# Cluster virial expansion for quark-nuclear matter and neutron star structure

*David.Blaschke@gmail.com*
*University of Wroclaw, Poland & JINR Dubna & MEPhI Moscow, Russia*

1. **Introduction: Beth-Uhlenbeck (BU) and Generalized BU**

2. **GBU from Φ-derivable approach: 2-loop approximation**

3. **GBU EoS for quark-hadron matter in (P)NJL-type models**

4. **Application to neutron stars: high-mass twin stars!**

Skobeltsyn Institute for Nuclear Physics, Moscow State U, 17.10. 2017

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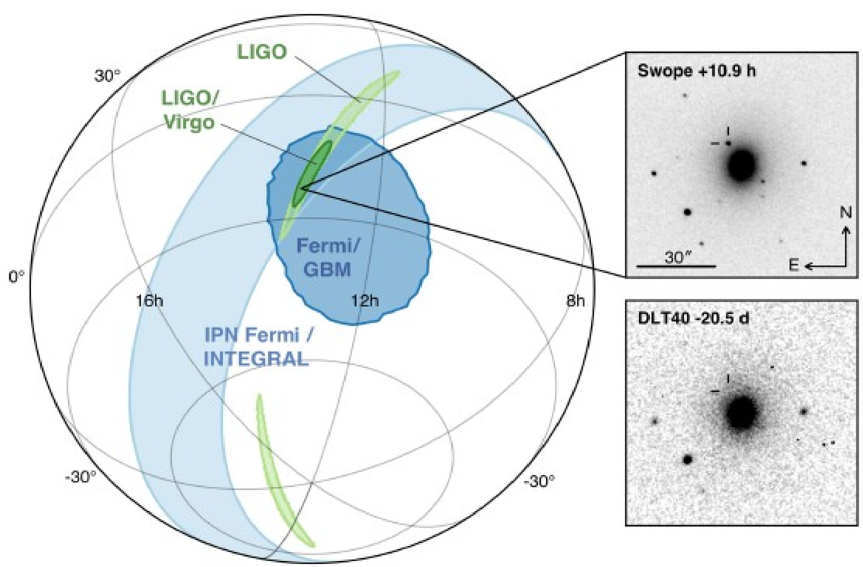
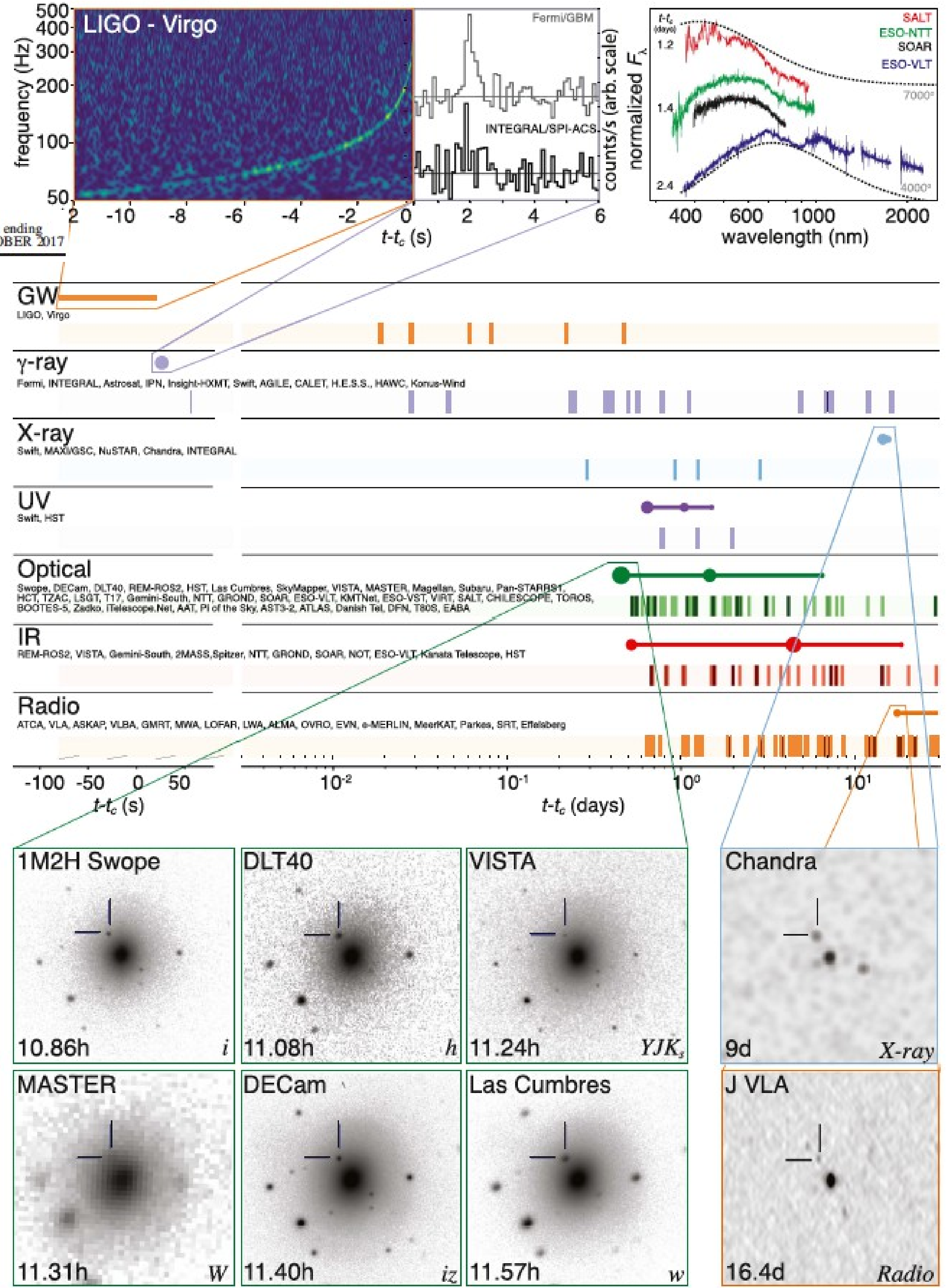
# <u>Breaking News:</u> First observation ... of NS – NS mergers in gravitational waves and all el-magn. spectrum

PRL 119, 161101 (2017) Selected for a Viewpoint in Physics PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

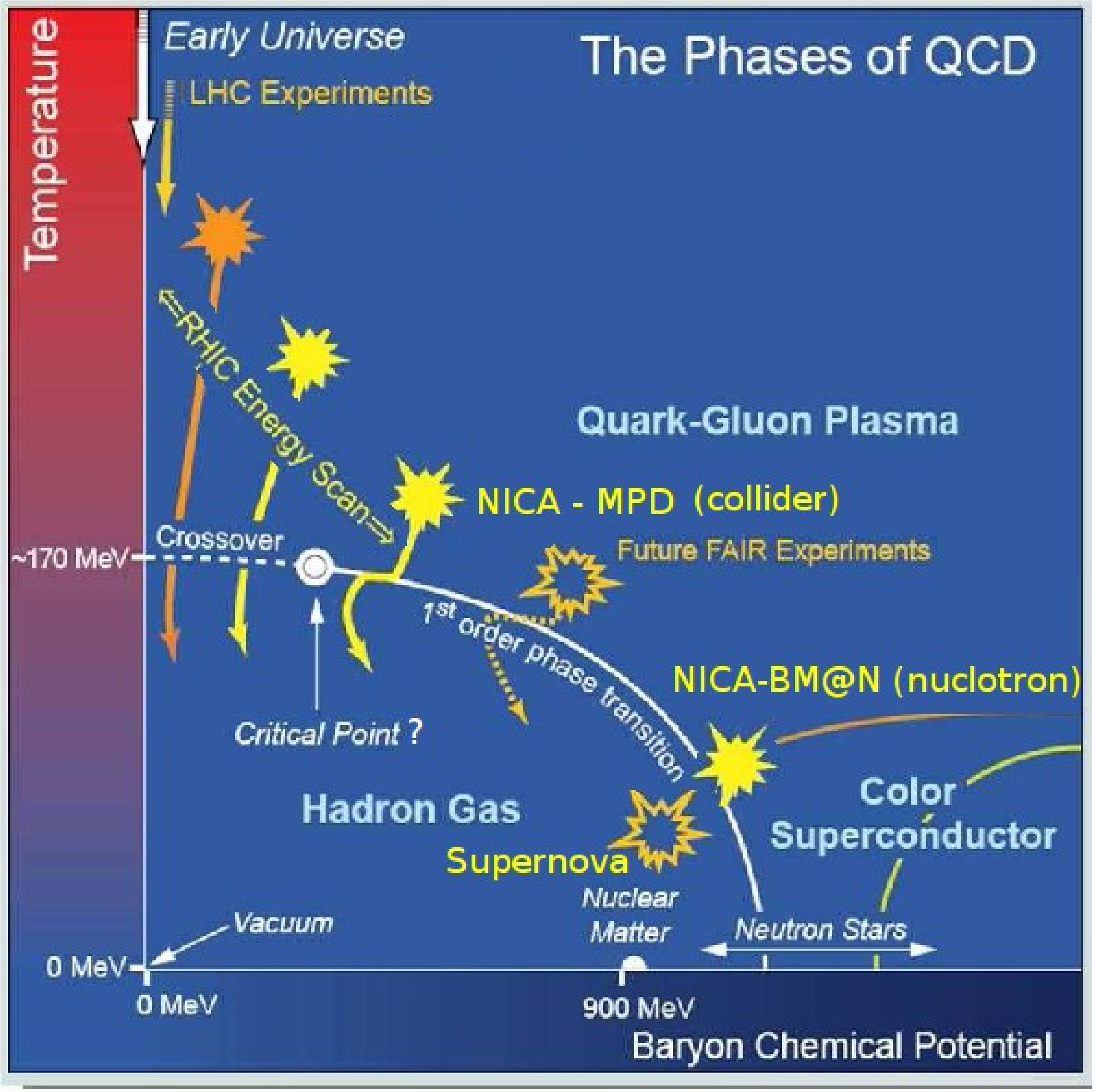
| | Low-spin priors ($|\chi| \leq 0.05$) | High-spin priors ($|\chi| \leq 0.89$) |
|---|---|---|
| Primary mass $m_1$ | 1.36–1.60 $M_\odot$ | 1.36–2.26 $M_\odot$ |
| Secondary mass $m_2$ | 1.17–1.36 $M_\odot$ | 0.86–1.36 $M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.188^{+0.004}_{-0.002} M_\odot$ | $1.188^{+0.004}_{-0.002} M_\odot$ |
| Mass ratio $m_2/m_1$ | 0.7–1.0 | 0.4–1.0 |
| Total mass $m_{tot}$ | $2.74^{+0.04}_{-0.01} M_\odot$ | $2.82^{+0.47}_{-0.09} M_\odot$ |
| Radiated energy $E_{rad}$ | $> 0.025 M_\odot c^2$ | $> 0.025 M_\odot c^2$ |
| Luminosity distance $D_L$ | $40^{+8}_{-14}$ Mpc | $40^{+8}_{-14}$ Mpc |

The Astrophysical Journal Letters, 848:L12 (59pp), 2017 October 20

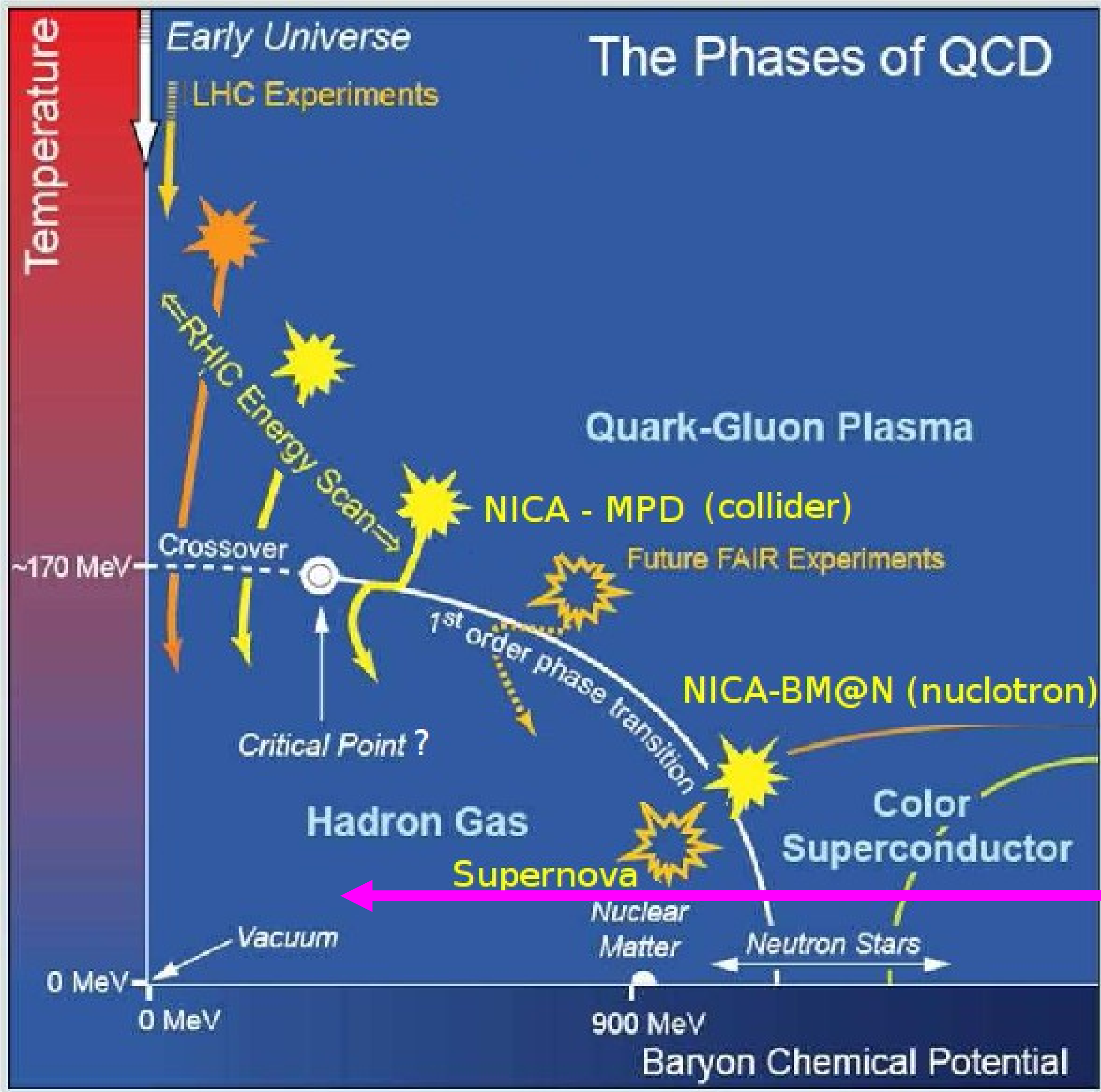


# The Goal: Theory of the QCD Phase Diagram



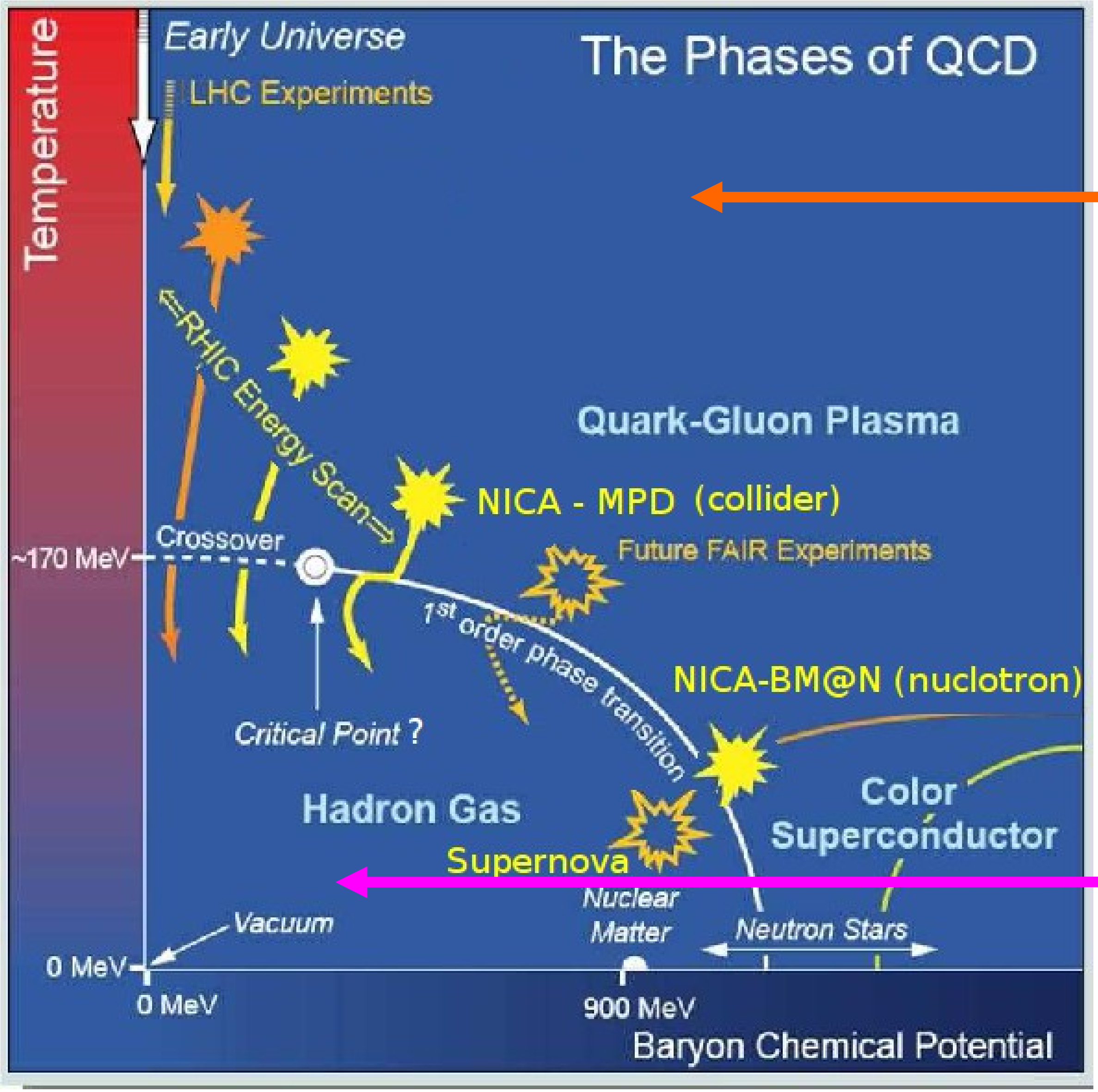
# The Goal: Theory of the QCD Phase Diagram




**Statistical Model of Hadron Resonance Gas**

Well established for Description of chemical freezeout

# The Goal: Theory of the QCD Phase Diagram



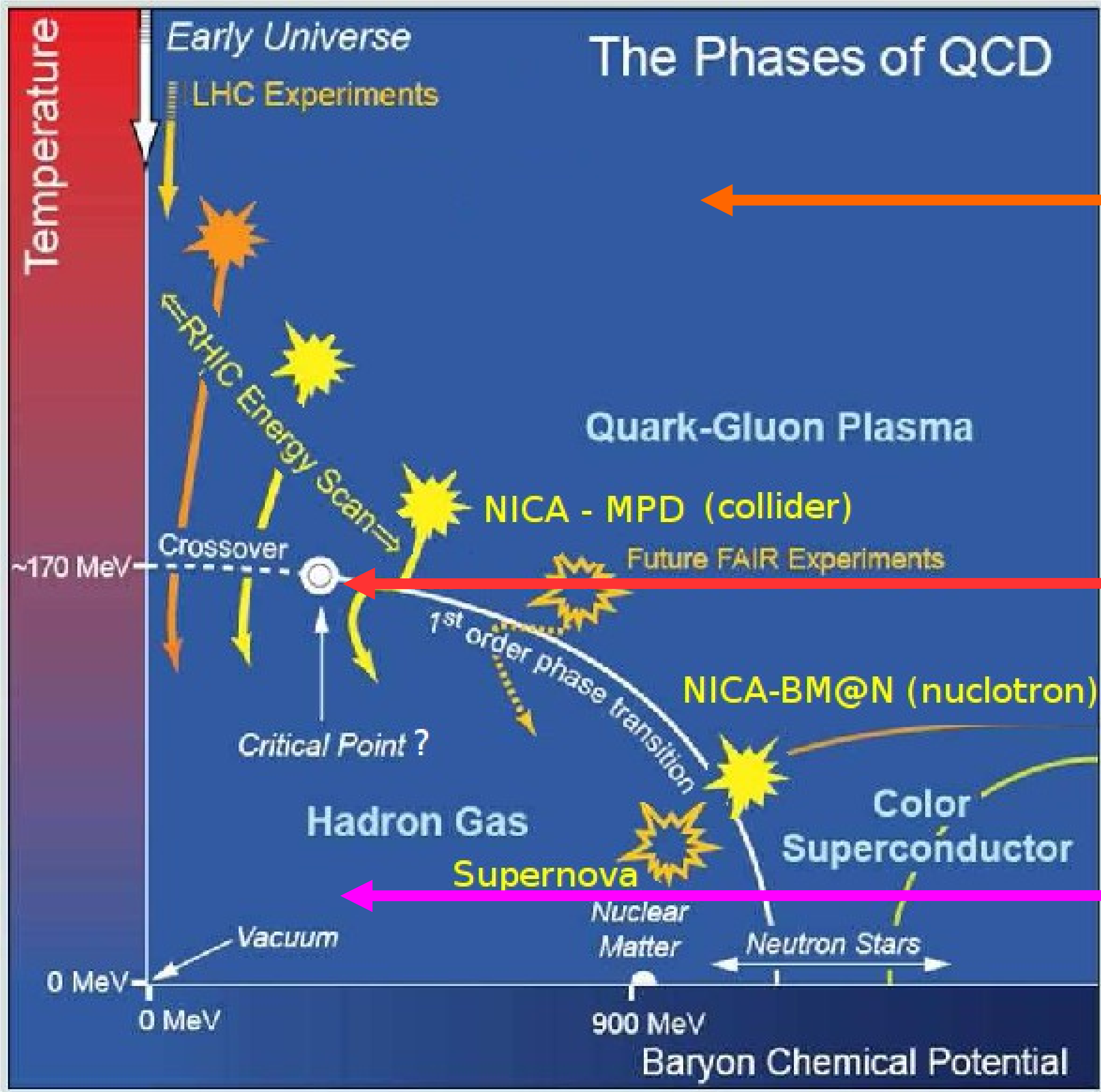

**Perturbative QCD**

Approximately selfconsistent HTL resummation ($T > 2.5\ T_c$ , $\mu > 1500$ MeV)

**Statistical Model of Hadron Resonance Gas**

Well established for Description of chemical freezeout

# The Goal: Theory of the QCD Phase Diagram




**Perturbative QCD**

Approximately selfconsistent HTL resummation
($T > 2.5$ Tc , $\mu > 1500$ MeV)

**QCD Phase transition(s)**

Mott dissociation of hadrons, Deconfinement, $\chi$SR

**Statistical Model of Hadron Resonance Gas**

Well established for Description of chemical freezeout

# Introduction: Beth-Uhlenbeck vs. Generalized BU

**Beth-Uhlenbeck: 2nd virial coefficient B(T)**

$$pV = NkT\left(1 + \frac{B(T)}{V} + \frac{C(T)}{V^2} + \dots\right)$$

**BU for virial expansion of density:**

$$n(\mu, T) = n_{\rm free}(\mu, T) + 2n_{\rm corr}(\mu, T)$$

$$n_{\rm free}(\mu, T) = 4\int \frac{d^3p}{h^3} e^{-(p^2/2m-\mu)/T} = \frac{4}{\lambda^3} e^{\mu/T}$$

$$n_{\rm corr}(\mu, T) = \int \frac{d^3\mathbf{P}}{h^3} e^{-(P^2/4m-2\mu)/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E)$$

$$= \frac{2^{3/2}}{\lambda^3} e^{2\mu/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E).$$

