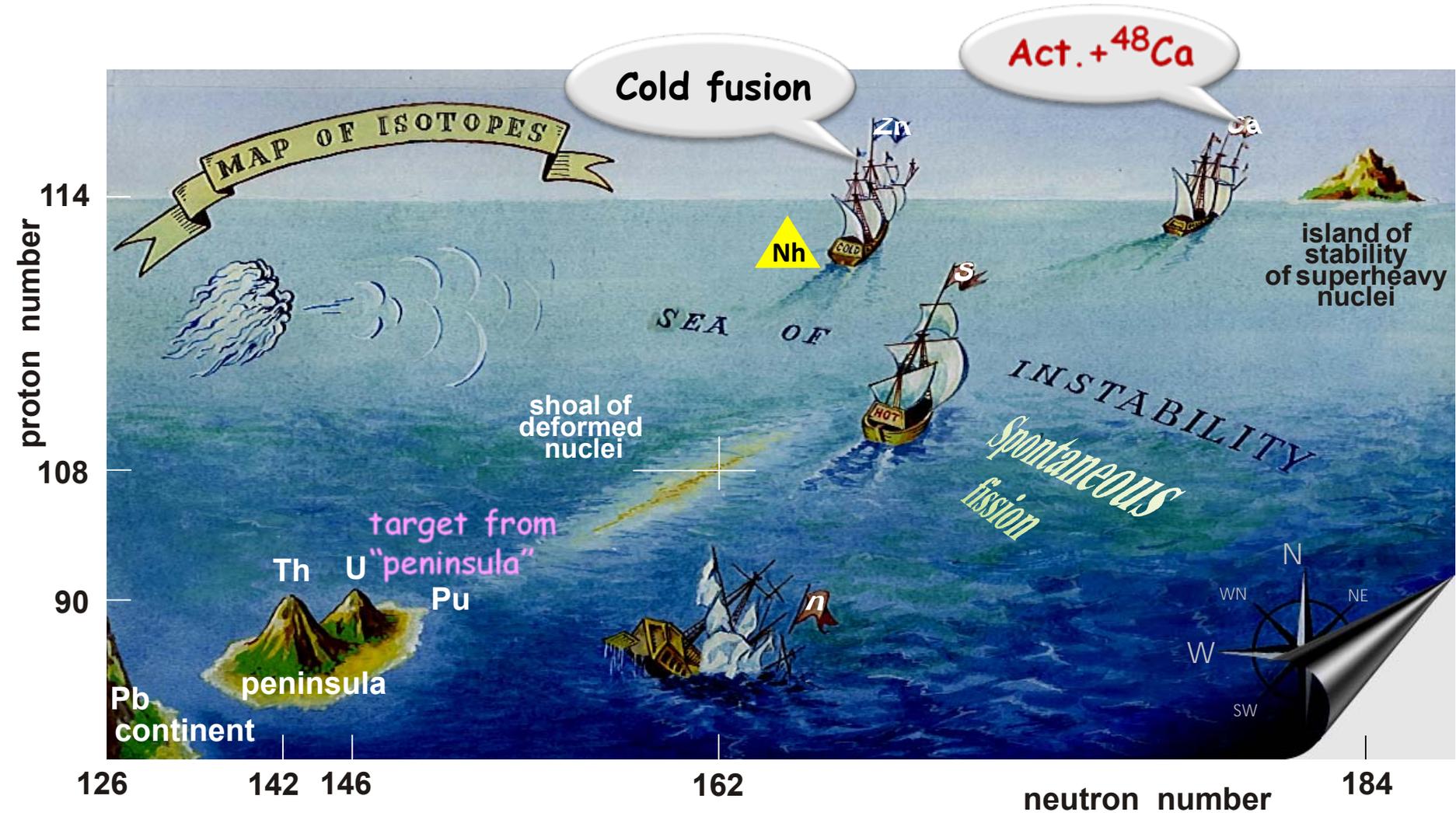
To have much patience

**To work for many years, being ready to rearrange on the way and improve  
the experimental technique**

# Reactions of synthesis

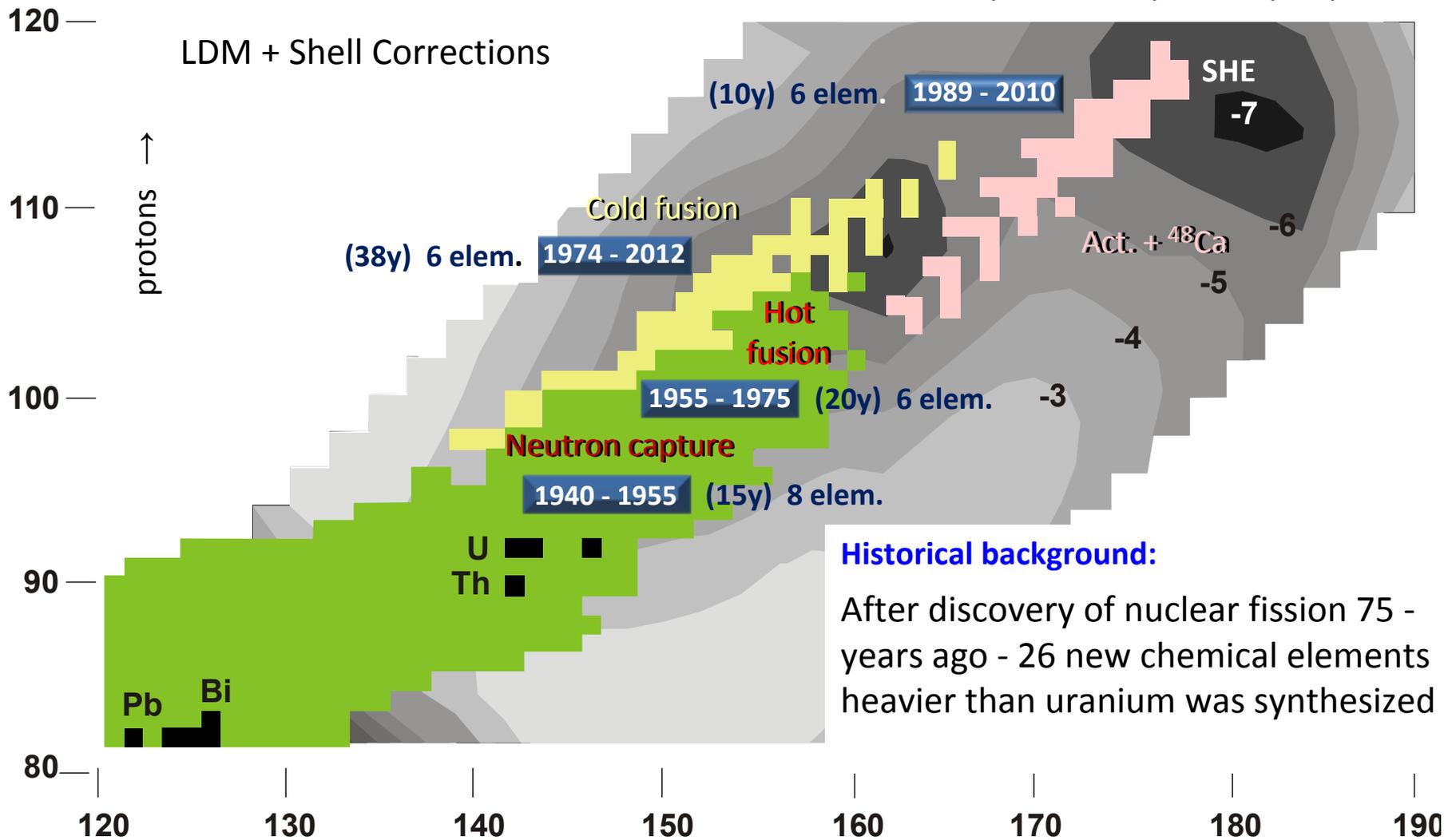


# Reactions of synthesis

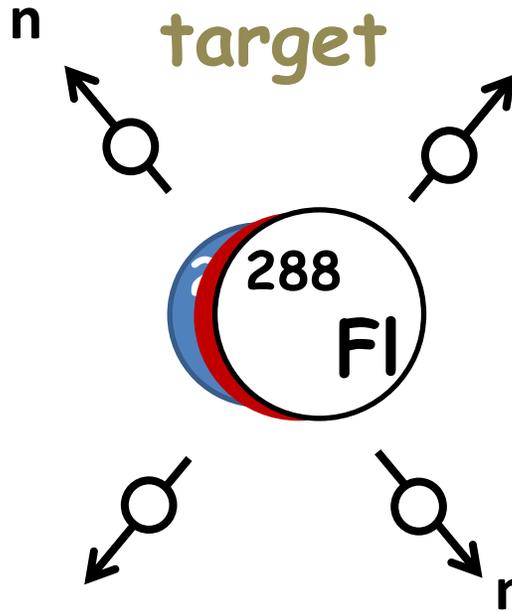


# Reactions of Synthesis

A. Sobiczewski, K. Pomorski, PPNP 58, 292, 2007



artificial element from  
flux nuclear reactor



to separator

from  
accelerator

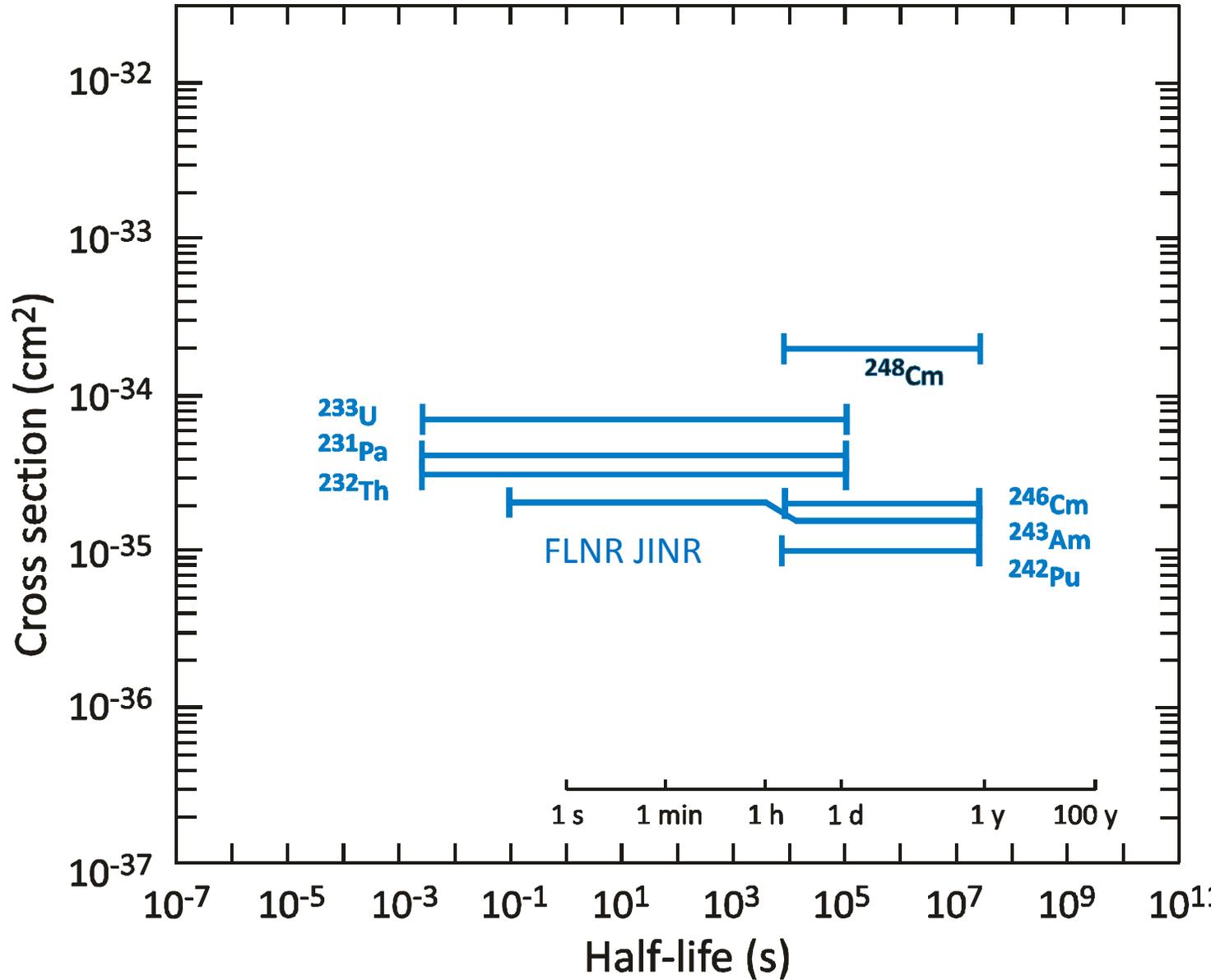


projectiles

rare and very  
expensive isotope of Ca

# Experiments on the synthesis of SHE with $^{48}\text{Ca}$ -beam at FLNR (JINR)

1977-1979

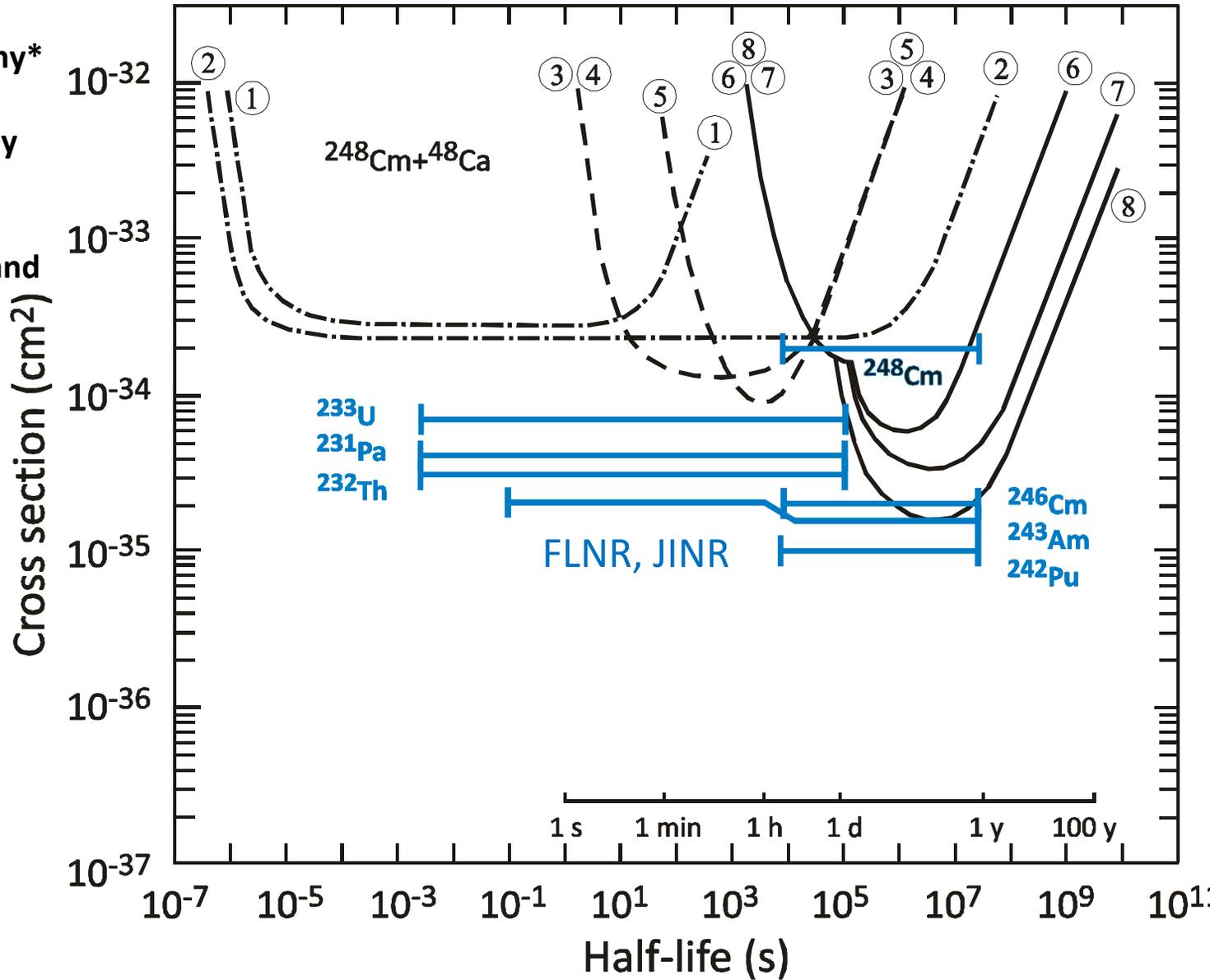


# Experiments on the synthesis of Z=116 nuclei in the reaction $^{248}\text{Cm}+^{48}\text{Ca}$

GSI, Darmstadt, Germany\*  
 LBL, UC Berkeley, CA  
 Univ. of Mainz, Germany  
 LANL, Los Alamos, NM  
 EIR, Würenlingen,  
 Switzerland

1985 →

1977-1979 →



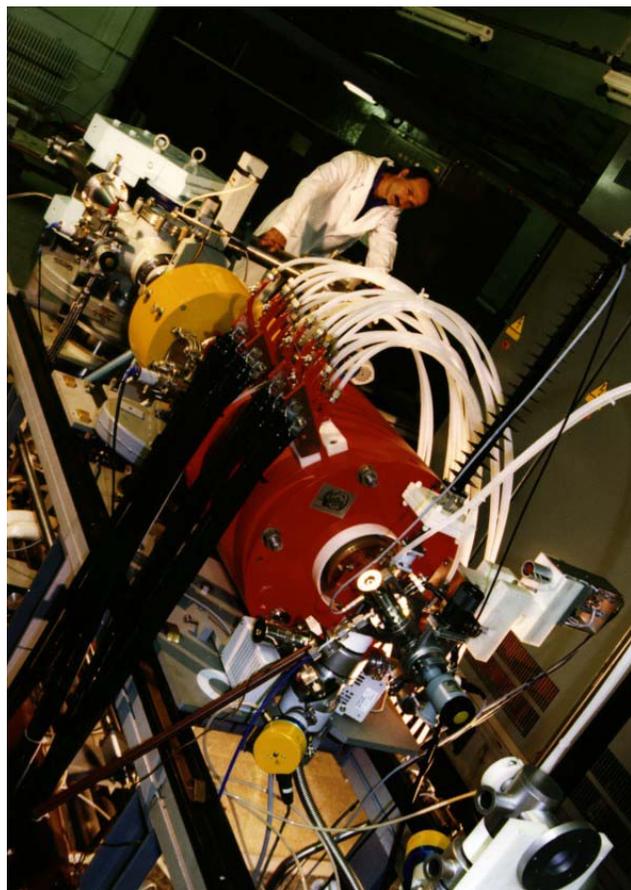
After this, an opinion arose that the “Stability Island” may exist but this cannot be verified...

