中国科学院高能物理研究所

On Two-photon exchange effects in the form factors

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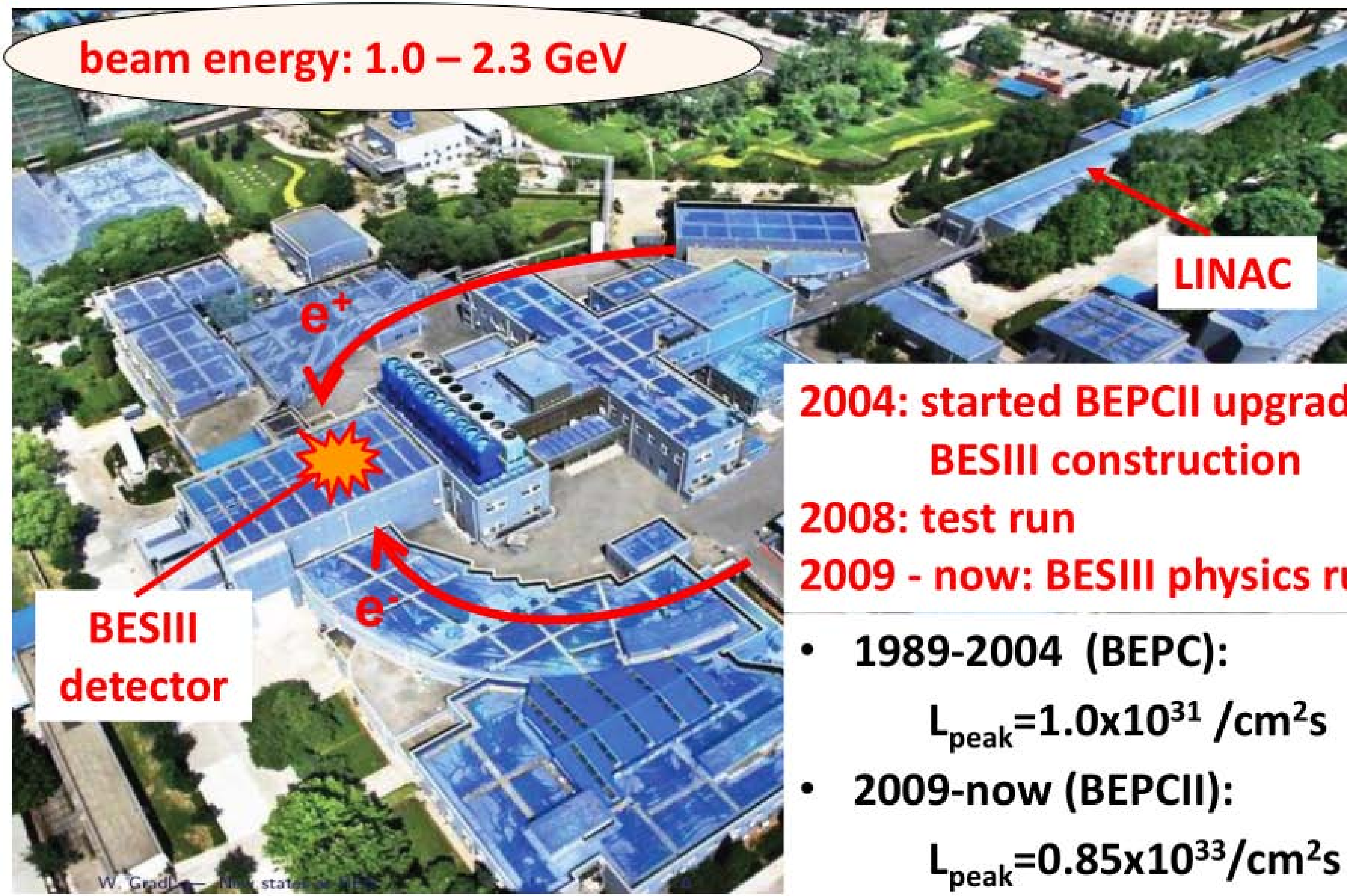
2016/9/27

MSU, Moscow

Beijing Electron Positron Collider (BEPC)



2004: started BEPCII upgrade, BESIII construction

2008: test run

2009 - now: BESIII physics run

- **1989-2004 (BEPC):**
  L_{peak}=1.0x10^{31} /cm^2s
- **2009-now (BEPCII):**
  L_{peak}=0.85x10^{33}/cm^2s

BESIII Collaboration

From Thailand

US (5)

Univ. of Hawaii
Carnegie Mellon Univ.
Univ. of Minnesota
Univ. of Rochester
Univ. of Indiana

Europe (13)

Germany: Univ. of Bochum, Univ. of Giessen, GSI
Univ. of Johannes Gutenberg Helmholtz Ins. In Mainz
Russia: JINR Dubna; BINP Novosibirsk
Italy: Univ. of Torino, Frascati Lab, Ferrara Univ.
Netherland: KVI/Univ. of Groningen
Sweden: Uppsala Univ.
Turkey: Turkey Accelerator Center

Mongolia (1)

Institute of phys. & Tech.

Korea (1)

Seoul Nat. Univ.

Japan (1)

Tokyo Univ.

Pakistan (2)

Univ. of Punjab
COMSAT CIIT

China(32)

IHEP, CCAST, GUCAS, Shandong Univ., Univ. of Sci. and Tech. of China
Zhejiang Univ., Huangshan Coll.
Huazhong Normal Univ., Wuhan Univ.
Zhengzhou Univ., Henan Normal Univ.
Peking Univ., Tsinghua Univ. ,
Zhongshan Univ.,Nankai Univ., Beihang Univ.
Shanxi Univ., Sichuan Univ., Univ. of South China
Hunan Univ., Liaoning Univ.
Nanjing Univ., Nanjing Normal Univ.
Guangxi Normal Univ., Guangxi Univ.
Suzhou Univ., Hangzhou Normal Univ.
Lanzhou Univ., Henan Sci. and Tech. Univ.
Univ. of Sci. & Tech. Liaoning

~400 members
from 55 institutions in 12 countries

Content

1, Motivation: Electromagnetic probes
Nucleon form factors

2, The two-photon exchange in the
proton form factors

3, Phenomenological approach,
deuteron form factors
two-photon exchange

4, Summary

I),Motivation:

EM probe to explore

- Electric and magnetic nucleon form factors
  (Pion and deuteron form factors)

- Nucleon-resonance transitions

$$eN -> N^*, \gamma p -> \Delta(S_{11}, D_{13}...)$$

- Nucleon spin structure
  Parton distribution functions

- Generalized parton distribution functions

History of Nucleon Structure Study

- 1933: *First (Indirect) Evidence of Proton Structure*

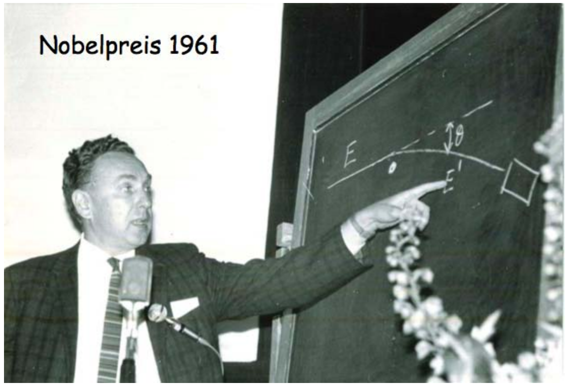
  magnetic moment of the proton: $\mu_p = e\hbar/2m_pc(1+\kappa_p)$!

  anomalous magnetic moment: κ_p= 1.5 +- 10%

Otto Stern
Nobel Prize 1943

- 1960s: Discovery: Proton Has Internal Structure

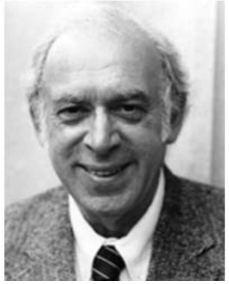
  elastic electron scattering



Robert Hofstadter,
Nobel Prize 1961

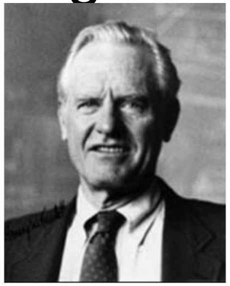
- 1970s: Discovery of Quarks (Partons)

  deep-inelastic scattering



J.T. Friedman

R. Taylor



H.W. Kendall

Nobel Prize 1990

- 1980s-1990s: Spin Structure
- 2000s: Multi-dimension Structure

Form factors of proton (from ep)

• Many experimental measurements have been carried out since last century:

* The fundamental properties of nucleon

Charge and magnetization distributions

* Test model calculations of the intrinsic structure of nucleon

Nearly all the measurements used Rosenbluth separation

The scaling behavior of the EM form factors of nucleon

$$G_E^p(Q^2) \approx G_M^p(Q^2)/\mu_p \approx G_M^n(Q^2)/\mu_n \approx G_D(Q^2)$$

• New and improved technique makes the measurement more precisely and can provide information more accurate

Form factors of Nucleon (from ep)

- Nucleon has its intrinsic structure *p(uud), n(udd)*
- Photon and nucleon interaction is not point-like

• One-photon exchange

•Electron-photon:

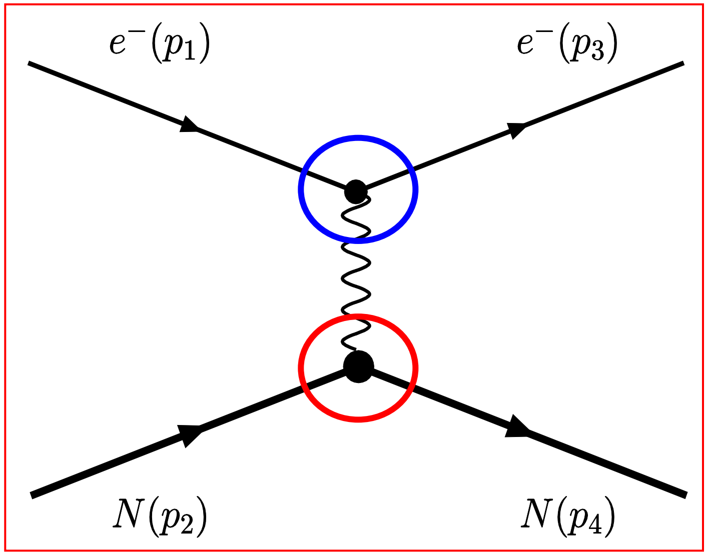
$$\Gamma_l^\mu = \gamma^\mu$$

•Nucleon EM vertex:

$$\Gamma_B^\mu = ??$$

$$\Gamma^\mu = F_1 \gamma^\mu + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{M}$$

$$\begin{aligned} G_M(Q^2) &= F_1(Q^2) + F_2(Q^2), \\ G_E(Q^2) &= F_1(Q^2) - \tau F_2(Q^2). \end{aligned}$$



Space like

Measurement (I)—Rosenbluth Separation (OPE)

Differential cross section:

$$\frac{d\sigma}{d\Omega} = \boxed{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} \frac{\boxed{\epsilon}}{\tau(1+\tau)} \left\{ \boxed{G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2)} \right\}$$

$$\boxed{\tau = \frac{Q^2}{4M^2}}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2 E' \cos^2(\theta/2)}{4E^3 \sin^4(\theta/2)}$$

$$\epsilon = [1 + 2(\tau + 1)\tan^2(\theta/2)]^{-1}$$

Reduced cross section: Rosenbluth separation

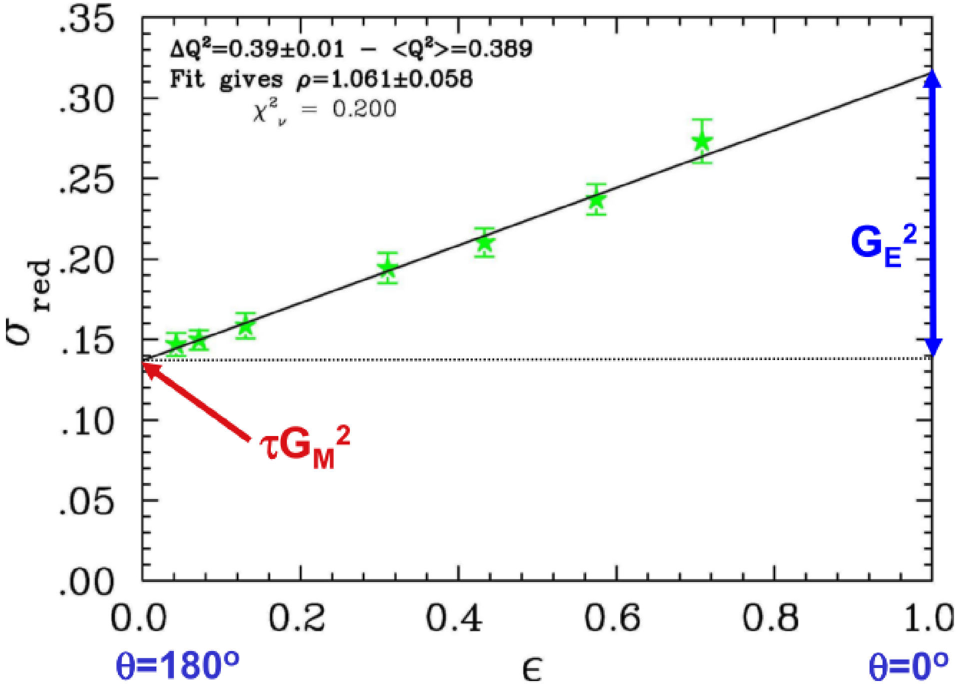
$$\sigma_R = G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2)$$

- **Proton form factor measurements from Rosenbluth separations**
  - G_{Mp} well measured to 10 GeV², data out to 30 GeV²
  - G_{Ep} well known to 1-2 GeV², data to ~6 GeV²

- The form factors are determined from the slope and intercept

- if Q^2 is large, large systematic error for G_E

- G_E is sensitive to the ε dependent-term

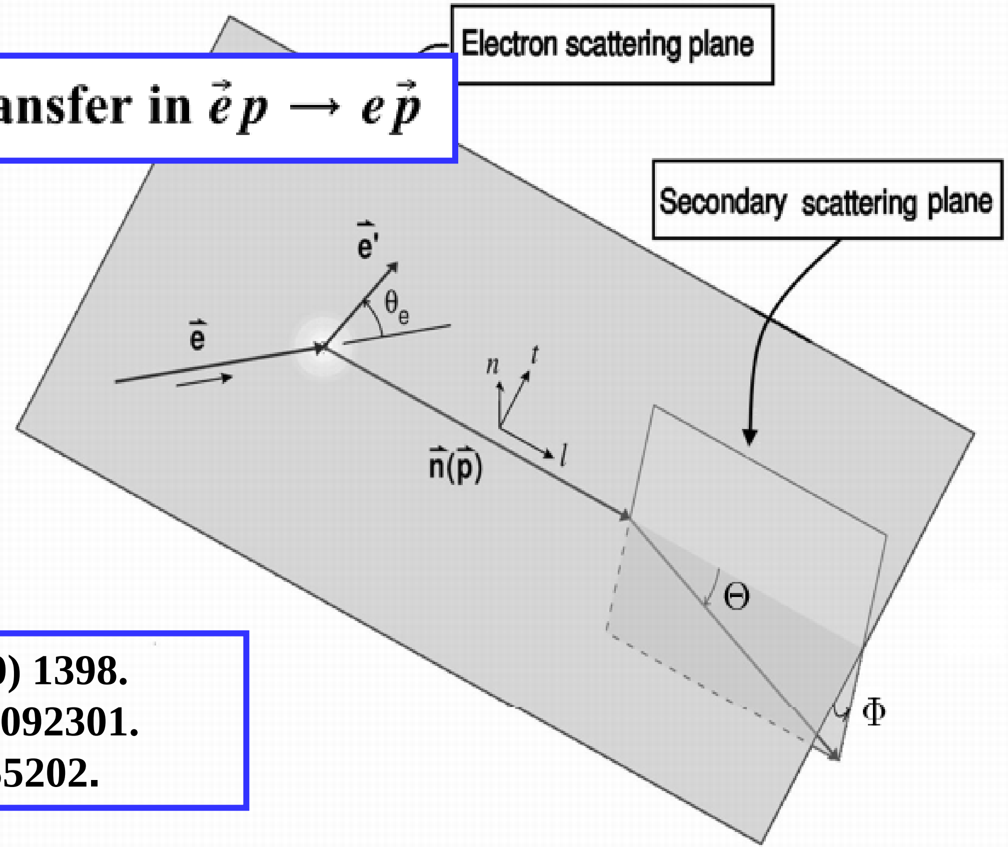


$$\epsilon = [1 + 2(\tau + 1)\tan^2(\theta/2)]^{-1}$$

- Mid ’90s brought measurements using improved techniques
  - High luminosity, highly polarized electron beams

Measurement（II）polarized Separation (precise measurement)

G_{E_p}/G_{M_p} Ratio by Polarization Transfer in $\vec{e}p \to e\vec{p}$



Initial: Electron polarized

$\lambda_e = \pm 1$;

ζ_4: Final proton polarization vector

M. K. Jones *et al*., Phys. Rev. Lett. 84 (2000) 1398.
O. Gayou *et al*., Phys. Rev. Lett. 88 (2002) 092301.
V. Punjabi *et al*., Phys. Rev. C 71 (2005) 055202.

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'}{4E^3 \sin^4(\theta_e/2)} \frac{\cos^2(\theta_e/2)}{(\tau+1)\epsilon} \Bigg\{ \left[\tau G_M^2 + \epsilon G_E^2\right] + \lambda_e \Bigg[\frac{E+E'}{M}$$

$$\times \sqrt{\tau(1+\tau)} G_M^2 \tan^2 \frac{\theta_e}{2} \vec{\zeta}_4 \cdot \hat{z} - 2\sqrt{\tau(1+\tau)} G_M G_E \tan \frac{\theta_e}{2} \vec{\zeta}_4 \cdot \hat{x} \Bigg] \Bigg\}$$

G_{E_p}/G_{M_p} Ratio by Polarization Transfer in $\vec{e}p \to e\vec{p}$

Final proton is longitudinal polarized (OPE):

$$P_z = \frac{E+E'}{MI_0}\sqrt{\tau(\tau+1)}G_M^2\tan^2(\frac{\theta_e}{2})$$

Final proton is transverse polarized (OPE):

$$P_x = -\frac{2}{I_0}\sqrt{\tau(\tau+1)}G_M G_E\tan(\frac{\theta_e}{2})$$

Ratio of electric and magnetic form factors (OPE):

$$\boxed{\frac{P_x}{P_z} = -\frac{2M}{E+E'}\frac{G_E}{G_M}\cot(\frac{\theta_e}{2})}$$

$$\Downarrow$$

$$\boxed{= -\frac{2M}{E+E'}R_{polarization}\cot(\frac{\theta_e}{2})}$$

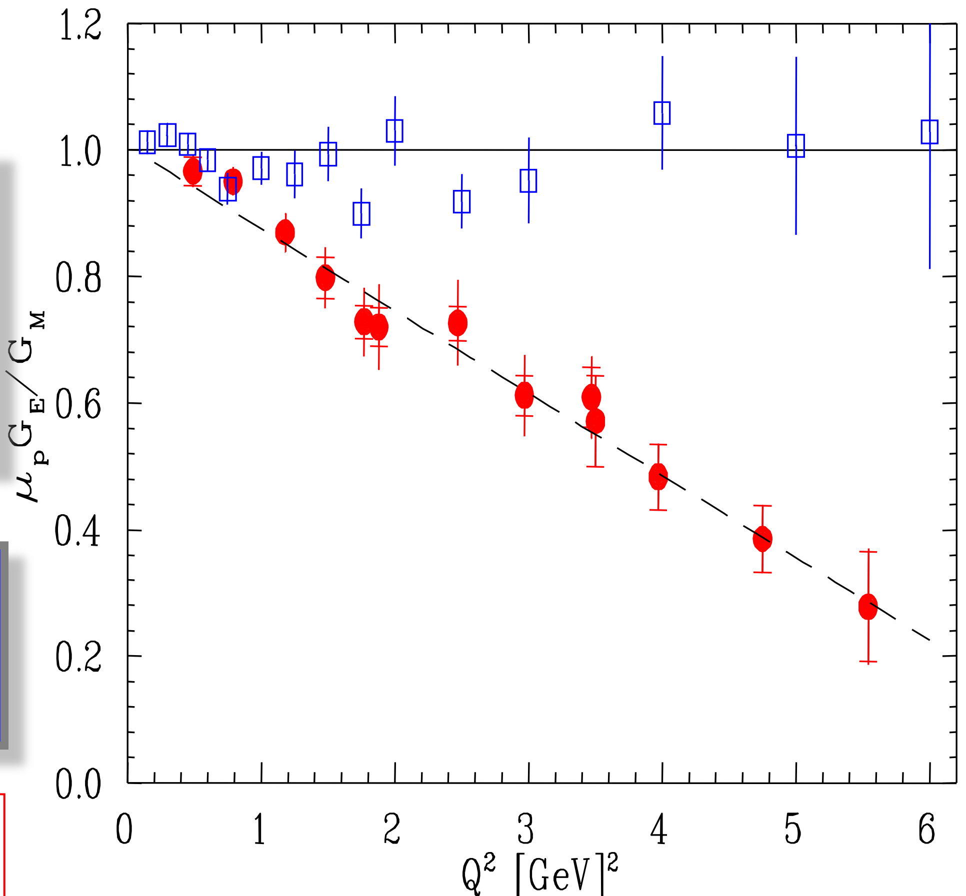
2, The two-photon exchange in the proton form factors

Measurements:

- $R_{Rosenbluth} \simeq 1$
- $R_p \approx 1 - 0.1306Q^2$

Since the two observations are based on the one-photon exchange, the divergence means something!!!

M. K. Jones *et al*., Phys. Rev. Lett. 84 (2000) 1398.
O. Gayou *et al*., Phys. Rev. Lett. 88 (2002) 092301.
V. Punjabi *et al*., Phys. Rev. C 71 (2005) 055202.



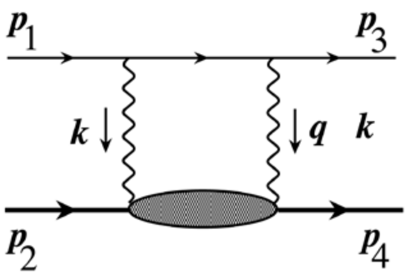
$$R_{Rusenbluth} = 1 - 0.0762Q^2 + 0.004896Q^4 + 0.001298Q^6.$$

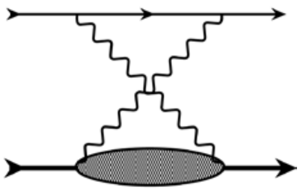
$$R_{polarization} = 1 - 0.1306Q^2 + 0.004174Q^4 - 0.000752Q^6.$$

Measurements of proton form factors

Comparing the measurements of Rosenbluth separation and polarization transfer, it is shown an unexpected and significant different dependence on Q^2 for G_p^E than on G_p^M.

This has been interpreted as indicating a difference between the spatial distributions of the charge and magnetization at short distances.





