LOCAL SPACE-TIME ANISOTROPY AND LORENTZ SYMMETRY VIOLATION WITHOUT VIOLATION OF RELATIVISTIC SYMMETRY

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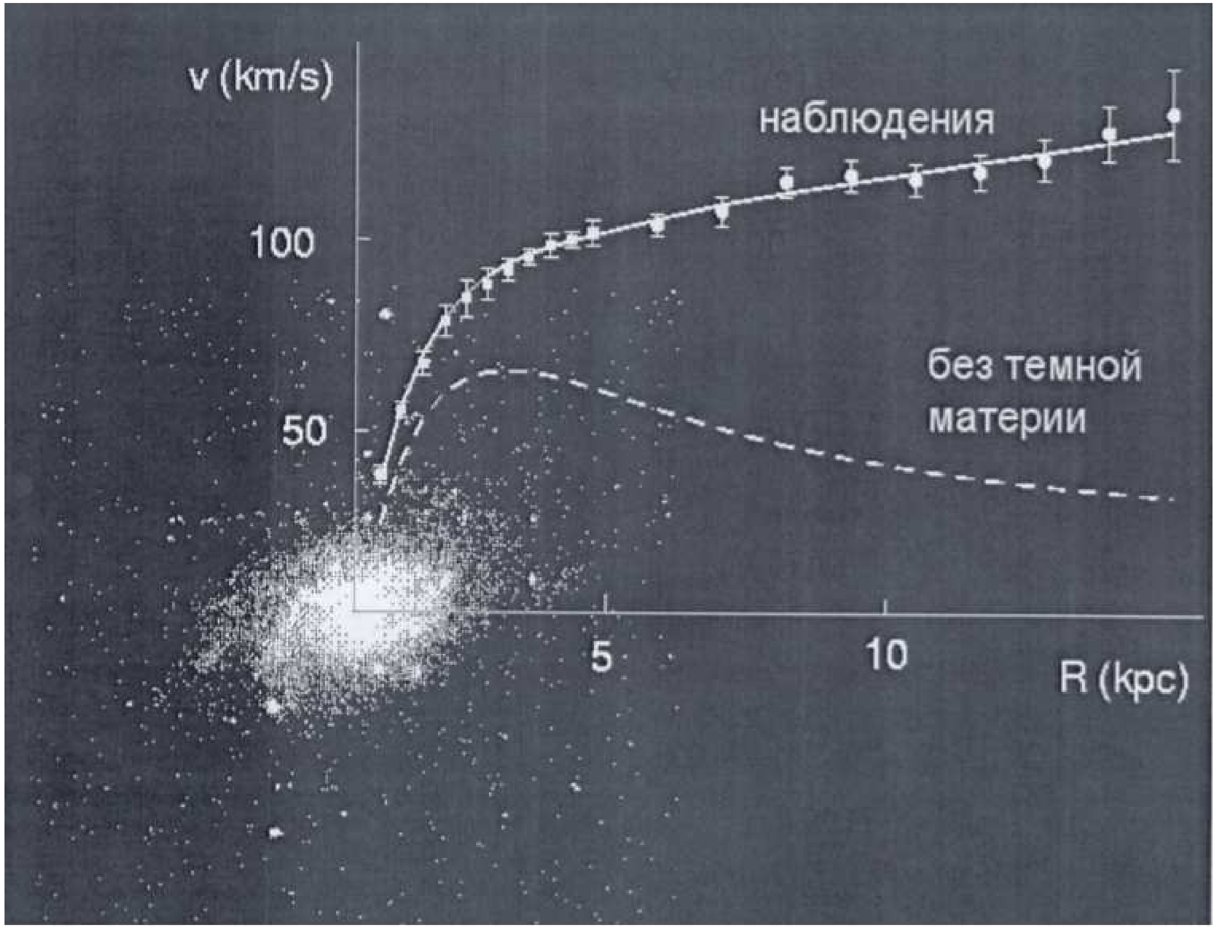
1 Introduction: i) A local space-time anisotropy, what does it mean? ii) Status and prospects of cosmology and field theory based on the idea of local space-time anisotropy

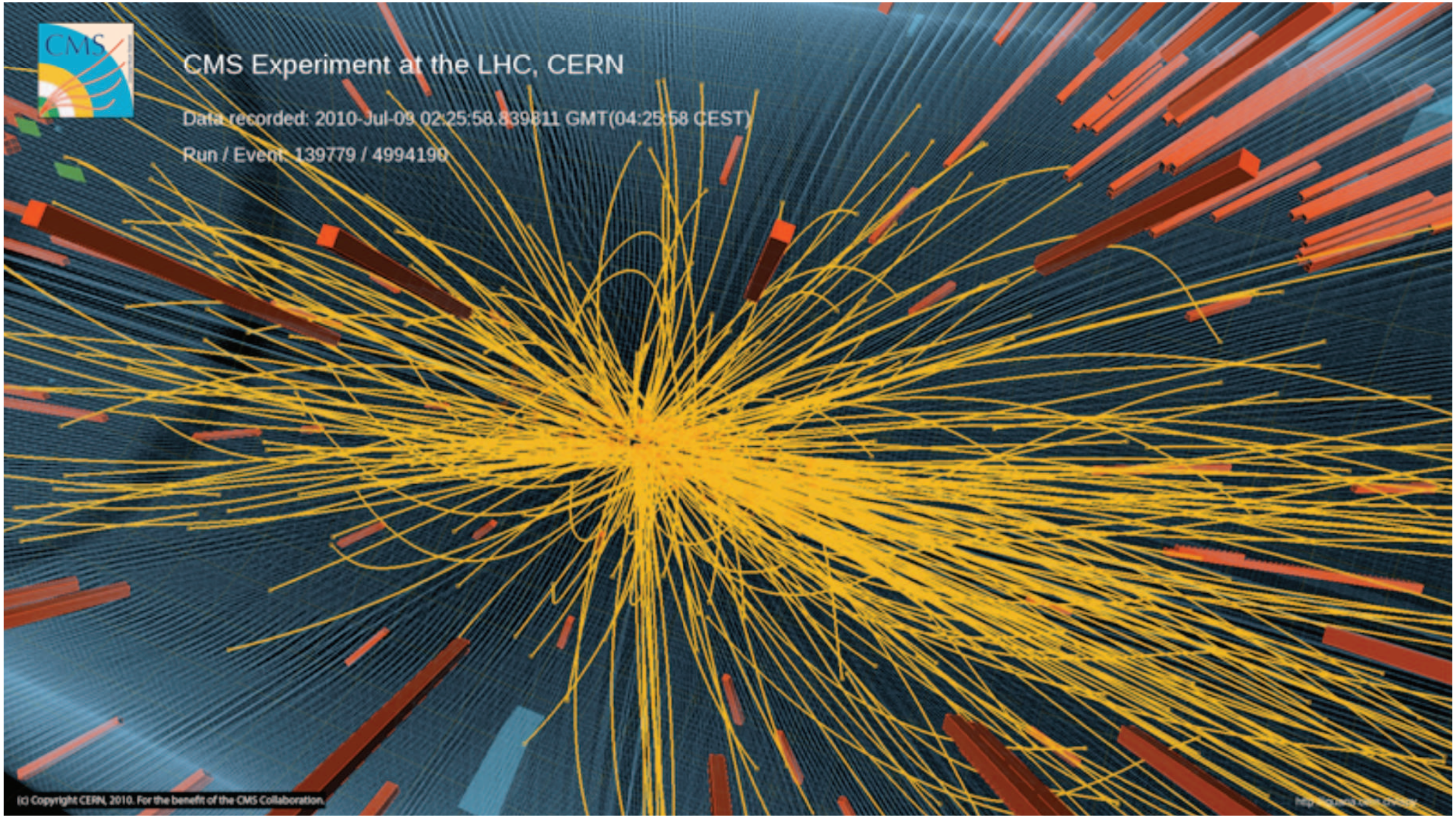
The gravitation theory, i.e. GR, was developed for and successfully applied at the scale of planetary systems. When applied to cosmological (galactic) scales in the way in which this is done now, it demands the introduction of corrections that are 25 times larger than the value of mass of the observable Universe and which are related to the existence of the new (still unknown) substances – dark matter and dark energy, which were not supposed to be present in the initial theory.

