Cluster virial expansion for quark-nuclear matter and neutron star structure

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1. Introduction: Beth-Uhlenbeck (BU) and Generalized BU

2. GBU from Φ-derivable approach: 2-loop approximation

3. GBU EoS for quark-hadron matter in (P)NJL-type models

4. Application to neutron stars: high-mass twin stars!

Skobeltsyn Institute for Nuclear Physics, Moscow State U, 17.10. 2017
Breaking News: First observation ... of NS – NS mergers in gravitational waves and all el-magn. spectrum
The Goal: Theory of the QCD Phase Diagram

The Phases of QCD

- Early Universe
- LHC Experiments
- RHIC Energy Scan
- Critical Point
- Crossover
- Hadron Gas
- Supernova
- Quark-Gluon Plasma
- Nuclear Matter
- Neutron Stars
- Color Superconductor
- NICA-MPD (collider)
- Future FAIR Experiments
- NICA-BM@N (nuclotron)

Baryon Chemical Potential

~170 MeV

0 MeV

Vacuum
The Goal: Theory of the QCD Phase Diagram

The Phases of QCD

Statistical Model of Hadron Resonance Gas
Well established for Description of chemical freezeout
The Goal: Theory of the QCD Phase Diagram

Statistical Model of Hadron Resonance Gas

Well established for Description of chemical freezeout

Perturbative QCD

Approximately selfconsistent HTL resummation
(T > 2.5 Tc , µ > 1500 MeV)
The Goal: Theory of the QCD Phase Diagram

- **Perturbative QCD**: Approximately selfconsistent HTL resummation ($T > 2.5 T_c$, $\mu > 1500$ MeV)
- **QCD Phase transition(s)**: Mott dissociation of hadrons, Deconfinement, $\chi_{SR}$
- **Statistical Model of Hadron Resonance Gas**: Well established for Description of chemical freezeout
Introduction: Beth-Uhlenbeck vs. Generalized BU

Beth-Uhlenbeck: $2^{\text{nd}}$ virial coefficient $B(T)$

BU for virial expansion of density:

$$n(\mu, T) = n_{\text{free}}(\mu, T) + 2n_{\text{corr}}(\mu, T)$$

$$n_{\text{free}}(\mu, T) = 4 \int \frac{d^3p}{h^3} e^{-(p^2/2m-\mu)/T} = \frac{4}{\lambda^3} e^{\mu/T}$$

$$n_{\text{corr}}(\mu, T) = \int \frac{d^3P}{h^3} e^{-(P^2/4m-2\mu)/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E)$$

$$= \frac{2^{3/2}}{\lambda^3} e^{2\mu/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E).$$

Example: Deuterons in nuclear matter

$$n = n_{\text{free}} + 2n_{\text{free}}^2 I(T)$$

$$I(T) = \lambda^3 \frac{2^{1/2}}{8} \left[ 3(e^{-E_d/T} - 1) + \int_0^\infty \frac{dE}{\pi T} e^{-E/T} \sum_x c_x \delta_x(E) \right].$$

For $T << E_d$:

$$n = n_{\text{free}} + 2n_{\text{deut}}, \quad n_{\text{deut}} = n_{\text{free}}^2 \lambda^3 \frac{2^{1/2}}{8} e^{-E_d/T}.$$
Introduction: Beth-Uhlenbeck vs. Generalized BU

Thermodynamic Greens function approach:

\[ n(1, \mu_1, T) = \int \frac{dE}{2\pi} f_1(E) A(1, E) \]
\[ A(1, E) = \frac{2 \Sigma_i(1, E - i0)}{(E - E(1) - \Sigma_R(1, E))^2 + \Sigma_i(1, E - i0)^2} = \frac{2\pi \delta(E - e(1))}{1 - (d/dz)((d/dz) \Sigma_R(1, z))|_{z = e(1) + i0}} \]