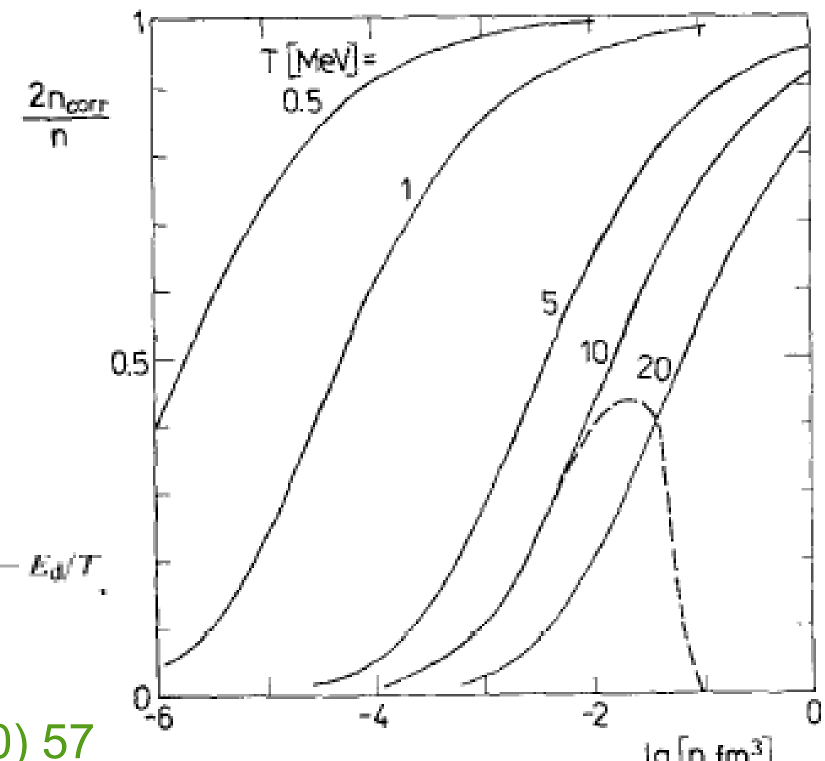
**Density of states: bound and scattering part**

$$D(E) = \sum_\alpha c_\alpha \left[\pi\delta(E - E_\alpha) + \frac{d}{dE}\delta_\alpha(E)\right],$$

**Example: Deuterons in nuclear matter**

$$n = n_{\rm free} + 2n_{\rm free}^2 I(T)$$

$$I(T) = \lambda^3 \frac{2^{1/2}}{8}\left[3(e^{-E_d/T} - 1) + \int_0^\infty \frac{dE}{\pi T} e^{-E/T} \sum_\alpha c_\alpha \delta_\alpha(E)\right].$$




**For T<<$E_d$ :** $n = n_{\rm free} + 2n_{\rm deut}\,,\quad n_{\rm deut} = n_{\rm free}^2 \lambda^3 3\,\frac{2^{1/2}}{8} e^{-E_d/T}.$


E. Beth and G.E. Uhlenbeck, Physica IV (1937) 915;
S. Schmidt, G. Roepke, H. Schulz, Ann. Phys. 202 (1990) 57

# Introduction: Beth-Uhlenbeck vs. Generalized BU

**Thermodynamic Greens function approach:**

$$n(1,\mu_1,T)=\int\frac{dE}{2\pi}f_1(E)\,A(1,E)$$

$$A(1,E)=\frac{2\Sigma_I(1,E-i0)}{(E-E(1)-\Sigma_R(1,E))^2+\Sigma_I(1,E-i0)^2}\approx\frac{2\pi\,\delta(E-e(1))}{1-((d/dz)\,\Sigma_R(1,z))|_{z=e(1)+i0}}-2\Sigma_I(1,E+i0)\frac{d}{dE}\frac{\mathrm{P}}{E-e(1)}$$

**Density formula**
**(free and corr. Quasiparticles):**

$$n(\mu,T)=n_{\text{free}}(\mu,T)+2n_{\text{corr}}(\mu,T),$$

$$\Sigma(1,z_\nu)=T\sum_2\sum_{z_{\nu'}}[T(1212,z_\nu+z_{\nu'})-\text{ex}]\,G(2,z_{\nu'})$$

$$n_{\text{corr}}(\mu,T)=\int\frac{dE}{2\pi}g(E)\,F(E)$$

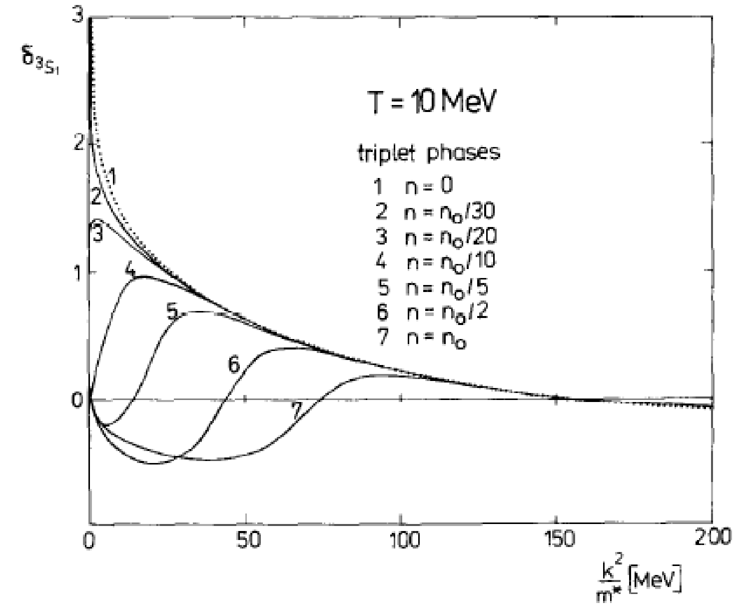
$$F(E)=F_{\text{deut}}(E)+\frac{2}{4\pi}\sum_\alpha c_\alpha F_\alpha(E),$$

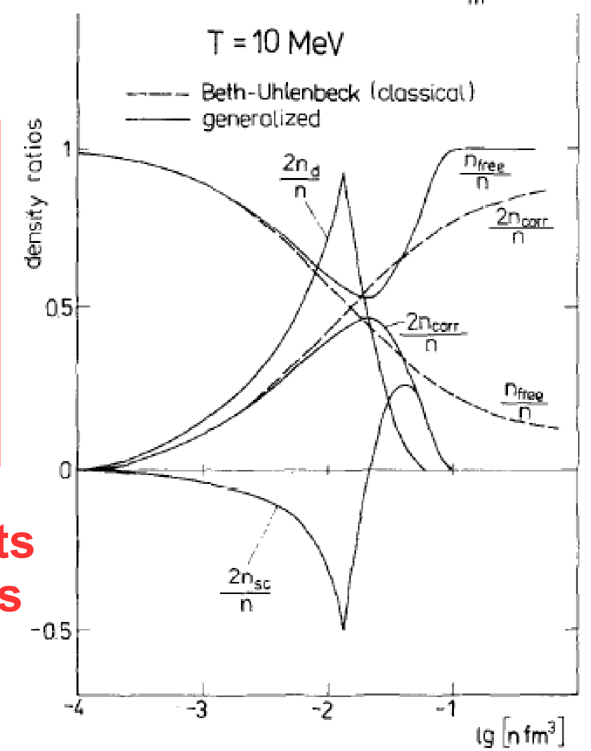
$$F_{\text{deut}}(E)=6\sum_{\mathbf{K}>\mathbf{K}^{\text{Mott}}}\pi\delta(E-E_b(K,\mu,T)).$$

$$F(E)=\sum_{12}[1-f(e(1))-f(e(2))]\cdot\left[(T_I(1212,E+i0)-\text{ex})\frac{d}{dE}\frac{\mathrm{P}}{e(1)+e(2)-E}-\pi\delta(E-e(1)-e(2))\frac{d}{dE}(T_R(1212,E+i0)-\text{ex})\right].$$

$$F_\alpha(E)=8\pi\sum_{\mathbf{K}}\sin^2\delta_\alpha(E,K,\mu,T)\frac{d}{dE}\delta_\alpha(E,K,\mu,T).$$

**The $\sin^2\delta$ term accounts for quasiparticle effects**



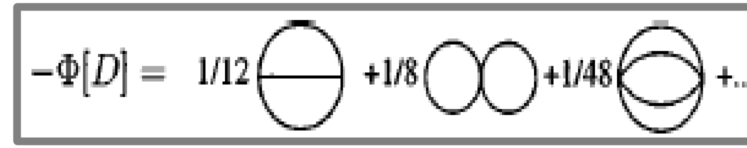

S. Schmidt, G. Röpke, H. Schulz, Ann. Phys. 202 (1990) 57

# Φ-derivable approach, 2-loop approximation

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

**Skeleton expansion for thermodynamic potential and entropy**

$$\beta\Omega[D] = -\log Z = \frac{1}{2}\mathrm{Tr}\log D^{-1} - \frac{1}{2}\mathrm{Tr}\,\Pi D + \Phi[D]$$




Inv. Temp: 1/T    trace in conf. Space    self-energy related to D

**Dyson equation:** $D^{-1} = D_0^{-1} + \Pi$    Free propagator Do is known

Essential property of Ω[D] is Stationarity under variation of D:  $\delta\,\Omega[D] / \delta D = 0$

This implies $\delta\,\Phi[D] / \delta D = 1/2\ \Pi$

Physical propagator and selfenergy are defined self-consistently !

**Self-consistent approximations are defined by the choice of Φ**

→ **Φ – derivable theories**

G. Baym, Phys. Rev. 127 (1962) 1391; Vanderheyden & Baym; J. Stat. Phys. 93, 843 (1998)

# Approximately selfconsistent thermodynamics

Matsubara summation:

$$\Omega/V = \int \frac{d^4k}{(2\pi)^4} n(\omega)[\mathrm{Im}\log(-\omega^2+k^2+\Pi) - \mathrm{Im}\,\Pi D] \;\; + T\Phi[D]/V$$

Analytic properties:

$$D(\omega,k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0,k)}{k_0-\omega}, \qquad \mathrm{Im}\,D(\omega,k) \equiv \mathrm{Im}\,D(\omega+i\epsilon,k) = \frac{\rho(\omega,k)}{2}.$$

Thermodynamics from entropy density: $\mathcal{S} = -\partial(\Omega/V)/\partial T$

$$\mathcal{S} = -\int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\log D^{-1}(\omega,k) \; + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\,\Pi(\omega,k) \mathrm{Re}\,D(\omega,k) + \mathcal{S}'$$

$$\mathcal{S}' \equiv -\left.\frac{\partial(T\Phi/V)}{\partial T}\right|_D + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Re}\,\Pi\,\mathrm{Im}\,D \longrightarrow 0$$

for two-loop skeleton diagrams

Loosely speaking: S' accounts for residual interactions of “independent quasiparticles”

$d/d\omega\ [\ \mathrm{Im}\log D^{-1} + \mathrm{Im}\Pi\ \mathrm{Re}D\ ] = 2\ \mathrm{Im}\ [\ D\ \mathrm{Im}\Pi\ (d/d\omega\ D^*)\ \mathrm{Im}\Pi] = 2 \sin^2\delta\ d\delta/d\omega$ , for $D = |D|e^{i\delta}$

D. B., in preparation (2016)

# Proof of cancellations resulting in S'=0 (I)

$$S' \equiv -\frac{\partial(T\Phi/V)}{\partial T}\bigg|_D + \int \frac{d^4k}{(2\pi)^4}\left\{\frac{\partial n(\omega)}{\partial T}\mathrm{Re}\Pi\,\mathrm{Im}D\right\}$$

First term

$$-\frac{T}{V}\Phi = \frac{g^2}{12}T^2 \sum_{\omega_1,\omega_2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^6} D(\omega_1,|k_1|)D(\omega_2,|k_2|)D(-\omega_1-\omega_2,|-k_1-k_2|)$$

Spectral representation

$$D(\omega,k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi}\frac{\rho(k_0,k)}{k_0-\omega}$$

Matsubara sums

$$-\frac{T}{V}\Phi = \frac{g^2}{12}T^2 \sum_{\omega_1,\omega_2}\int \frac{d^4k d^4k' d^4k"}{(2\pi)^9}\delta^{(3)}(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')\rho(k)\rho(k')\rho(k'')\frac{-1}{\omega_1-k_0}\frac{-1}{\omega_2-k_0'}\frac{1}{\omega_1+\omega_2+k_0''}$$

Partial fraction decomposition of the three energy denominators and Matsubara summation over $\omega_1$, $\omega_2$ yields:

$$\frac{1}{k_0+k_0'+k_0''}\left\{[n(k_0'')+1][n(k_0)+n(k_0')+1]+n(k_0)n(k_0')\right\}$$

Temperature derivative and renaming variables under the integrals

$$\partial_T\left[n(k_0+n(k_0')+n(k_0'')+n(k_0')n(k_0)+n(k_0')n(k_0'')+n(k_0)n(k_0'')\right] \to 3\partial_T n(k_0)\left[1+n(k_0')+n(k_0'')\right]$$

# Proof of cancellations resulting in S'=0 (II)

Second term:

$$\begin{aligned}\mathrm{Re}\Pi(\omega,q) &= -\frac{g^2}{2}\int\frac{d^3k}{(2\pi)^3}\int\frac{dk_0}{2\pi}\int\frac{dk_0'}{2\pi}\rho(k_0,|k|)\rho(k_0',|\mathbf{k}+\mathbf{q}|)\sum_{\omega_1}\frac{1}{\omega_1-k_0}\,\frac{1}{\omega_1+\omega-k_0'}\\ &= -\frac{g^2}{2}\int\frac{d^3k}{(2\pi)^3}\int\frac{dk_0}{2\pi}\int\frac{dk_0'}{2\pi}\rho(k_0,|k|)\rho(k_0',|\mathbf{k}+\mathbf{q}|)\frac{1+n(k_0)+n(k_0')}{\omega+k_0+k_0'}\end{aligned} \tag{7}$$

$$\begin{aligned}&\int\frac{d^4q}{(2\pi)^4}\frac{\partial n(k_0)}{\partial T}\mathrm{Re}\Pi(\omega,q)\mathrm{Im}D(\omega,q) =\\ &= -\frac{g^2}{2\cdot 2}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^k}{(2\pi)^4}\int\frac{d^4k'}{2\pi}\delta^{(3)}(\mathbf{q}+\mathbf{k}+\mathbf{k}')\rho(q)\rho(k)\rho(k')\partial_T n(q_0)\left[1+n(k_0)+n(k_0')\right]\frac{1}{q_0+k_0+k_0'}\end{aligned} \tag{8}$$

This proves the cancellation of $\mathcal{S}'$ for the scalar theory with cubic selfinteraction in the 2-loop approximation (sunset diagram) for the $\Phi-$ functional.

This cancellation holds as well for the pressure and the density!

For the pressure we obtain

$$p(T) = -\int\frac{d^4q}{(2\pi)^4}n(q_0)\left[\delta(q)-\sin\delta(q)\cos\delta(q)\right] = -\int\frac{d^4q}{(2\pi)^4}T\ln\left(1-\mathrm{e}^{-q_0/T}\right)\frac{\partial\delta(q)}{\partial q_0}\,2\sin^2\delta(q) \tag{9}$$

Note that in the approximation $\delta(q_0,q) = -\arctan[\omega\gamma/(q_0^2-\omega^2)]$ the "spectral distribution" does not correspond to a Lorentzian (Breit-Wigner) function as naïvely expected, but to a "squared Lorentzian"

$$\frac{q_0(\omega\gamma)^3}{[(q_0^2-\omega^2)^2+(\omega\gamma)^2]^2} \tag{10}$$

See, e.g., Vanderheyden & Baym (1998); Morozov & Röpke, Ann. Phys. 324 (2009) 1261

# Approximately selfconsistent HTL resumm. QCD

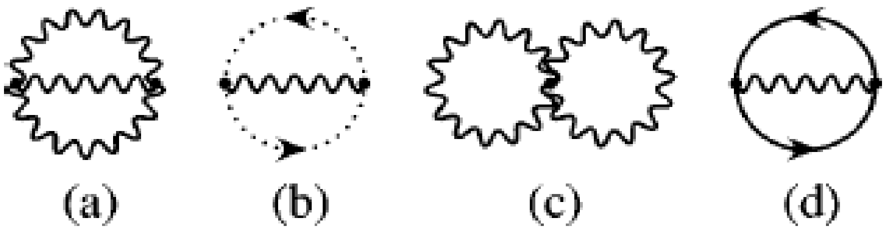
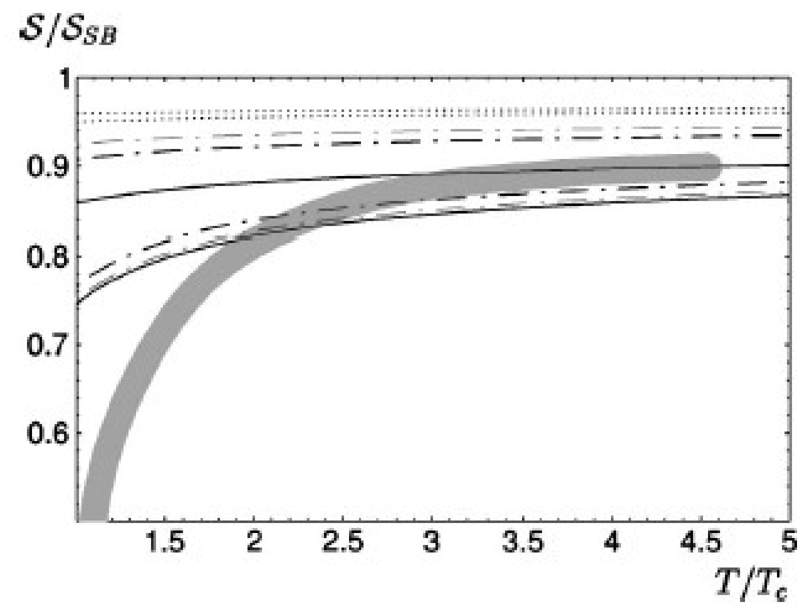



FIG. 3. Diagrams for Φ at 2-loop order in QCD. Wiggly, plain, and dotted lines refer respectively to gluons, quarks, and ghosts.




In ghost-free gauge, HTL resummed QCD thermodyn.

$$\mathcal{S}_2 = -\frac{g^2 N_g T}{48}\left\{\frac{4N+5N_f}{3}T^2 + \frac{3N_f}{\pi^2}\mu^2\right\},$$

$$\mathcal{N}_2 = -\frac{g^2 \mu N_g N_f}{16\pi^2}\left(T^2 + \frac{\mu^2}{\pi^2}\right),$$

$$P_2 = -\frac{g^2 N_g}{32}\left\{\frac{4N+5N_f}{18}T^4 + \frac{N_f}{\pi^2}\mu^2 T^2 + \frac{N_f}{2\pi^4}\mu^4\right\}$$

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

# Generalized Optical Theorems

See derivations for T-matrices by R. Zimmermann & H. Stolz, pss (b) 131, 151 (1985)
Here we consider the analogue of $T^{-1} = V^{-1} - G_2^0$, the propagator $S^{-1} = G^{-1} - \Pi$, G real, static

Assuming the inverse exists we have two identities: $S = S^* {S^*}^{-1} S$ and $S^* = S^* S^{-1} S$

$$\begin{aligned} S_R + \mathrm{i} S_I &= S^*(S_R^{-1} - \mathrm{i} S_I^{-1}) S \,, \\ S_R - \mathrm{i} S_I &= S^*(S_R^{-1} + \mathrm{i} S_I^{-1}) S \,. \end{aligned} \longrightarrow \begin{aligned} S_R &= S^* S_R^{-1} S \,, \\ S_I &= -S^* S_I^{-1} S \,, \end{aligned}$$

With definition $S^{-1} = G^{-1} - \Pi$ follows off-shell optical theorem: $S_I = S^* \Pi_I S = S \Pi_I S^*$

Using the fact that G is a real constant, we have: $(S_R^{-1})' = -\Pi'_R$ and $S_I^{-1} = -\Pi_I$

$$\begin{aligned} S'_R &= S^{*\prime} S_R^{-1} S + S^*(S_R^{-1})' S + S^* S_R^{-1} S' \\ &= S^{*\prime}(\underbrace{S_R^{-1} + \mathrm{i} S_I^{-1}}_{S^{-1}} - \mathrm{i} S_I^{-1}) S + S^*(S_R^{-1})' S + S^*(\underbrace{S_R^{-1} - \mathrm{i} S_I^{-1}}_{S^{*-1}} + \mathrm{i} S_I^{-1}) S' \\ &= \underbrace{S^{*\prime} + S'}_{2S'_R} - \mathrm{i} S^{*\prime} S_I^{-1} S + \mathrm{i} S^* S_I^{-1} S' + S^*(S_R^{-1})' S \\ &= S^* \Pi'_R S - \mathrm{i} S^{*\prime} \Pi_I S + \mathrm{i} S^* \Pi_I S' \,, \end{aligned}$$

Derivative optical theorem:

$$S'_R \Pi_I = \underbrace{S^* \Pi'_R S \Pi_I}_{\Pi'_R S_I} + \underbrace{\mathrm{i} S^* \Pi_I S' \Pi_I - \mathrm{i} S^{*\prime} \Pi_I S \Pi_I}_{2\,\mathrm{Im}[\Pi_I S \Pi_I S^{*\prime}]} \,, \longrightarrow S'_R \Pi_I - \Pi'_R S_I = 2\,\mathrm{Im}\,[\Pi_I S \Pi_I S^{*\prime}]$$

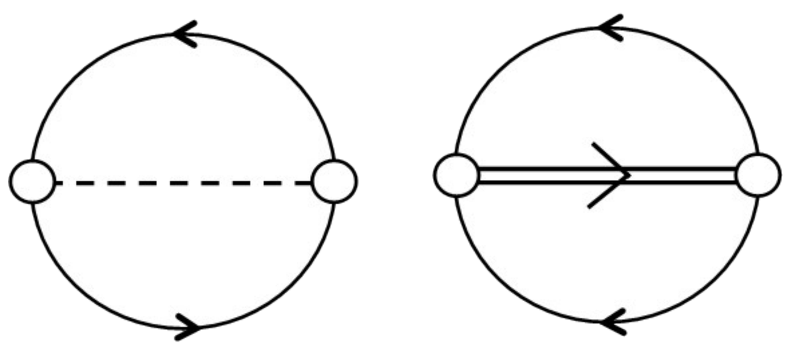
# Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i \, \mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i \Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \ ,$$

$$S_i^{-1}(\mathrm{i}z_n, \mathbf{q}) = S_{i,0}^{-1}(\mathrm{i}z_n, \mathbf{q}) - \Pi_i(\mathrm{i}z_n, \mathbf{q}) \ , \qquad \frac{\delta\Omega}{\delta S_i} = 0 \ , \quad \text{if} \quad \Pi_i = \frac{\delta\Omega}{\delta S_i} \ .$$

$$\Omega = \frac{1}{2}T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega) \, \mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\, S_i \, \mathrm{Im}\, \Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} = \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega) \, \mathrm{Tr}\left\{\left[\mathrm{Im}\, S_i \, \mathrm{Re}\, \Pi_i\right]\right\} \ ,$$

$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \ .$$

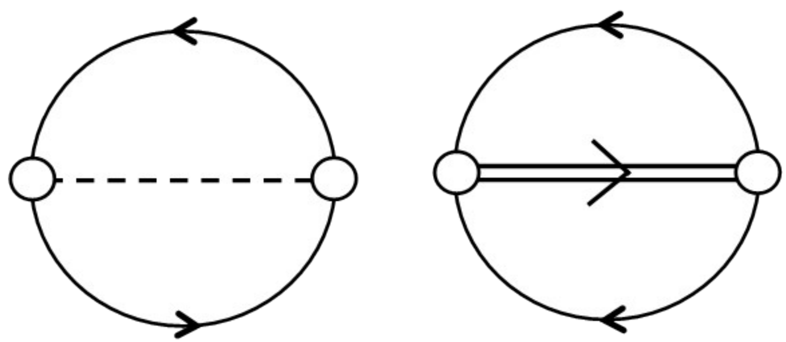
# Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i \,\mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i \Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \ ,$$

$$S_i^{-1}(iz_n, \mathbf{q}) = S_{i,0}^{-1}(iz_n, \mathbf{q}) - \Pi_i(iz_n, \mathbf{q}) \ , \qquad \frac{\delta\Omega}{\delta S_i} = 0 \ , \quad \text{if} \quad \Pi_i = \frac{\delta\Omega}{\delta S_i} \ .$$

$$\Omega = \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} f_i(\omega)\,\mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\,S_i\,\mathrm{Im}\,\Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} = \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} f_i(\omega)\,\mathrm{Tr}\left\{\left[\mathrm{Im}\,S_i\,\mathrm{Re}\,\Pi_i\right]\right\} \ ,$$

$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \, .$$

# Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\left(\mathrm{Im}\ln S^{-1}\right)' = -\mathrm{Im}\,(S\Pi') = \underbrace{S_R'\Pi_I - S_I\Pi_R'}_{2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I)} - \underbrace{(\Pi_I S_R' + S_R\Pi_I')}_{(\Pi_I S_R)'} ,$$

Use optical theorems ...

$$S\Pi_I = \sin\delta\, \mathrm{e}^{\mathrm{i}\delta} , \quad S^{*\prime}\Pi_I = -\mathrm{i}\delta'\sin\delta\,\mathrm{e}^{-\mathrm{i}\delta} , \quad 2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I) = -2\delta'\sin^2\delta .$$