For instance, important observations that make simple sense, have sufficient value and statistical validity, but contradict classical GR, are the rotation curves of spiral galaxies.



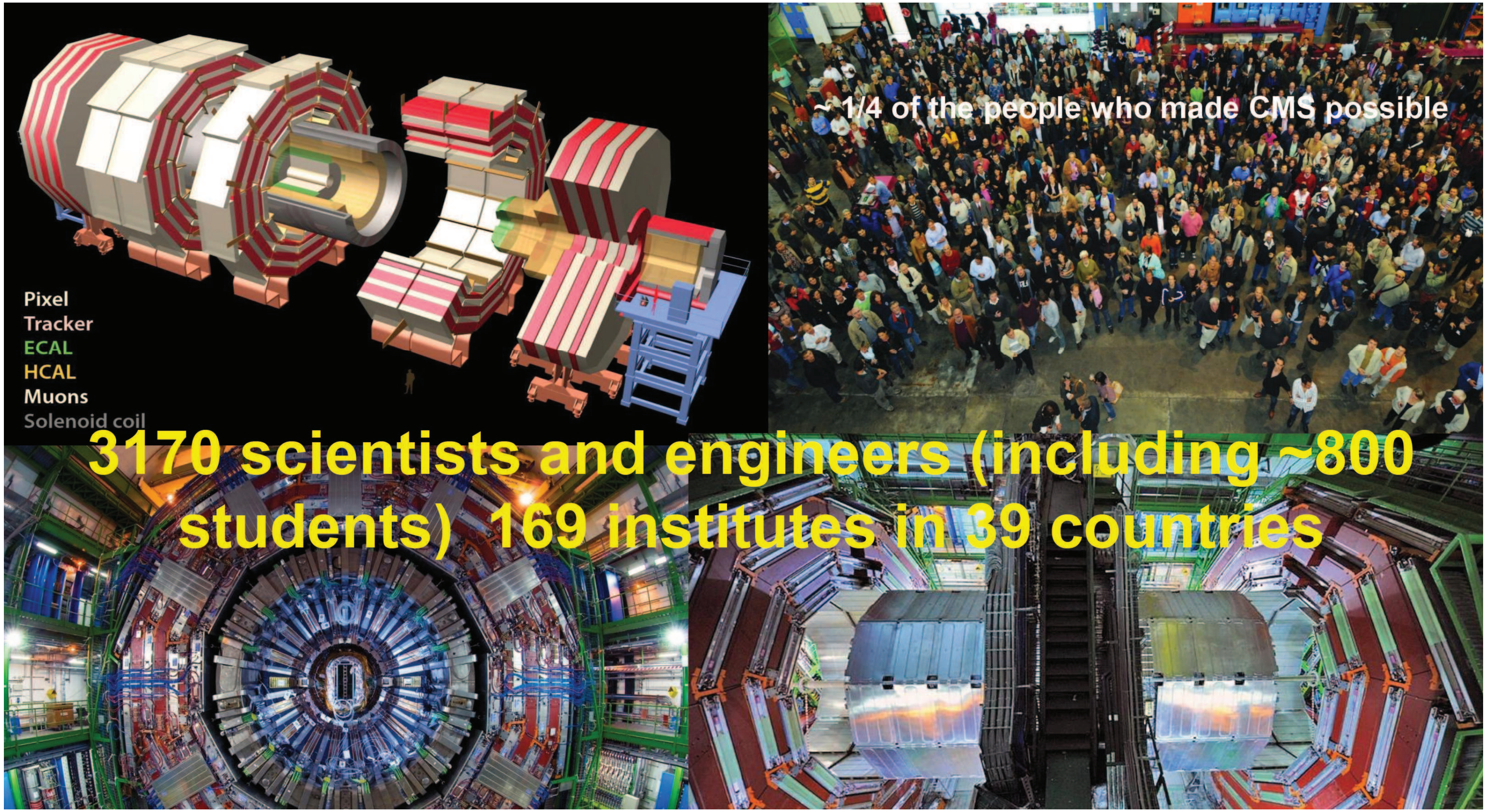
Alongside with the phenomena, for explanation of which the concept of dark matter or dark energy is commonly used, there are the data of astrophysical observations and particle physics, indirectly indicating the existence of a local space-time anisotropy in the Universe. In this respect one should mention, first of all, the anisotropy of the acceleration of the Universe expansion, the anisotropy of relic radiation, the baryonic asymmetry of the Universe and a breaking of the discrete space-time symmetries in weak interactions.

Actually, the first direct evidence of the existence of local space-time anisotropy was obtained by the CMS collaboration at LHC within the framework of the so-called Ridge/CMS-effect. Note in passing that the Ridge/CMS-effect is characteristic only to the high multiplicity events. Such events take place in case of the central collision of the initial protons where the energy density at the moment of the collision is comparable to the energy density shortly after the Big Bang, when instead of hadrons there was quark-gluon plasma.