Density formula
(free and corr. Quasiparticles):

\[ n(\mu, T) = n_{\text{free}}(\mu, T) + 2n_{\text{corr}}(\mu, T), \]
\[ \Sigma(1, z_\nu) = T \sum \sum [T(1212, z_\nu + z_\nu') - \text{ex}] G(2, z_\nu'). \]
\[ n_{\text{corr}}(\mu, T) = \int \frac{dE}{2\pi} g(E) F(E) \]
\[ F(E) = F_{\text{deut}}(E) + \frac{2}{4\pi} \sum_{\mathbf{k}} c_\mathbf{K} F_\mathbf{x}(\mathbf{E}), \]
\[ F_{\text{deut}}(E) = 6 \sum_{\mathbf{k} > \mathbf{K}_{\text{mol}}} \pi \delta(E - E_b(\mathbf{K}, \mu, T)). \]

\[ F_\mathbf{x}(E) = 8\pi \sum_{\mathbf{K}} \sin^2 \delta_\mathbf{x}(E, \mathbf{K}, \mu, T) \frac{d}{dE} \delta_\mathbf{x}(E, \mathbf{K}, \mu, T). \]

The $\sin^2\delta$ term accounts for quasiparticle effects.
Φ-derivable approach, 2-loop approximation


Skeleton expansion for thermodynamic potential and entropy

\[ \beta \Omega[D] = -\log Z = \frac{1}{2} \text{Tr} \log D^{-1} - \frac{1}{2} \text{Tr} \Pi D + \Phi[D] \]

Inv. Temp: 1/T \hspace{1cm} \text{trace in conf. Space} \hspace{1cm} \text{self-energy related to } D

Dyson equation:

\[ D^{-1} = D_0^{-1} + \Pi \]

Free propagator \( D_0 \) is known

Essential property of \( \Omega[D] \) is Stationarity under variation of \( D \):

\[ \frac{\delta \Omega[D]}{\delta D} = 0 \]

This implies \( \frac{\delta \Phi[D]}{\delta D} = \frac{1}{2} \Pi \)

Physical propagator and selfenergy are defined self-consistently!

Self-consistent approximations are defined by the choice of \( \Phi \)

Φ – derivable theories

Approximately selfconsistent thermodynamics

Matsubara summation:

\[ \frac{\Omega}{V} = \int \frac{d^4k}{(2\pi)^4} n(\omega) [\text{Im} \log(-\omega^2 + k^2 + \Pi) - \text{Im} \Pi D] + T\Phi[D]/V \]

\[ D(\omega,k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(\omega,k)}{k_0 - \omega} , \quad \text{Im} D(\omega,k) = \text{Im} D(\omega + i\epsilon,k) = \frac{\rho(\omega,k)}{2} . \]

Analytic properties:

Thermodynamics from entropy density:

\[ S = -\frac{\partial (\Omega/V)}{\partial T} \]

\[ S' = -\frac{\partial (T\Phi/V)}{\partial T} \Bigg|_{D} + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \text{Re} \Pi \text{Im} D \]

Loosely speaking: \( S' \) accounts for residual interactions of “independent quasiparticles”

\[ \frac{d}{d\omega} \left[ \text{Im} \log D^{-1} + \text{Im} \Pi \text{Re} D \right] = 2 \text{Im} \left[ D^-1 (d/d\omega D^*) \text{Im} \Pi \right] = 2 \sin^2 \delta \frac{d\delta}{d\omega} , \text{ for } D = |D|e^{i\delta} \]

D. B., in preparation (2016)
Proof of cancellations resulting in $S' = 0$

$$S' \equiv -\left. \frac{\partial (T \Phi / V)}{\partial T} \right|_D + \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{\partial n(\omega)}{\partial T} \text{Re}\Pi \text{Im}D \right\}$$

First term

$$-\frac{T}{V} \Phi = \frac{g^2}{12} T^2 \sum_{\omega_1, \omega_2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} D(\omega_1, |k_1|) D(\omega_2, |k_2|) D(-\omega_1 - \omega_2, |k_1 - k_2|)$$

Spectral representation

$$D(\omega, k) = \int_{-\infty}^{\infty} \frac{dk_0 \rho(k_0, k)}{2\pi k_0 - \omega}$$

Matsubara sums

$$-\frac{T}{V} \Phi = \frac{g^2}{12} T^2 \sum_{\omega_1, \omega_2} \int \frac{d^4k d^4k' d^4k''}{(2\pi)^9} \delta^{(3)}(k + k' + k'') \rho(k) \rho(k') \rho(k'') \frac{-1}{\omega_1 - k_0} \frac{-1}{\omega_2 - k_0'} \frac{1}{\omega_1 + \omega_2 + k_0''}$$