## Here our opinions have diverged

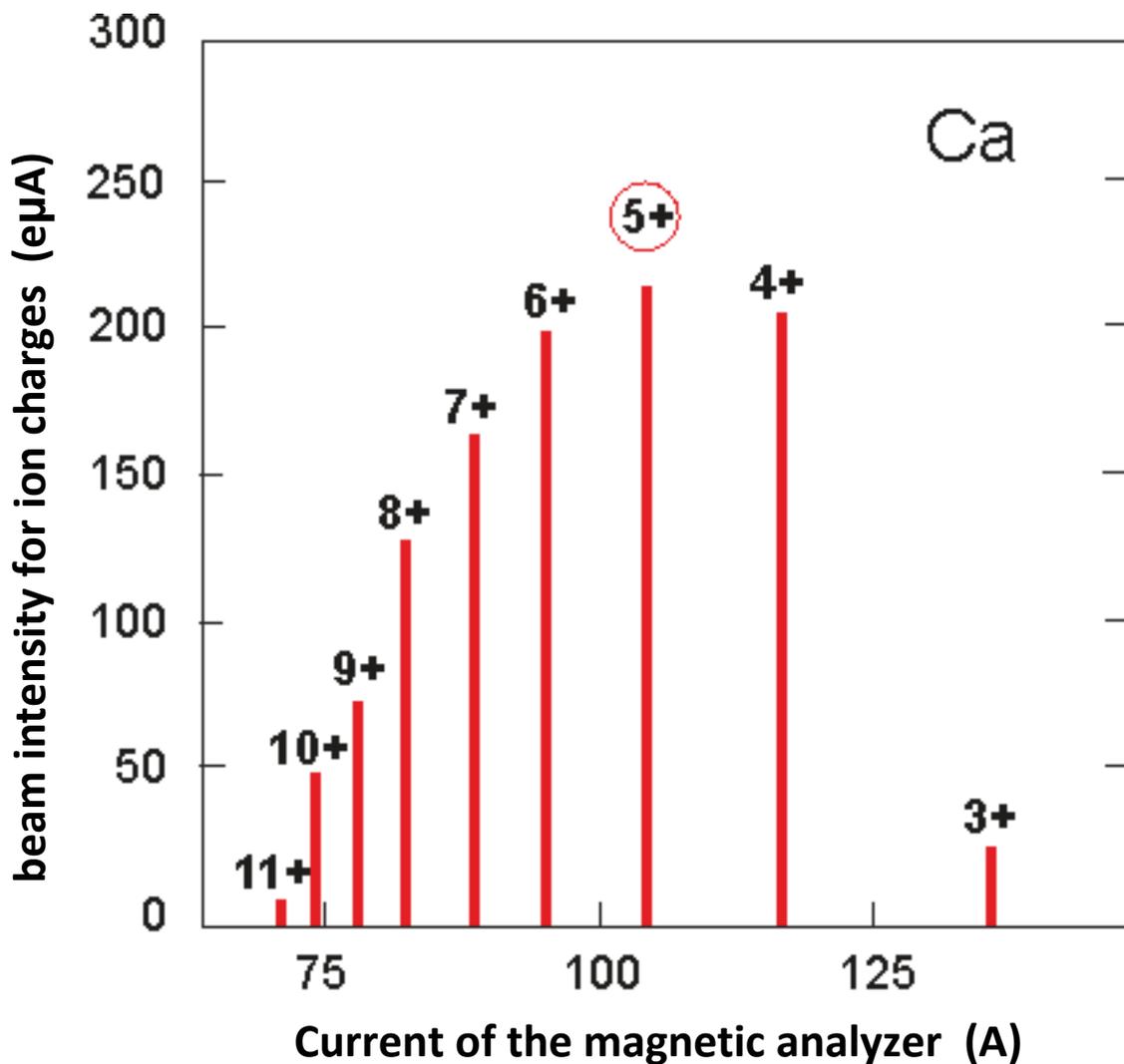
We did not share this view and, since the beginning of 1990, we started preparing:

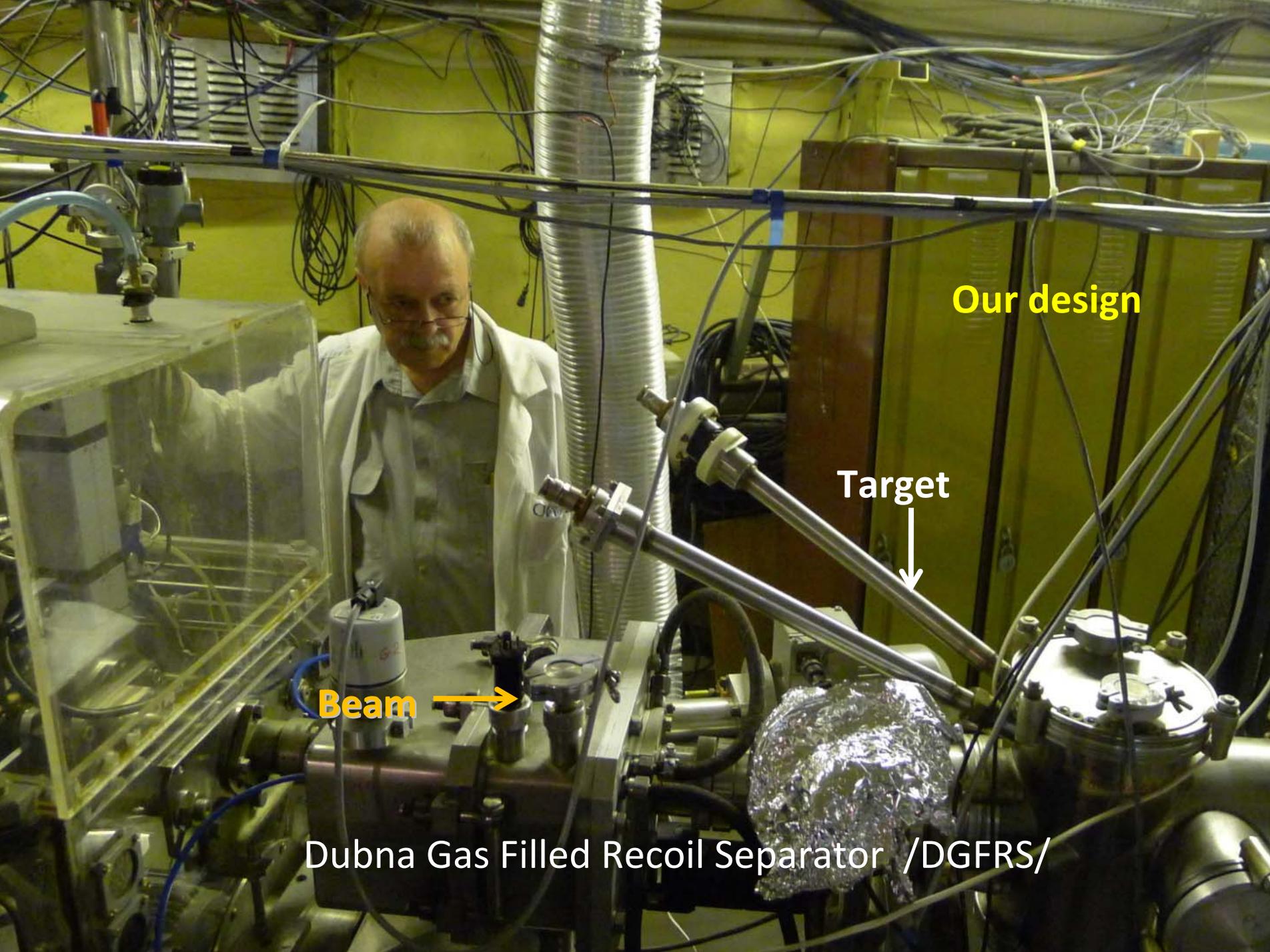
- development of a separator of superheavy nuclei in 1989  
(gas-filled separator of recoil nuclei)
- search for the target materials and manufacturing targets, 1990  
(collaboration with nuclear centers of Russia and USA)
- production of intense beam of Ca-48 in 1996  
(new type of ion source ECR-4M)
- getting ready to long-term experiments, 1990-97.

## Ion charge distribution of Ca-48 extracted from 14 GHz ECR-ion source



Our modification of  
GANIL source



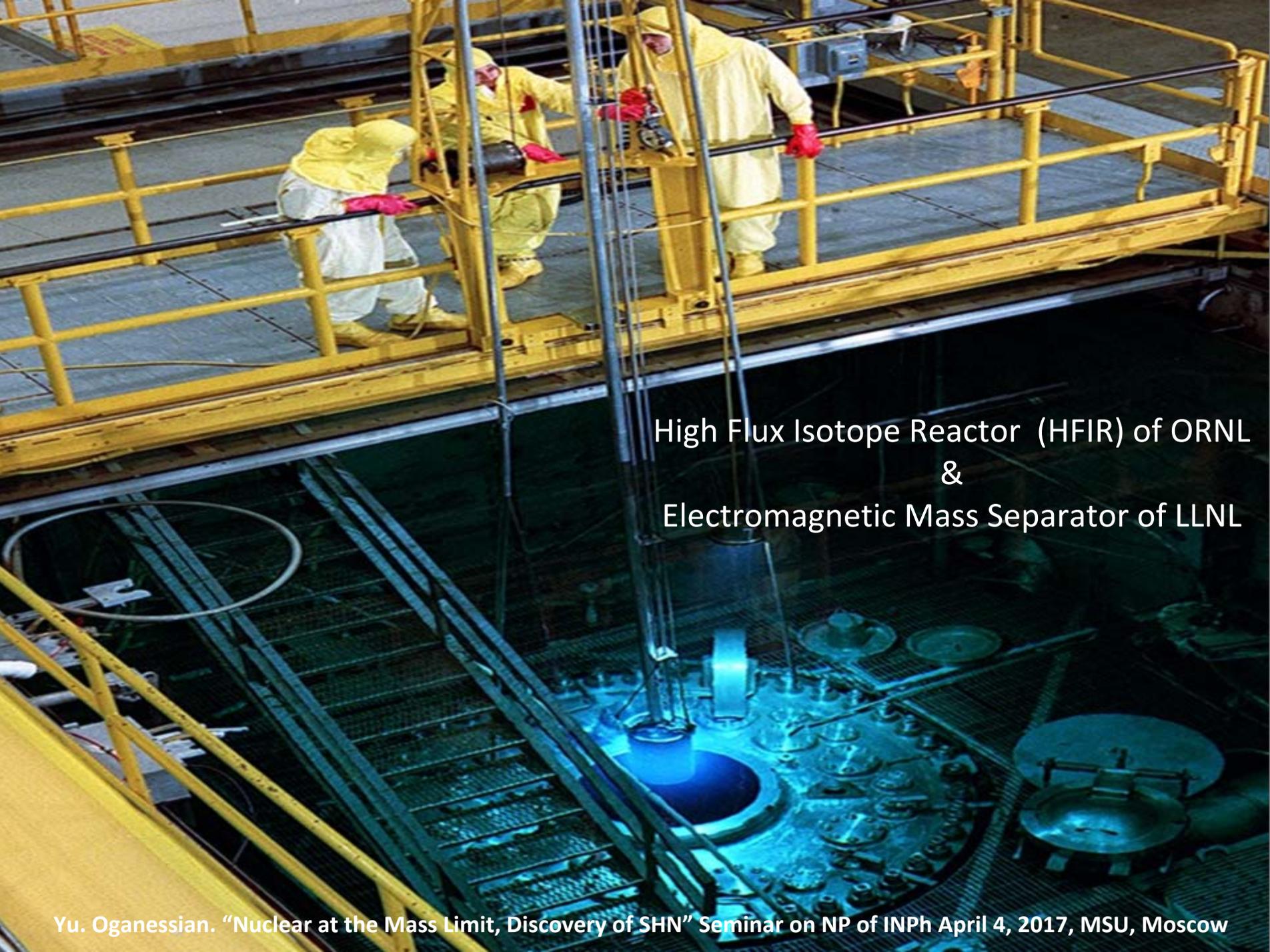


**Our design**

**Target**

**Beam** →

**Dubna Gas Filled Recoil Separator /DGFRS/**



High Flux Isotope Reactor (HFIR) of ORNL  
&  
Electromagnetic Mass Separator of LLNL

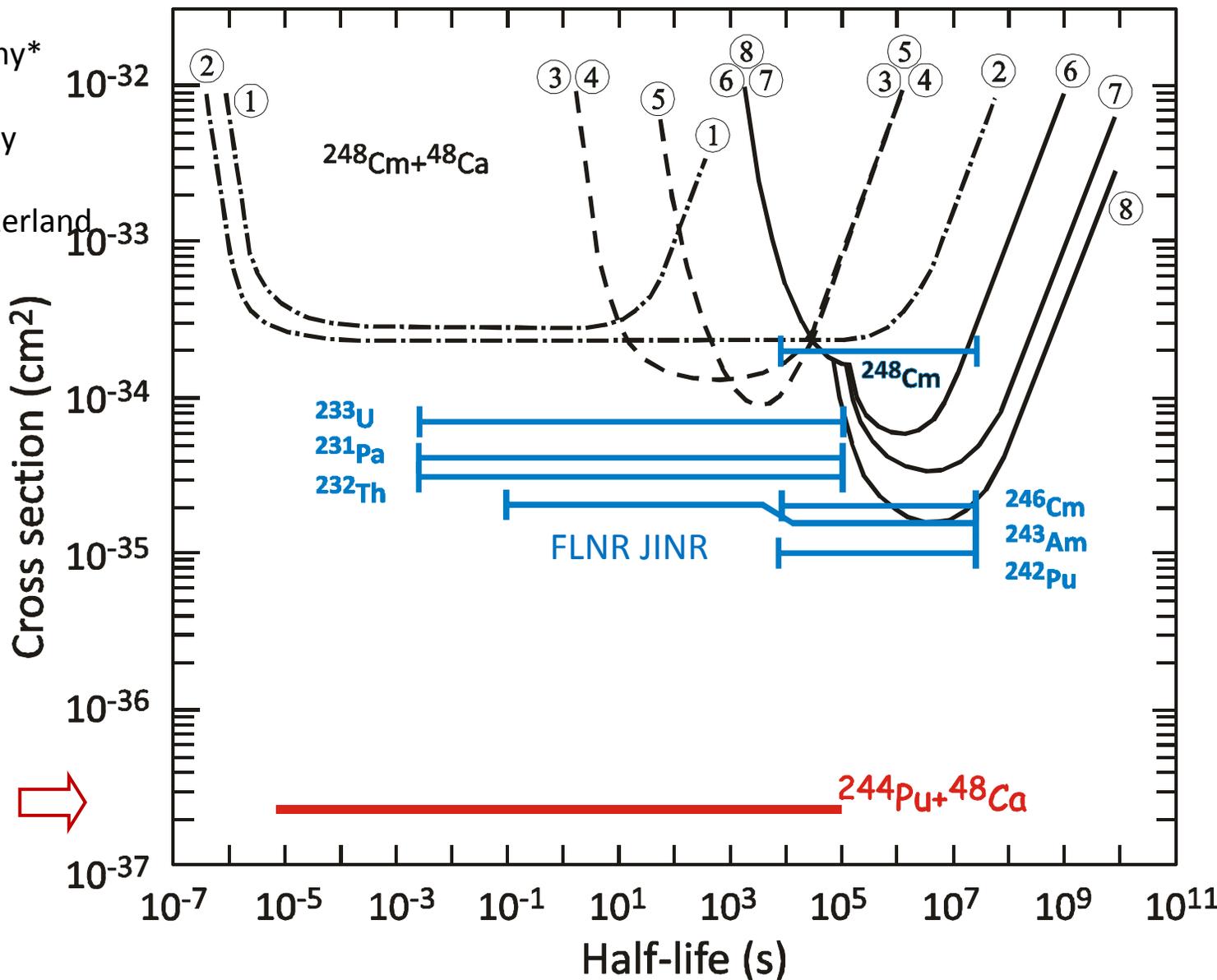
# Experiments on the synthesis of SHE in $^{48}\text{Ca}$ induced reaction

GSI, Darmstadt, Germany\*  
 LBL, UC Berkeley, CA  
 Univ. of Mainz, Germany  
 LANL, Los Alamos, NM  
 EIR, Würenlingen, Switzerland