## Generalized Beth-Uhlenbeck EoS

$$\Omega = -\sum_{i=Q,M,D} d_i \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, T\ln[1-\mathrm{e}^{-(\omega-\mu_i)/T}]\, \sin^2\delta_i(\omega,\mathbf{q})\frac{\partial\delta_i(\omega,\mathbf{q})}{\partial\omega}$$

Effect of the sin^2 term ... example: Breit-Wigner ...

$$\delta_i(\omega) = -\arctan\left[\frac{\omega_i\Gamma_i}{\omega^2-\omega_i^2}\right] , \quad \frac{\partial\delta_i(\omega)}{\partial\omega} = \frac{2\omega\omega_i\Gamma_i}{(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2} ,$$

$$\sin^2\delta_i(\omega)\frac{\partial\delta_i(\omega)}{\partial\omega} = \frac{2\omega(\omega_i\Gamma_i)^3}{[(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2]^2} .$$

"Squared Lorentzian" ...
Vanderheyden & Baym (1998)
Morozov & Roepke (2009)

# Cluster expansion in the 2PI formalism

- $\Phi-$ derivable approach to the grand canonical thermodynamic potential
  [Baym, Phys. Rev. 127 (1962) 139]

$$J = -\mathrm{Tr}\left\{\ln(-G_1)\right\} - \mathrm{Tr}\{\Sigma_1 G_1\} + \mathrm{Tr}\left\{\ln(-G_2)\right\} + \mathrm{Tr}\{\Sigma_2 G_2\} + \Phi[G_1, G_2]$$

with full propagators:
$G_1^{-1}(1,z) = z - E_1(p_1) - \Sigma_1(1,z)$; $G_2^{-1}(12,1'2',z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12,1'2',z)$
and selfenergies

$$\Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}\ ; \Sigma_2(12,1'2',z) = \frac{\delta\Phi}{\delta G_2^{-1}(12,1'2',z)}.$$

Because of stationarity equivalent to

$$n = -\frac{1}{\Omega}\frac{\partial J}{\partial\mu} = \frac{1}{\Omega}\sum_1\int_{-\infty}^{\infty}\frac{d\omega}{\pi}f_1(\omega)S_1(1,\omega)\,,$$

(baryon number conservation)

- Generalization to A-nucleon clusters in nuclear matter

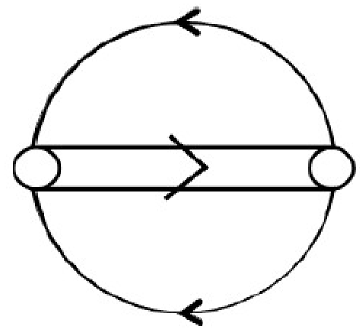
$$\Omega \;=\; \sum_A(-1)^A\left[\mathrm{Tr}\ln\left(-G_A^{-1}\right) + \mathrm{Tr}\left(\Sigma_A\, G_A\right)\right] + \Phi\,,$$

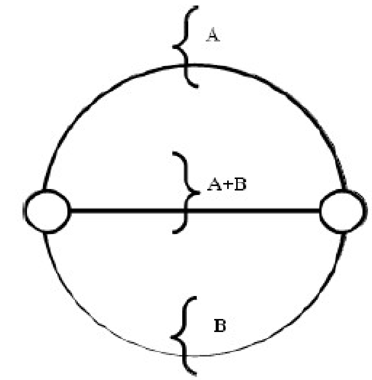
$$G_A^{-1} \;=\; G_A^{(0)-1} - \Sigma_A\,,\;\; \Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}\,.$$

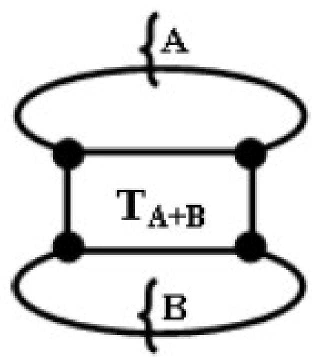
# Cluster expansion in the 2PI formalism

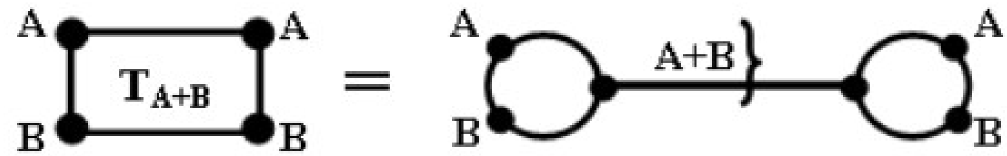
**A) Choice of the Φ-functional:**

- 2-particle irreducible diagrams
- closed 2-loop diagram involving 3 cluster propagators (A, B, A+B) and 2 vertices
- equivalent to 1 T-matrix + 2 propagators






**B) Ansatz for thermodynamic potential:**

$$\Omega = \sum_A (-1)^A \left[\mathrm{Tr}\ln\left(-G_A^{-1}\right) + \mathrm{Tr}\left(\Sigma_A\, G_A\right)\right] + \sum_{A,B} \Phi[G_A, G_B, G_{A+B}] \,,$$

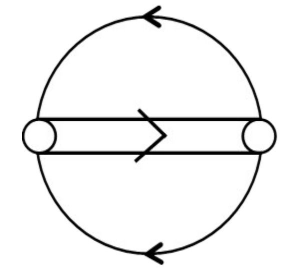
$$G_A^{-1} = G_A^{(0)^{-1}} - \Sigma_A \,, \;\; \Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)} \,.$$

**C) Check: conservation laws, e.g.:**
(correspondence to GF formalism)

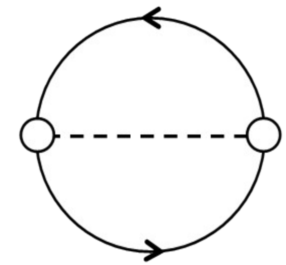
$$n = -\frac{1}{V}\frac{\partial\Omega}{\partial\mu} = \frac{1}{V}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{\pi} f_1(\omega) A_1(1,\omega)$$

# Cluster virial expansion in the 2PI formalism, Examples:

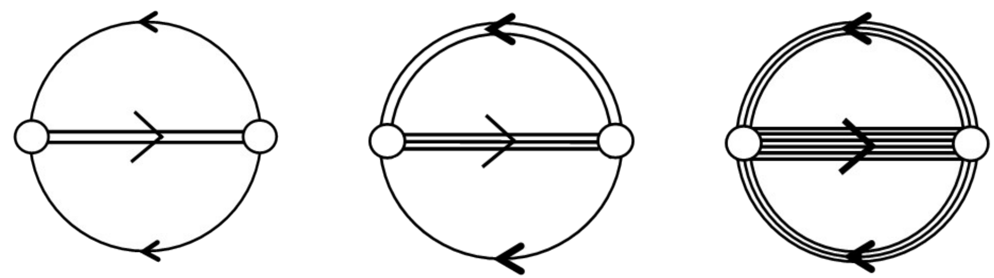
**A) Deuterons in nuclear matter:**



**B) Mesons in quark matter:**

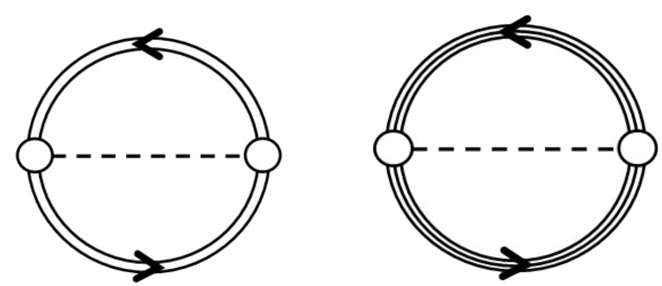


**C) Nucleons in quark matter:**

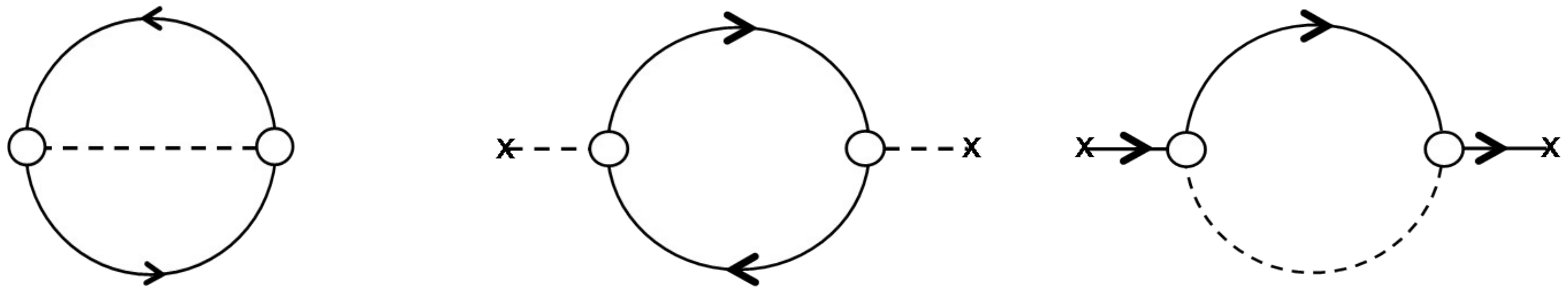


**D) Nucleons and mesons (hadron resonance gas) in quark matter:**

B) + C) +



# Example B: Mesons in quark matter




$$T_M^{-1}(q,\omega+i\eta) = G_S^{-1} - \Pi_M(q,\omega+i\eta) = |T_M(q,\omega)|^{-1}\mathrm{e}^{-i\delta_M(q,\omega)} , \quad \delta_M(q,\omega) = \arctan(\Im T_M/\Re T_M)$$

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M , \qquad \sigma_{\mathrm{MF}} = 2N_f N_c G_S \int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}[1 - f_-(E_p) - f_+(E_p)] ,$$

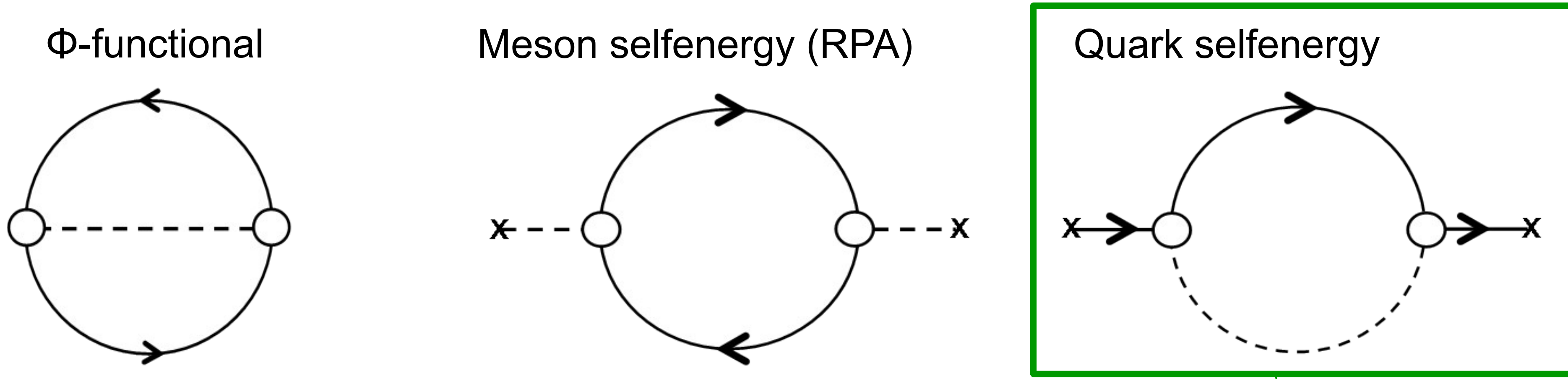
$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4G_S} - 2N_c N_f \int \frac{d^3p}{(2\pi)^3}\left[E_p + T\ln\left(1+\mathrm{e}^{-(E_p-\Sigma_+-\mu)/T}\right) + T\ln\left(1+\mathrm{e}^{-(E_p+\Sigma_-+\mu)/T}\right)\right] ,$$

$$\Omega_M = d_M \int \frac{d^3k}{(2\pi)^3}\int\frac{d\omega}{2\pi}\left\{\omega + 2T\ln\left[1-\mathrm{e}^{-\omega/T}\right]2\sin^2\delta_M(k,\omega)\,\frac{\delta_M(k,\omega)}{d\omega}\right\} ,$$

$$\Sigma_M(0,p_0) = d_M\int\frac{d^4q}{(2\pi)^4}\pi\rho_M(\mathbf{q},q_0)\left\{\frac{(\gamma_0+m/E_q)[1+g(q_0)-f_-(E_q)]}{q_0-p_0+E_q-\mu-i\eta} + \frac{(\gamma_0-m/E_q)[g(q_0)+f_+(E_q)]}{q_0-p_0-E_q-\mu-i\eta}\right\} ,$$

D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228
D. Blaschke, PoS Baldin ISHEPXXII (2015) 113; arxiv:1502.06279

# Example B: Mesons in quark matter




$$T_M^{-1}(q,\omega+i\eta) = G_S^{-1} - \Pi_M(q,\omega+i\eta) = |T_M(q,\omega)|^{-1}\mathrm{e}^{-i\delta_M(q,\omega)}\,, \quad \delta_M(q,\omega) = \arctan(\Im T_M/\Re T_M)$$

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M\,, \qquad \sigma_{\mathrm{MF}} = 2N_f N_c G_S \int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}[1 - f_-(E_p) - f_+(E_p)]\,,$$

$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4G_S} - 2N_c N_f \int \frac{d^3p}{(2\pi)^3}\left[E_p + T\ln\left(1+\mathrm{e}^{-(E_p-\Sigma_+-\mu)/T}\right) + T\ln\left(1+\mathrm{e}^{-(E_p+\Sigma_-+\mu)/T}\right)\right]\,,$$

$$\Omega_M = d_M \int \frac{d^3k}{(2\pi)^3}\int \frac{d\omega}{2\pi}\left\{\omega + 2T\ln\left[1-\mathrm{e}^{-\omega/T}\right]\boxed{2\sin^2\delta_M(k,\omega)}\frac{\delta_M(k,\omega)}{d\omega}\right\}\,,$$

**new !**

$$\Sigma_M(0,p_0) = d_M \int \frac{d^4q}{(2\pi)^4}\pi\rho_M(\mathbf{q},q_0)\left\{\frac{(\gamma_0+m/E_q)[1+g(q_0)-f_-(E_q)]}{q_0-p_0+E_q-\mu-i\eta} + \frac{(\gamma_0-m/E_q)[g(q_0)+f_+(E_q)]}{q_0-p_0-E_q-\mu-i\eta}\right\}\,,$$

D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228
D. Blaschke, PoS Baldin ISHEPXXII (2015) 113; arxiv:1502.06279

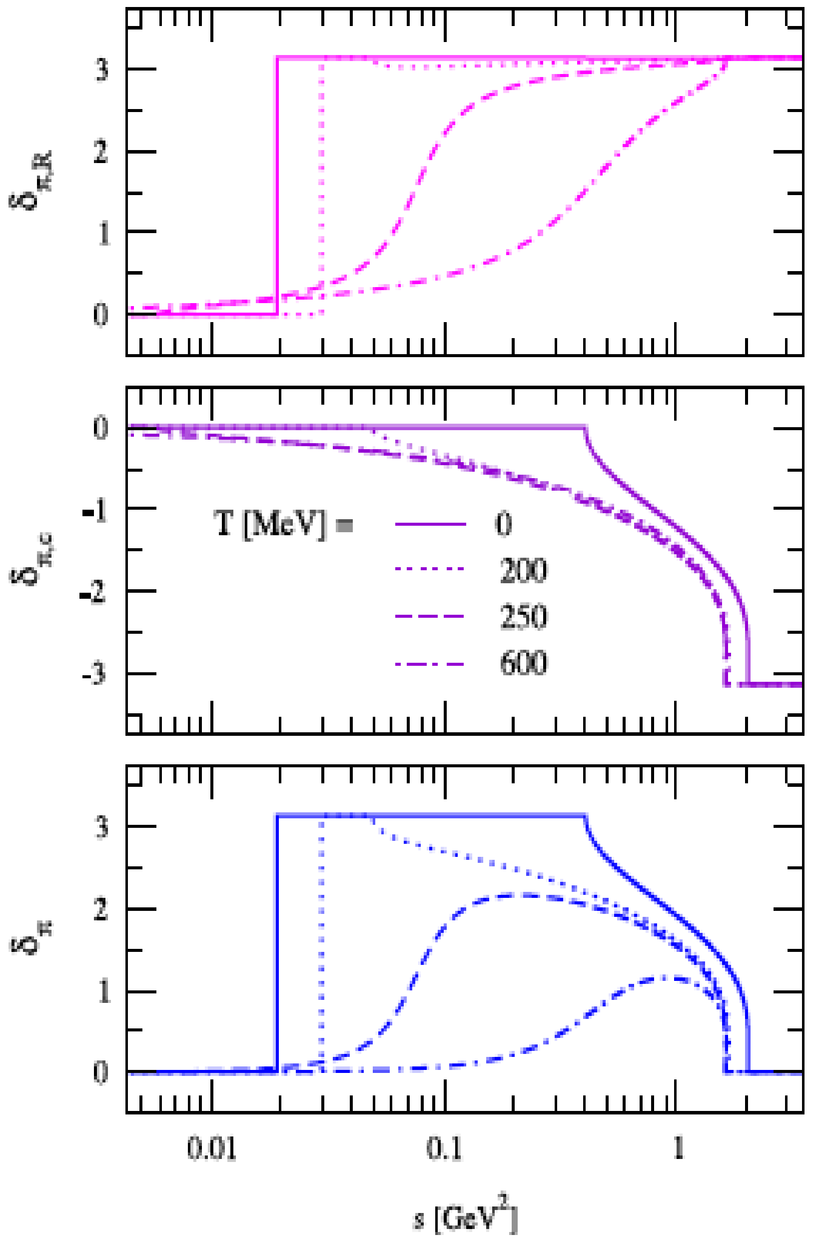
# Example B: Mesons in quark matter

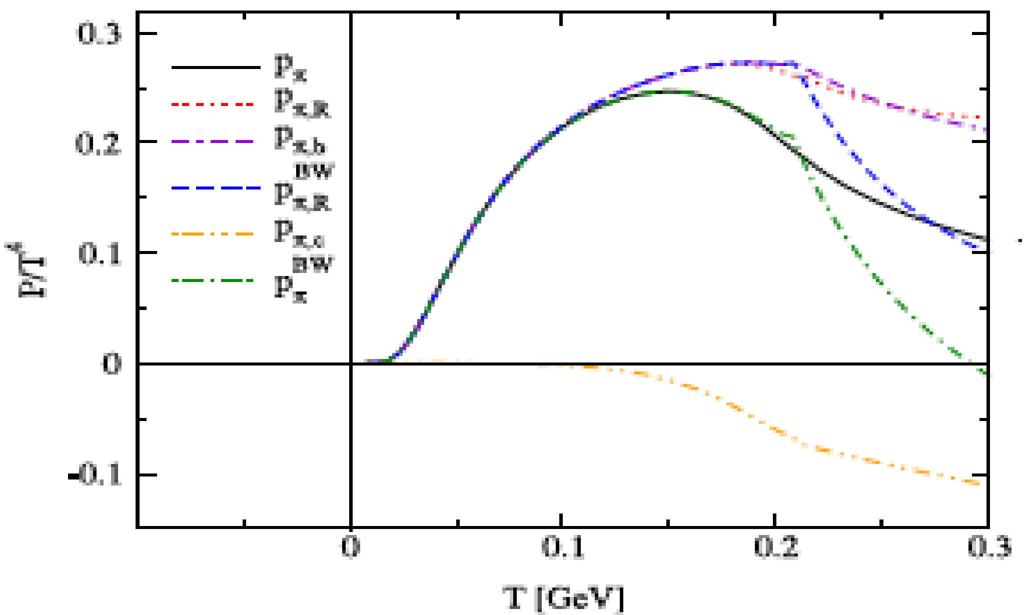
$$\Omega_X(T,\mu) = -d_X \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} n_X^-(\omega)\delta_X(\omega,\mathbf{q}),$$

$$\int_0^\infty d\omega \frac{1}{\pi}\frac{d\delta_X(\omega;T)}{d\omega} = 0 = \underbrace{\int_0^{\omega_{thr}(T)} d\omega \frac{1}{\pi}\frac{d\delta_X(\omega;T)}{d\omega}}_{n_{B,X}(T)} + \underbrace{\frac{1}{\pi}\int_{\omega_{thr}(T)}^\infty d\omega \frac{d\delta_X(\omega;T)}{d\omega}}_{\frac{1}{\pi}[\delta_X(\infty;T)-\delta_X(\omega_{thr};T)]},$$

$$p_\pi(T) = -d_\pi T \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \ln\left(1-e^{-\omega/T}\right) \frac{d\delta_\pi(\omega,\mathbf{q})}{d\omega}$$

$$\delta_\pi \doteq \delta_{\pi,c} + \delta_{\pi,R}$$




D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228

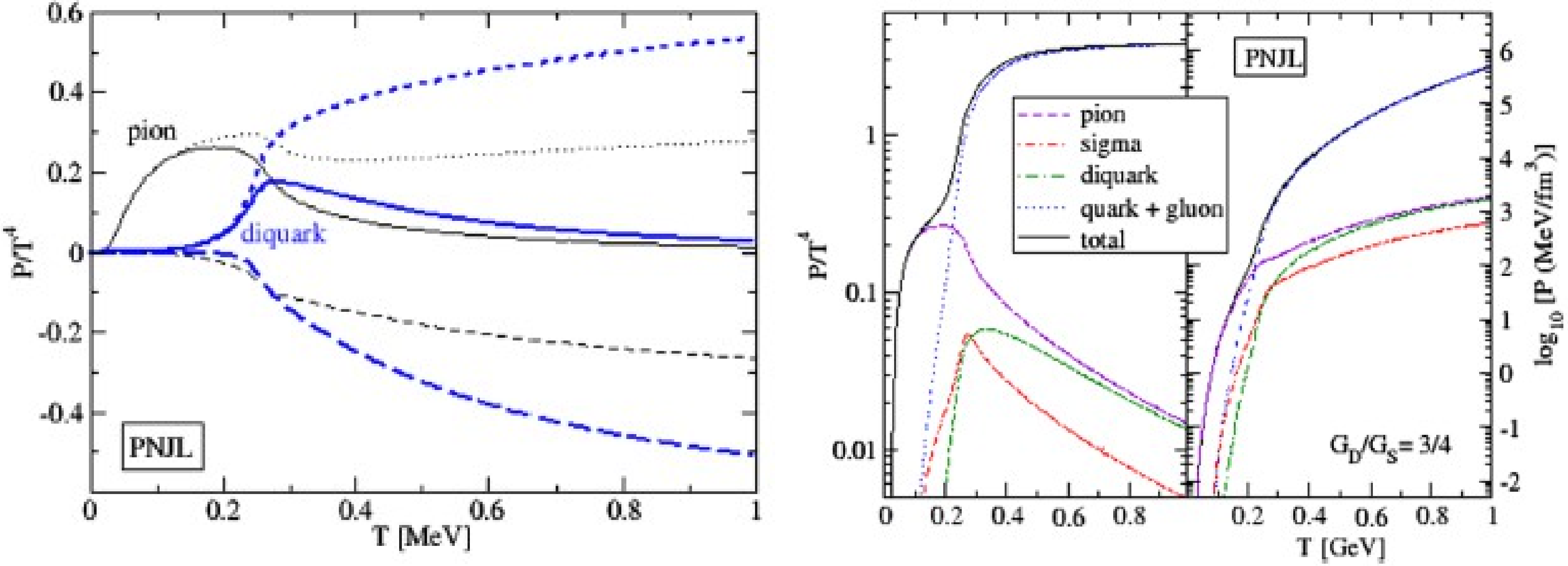
# Example B*: Mesons+diquarks in quark matter

$$\Omega_Q = -\frac{2N_cN_f}{3}\int \frac{dp}{2\pi^2}\frac{p^4}{E_p}[f_\Phi^+(E_p)+f_{\bar\Phi}^-(E_p)],\ \ f_\Phi^+(E_p) = \frac{(\bar\Phi+2\Phi Y)Y+Y^3}{1+3(\bar\Phi+\Phi Y)Y+Y^3},\ \ Y = e^{-(E_p-\mu)/T}$$

$$\Omega_D = -3\int\frac{d^3p}{(2\pi)^3}\int\frac{d\omega}{2\pi}[g_\Phi^+(\omega)+g_{\bar\Phi}^-(\omega)]\delta_D(\omega),\ \ g_\Phi^+(\omega) = \frac{(\Phi-2\bar\Phi X)X+X^3}{1-3(\Phi-\bar\Phi X)X-X^3}\qquad X = e^{-(\omega-2\mu)/T}$$