Partial fraction decomposition of the three energy denominators and Matsubara summation over $\omega_1, \omega_2$ yields:

$$\frac{1}{k_0 + k_0' + k_0''} \{ [n(k_0'') + 1][n(k_0) + n(k_0') + 1] + n(k_0)n(k_0'') \}$$

Temperature derivative and renaming variables under the integrals

$$\partial_T [n(k_0 + n(k_0') + n(k_0'') + n(k_0)n(k_0'') + n(k_0)n(k_0'')]) \rightarrow 3\partial_T n(k_0) [1 + n(k_0') + n(k_0'')]$$
Proof of cancellations resulting in $S' = 0$

Second term:

\[
\text{Re} \Pi(\omega, q) = -\frac{g^2}{2} \int \frac{d^3 k}{(2\pi)^3} \int \frac{dk_0}{2\pi} \int \frac{dk_0'}{2\pi} \rho(k_0, |k|)\rho(k_0', |k + q|) \sum_{\omega_1} \frac{1}{\omega_1 - k_0} \frac{1}{\omega_1 + \omega - k_0'} \\
= -\frac{g^2}{2} \int \frac{d^3 k}{(2\pi)^3} \int \frac{dk_0}{2\pi} \int \frac{dk_0'}{2\pi} \rho(k_0, |k|)\rho(k_0', |k + q|) \frac{1 + n(k_0) + n(k_0')}{\omega + k_0 + k_0'}
\]  

(7)

\[
\int \frac{d^4 q}{(2\pi)^4} \frac{\partial n(k_0)}{\partial T} \text{Re} \Pi(\omega, q) \text{Im} D(\omega, q) = \\
= -\frac{g^2}{2 \cdot 2} \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \int \frac{d^4 k'}{2\pi} \delta^{(3)}(q + k + k')\rho(q)\rho(k)\rho(k') \partial_T n(q_0) [1 + n(k_0) + n(k_0')] \frac{1}{q_0 + k_0 + k_0'}
\]  

(8)

This proves the cancellation of $S'$ for the scalar theory with cubic selfinteraction in the 2-loop approximation (sunset diagram) for the $\Phi^4$ functional.

This cancellation holds as well for the pressure and the density!

For the pressure we obtain

\[
p(T) = -\int \frac{d^4 q}{(2\pi)^4} n(q_0) [\delta(q) - \sin \delta(q) \cos \delta(q)] = -\int \frac{d^4 q}{(2\pi)^4} T \ln \left(1 - e^{-q_0/T}\right) \frac{\partial \delta(q)}{\partial q_0} 2 \sin^2 \delta(q)
\]  

(9)

Note that in the approximation $\delta(q_0, q) = -\arctan[\omega/(q_0^2 - \omega^2)]$ the ”spectral distribution” does not correspond to a Lorentzian (Breit-Wigner) function as naively expected, but to a ”squared Lorentzian”

\[
\frac{q_0(\omega\gamma)^3}{[(q_0^2 - \omega^2)^2 + (\omega\gamma)^2]^2}
\]  

(10)

In ghost-free gauge, HTL resummed QCD thermodyn.

\[
S_2 = -\frac{g^2 N_g T}{48} \left\{ \frac{4N + 5N_f}{3} T^2 + \frac{3N_f}{\pi^2} \mu^2 \right\},
\]

\[
N_2 = -\frac{g^2 \mu N_g N_f}{16 \pi^2} \left( T^2 + \frac{\mu^2}{\pi^2} \right),
\]

\[
P_2 = -\frac{g^2 N_g}{32} \left\{ \frac{4N + 5N_f}{18} T^4 + \frac{N_f}{\pi^2} \mu^2 T^2 + \frac{N_f}{2 \pi^4} \mu^4 \right\}
\]

Generalized Optical Theorems

See derivations for T-matrices by R. Zimmermann & H. Stolz, pss (b) 131, 151 (1985)

Here we consider the analogue of $T^{-1} = V^{-1} - G_2^0$, the propagator $S^{-1} = G^{-1} - \Pi$, $G$ real, static.