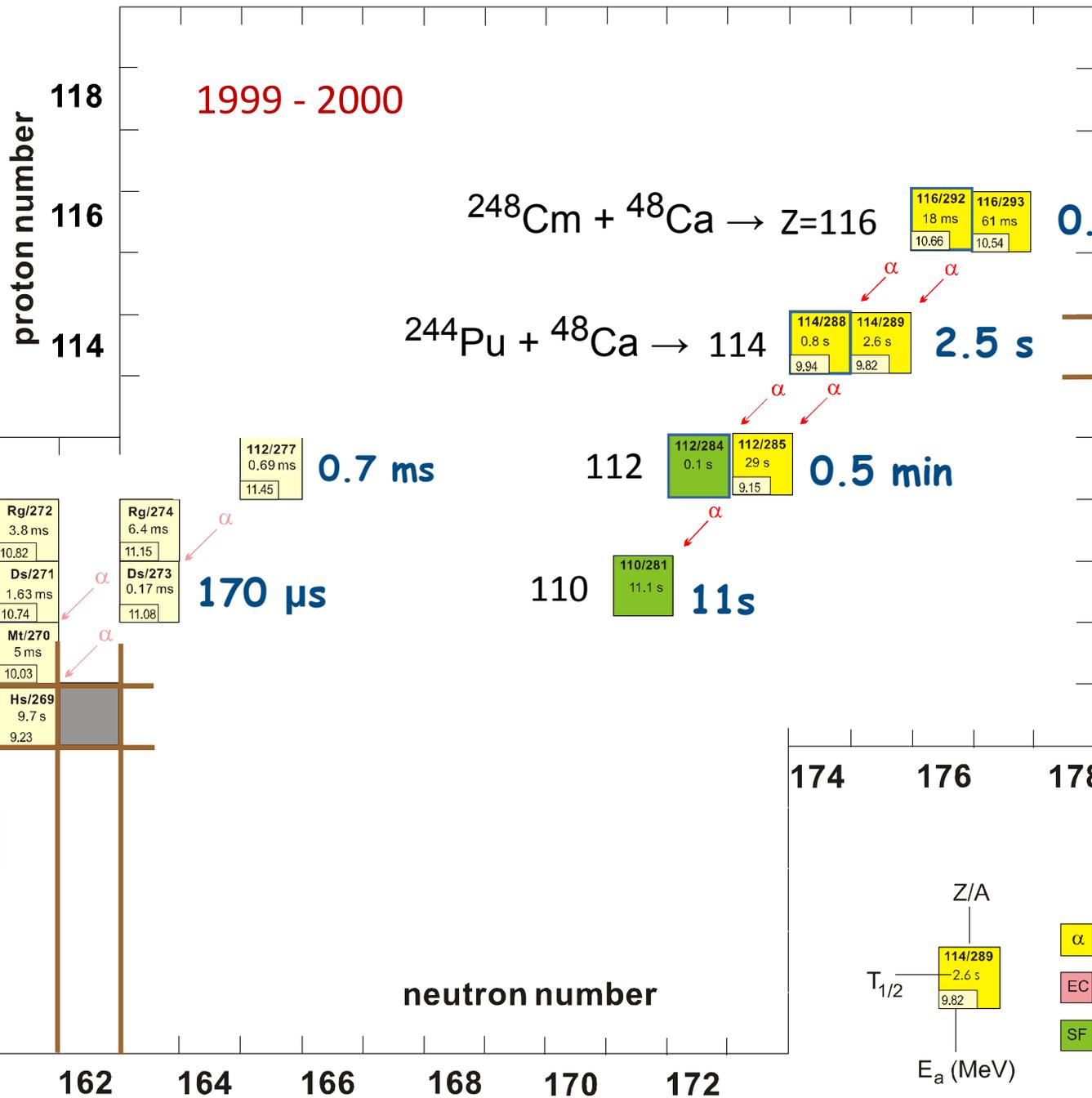
1985  $\Rightarrow$

1977-1979  $\Rightarrow$

1999  $\Rightarrow$

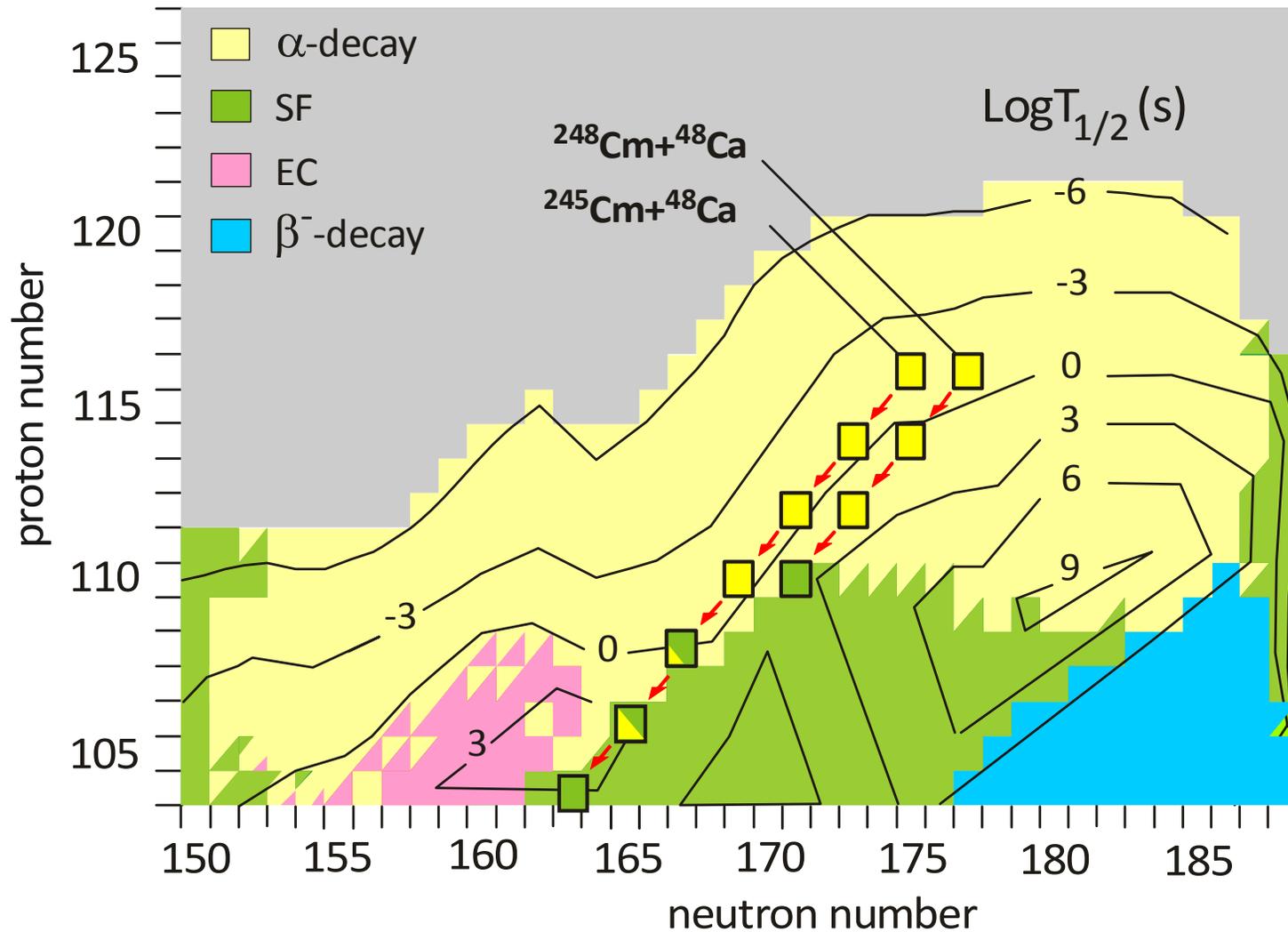


# Decay chains



184

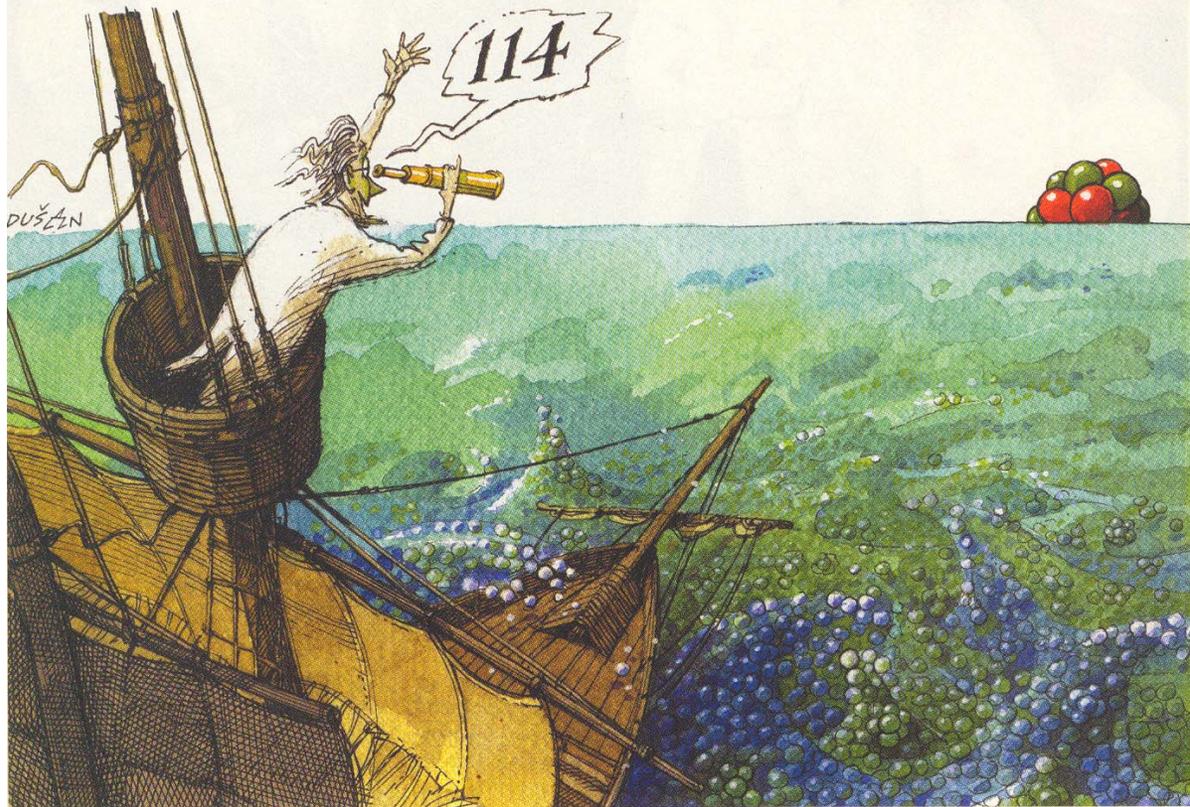
## Decay chains of the isotopes of Element 116: $^{291}\text{Lv}$ and $^{293}\text{Lv}$



an Article from | **SCIENTIFIC  
AMERICAN**

JANUARY 2000 VOL. 282 NO. 1

# Voyage to SUPERHEAVY Island

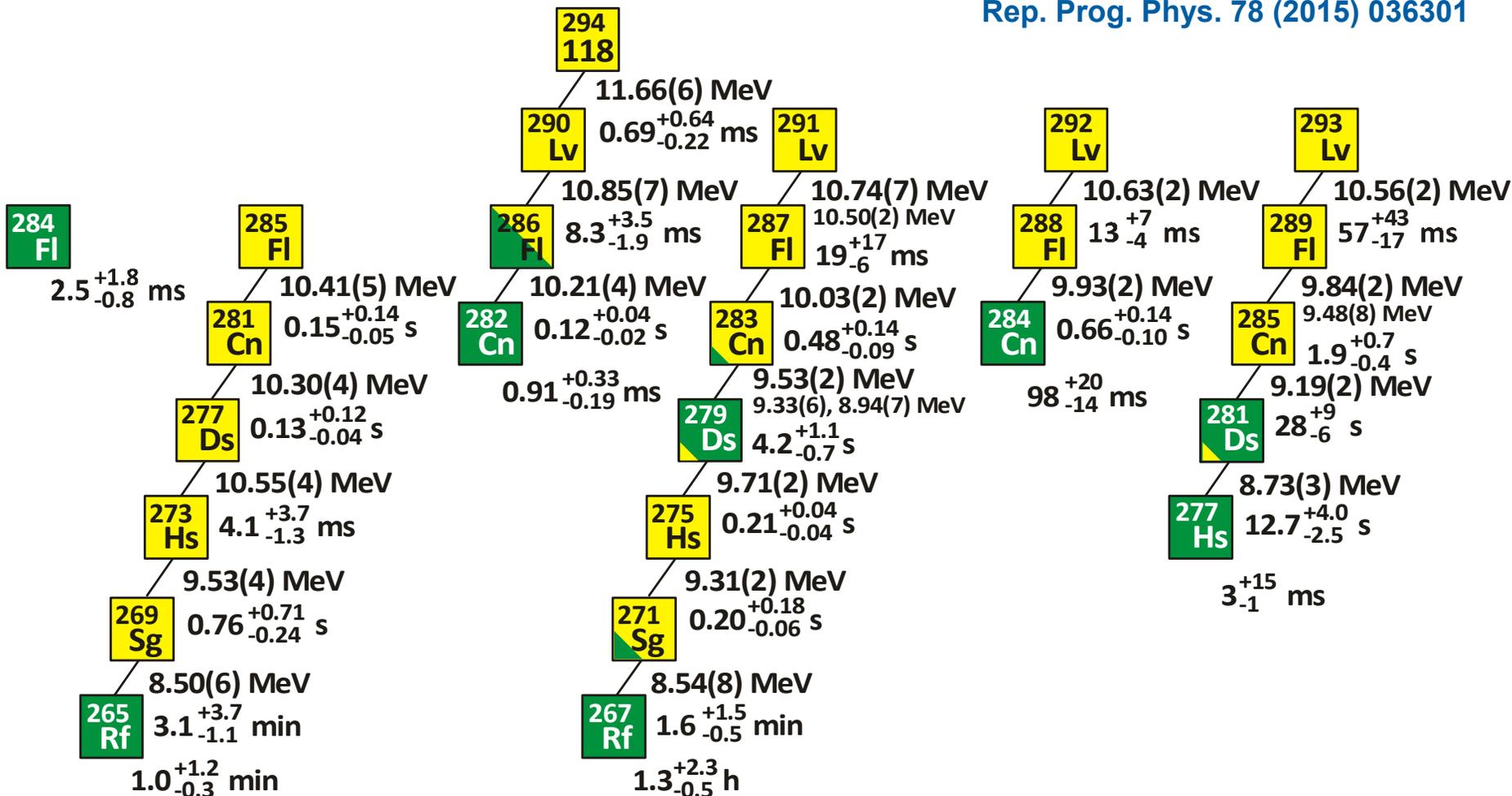


*The synthesis of element 114 confirmed decades-old theoretical predictions of a little patch of nuclear stability in a sea of short-lived superheavy nuclei*

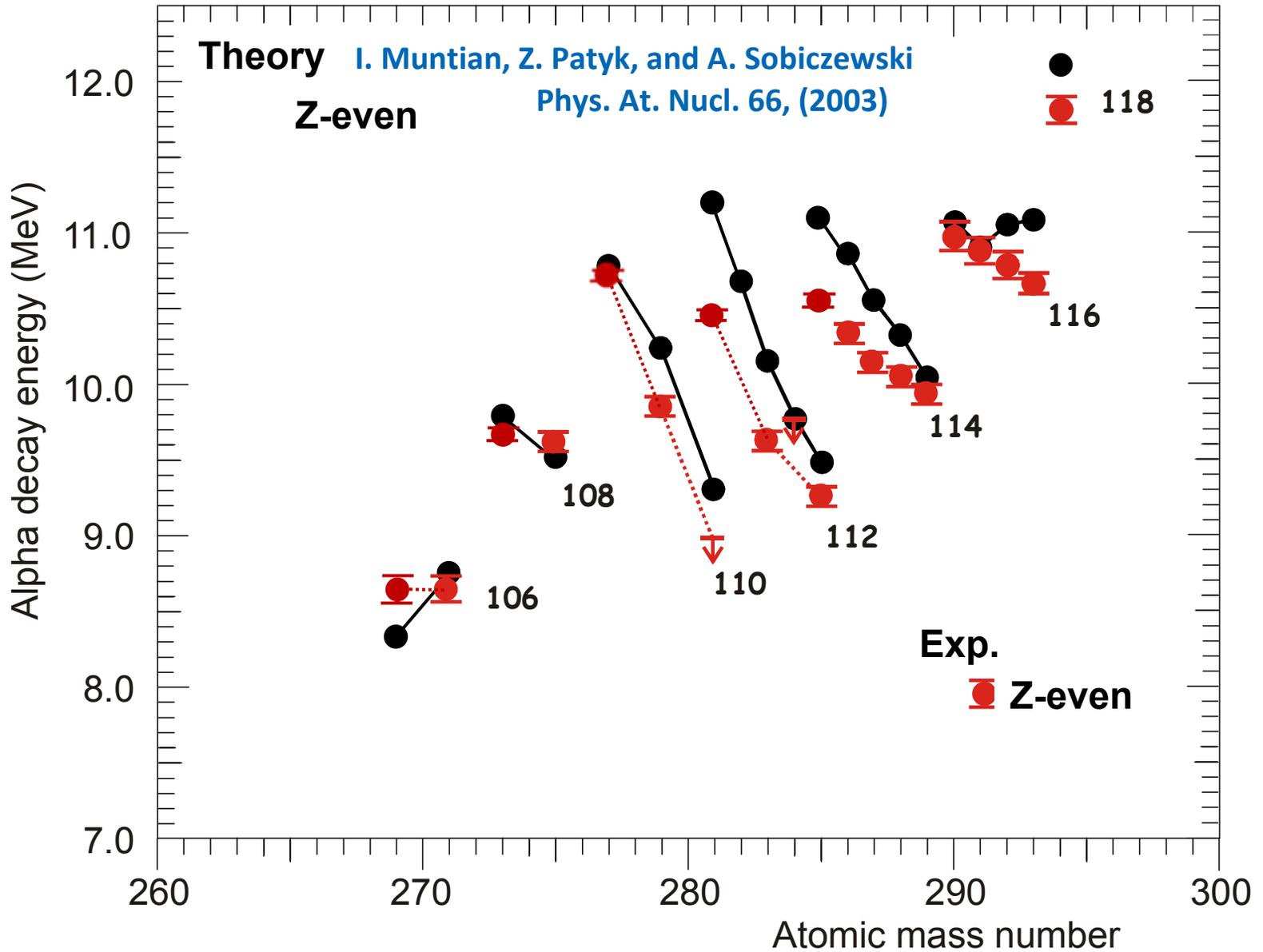
Summary decay properties of the isotopes of elements 112, 114, 116 and 118 observed in  $^{238}\text{U}$ ,  $^{240,242,244}\text{Pu}$ ,  $^{245,248}\text{Cm}$  and  $^{249}\text{Cf} + ^{48}\text{Ca}$  reactions