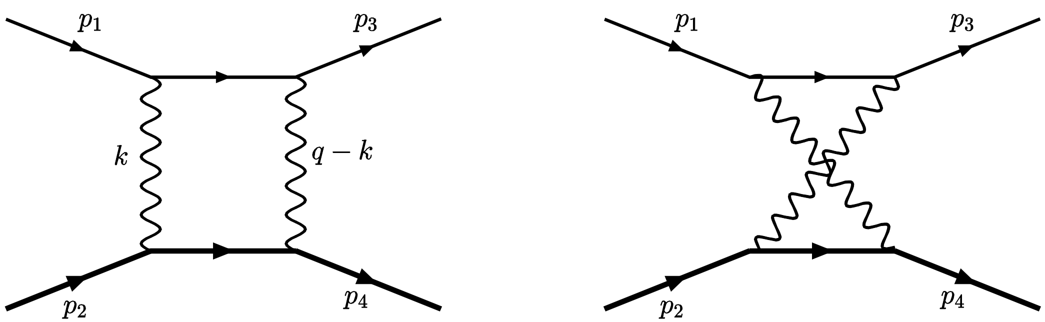
Two-photon exchange

corrections believed to explain the discrepancy

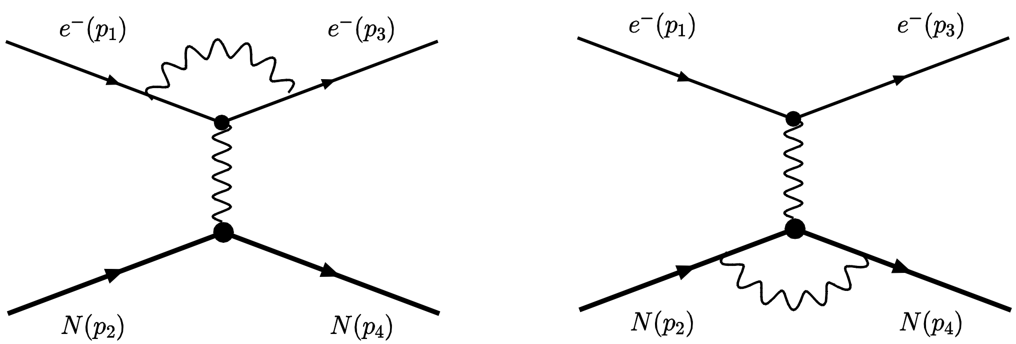
P.A.M.Guichon and M.Vanderhaeghen, PRL 91, 142303 (2003)

MT Corrections : (L. M. Mo and Y. S. Tsai, in 60s)

Two-photon exchange process



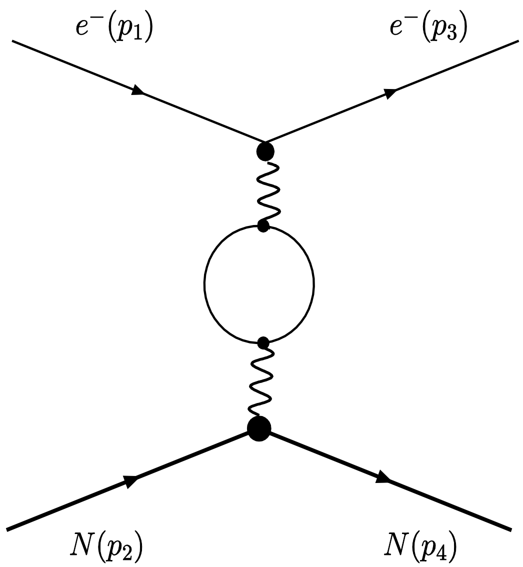
Vertex corrections



(c) (d)

Elastic scattering:

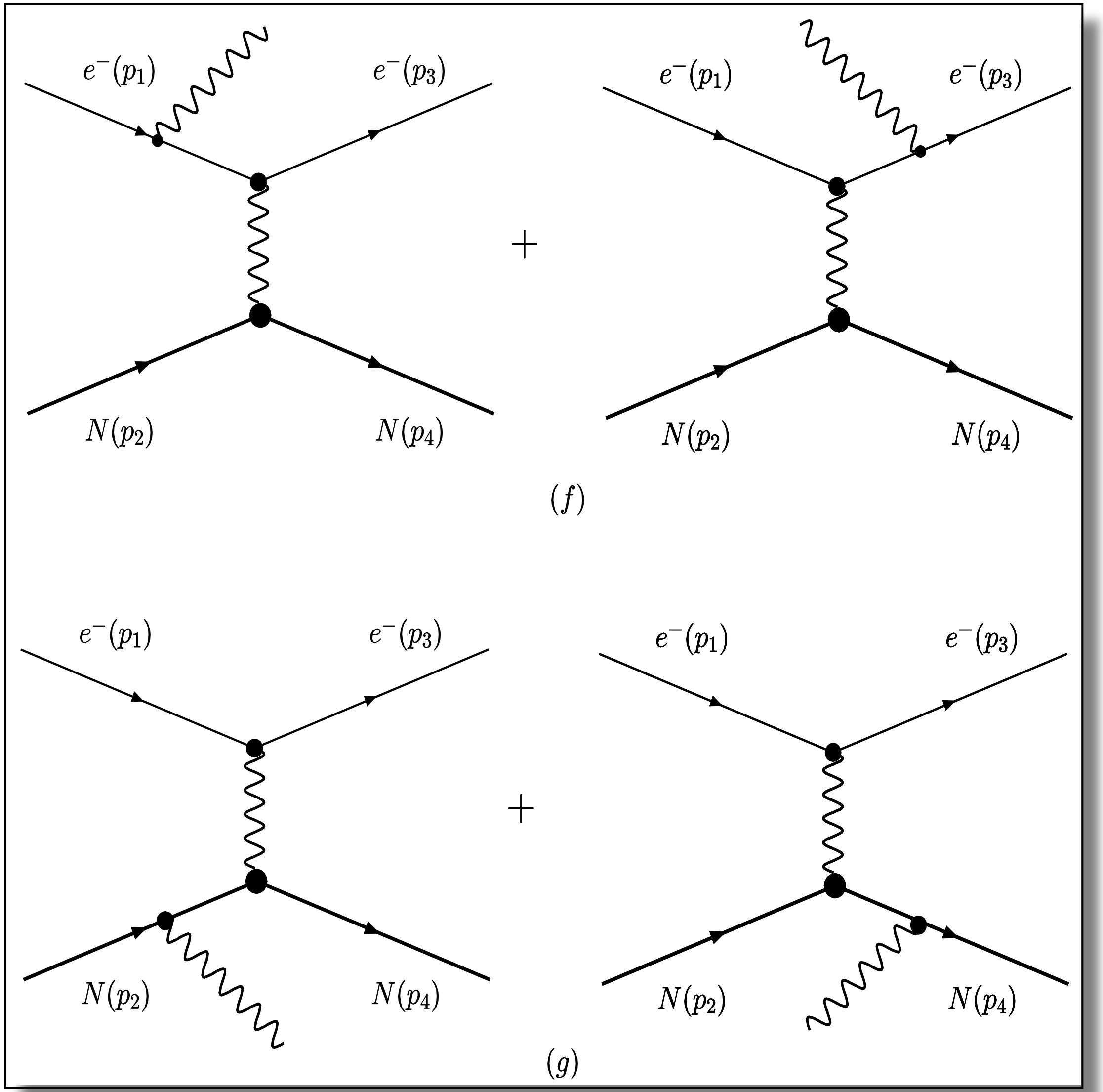
Vacuum polarization



(e)

Inelastic process:

Bremsstrahlung



MT corrections

High order elastic contributions:

$$\mathcal{M}_1 = f(Q^2, \epsilon)\mathcal{M}_0 + \overline{\mathcal{M}_1}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Expt.} = \left(\frac{d\sigma}{d\Omega}\right)_{elastic} + \left(\frac{d\sigma}{d\Omega}\right)_{inelastic} = \left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} (1+\delta).$$

$\mathcal{M}_0$:

- Vertex corrections for electron-photon coupling
- Vacuum polarization
- Diverge part of proton-photon coupling
- Diverge part of two-photon exchange

$\overline{\mathcal{M}_1}$:

- Finite part of two-photon exchange
- Finite part of vertex corrections of proton-photon

Independent on ε, contribute a common factor

Need to be study further

Contribution is much smaller

(1). Two-photon exchange amplitudes

- Intermediate state: Nucleon
- The finite contribution is simply ignored
- Momentum of one of photons in the denominator and numerator sets to 0

(2). Vertex correction for electron-photon is the same as Møller scattering

(3). Vertex correction for proton-photon is the same as (1)

(4). Vacuum polarization is the same as Møller scattering

Two-photon exchange (proton form factors):

• Baryonic level:

•Intermediate states are baryon resonances;

P. A. M. Guichon and M. Vanderhaeghen, PRL 91, 142303, 03

L. C. Maximon *et al.*, Phys. Rev. C **62** (2000) 054320.

P. G. Blunden *et al.*, Phys. Rev. Lett. **91** (2003) 142304.

•Quark level:

•To consider TPE on quark level ;

•To set a simple connection between the e-q and e-N

•Generalized parton distribution (GPD) PRL 93

•Constituent quark model NPA782

General expression of the matrix element: including TPE

$$\mathcal{M} = -\frac{ie^2}{q^2}\Big\{\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\left[\tilde{F}_1\gamma^\mu + \tilde{F}_2\frac{i\sigma^{\mu\nu}q_\nu}{2M}\right]u(p_2) + \bar{u}(p_3)\gamma_\mu\gamma_5 u(p_1)\bar{u}(p_4)\gamma^\mu\gamma^5\tilde{G}_A u(p_2)\Big\}.$$

$$\mathcal{M} = -\frac{ie^2}{q^2}\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\left[\tilde{F}_1\gamma^\mu + \tilde{F}_2\frac{i\sigma^{\mu\nu}q_\nu}{2M} + \tilde{F}_3\frac{\gamma\cdot K P^\mu}{M^2}\right]u(p_2).$$

$$\tilde{G}_E(Q^2,\varepsilon) = G_E(Q^2)_{OPE} + \Delta G_E(Q^2,\varepsilon)$$
$$\tilde{G}_M(Q^2,\varepsilon) = G_M(Q^2)_{OPE} + \Delta G_M(Q^2,\varepsilon)$$

There are three form factors. They are the functions of Q^2, ε , and are complex numbers

$$\sigma_R \simeq |\tilde{G}_M|^2 \left\{ 1 + \frac{\epsilon}{\tau} \left[\frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}) \mathcal{R}(\frac{\nu \tilde{F}_3}{M^2 |\tilde{G}_M|}) \right] \right\},$$

$$\frac{p_x}{p_z} \simeq -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \left\{ \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + (1 - \frac{2\epsilon}{1+\epsilon} \frac{|\tilde{G}_E|}{|\tilde{G}_M|}) \mathcal{R}(\frac{\nu \tilde{F}_3}{M^2 |\tilde{G}_M|}) \right\}.$$

$$\begin{aligned} \tilde{G}_M &= \tilde{F}_1 + \tilde{F}_2, \\ \tilde{G}_E &= \tilde{F}_1 - \tau \tilde{F}_2, \\ Y_{2\gamma} &= \mathcal{R}\left(\frac{\nu \tilde{F}_3}{M^2 |\tilde{G}_M|^2} \right), \quad \nu = (s-u)/4 \end{aligned}$$

$$\boxed{\begin{aligned} \left(R^{exp}_{Rosenbluth}\right)^2 &= \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2\left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right) Y_{2\gamma}, \\ R^{exp}_{polarization} &= \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\epsilon}{1+\epsilon} \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right) Y_{2\gamma}. \end{aligned}}$$

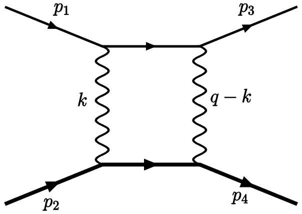
Two-photon exchange amplitudes of electron-proton scattering

$$M^{2\gamma}=e^4\int\frac{d^4k}{(2\pi)^4}\Big[\frac{N_a(k)}{D_a(k)}+\frac{N_b(k)}{D_b(k)}\Big].$$

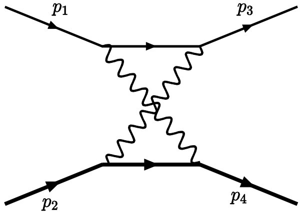
Leptonic tensor and hadronic tensor

$$\begin{aligned}
N_a(k) &= L^{(a)}_{\mu\nu}(k)H^{(a)\mu\nu}(k),\\
L^{(a)}_{\mu\nu} &= \bar{u}(p)_3\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\nu u(p_1),\\
H^{(a)}_{\mu\nu} &= \bar{u}(p_4)\Gamma_\mu(q-k)(\hat{p}_2+\hat{k}+M)\Gamma_\nu(k)u(p_2),
\end{aligned}$$

$$\begin{aligned}
N_b(k) &= L^{(b)}_{\mu\nu}(k)H^{(b)\mu\nu}(k),\\
L^{(b)}_{\mu\nu} &= \bar{u}(p)_3\gamma_\mu(\hat{p}_3+\hat{k})\gamma_\nu u(p_1),\\
H^{(b)}_{\mu\nu} &= \bar{u}(p_4)\Gamma_\mu(q-k)(\hat{p}_2+\hat{k}+M)\Gamma_\nu(k)u(p_2).
\end{aligned}$$



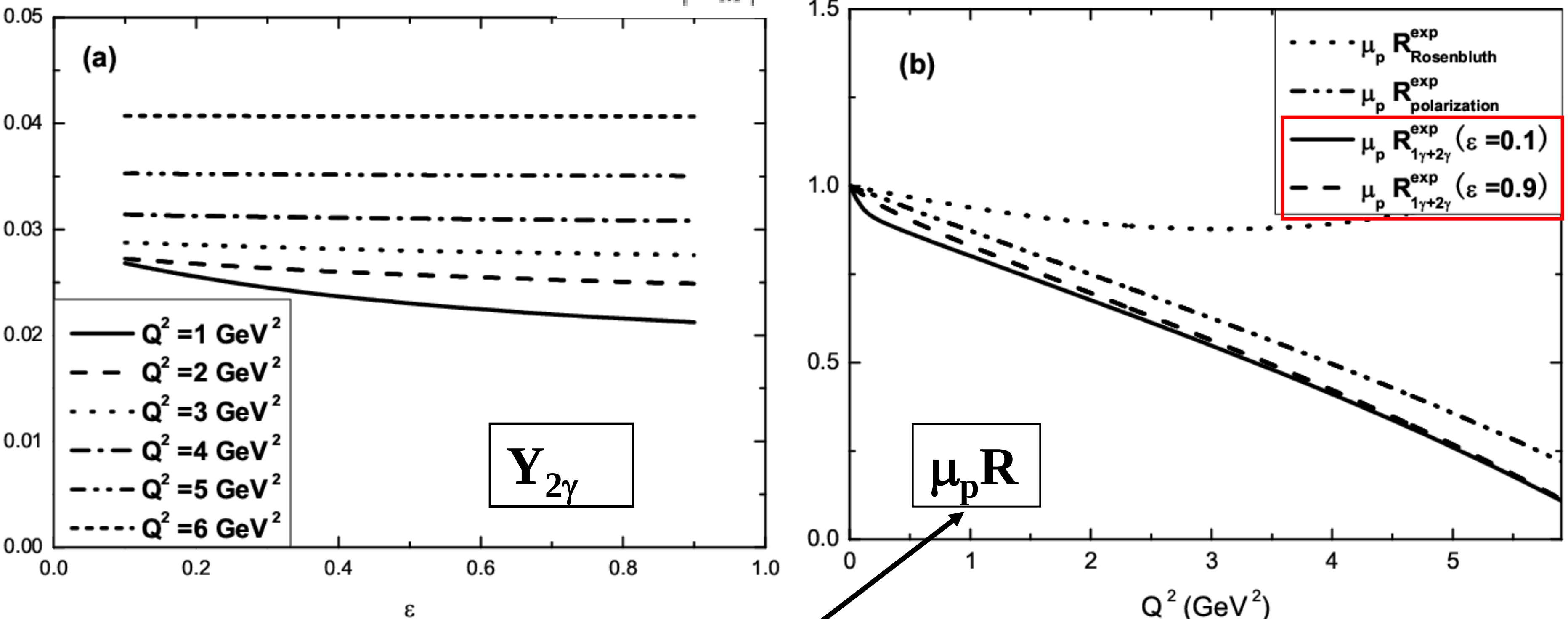
(a)



(b)

Results：

$\mathbf{Y_{2\gamma}}$ $\mathcal{R}\left(\frac{\nu\tilde{F}_3}{M^2|\tilde{G}_M|^2}\right)$



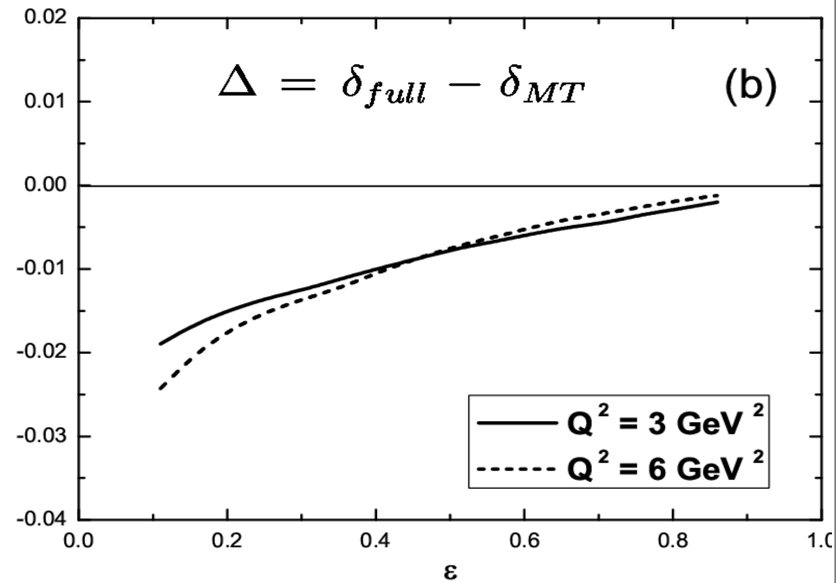
$$\left(R^{exp}_{Rosenbluth}\right)^2 = \boxed{\frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2}} + 2\left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)Y_{2\gamma}$$

P. A. M. Guichon, M. Vanderhaeghen, Phys. Rev. Lett. **91** (2003) 142303.

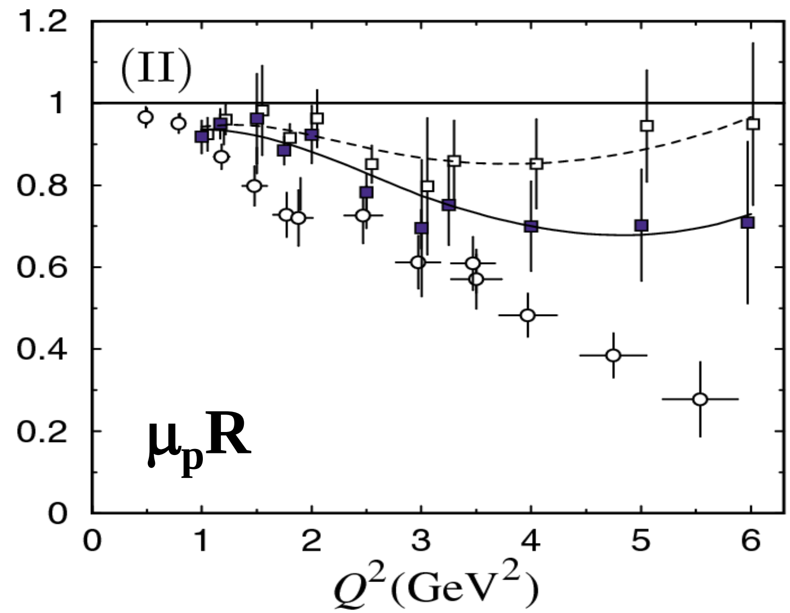
TPE contributions

P. G. Blunden *et al.*, Phys. Rev. Lett. **91** (2003) 142304.

Contribution to cross section



Contribution to the ratio



$$\sigma_R = \sigma_R^{1\gamma}(1+\Delta)$$

$$\approx \sigma_R^{1\gamma}(1+b\varepsilon)$$

$$= G_M^2\left[1+\frac{\varepsilon}{\tau}\mu_p^2\boxed{\left(R^2+b\mu_p^2\tau\right)}\right]$$

$$R^2 = R_{\exp}^2 - b\mu_p^2\tau$$

$$b \approx 3\%, \quad b\mu_p^2\tau = 0.39$$

$$R^{\exp} = 1, \quad R = 0.78$$

MSU, Moscow

TPE in SL (contributed by other nucleon resonances)

$$\mathcal{M}_{2\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \left[\boxed{\frac{N_a(k)}{D_a(k)} + \frac{N_b(k)}{D_b(k)}} + \boxed{\frac{N_c(k)}{D_c(k)} + \frac{N_d(k)}{D_d(k)}} \right]$$

N-Intermediate

Δ-Intermediate

$$\begin{aligned}
\Gamma^{\nu\alpha}_{\gamma R\to N}(p,q) &= i\frac{eF_R(q^2)}{2M_R^2}\{g_1^R[g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q \\
&\quad + \gamma^\nu\not{p}q^\alpha] + g_2^R[p^\nu q^\alpha - g^{\nu\alpha}p\cdot q] + (g_3^R/M_R) \\
&\quad \times [q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q)]\}P_R I_R,
\end{aligned}$$

where p^α and q^ν are the four-momenta of the resonance and photon, respectively, and $g^R_{1,2,3}$ are coupling constants discussed below. The Lorentz factor $P_R = \gamma_5$ if $R = P33$, and $P_R = 1$ if $R = D13$ or $R = D33$; and the isospin factor $I_R = T_3$ if $R = P33$ or $R = D33$, and $I_R = 1$ if $R = D13$.

The vertices of the spin 1/2 resonances read

$$\Gamma^\mu_{\gamma R\to N}(q) = -\frac{eg^R F_R(q^2)}{2M}\sigma^{\mu\nu}q_\nu P_R I_R,$$

where for $R = P11$, $P_R = 1$ and $I_R = 1$; for $R = S11$, $P_R = \gamma_5$ and $I_R = 1$; and for $R = S31$, $P_R = \gamma_5$ and $I_R = T_3$.

TPE in SL (contributed by other nucleon resonances)

PRC75

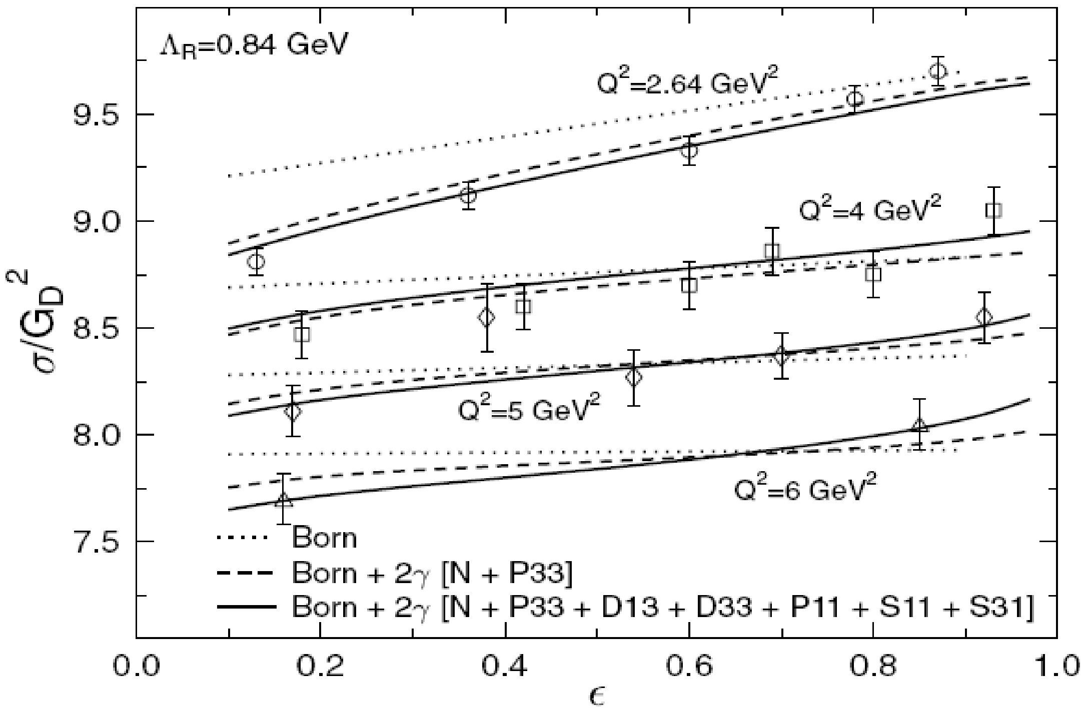


FIG. 1. Effect of adding the two-photon exchange correction to the Born cross section, the latter evaluated with the nucleon form factors from the polarization transfer experiment [1]. The intermediate state includes a nucleon and indicated hadron resonances. We show the reduced cross section divided by the square of the standard dipole form factor $G_D^2(Q^2) = 1/[1 + Q^2/(0.84\,\text{GeV})^2]^4$. The data points at four fixed momentum transfers are taken from Refs. [2,3].