Assuming the inverse exists we have two identities:

$$S = S^*S^{-1}S \text{ and } S^* = S^*S^{-1}S$$

$$S_R = S^*S^{-1}_R S, \quad S_I = -S^*S^{-1}_I S,$$

With definition $S^{-1} = G^{-1} - \Pi$ follows off-shell optical theorem:

$$(S^{-1}_R)' = -\Pi'_R \quad \text{and} \quad S^{-1}_I = -\Pi_I$$

Using the fact that $G$ is a real constant, we have:

$$S'_R = S^*S^{-1}_R S + S^*(S^{-1}_R)' S + S^*S^{-1}_R S'$$

$$= S^*\left(\frac{S^{-1}_R + iS^{-1}_I}{S^{-1}} - iS^{-1}_I\right)S + S^*(S^{-1}_R)' S + S^*(\frac{S^{-1}_R - iS^{-1}_I}{S^*}) S'$$

$$= S^*\left(2S'_R - iS^*\Pi IS' + iS^*\Pi IS'\right),$$

$$S'_R \Pi I = S^*\Pi' R S I + iS^*\Pi IS' I - iS'^*\Pi I S' I, \quad 2\Im[\Pi I S' IS' I]$$
Φ-derivable Q-M-D PNJL model, 2-loop approximation

\[ \Omega = \frac{1}{2} T \frac{1}{V} \sum_{i=Q,M,D} c_i \text{Tr} \{ \ln [S_i^{-1}] + [S_i \Pi_i] \} + \Phi [S_Q, S_M, S_D] , \]

\[ S_i^{-1}(iz_n, q) = S_i^{-1,0}(iz_n, q) - \Pi_i(iz_n, q) , \quad \frac{\delta \Omega}{\delta S_i} = 0 , \quad \text{if} \quad \Pi_i = \frac{\delta \Omega}{\delta S_i} . \]

\[ \tilde{\Omega} = \frac{1}{2} T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega) \text{Tr} \{ \text{Im} \ln [S_i^{-1}] + [\text{Re} S_i \text{Im} \Pi_i] \} + \tilde{\Omega} , \]

\[ S = -\frac{\partial \Omega}{\partial T} = \sum_i S_i + \tilde{S} , \]

\[ N = -\frac{\partial \Omega}{\partial \mu} = \sum_i N_i + \tilde{N} . \]
Φ-derivable Q-M-D PNJL model, 2-loop approximation

\[ \Omega = \frac{1}{2V} \sum_{i=Q,M,D} c_i \text{Tr} \{ \ln [S_i^{-1}] + [S_i \Pi_i] \} + \Phi [S_Q, S_M, S_D] , \]

\[ S_i^{-1} (iz_n, \mathbf{q}) = S_{i,0}^{-1} (iz_n, \mathbf{q}) - \Pi_i (iz_n, \mathbf{q}) , \quad \frac{\delta \Omega}{\delta S_i} = 0 , \text{ if } \Pi_i = \frac{\delta \Omega}{\delta S_i} . \]

\[ \tilde{\Omega} = \frac{1}{2} T \sum_{i=Q,M,D} \int \frac{d^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega) \text{Tr} \{ \text{Im} \ln [S_i^{-1}] + [\text{Re} S_i \text{Im} \Pi_i] \} + \tilde{\Omega} \]

\[ S = -\frac{\partial \Omega}{\partial T} = \sum_i S_i + \delta \]

\[ N = -\frac{\partial \Omega}{\partial \mu} = \sum_i N_i + \Lambda . \]
Φ-derivable Q-M-D PNJL model, 2-loop approximation

\[(\text{Im} \ln S^{-1})' = -\text{Im} (S \Pi' I) = \frac{S' R \Pi_I - S_I \Pi'_ R - (\Pi_I S'_ R + S_R \Pi'_ I)}{2 \text{Im}(S \Pi I S^* \Pi I)},\]

Use optical theorems...

\[S \Pi_I = \sin \delta e^{i \delta'}, \quad S^* \Pi_I = -i \delta' \sin \delta e^{-i \delta'}, \quad 2 \text{Im}(S \Pi I S^* \Pi I) = -2 \delta' \sin^2 \delta.\]

Generalized Beth-Uhlenbeck EoS

\[\Omega = -\sum_{i=Q,M,D} d_i \int \frac{\text{d}^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\text{d} \omega}{2\pi} T \ln[1 - e^{-(\omega - \mu_i)/T}] \sin^2 \delta_i(\omega, q) \frac{\partial \delta_i(\omega, q)}{\partial \omega},\]

Effect of the \(\sin^2\) term ... example: Breit-Wigner ...