2015

Yu Ts Oganessian and V K Utyonkov,  
Rep. Prog. Phys. 78 (2015) 036301

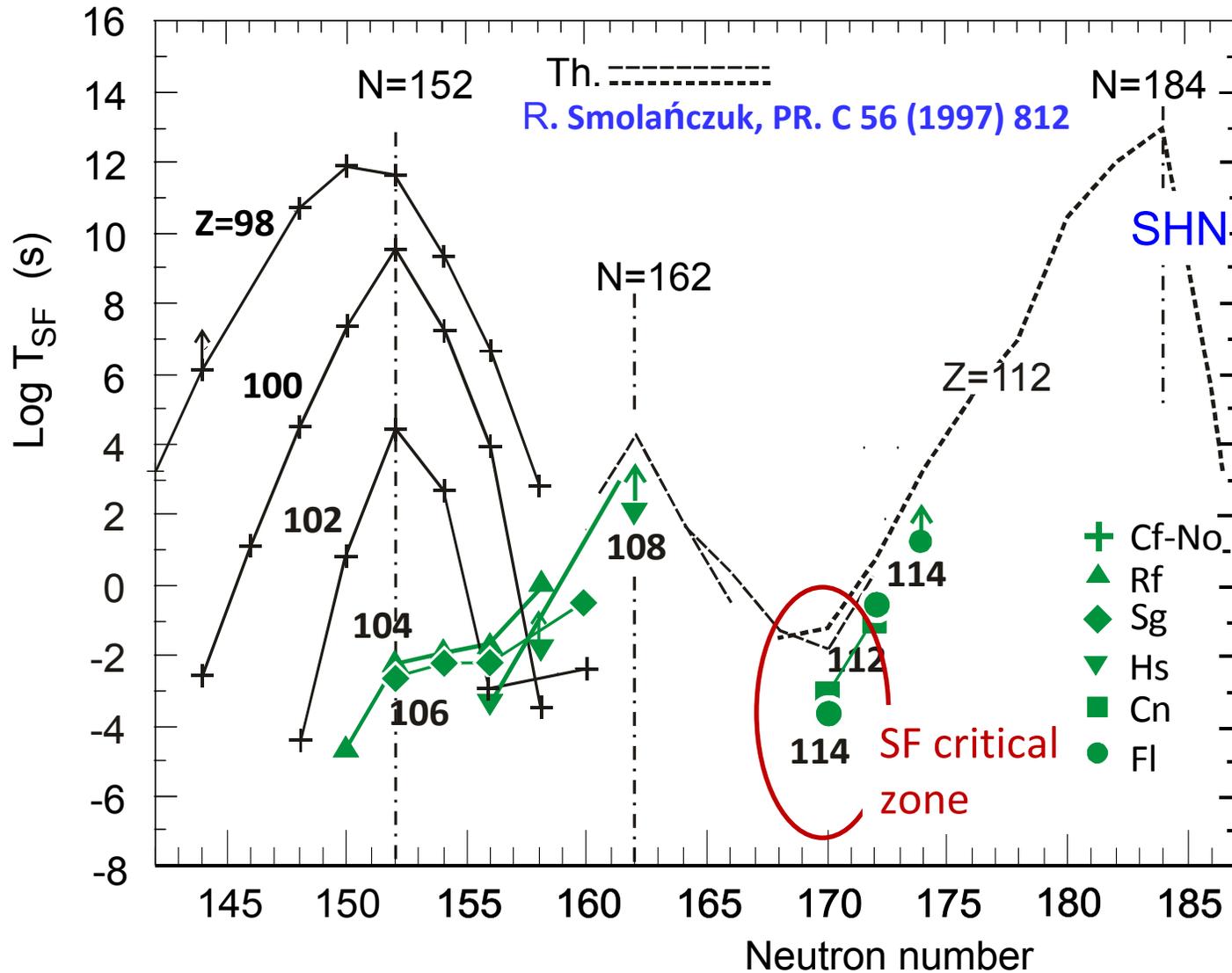


# Alpha - decay



# Spontaneous fission

even-even isotopes



# Cross sections

## hot fusion



$$E_x = 40 - 45 \text{ MeV}$$

$$x = 4 - 5$$

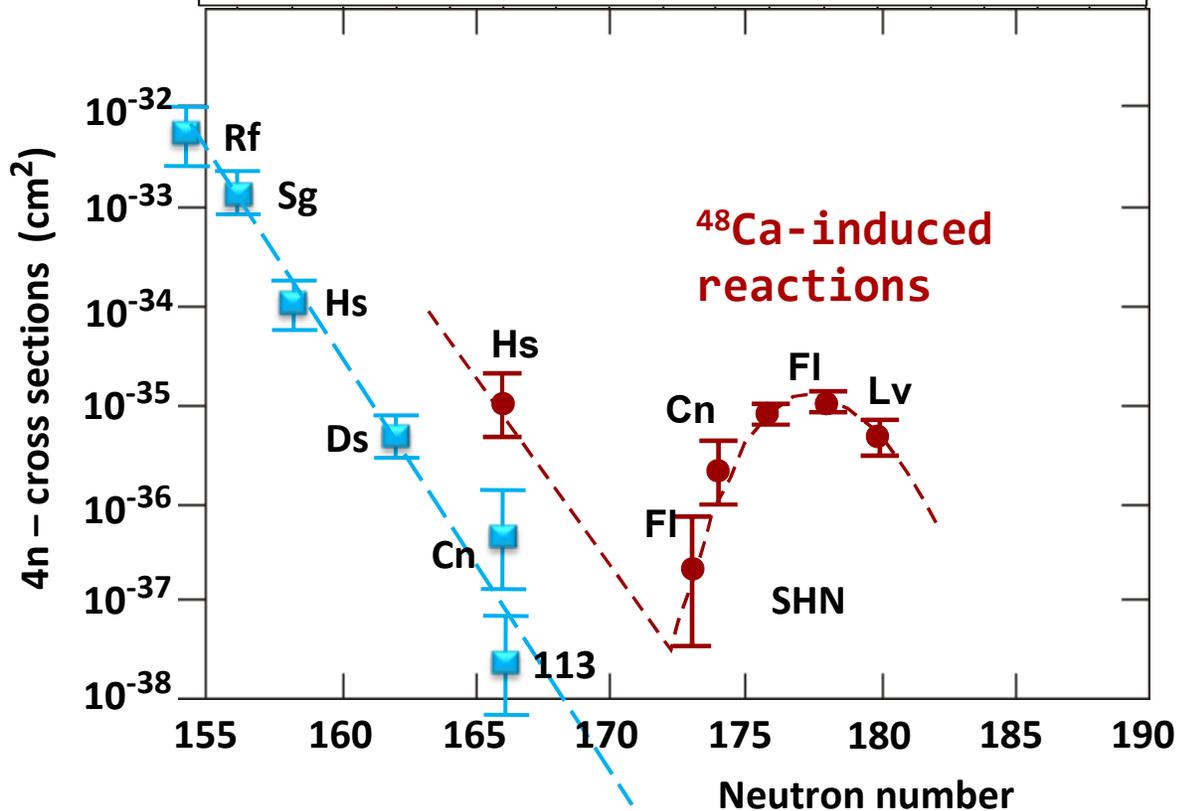
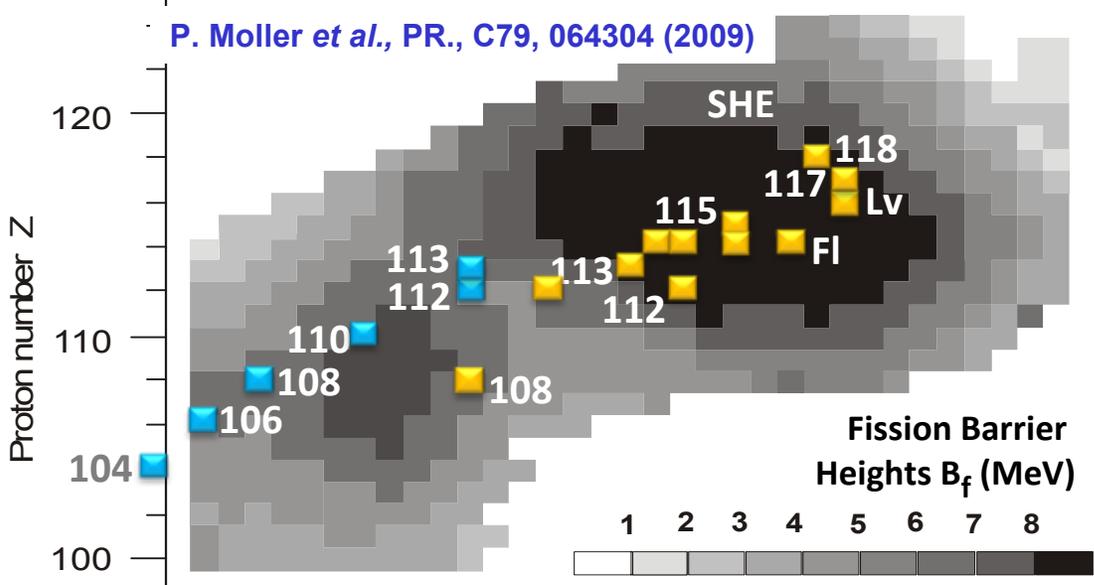
## cold fusion



$$E_x = 12 - 15 \text{ MeV}$$

$$x = 1$$

P. Moller et al., PR., C79, 064304 (2009)



# Odd-Z Superheavy Nuclei

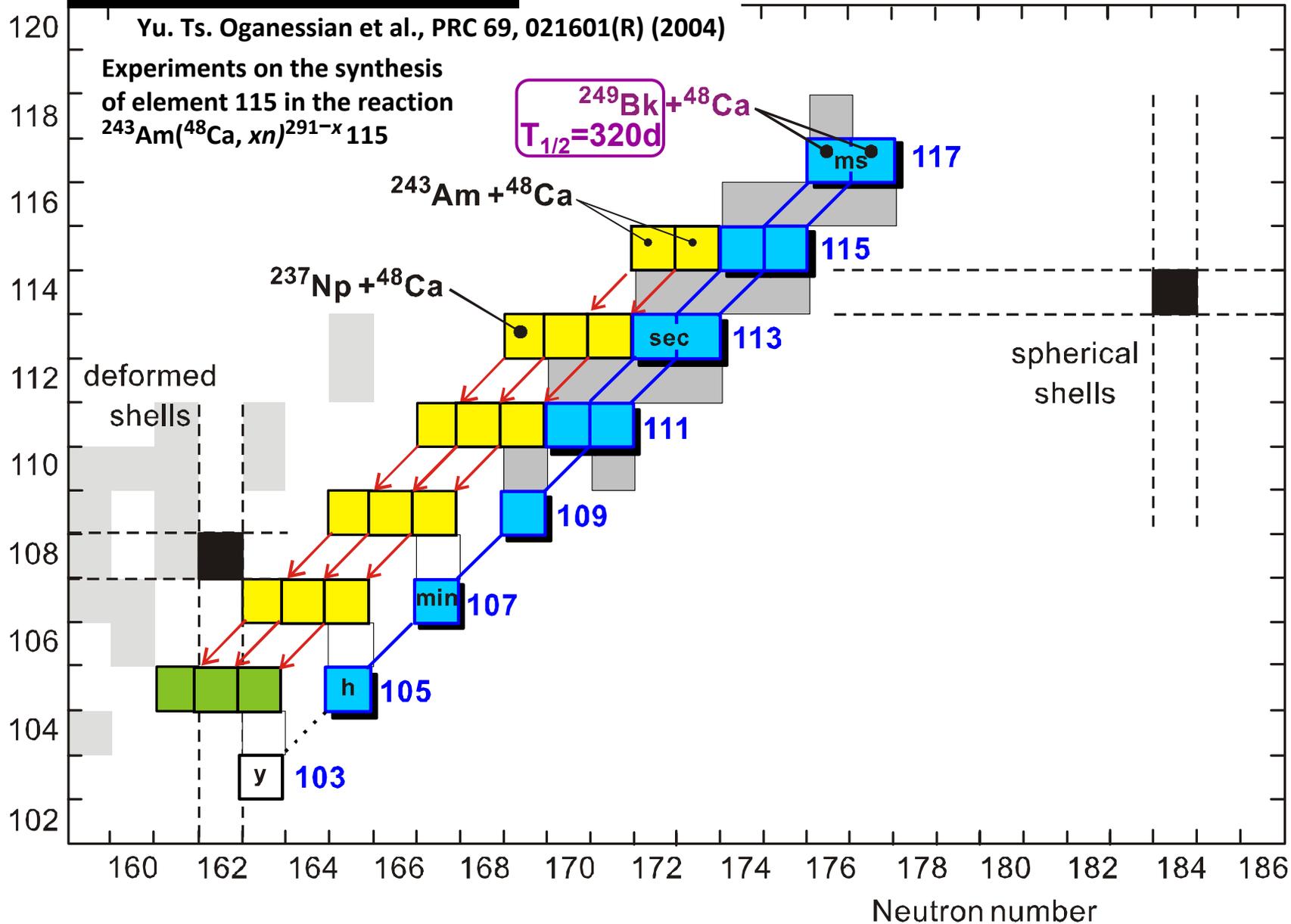
**Synthesis of the Isotopes  
with  $Z = 113, 115$  and  $117$**

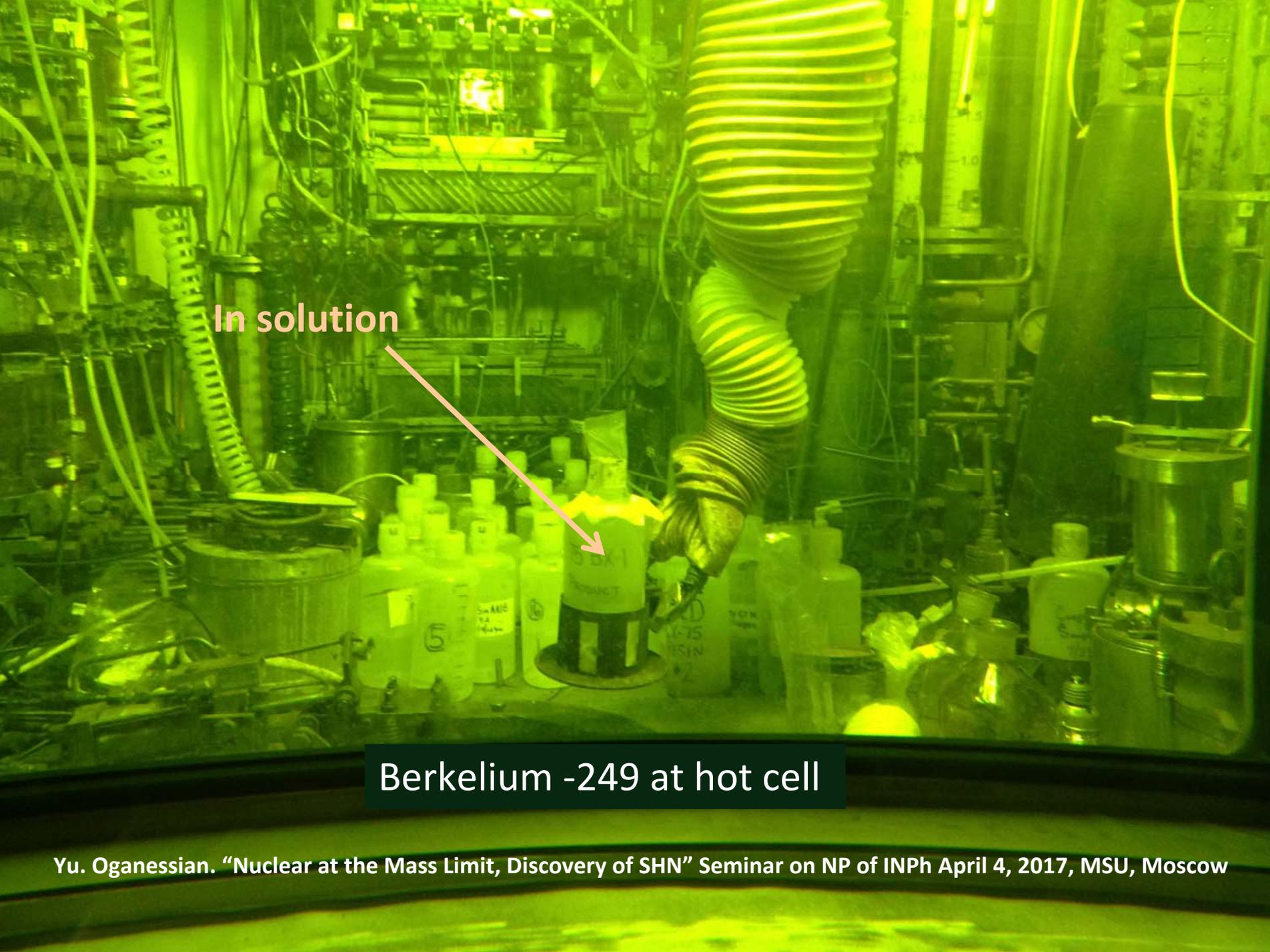
# RAPID COMMUNICATIONS

Yu. Ts. Oganessian et al., PRC 69, 021601(R) (2004)

Experiments on the synthesis  
of element 115 in the reaction  
 $^{243}\text{Am}(^{48}\text{Ca}, xn)^{291-x}\text{115}$

Proton number

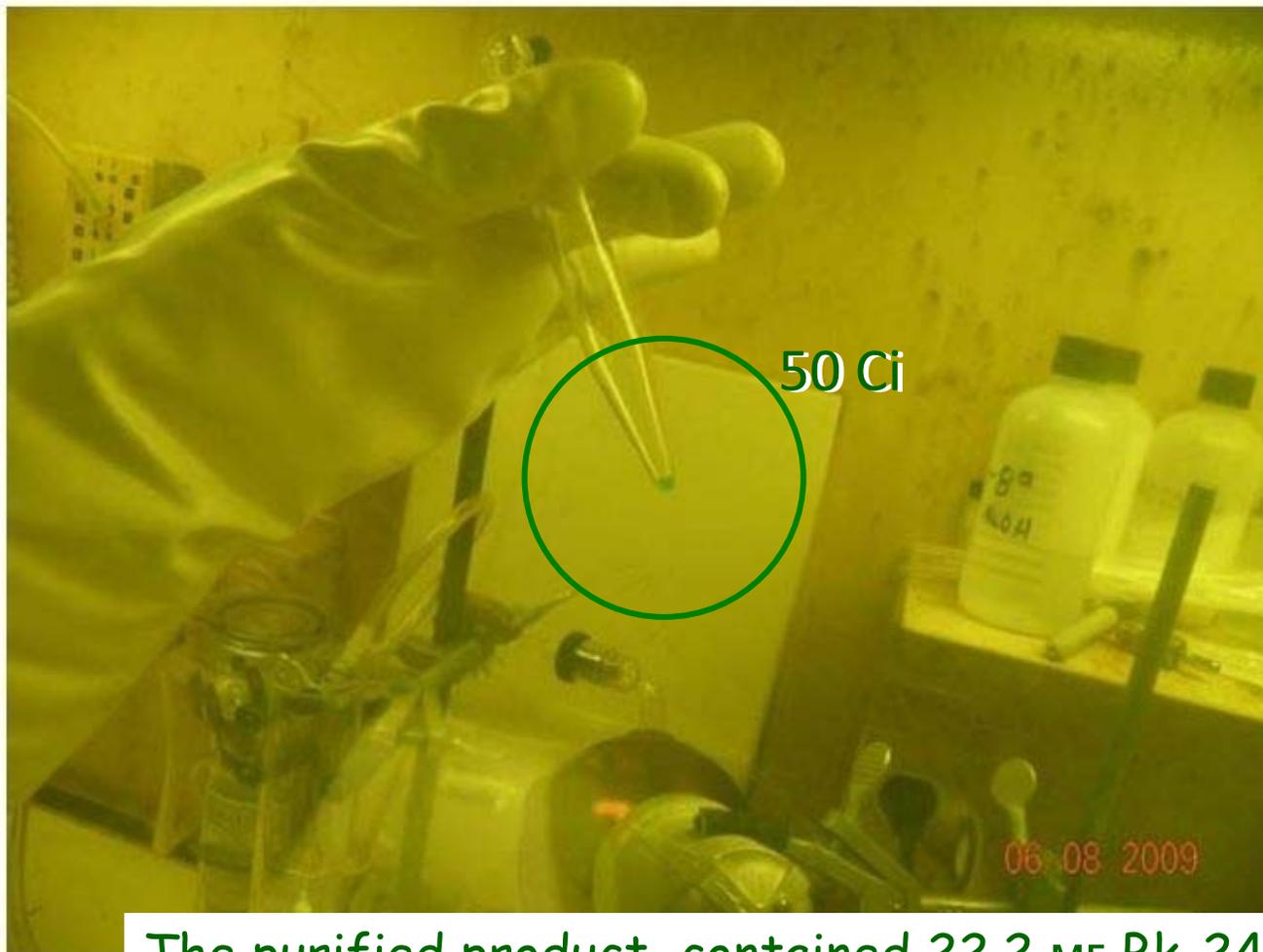




In solution

Berkelium -249 at hot cell

Yu. Oganessian. "Nuclear at the Mass Limit, Discovery of SHN" Seminar on NP of INPh April 4, 2017, MSU, Moscow