Suppression of colored states by Polyakov-loop Φ

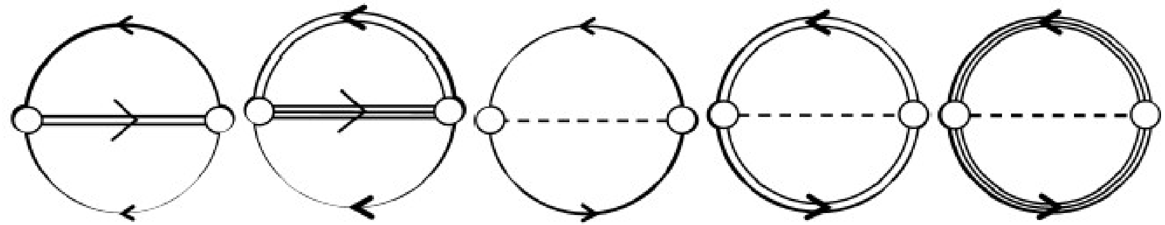
Confinement: Φ=0



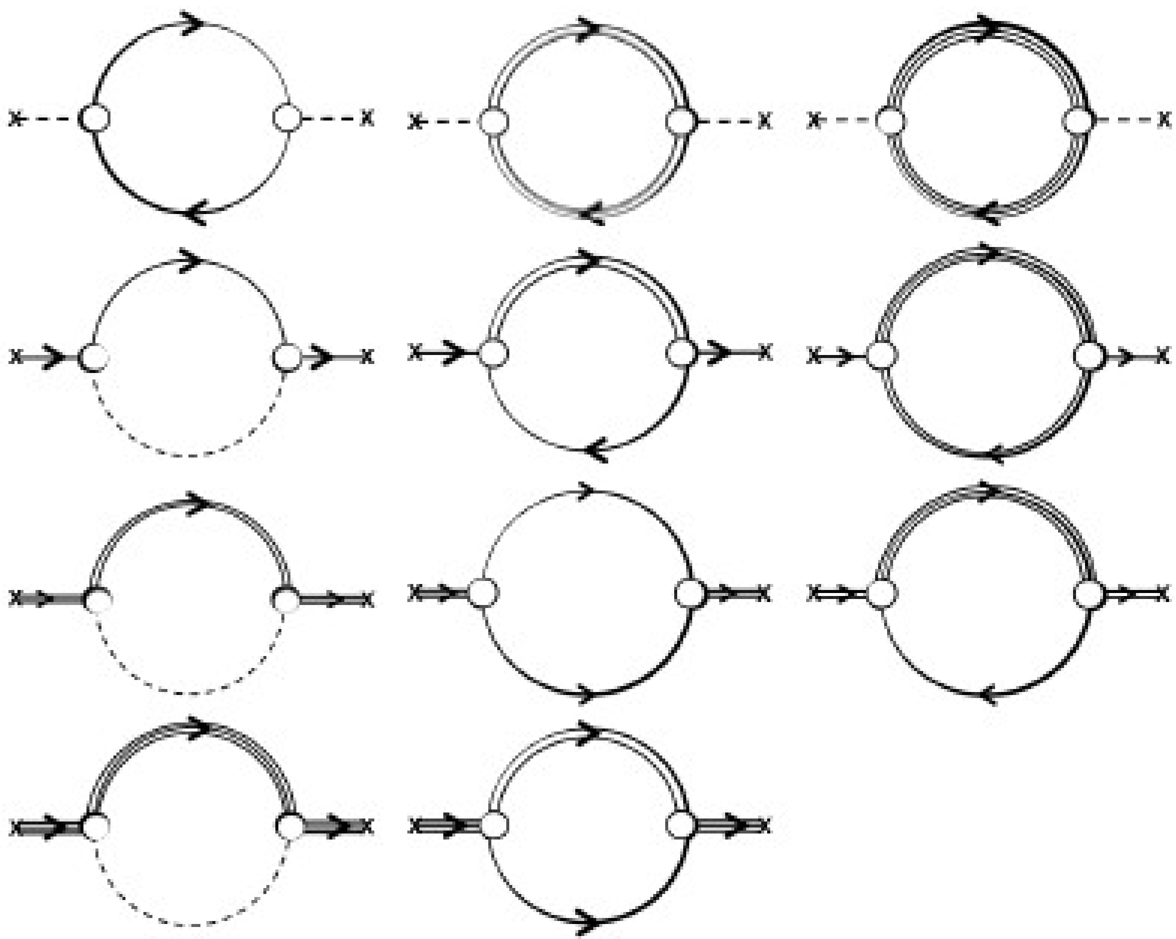

D. Blaschke, A. Dubinin, M. Buballa: Phys. Rev. D 91 (2015) 125040

# Example D: Hadron resonance gas – effect. model

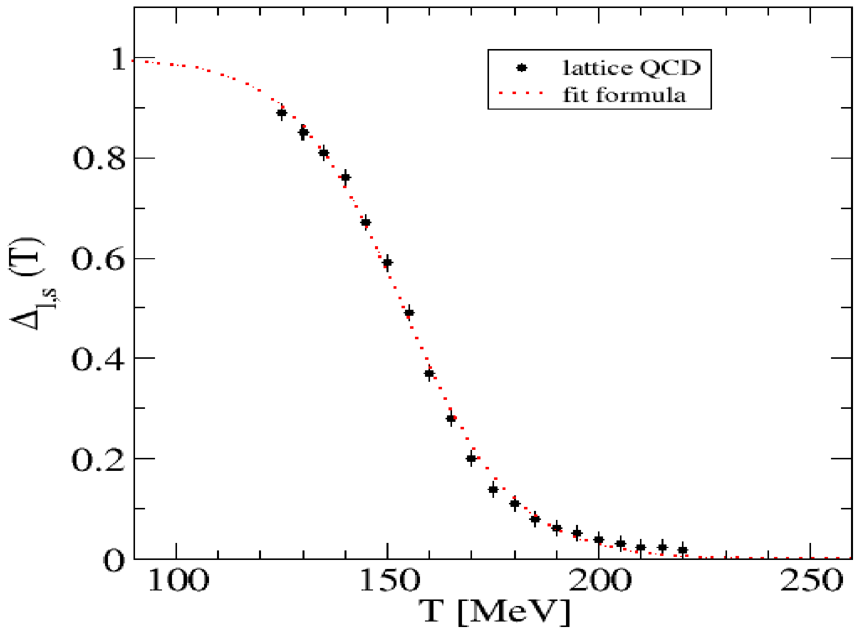
Φ-functional:

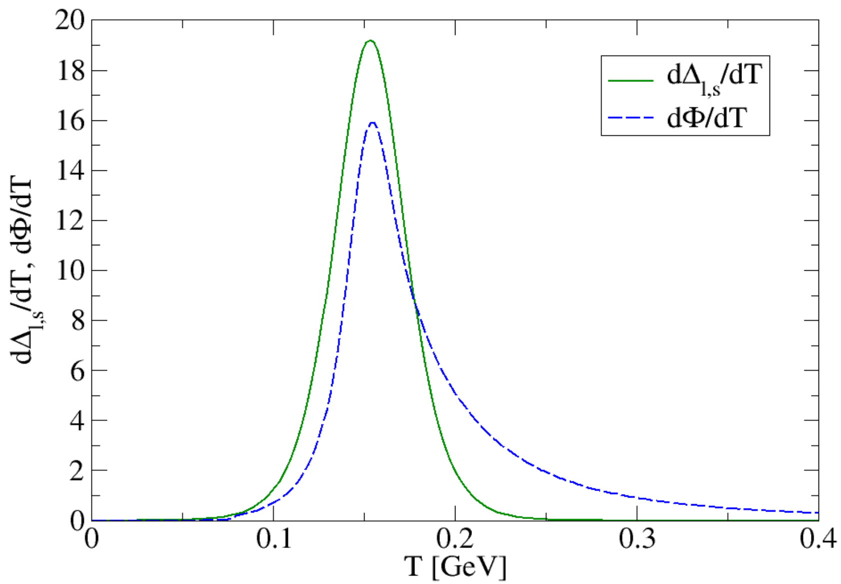


Selfenergies:



# Example D: Mott HRG / PNJL – effective model




$$P_{\rm PNJL}(T) = P_{\rm FG}(T) + \mathcal{U}[\Phi;T] \; ,$$

$$P_{\rm FG}(T) = 4 \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} T \ln\left[1 + 3\Phi(Y + Y^2) + Y^3\right]$$

$$Y(E_p) \; = \; \exp(-E_p/T)$$

$$\mathcal{U}[\Phi;T] = -\frac{a(T)}{2}\Phi^2 + b(T)\ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4)$$

T-dependent quark masses from fit to LQCD

$$m(T) = [m(0) - m_0]\Delta_{l,s}(T) + m_0 \; ,$$

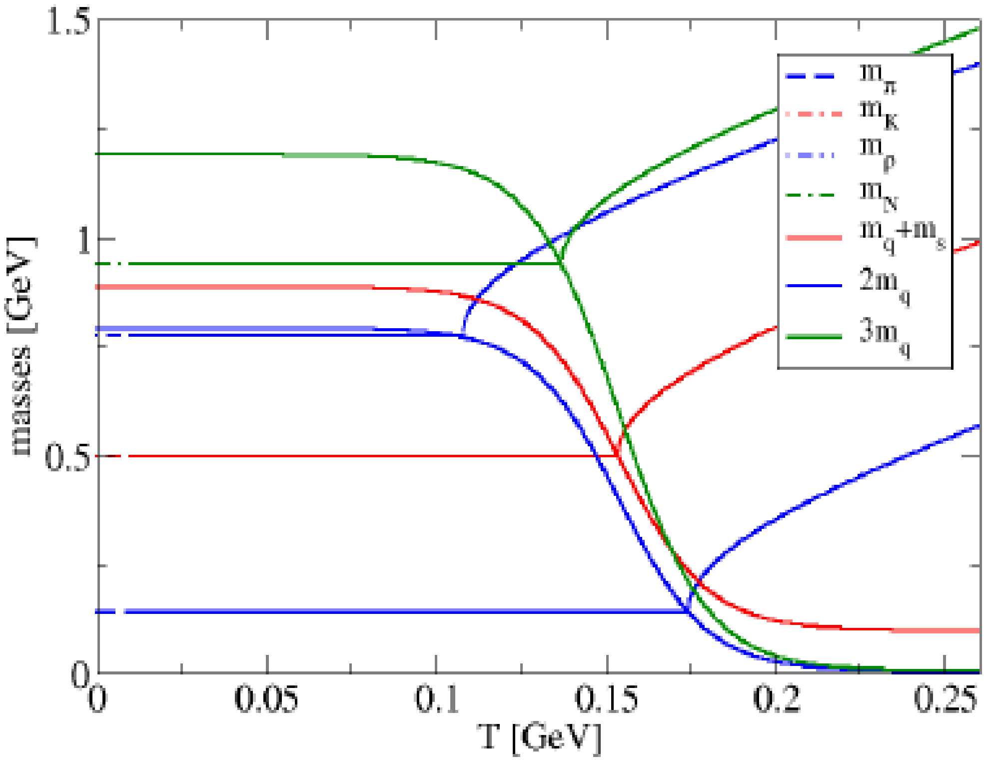
$$m_s(T) = m(T) + m_s - m_0 \; ,$$

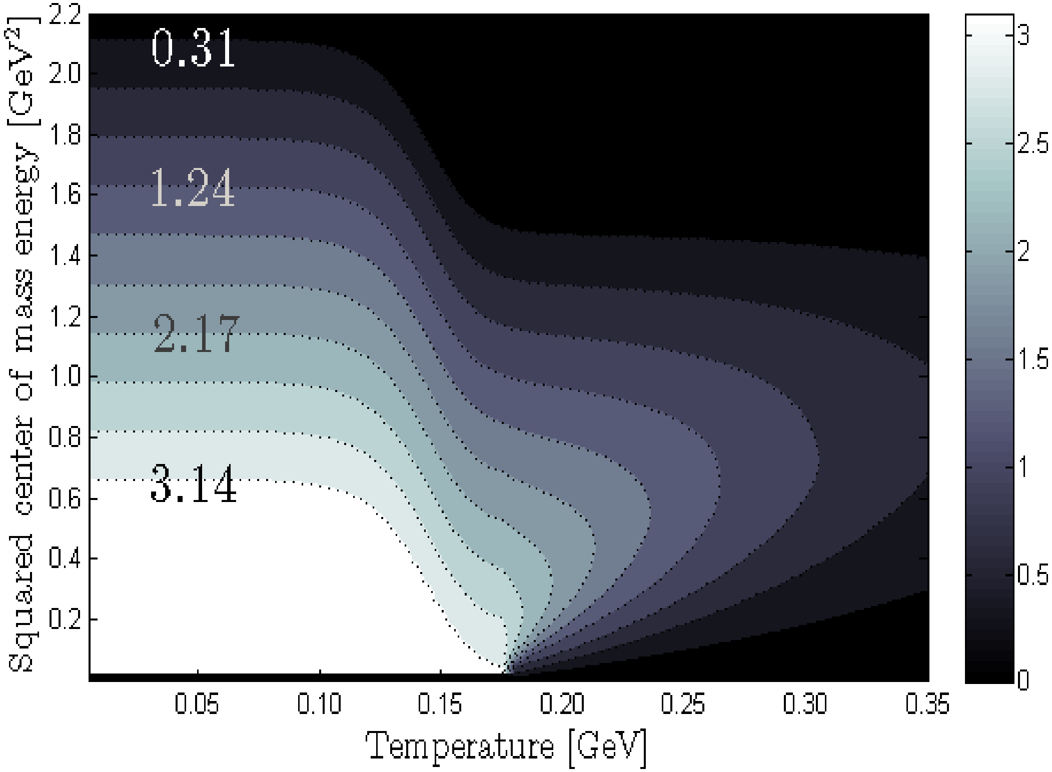
$$\Delta_{l,s}(T) = \frac{1}{2}\left[1 - \tanh\left(\frac{T - T_c}{\delta_T}\right)\right]$$

$$T_c = 154 \text{ MeV} \qquad \delta_T = 26 \text{ MeV}$$

D. Blaschke, A. Dubinin, L. Turko, in prep. (2015)

# Example D: Mott HRG / PNJL – effective model




Hadrons + Mott effect

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp\, p^2}{2\pi^2} \int_0^\infty dM\, T \ln\left(1 \mp e^{-\sqrt{p^2+M^2}/T}\right) \frac{2}{\pi} \sin^2 \delta_i(M^2;T) \frac{d\delta_i(M^2;T)}{dM}$$

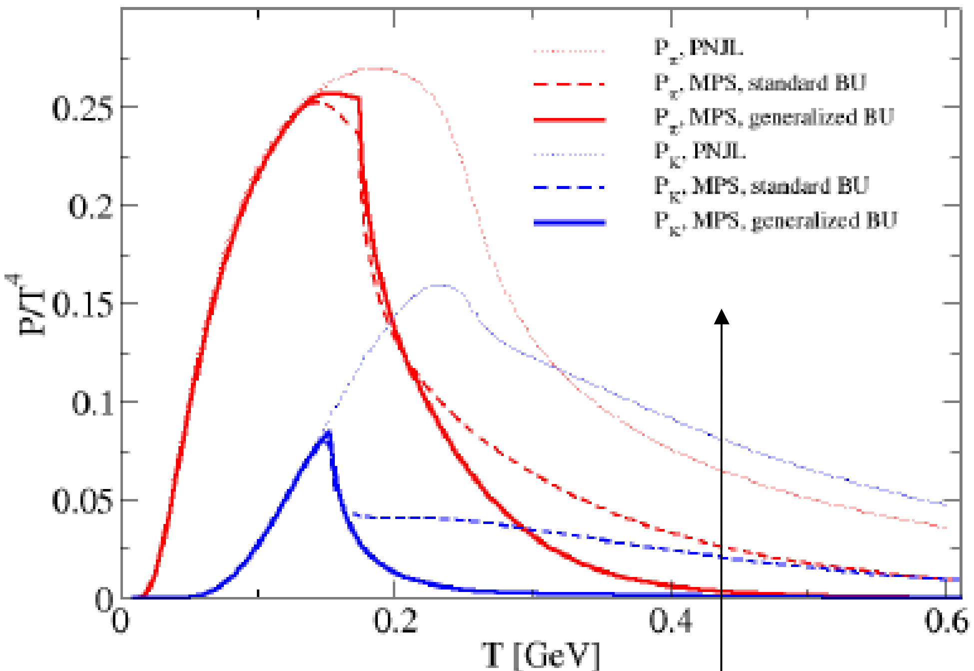
Quarks + rescattering effects

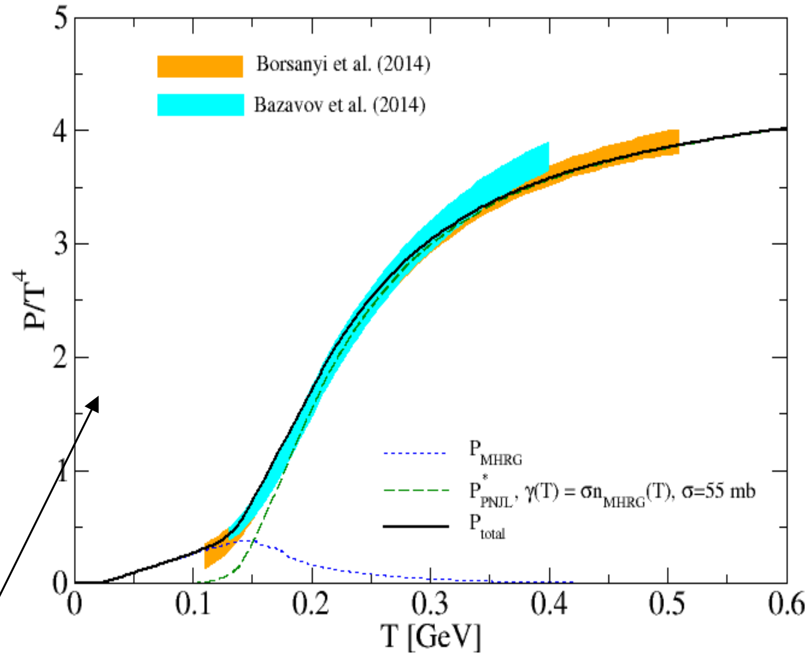
$$f_\Phi(\omega) = \frac{\Phi(1+2Y)Y + Y^3}{1 + 3\Phi(1+Y)Y + Y^3},$$

$$P^*_{\rm FG}(T) = 4N_c \sum_{q=u,d,s} \int \frac{dp\, p^2}{2\pi^2} \int \frac{d\omega}{\pi} f_\Phi(\omega) \delta_q(\omega;\gamma),$$

$$\delta_q(\omega;\gamma) = \frac{\pi}{2} + \arctan\left[\frac{\omega - \sqrt{p^2+m_q^2}}{\gamma}\right]$$

# Example D: Mott HRG / PNJL – effective model

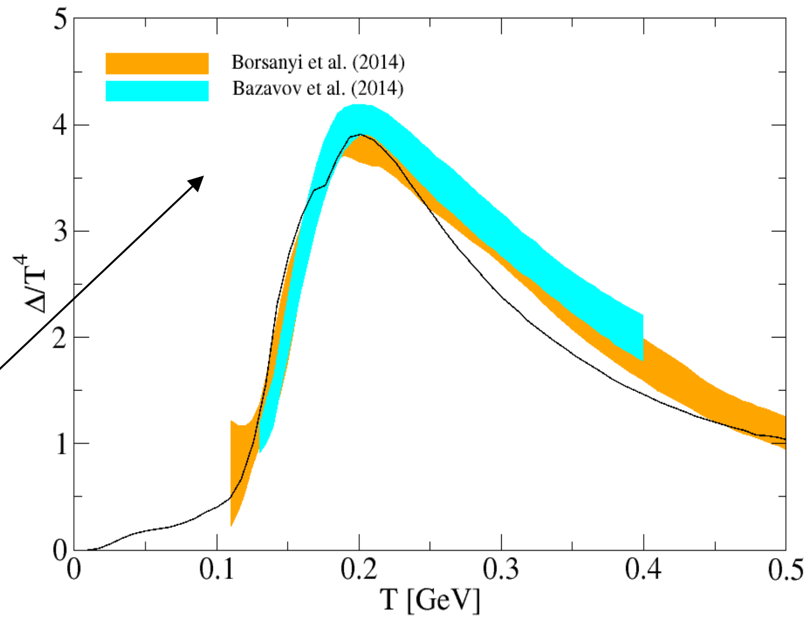



- Mott dissociation of hadrons (here pi, K) at the Chiral rstoration temperature T_c = 153 MeV

- Asymptotic behaviour of quark-gluon Pressure can be adjusted with rescattering Parameter gamma

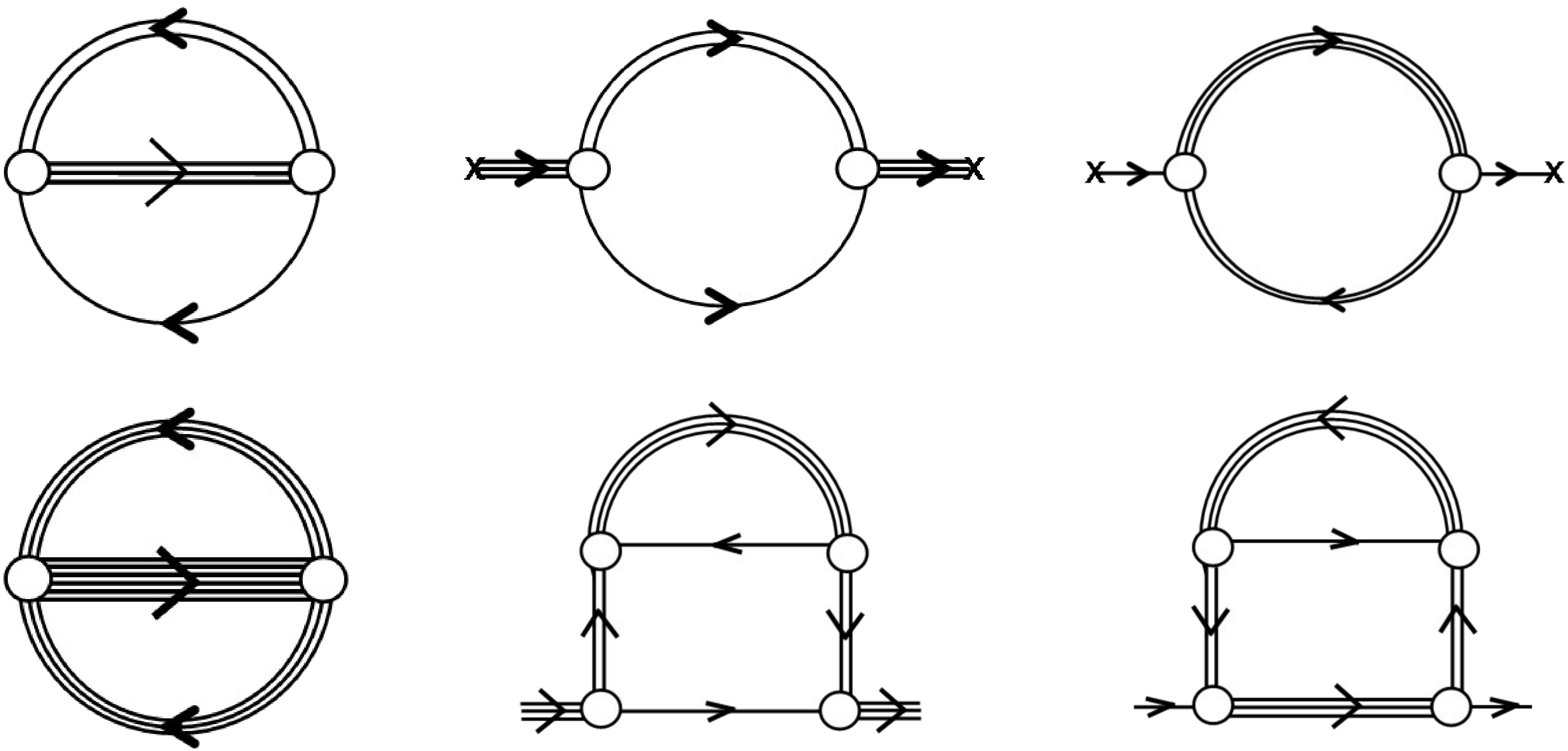
- Very good correspondence between lattice QCD Thermodynamics and improved MHRG/PNJL model; Hadronic and partonic contributions quantified