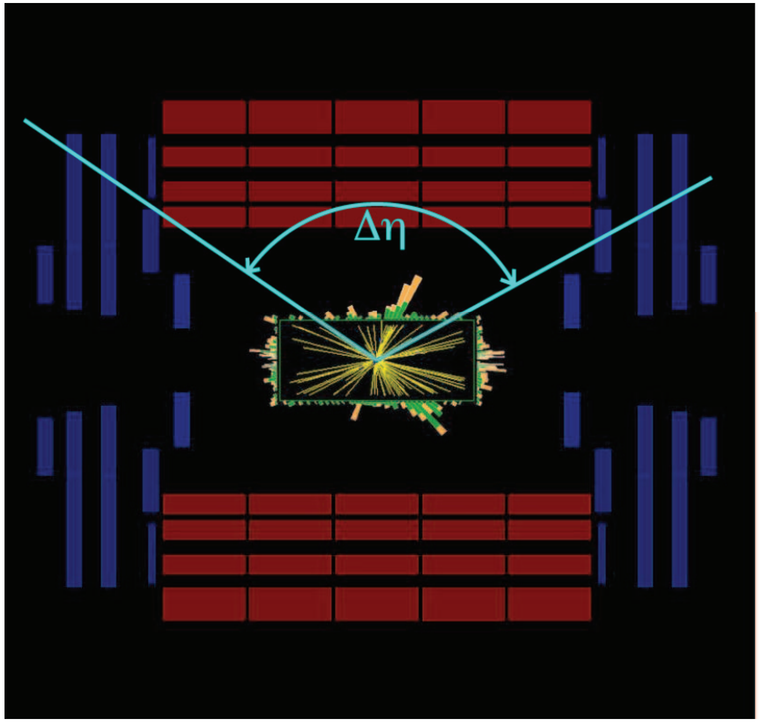


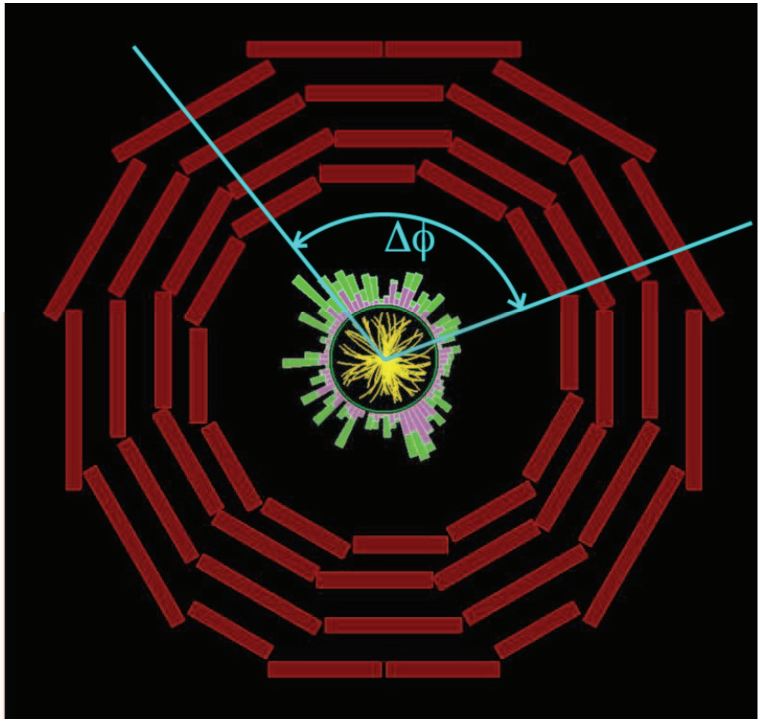
Example image showing a 7 TeV proton-proton collision in CMS producing more than 100 charged particles

The CMS Collaboration



In the CMS experiment, all pairs of charged particles in a collision were selected and the differences $\Delta\eta, \Delta\phi$ in the directions of the two particles measured. The former is a measure of the angle between two tracks in the longitudinal plane – shown below left. The latter is a measure of the angle between two tracks in the transverse plane, shown below right.

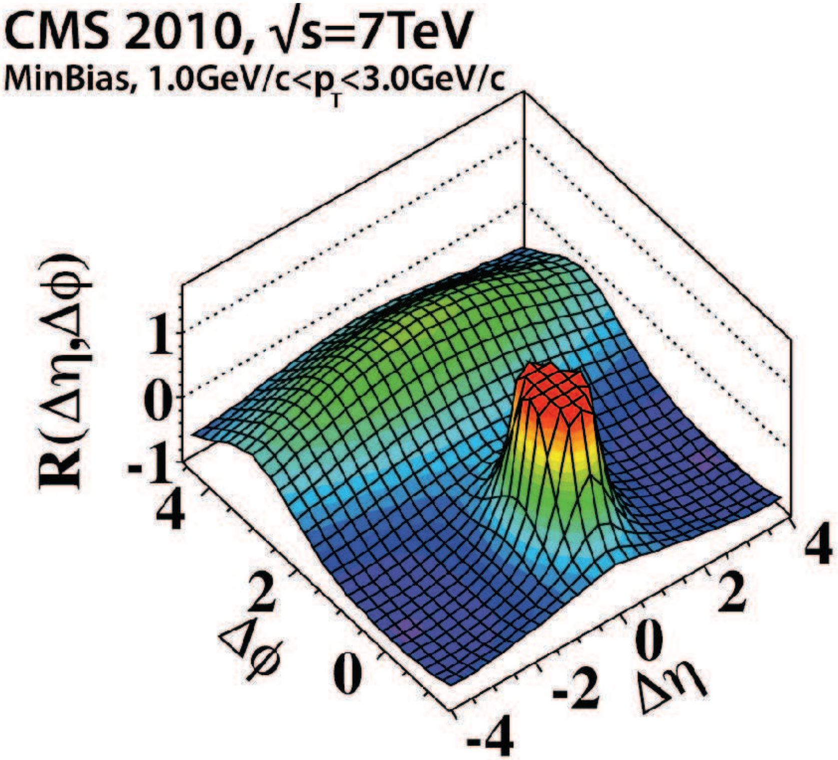


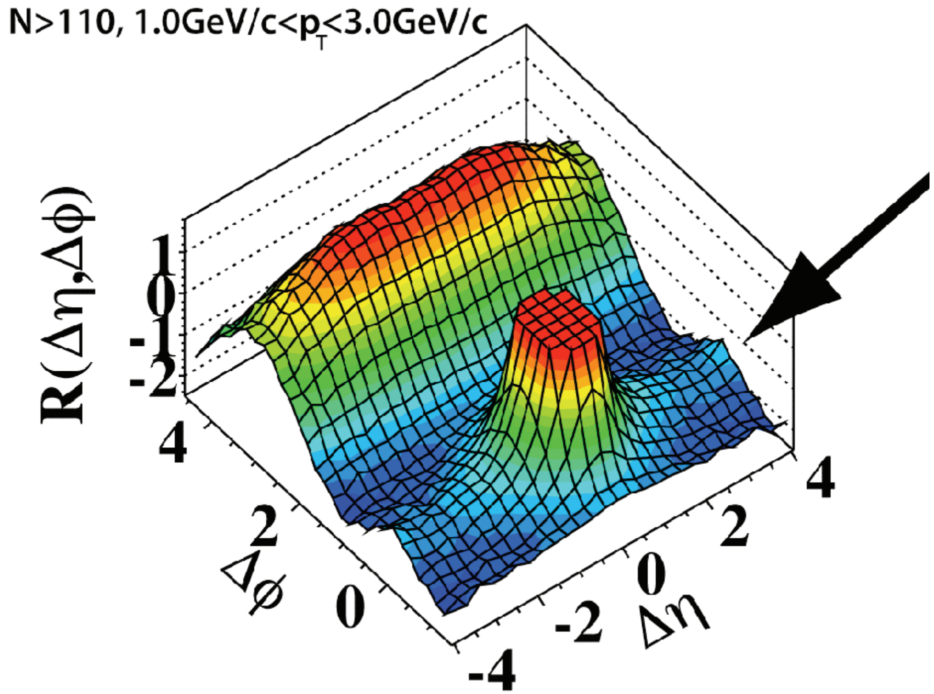


The variation of a correlation function R with $\Delta\eta$ and $\Delta\phi$, for proton-proton collisions in CMS.