Inclusion of the excited state resonance contributions reduces the nucleon elastic TPE by ~15% at Q^2 around 4 GeV^2

Summary for TPE on proton form factors

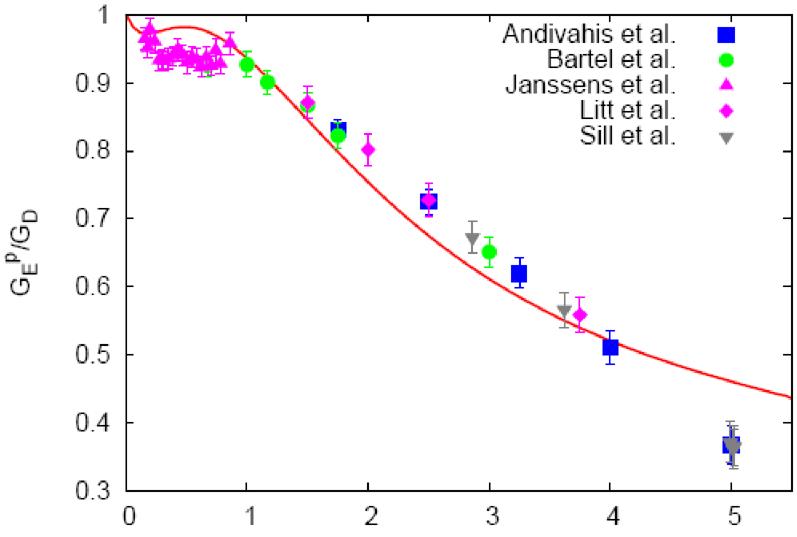
- Two-photon exchange effect can explain the difference between the results of the two measurements
  It should be taken into account.

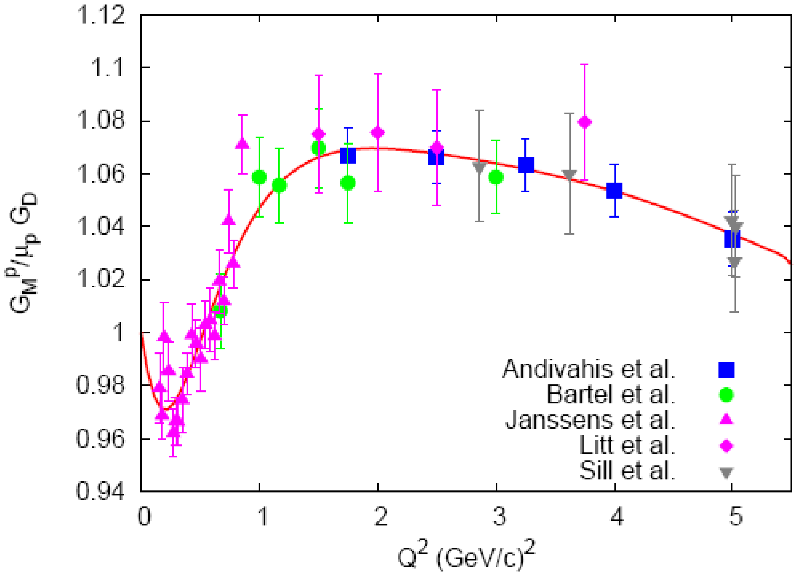
- The TPE effect is small, it changes with respect to ε

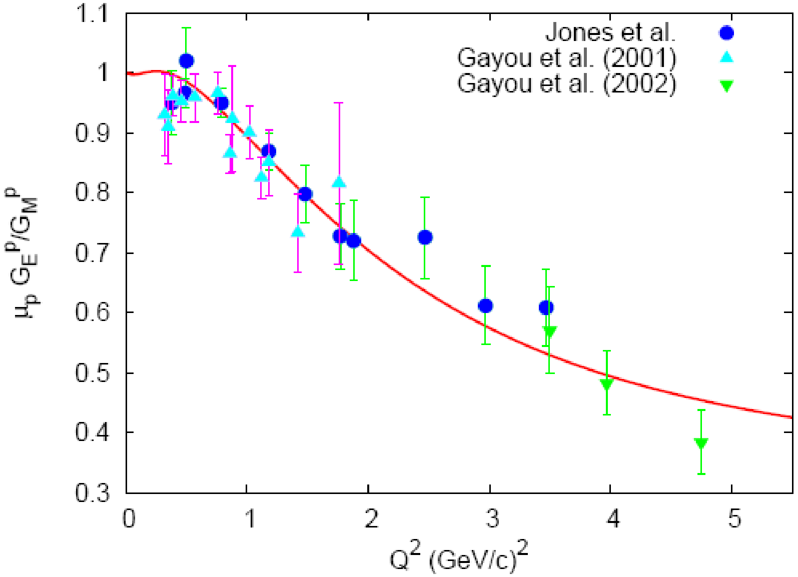
  The contribution of the higher excitations to TPE is not very clear.

Other interpretations *TPE in SL*

Quark model interpretations: relativistic effect（hCQM）

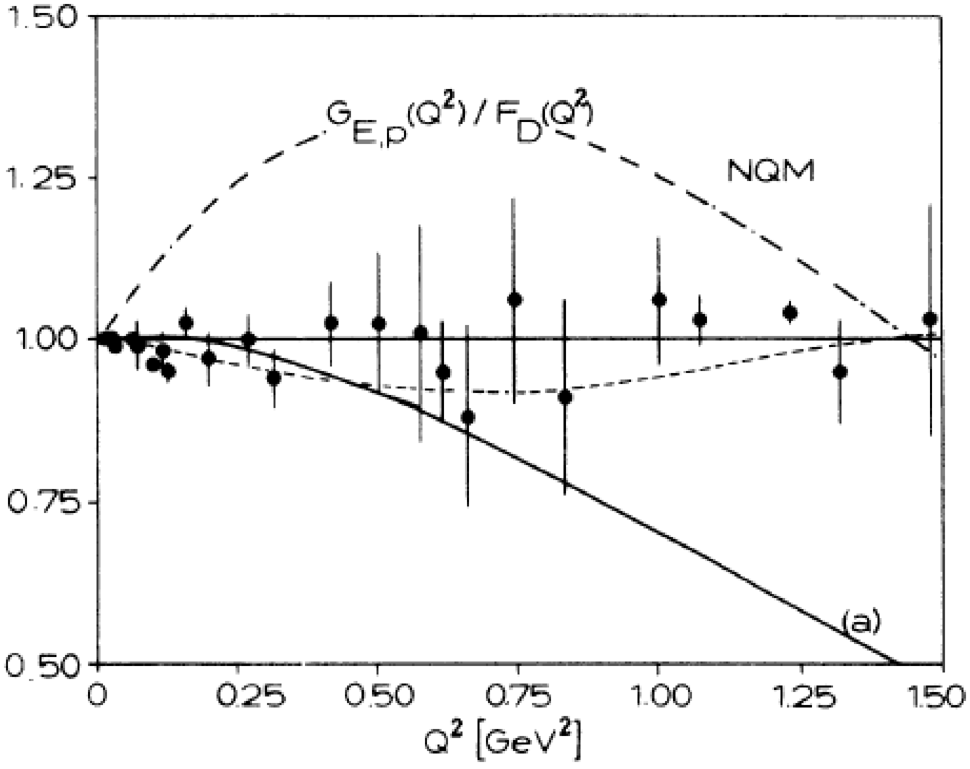






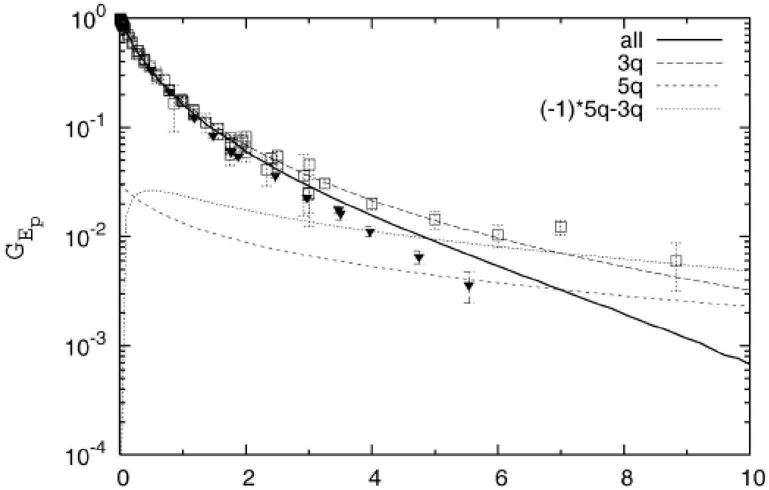
1, Constituent quark model
2, Hyper-central potential (three-body interaction)
3, Relativistic corrections

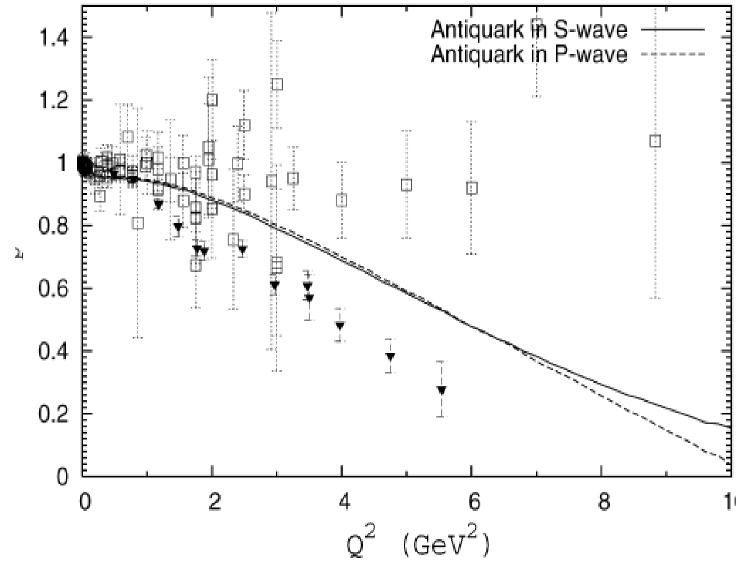
Quark model interpretations: light-cone relativistic QM by H. J. Weber



Quark model interpretations:

QM with 5-quark components





calculated proton electric form factors to the dipole form in the presence of a $qqqq\bar{q}$ contribution the S- (solid curve) and P-states (dashed curve). The data points correspond to those in Fig. 3.

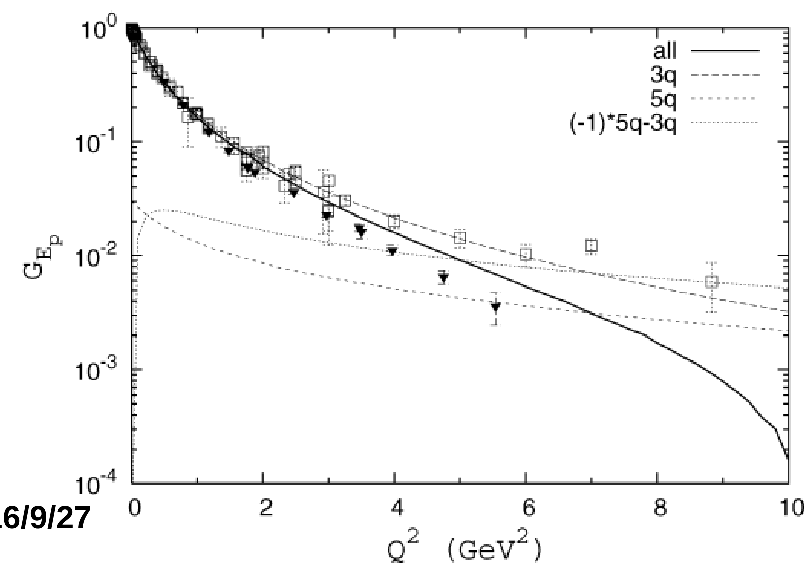


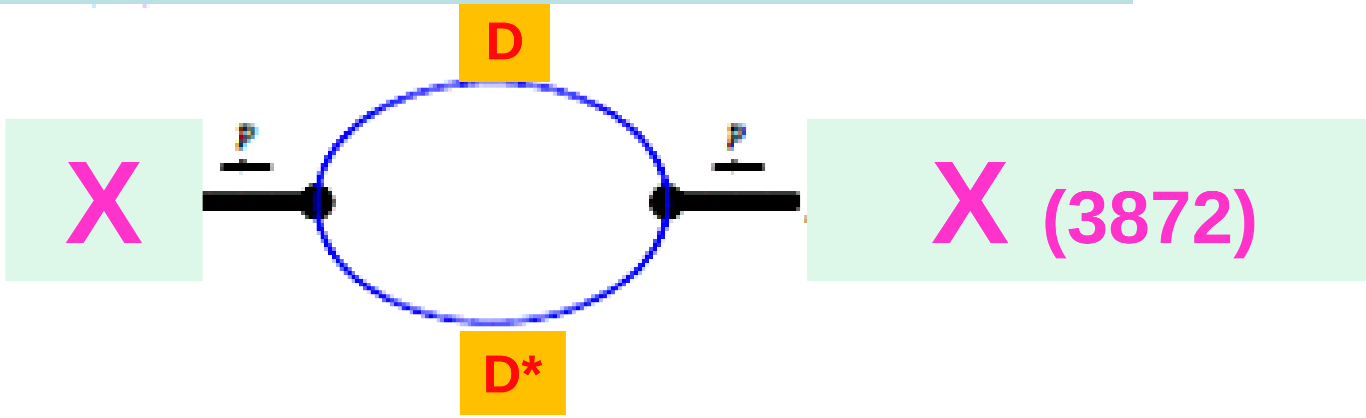
Fig. 4. Calculated proton electric form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the P-state.

3, Phenomenological approach, deuteron form factors two-photon exchange

Our Phenomenological approach:

Molecule scenario PRD77,094013+…

The mass operator represented by $\widetilde{\Pi}(p^2)$



$$L_{XDD} = X_\mu J^\mu$$

in Collaboration with Amand Faessler, Thomas Gutsche, and V. E. Lyubovitskij

$$= \frac{g_X}{\sqrt{2}} X_\mu \int d^4y\, \Phi_X(y^2)[D(x+y/2)\bar{D}^{*\mu}(x-y/2)+\bar{D}(x+y/2)D^{*\mu}(x-y/2)]$$

Correlation function

Two fields

Compositeness condition:

Bound state description of hadronic molecules in QFT based on compositeness condition: Weinberg,PR1963;Salam, Nuov.Cim. 1962
Heyashi et al.,Fortsch. Phys. 1967

The coupling g is determined by the condition

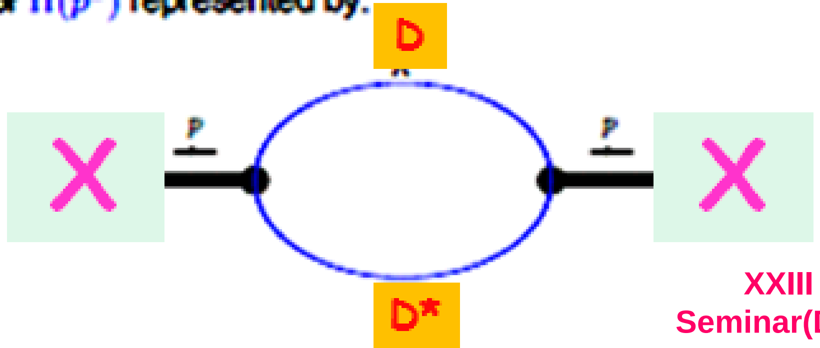
$$\boxed{Z_M = 1 - \Sigma'_M(m_M^2) = 0}$$

with the derivative of the mass operator

Exp. input

$$\Sigma'_M(m_M^2) = g_M^2 \Pi'_M(m_M^2) = g_M^2 \frac{d\Pi_M(p^2)}{dp^2}\Big|_{p^2=m_M^2}$$

with the mass operator $\tilde{\Pi}(p^2)$ represented by:



Vertex function

Characterize the finite size of the hadron
the distributions in the hadron

Gaussian-type is chosen for the function

$$\Phi_M(y^2) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot y} \tilde{\Phi}(-k^2), \qquad \tilde{\Phi}(-k_E^2) = exp\left(-k_E^2/\Lambda_M^2\right)$$

local limit $\Phi(y^2) \to \delta^{(4)}(y)$

Parameter: Gaussian with free size parameter Λ

Four-dimensional covariant calculation

Applications

- 1), Hadronic molecules: old -
- renewed interest in heavy mesons
- 2), Effective approach is applied
- to the states (Compositeness)
- 3), Hadronic loop is considered
- 4), Decay modes: some c\bar{c}
- +dominate hadronic picture

1), Open charmed mesons: Ds(2317)

Other applications:

2), X(3872)

3), Y-type: Y(4260), Y(3940);

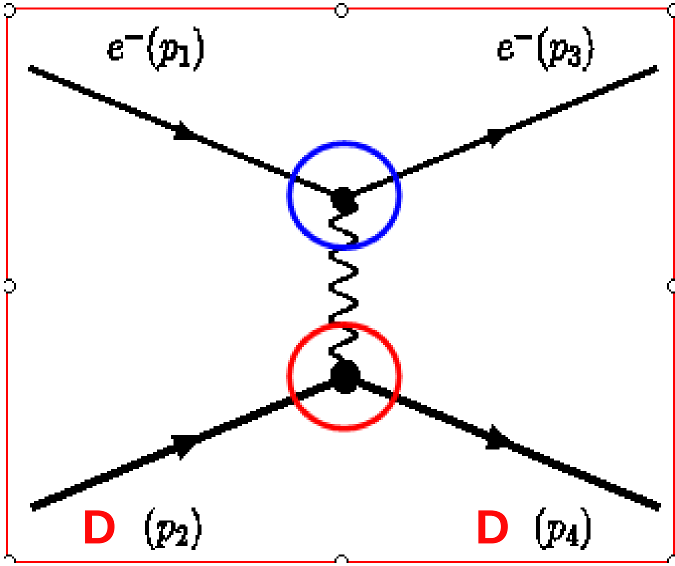
Z-type: Z(4430), Zc(3900); Zb(10610), Zb(10650)

4), $\Lambda_c(2940)$, $\Sigma_c(2800)$

Deuteron Electromagnetic form factors eD (S=1)

Electron-deueron elastic scattering *(OPE)*

$$
\begin{aligned}
&\langle p'_D, \lambda' | J_\mu(0) | p_D, \lambda \rangle \\
&= -e_D \Bigg\{ \Bigg[G_1(Q^2) \xi'^*(\lambda') \cdot \xi(\lambda) \\
&\quad - G_3(Q^2) \frac{(\xi'^*(\lambda') \cdot q)(\xi(\lambda) \cdot q)}{2M_D^2} \Bigg] \cdot P_\mu \\
&\quad + G_2(Q^2) [\xi_\mu(\lambda)(\xi'^*(\lambda') \cdot q) - \xi'^*_\mu(\lambda')(\xi(\lambda) \cdot q)] \Bigg\},
\end{aligned}
$$



Deuteron: spin-1 particle

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{\text{Mott}} I_0(\text{OPE}),$$

$$I_0(\text{OPE}) = A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2},$$

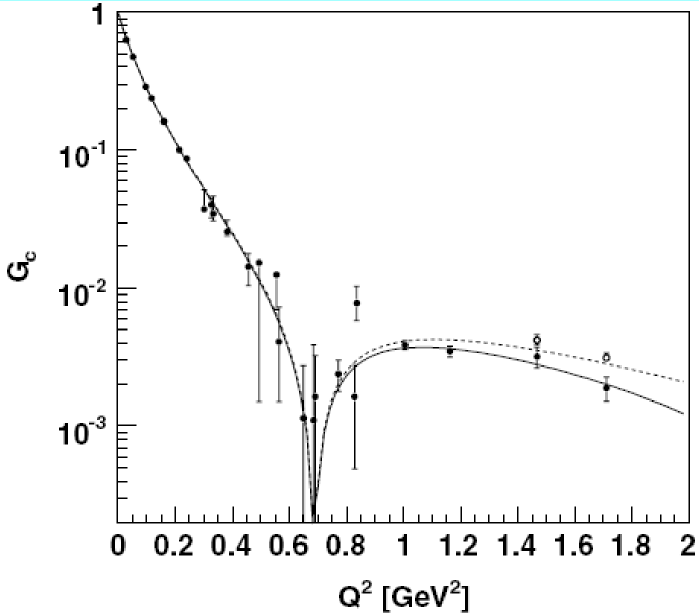
$$A(Q^2) = G_C^2(Q^2) + \tfrac{2}{3}\tau_D G_M^2(Q^2) + \tfrac{8}{9}\tau_D^2 G_Q^2(Q^2),$$

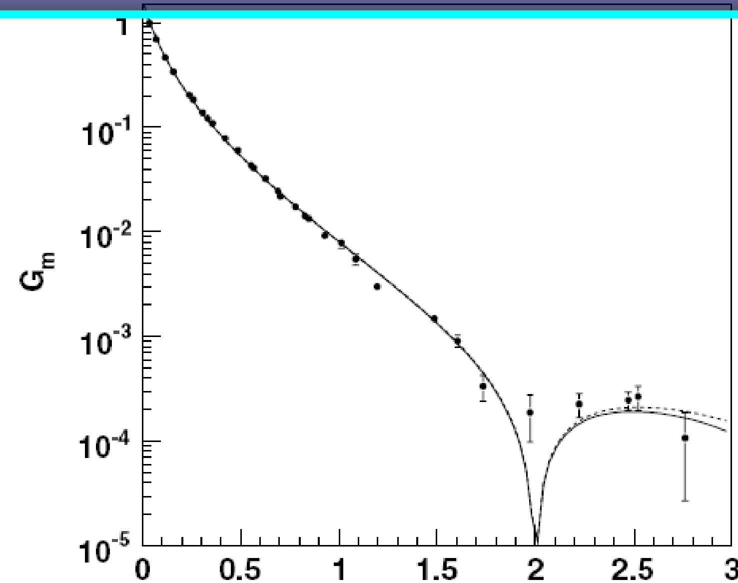
$$B(Q^2) = \tfrac{4}{3}\tau_D(1+\tau_D)G_M^2(Q^2).$$