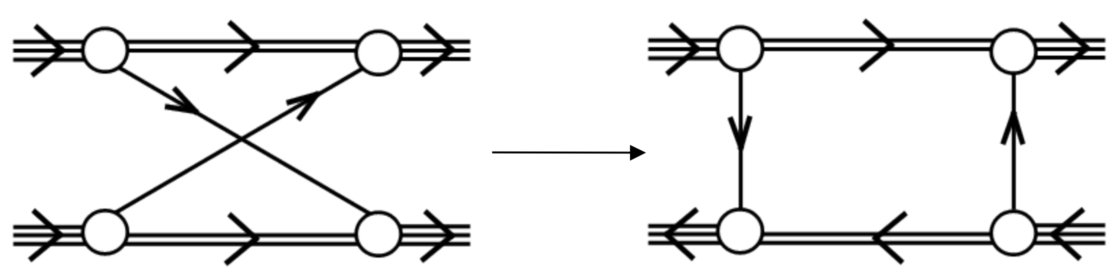

D.B., A. Dubinin, L. Turko, in prep. (2016)

# Example C: Nucleons in quark matter

Φ-functional | nucleon selfenergy | quark selfenergy

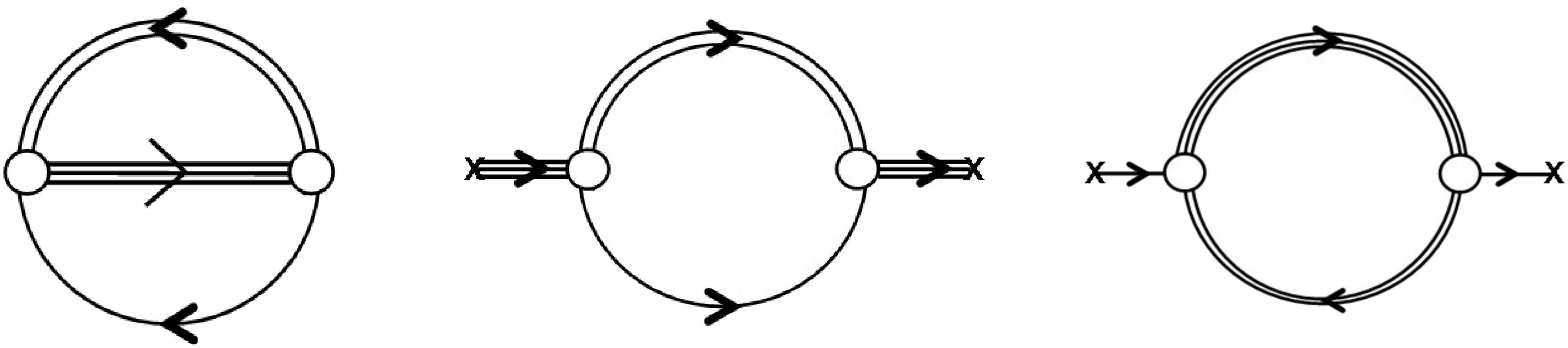


quark exchange interaction
between nucleons:

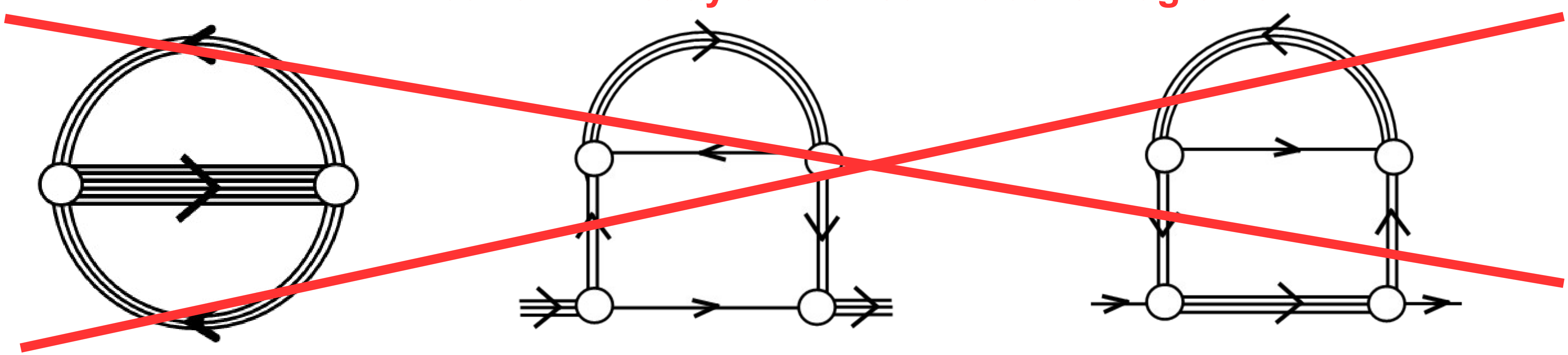


# Example C: Nucleons in quark matter

Φ-functional | nucleon selfenergy | quark selfenergy




**Not new! Already contained in above diagrams!**



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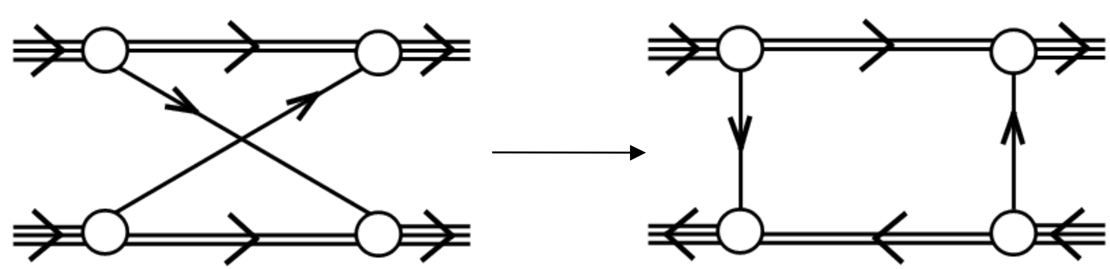
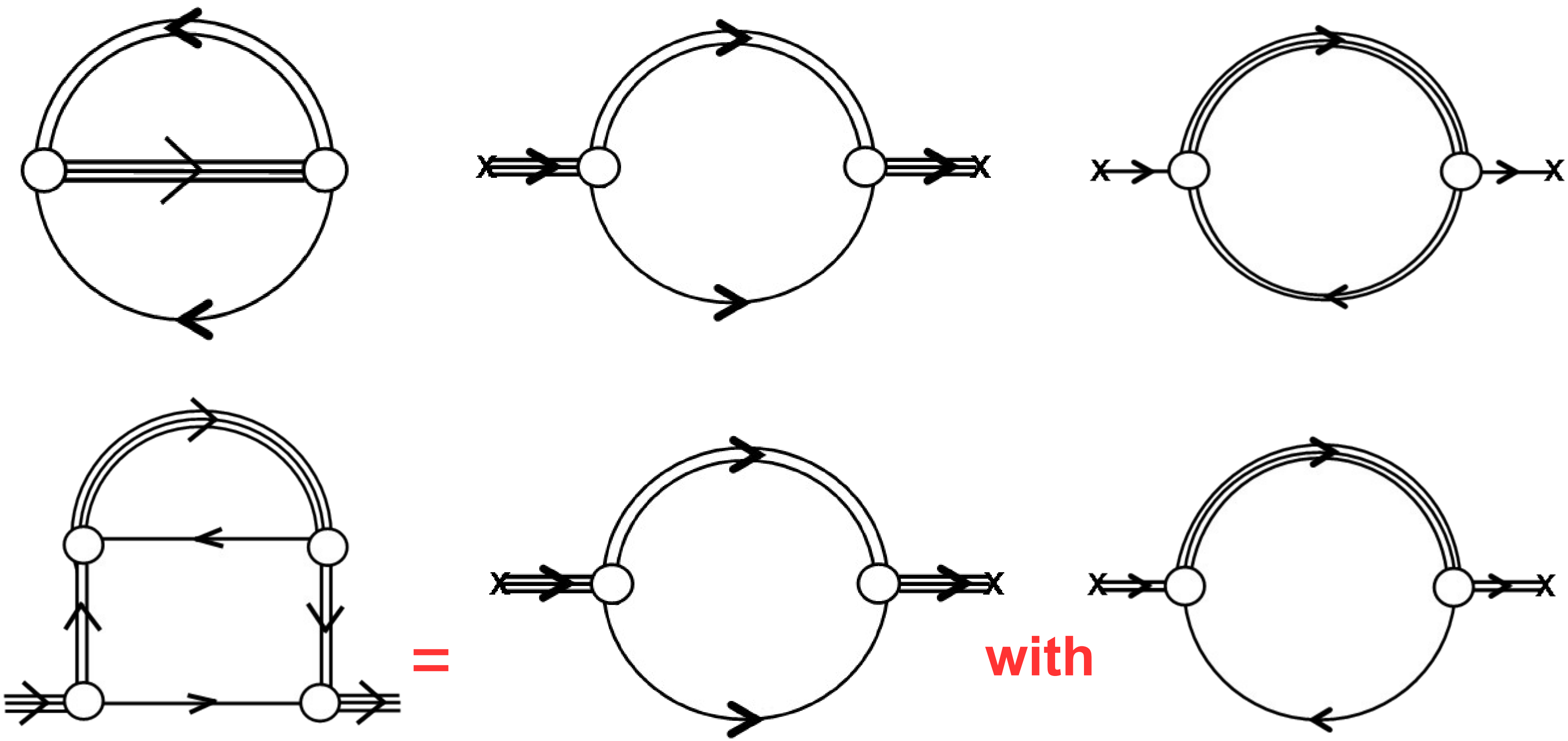
quark exchange interaction
between nucleons:

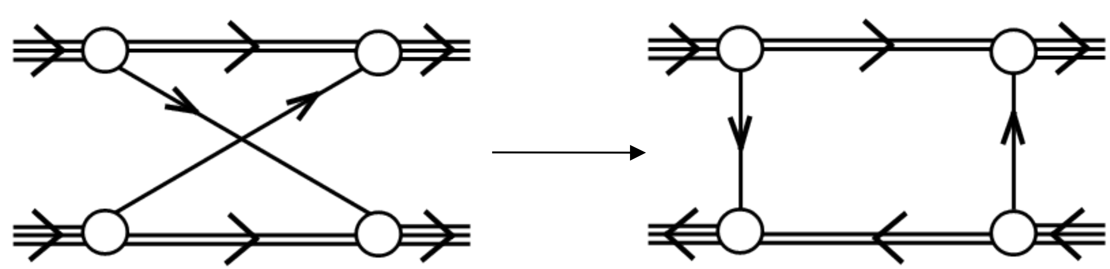


# Example C: Nucleons in quark matter

Φ-functional | nucleon selfenergy | quark selfenergy




quark exchange interaction between nucleons:



# Intermezzo: Structure of the baryon?

12-Apostle Church, Kars

# Intermezzo: Structure of the baryon?

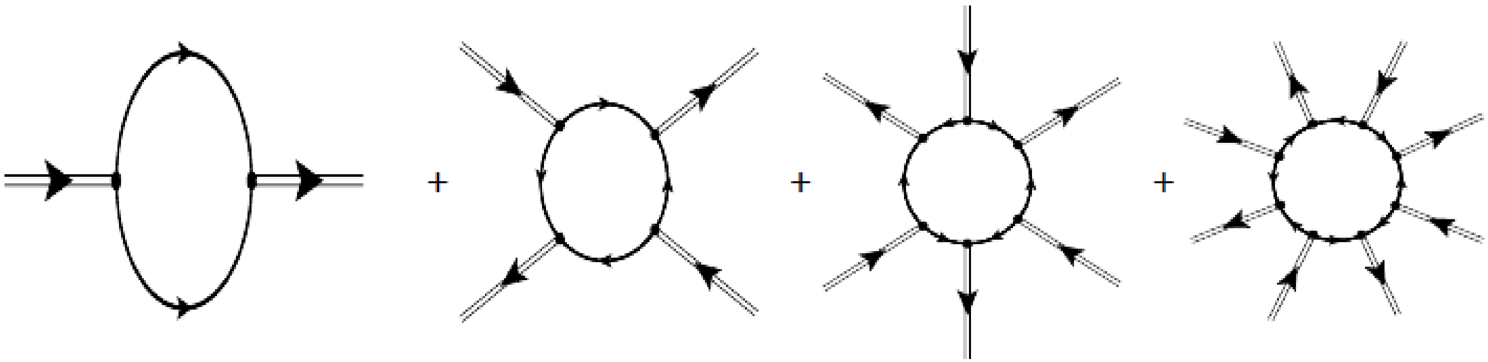
12-Apostle Church, Kars

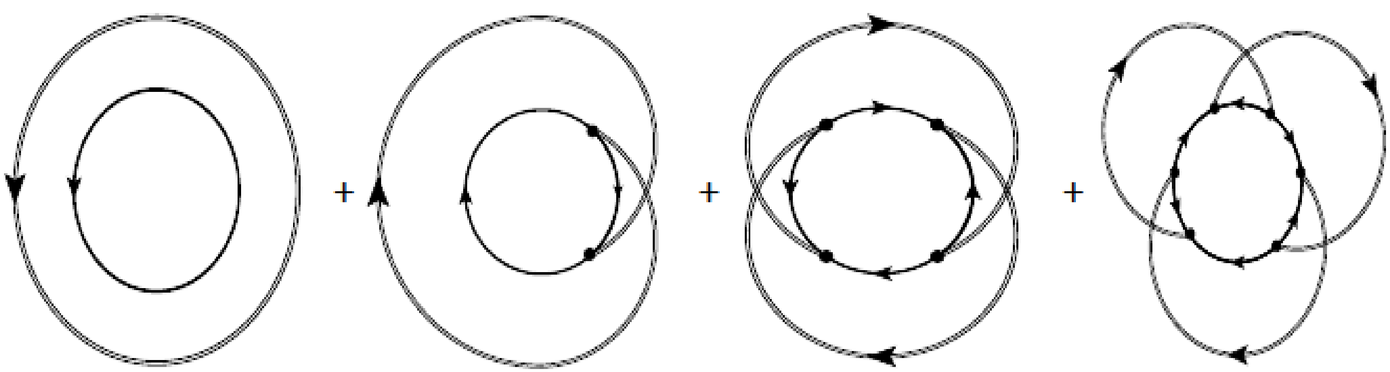
# Intermezzo: Structure of the baryon?

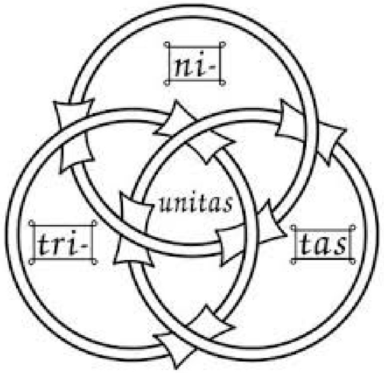
Borromean ? !!

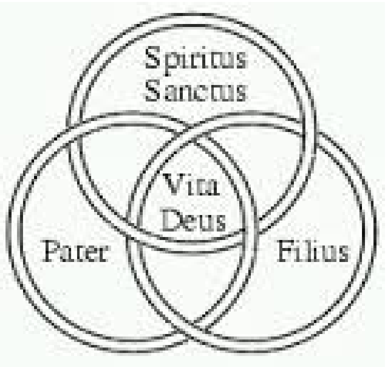
$$Z_{\text{fluct}} = \int D\Delta^\dagger D\Delta D\phi \exp\{-\frac{|\Delta|^2}{4G_D} - \frac{\phi^2}{4G} - Tr\, lnS^{-1}[\Delta, \Delta^\dagger, \phi]\}$$

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29

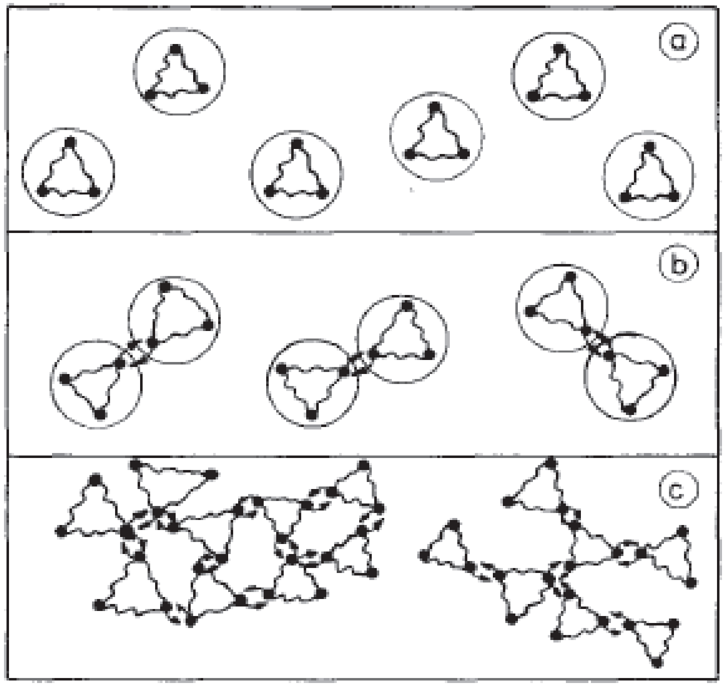






# Example C: Pauli blocking among baryons

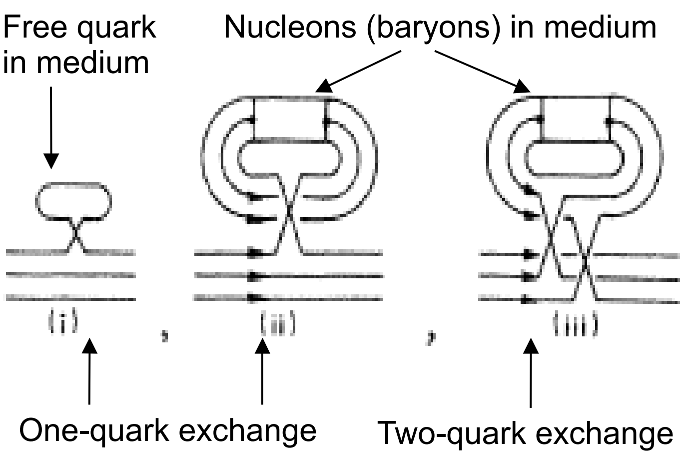



a) Low density: Fermi gas of nucleons (baryons)

b) ~ saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)

Roepke & Schulz, Z. Phys. C 35, 379 (1987); Roepke, DB, Schulz, PRD 34, 3499 (1986)




Nucleon (baryon) self-energy --> Energy shift

$$\Delta E_{\nu P}^{\rm Pauli} = \sum_{123} |\psi_{\nu P}(123)|^2 [E(1)+E(2)+E(3)-E_{\nu P}^0][f_{\alpha_1}(1)+f_{\alpha_2}(2)+f_{\alpha_3}(3)]$$

$$+\sum_{123}\sum_{456}\sum_{\nu' P'} \psi_{\nu P}^*(123)\psi_{\nu' P'}(456) f_3(E_{\nu' P'}^0)\{\delta_{36}\psi_{\nu P}(123)\psi_{\nu' P'}^*(456) - \psi_{\nu P}(453)\psi_{\nu' P'}^*(126)\}$$

$$\times [E(1)+E(2)+E(3)+E(4)+E(5)+E(6)-E_{\nu P}^0 - E_{\nu' P'}^0]$$

$$= \Delta E_{\nu P}^{\rm Pauli,free} + \Delta E_{\nu P}^{\rm Pauli,bound} .$$

PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

# Pauli quenching effects in a simple string model of quark/nuclear matter

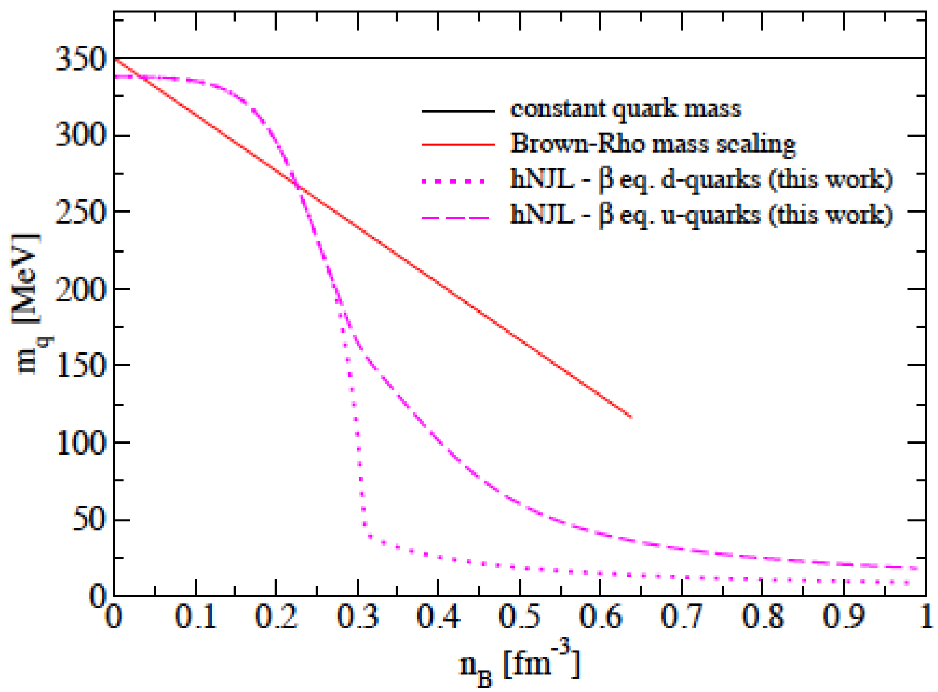
G. Röpke and D. Blaschke
*Department of Physics, Wilhelm-Pieck-Universtiy, 2500 Rostock, German Democratic Republic*

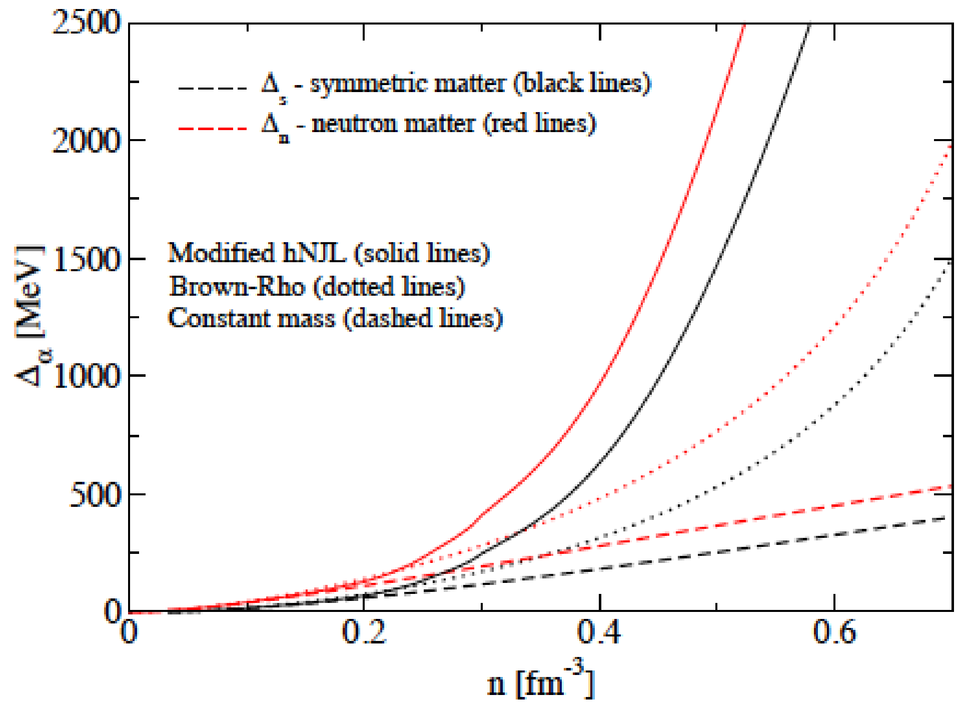
H. Schulz
*Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic*
*and The Niels Bohr Institute, 2100 Copenhagen, Denmark*

(Received 16 December 1985)