Left: for **Minimum Bias** collisions; Right: for collisions that produced at least 110 charged particles

(Minimum Bias: events that have just the most basic selection criteria)





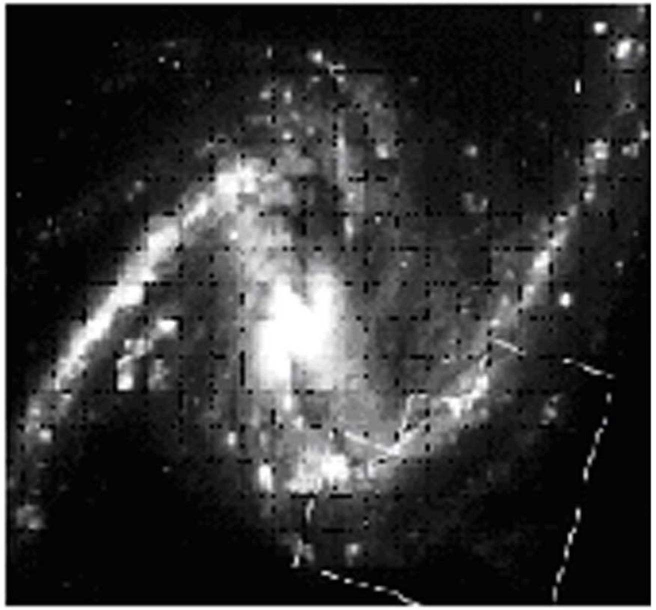
The most notable difference between the two images is the addition of an elongated ridge at $\Delta\phi = 0$ for all $\Delta\eta$. This means that there exists a set of selected pairs, in which case each pair of the particle tracks lies in its own plane. However all such planes have the only (common) line of their intersection, namely the collision axis of the initial protons. This resembles a situation with elastic collision of a moving particle and a particle at rest. Due to the law of momentum conservation, all possible track planes have the only line of their intersection, namely the track of initial moving particle. In contrast to the elastic collision, the initial total momentum within the CMS experiment is equal to zero. Therefore, the appearance of the preferred direction coinciding with the protons collision axis speaks of vacuum rearrangement with the appearance of axially-symmetric anisotropic fermion-antifermion condensate. On the one hand, quantum-field vacuum, that includes the anisotropic condensate, is the physical carrier of the local anisotropy of space-time, and it can be regarded as an anisotropic quintessence, on the other – it imparts all the particles the properties of quasi-particles in the crystalline environment.

In particular, in addition to the rest energy, all emitted massive particles acquire a rest momentum directed along the protons collision axis .

What is said above is far from ordinary speculations and based on Finslerian (anisotropic) extension of relativity theory. Below we shall discuss the specific features of such a generalized theory, but here (as some general remarks) note the following.

The approach based on Finslerian extension of GR is consistent with observations at the galactic scale, and does not require the introduction of dark matter. In particular, Finslerian simulation of a spiral galaxy, leads to the expression $v_{orb} \sim const$ for the orbital velocity corresponding to the observed flat rotation curve, and to the empirical Tully-Fisher law $v_{orb} \sim L_{lum}{}^{1/4}$, which has no explanation in general relativity. Within the framework of the same approach one can explain the observed substantial excess of deflection in some grav. lenses over the theoretical calculations. Besides, in addition to the known convex gravitational lenses, the theory predicts the existence concave gravitational lenses.

Finally, the following Figures allow us to compare the specific structures observed in most of spiral galaxies with the corresponding Finslerian calculations.



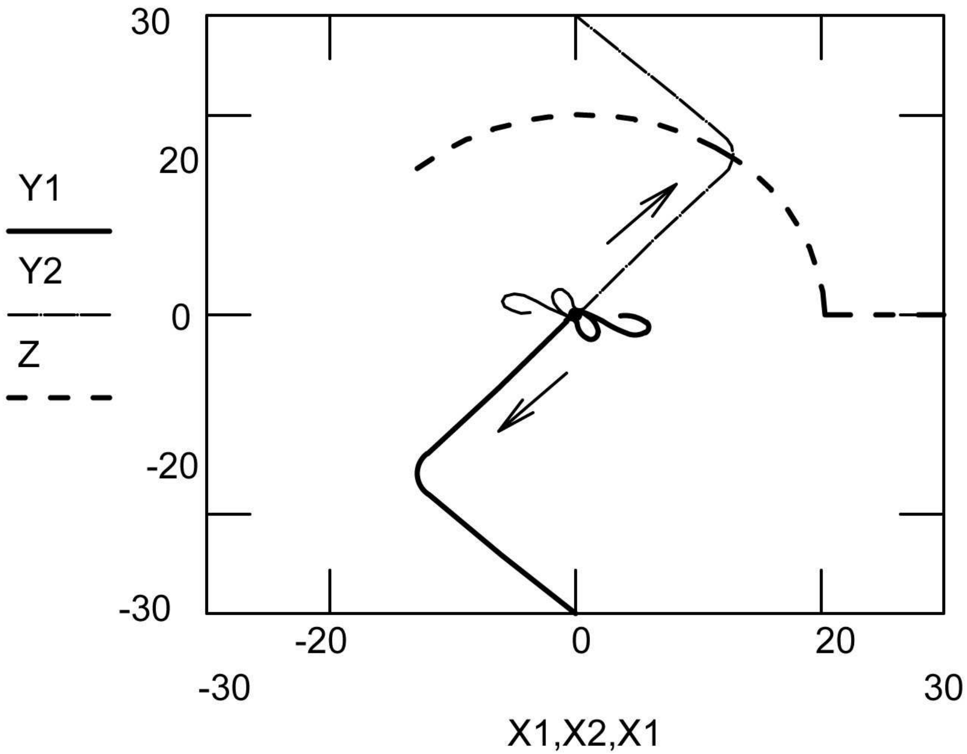


Fig. 1 -left : Galaxy NGC-1365 (Hubble telescope image, NASA/ESA).

Fig. 1 -right : Finslerian calculation (the exact view of the central details depends on the step of the calculation but they remain always present).