\[\delta_i(\omega) = -\arctan \left[ \frac{\omega_i \Gamma_i}{\omega^2 - \omega_i^2} \right], \quad \frac{\partial \delta_i(\omega)}{\partial \omega} = \frac{2 \omega \omega_i \Gamma_i}{(\omega^2 - \omega_i^2)^2 + \omega_i^2 \Gamma_i^2},\]

\[\sin^2 \delta_i(\omega) \frac{\partial \delta_i(\omega)}{\partial \omega} = \frac{2 \omega (\omega_i \Gamma_i)^3}{[(\omega^2 - \omega_i^2)^2 + \omega_i^2 \Gamma_i^2]}.\]

Cluster expansion in the 2PI formalism

- Derivable approach to the grand canonical thermodynamic potential
  \[ J = -\text{Tr}\{\ln(-G_1)\} - \text{Tr}\{\Sigma_1 G_1\} + \text{Tr}\{\ln(-G_2)\} + \text{Tr}\{\Sigma_2 G_2\} + \Phi[G_1, G_2] \]

with full propagators:
\[ G_1^{-1}(1, z) = z - E_1(p_1) - \Sigma_1(1, z); \quad G_2^{-1}(12, 1'2', z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12, 1'2', z) \]
and selfenergies
\[ \Sigma_1(1, 1') = \frac{\delta\Phi}{\delta G_1(1, 1')}; \quad \Sigma_2(12, 1'2', z) = \frac{\delta\Phi}{\delta G_2^{-1}(12, 1'2', z)}. \]

Because of stationarity equivalent to
\[ n = -\frac{1}{\Omega} \frac{\partial J}{\partial \mu} = \frac{1}{\Omega} \sum_1^{\infty} \int_0^{\infty} \frac{d\omega}{\pi} f_1(\omega) S_1(1, \omega), \]
(baryon number conservation)

- Generalization to A-nucleon clusters in nuclear matter
\[ \Omega = \sum_A (-1)^A \left[ \text{Tr} \ln (-G_A^{-1}) + \text{Tr} (\Sigma_A G_A) \right] + \Phi, \]
\[ G_A^{-1} = G_A^{(0)-1} - \Sigma_A, \quad \Sigma_A(1 \ldots A, 1' \ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1 \ldots A, 1' \ldots A', z_A)}. \]
Cluster expansion in the 2PI formalism

**A) Choice of the  Φ-functional:**
- 2-particle irreducible diagrams
- closed 2-loop diagram involving 3 cluster propagators (A, B, A+B) and 2 vertices
- equivalent to 1 T-matrix + 2 propagators

![Diagram of 2-loop diagram involving cluster propagators](image)

**B) Ansatz for thermodynamic potential:**
\[
\Omega = \sum_A (-1)^A \left[ \text{Tr} \ln (-G_A^{-1}) + \text{Tr} (\Sigma_A G_A) \right] + \sum_{A,B} \Phi[G_A, G_B, G_{A+B}],
\]

\[
G_A^{-1} = G_A^{(0)-1} - \Sigma_A, \quad \Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta \Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}.
\]

**C) Check: conservation laws, e.g.:**
(correspondence to GF formalism)

\[
n = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu} = \frac{1}{V} \sum_1^{\infty} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} f_1(\omega) A_1(1, \omega)
\]
Cluster virial expansion in the 2PI formalism, Examples:

A) Deuterons in nuclear matter:

B) Mesons in quark matter:

C) Nucleons in quark matter:

D) Nucleons and mesons (hadron resonance gas) in quark matter:

B) + C) +
Example B: Mesons in quark matter

\[ T_M^{-1}(q, \omega + i\eta) = G_S^{-1} - \Pi_M(q, \omega + i\eta) = |T_M(q, \omega)|^{-1} e^{-i\delta_M(q, \omega)}, \quad \delta_M(q, \omega) = \arctan(\Im T_M/\Re T_M) \]

\[ \Omega = \Omega_{MF} + \Omega_M, \quad \sigma_{MF} = 2N_f N_c G_S \int \frac{d^3 p}{(2\pi)^3} \frac{m}{E_p} [1 - f_-(E_p) - f_+(E_p)], \]

\[ \Omega_M = d_M \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left\{ \omega + 2T \ln \left[ 1 - e^{-\omega/T} \right] 2\sin^2 \delta_M(k, \omega) \frac{\delta_M(k, \omega)}{d\omega} \right\}, \]

\[ \Sigma_M(0, p_0) = d_M \int \frac{d^4 q}{(2\pi)^4} \pi \rho_M(q, q_0) \left\{ \frac{(\gamma_0 + m/E_q)[1 + g(q_0) - f_-(E_q)]}{q_0 - p_0 - E_q - \mu - i\eta} + \frac{(\gamma_0 - m/E_q)[g(q_0) + f_+(E_q)]}{q_0 - p_0 - E_q - \mu - i\eta} \right\}, \]