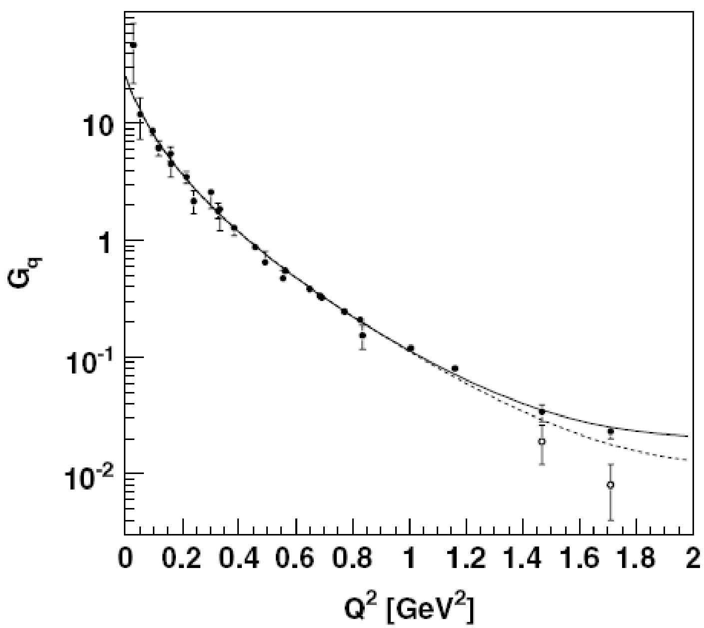
$$G_M = G_2, \quad G_Q = G_1 - G_2 + (1+\tau_D)G_3,$$

$$G_C = G_1 + \tfrac{2}{3}\tau_D G_Q.$$

Deuteron form factors (parameterizations, PRC73, (2006))







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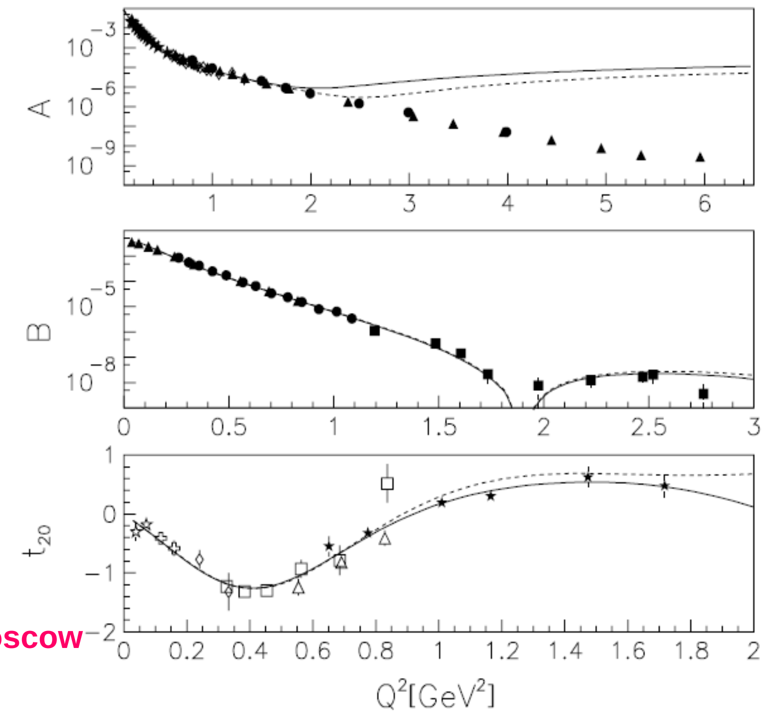


Electron-deueron elastic scattering (TPE)

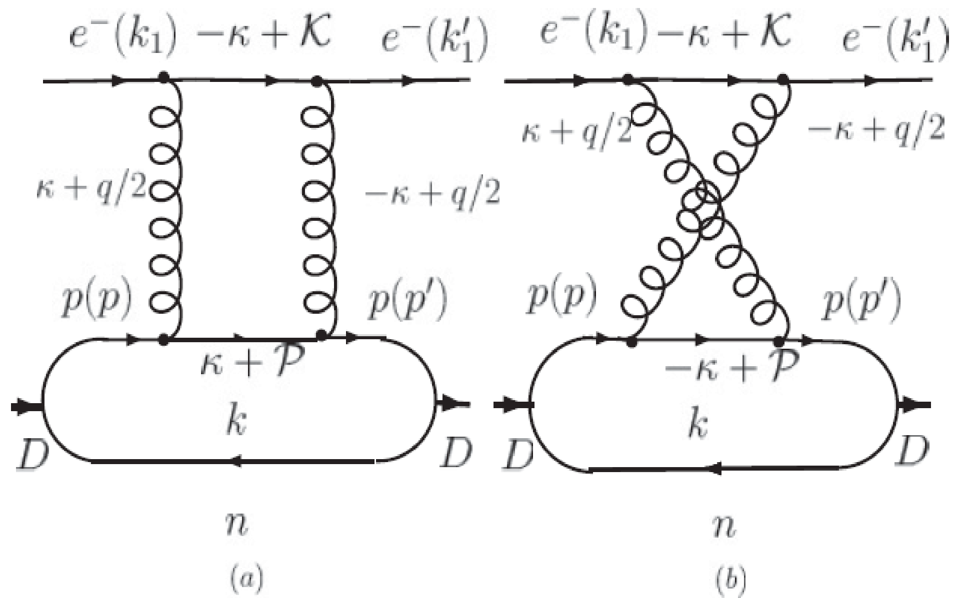
$$\mathcal{M}^{eD} = \frac{e^2}{Q^2}\bar{u}(k_1', s_3)\gamma_\mu u(k_1, s_1)\sum_{i=1}^{6} G_i' M_i^\mu,$$

where

$$M_1^\mu = (\xi'^* \cdot \xi)P^\mu,$$
$$M_2^\mu = [\xi^\mu(\xi'^* \cdot q) - (\xi \cdot q)\xi'^{*\mu}],$$
$$M_3^\mu = -\frac{1}{2M_D^2}(\xi \cdot q)(\xi'^* \cdot q)P^\mu,$$

and

$$M_4^\mu = \frac{1}{2M_D^2}(\xi \cdot K)(\xi'^* \cdot K)P^\mu,$$
$$M_5^\mu = [\xi^\mu(\xi'^* \cdot K) + (\xi \cdot K)\xi'^{*\mu}],$$
$$M_6^\mu = \frac{1}{2M_D^2}[(\xi \cdot q)(\xi'^* \cdot K) - (\xi \cdot K)(\xi'^* \cdot q)]P^\mu,$$



PRC74, 064006

• By considering Two-photon exchange corrections

$$\tilde{G}_i(s, Q^2) = G_i(Q^2) + G_i^{(2)}(s, Q^2),$$

where G_i's correspond to the contributions arising from the one-photon exchange and $G_i^{(2)}$'s stand for the rest which would come mostly from the TPE. In the OPE approximation, $G_4 = G_5 = G_6 = 0$. It is easy to see that G_i $(i = 1, 2, 3)$ is of order of $(\alpha)^0$ and $G_i^{(2)}(i = 1, \ldots, 6)$ are of order α.

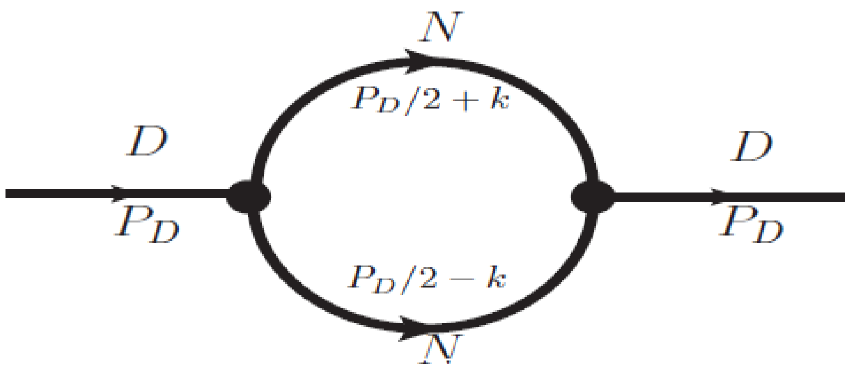
$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} I_0 \\
&= \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} \left\{\left[(A + \Delta A) + (B + \Delta B)\tan^2\frac{\theta}{2}\right]\right. \\
&\quad \left. + \Delta\sigma(\theta, Q^2)\right\} \\
&= \sigma_0 \left\{\left[(A + \Delta A)\cot^2\frac{\theta}{2} + (B + \Delta B)\right]\right. \\
&\quad \left. + \Delta\sigma(\theta, Q^2)\cot^2\frac{\theta}{2}\right\}
\end{aligned}$$

$$\begin{aligned}
\Delta\sigma(\theta, Q^2) = \frac{2}{3}\Big\{& 2\tau\cot^2\frac{\theta}{2}[(2\tau - 1)G_1 - 2\tau G_2 + 2\tau^2 \\
&\times \text{Re}\left(G_4^{(2)*}\right) + \frac{K_0}{M_D}\Big[\Big((2\tau - 1)G_1 - 2\tau \\
&+ 2\tau^2 G_3 - 2\tau\tan^2\frac{\theta}{2}G_2\Big)\text{Re}\left(G_5^{(2)*}\right) \\
&+ 2\tau\big((2\tau + 1)G_1 - (2\tau + 1)G_2 \\
&+ 2\tau(\tau + 1)G_3\big)\text{Re}\left(G_6^{(2)*}\right)\Big]\Big\},
\end{aligned}$$

$$\begin{aligned}
\Delta A = 2\Big[& G_c\text{Re}\left(G_C^{(2)*}\right) + \frac{2}{3}\tau G_M\text{Re}\left(G_M^{(2)*}\right) \\
&+ \frac{8}{9}\tau^2 G_Q\text{Re}\left(G_Q^{(2)*}\right)\Big] \\
&+ \frac{4\tau^2}{3}[(2\tau + 1)G_1 - 2(\tau + 1)G_2 + 2\tau(\tau + 1)G_3] \\
&\times \text{Re}\left(G_4^{(2)*}\right), \\
\Delta B = {}& \frac{8}{3}\tau(1 + \tau)G_M\text{Re}\left(G_M^{(2)*}\right),
\end{aligned}$$

PRC74, 064006

Approach (phenomenological)

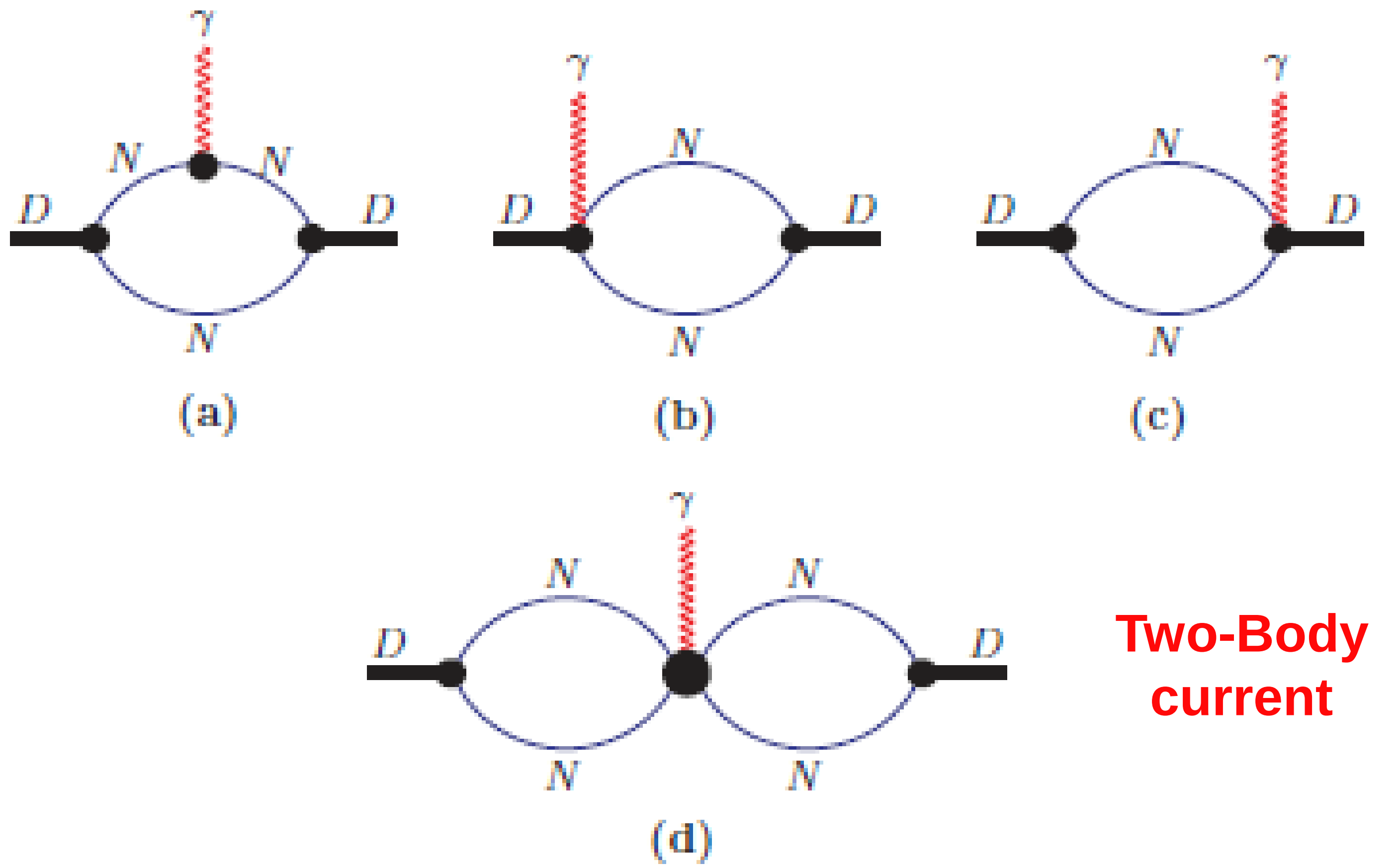


$$\mathcal{L}_D(x) = g_D D^{\dagger}_{\mu}(x) \int dy \Phi_D(y^2) p(x+y/2) C\gamma^{\mu} n(x-y/2) + \mathrm{H.c.},$$

$$\mathcal{L}_D(x) = g_D D^{\dagger}_{\mu}(x) \int dy \bar{p}^c(x+y/2) \boxed{\Phi_D(y^2)\Gamma^{\mu}} n(x-y/2) + H.c.,$$

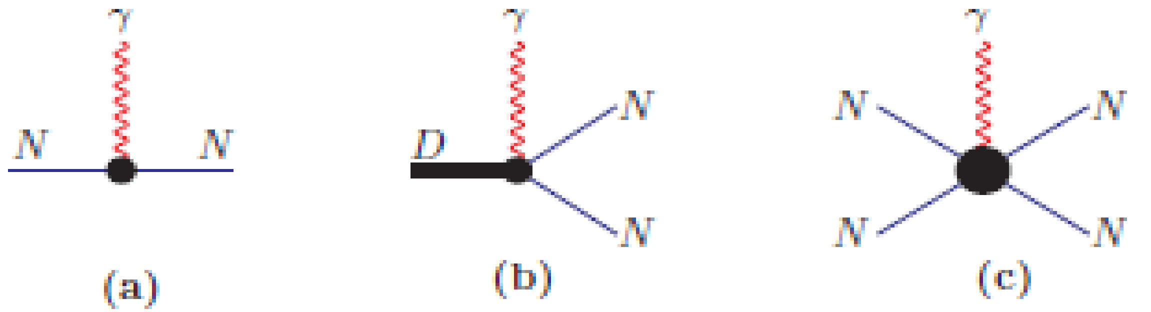
Correlation function (Cut-off) *PRC78, 035205*

Deuteron EM form factors (vertex)



1), Loop
2), Nucleon form factors

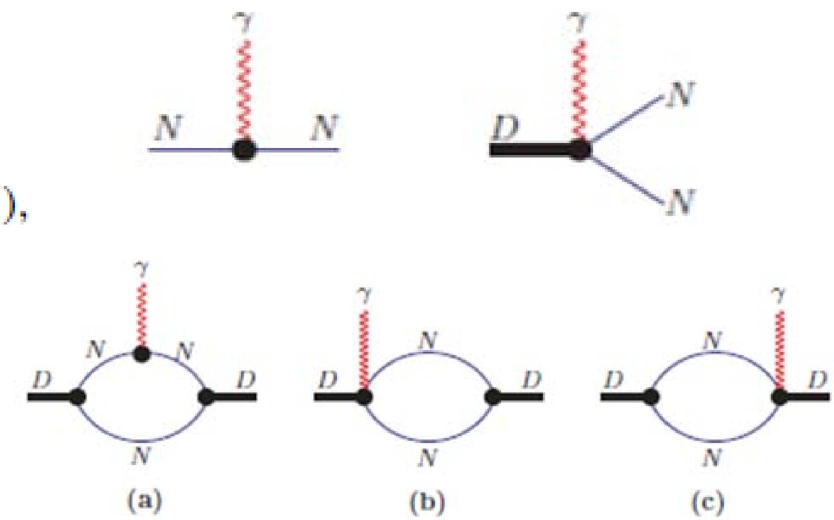
Two-Body current



One-Body

$$J_\mu^{NN}(q) = \int d^4x e^{-iqx} \bar{N}(x)\left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N} F_2^N(q^2)\right] N(x),$$

$$J_\mu^{DNN}(q) = -ig_D \int d^4x d^4y D_\nu^\dagger(x)\Phi_D(y^2)p(x+y/2) \times C\gamma^\nu n(x-y/2)\int_x^{x+y/2} dz_\mu e^{-iqz} + \text{H.c.}$$



Two-Body

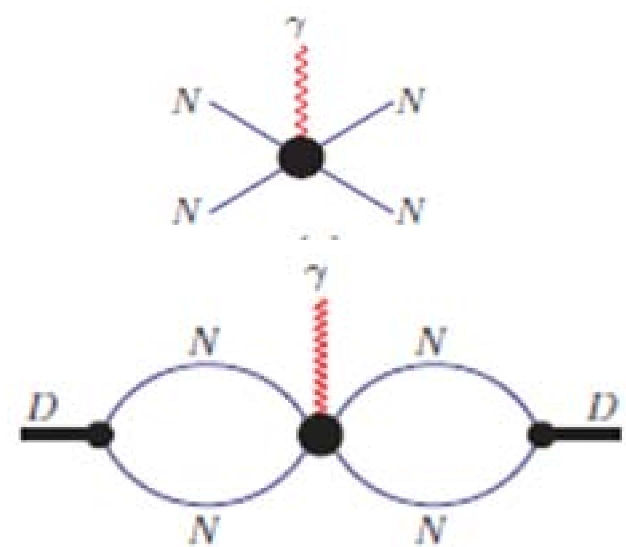
$$J_\mu^{4N}(q) = J_\mu^{4N;1}(q) + J_\mu^{4N;2}(q) + J_\mu^{4N;3}(q),$$

$$J_\mu^{4N;1}(q) = \int d^4x e^{-iqx} g_1 F_1^{NN}(q^2)\bar{n}(x)\gamma^\alpha C\bar{p}(x) \times p(x)C\gamma_\alpha i\sigma_{\mu\nu}q^\nu n(x) + \text{H.c.},$$

$$J_\mu^{4N;2}(q) = \int d^4x e^{-iqx} g_2 F_2^{NN}(q^2)\bar{n}(x)\not{q}C\bar{p}(x) \times p(x)Ci\sigma_{\mu\nu}q^\nu n(x) + \text{H.c.},$$

$$J_\mu^{4N;3}(q) = \int d^4x e^{-iqx} g_3 F_3^{NN}(q^2)\,[\bar{n}(x)\gamma^\alpha C\bar{p}(x)] \times i(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)[p(x)C\gamma_\alpha n(x)],$$

There is a number of possible contributions to the two-body operator $J_\mu^{(2)}(q) = J_\mu^{4N}(q)$. We restrict to the three simplest terms with the smallest number of derivatives



(d)

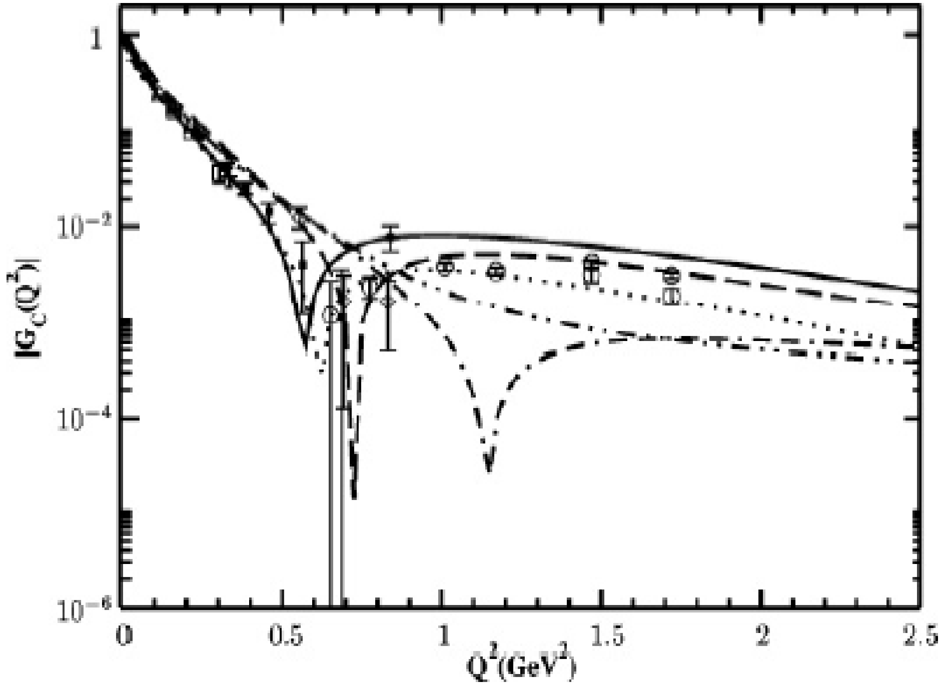


FIG. 4. Form factor $|G_C(Q^2)|$. The solid curve is the result of the TGA parametrization. The double dash-dotted and dashed lines are our results with the MMD [21] parametrization restricting to one-body and including two-body electromagnetic currents, respectively. The double dot-dashed and dotted lines are our results with the Kelly [22] parametrization restricting to one-body and including two-body electromagnetic currents, respectively. The data are from Ref. [27]

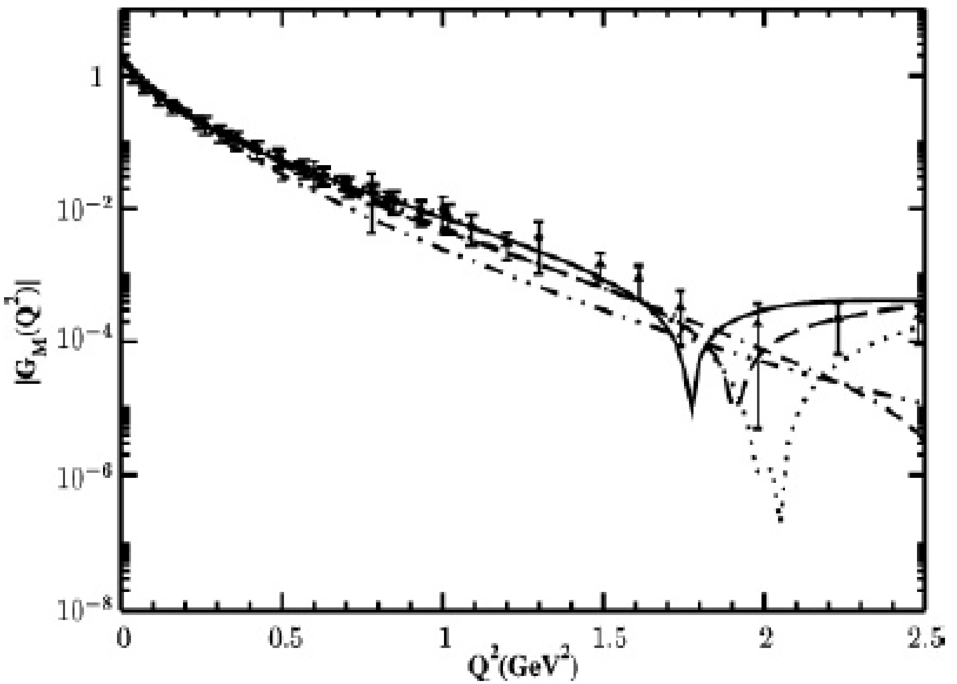


FIG. 6. Form factor $|G_M(Q^2)|$. The solid curve is the result of the TGA parametrization. The double dash-dotted and dashed lines are our results with the MMD [21] parametrization restricting to one-body and including two-body electromagnetic currents, respectively. The double dot-dashed and dotted lines are our results with the Kelly [22] parametrization restricting to one-body and including two-body electromagnetic currents, respectively. The data are quoted from Ref.