# Example C: Pauli blocking in NM – details

## New aspect: chiral restoration --> dropping quark mass

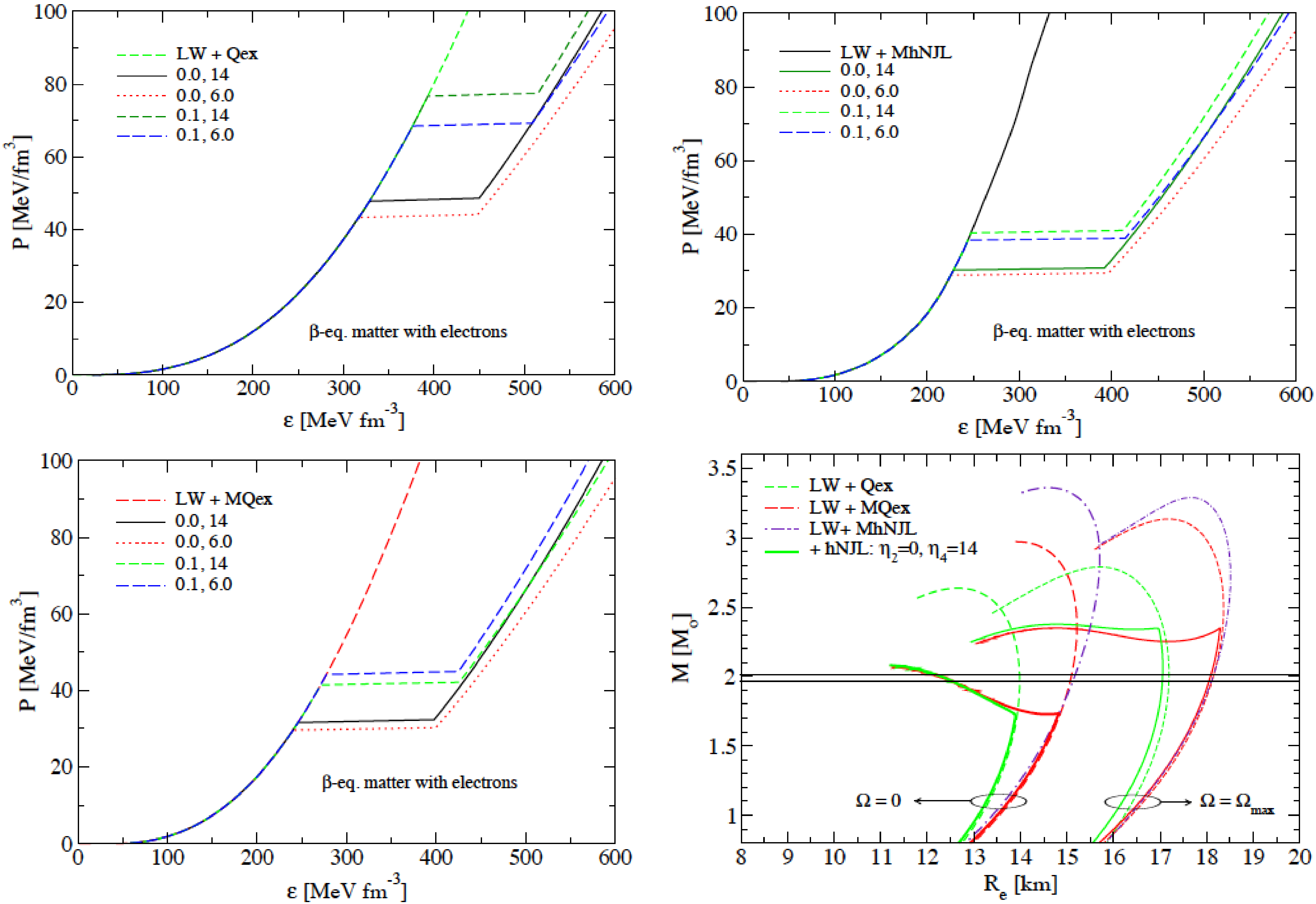



## Increased baryon swelling at supersaturation densities: --> dramatic enhancement of the Pauli repulsion !!

D.B., H. Grigorian, G. Roepke: "Quark exchange effects in dense nuclear matter", in prep. (2016)

# Example C: Pauli blocking in NM – results



# Example C: Pauli blocking in NM – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:
- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high n
- quark exchange among baryons -> six-quark wavefunction -> “bag melting” -> deconfinement

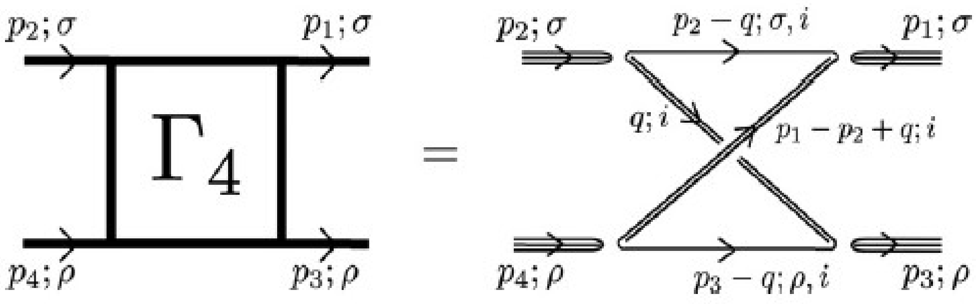
Chiral stiffening of nuclear matter --> reduces onset density for deconfinement

Hybrid EoS:
Convenient generalization of RMF models,
Take care: eventually aspects of quark exchange already in density dependent vertices!

Other baryons:
- hyperons
- deltas

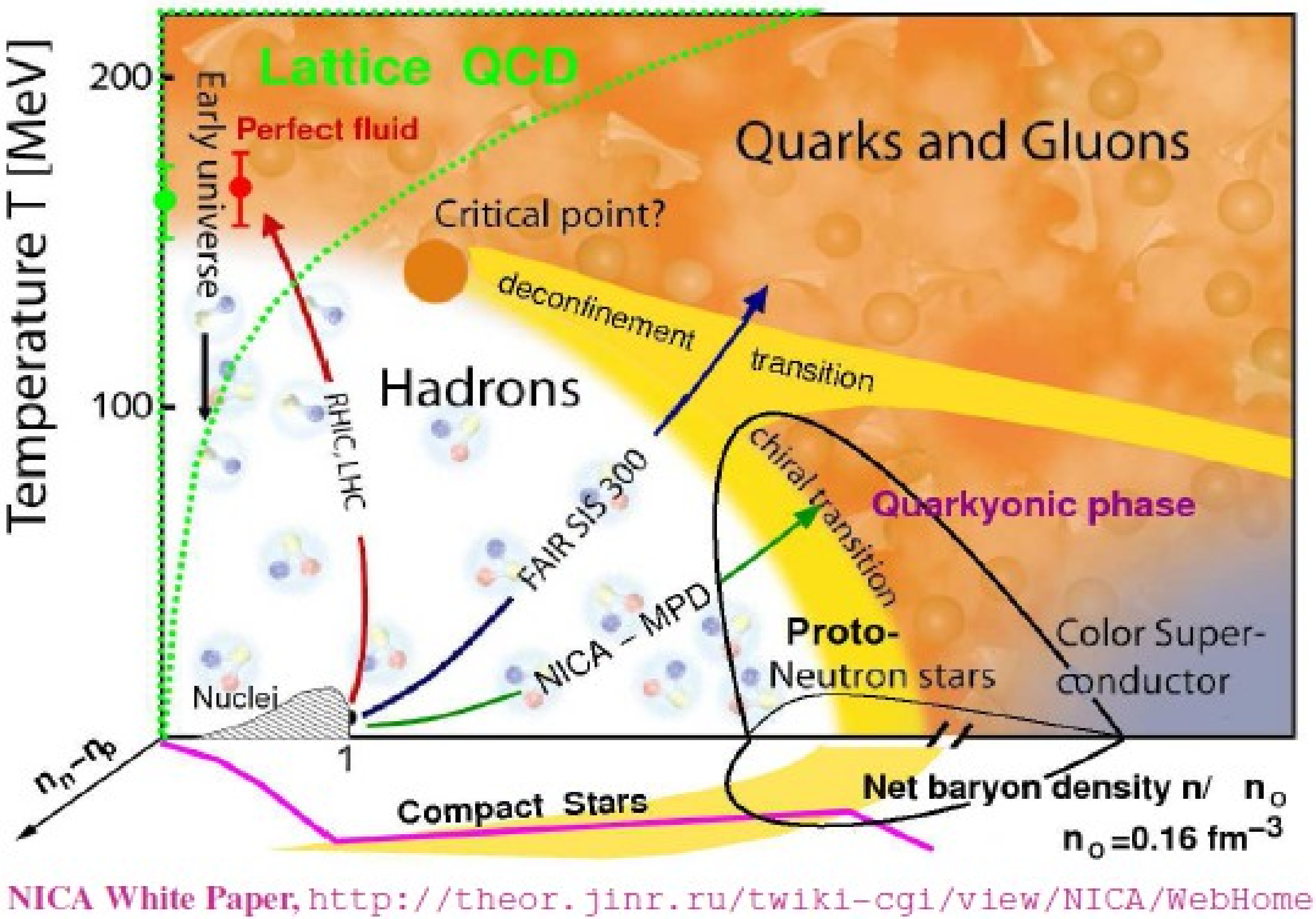
Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

Relativistic generalization:
Box diagrams of quark-diquark model ...




K. Maeda, Ann. Phys. 326 (2011) 1032

# Support a CEP in QCD phase diagram with Astrophysics?

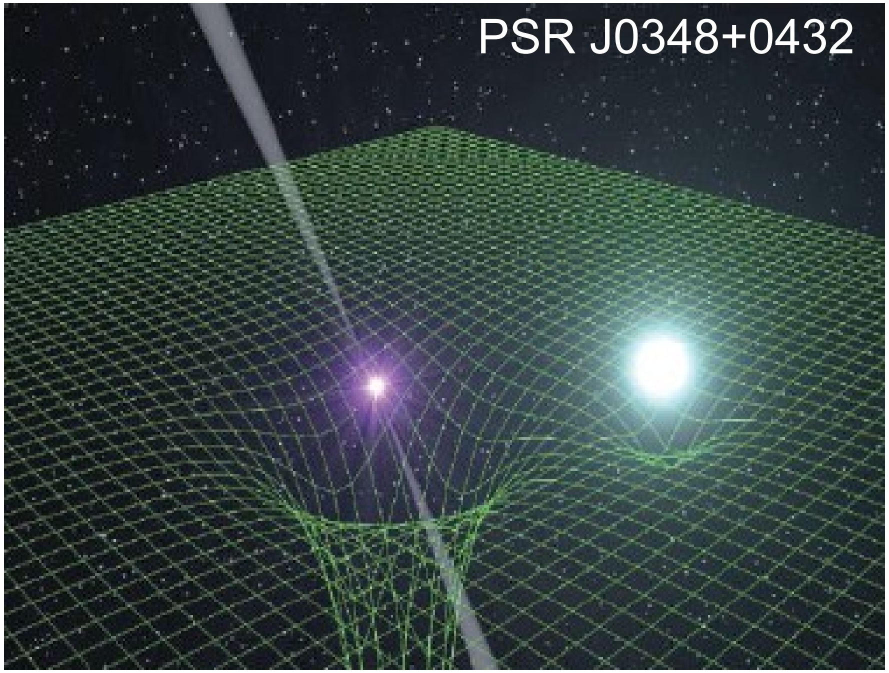



S. Benic et al., A&A 577, A40 (2015)

**Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!**

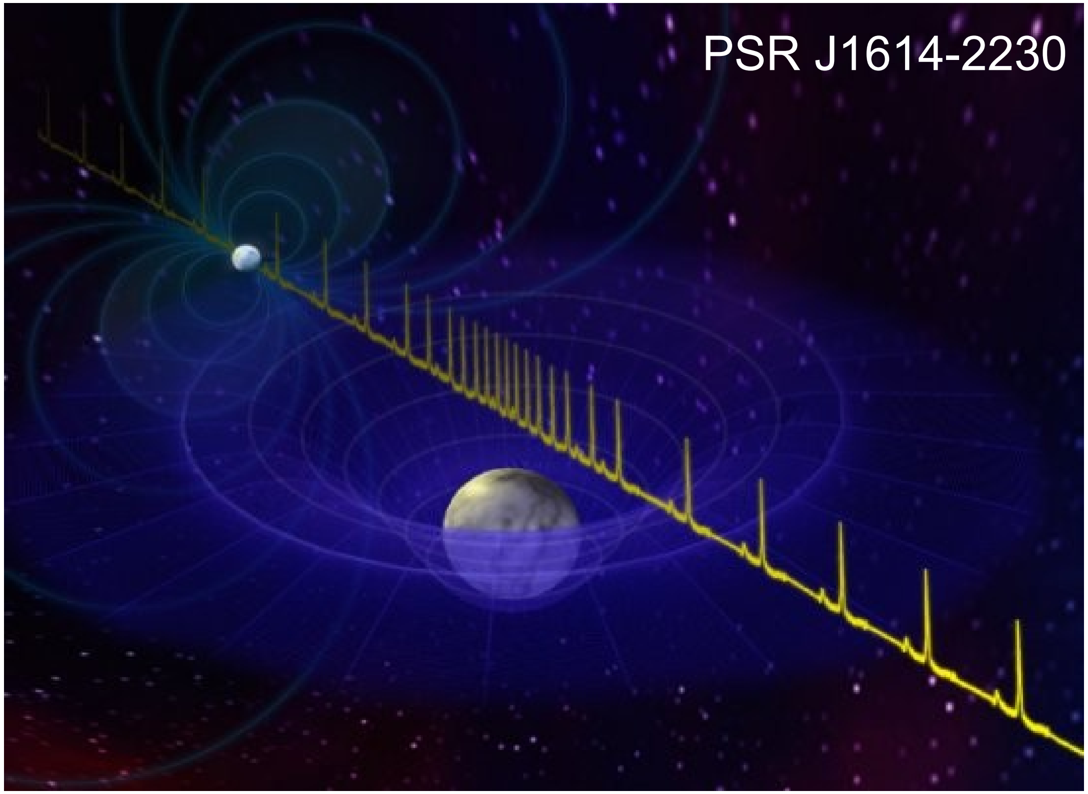
# Two high-mass pulsars with M ~ $2M_{sun}$

M=2.01 +/- 0.04 Msun




Antoniadis et al., Science 340 (2013) 448
Demorest et al., Nature 467 (2010) 1081
Fonseca et al., arxiv:1603.00545

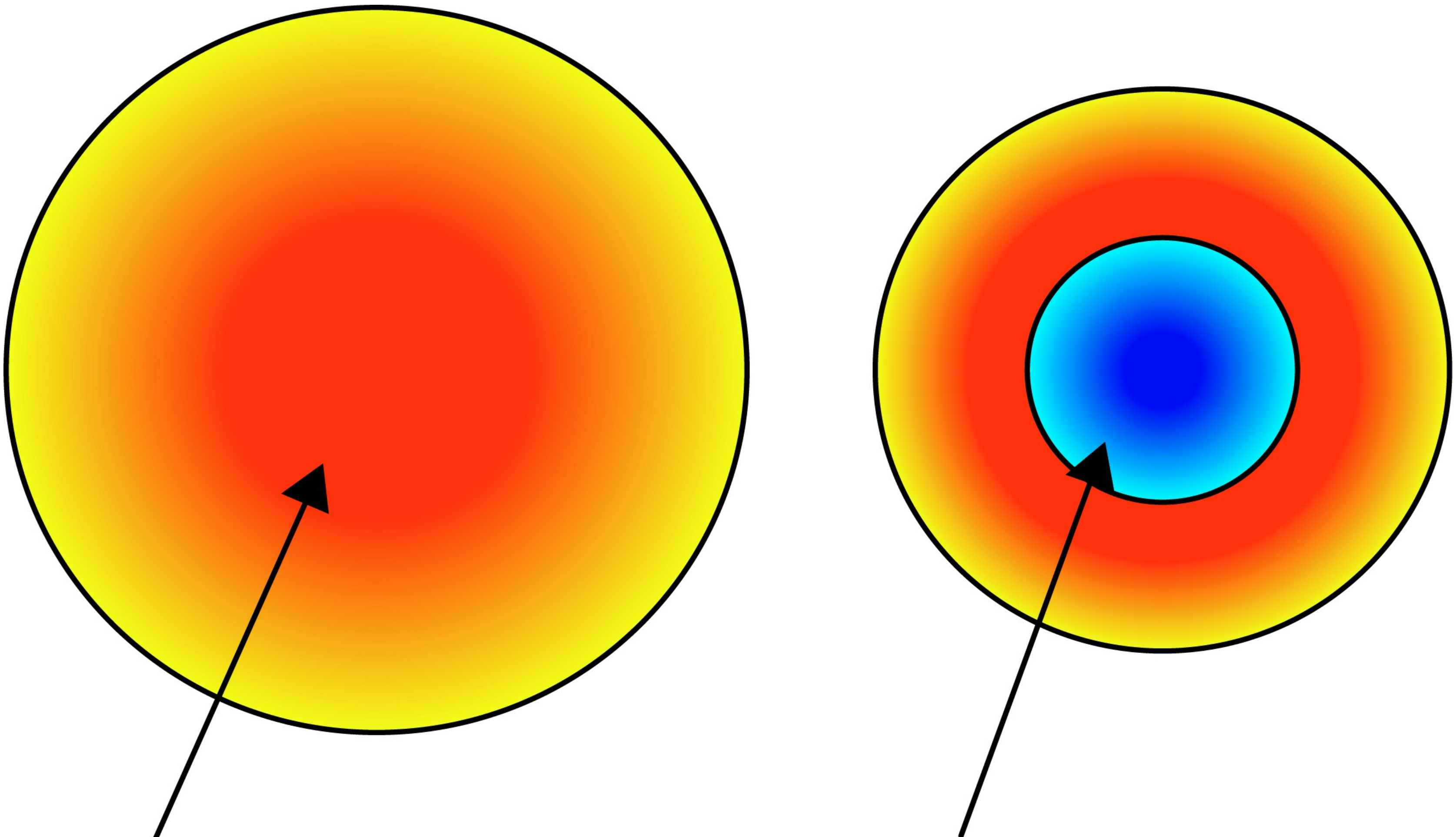
M=1.928 +/- 0.017 Msun




**What if they were high-mass twin stars?**

→ radius measurement required ! → NICER (2017)

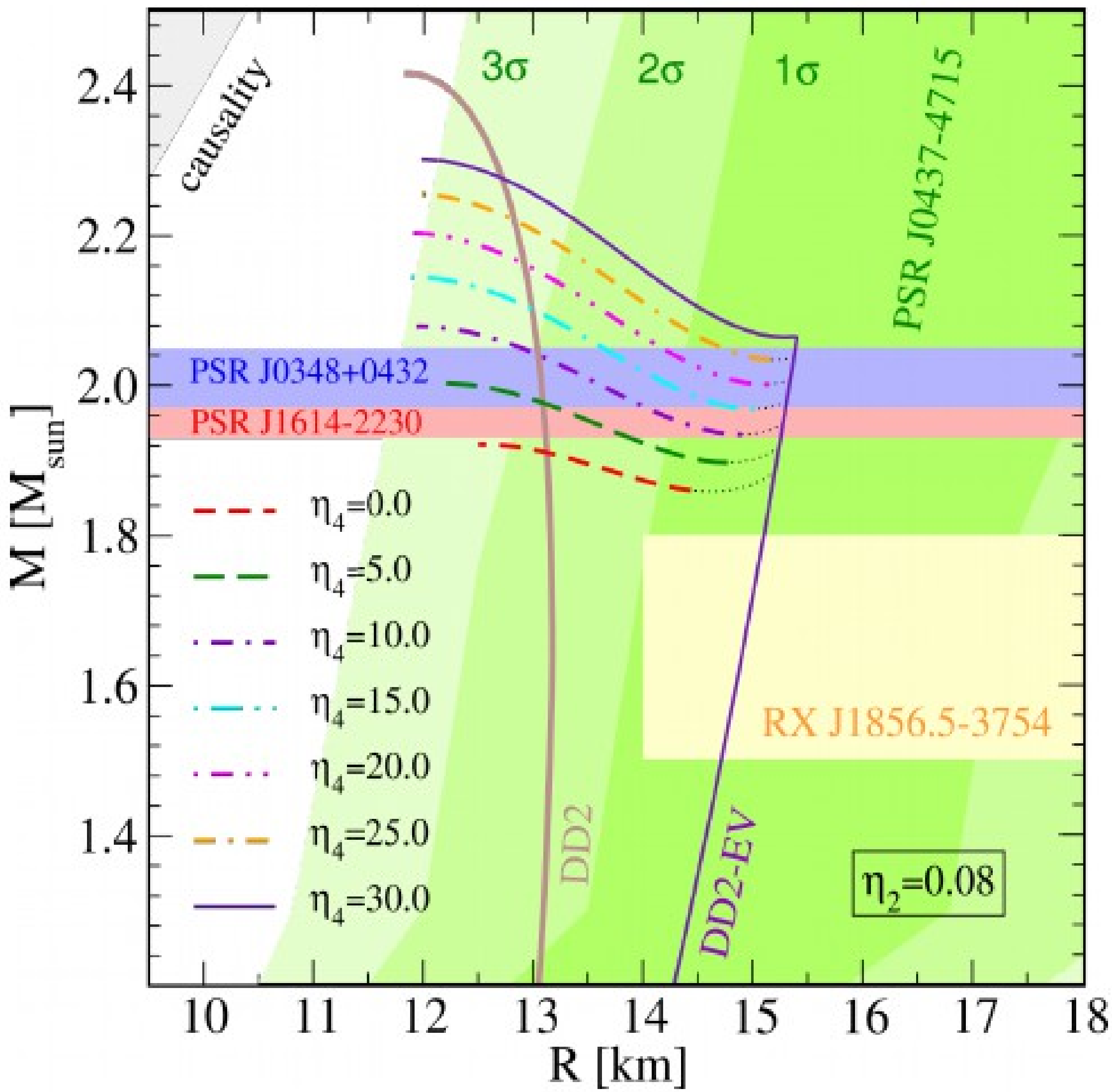
# Two high-mass pulsars with $M \sim 2M_{sun}$



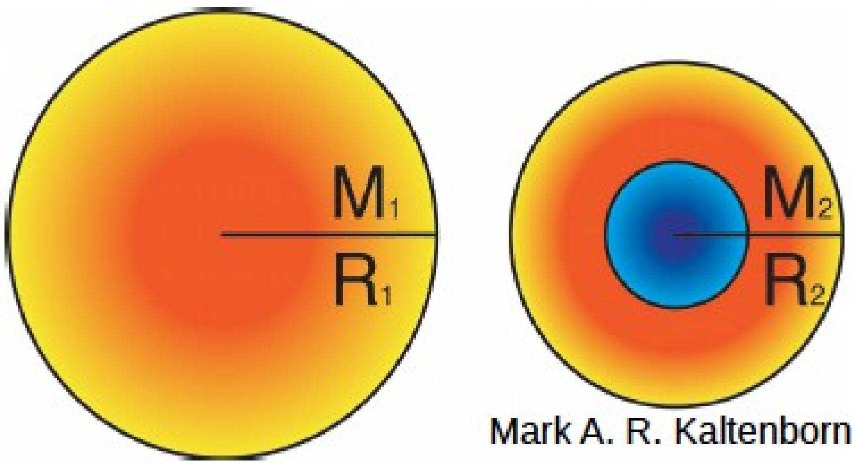
Neutron Star
Hadronic matter
$M_{star}$ = 2.0 $M_o$
$R_{star}$ = 13.9 km

Hybrid Star
Hadronic and Quark matter
$M_{star}$ = 2.0 $M_o$
$R_{star}$ = 11.1 km
$R_{quark\text{-}core}$ = 7.36 km