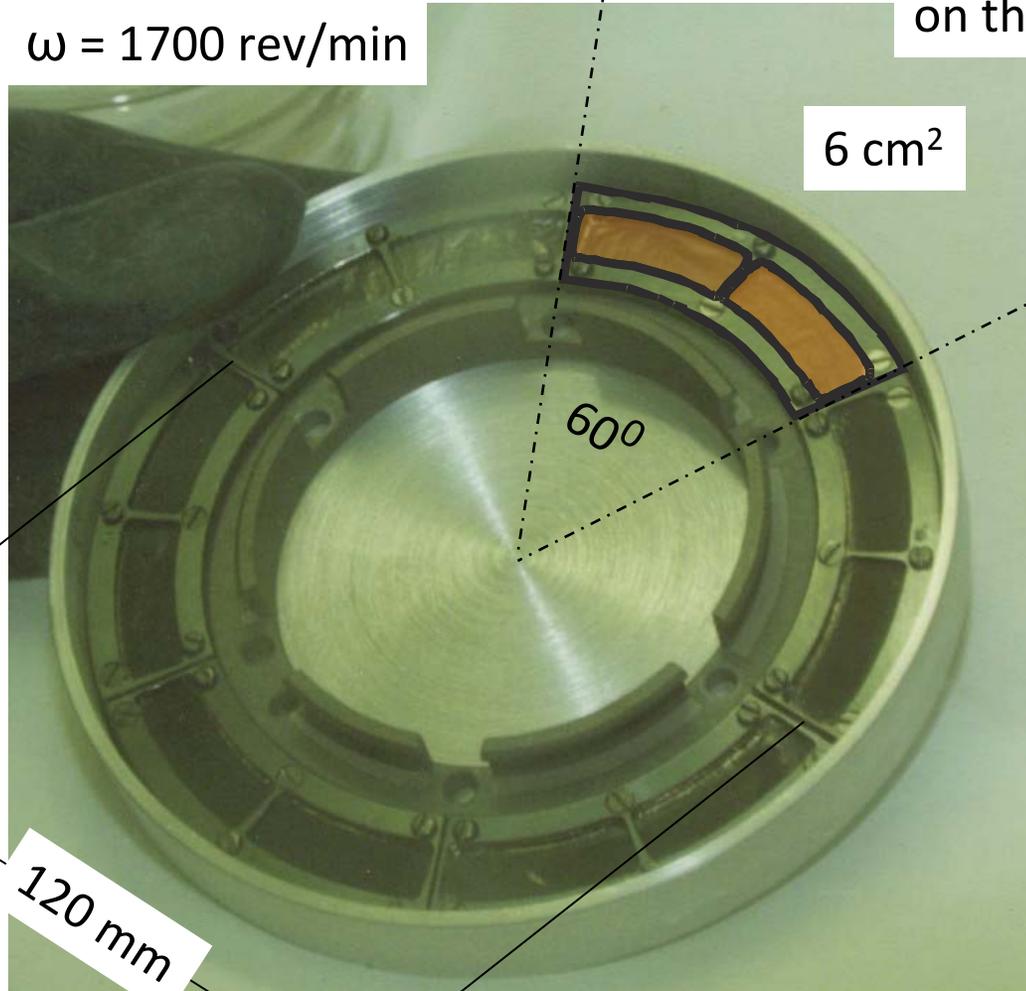
The purified product contained 22.2 mg Bk-249

$\text{Bk}(\text{NO}_3)_3$  Product

# Target

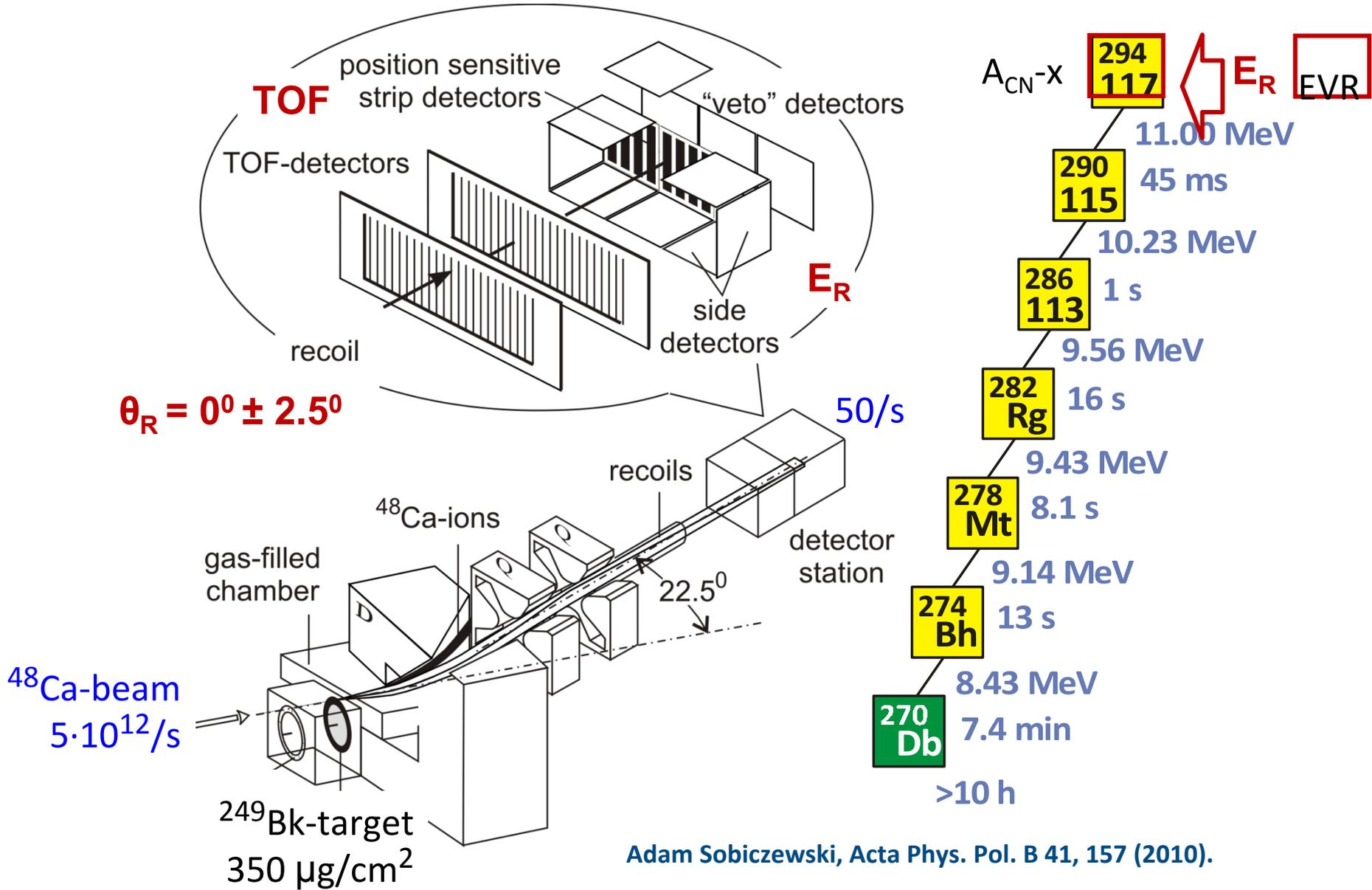
$\omega = 1700 \text{ rev/min}$

$350 \text{ mg/cm}^2$  deposited  
on the  $1.5 \mu\text{m-Ti}$  foil

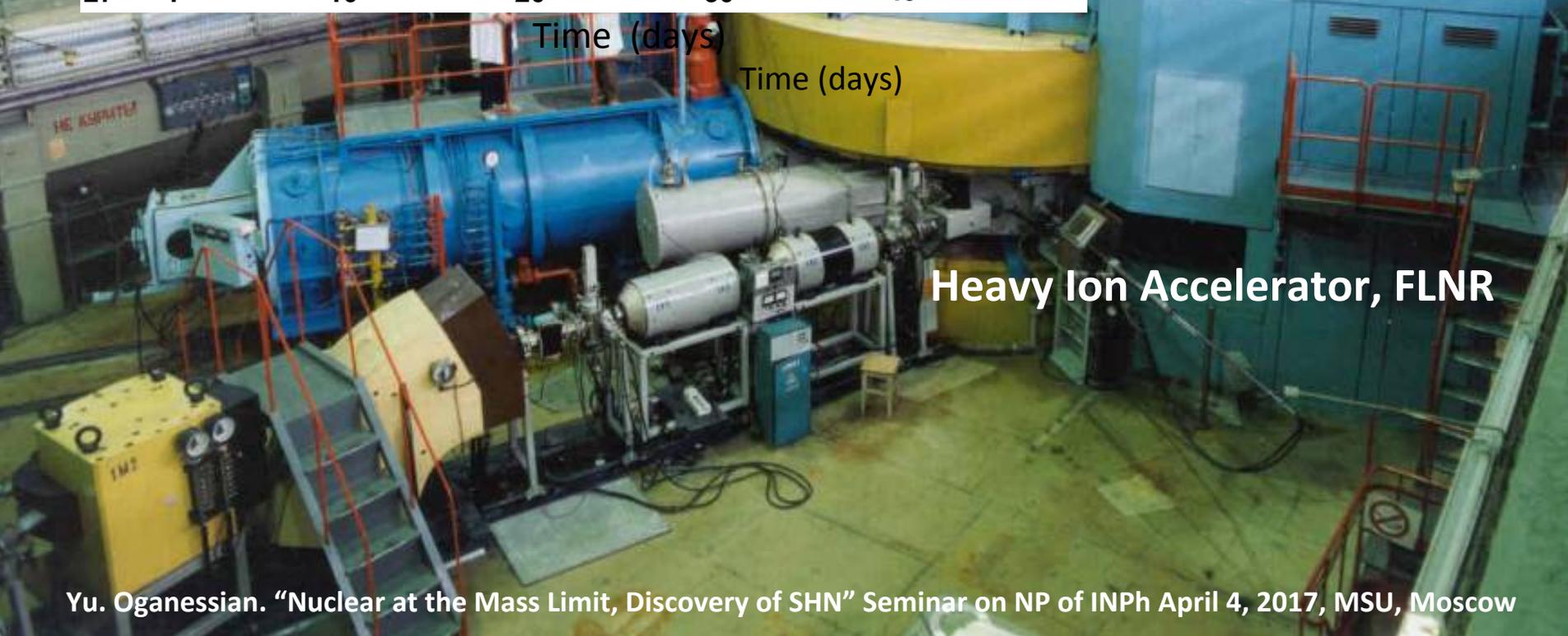
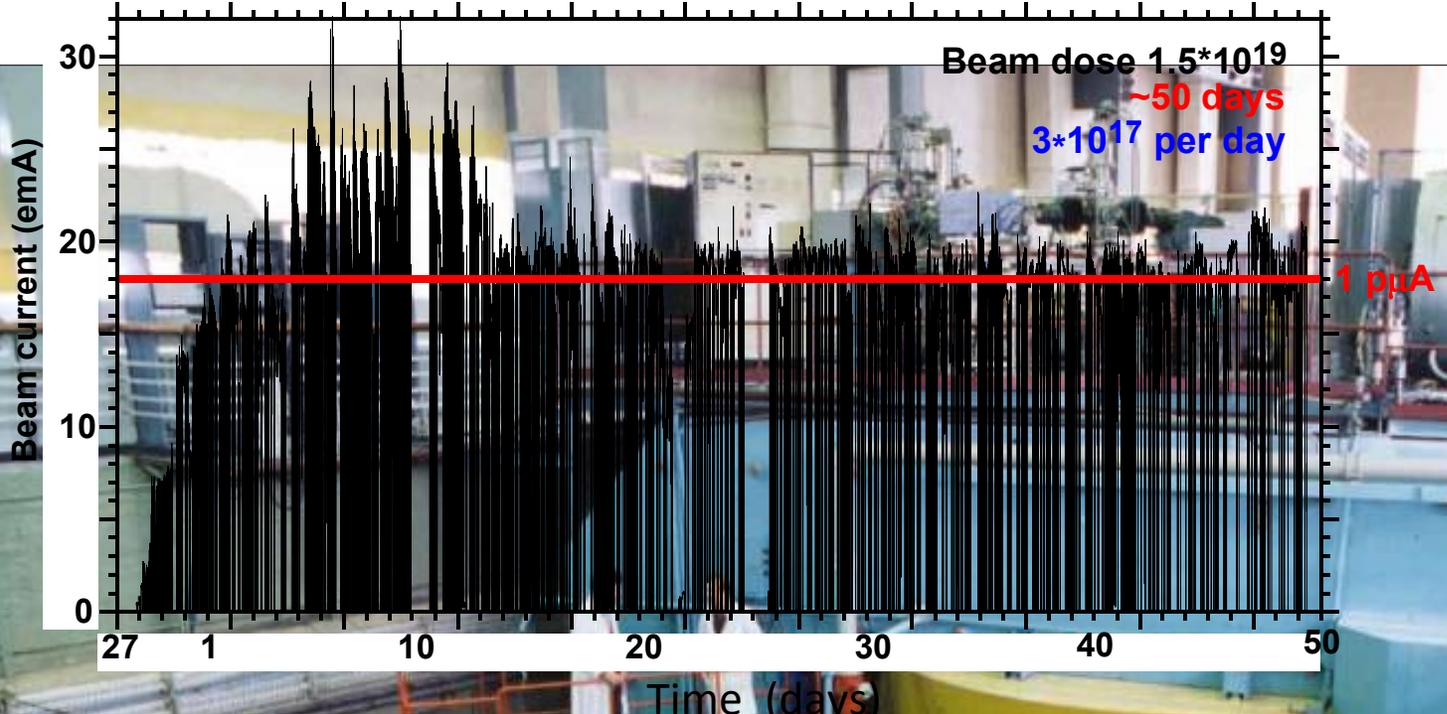


Yu. Oganessian 2010

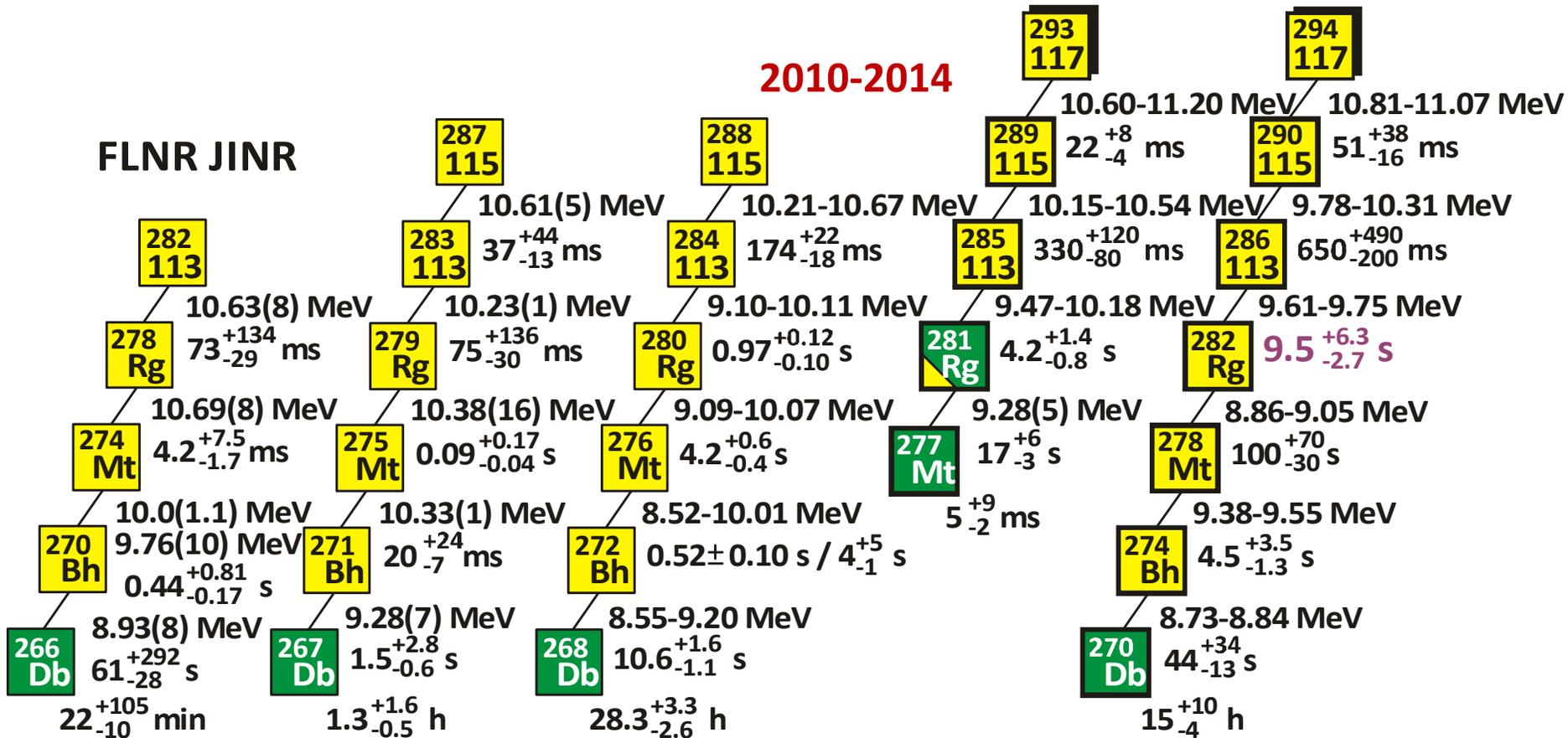
# Dubna Gas-Filled Recoil Separator DGFRS



Adam Sobiczewski, Acta Phys. Pol. B 41, 157 (2010).