Parametrizations

(MMD): **Mergell-Meissner-Drechsel**

Kelly parametrization

TGA: Tomasi-Gake-Adamuscin

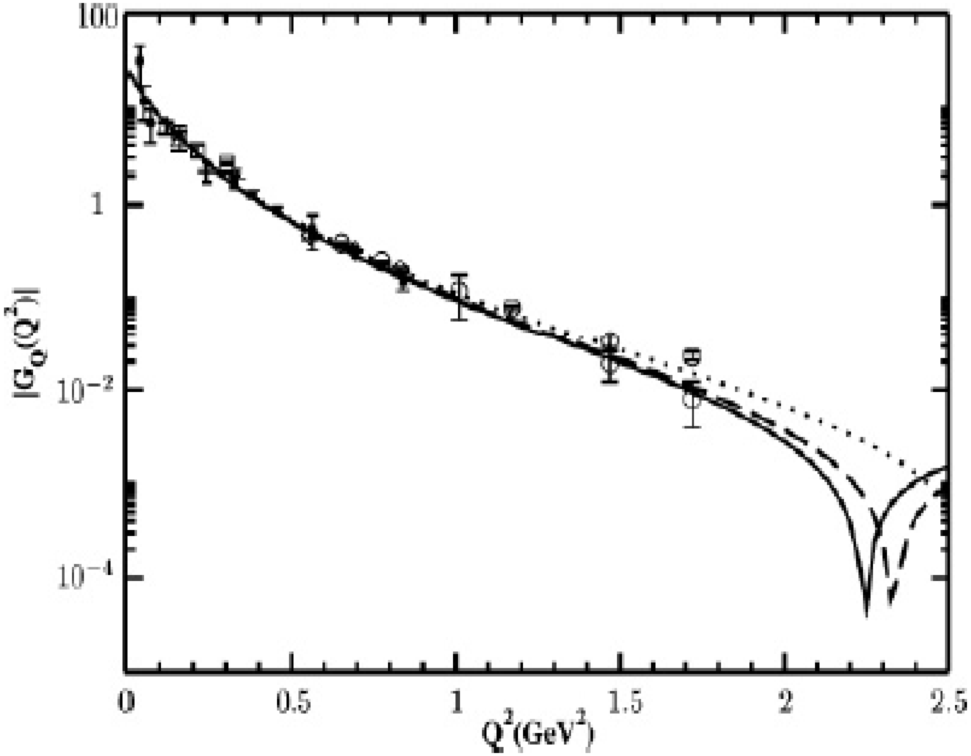


FIG. 5. Form factor $|G_Q(Q^2)|$. Notations are the same as described in the caption to Fig. 4.

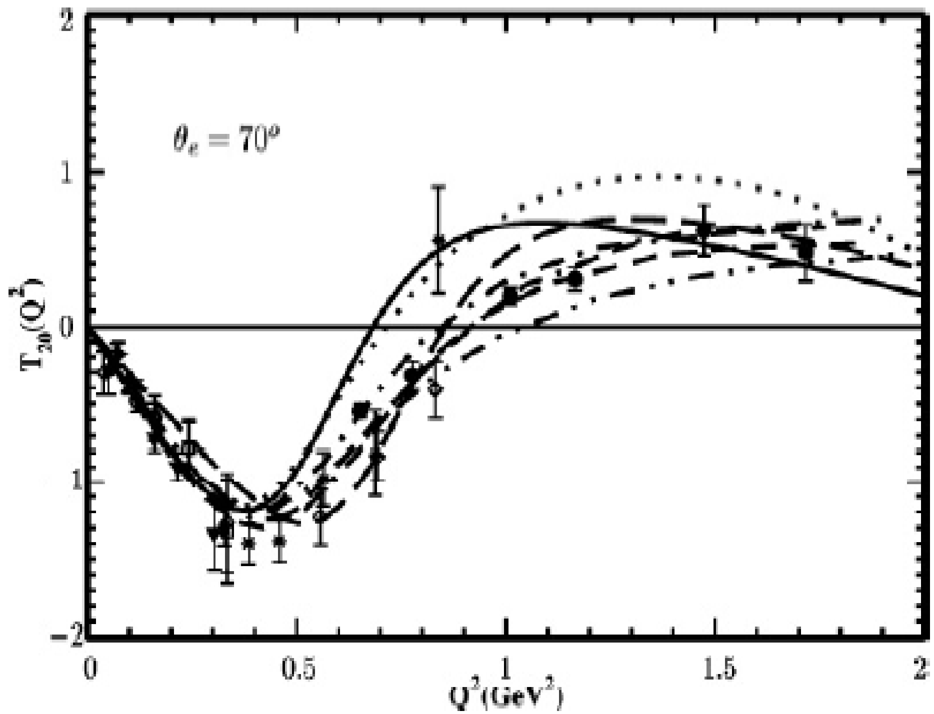


FIG. 18. Deuteron polarization tensor $T_{20}(Q^2)$ at $\theta_e = 70°$. The solid curve is the result of the TGA parametrization. The dashed and dotted lines are our results with the MMD [21] and Kelly [22] parametrizations, respectively. The data are quoted from Ref. [31]

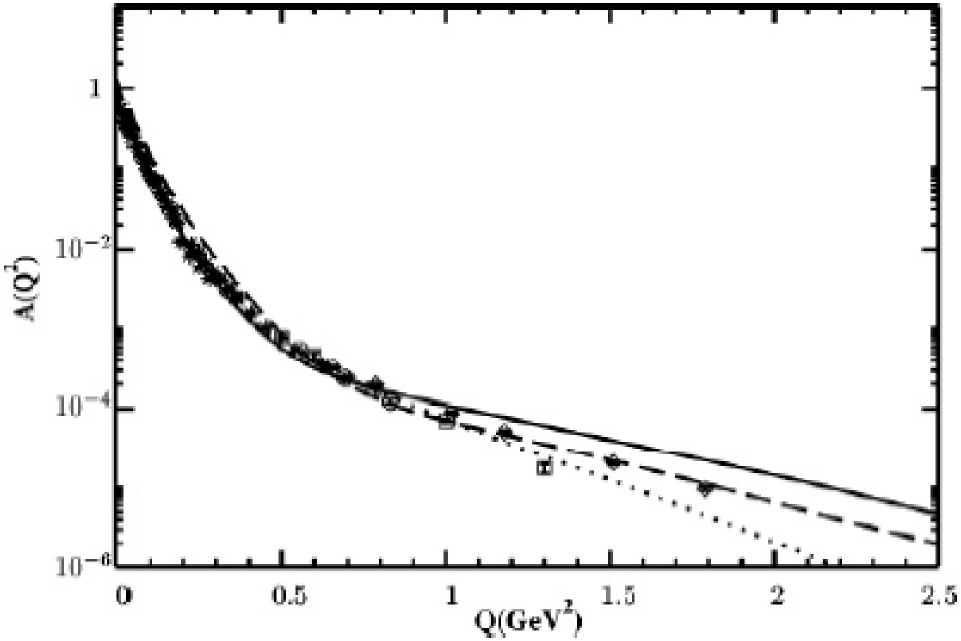


FIG. 16. Form factor $A(Q^2)$. The solid curve is the result of the TGA parametrization. The dashed and dotted lines are our results with the MMD [21] and Kelly [22] parametrizations, respectively.

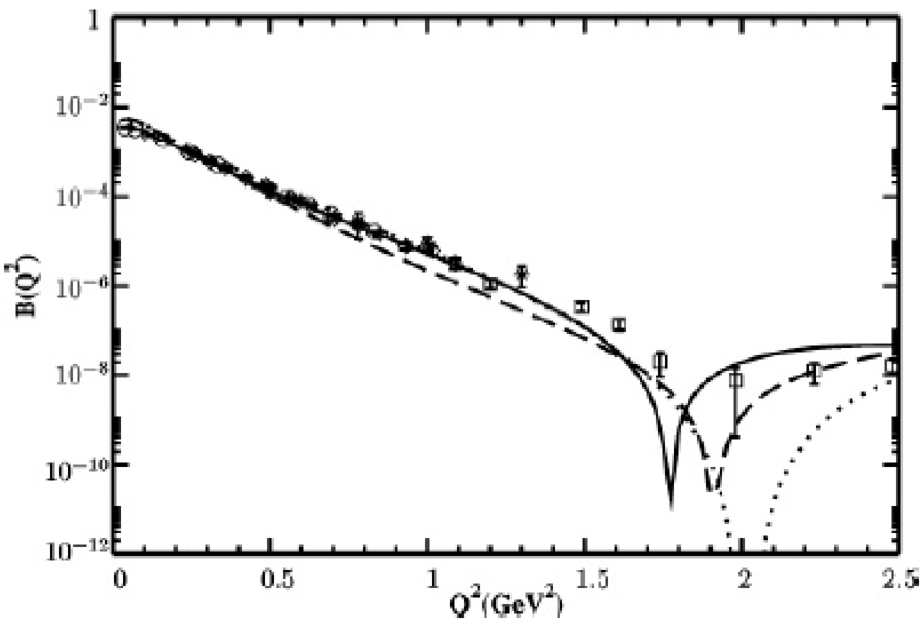
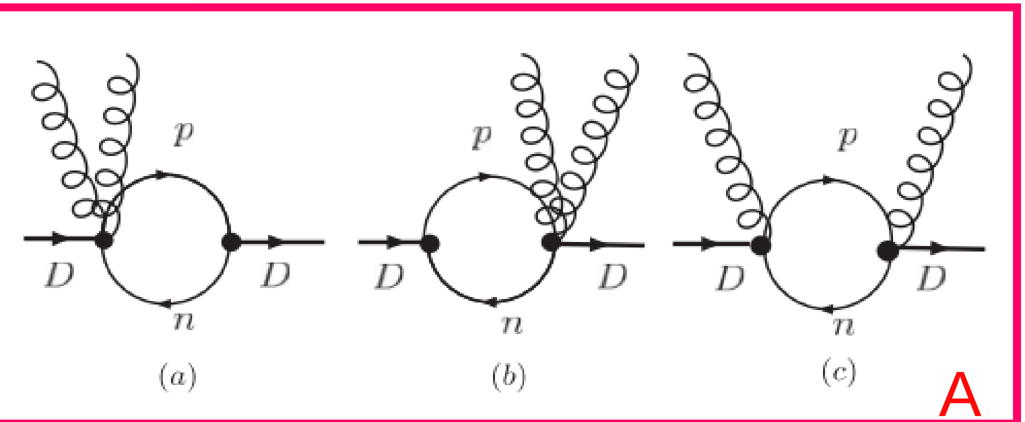
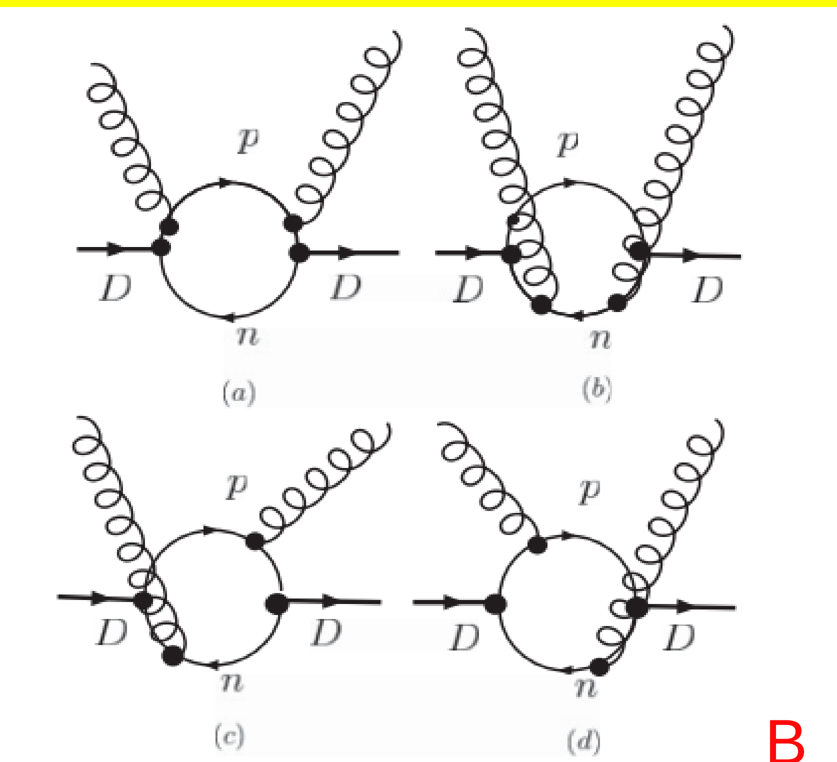


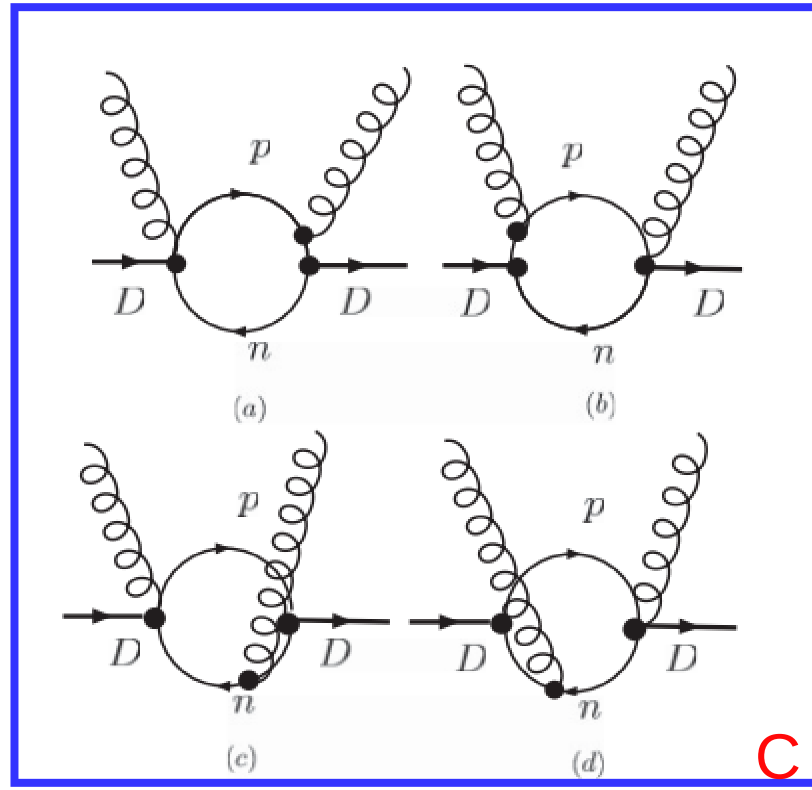
FIG. 17. Form factor $B(Q^2)$. Notations are the same as described in the caption to Fig. 16.

Hadronic part *TPE in eD*

• Two-photon exchange corrections







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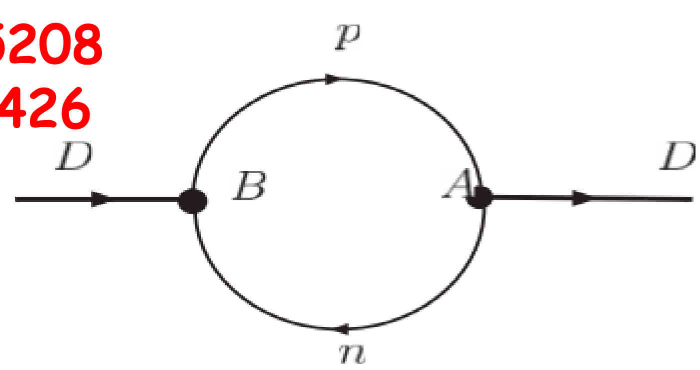
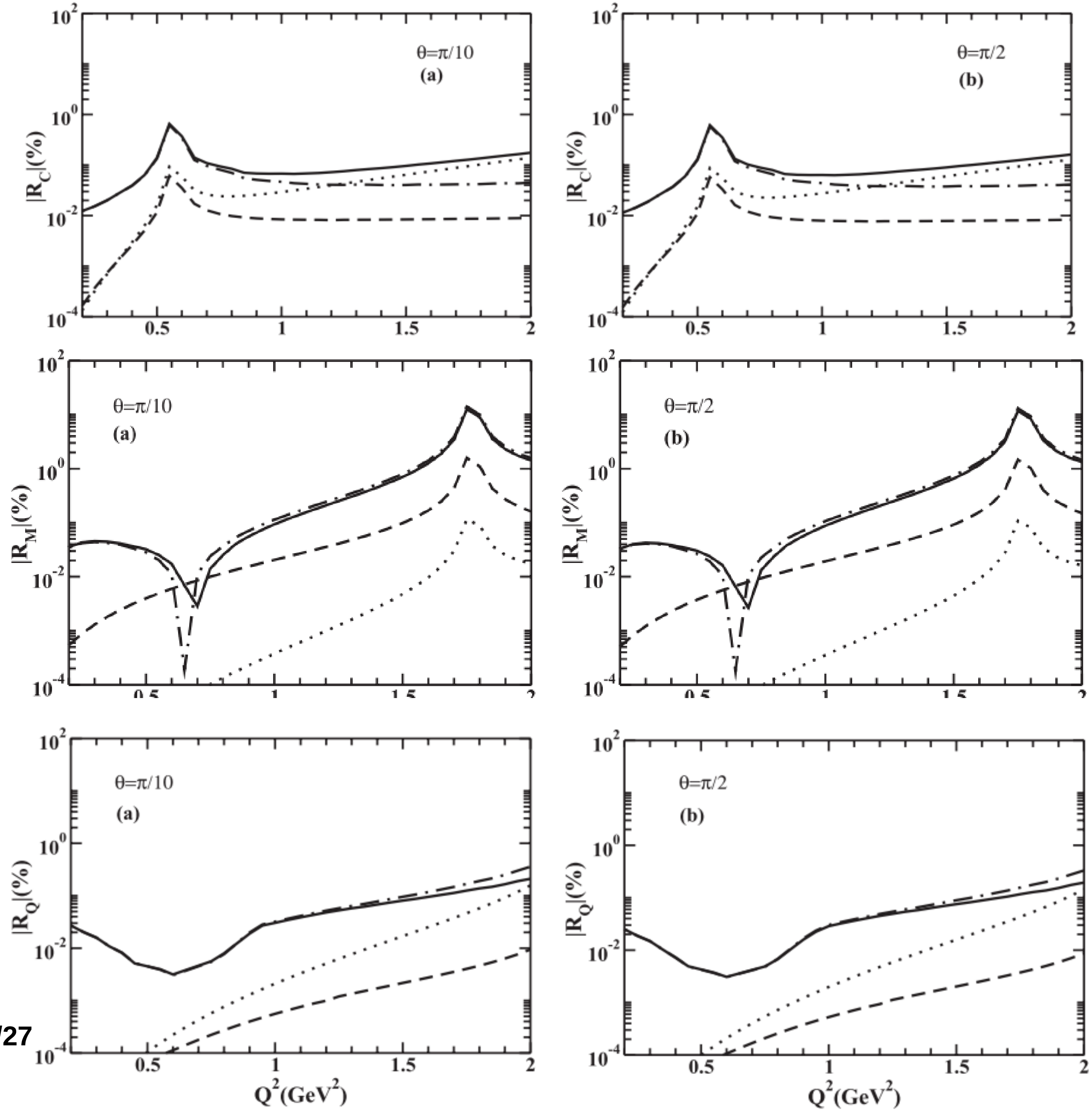


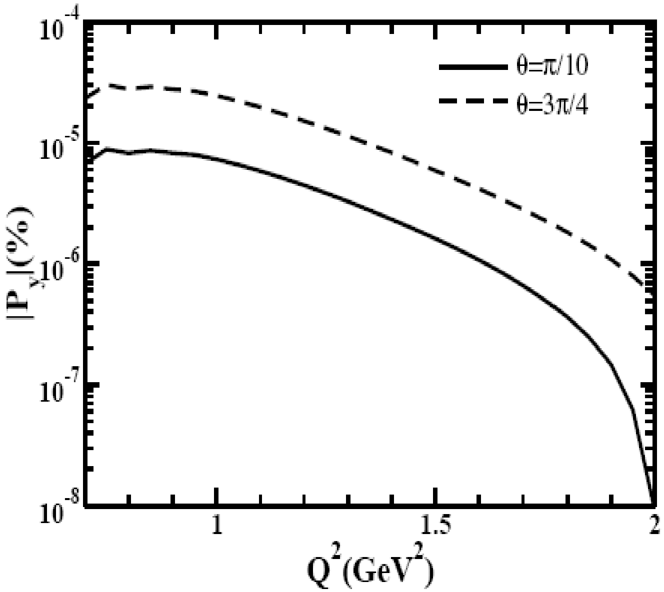


Solid-Total
A→dotted
B→dashed
C→dotted-dashed

Electron-deueron elastic scattering (TPE)

P_y results from the vector polarized final deuteron along the y direction which is perpendicular to the scattering plane. In OPE , P_y =0

$$
\begin{aligned}
P_y^{(a)} &= \frac{2}{3}\tan\frac{\theta}{2}\frac{K_0}{M_D}\Big[-(\tau+1)\Big(G_1 Im(G_2^{(2)*})+G_2 Im(G_1^{(2)*})\Big) \\
&-\tau(\tau+1)\Big(G_2 Im(G_3^{(2)*})+G_3 Im(G_2^{(2)*})\Big) \\
&+\tau\Big(\cot^2\frac{\theta}{2}(2G_1-G_2-2\tau G_3)+\tau G_2\Big)Im(G_4^{(2)*})\Big] \\
P_y^{(b)} &= \frac{2}{3}\tan\frac{\theta}{2}\Big[2\tau cot^2\frac{\theta}{2}(2G_1-G_2-2\tau G_3) \\
&+\tau\Big(2(\tau+1)G_1+G_2-2\tau(\tau+1)G_3\Big)\Big]Im(G_5^{(2)*}) \\
P_y^{(c)} &= \frac{2}{3}4\tau(\tau+1)\tan\frac{\theta}{2}\Big[cot^2\frac{\theta}{2}(G_1-\tau G_3)+\tau G_2\Big]Im(G_6^{(2)*})\Big|,
\end{aligned}
$$



PRC82,068202

He-3 (spin=1/2; form factors):