# Motivation – Neutron stars (Twins?)




- Star configurations with same masses, but different radii

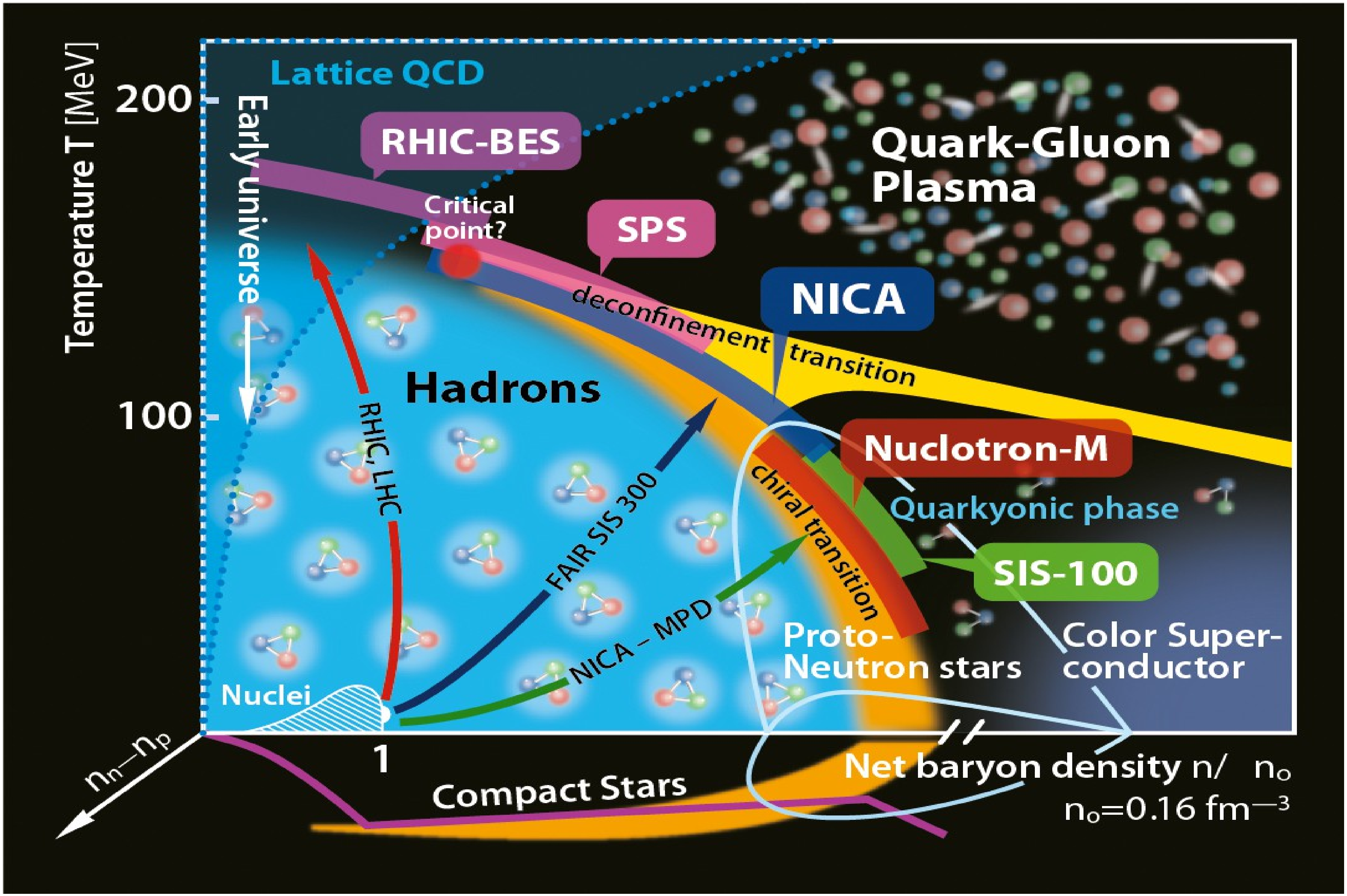



- **New class of EOS, that features high mass twins**
- NASA NICER mission: radii measurements ~ 0.5 km
- Existence of twins implies 1st order phase-transition and hence a critical point



Benic, Blaschke, Alvarez-Castillo, Fischer, Typel, A&A 577, A40 (2015)

# Support a CEP in QCD phase diagram with Astrophysics?



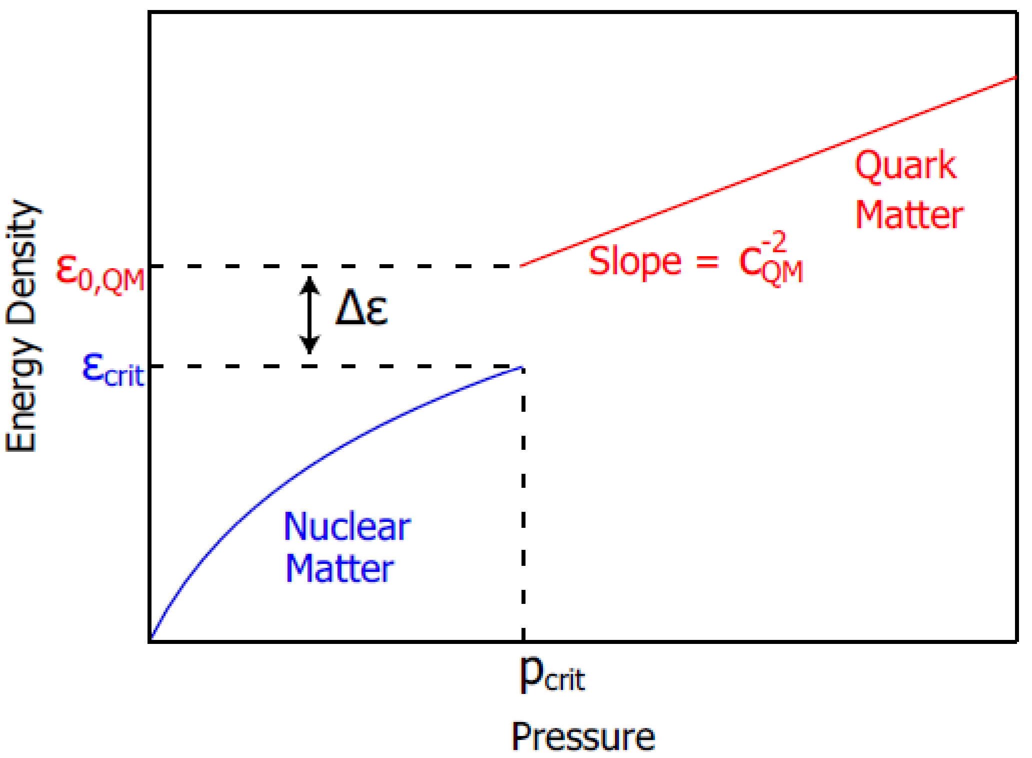

S. Benic et al., A&A 577, A40 (2015)

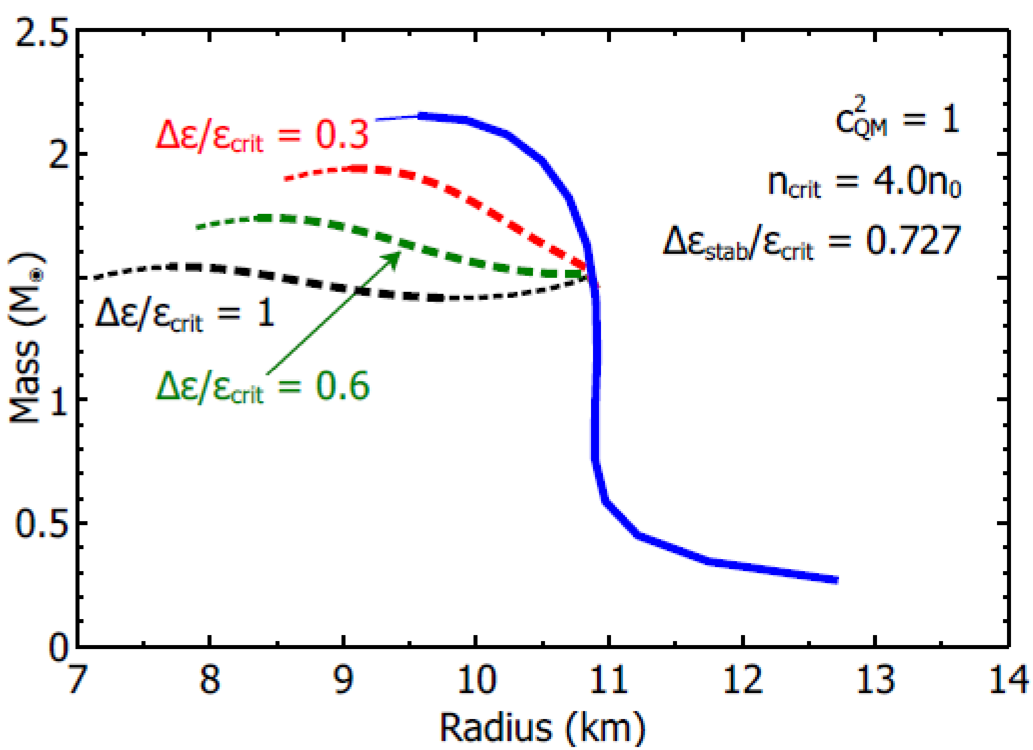
**Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!**

# Constant speed of sound (css) model

**Alford, Han, Prakash, arxiv:1302.4732**

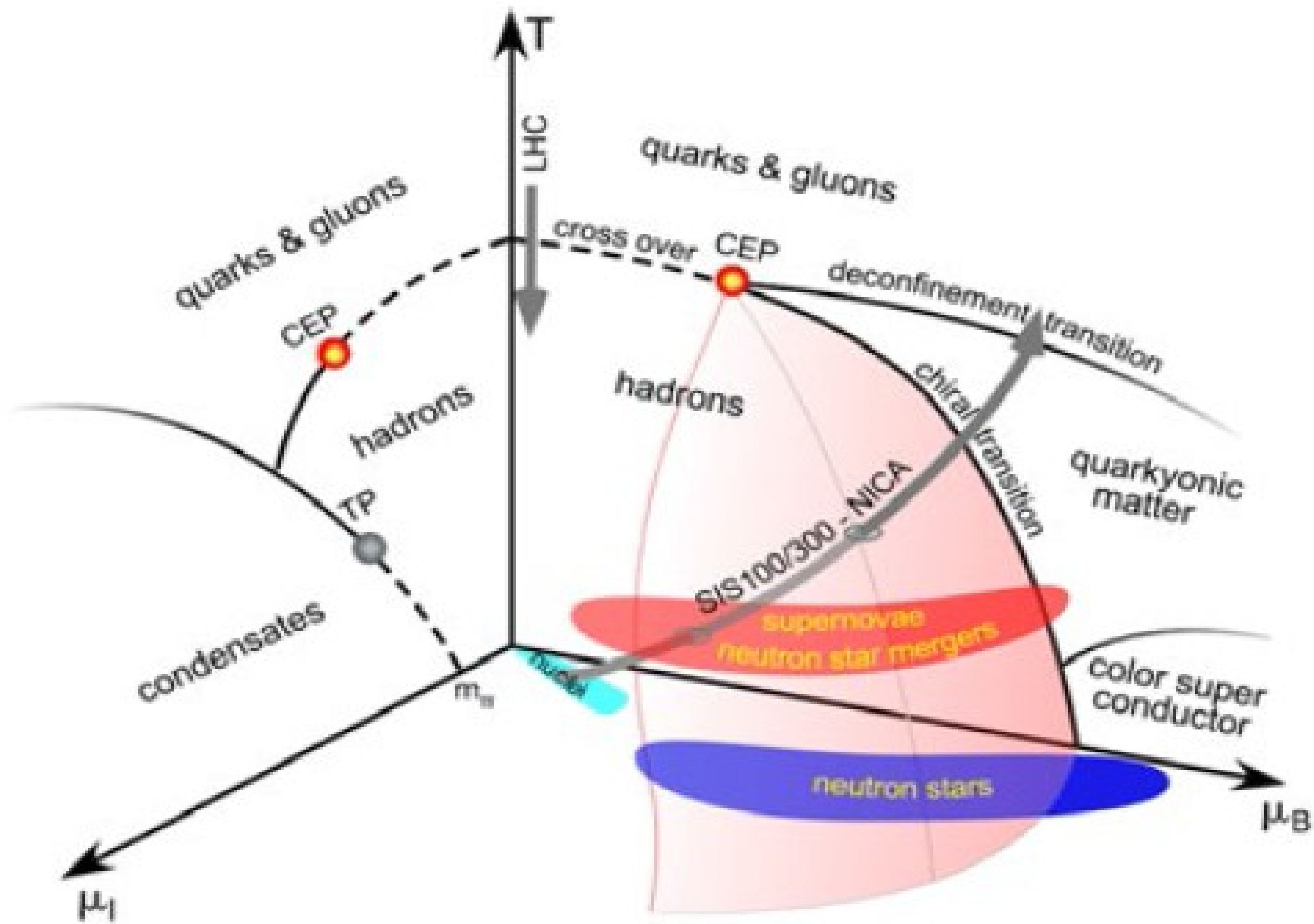
First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the "latent heat" (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → **"third family of CS"**.




Measuring two **disconnected populations** of compact stars in the M-R diagram would be the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

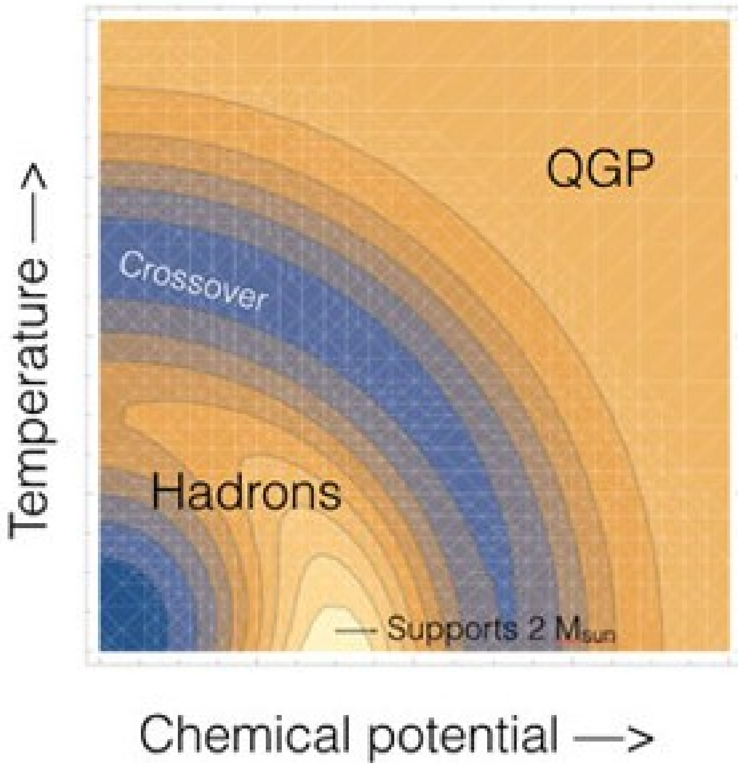
# CEP in the QCD phase diagram: HIC vs. Astrophysics

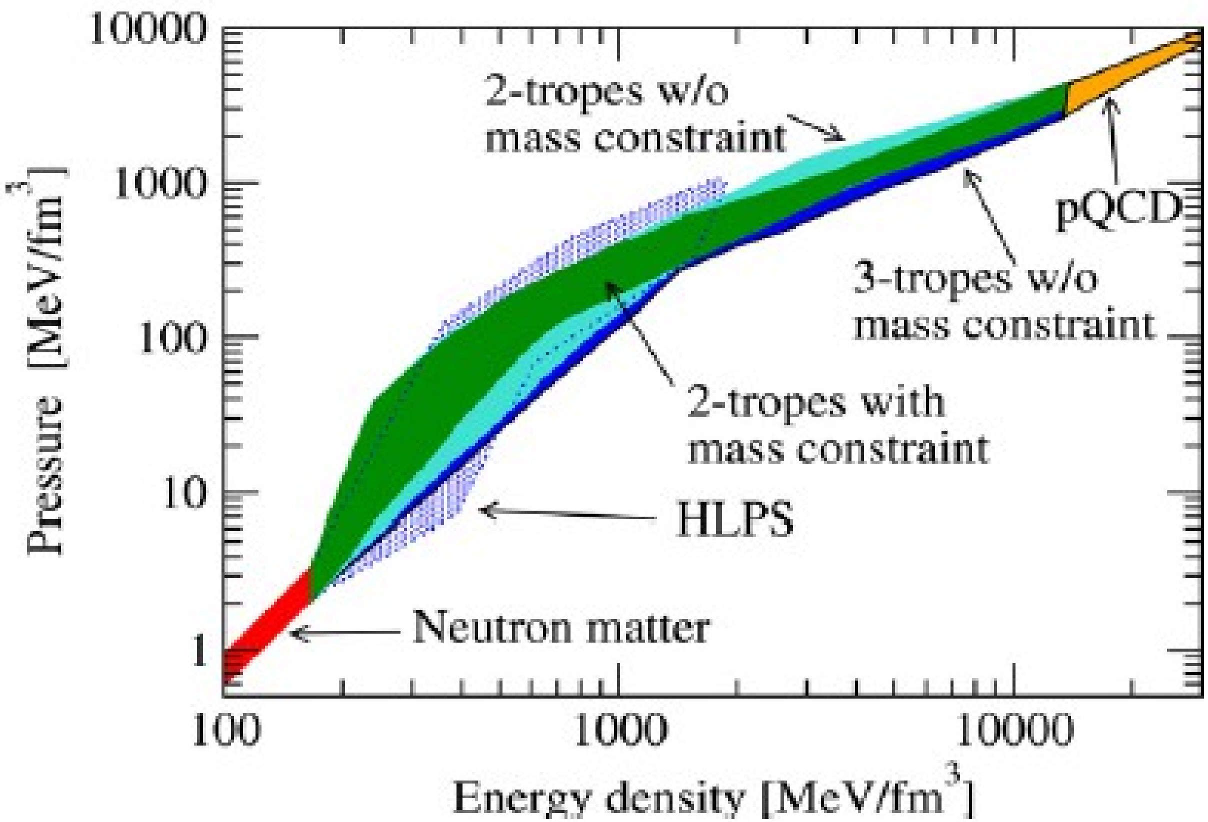



NuPECC Long Range Plan 2017; http://www.nupecc.org

A. Andronic, D. Blaschke, et al., “Hadron production ...”, Nucl. Phys. A 837 (2010) 65 - 86

# Towards "measuring" the EoS in the T – mu plane (QCD phase diagram)



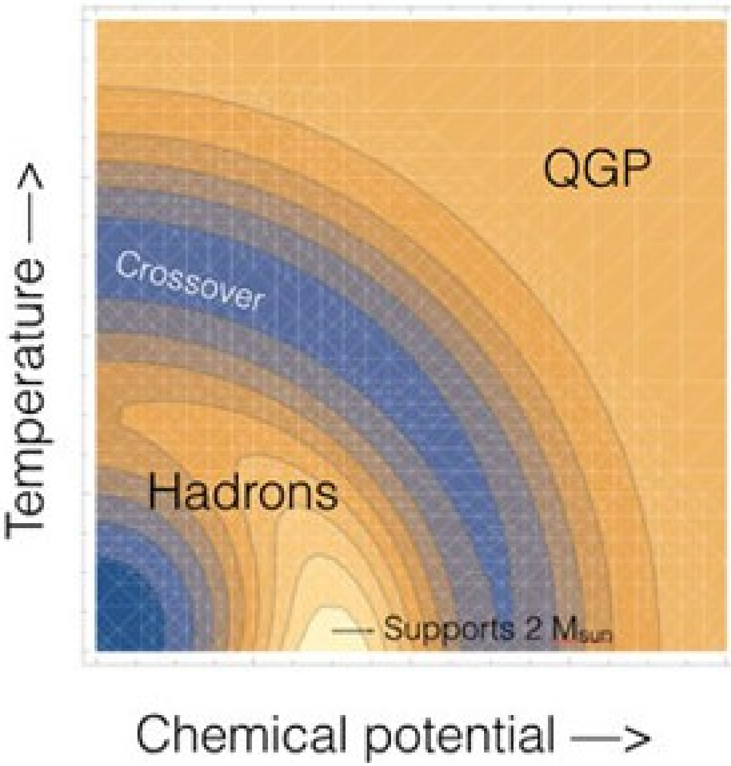



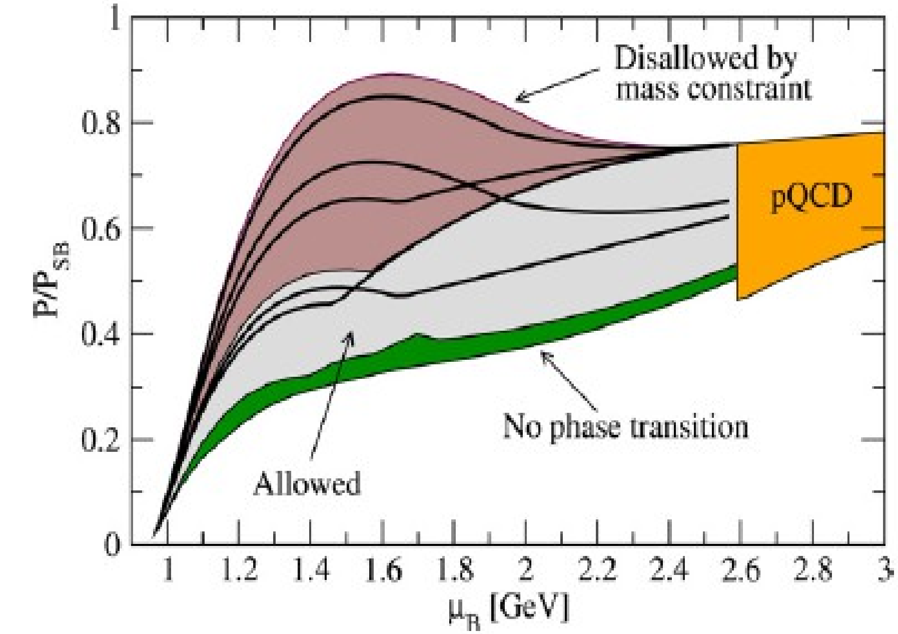

Speed-of-sound diagram from the INT program in Seattle, Summer 2016

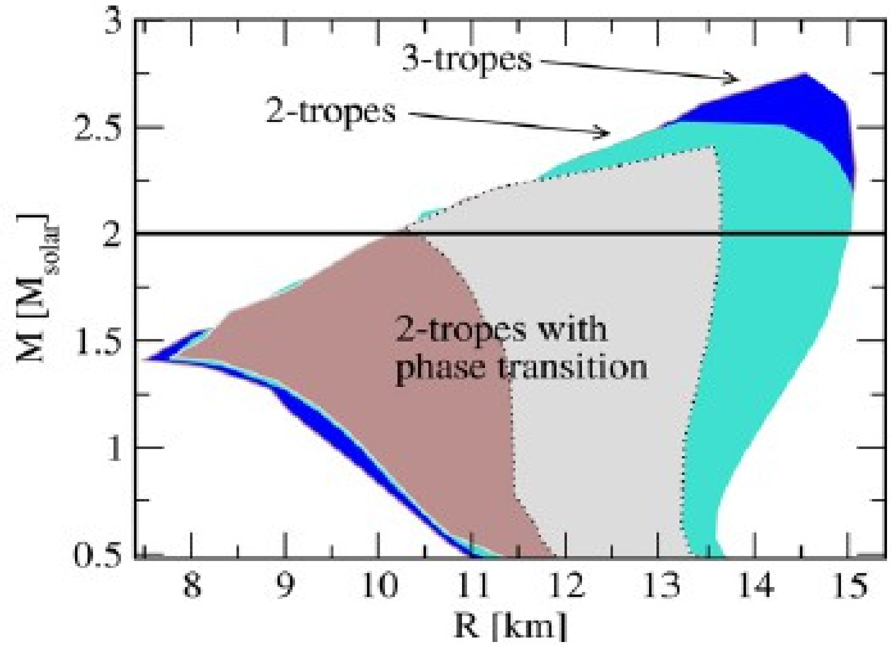
Interpolation between lattice QCD and Compact star physics (2 M_sun)

A. Kurkela, E. Fraga, J. Schaffner-Bielich, A. Vuorinen, Astrophys. J. 789 (2014) 127

# Towards “measuring” the EoS in the T – mu plane (QCD phase diagram)

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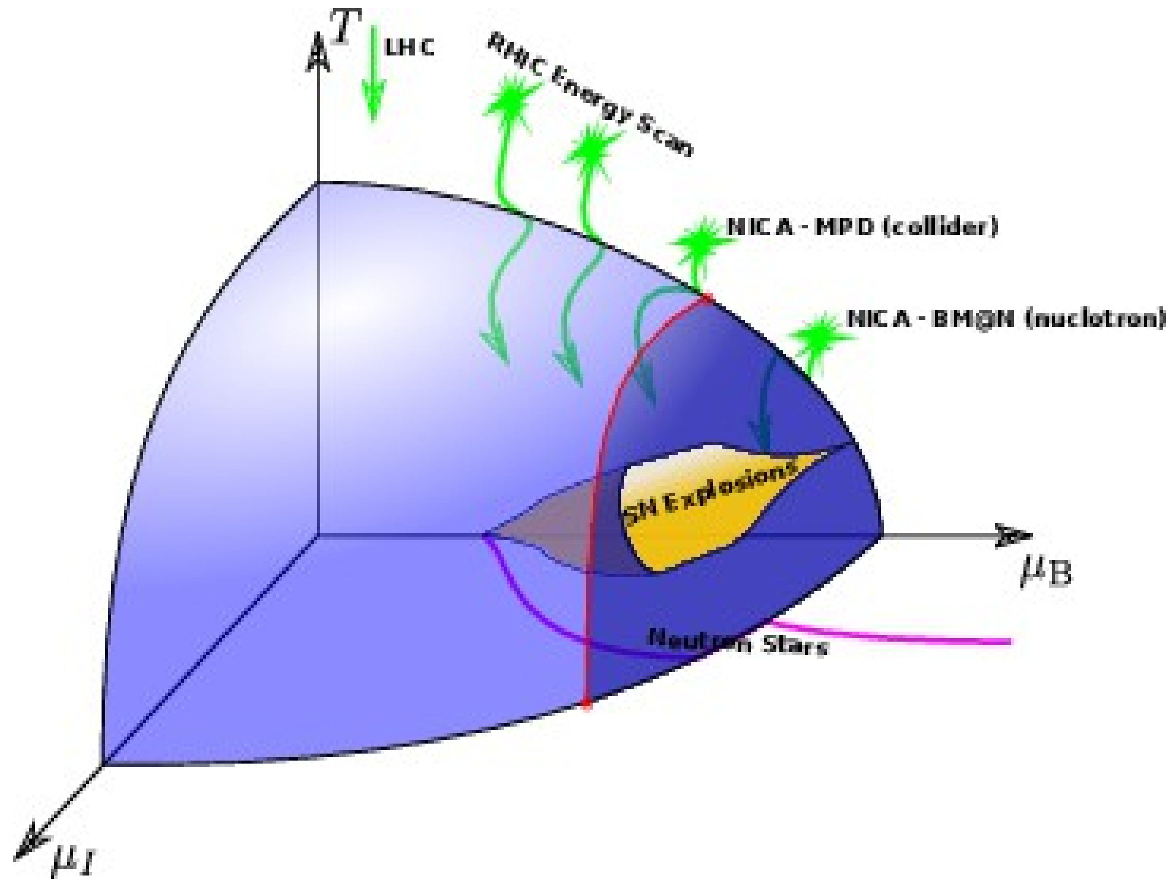
# Conclusion:

High-mass twins (HMTs) with quark matter cores can be obtained within different hybrid star EoS models, e.g.,
- constant speed of sound
- higher order NJL
- piecewise polytrope
- density functional

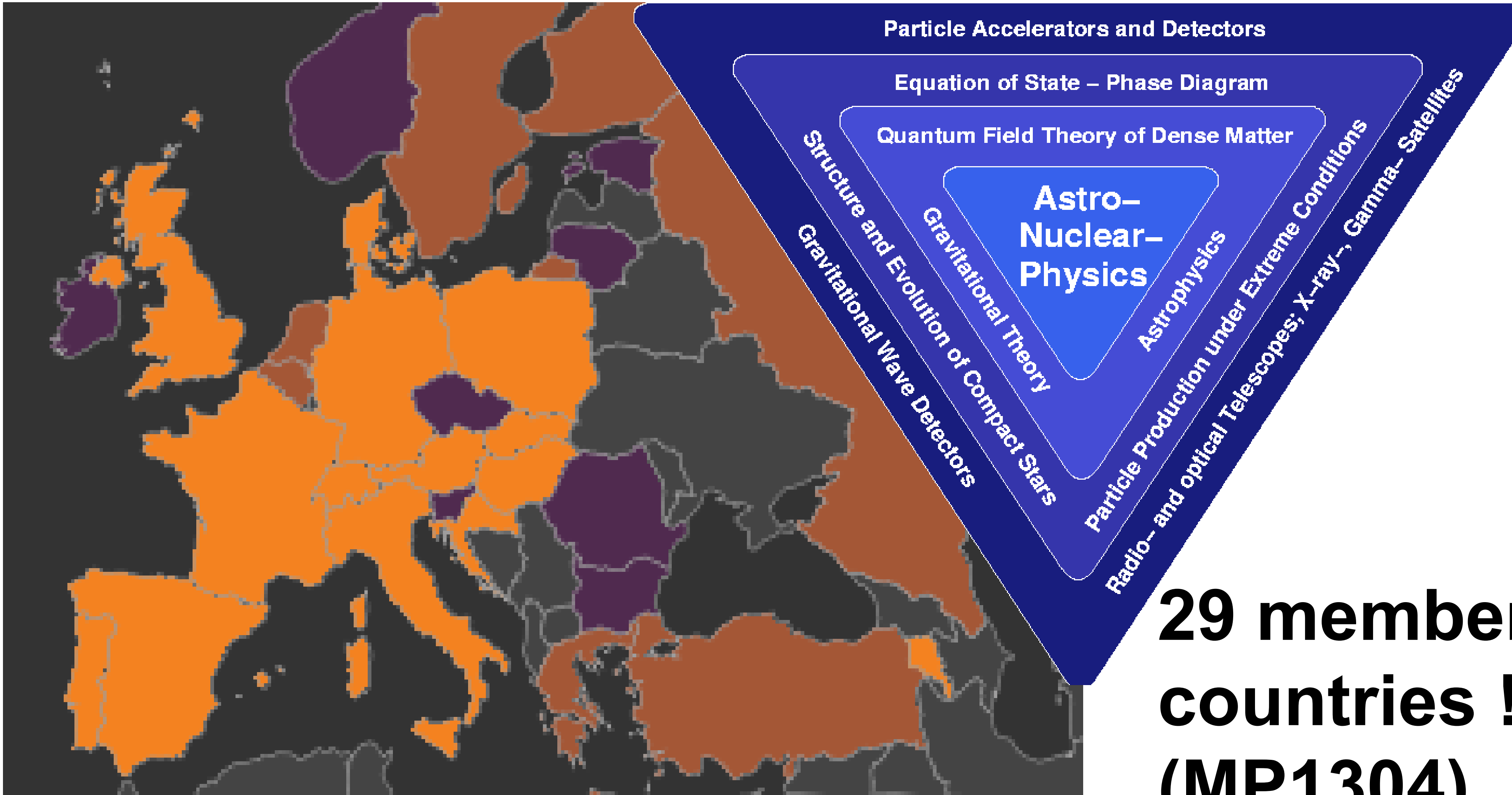



HMTs require stiff hadronic and quark matter EoS with a strong phase transition (PT)

Existence of HMTs can be verified, e.g., by precise compact star mass and radius observations (and a bit of good luck) → Indicator for strong PT !!

Extremely interesting scenarios possible for dynamical evolution of isolated (spin-down and accretion) and binary (NS-NS merger) compact stars

**Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics**




**29 member countries !! (MP1304)**

**New** comp star **!**

cost
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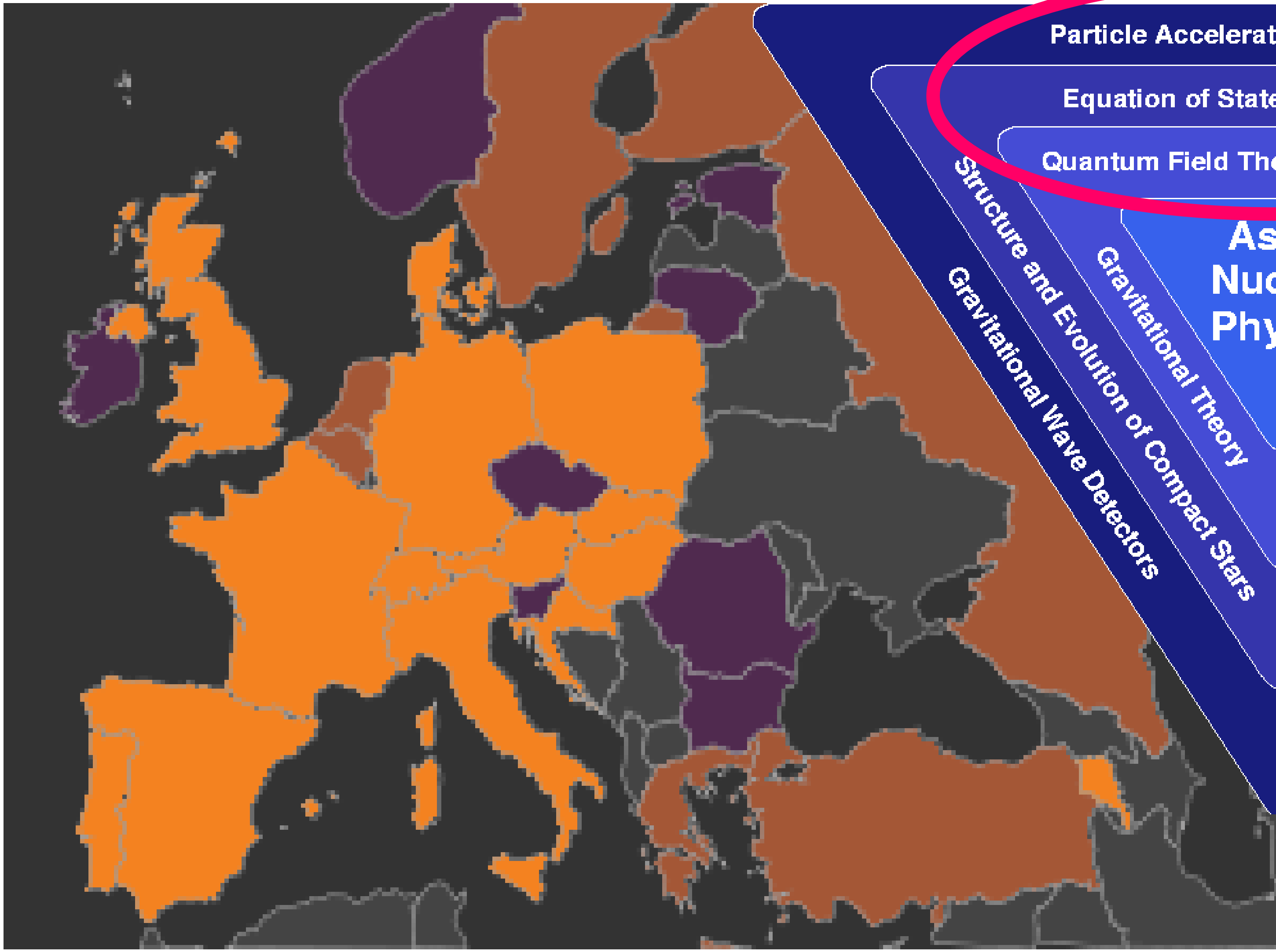
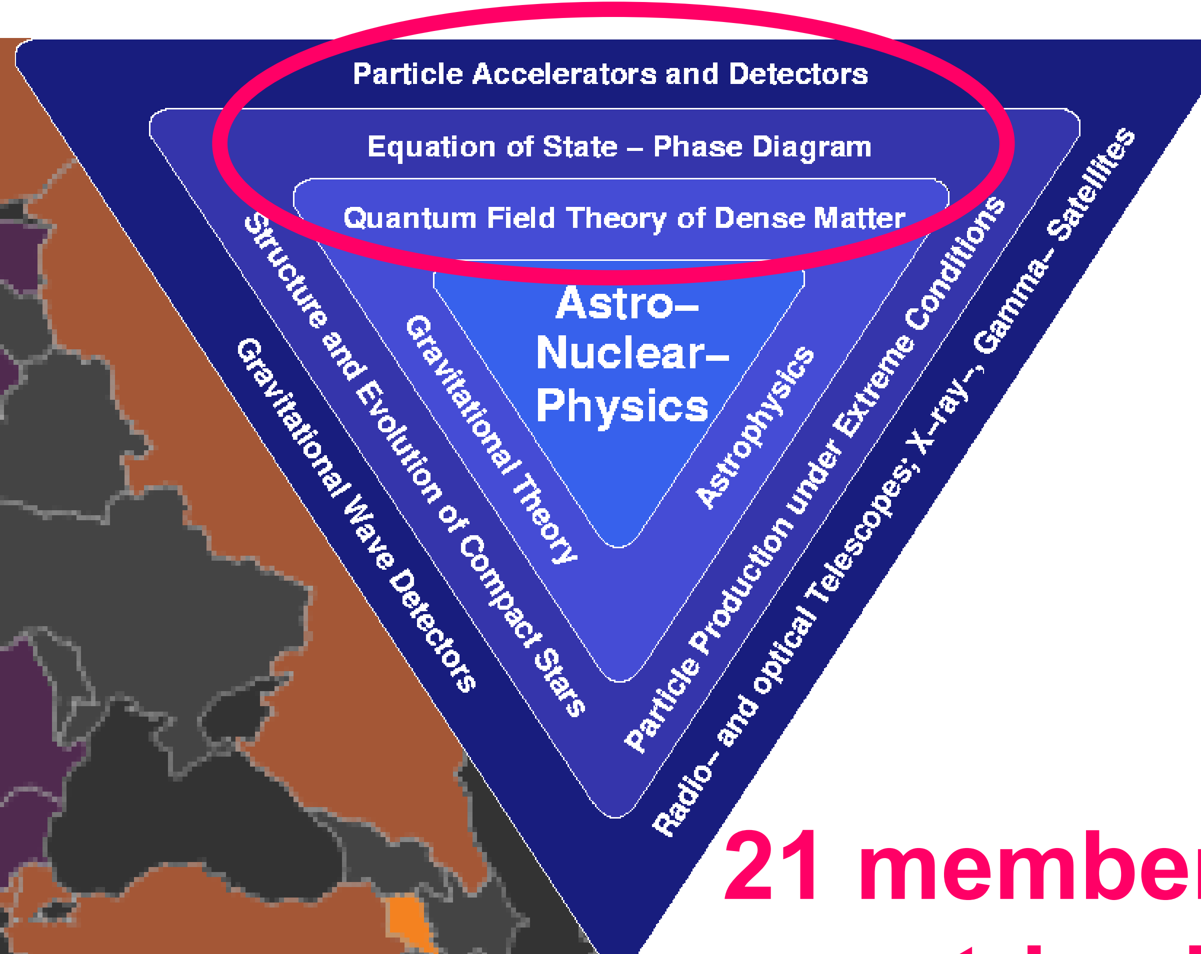
**Kick-off: Brussels, November 25, 2013**





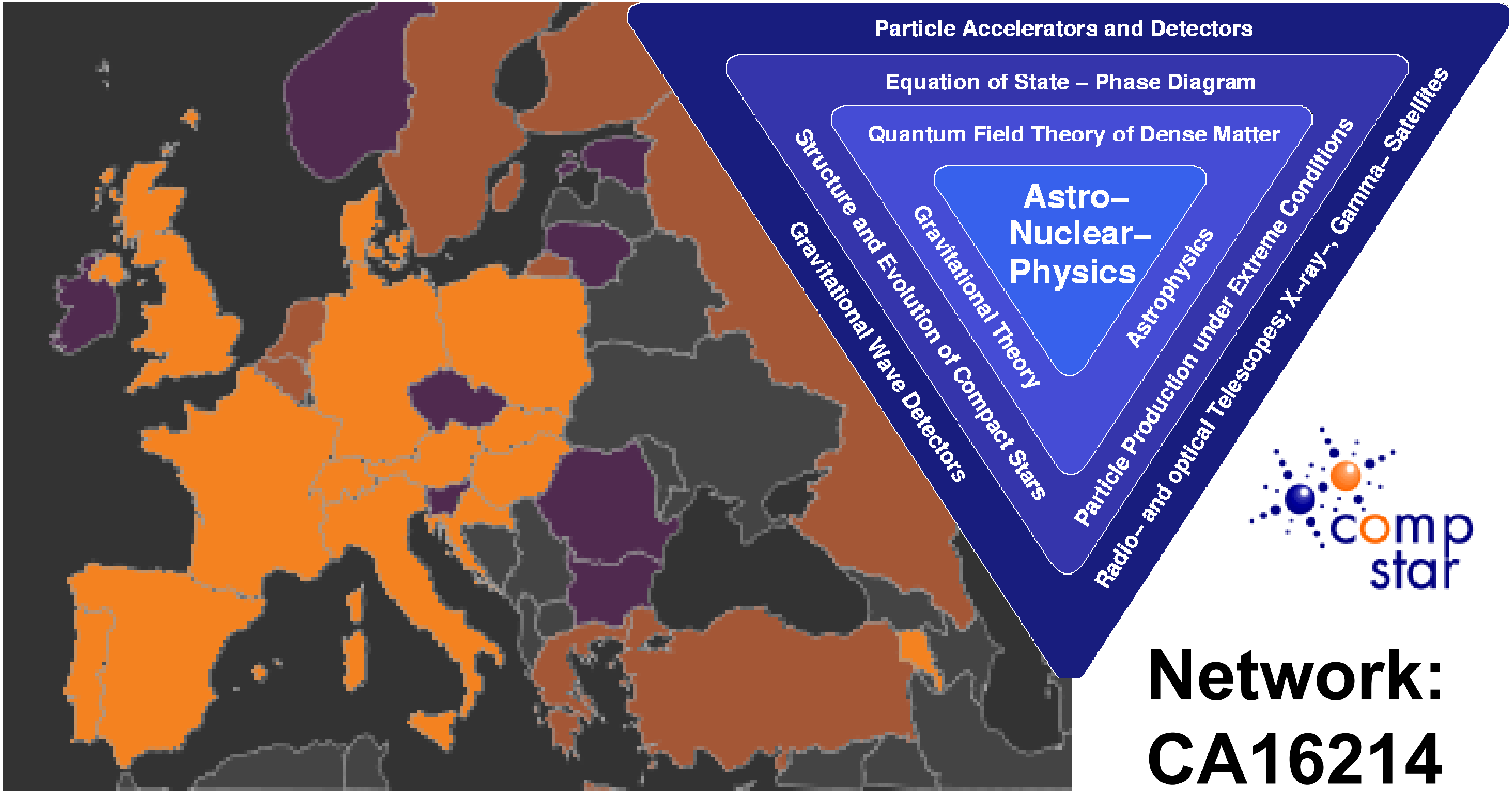

**21 member countries ! (CA15213)**

"Theory of HOt Matter in Relativistic Heavy-Ion Collisions"

**New: THOR !**

EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

**Kick-off: Brussels, October 17, 2016**




comp star

**Network: CA16214**

**Newest: PHAROS**

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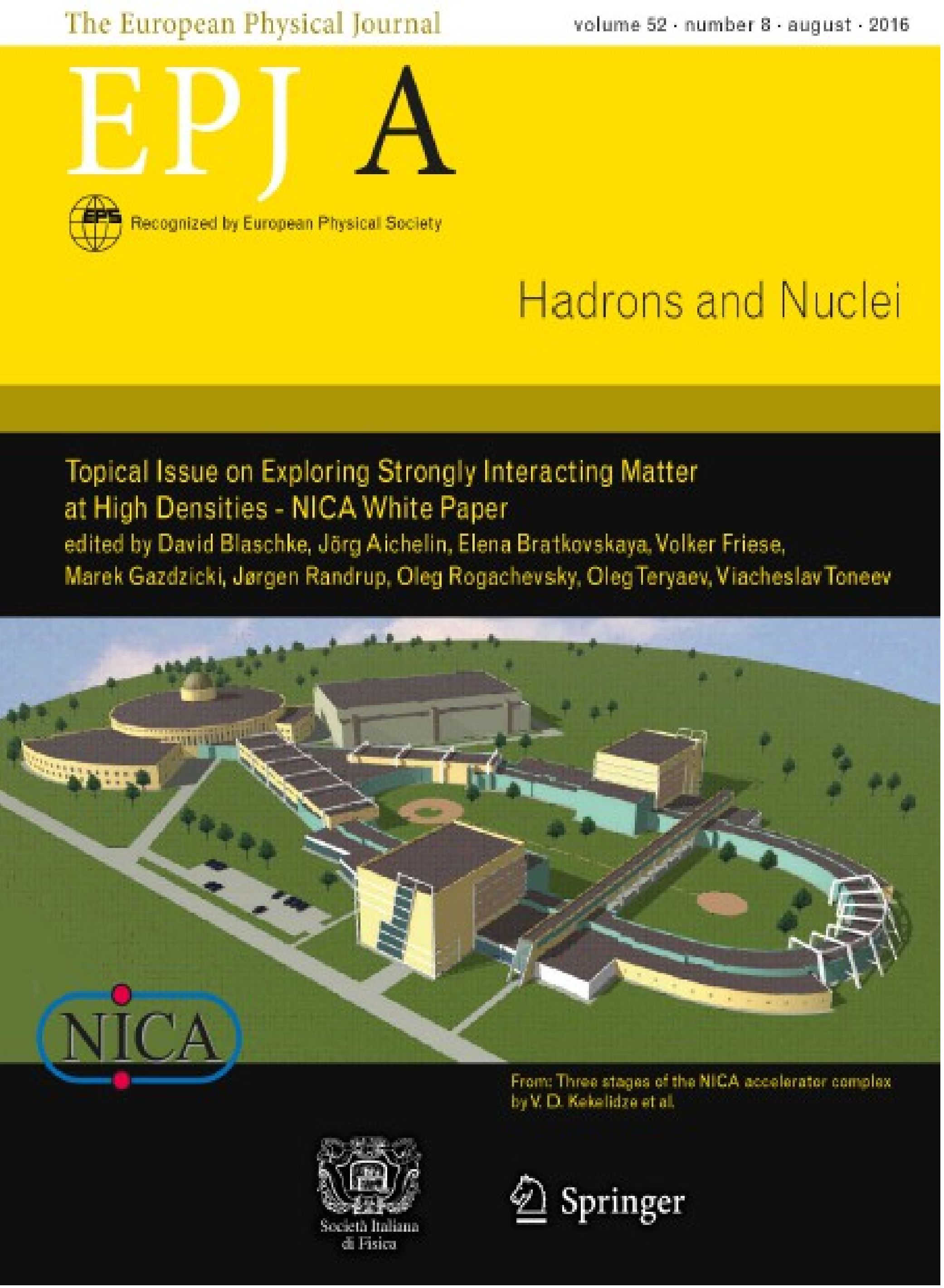
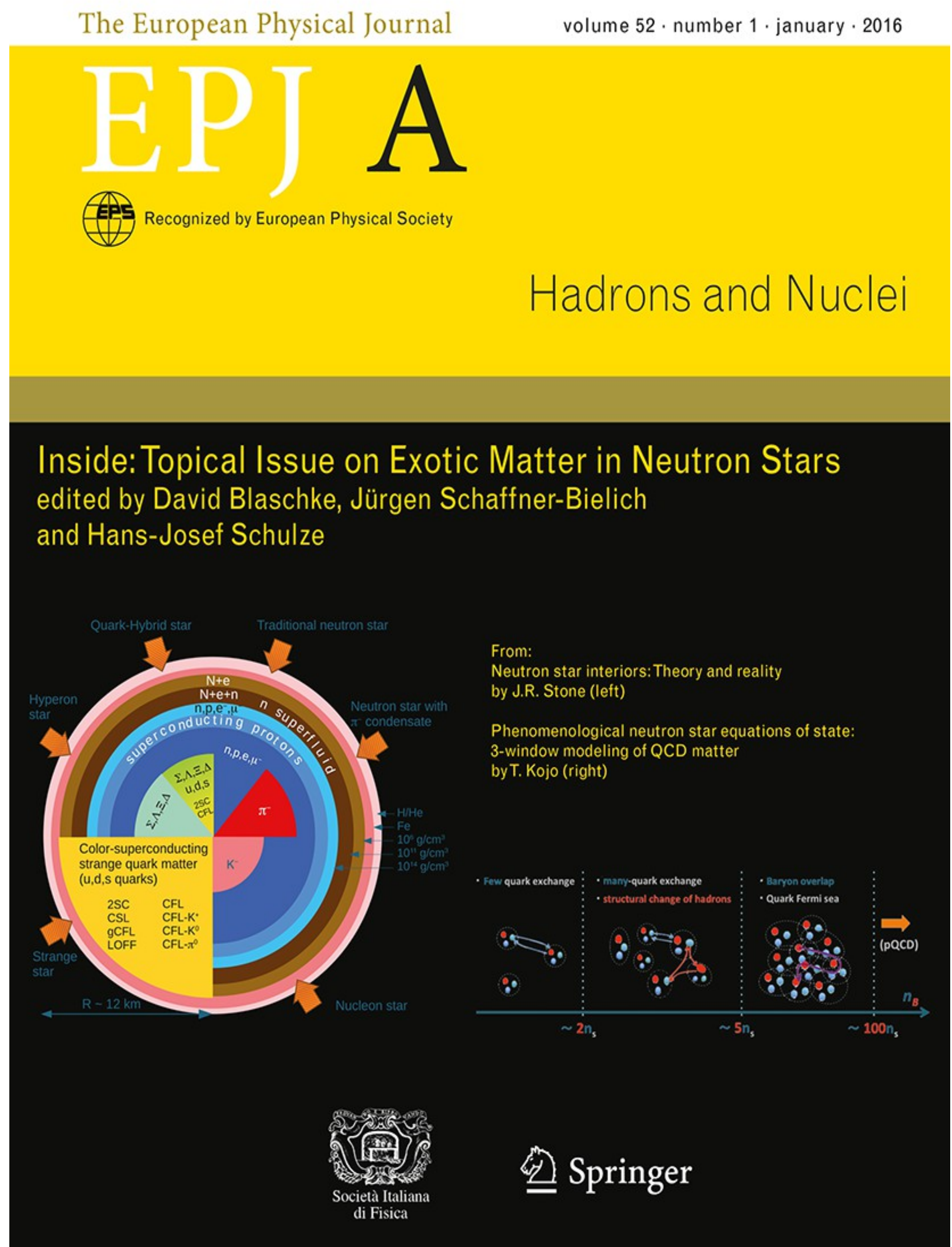
http://www.cost.eu/COST_Actions/ca/CA16214

**Kick-off: Brussels, late 2017**

International Conference “Critical Point and Onset of Deconfinement”
University of Wroclaw, May 29 – June 4, 2016

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