Summary decay properties of the isotopes of elements 113, 115, and 117 observed in  $^{237}\text{Np}$ ,  $^{243}\text{Am}$  and  $^{249}\text{Bk} + ^{48}\text{Ca}$  reactions



## Confirmations of DGFRS data **2007 - 2014**

<b>A/Z</b>	<b>Setup</b>	<b>Laboratory</b>	<b>Publications</b>
<b><math>^{283}_{112}</math></b>	<b>SHIP</b>	<b>GSI Darmstadt</b>	<b>Eur. Phys. J. A32, 251 (2007)</b>
<b><math>^{283}_{112}</math></b>	<b>COLD</b>	<b>PSI-FLNR (JINR)</b>	<b>NATURE 447, 72 (2007)</b>
<b><math>^{286, 287}_{114}</math></b>	<b>BGS</b>	<b>LBNL (Berkeley)</b>	<b>P.R. Lett. 103, 132502 (2009)</b>
<b><math>^{288, 289}_{114}</math></b>	<b>TASCA</b>	<b>GSI – Mainz</b>	<b>P.R. Lett. 104, 252701 (2010)</b>
<b><math>^{292, 293}_{116}</math></b>	<b>SHIP</b>	<b>GSI Darmstadt</b>	<b>Eur. Phys. J. A48, 62 (2012)</b>
<b><math>^{287, 288}_{115}</math></b>	<b>TASCA</b>	<b>GSI – Mainz</b>	<b>P.R. Lett. 111, 112502 (2013)</b>
<b><math>^{293, 294}_{117}</math></b>	<b>TASCA</b>	<b>GSI – Mainz</b>	<b>P.R. Lett. 112, 172501 (2014)</b>
<b><math>^{292, 293}_{116}</math></b>	<b>GARIS</b>	<b>RIKEN Tokyo</b>	<b>Accelerator Progress Rep. (2013)</b>

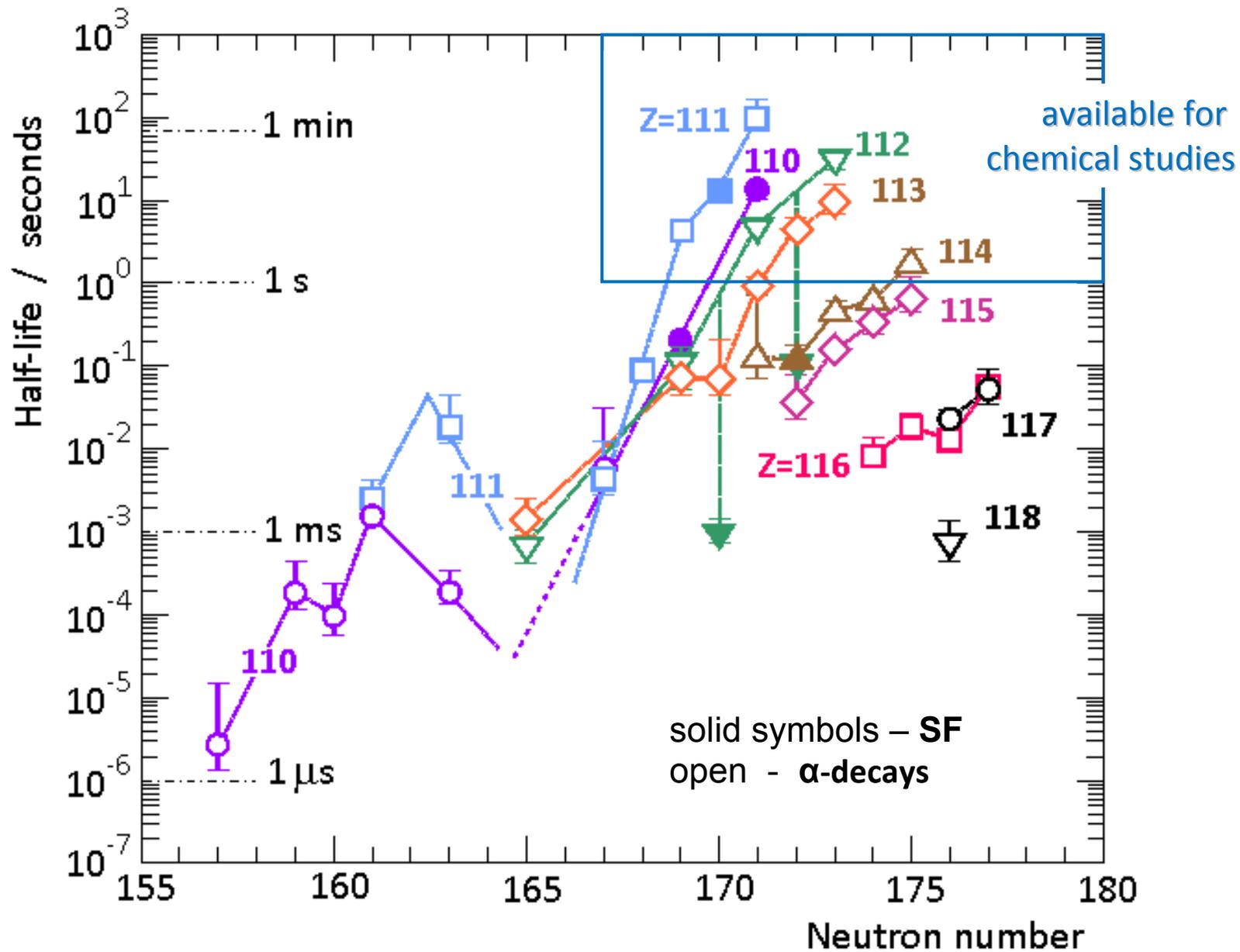
With  $Z > 40\%$  larger than that of Bi, the heaviest stable element we see an impressive extension in nuclear survivability.

Although SHN are at the limits of Coulomb stability,

- shell stabilization lowers ground-state energy,
- creates a fission barrier,
- and thereby enables SHN to exist.

.

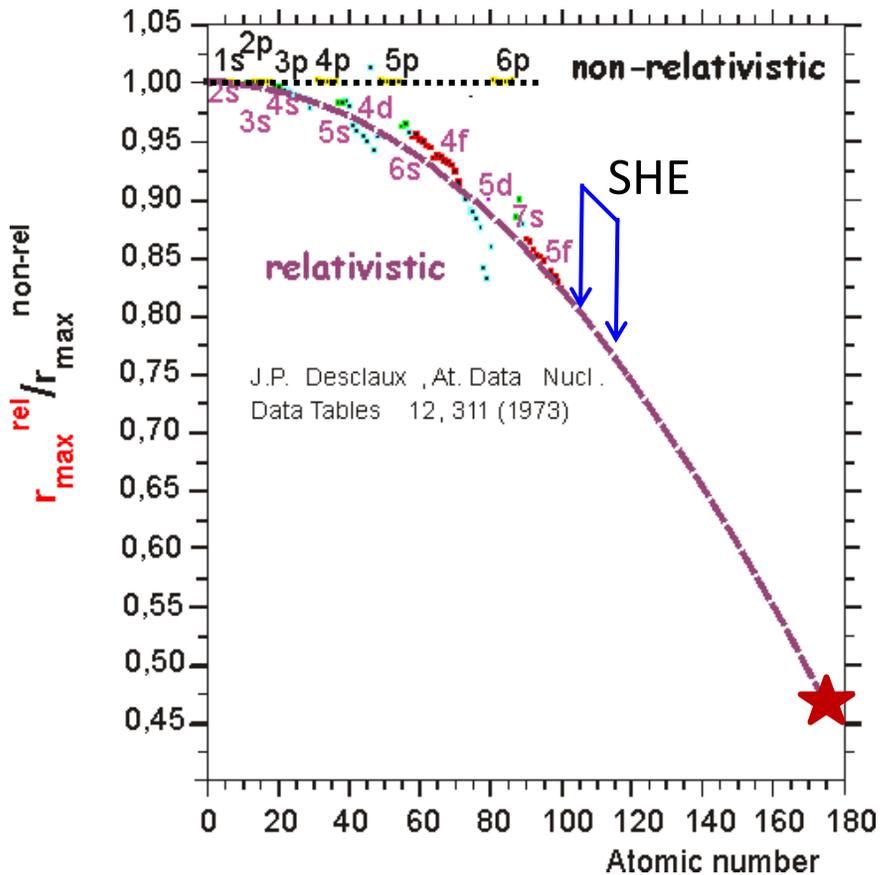
The fundamentals of the modern theory concerning the mass limits of nuclear matter have obtained experimental verification



# Super Heavy Atoms

## Chemistry of the SHE

## Table of Elements от Z=1 до Z=172



Relativistic Contraction

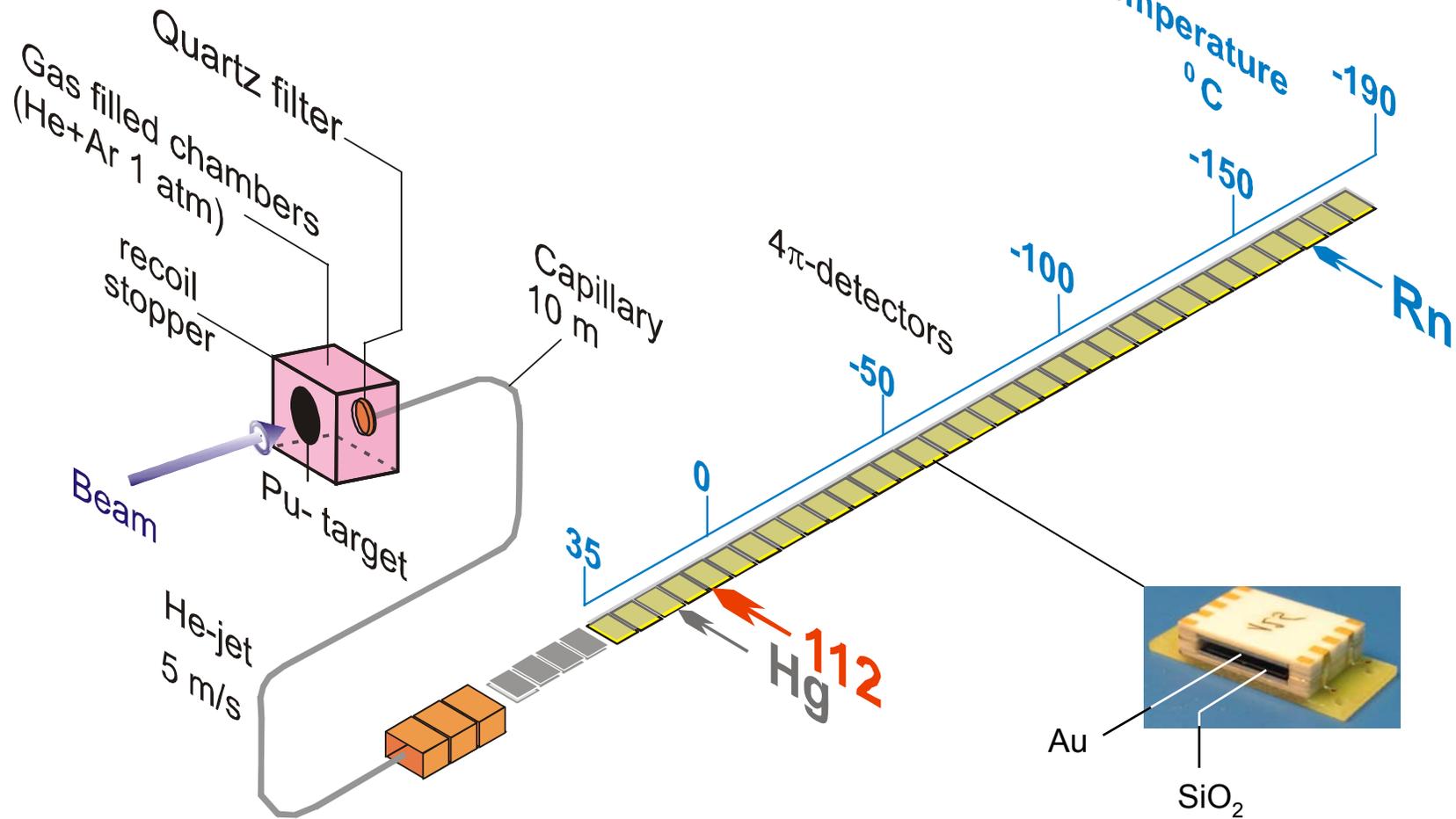
Period	Orbitals																18						
1	1	2															17	18	1s				
2	3	4															13	14	15	16	17	18	2s2p
3	11	12	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	3s3p				
4	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	4s3d4p				
5	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	5s4d5p				
6	55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	6s5d6p				
7	87	88	89-103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	7s6d7p				
8	119	120	121	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	8s7d8p		
9	165	166															167	168	9s9p	9s9p			

6	LANTANIDES															4f			
7	ACTINIDES															5f			
8	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	6f			
8	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	5g

Calculated by P. Dirac and V. Fock  
in non-relativistic approximation

Reaction:

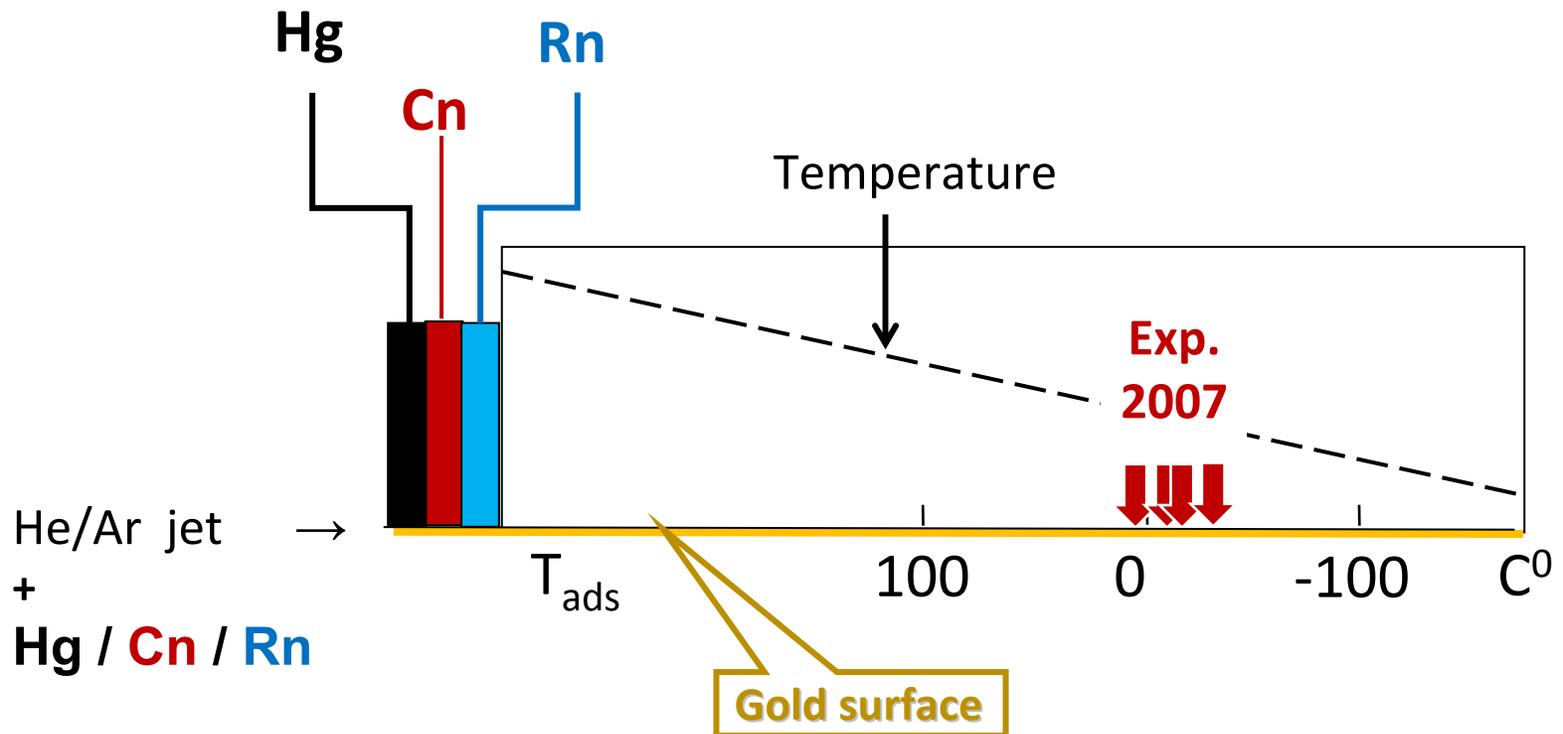
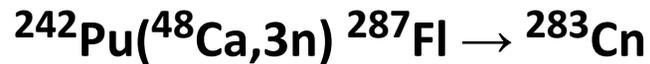


# Relativistic Effect in Chemistry of the Element 112

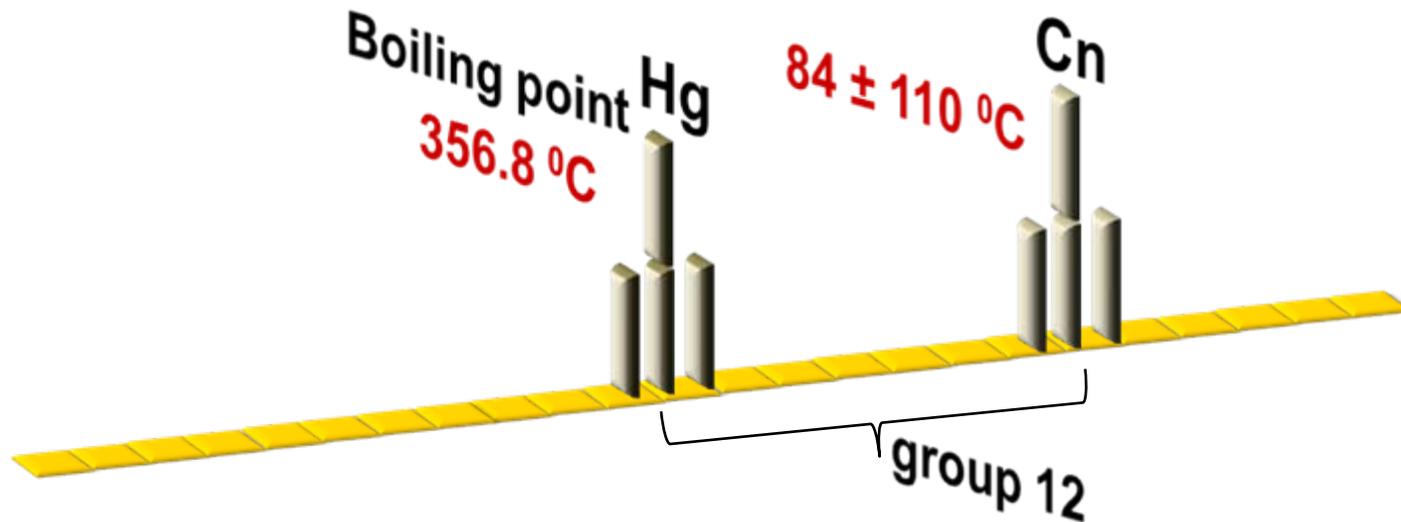
FLNR, JINR 2007

Adsorption of Hg and Cn (Z=112) on the Gold Surface

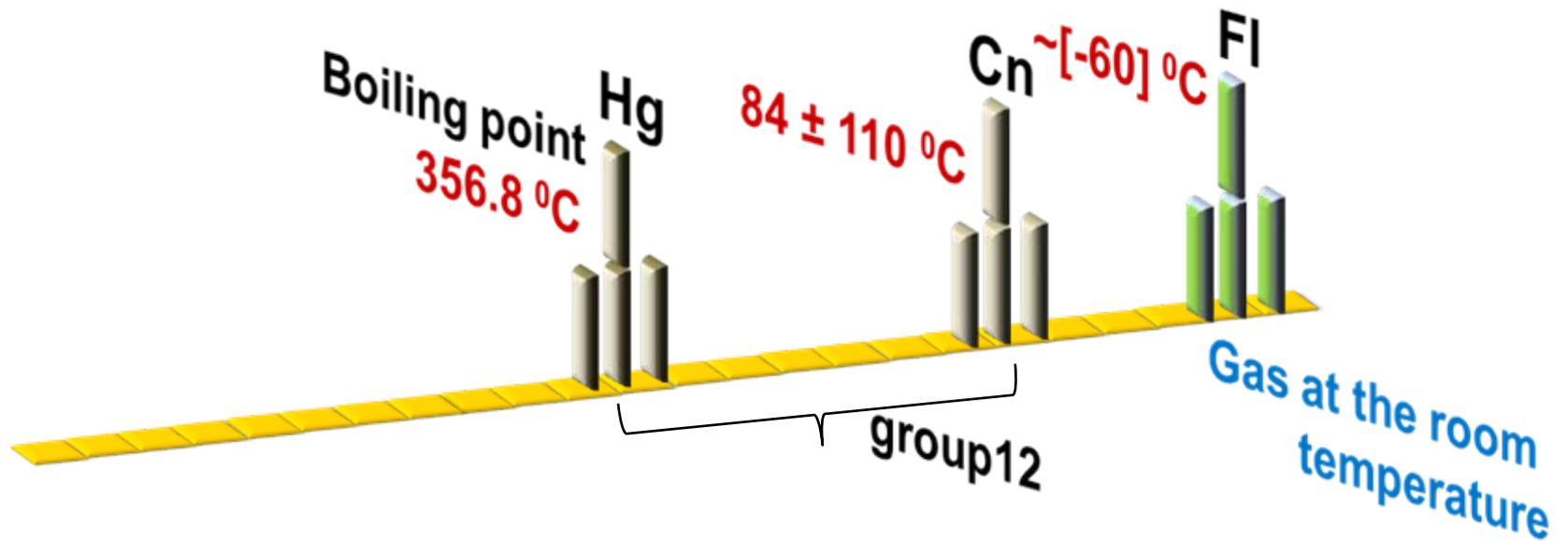
V. Pershina, 2006 (theory)