Fig. 2 : Details discovered by Herschel orbital observatory in the center of Milky Way (to appear in ApJ)

2 From Minkowski isotropic event space to the flat anisotropic event space: Anisotropic (Finslerian) Special Relativity

In order to arrive naturally at the flat relativistically invariant anisotropic space-time we first confine ourselves (for the sake of simplicity) to a two-dimensional space and show that it is possible to generalize the Lorentz transformations

$$\begin{cases} x_0' = x_0 \cosh\alpha - x\sinh\alpha \\ x' = -x_0\sinh\alpha + x\cosh\alpha\,; \qquad \tanh\alpha = v/c \end{cases}$$

so that the new linear transformations will also form a group with a single parameter α and will keep invariance of the wave equation.

Guided by the conformal invariance of the electrodynamic equations, we insert the additional scale transformations $e^{-r\alpha}$ into the standard Lorentz ones. As a result we obtain the generalized Lorentz transformations in the form

$$\begin{cases} x_0' = e^{-r\alpha}\,(x_0\cosh\alpha - x\sinh\alpha) \\ x' = e^{-r\alpha}\,(-x_0\sinh\alpha + x\cosh\alpha)\,, \end{cases}$$

where r is a dimensionless parameter of the scale transformations. Since the relation of the group parameter α to the velocity v of the primed frame remains unchanged, i.e. $\tanh\alpha = v/c$, the generalized Lorentz transformations can be rewritten as follows

$$\begin{cases} x_0' = \left(\frac{1-v/c}{1+v/c}\right)^{r/2} \frac{x_0-(v/c)x}{\sqrt{1-v^2/c^2}} \\ x' = \left(\frac{1-v/c}{1+v/c}\right)^{r/2} \frac{x-(v/c)x_0}{\sqrt{1-v^2/c^2}}\,. \end{cases}$$

Obviously, in contrast to the standard Lorentz transformations, these generalized ones do not leave invariant the Minkowski metric $ds^2 = dx_0^2 - dx^2$ but conformally modify it. Therefore, the question arises as to what the metric of an event space invariant under such generalized Lorentz transformations is. The rigorous solution to this problem is

$$ds^2 = \left[\frac{(dx_0 - dx)^2}{dx_0^2 - dx^2}\right]^r (dx_0^2 - dx^2)\,.$$

Not being a quadratic form but a homogeneous function of the coordinate differentials of degree two, this metric falls into the category of Finslerian metrics. It describes a flat but anisotropic event space.
Proceeding from this 2D metric and using the substitution

$$(dx_0^2 - dx^2) \to (dx_0^2 - d\boldsymbol{x}^{\,2})\,; \quad (dx_0 - dx) \to (dx_0 - \boldsymbol{\nu} d\boldsymbol{x})\,,$$

we arrive at the corresponding 4D metric

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2).$$

The unit vector $\boldsymbol{\nu}$ indicates a preferred direction in 3D space while the parameter r determines the magnitude of space anisotropy, characterizing the degree of deviation of this metric from Minkowski one. Thus, the flat Finslerian (anisotropic) event space

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)$$

proves to be a generalization of the isotropic Minkowski space of conventional special relativity theory. For the first time this metric was proposed in

G. Yu. Bogoslovsky,
"On a special relativistic theory of anisotropic space-time",
Dokl. Akad. Nauk SSSR **213** (1973) 1055.

As to the transformations that relate the various inertial frames to each other, an analog of the ordinary Lorentz boosts has in our case the form

$$x'^i = D(\boldsymbol{v},\boldsymbol{\nu})R^i_j(\boldsymbol{v},\boldsymbol{\nu})L^j_k(\boldsymbol{v})x^k\,,$$

where $\boldsymbol{v}$ denotes the velocities of moving (primed) inertial reference frames, the matrices $L^j_k(\boldsymbol{v})$ represent the ordinary Lorentz boosts, the matrices $R^i_j(\boldsymbol{v},\boldsymbol{\nu})$ represent additional rotations of the spatial axes of the moving frames around the vectors $[\boldsymbol{v}\boldsymbol{\nu}]$ through the angles

$$\varphi = \arccos\left\{1 - \frac{(1-\sqrt{1-\boldsymbol{v}^2/c^2})[\boldsymbol{v}\boldsymbol{\nu}]^2}{(1-\boldsymbol{v}\boldsymbol{\nu}/c)\boldsymbol{v}^2}\right\}$$

of relativistic aberration of $\boldsymbol{\nu}$, and the diagonal matrices

$$D(\boldsymbol{v},\boldsymbol{\nu}) = \left(\frac{1-\boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1-\boldsymbol{v}^2/c^2}}\right)^r I$$

stand for the additional dilatational transformations of the event coordinates.

If $r = 0$, the Finslerian metric

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)$$

reduces to the Minkowski one

$$ds^2 = dx_0^2 - d\boldsymbol{x}^2 \,.$$

However the respective transformations of the relativisic symmetry of the Finslerian space, i.e. transformations

$$x'^i = D(\boldsymbol{v}, \boldsymbol{\nu}) R^i_j(\boldsymbol{v}, \boldsymbol{\nu}) L^j_k(\boldsymbol{v}) x^k \,,$$

in which

$$D(\boldsymbol{v}, \boldsymbol{\nu}) = \left(\frac{1 - \boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1 - \boldsymbol{v}^2/c^2}}\right)^r I \,,$$

do not reduce to the ordinary Lorentz boosts

$$x'^i = L^i_k(\boldsymbol{v}) x^k \,.$$

At $r = 0$, i.e. in the case of Minkowski space where all directions in 3D space are equivalent, $\boldsymbol{\nu}$ has no physical meaning. In this case, each of the transformations

$$x'^i = R^i_j(\boldsymbol{v}, \boldsymbol{\nu}) L^j_k(\boldsymbol{v}) x^k$$

is differed from the respective Lorentz boost

$$x'^i = L^i_k(\boldsymbol{v}) x^k$$

by the additional rotation

$$x'^i = R^i_k(\boldsymbol{v}, \boldsymbol{\nu}) x^k$$

of the spatial axes. This additional rotation is adjusted in such a way that if a ray of light has the the direction $\boldsymbol{\nu}$ in one frame, then it will have the same direction in all the frames.

Thus, at $r = 0$, i.e. within the framework of conventional special relativity, the transformations

$$x'^i = R^i_j(\boldsymbol{v}, \boldsymbol{\nu}) L^j_k(\boldsymbol{v}) x^k \,,$$

represent an alternative to the Lorentz boosts, however, in contrast to the boosts, they constitute a 3-parameter noncompact group.
Physically such noncompact transformations are realized as follows. First choose as $\boldsymbol{\nu}$ a direction towards a preselected star and then perform an arbitrary Lorentz boost by complementing it with such a turn of the spatial axes that in a new reference frame the direction towards the star remains unchanged. The set of the transformations described has just been given by the above-displayed relation.