Example B: Mesons in quark matter

\[ T_M^{-1}(q, \omega + i\eta) = G_S^{-1} - \Pi_M(q, \omega + i\eta) = |T_M(q, \omega)|^{-1} e^{-i\delta_M(q, \omega)}, \quad \delta_M(q, \omega) = \arctan(\Im T_M/\Re T_M) \]

\[ \Omega = \Omega_{MF} + \Omega_M, \quad \sigma_{MF} = 2N_f N_c G_S \int \frac{d^3 p}{(2\pi)^3} \frac{m}{E_p} [1 - f_-(E_p) - f_+(E_p)], \]

\[ \Omega_{MF} = \frac{\sigma_{MF}^2}{4G_s} - 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left[ E_p + T \ln \left(1 + e^{-\frac{(E_p - \Sigma - \mu)}{T}}\right) + T \ln \left(1 + e^{-\frac{(E_p + \Sigma - \mu)}{T}}\right)\right], \]

\[ \Omega_M = d_M \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left\{ \omega + 2T \ln \left[1 - e^{-\frac{\omega}{T}}\right] \left[ 2\sin^2 \frac{\delta_M(k, \omega)}{d\omega} \right] \right\} \]

\[ \Sigma_M(0, p_0) = d_M \int \frac{d^4 q}{(2\pi)^4} \pi \rho_M(q, q_0) \left\{ \frac{(\gamma_0 + m/E_q)[1 + g(q_0) - f_-(E_q)]}{q_0 - p_0 + E_q - \mu - i\eta} + \frac{(\gamma_0 - m/E_q)[g(q_0) + f_+(E_q)]}{q_0 - p_0 - E_q - \mu - i\eta} \right\} \]

Example B: Mesons in quark matter

\[ \Omega_X(T, \mu) = -d_X \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} n_X^{-}(\omega) \delta_X(\omega, \mathbf{q}), \]

\[ \int_0^{\infty} \frac{1}{\pi} \frac{d\delta_X(\omega; T)}{d\omega} = 0 = \int_0^{\omega_{\text{thr}}(T)} \frac{1}{\pi} \frac{d\delta_X(\omega; T)}{d\omega} + \frac{1}{\pi} \int_{\omega_{\text{thr}}(T)}^{\infty} \frac{d\delta_X(\omega; T)}{d\omega}, \]

\[ p_\pi(T) = -d_\pi T \int \frac{d^3q}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{\pi} \ln \left(1 - e^{-\omega/T}\right) \frac{d\delta_\pi(\omega, \mathbf{q})}{d\omega}, \]

Example B*: Mesons+diquarks in quark matter

$$\Omega_Q = -\frac{2N_c N_f}{3} \int \frac{dp}{2\pi^2 E_p} [f_\Phi^+(E_p) + f_\Phi^-(E_p)], \quad f_\Phi^+(E_p) = \frac{(\Phi + 2\Phi Y)Y + Y^3}{1 + 3(\Phi + \Phi Y)Y + Y^3}, \quad Y = e^{-(E_p - \mu)/T}$$

$$\Omega_D = -3 \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} [g_{\Phi^+}(\omega) + g_{\Phi^-}(\omega)] \delta_D(\omega), \quad g_{\Phi^+}(\omega) = \frac{(\Phi - 2\Phi X)X + X^3}{1 - 3(\Phi - \Phi X)X - X^3}$$

Suppression of colored states by Polyakov-loop $\Phi$

Confinement: $\Phi=0$

Example D: Hadron resonance gas – effect. model

Φ-functional:

Selfenergies:
Example D: Mott HRG / PNJL – effective model

\[ P_{\text{PNJL}}(T) = P_{\text{FG}}(T) + \mathcal{V}[\Phi; T] , \]

\[ P_{\text{FG}}(T) = 4 \sum_{u,d,s} \frac{d^3p}{(2\pi)^3} T \ln \left[ 1 + 3\Phi(Y + Y^2) + Y^3 \right] \]

\[ Y(E_p) = \exp\left(-\frac{E_p}{T}\right) \]

\[ \mathcal{V}[\Phi; T] = -\frac{a(T)}{2} \Phi^2 + b(T) \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4) \]

T-dependent quark masses from fit to LQCD

\[ m(T) = [m(0) - m_0] \Delta_{l,s}(T) + m_0 , \]

\[ m_s(T) = m(T) + m_s - m_0 , \]

\[ \Delta_{l,s}(T) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{T - T_c}{\delta_T} \right) \right] \]

\[ T_c = 154 \text{ MeV} \quad \delta_T = 26 \text{ MeV} \]

Example D: Mott HRG / PNJL – effective model

Hadrons + Mott effect

\[ P_i(T) = \mp d_i \frac{p^2}{2\pi^2} \int_0^\infty dM \ln \left( 1 \mp e^{-\frac{\sqrt{p^2+M^2}}{T}} \right) \frac{2}{\pi} \sin^2 \delta_i(M^2; T) \frac{d\delta_i(M^2; T)}{dM} \]

Quarks + rescattering effects

\[ P_{FG}^*(T) = 4N_c \sum_{q=u,d,s} \int \frac{dp}{2\pi^2} \int \frac{d\omega}{\pi} f_\Phi(\omega) \delta_q(\omega; \gamma), \]

\[ f_\Phi(\omega) = \frac{\Phi(1 + 2Y)Y + Y^3}{1 + 3\Phi(1 + Y)Y + Y^3}, \]

\[ \delta_q(\omega; \gamma) = \frac{\pi}{2} + \arctan \left[ \frac{\omega - \sqrt{p^2 + m_q^2}}{\gamma} \right] \]
Example D: Mott HRG / PNJL – effective model

- Mott dissociation of hadrons (here pi, K) at the Chiral restoration temperature $T_c = 153$ MeV

- Asymptotic behaviour of quark-gluon Pressure can be adjusted with rescattering Parameter gamma

- Very good correspondence between lattice QCD Thermodynamics and improved MHRG/PNJL model; Hadronic and partonic contributions quantified

Example C: Nucleons in quark matter

Φ-functional

nucleon selfenergy

quark selfenergy

quark exchange interaction between nucleons:
Example C: Nucleons in quark matter

- $\Phi$-functional
- Nucleon selfenergy
- Quark selfenergy

Not new! Already contained in above diagrams!

Quark exchange interaction between nucleons:
Example C: Nucleons in quark matter

\[ \Phi \text{-functional} \quad \text{nucleon selfenergy} \quad \text{quark selfenergy} \]

\[ = \quad \text{with} \]

quark exchange interaction between nucleons:
Intermezzo: Structure of the baryon?

12-Apostle Church, Kars
Intermezzo: Structure of the baryon?

12-Apostle Church, Kars
Intermezzo: Structure of the baryon?

\[ Z_{\text{fluct}} = \int D\Delta \, D\Delta \, D\phi \exp \left\{ -\frac{|\Delta|^2}{4G_D} - \frac{\phi^2}{4G} - Tr \ln S^{-1}[\Delta, \Delta^\dagger, \phi] \right\} \]

Cahill, ibid. 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29

Borromean ?!!
Example C: Pauli blocking among baryons

a) Low density: Fermi gas of nucleons (baryons)

b) \(\sim\) saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)


Nucleon (baryon) self-energy --> Energy shift

\[
\Delta E_{\text{Pauli}}^{\nu P} = \sum_{123} |\psi_{\nu P}(123)|^2 \left[ E(1) + E(2) + E(3) - E_{\nu P}^0 \right] \left[ f_{\alpha_1}(1) + f_{\alpha_2}(2) + f_{\alpha_3}(3) \right] \\
+ \sum_{123} \sum_{456} \sum_{\nu P'} \langle \psi_{\nu P}(123) | \psi_{\nu P'}(456) \rangle f_{\beta}(E_{\nu P}^0) \left[ \delta_{\beta\beta} \psi_{\nu P}(123) \psi_{\nu P'}^*(456) - \psi_{\nu P}(453) \psi_{\nu P'}^*(126) \right] \\
\times \left[ E(1) + E(2) + E(3) + E(4) + E(5) + E(6) - E_{\nu P}^0 - E_{\nu P'}^0 \right] \\
= \Delta E_{\text{Pauli, free}}^{P,\nu} + \Delta E_{\text{Pauli, bound}}^{P,\nu}
\]
Pauli quenching effects in a simple string model of quark/nuclear matter

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(Received 16 December 1985)
Example C: Pauli blocking in NM – details

New aspect: chiral restoration --> dropping quark mass

Increased baryon swelling at supersaturation densities:
--> dramatic enhancement of the Pauli repulsion!!