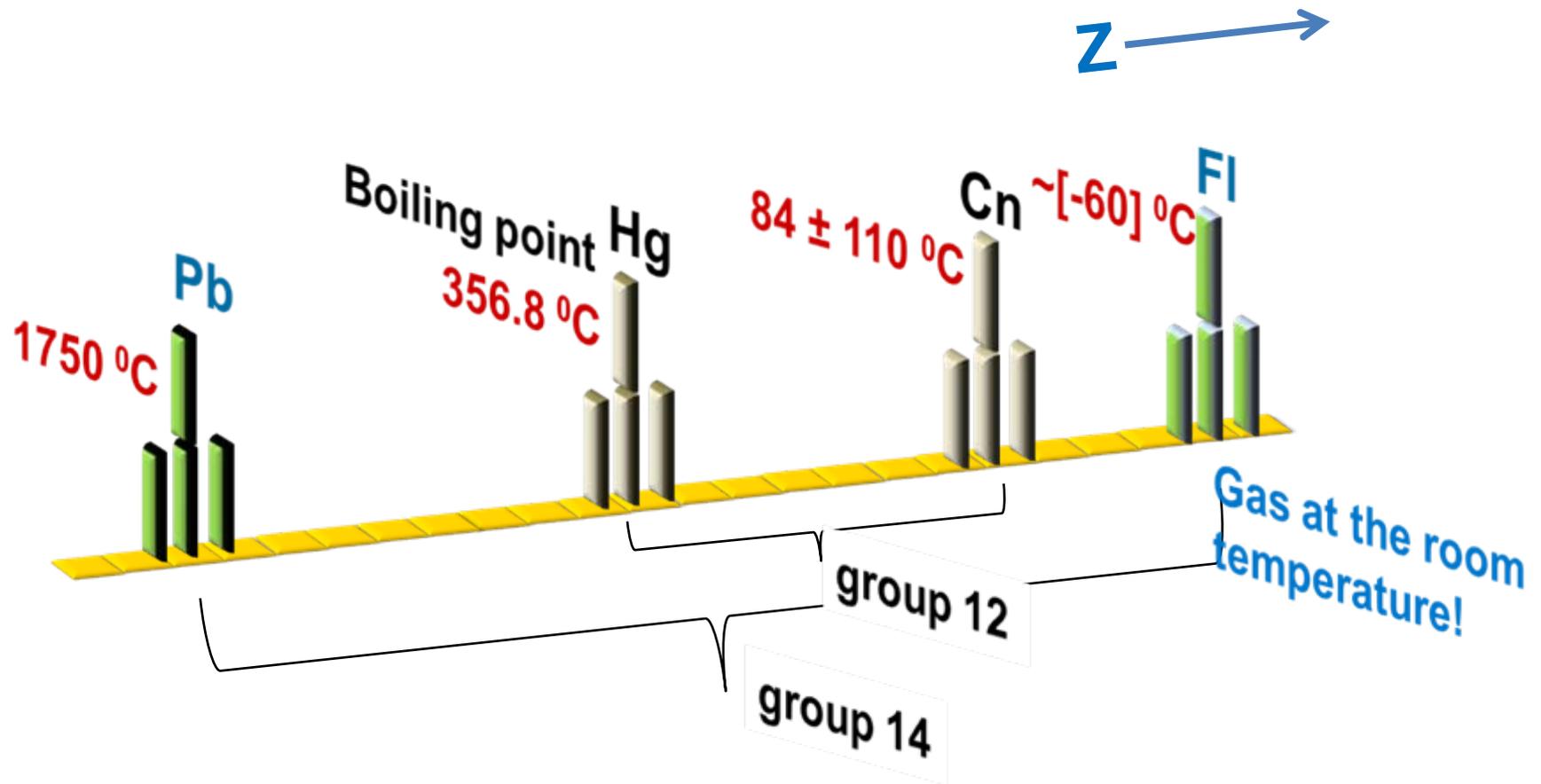
# Thermo chromatography of the SHE and its light homologies on the gold surface



# Thermo chromatography of the SHE and its light homologues on the gold surface



# Thermo chromatography of the SHE and its light homologies on the gold surface



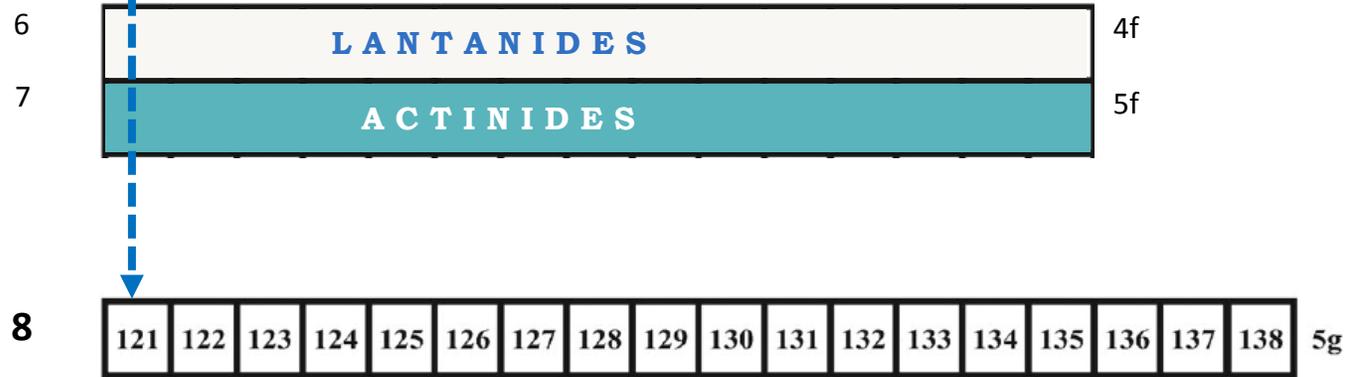
**Periodic Table Z=1-138**

Calculated in non-relativistic approximation of Dirac - Fok

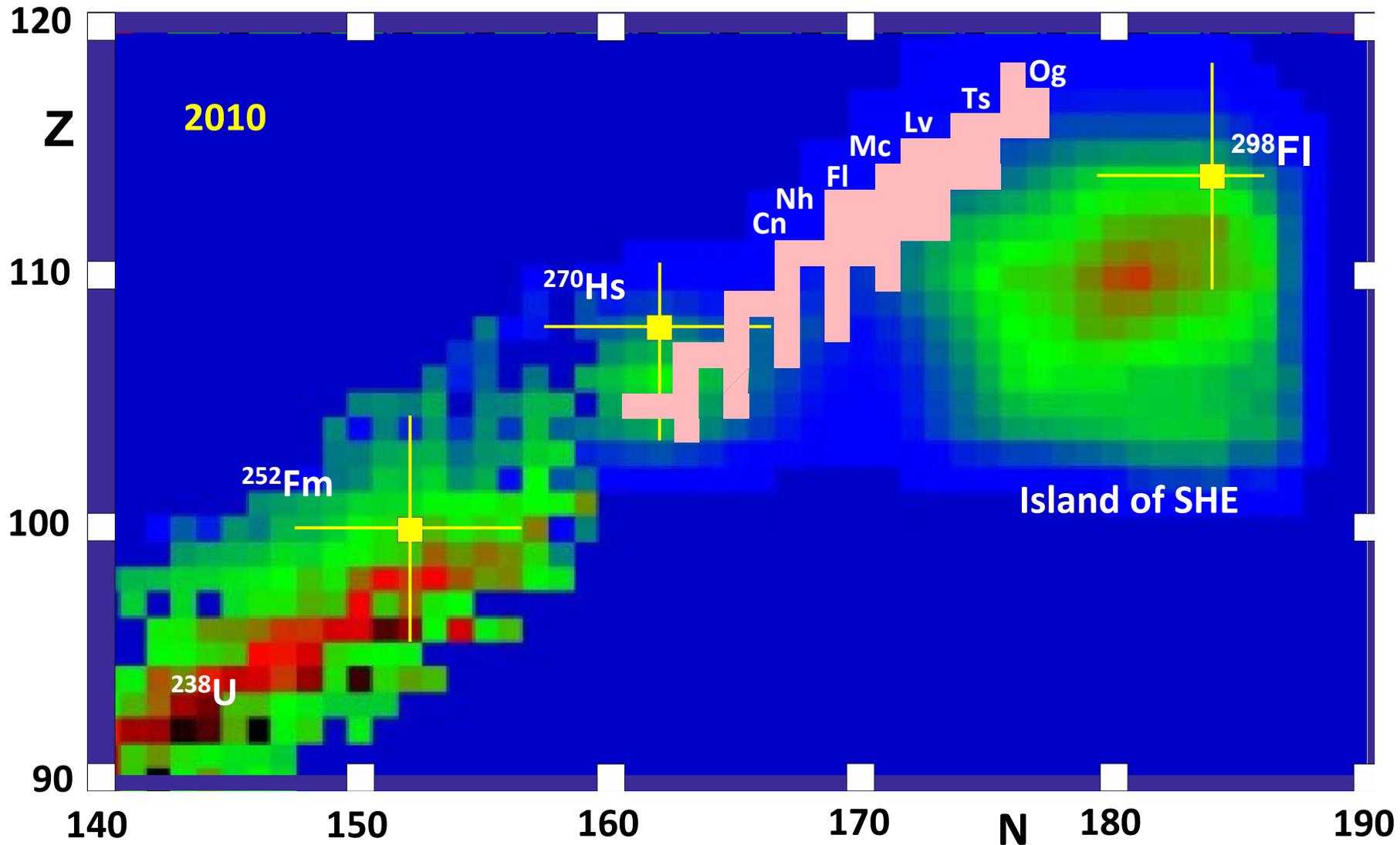
Period 1	1	2											13	14	15	16	17	18	Orbitals		
1	1 H	2 He																	1s s		
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	2s2p		
3	11 Na	12 Mg	3	4	5	6	7	8	9	10	11	12	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	3s3p		
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	4s3d4p		
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	5s4d5p		
6	55 Cs	56 Ba	57-71 Lanthanides	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn	6s5d6p		
7	87 Fr	88 Ra	89-103 Actinides	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og	7s6d7p		
8	119	120	121-																		

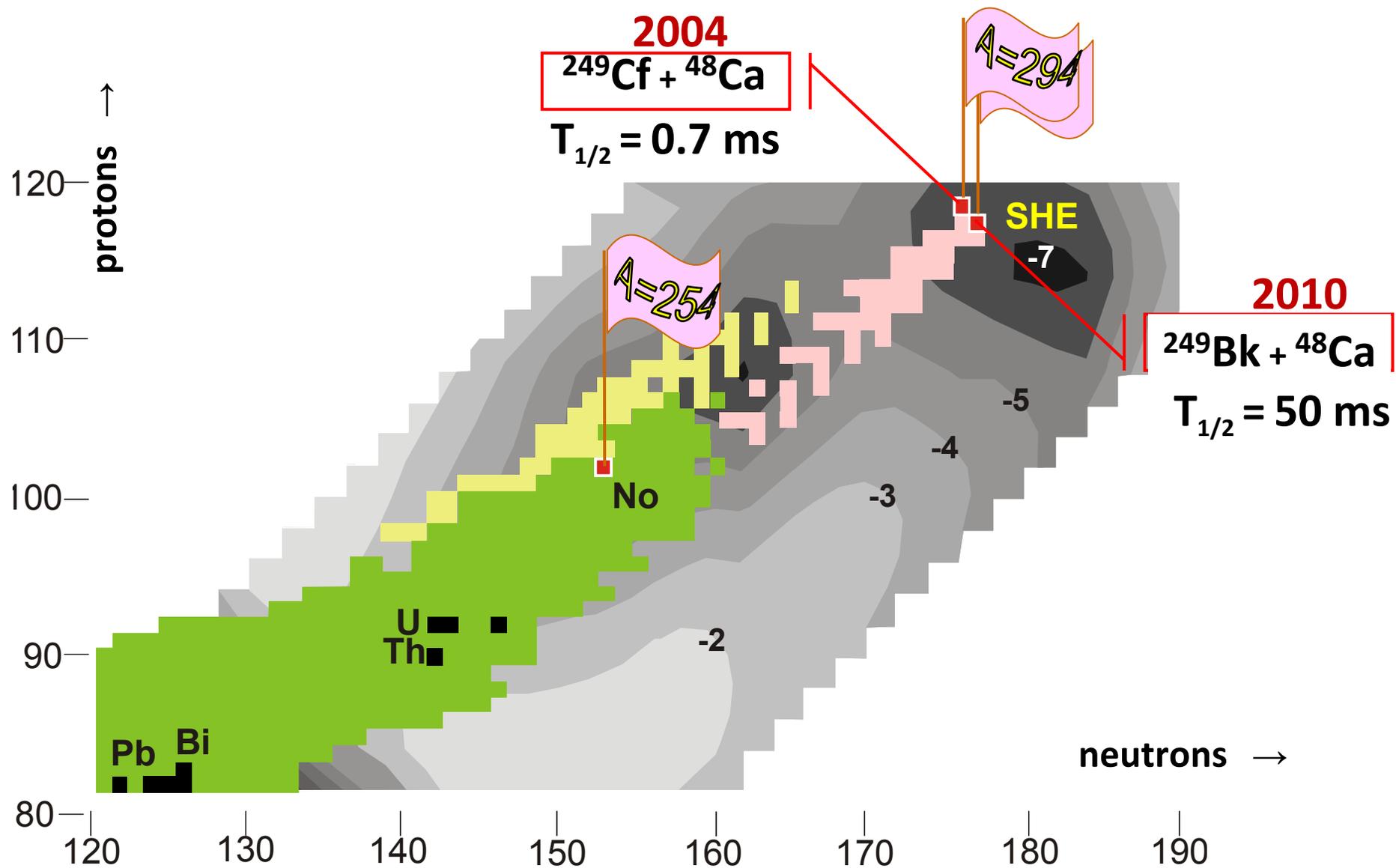
Completion of the 7-th row

Is Element 118 – noble gas?

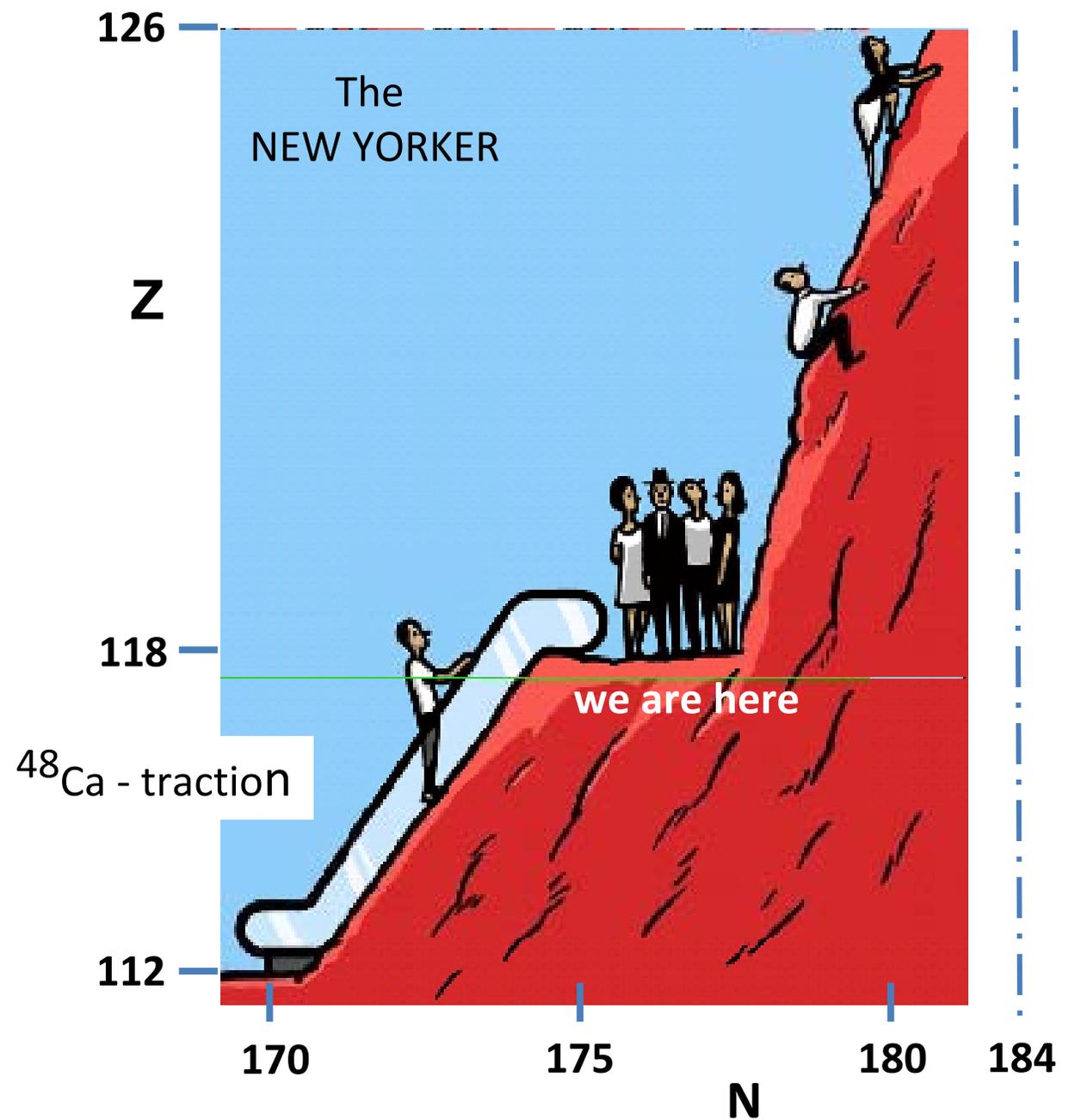


52 neutron-rich isotopes of the 15 heaviest elements  
were synthesized in  $^{48}\text{Ca}$ -induced reactions