Let us consider an inhomogeneous group of isometries of Finslerian space-time

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)\,.$$

Since the respective homogeneous non-compact group of generalized Lorentz boosts

$$x'^i = D(\boldsymbol{v},\boldsymbol{\nu}) R^i_j(\boldsymbol{v},\boldsymbol{\nu}) L^j_k(\boldsymbol{v}) x^k$$

is 3-parameteric, with inclusion of 1-parameter group of rotations around the preferred direction $\boldsymbol{\nu}$ and 4-parameter translation group, the inhomogeneous group of isometries, or in other words, inhomogeneous group of relativistic symmetry of the Finslerian space-time appears to have 8-parameters. To obtain the simplest representation for its generators, it is enough to send the third spatial axis along $\boldsymbol{\nu}$ and rewrite the above-written homogeneous transformations in the infinitesimal form.

As a result, we come to the following eight generators

$$
\begin{aligned}
X_1 &= -(x^1p_0 + x^0p_1) - (x^1p_3 - x^3p_1)\,,\\
X_2 &= -(x^2p_0 + x^0p_2) + (x^3p_2 - x^2p_3)\,,\\
X_3 &= -rx^ip_i - (x^3p_0 + x^0p_3)\,,\\
R_3 &= x^2p_1 - x^1p_2\,; \qquad\qquad p_i = \partial/\partial x^i\,.
\end{aligned}
$$

These generators satisfy the commutation relations

$$[X_1X_2] = 0\,, \qquad [R_3X_3] = 0\,,$$
$$[X_3X_1] = X_1\,, \qquad [R_3X_1] = X_2\,,$$
$$[X_3X_2] = X_2\,, \qquad [R_3X_2] = -X_1\,;$$

$$[p_ip_j] = 0\,;$$

$$[X_1p_0] = p_1\,, \quad [X_2p_0] = p_2\,, \quad [X_3p_0] = rp_0 + p_3\,, \quad [R_3p_0] = 0\,,$$
$$[X_1p_1] = p_0 + p_3\,, \quad [X_2p_1] = 0\,, \quad [X_3p_1] = rp_1\,, \quad [R_3p_1] = p_2\,,$$
$$[X_1p_2] = 0\,, \quad [X_2p_2] = p_0 + p_3\,, \quad [X_3p_2] = rp_2\,, \quad [R_3p_2] = -p_1\,,$$
$$[X_1p_3] = -p_1\,, \quad [X_2p_3] = -p_2\,, \quad [X_3p_3] = rp_3 + p_0\,, \quad [R_3p_3] = 0\,.$$

It is clear that the homogeneous isometry group of the flat Finslerian space with partially broken 3D isotropy contains four parameters (generators X_1 , X_2 , X_3 and R_3). It is a subgroup of the 11-parametric Similitude (Weyl) group, and it is isomorphic to the corresponding 4-parametric subgroup (with generators X_1 , X_2 , $X_3|_{r=0}$ and R_3) of the homogeneous Lorentz group. Since the 6-parametric homogeneous Lorentz group does not have any 5-parametric subgroup, while its 4-parametric subgroup is unique up to isomorphisms, the passage from Minkowski space to the Finslerian space with partially broken 3D isotropy implies a minimum possible violation of Lorentz symmetry. With this, the relativistic symmetry represented now by generalized Lorentz boosts,

$$x'^i = D(\boldsymbol{v},\boldsymbol{\nu})R^i_j(\boldsymbol{v},\boldsymbol{\nu})L^j_k(\boldsymbol{v})x^k$$

remains valid. Besides, in spite of a new geometry of event space, the relativistic law of addition of 3-velocities remains unchanged.

The 8-parameter inhomogeneous group of isometries (group of motions of the partially anisotropic event space) and its Lie algebra were scrutinized in

G. Yu. Bogoslovsky,
"A special relativistic theory of the locally anisotropic space-time.
I. The metric and group of motions of the anisotropic space of events ;
II. Mechanics and electrodynamics in the anisotropic space",
Nuovo Cimento B **40** (1977) 99 ; 116.

"Subgroups of the group of generalized Lorentz transformations and their geometric invariants",
SIGMA **1** (2005), 017.

Notice that the above-mentioned results are mostly reproduced with the help of a different method in

G. W. Gibbons, Joaquim Gomis, C. N. Pope,
"General very special relativity is Finsler geometry",
Phys. Rev. D **76** (2007), 081701(R).

Notice also that the authors used in this paper a different relevant notation. In particular, the symbol b was used instead of the original symbol r, while the group of motions of the flat Finslerian event space

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)$$

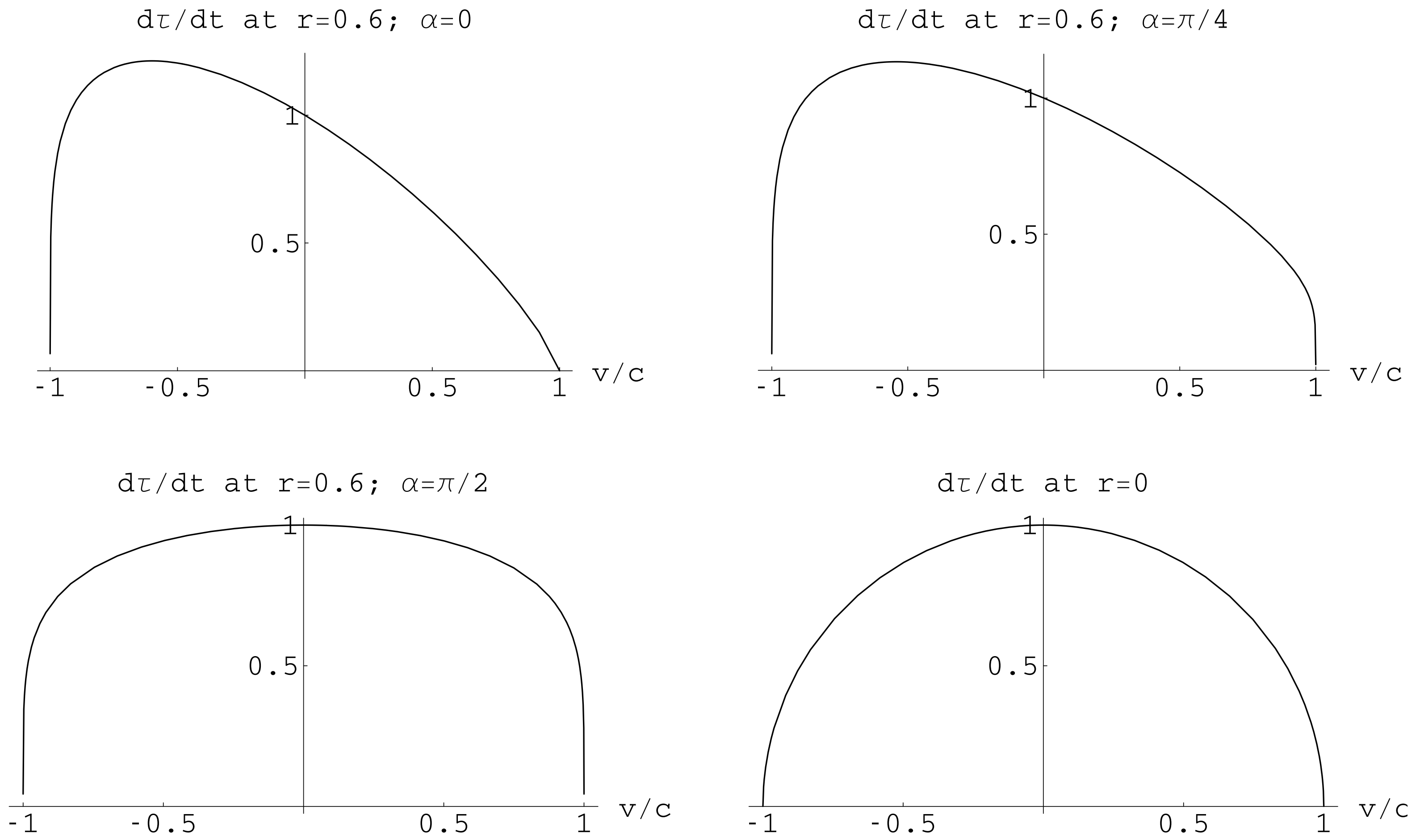
was called DISIM$_b$(2), i.e. Deformed Inhomogeneous SIMilitude group that includes 2-parameter Abelian homogeneous noncompact subgroup. Nevertheless, hereafter we shall hold on to our original symbols.