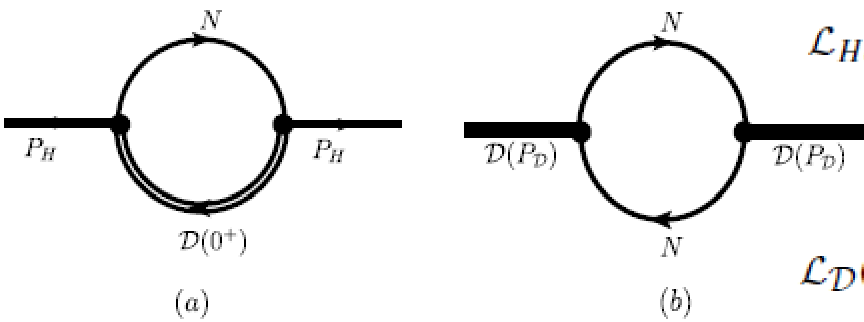
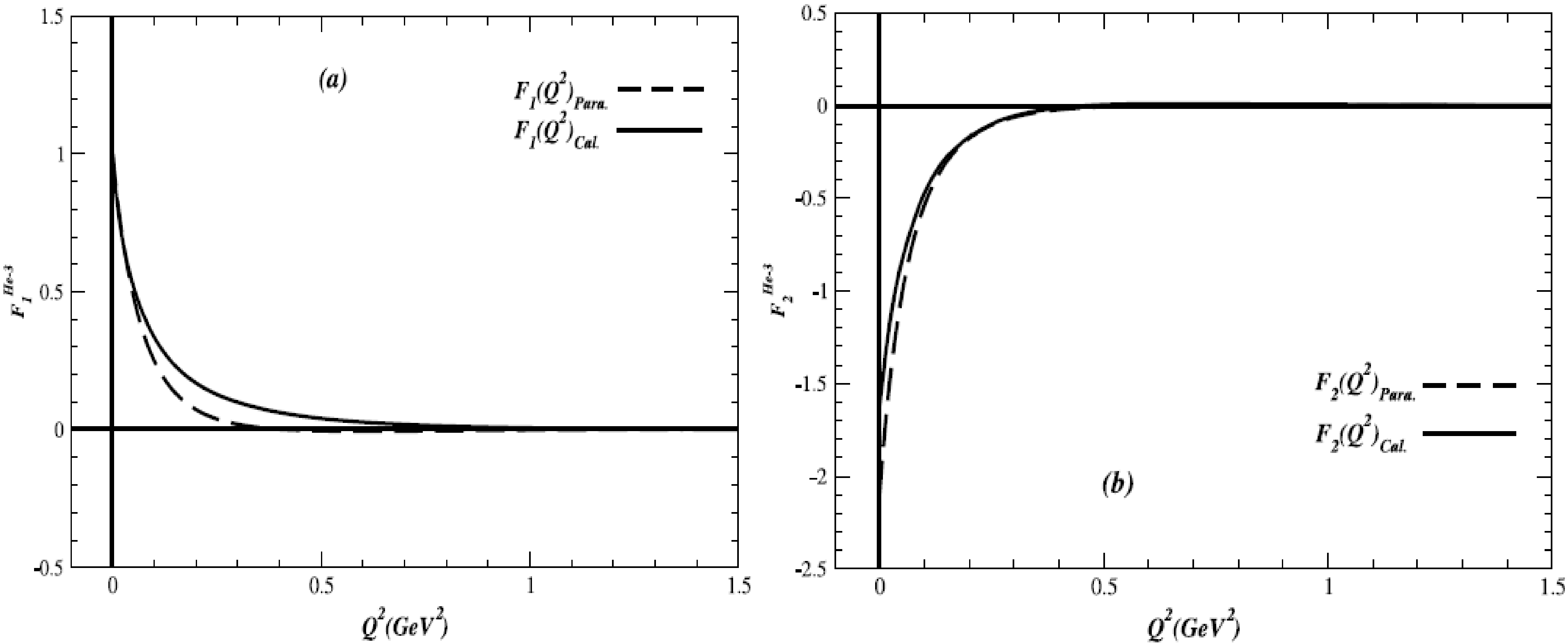


Fig. 1. Mass operators, (a) for He-3 and (b) for the dibaryon.

$$\mathcal{L}_{He}(x) = g_H \bar{H}(x) \int dy\, \Phi_H(y^2) n(x + \omega_\mathcal{D} y) \mathcal{D}(x - \omega y) + \text{H.c.},$$

$$\mathcal{L}_\mathcal{D}(x) = g_\mathcal{D} \mathcal{D}^*(x) \int dy\, \Phi_\mathcal{D}(y^2) \bar{p}^c(x + y/2) p(x - y/2) + \text{H.c.},$$

2p+n **NPA918,25**



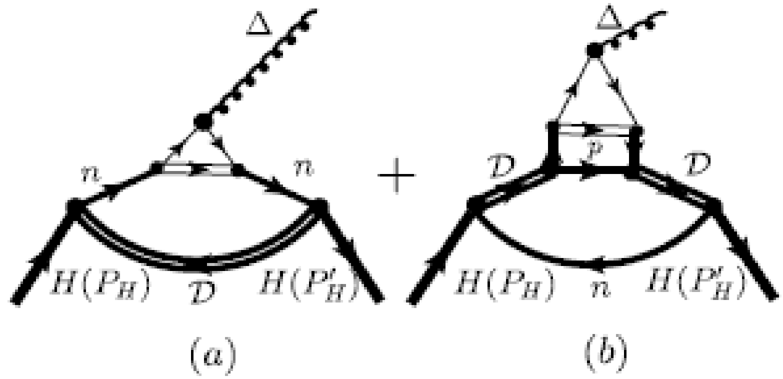
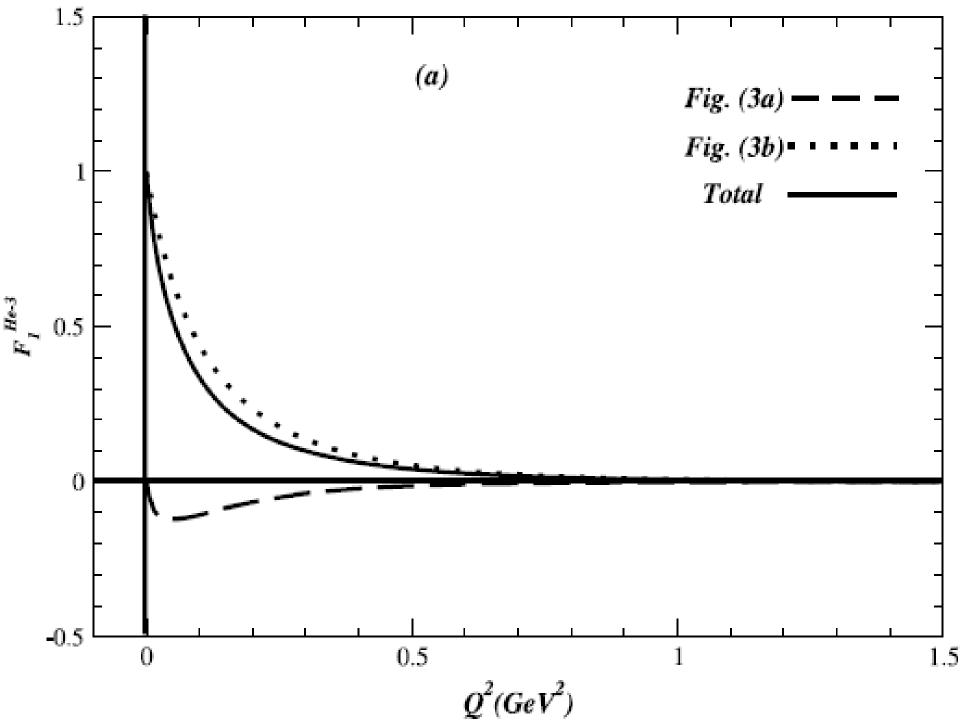
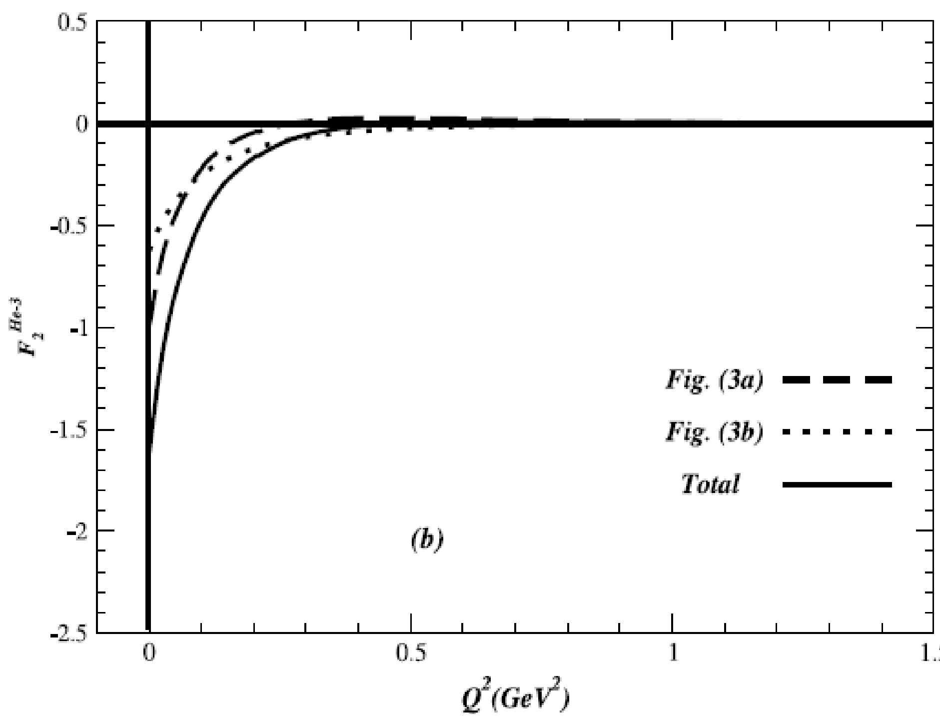


Fig. 3. Effective diagram for calculating the generalized parton distribution functions of He-3 in our approach, where the current interacting (a) on the odd nucleon and (b) on the dibaryon, respectively.



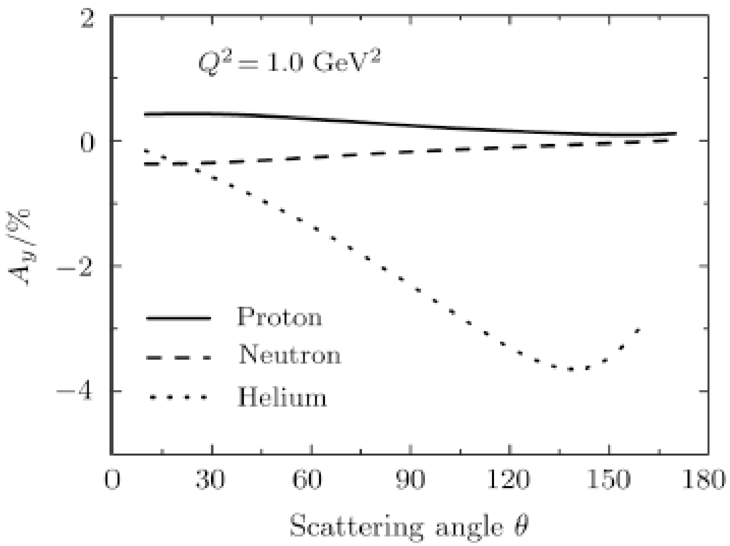


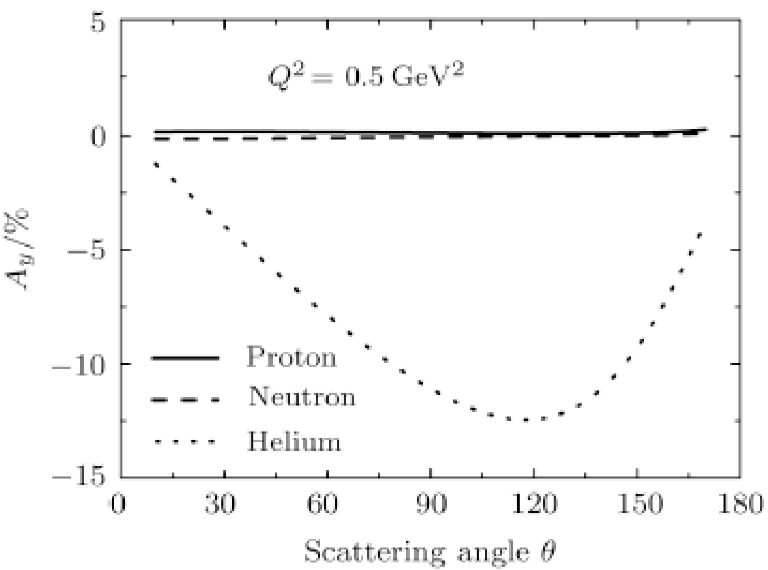
TPE on He-3 (form factors):

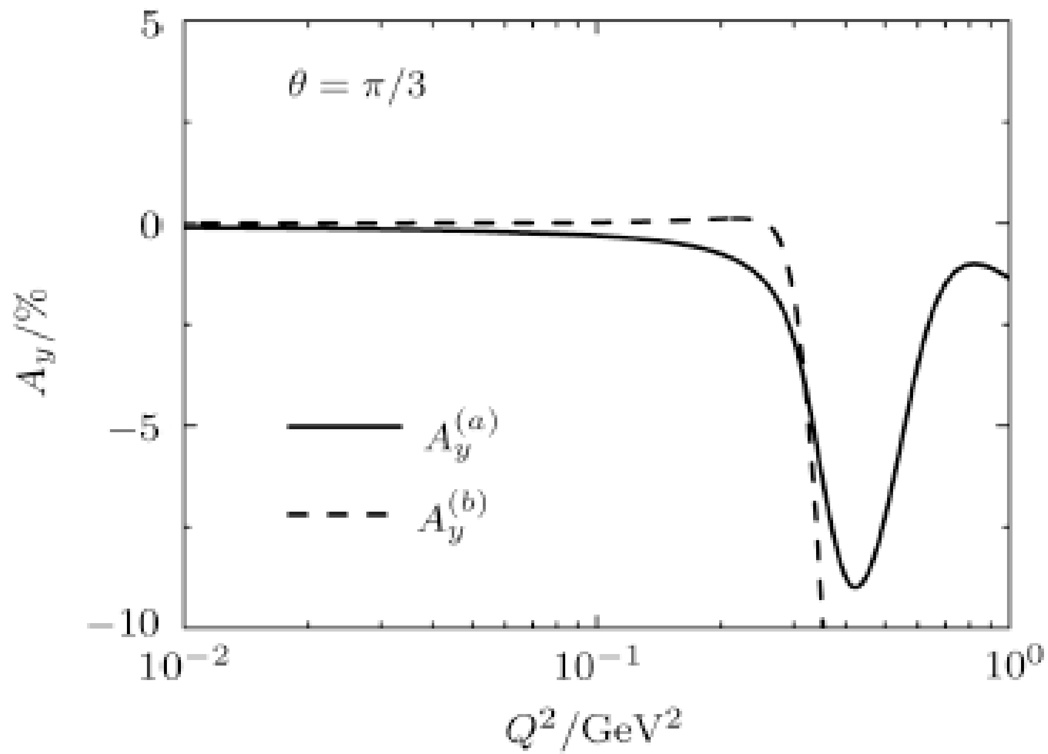
$$\Gamma^\mu = \tilde{F}_1\gamma^\mu + \tilde{F}_2\frac{i\sigma^{\mu\nu}q_\nu}{2M} + \tilde{F}_3\frac{\gamma\cdot K P^\mu}{M^2},$$

$$A_y = \sqrt{\frac{2\epsilon(1+\epsilon)}{\tau}}\frac{1}{\sigma_R}\Big\{G_E G_M\Big[\frac{\mathcal{I}(\Delta G_M)}{G_M} - \frac{\mathcal{I}(\Delta G_E)}{G_E}\Big] + G_M\Big[\frac{2\epsilon}{1+\epsilon}G_E - G_M\Big]\mathcal{I}(Y_{2\gamma})\Big\}.$$

$$A_y^{^3\mathrm{He}} = \frac{\sigma^n}{\sigma^{^3\mathrm{He}}}P_n A_y^n + 2\frac{\sigma^p}{\sigma^{^3\mathrm{He}}}P_p A_y^p,$$







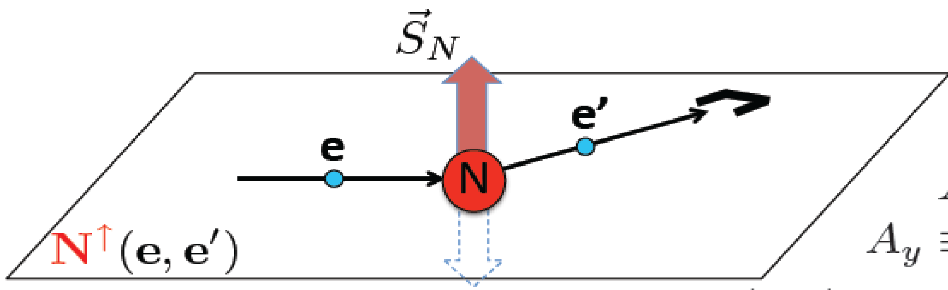
Highlights of New Physics Results

a new paper on Physical Review Letters

- New paper on Phys. Rev. Lett. (in print, arXiv:1502.02636).
- First measurement of a new observable. A new precision tool for the studies of nucleon structure.
- 10x improvements over the last measurement (SLAC-1970).

Experiment E05-015 @ Jefferson Lab Hall A

Target Single-Spin Asymmetry in $\mathbf{N}^{\uparrow}(\mathbf{e},\mathbf{e}')$



$$A_y = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

$$A_y \propto (\vec{e} \times \vec{e'}) \cdot \vec{S}_N$$

$$A_y \equiv 0 \text{ under } 1\gamma \text{ exchange}$$

$$A_y \neq 0 \text{ with } 1\gamma \otimes 2\gamma \text{ interference}$$

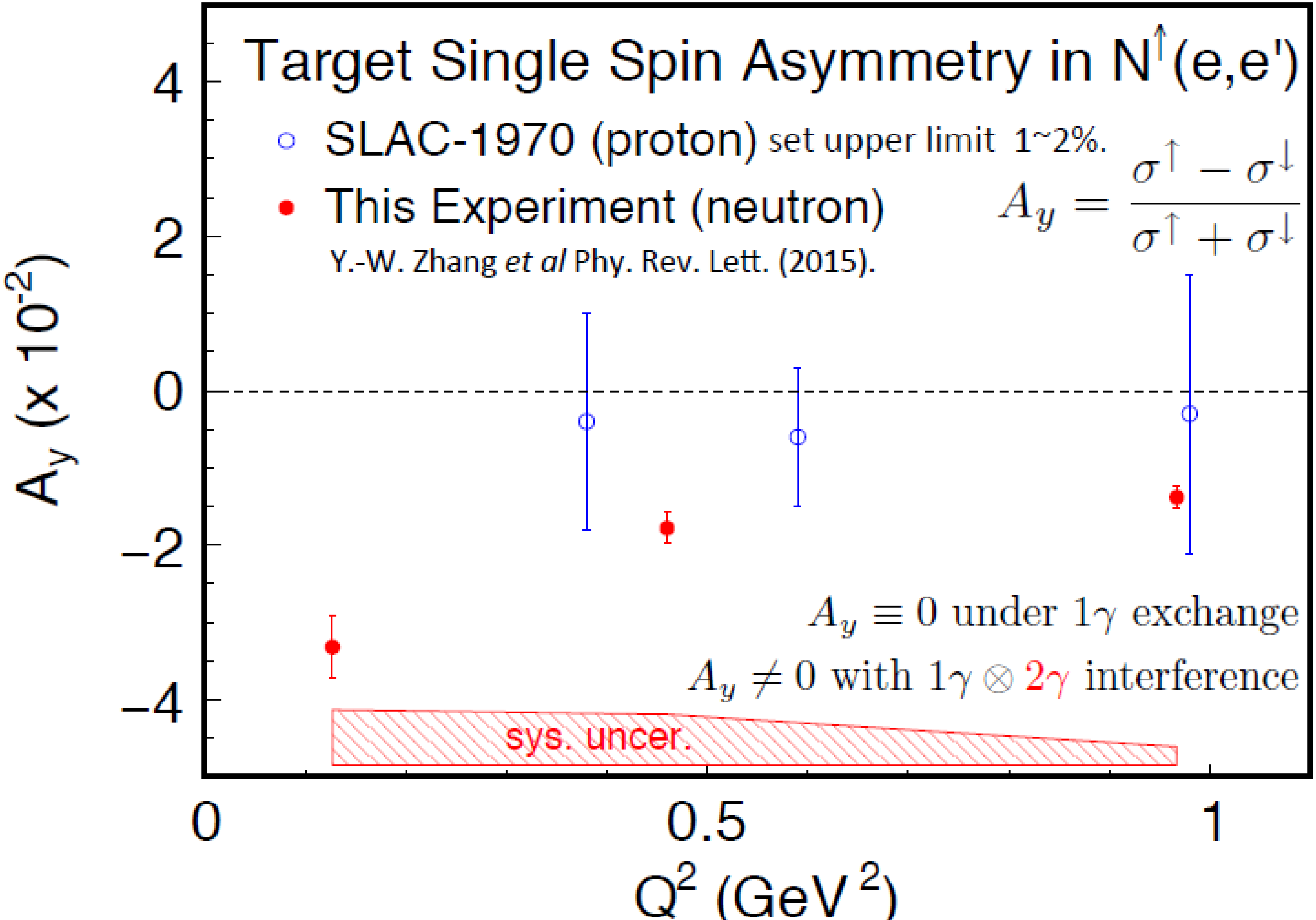
Q: any difference in electron's scattering probability between target spin-Up vs spin-Down ?

A: **Yes. Definitely !!!**

- Time-Reversal Odd observable, forbidden at the leading-order.
- Non-zero A_y has never been measured.
- New observable to study the fundamental sub-structure of nucleon, provides access to the moments of nucleon's Generalized-Parton-Distributions (GPDs).

2016/9/27

- The last experiment was in 1970, set an upper limit of 1~2%. T. Powell *et al*, PRL 753, **24** (1970).
- Jefferson Lab E05-015: Y.-W. Zhang Ph.D. Rutgers (2013). Co-PI: T. Averett (W&M), J.-P. Chen (JLab), X. Jiang (LANL).



- First observation of a non-zero target single-spin asymmetry in $\mathrm{N}^\uparrow(\mathrm{e},\mathrm{e}')$
- The last measurement was SLAC-1970, led by O. Chamberlain (Nobel 1959, discovered $\overline{P}$).
- Polarized ^{3}He as an effective polarized neutron target, in quasi-elastic kinematics.

4, Summary (form factors):

- The nucleon form factor
- The two photon exchange effect is addressed

- A phenomenological approach is introduced.
- The deuteron form factor and GPDs can be expressed in terms of the nucleon form factors and GPDs as well as loop integral, and its tensor structure function is discussed.