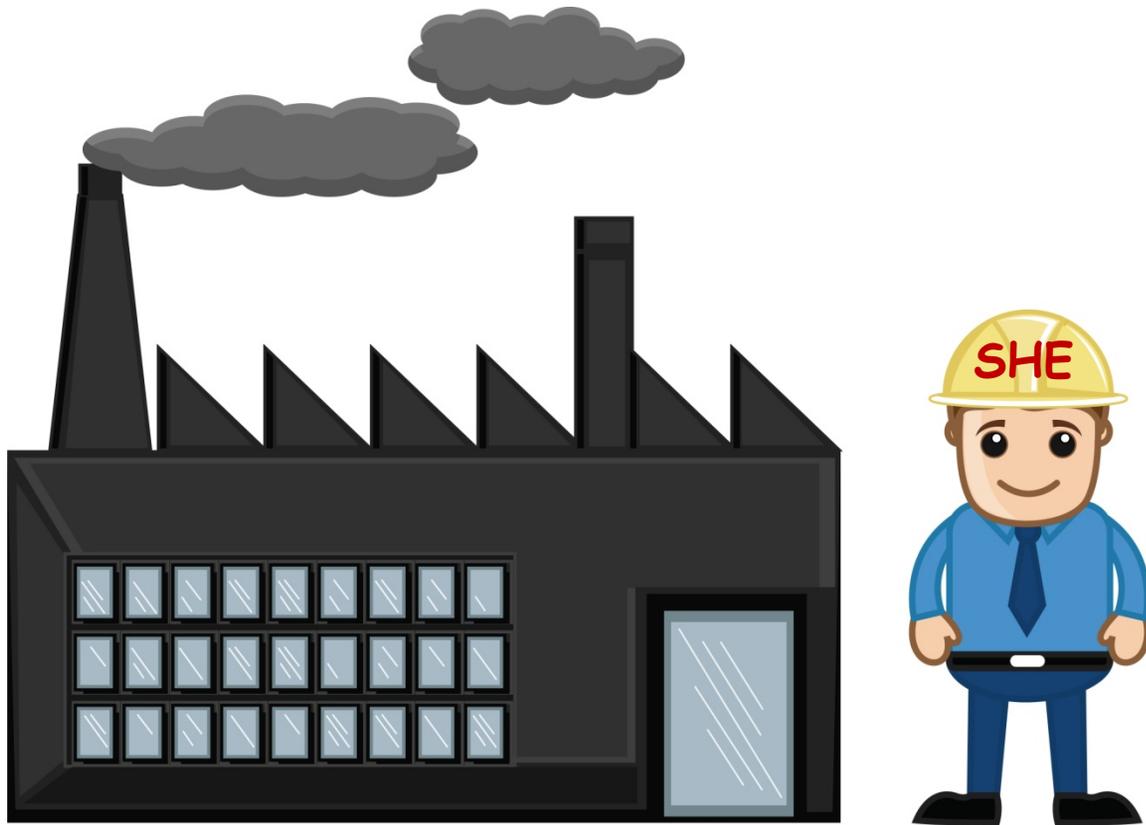




Yuri Oganessian. Discovery of SHN, Seminar of RIKEN, March 13, 2017, Tokyo, Japan



# Outcomes and the future



SHE-Factory

# SHE-Factory

joining of efforts

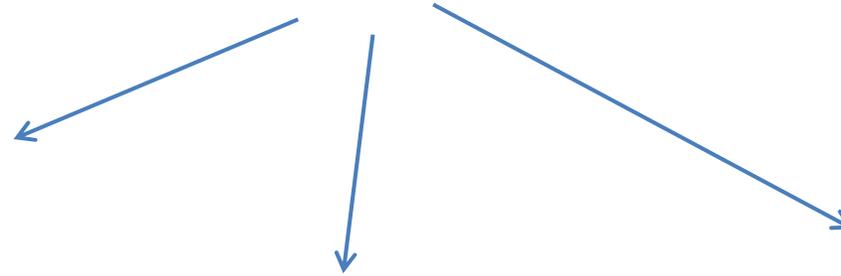
Isotope production:

Cm-248

Bk-249

Cf-251

**To be increased  
10 times**



New accelerator

High beam

dose of : Ca-48

**Factor 10-20**

Ti-50

Ni-64

**Depend of  
target durability**

SC- recoil separator

equipped with

Gas Catcher

On-line separator

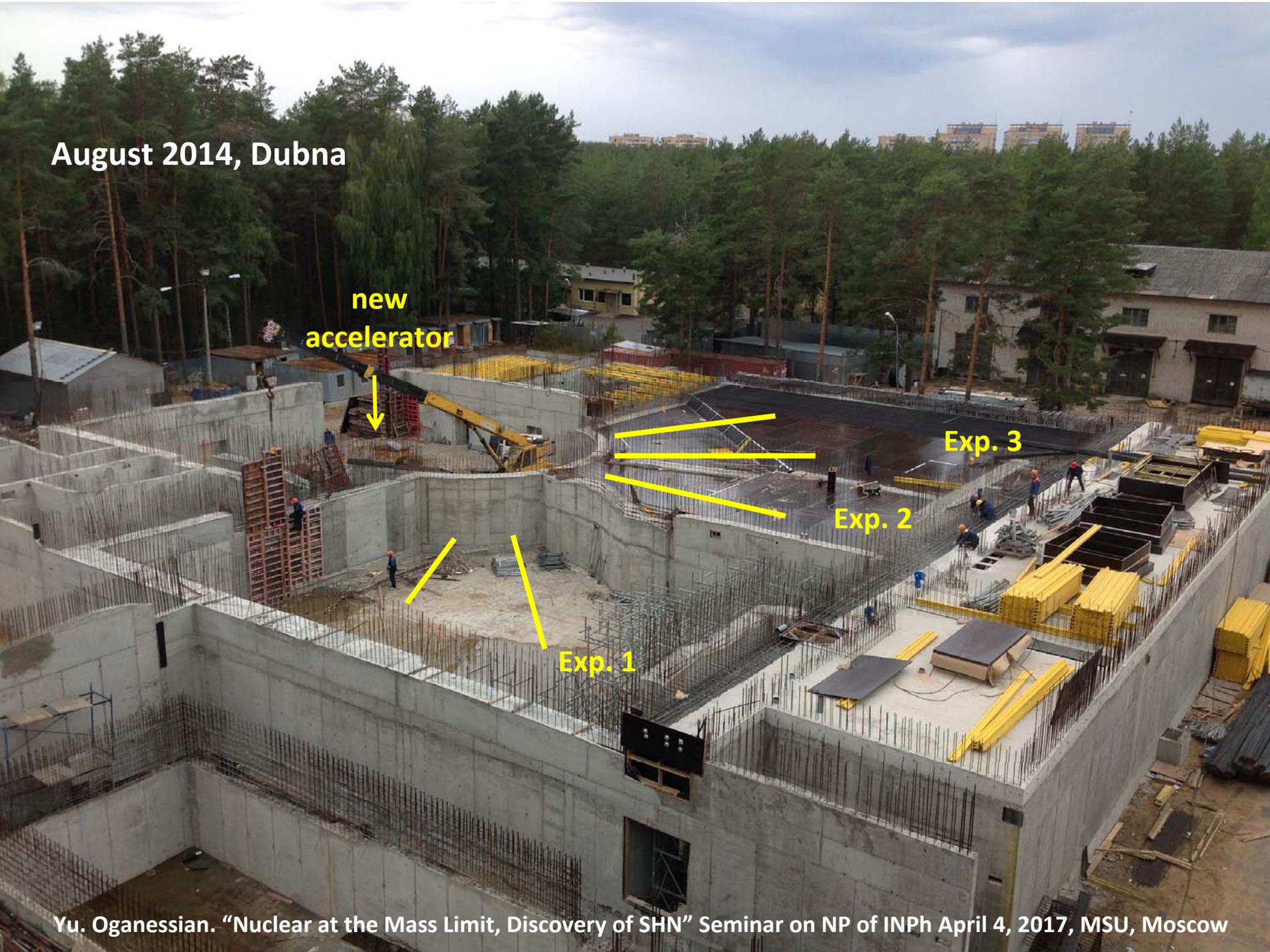
& sophisticated

Detectors

**Factor 3-5**

**is closely linked  
to the intellect**

August 2014, Dubna

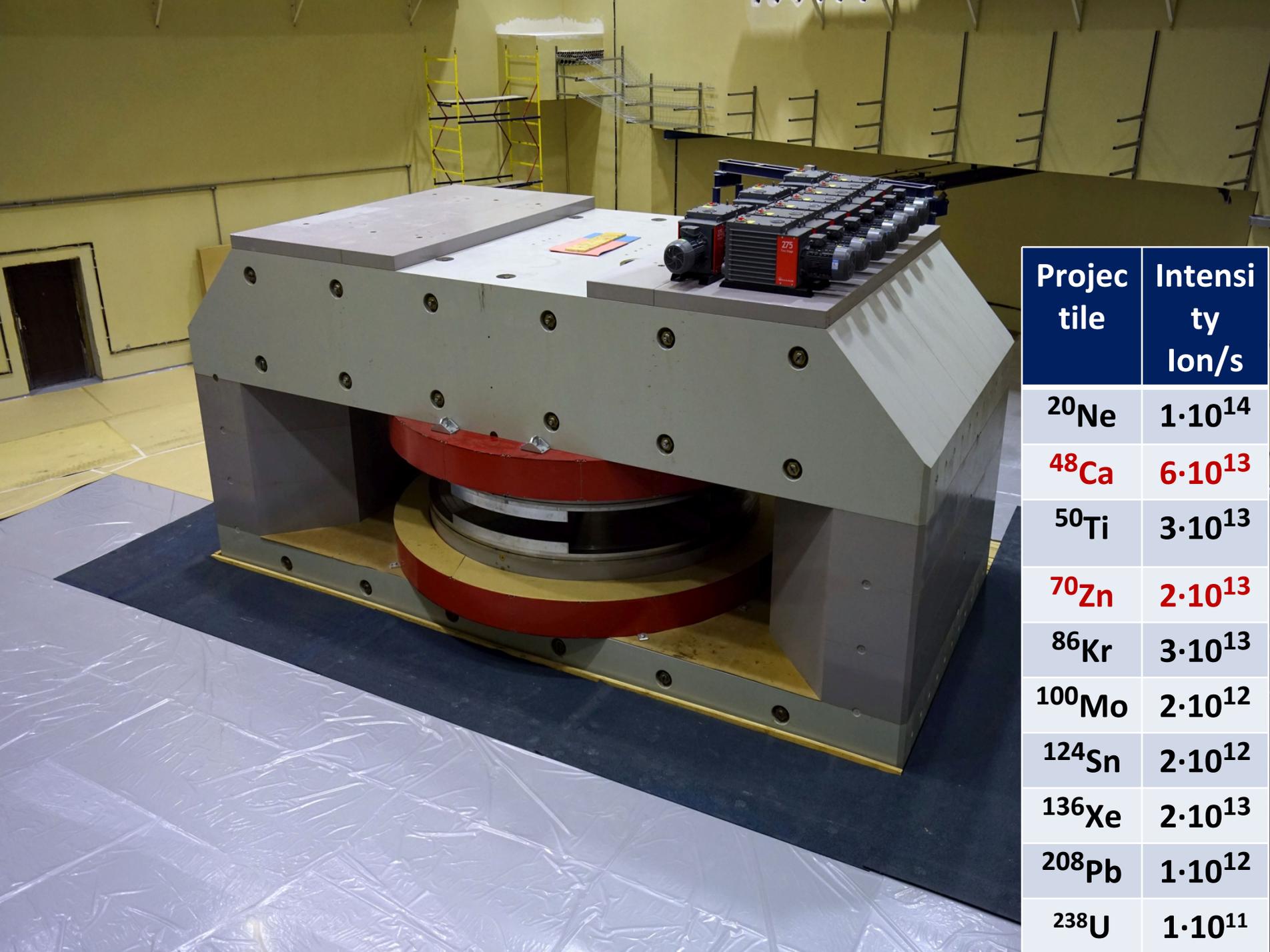


new  
accelerator

Exp. 3

Exp. 2

Exp. 1



Projec tile	Intensi ty Ion/s
$^{20}\text{Ne}$	$1 \cdot 10^{14}$
$^{48}\text{Ca}$	$6 \cdot 10^{13}$
$^{50}\text{Ti}$	$3 \cdot 10^{13}$
$^{70}\text{Zn}$	$2 \cdot 10^{13}$
$^{86}\text{Kr}$	$3 \cdot 10^{13}$
$^{100}\text{Mo}$	$2 \cdot 10^{12}$
$^{124}\text{Sn}$	$2 \cdot 10^{12}$
$^{136}\text{Xe}$	$2 \cdot 10^{13}$
$^{208}\text{Pb}$	$1 \cdot 10^{12}$
$^{238}\text{U}$	$1 \cdot 10^{11}$

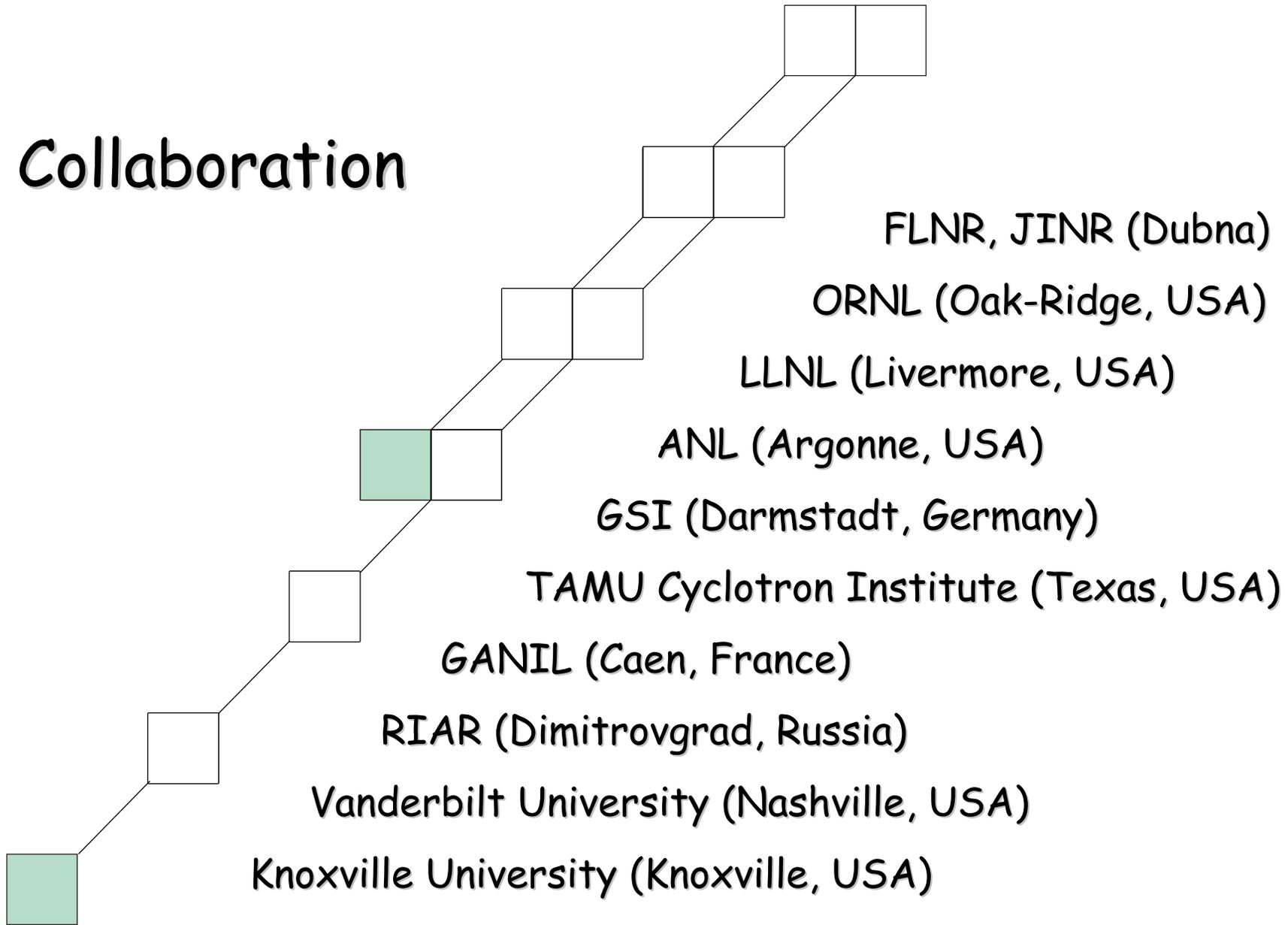
1. Radiochemical Engineering Development Center
2. High-Flux Isotope Reactor
3. Nuclear Science and Technology Division
4. Chemical Sciences Division
5. Transportation and
6. Packaging Division

Target Team at Oak Ridge

**More than 50 ORNL of staff contributed to the Bk-Cf production and separation**



# Collaboration



113	114	115	116	117	118
Nh Nihonium (284)	Fl Flerovium (289)	Mc Moscovium (288)	Lv Livermorium (293)	Ts Tennessine (294)	Og Oganesson (294)
112	111	110	109	108	107
Cn Copernicium (285)	110	109	108	107	106
108	107	106	105	104	103
Hg Mercury (200.59)	Tl Thallium (204.380)	Pb Lead (207.2)	Bi Bismuth (208.980)	Po Polonium (209)	At Astatine (210)
106	105	104	103	102	101
105	104	103	102	101	100
104	103	102	101	100	99
103	102	101	100	99	98
102	101	100	99	98	97
101	100	99	98	97	96
100	99	98	97	96	95
99	98	97	96	95	94
98	97	96	95	94	93
97	96	95	94	93	92
96	95	94	93	92	91
95	94	93	92	91	90
94	93	92	91	90	89
93	92	91	90	89	88
92	91	90	89	88	87
91	90	89	88	87	86
90	89	88	87	86	85
89	88	87	86	85	84
88	87	86	85	84	83
87	86	85	84	83	82
86	85	84	83	82	81
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84	83	82	81	80	79
83	82	81	80	79	78
82	81	80	79	78	77
81	80	79	78	77	76
80	79	78	77	76	75
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72	71	70	69	68	67
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69	68	67	66	65	64
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14	13	12	11	10	9
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11	10	9	8	7	6
10	9	8	7	6	5
9	8	7	6	5	4
8	7	6	5	4	3
7	6	5	4	3	2
6	5	4	3	2	1
5	4	3	2	1	0
4	3	2	1	0	-1
3	2	1	0	-1	-2
2	1	0	-1	-2	-3
1	0	-1	-2	-3	-4
0	-1	-2	-3	-4	-5

Thank you!