Although in the 3D space there is a preferred direction $\boldsymbol{\nu}$, its geometry remains Euclidean. But, what does the anisotropy physically manifest itself in?

First of all, it affects the dependence of proper time of a moving clock by including the *direction* of its velocity in addition to the magnitude : the interval $d\tau$ of proper time read by the clock moving with a velocity $\boldsymbol{v}$, is related to the time interval dt read by a clock at rest by the relation $d\tau = (d\tau/dt)\, dt$, where

$$\frac{d\tau}{dt} = \left(\frac{1 - \boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1 - \boldsymbol{v}^{\,2}/c^2}}\right)^r \sqrt{1 - \boldsymbol{v}^{\,2}/c^2}\ .$$

Thus, in contrast to Minkowski space (for which: $r = 0\,,\ (d\tau/dt)\,|_{r=0} = \sqrt{1 - v^2/c^2} \leq 1$ and, hence, the moving clock is always slow in comparison with the clock at rest), in the anisotropic space the time dilatation factor $(d\tau/dt)\,|_{r>0}$ can take on values greater than unity (see Figure).



These plots demonstrate the specific features of the behaviour of the anisotropic factor of time dilatation $(d\tau/dt)|_{r>0}$ in comparison with the behaviour of the isotropic (Minkowskian) factor $(d\tau/dt)|_{r=0}$.

Along with the time dilatation factor the anisotropy of space also affects the Doppler shift. In place of the usual relativistic formula, now the modified relation holds:

$$\omega = \omega' \frac{\sqrt{1 - \boldsymbol{v}^2/c^2}}{1 - \boldsymbol{v}\boldsymbol{e}/c} \left(\frac{1 - \boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1 - \boldsymbol{v}^2/c^2}} \right)^r ,$$

where r is the magnitude of space anisotropy, $\boldsymbol{v}$ the velocity of a moving frame, ω' the frequency of a ray with respect to it, and ω , $\boldsymbol{e}$ and $\boldsymbol{\nu}$ are the frequency, direction of the ray and the preferred direction in an initial frame.

In connection with this relation we propose a Lab. experiment aimed at seeking and measuring the local space anisotropy. The experiment consists in measuring a relative frequency shift $\Delta\omega/\omega = (\omega_a - \omega_s)/\omega_s$ between a Mössbauer source (s) and an absorber (a) placed at equal and diametrically opposite distances from the center of a rapidly rotating rotor.

For the quantity $\Delta\,\omega/\omega$, the ordinary special relativity (SR) and the relativistic theory of locally anisotropic space (AR), respectively, give the following predictions to within v^2/c^2

$$(\Delta\,\omega/\omega)^{SR} = 0\,, \qquad (\Delta\,\omega/\omega)^{AR} = 2rc\boldsymbol{\nu}\boldsymbol{v}_a/c^2$$

where $\boldsymbol{v}_a = -\boldsymbol{v}_s$. The present day use of the radically new rotors ($n \geq 6 \times 10^5$ turns/min) developed by the ITEP team (Moscow), as well as of the corresponding Mössbauer sources has made it possible to detect the space anisotropy at the level $r \sim 10^{-13}$.

3 A rest momentum in addition to the rest energy

In order to generalize conventional relativistic point mechanics for the flat partially anisotropic space it is sufficient in the action integral $S = -mc\int\limits_a^b ds$ to replace the Minkowski line element $ds = \sqrt{dx_0^2 - d\boldsymbol{x}^2}$ with the Finslerian one $ds = \left(\frac{dx_0 - \boldsymbol{\nu} d\boldsymbol{x}}{\sqrt{dx_0^2 - d\boldsymbol{x}^2}}\right)^r \sqrt{dx_0^2 - d\boldsymbol{x}^2}$. As a result, the Lagrangian function corresponding to a free particle in the locally anisotropic space, takes the form

$$L = -mc^2 \left(\frac{1 - \boldsymbol{v\nu}/c}{\sqrt{1 - \boldsymbol{v}^{\,2}/c^2}}\right)^r \sqrt{1 - \boldsymbol{v}^{\,2}/c^2}\,.$$

Hence it appears that

$$E = \frac{mc^2}{\sqrt{1-\boldsymbol{v}^2/c^2}} \left(\frac{1-\boldsymbol{v\nu}/c}{\sqrt{1-\boldsymbol{v}^2/c^2}} \right)^r \left[1-r+r\,\frac{1-\boldsymbol{v}^2/c^2}{1-\boldsymbol{v\nu}/c} \right],$$

i.e the energy E of a free particle in the anisotropic space depends on both the magnitude and the direction of its velocity $\boldsymbol{v}$. At $\boldsymbol{v}=0$ the energy reaches its absolute minimum, i.e a rest energy $E_0 = mc^2$.

As regards the momentum

$$\boldsymbol{p} = \frac{m}{\sqrt{1-\boldsymbol{v}^2/c^2}} \left(\frac{1-\boldsymbol{v\nu}/c}{\sqrt{1-\boldsymbol{v}^2/c^2}} \right)^r \left[(1-r)\boldsymbol{v}+rc\boldsymbol{\nu}\,\frac{1-\boldsymbol{v}^2/c^2}{1-\boldsymbol{v\nu}/c} \right],$$

its direction does not coincide with the direction of the velocity $\boldsymbol{v}$ of a massive particle. Even in the case $\boldsymbol{v}=0$ the momentum of a particle does not vanish; there remains a rest momentum $\boldsymbol{p}_0 = rmc\boldsymbol{\nu}$.