- The application of this approach to other light nucleus is going, which relates to the experiments in future Jlab. Or EIC

Thank you for your attention

Generalized parton distributions (Nucleon)

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+ q(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

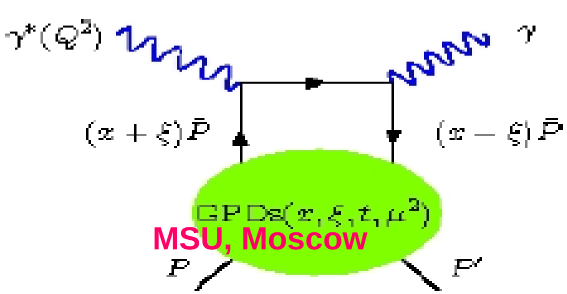
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{q}(-\tfrac{1}{2}z)\gamma^+\gamma_5 q(\tfrac{1}{2}z)|p\rangle\Big|_{z^+=0,\mathbf{z}=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2m}u(p)\right],$$

Deeply virtual Compton scattering

$$\boxed{\gamma^* p \to \gamma p}$$



FFs and GPDs of nucleon

$$\boxed{\Gamma^\mu = F_1 \gamma^\mu + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{M}}$$

Properties of GPDs

1) Forward limit (ξ =t=0):

$$H^q(x,0,0) = q(x)\,, \;\; x > 0$$

$$H^q(x,0,0) = -\bar{q}(-x)\,, \;\; x > 0$$

- GPD H reduces to usual PDFs

2) Connection to elastic form factors:

$$\int_{-1}^{1} dx H^q(x,\xi,t) = F_1^q(t) \qquad \text{Dirac}$$

$$\int_{-1}^{1} dx E^q(x,\xi,t) = F_2^q(t) \qquad \text{Pauli}$$

GPDs of deuteron, and tensor SF

$$V_{\lambda'\lambda} = \int \frac{d\kappa}{2\pi} e^{ix\kappa P\cdot n} \langle p', \lambda' | \bar{\psi}(-\kappa n) \gamma \cdot n \psi(\kappa n) | p, \lambda \rangle = \sum_i \epsilon'^{*\beta} V^{(i)}_{\beta\alpha} \epsilon^{\alpha} H_i(x, \xi, t),$$

$$A_{\lambda'\lambda} = \int \frac{d\kappa}{2\pi} e^{ix\kappa P\cdot n} \langle p', \lambda' | \bar{\psi}(-\kappa n) \gamma \cdot n \gamma_5 \psi(\kappa n) | p, \lambda \rangle = \sum_i \epsilon'^{*\beta} A^{(i)}_{\beta\alpha} \epsilon^{\alpha} \tilde{H}_i(x, \xi, t)$$

Decomposition

$$V_{\lambda'\lambda} = -(\epsilon'^{*} \cdot \epsilon) H_1 + \frac{(\epsilon \cdot n)(\epsilon'^{*} \cdot P) + (\epsilon'^{*} \cdot n)(\epsilon \cdot P)}{P \cdot n} H_2 - \frac{(\epsilon \cdot P)(\epsilon'^{*} \cdot P)}{2M^2} H_3$$
$$+ \frac{(\epsilon \cdot n)(\epsilon'^{*} \cdot P) - (\epsilon'^{*} \cdot n)(\epsilon \cdot P)}{P \cdot n} H_4 + \left\{ 4M^2 \frac{(\epsilon \cdot n)(\epsilon'^{*} \cdot n)}{(P \cdot n)^2} + \frac{1}{3}(\epsilon'^{*} \cdot \epsilon) \right\} H_5,$$

$$A_{\lambda'\lambda} = -i \frac{\epsilon_{\mu\alpha\beta\gamma} n^{\mu} \epsilon'^{*\alpha} \epsilon^{\beta} P^{\gamma}}{P \cdot n} \tilde{H}_1 + i \frac{\epsilon_{\mu\alpha\beta\gamma} n^{\mu} \Delta^{\alpha} P^{\beta}}{P \cdot n} \frac{\epsilon^{\gamma}(\epsilon'^{*} \cdot P) + \epsilon'^{*\gamma}(\epsilon \cdot P)}{M^2} \tilde{H}_2$$
$$+ i \frac{\epsilon_{\mu\alpha\beta\gamma} n^{\mu} \Delta^{\alpha} P^{\beta}}{P \cdot n} \frac{\epsilon^{\gamma}(\epsilon'^{*} \cdot P) - \epsilon'^{*\gamma}(\epsilon \cdot P)}{M^2} \tilde{H}_3 + i \frac{\epsilon_{\mu\alpha\beta\gamma} n^{\mu} \Delta^{\alpha} P^{\beta}}{P \cdot n} \frac{\epsilon^{\gamma}(\epsilon'^{*} \cdot n) + \epsilon'^{*\gamma}(\epsilon \cdot n)}{P \cdot n} \tilde{H}_4.$$

GPDs and PDFs (forward limit), deuteron

$$H_1 = \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3},$$

$$b_1 = \frac{1}{2}\sum_i e_i^2 \left(\delta_T q_i + \delta_T \bar{q}_i\right) \qquad \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

$$\boxed{H_5 = q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2},}$$

~~b_1

$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$

$$\tilde{H}_1 = q_\uparrow^1(x) - q_\uparrow^{-1}(x)$$

Here $q^{\lambda}_{\uparrow(\downarrow)}(x)$ represents the probability to find a quark with momentum fraction x and positive (negative) helicity in a deuteron target of helicity λ. The unpolarized quark densities q^{λ} are defined as $q^{\lambda}(x) = q_\uparrow^{\lambda}(x) + q_\downarrow^{\lambda}(x)$.

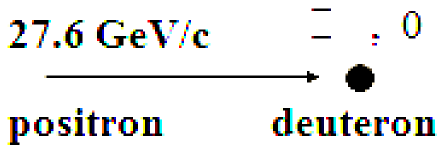
Sum rule of Tensor structure function

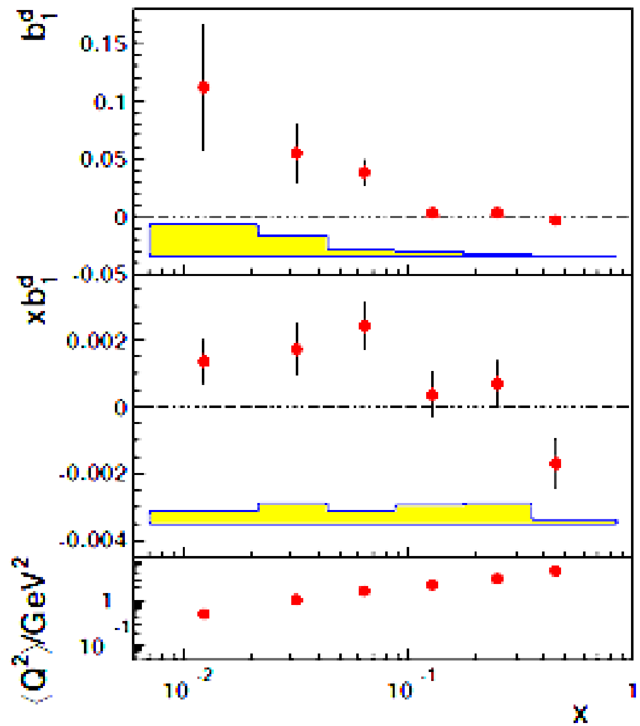
$$0 = \int_{-1}^{1} dx\, H_5(x,0,0)$$

$$= \int_0^1 dx \left[q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2}\right] - \{q \to \bar{q}\}$$

HERMES measurements on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.





b_1 measurements in the kinematical region

$0.01 < x < 0.45, \;\; 0.5\ \mathrm{GeV}^2 < Q^2 < 5\ \mathrm{GeV}^2$

TABLE II. Measured values (in 10^{-2} units) of the tensor asymmetry A_{zz}^d and the tensor structure function b_1^d. Both the corresponding statistical and systematic uncertainties are listed as well.

$\langle x \rangle$	$\langle Q^2 \rangle$ [GeV^2]	A_{zz}^d [10^{-2}]	$\pm\delta\mathcal{A}_{zz}^{\rm stat}$ [10^{-2}]	$\pm\delta\mathcal{A}_{zz}^{\rm sys}$ [10^{-2}]	b_1^d [10^{-2}]	$\pm\delta b_1^{\rm stat}$ [10^{-2}]	$\pm\delta b_1^{\rm sys}$ [10^{-2}]
0.012	0.51	−1.06	0.52	0.26	11.20	5.51	2.77
0.032	1.06	−1.07	0.49	0.36	5.50	2.53	1.84
0.063	1.65	−1.32	0.38	0.21	3.82	1.11	0.60
0.128	2.33	−0.19	0.34	0.29	0.29	0.53	0.44
0.248	3.11	−0.39	0.39	0.32	0.29	0.28	0.24
0.452	4.69	1.57	0.68	0.13	−0.38	0.16	0.03

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

Nucleon PDFs and GPDs : M. Guidal et al., PRD

$$\mathcal{H}^{q}_{R2}(x,t) = q_v(x)\, x^{-\alpha'(1-x)t},$$

$$\mathcal{E}^{q}(x,t) = \mathcal{E}^{q}(x)\, x^{-\alpha'(1-x)t},$$

$$\mathcal{E}^{u}(x) = \frac{\kappa_u}{N_u}(1-x)^{\eta_u} u_v(x)$$

$$\mathcal{E}^{d}(x) = \frac{\kappa_d}{N_d}(1-x)^{\eta_d} d_v(x),$$

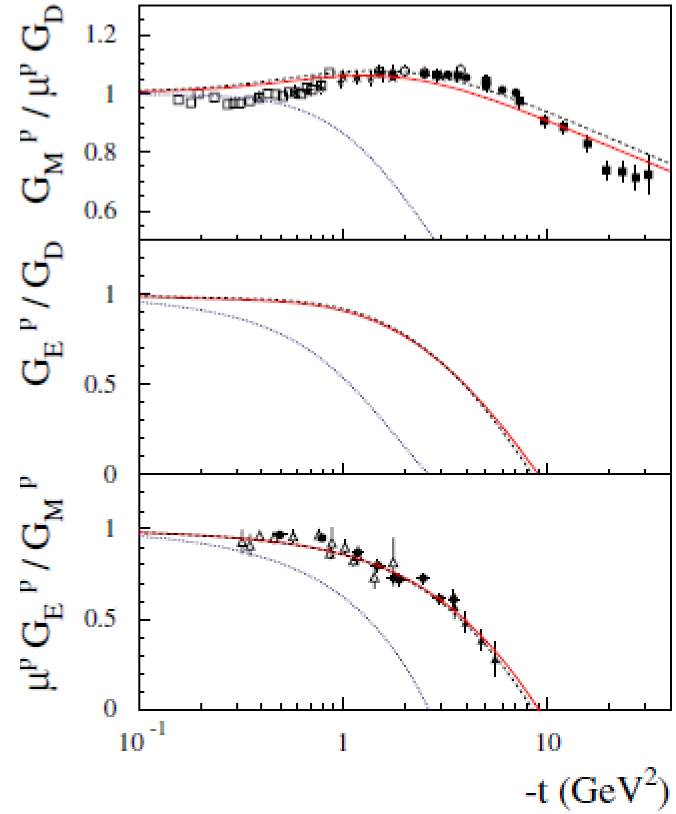
$$N_u = \int_0^1 dx (1-x)^{\eta_u} u_v(x),$$

$$xu_V = 0.262x^{0.31}(1-x)^{3.50} \times (1+3.83x^{0.5}+37.65x),$$

$$xd_V = 0.061x^{0.35}(1-x)^{4.03} \times (1+49.05x^{0.5}+8.65x),$$

$$xS = 0.759x^{-0.12}(1-x)^{7.66} \times (1-1.34x^{0.5}+7.40x),$$

$$xg = 0.669x^{0.00}(1-x)^{3.96} \times (1+6.98x^{0.5}-3.63x) - 0.23x^{-0.27}(1-x)^{8.7}.$$

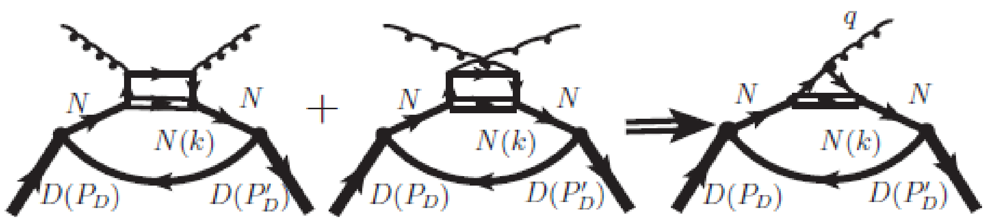


The flavour structure of the light quark sea is taken to be

$$2\bar{u}, 2\bar{d}, 2\bar{s} = 0.4S - \Delta, \qquad 0.4S + \Delta, \qquad 0.2S \quad (5)$$

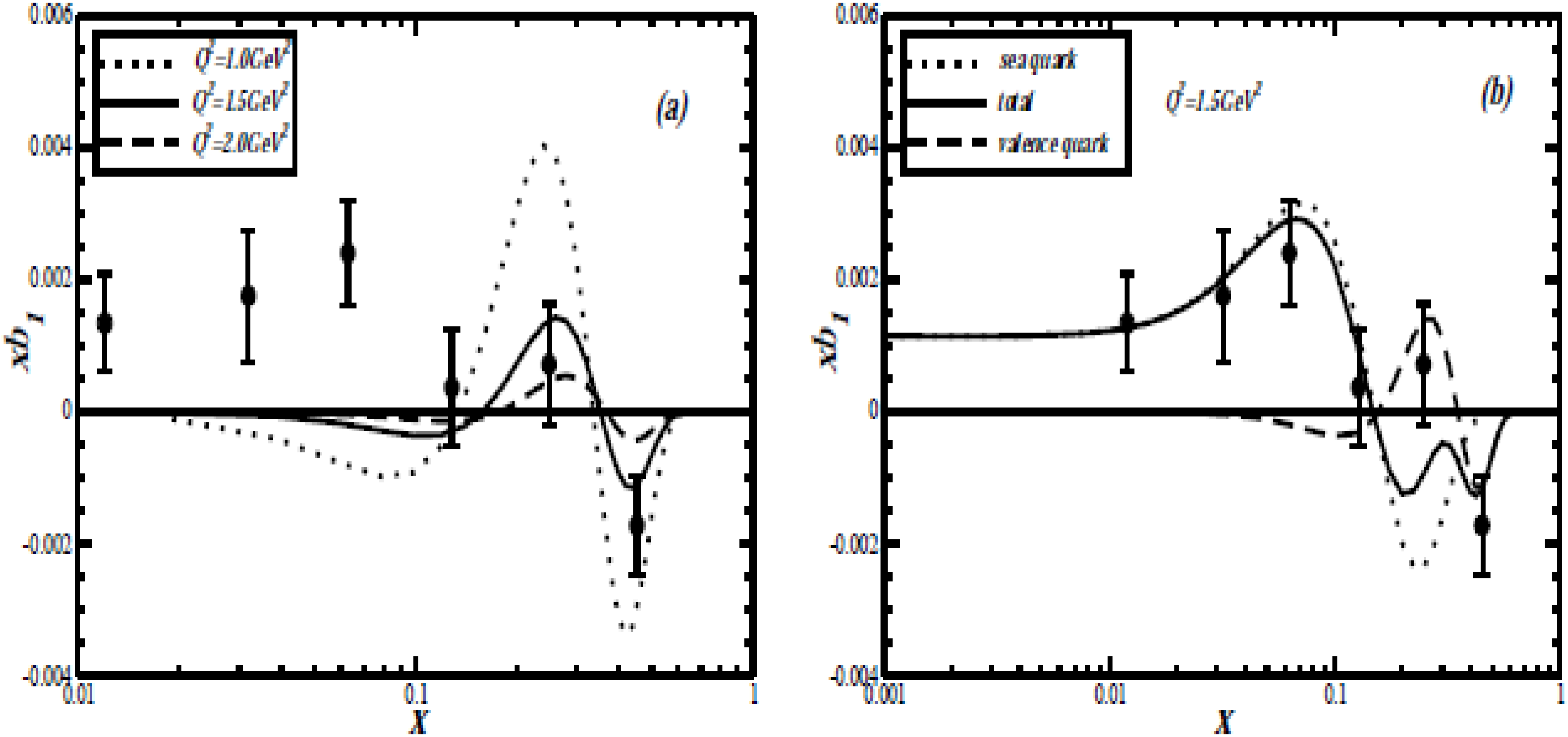
with $s = \bar{s}$, as implied by the NuTeV data [19], and where

$$x\Delta = x(\bar{d} - \bar{u}) = 1.432x^{1.24}(1-x)^{9.66}(1+9.86x-29.04x^2). \quad (6)$$



Effective diagram for calculating the generalized parton distribution functions of the deuteron

Discussions for tensor structure function



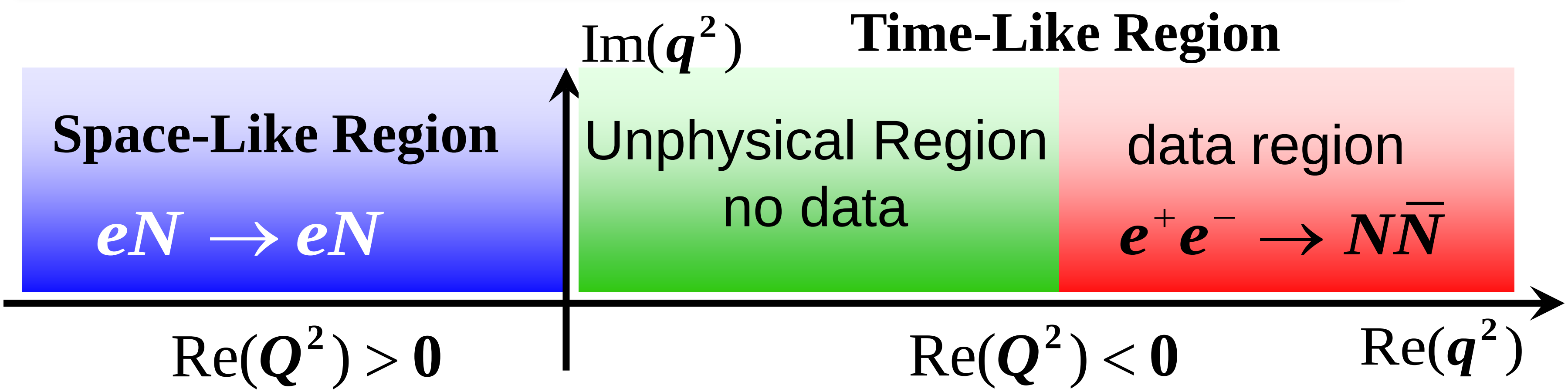
- Deuteron Tensor structure function measured in
  - DESY is sensitive to the small x-region .

Motivations

- Time-like form factors is essential for the form factors in the whole region

- **TPE effect on the Time-Like FFs is expected**

- **TPE effect provides complex amplitude**

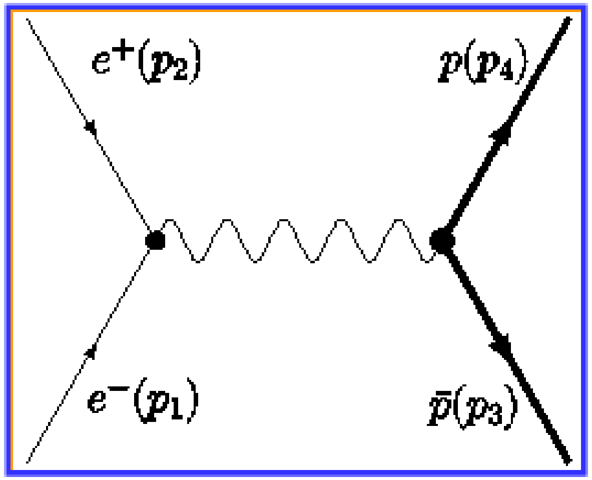


Measurement:

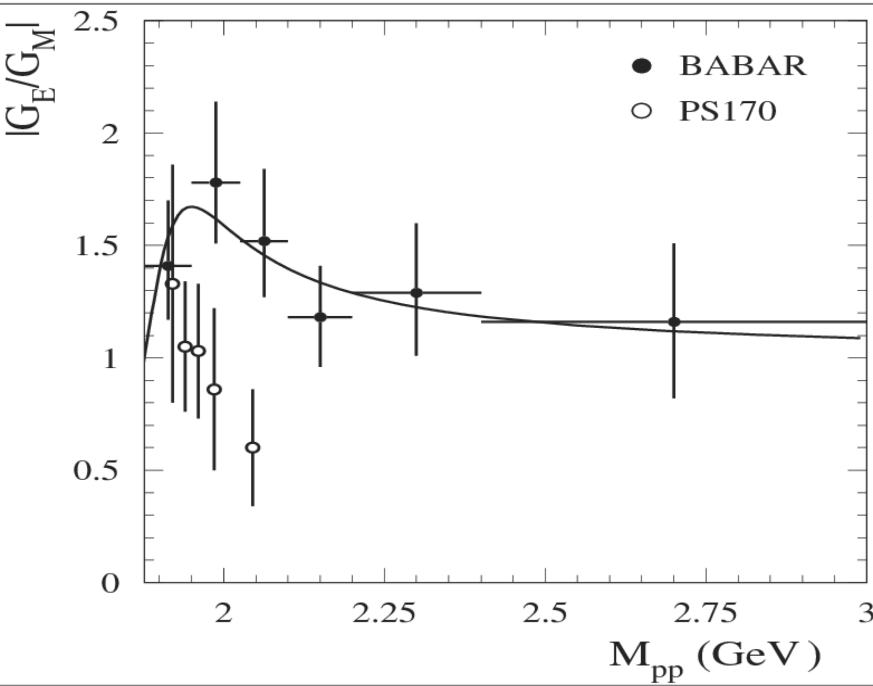
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4s}\left\{|G_M^p(s)|^2(1+\cos^2\theta) + \frac{4M^2}{s}|G_E^p(s)|^2(1-\cos^2\theta)\right\}$$

$$\beta = \sqrt{1-4M^2/s}$$

$$C \sim y/(1-e^{-y}) \,,\; y = \pi\alpha M/(\beta q)$$

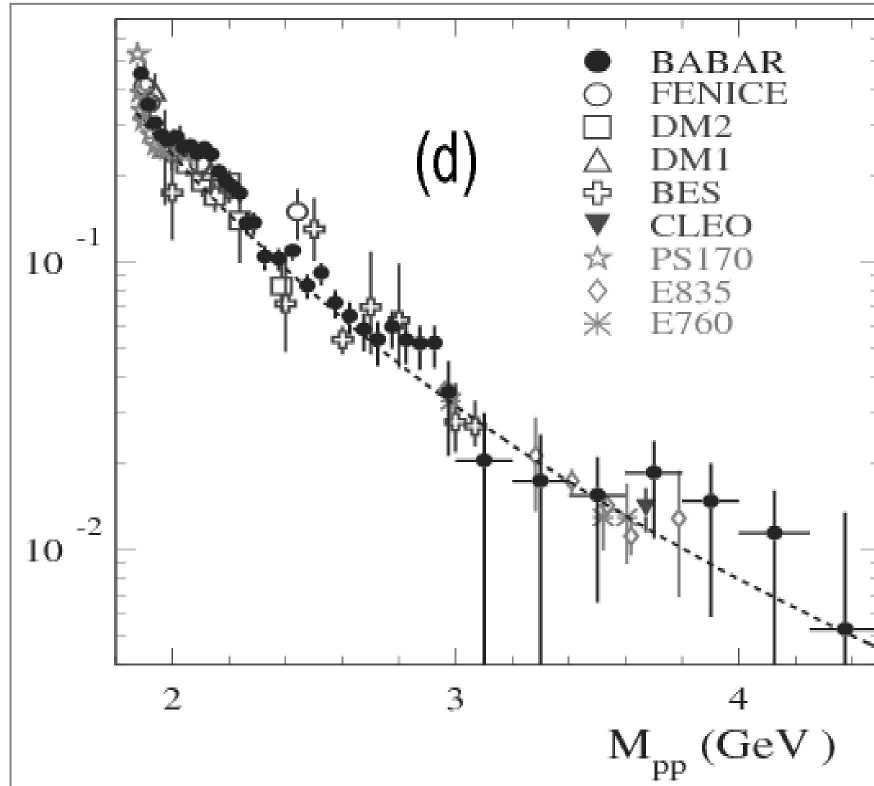


$$\boxed{f(\cos\theta) = \tau\mu_p^2 A^2(1+\cos^2\theta) + B^2(1-\cos^2\theta)}$$



G. Bardin *et al.*, PS170 Collaboration, Nucl. Phys. B **411** (1994) 3.

F Anulli *et al.*, BaBar Collaboration, J. Phys. : Conf. Ser.**69** 012014.