Example C: Pauli blocking in NM – results
Example C: Pauli blocking in NM – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:
- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high n
- quark exchange among baryons -> six-quark wavefunction -> “bag melting” -> deconfinement

Chiral stiffening of nuclear matter --> reduces onset density for deconfinement

Hybrid EoS:
Convenient generalization of RMF models,
Take care: eventually aspects of quark exchange already in density dependent vertices!

Other baryons:
- hyperons
- deltas
Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

Relativistic generalization:
Box diagrams of quark-diquark model ...

Support a CEP in QCD phase diagram with Astrophysics?

Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!
Two high-mass pulsars with $M \sim 2M_{\text{sun}}$

$M=2.01 \pm 0.04 \text{ Msun}$  
$M=1.928 \pm 0.017 \text{ Msun}$

PSR J0348+0432  
PSR J1614-2230

What if they were high-mass twin stars?  
→ radius measurement required!  
→ NICER (2017)
Two high-mass pulsars with $M \sim 2M_{\text{sun}}$

Neutron Star
Hadronic matter
$M_{\text{star}} = 2.0 \ M_{\odot}$
$R_{\text{star}} = 13.9 \text{ km}$

Hybrid Star
Hadronic and Quark matter
$M_{\text{star}} = 2.0 \ M_{\odot}$
$R_{\text{star}} = 11.1 \text{ km}$
$R_{\text{quark-core}} = 7.36 \text{ km}$
Motivation – Neutron stars (Twins?)

- Star configurations with same masses, but different radii
- New class of EOS, that features high mass twins
- NASA NICER mission: radii measurements ~ 0.5 km
- Existence of twins implies 1st order phase-transition and hence a critical point

Support a CEP in QCD phase diagram with Astrophysics?

Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!
First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “third family of CS”.

Measuring two disconnected populations of compact stars in the M-R diagram would be the detection of a first order phase transition in compact star matter and thus the indirect proof for the existence of a critical endpoint (CEP) in the QCD phase diagram!
CEP in the QCD phase diagram: HIC vs. Astrophysics

Towards “measuring” the EoS in the T – mu plane
(QCD phase diagram)

Speed-of-sound diagram from the INT program in Seattle, Summer 2016

Interpolation between lattice QCD and Compact star physics (2 M_sun)

Towards “measuring” the EoS in the T – mu plane (QCD phase diagram)

Speed-of-sound diagram from the INT program in Seattle, Summer 2016

Interpolation between lattice QCD and Compact star physics (2 M_sun)
Conclusion:

High-mass twins (HMTs) with quark matter cores can be obtained within different hybrid star EoS models, e.g.,
- constant speed of sound
- higher order NJL
- piecewise polytrope
- density functional

HMTs require stiff hadronic and quark matter EoS with a strong phase transition (PT)

Existence of HMTs can be verified, e.g., by precise compact star mass and radius observations (and a bit of good luck) → Indicator for strong PT !!

Extremely interesting scenarios possible for dynamical evolution of isolated (spin-down and accretion) and binary (NS-NS merger) compact stars

Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics
29 member countries!! (MP1304)
"Theory of HOt Matter in Relativistic Heavy-Ion Collisions"

**New: THOR!**

**21 member countries! (CA15213)**

**Kick-off: Brussels, October 17, 2016**
Network: CA16214

Newest: PHAROS

http://www.cost.eu/COST_Actions/ca/CA16214

Kick-off: Brussels, late 2017
Inside: Topical Issue on Exotic Matter in Neutron Stars
edited by David Blaschke, Jürgen Schaffner-Bielich and Hans-Josef Schulze

From:
Neutron star interiors: Theory and reality by J.R. Stone (left)

Phenomenological neutron star equations of state: 3-window modeling of QCD matter by T. Kojio (right)