Massless particles have no such property; for them, as in conventional special relativity, $v=c$ and $E^2/c^2 - \boldsymbol{p}^2 = 0$.

The rest momentum $\boldsymbol{p}_0 = rmc\boldsymbol{\nu}$ of a particle (unequal to zero) means that the particle resides in an anisotropic physical vacuum. Such a vacuum is actually filled with an anisotropic condensate or, in other words, an anisotropic quintessence which in turn provides a flat event space with Finslerian geometry

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu}d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2) \,.$$

In contrast to the conventional Minkowski space with its constant quintessence, the flat Finslerian space together with its anisotropic quintessence are able to change their anisotropy if r is changeable.
Rather more detailed consideration consists in the following.

It easy to see that, at $r \to 1$, the Finslerian metric

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^{\,2}}\right]^r (dx_0^2 - d\boldsymbol{x}^{\,2})$$

degenerates into the total differential

$$ds = dx_0 - \boldsymbol{\nu} d\boldsymbol{x}\,,$$

in which case the notion of spatial extension disappeares and in the space-time there remains the single physical characteristic, namely, time duration and it should be regarded as an interval of absolute time. Besides that, at $r \to 1$, in accordance with the relation

$$m_{\alpha\beta} = m(1 - r)(\delta_{\alpha\beta} + r\nu_\alpha \nu_\beta)\,,$$

there also disappeares the inertial mass of any particle.

This suggests that absolute time, where the very notions of spatial extension and inertial mass together with unobservable primordial quintessence become meaningless, is not a stable degenerate state of space-time. As a result of geometric phase transition, which accompanies a spontaneous breaking of the original gauge symmetry, such a primordial space-time may turn into anisotropic space-time whose anisotropy and, respectively, quintessence decrease with the Universe's accelerated expansion. If this scenario proves to be correct, then the question posed by G. W. Gibbons, Joaquim Gomis, C. N. Pope,
namely, why are the observed anisotropy and Λ-term simultaneously so small, may be answered.

4 Towards Finslerian extension of General Relativity and the field theory

Let us rewrite the flat Finslerian metric so that it is expressed through the four-dimensional quantities :

$$ds = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^{r/2} \sqrt{dx_0^2 - d\boldsymbol{x}^2} = \left[\frac{(\nu_i dx^i)^2}{\eta_{ik} dx^i dx^k}\right]^{r/2} \sqrt{\eta_{ik} dx^i dx^k}\,.$$

Since $\boldsymbol{\nu}^2 = 1$, it is clear that here we have

$$\nu_i = \{1, -\boldsymbol{\nu}\}\,, \quad \eta_{ik} = diag\{1, -1, -1, -1\}\,, \quad \nu^i = \{1, \boldsymbol{\nu}\}\,, \quad \nu_i \nu^i = 0.$$

The key point in the generalization of the flat $DISIM_b(2)$ -invariant Finslerian metric to a Finslerian metric, which describes the corresponding curved locally anisotropic space-time is the following. If the constant values on which the flat metric depends, namely a scalar r, null-vector ν_i and tensor $\eta_{ik} = diag\{1, -1, -1, -1\}$, are replaced by the corresponding conventional fields defined on the space-time manifold, i.e. in the flat metric the substitutions $r \to r(x)$, $\nu_i \to \nu_i(x)$, $\eta_{ik} \to g_{ik}(x)$ are performed, then the result will be the curved Finslerian metric of the following form

$$ds = \left[\frac{(\nu_i dx^i)^2}{g_{ik}dx^i dx^k}\right]^{r/2} \sqrt{g_{ik}dx^i dx^k}\,,$$

where: $g_{ik} = g_{ik}(x)$ is the Riemannian metric tensor associated with the gravitational field, $r = r(x)$ is a scalar field, which characterizes the magnitude of the local space-time anisotropy and $\nu_i = \nu_i(x)$ is a null-vector field that indicates the locally preferred directions in the space-time.

At any point of the curved Finslerian space, the corresponding flat tangent Finslerian space has its own values of the parameters r and $\boldsymbol{\nu}$. These values are nothing but the values of the fields $r(x)$ and $\boldsymbol{\nu}(x)$ at the point of tangency.

Obviously, the dynamics of curved Finslerian space

$$ds = \left[\frac{(\nu_i dx^i)^2}{g_{ik}dx^i dx^k}\right]^{r/2}\sqrt{g_{ik}dx^i dx^k}$$

is completely determined by the dynamics of the interacting fields $g_{ik}(x)$, $r(x), \nu_i(x)$, and these fields together with fields of matter form a unified dynamic system. Therefore, in contrast to the existing purely geometric approaches to the Finslerian generalization of Einstein's equations, our approach to this problem is based on the use of methods of the conventional theory of interacting fields.

The fact that during the transition from the flat $DISIM_b(2)$ -invariant Finslerian metric to the curved Finslerian metric, we replaced tensor $\eta_{ik} = diag\{1, -1, -1, -1\}$ and null-vector ν_i by the conventional fields, became the property of invariance of the curved metric with regard to the following local transformations

$$g_{ik} \rightarrow e^{2\sigma(x)} g_{ik}\,, \qquad \nu_i \rightarrow e^{(r-1)\sigma(x)/r} \nu_i\,, \qquad r \rightarrow r\,,$$

where $\sigma(x)$ is an arbitrary function.

In addition to the curved metric, these local transformations leave invariant all the observables. Therefore, in the theory of gravitation based on the group $DISIM_b(2)$, the above-given transformations have the meaning of local gauge transformations. For example, the action

$$S = -\frac{1}{c}\int \mu^* \left(\frac{\nu_i\, v^i}{\sqrt{g_{ik}\, v^i\, v^k}}\right)^{4r} \sqrt{-g}\; d^4x$$

for a compressible fluid in the curved Finslerian space is gauge invariant.

In connection with the above-mentioned local gauge invariance, the dynamical system consisting of the fields $g_{ik}\,,r\,,\nu_i$ and a compressible fluid must be supplemented by two vector gauge fields A_i and B_i , that under local gauge transformations are transformed in the corresponding gradient manner. The A_i field for a certain class of problems is a pure gauge field, and the B_i field, whose gauge transformation has the form

$$B_i \;\rightarrow\; B_i + b\,[\,(\,r-1\,)\,\sigma(\,x\,)\,/\,r\,]_{;\,i}\;,$$

where b is a constant with the dimensionality of length, interacts with the conserved rest mass current j^i , adding the term proportional to $B_i j^i$ to the full gauge invariant Lagrangian.

Finally, let us demonstrate the $DISIM_b(2)$ -invariant generalized Dirac Lagrangian

$$\mathcal{L} = \frac{i}{2}\left(\bar\psi\gamma^\mu\partial_\mu\psi - \partial_\mu\bar\psi\gamma^\mu\psi\right) - m\left[\left(\frac{\nu_\mu\bar\psi\gamma^\mu\psi}{\bar\psi\psi}\right)^2\right]^{r/2}\bar\psi\psi\,.$$