- Less precise data
- Contribution of G_E small
- Assumption:$G_E(s)=G_M(s)$

$$e^-(p_1)+e^+(p_2)\to p(p_3)+\bar{p}(p_4)$$

- **Current operators**

$$\Gamma_\mu = \widetilde{F}_1(s,t)\gamma_\mu + i\frac{\widetilde{F}_2(s,t)}{2m_N}\sigma_{\mu\nu}q^\nu + \boxed{\widetilde{F}_3(s,t)\frac{\gamma\cdot KP_\mu}{m_N^2}}$$

$$P=\frac{1}{2}(p_4-p_2),\ K=\frac{1}{2}(p_1-p_3)$$

- **Form factors**

$$\begin{aligned}\widetilde{G}_E &= \widetilde{F}_1+\tau\widetilde{F}_2 = G_E(q^2)+\Delta G_E(q^2,\cos\theta)\\ \widetilde{G}_M &= \widetilde{F}_1+\widetilde{F}_2 = G_M(q^2)+\Delta G_M(q^2,\cos\theta)\end{aligned}$$

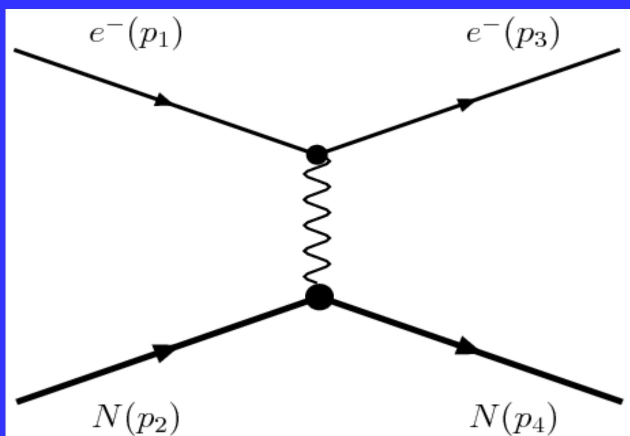
OPE TPE

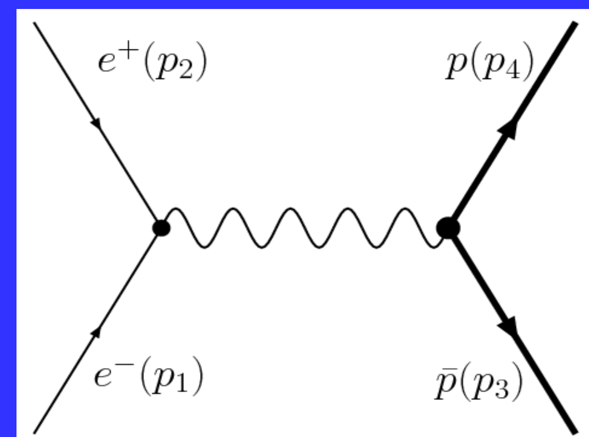
Crossing symmetry

$$\overline{|\mathcal{M}(e^-p \to e^-p)|^2} = f(s,t) = \overline{|\mathcal{M}(e^+e^- \to p\bar{p})|^2}.$$

Scattering

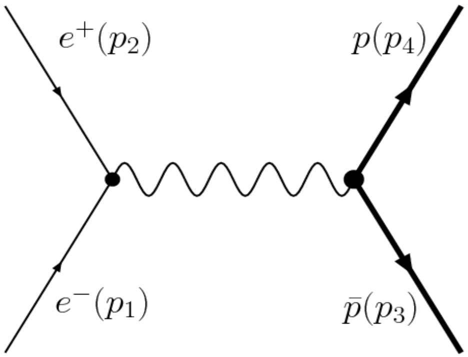
Annihilation





$$\begin{aligned} s &= (p_1+p_2)^2 \\ t &= (p_1-p_3)^2 \end{aligned}$$

$$\begin{aligned} s &= (p_1-p_3)^2 \\ t &= (p_1+p_2)^2 \end{aligned}$$



One-photon exchange

Quantum number: $\mathcal{J}^p = 1^-, \quad C(1\gamma) = -1$

Final state: $S = 1,\ \ell = 0 \quad S = 1,\ \ell = 2$

$$\boxed{\frac{d\sigma^{1\gamma}}{d\Omega} = a(t) + b(t)\cos^2\theta.}$$

Two-photon exchange

$\mathcal{J}^p = ?? \qquad C(2\gamma) = +1$

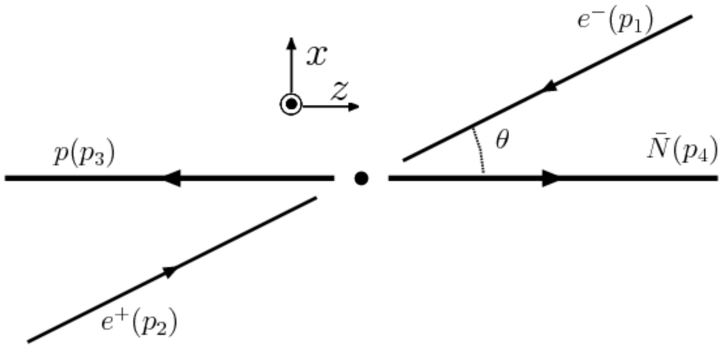
Final state: $\ell = 1, 3, 5, \dots \dots$

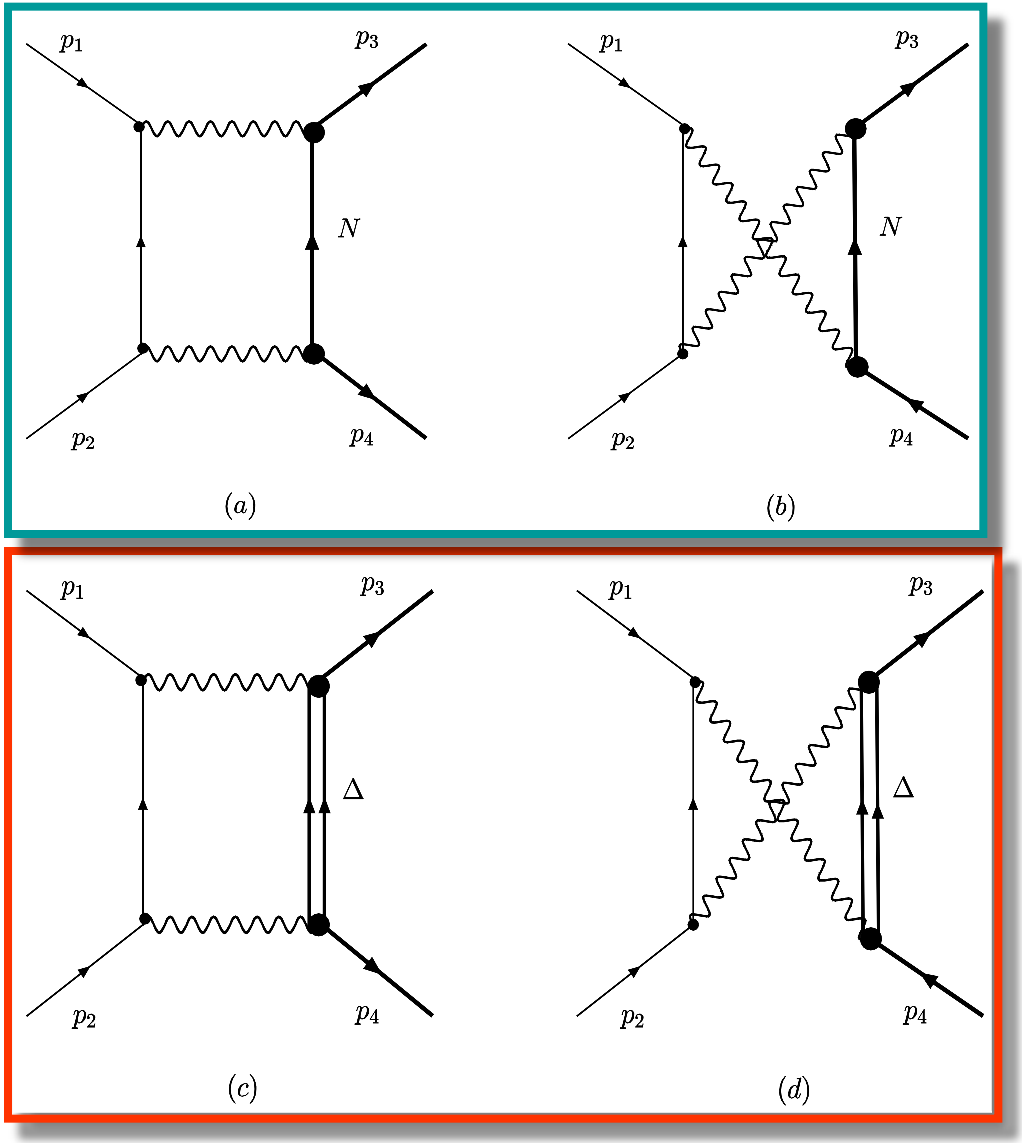
$$\boxed{\frac{d\sigma^{int}}{d\Omega} = \cos\theta\left[c_0(t) + c_1(t)\cos^2\theta + c_2(t)\cos^4\theta + \dots\right].}$$

Feynman Diagrams

Intermediate state: nucleon

Intermediate state:Δ(1232)





Unpolarized cross section

$$\frac{d\sigma_{un}}{d\Omega} = \frac{\alpha^2}{4q^2}\sqrt{\frac{\tau}{\tau-1}}\, D$$

$$\begin{aligned} D \quad = \quad & \boxed{|G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2\sin^2\theta} \\ & \boxed{\begin{aligned} &+2Re[G_M \Delta G_M^*](1+\cos^2\theta) + \frac{2}{\tau} Re[G_E \Delta G_E^*]\sin^2\theta \\ &-2\sqrt{\tau(\tau-1)}Re[(G_M - \frac{1}{\tau}G_E)\widetilde{F}_3^*]\sin^2\theta\cos\theta. \end{aligned}} \end{aligned}$$

Properties of TPE:

$$\begin{aligned} \Delta G_E(q^2,+\theta) &= -\Delta G_E(q^2,-\theta) \\ \Delta G_M(q^2,+\theta) &= -\Delta G_M(q^2,-\theta) \\ \widetilde{F}_3(q^2,+\theta) &= \widetilde{F}_3(q^2,-\theta). \end{aligned}$$

TPE amplitudes

$$\mathcal{M}_{2\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \left[\boxed{\frac{N_a(k)}{D_a(k)} + \frac{N_b(k)}{D_b(k)}} + \boxed{\frac{N_c(k)}{D_c(k)} + \frac{N_d(k)}{D_d(k)}} \right]$$

N-Intermediate

N-Intermediate

Δ-Intermediate

$$N_i(k) = j^i_{\mu\nu}(k) J^{\mu\nu}_i(k) \quad \{i = a, b, c, d\}$$

$$\Gamma_\mu(q) = F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q^\nu$$

$$j^{\mu\nu}_a = \bar{u}(-p_2)\gamma^\mu(\hat{p}_1 - \hat{k})\gamma^\nu u(p_1),$$

$$J^{\mu\nu}_a = \bar{u}(p_4)\boxed{\Gamma^\mu(p_1 + p_2 - k)}(\hat{k} - \hat{p}_3 - m_N)\boxed{\Gamma^\nu(k)}u(-p_3),$$

$$D_a(k) = [k^2 - \lambda^2][(p_1 + p_2 - k)^2 - \lambda^2][(p_1 - k)^2 - m_e^2][(k - p_3)^2 - m_N^2]$$

Δ contribution

$$j^{\mu\nu}_c = \bar{u}(-p_2)\gamma^\mu(\hat{p}_1 - \hat{k})\gamma^\nu u(p_1),$$

$$J^{\mu\nu}_c = \bar{u}(p_4)\boxed{\Gamma^{\mu\alpha}_{\gamma\Delta\to N}(p_1 + p_2 - k)}(\hat{k} - \hat{p}_3 - m_\Delta)\boxed{P^{3/2}_{\alpha\beta}}\boxed{\Gamma^{\nu\beta}_{\gamma\to\bar{N}\Delta}(k)}u(-p_3)$$

$$D_c(k) = [k^2 - \lambda^2][(p_1 + p_2 - k)^2 - \lambda^2][(p_1 - k)^2 - m_e^2][(k - p_3)^2 - m_\Delta^2]$$

TPE to differential cross section

$$\delta_{2\gamma} = 2\frac{Re\{\overline{\mathcal{M}_{2\gamma}\mathcal{M}_0^\dagger}\}}{\overline{|\mathcal{M}_0|^2}}$$

$$\frac{d\sigma}{d\Omega} \propto \overline{|\mathcal{M}|^2} = \overline{|\mathcal{M}_0|^2}\,(1+\delta_{2\gamma})$$

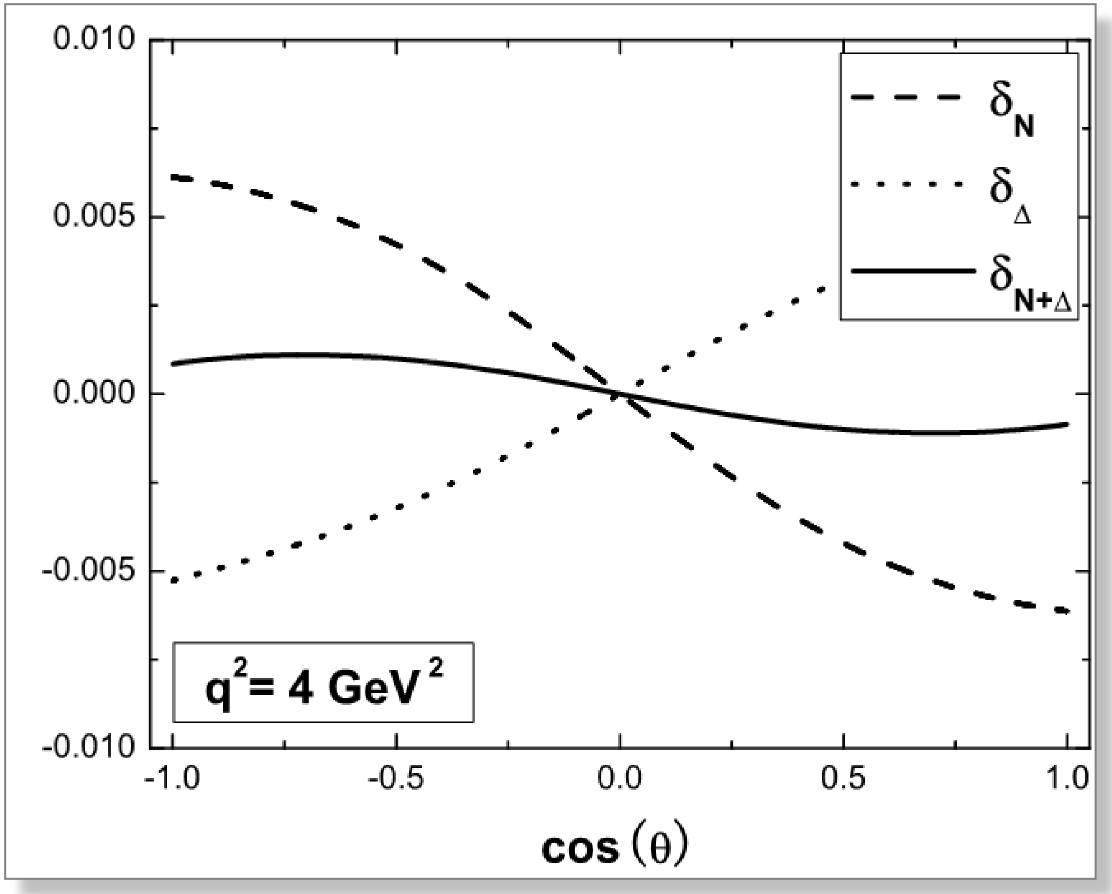
1: Odd function of $\cos\theta$;

2: Opposite contributions of N and Δ

3: Total contribution small

4: effect increasing as q^2 increasing

5: May hard to be detected



TPE on P_x

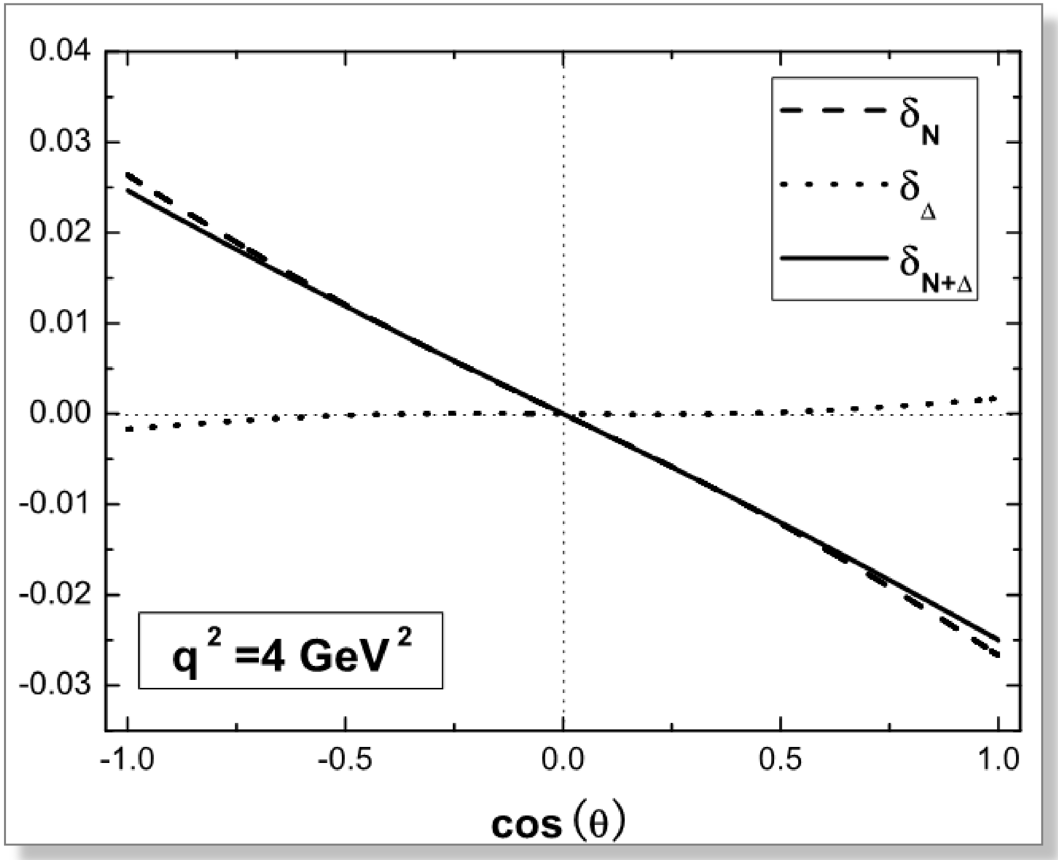
$$\delta(P_x) = \frac{P_x^{1\gamma\otimes 2\gamma}}{P_x^{1\gamma}}$$

$$P_x = -\frac{2\sin\theta}{D\sqrt{\tau}}\left\{Re\boxed{[G_M G_E^*}+\boxed{G_M \Delta G_E^* + \Delta G_M G_E^*] + Re[G_M \tilde{F}_3^*]\sqrt{\tau(\tau-1)}\cos\theta}\right\}$$

1: To be maximum at $\cos\theta = \pm 1$

2: $P_x^{1\gamma} \propto \sin\theta$, denominator is small

3: Hard to be detected



TPE on P_z

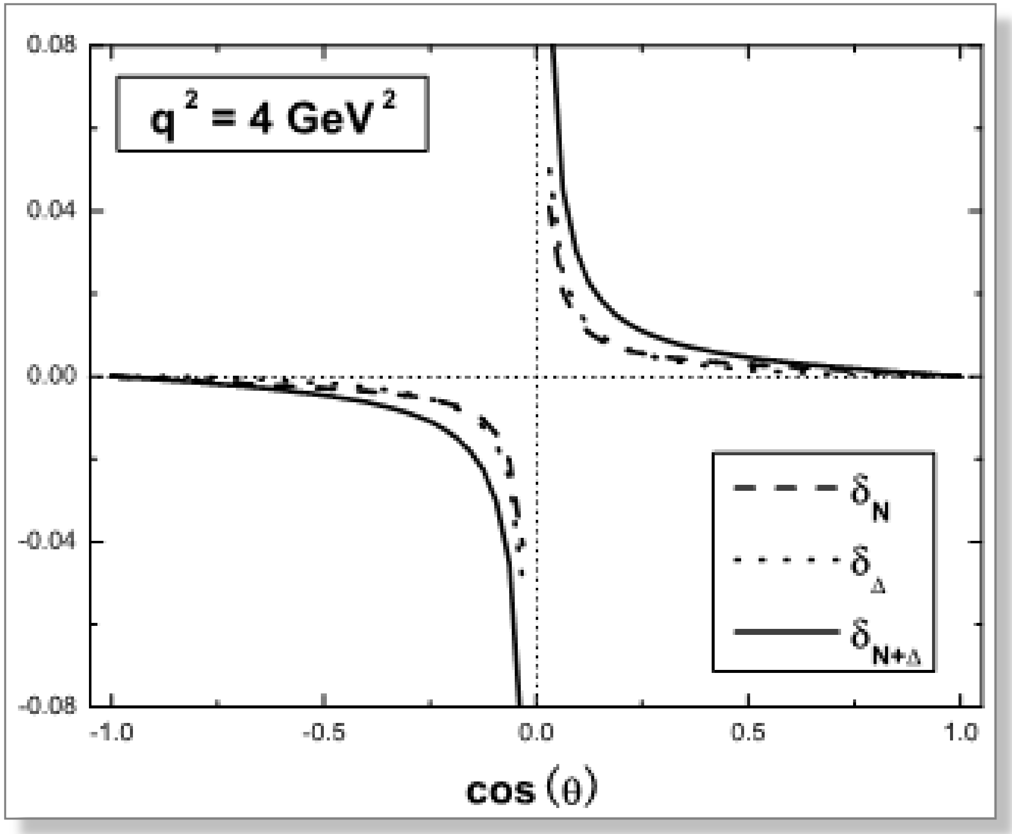
$$\delta(P_z) = \frac{P_z^{1\gamma\otimes 2\gamma}}{P_z^{1\gamma}}$$

$$P_z = \frac{2}{D}\left\{\boxed{|G_M|^2\cos\theta} + \boxed{2Re[G_M\Delta G_M]\cos\theta - Re[G_M\widetilde{F}_3^*]\sqrt{\tau(\tau-1)}\sin^2\theta}\right\}$$

1: $P_z^{1\gamma}(\pi/2) = 0,$ $P_z^{2\gamma}(\pi/2) \neq 0.$

2: To be maximum: π/2

3: Possible evidence



Summary (TPE on TL)

- **TPE effect should not be ignored**
- **TPE effect increases as q^2 increasing**
- **TPE effect is more sizeable in some polarizations**
- **at $\pi/2$, non-vanishing P_z may be an evidence for TPE.**

Approach (phenomenological)

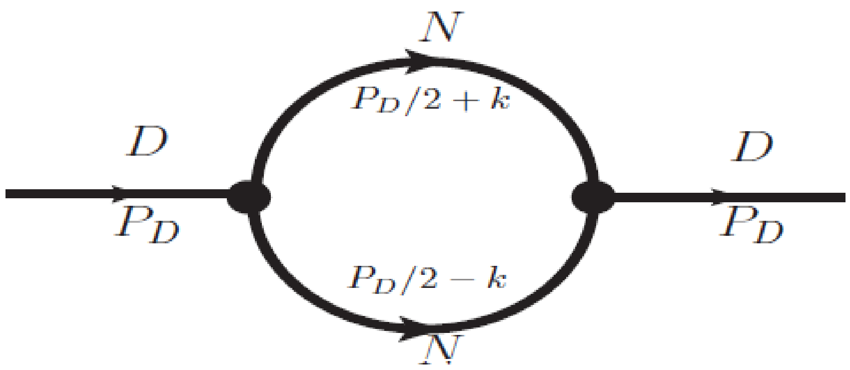


FIG. 1: The mass operator of the deuteron

$$\mathcal{L}_D(x) = g_D D^\dagger_\mu(x) \int dy \Phi_D(y^2) p(x+y/2) C\gamma^\mu n(x-y/2) + \mathrm{H.c.},$$

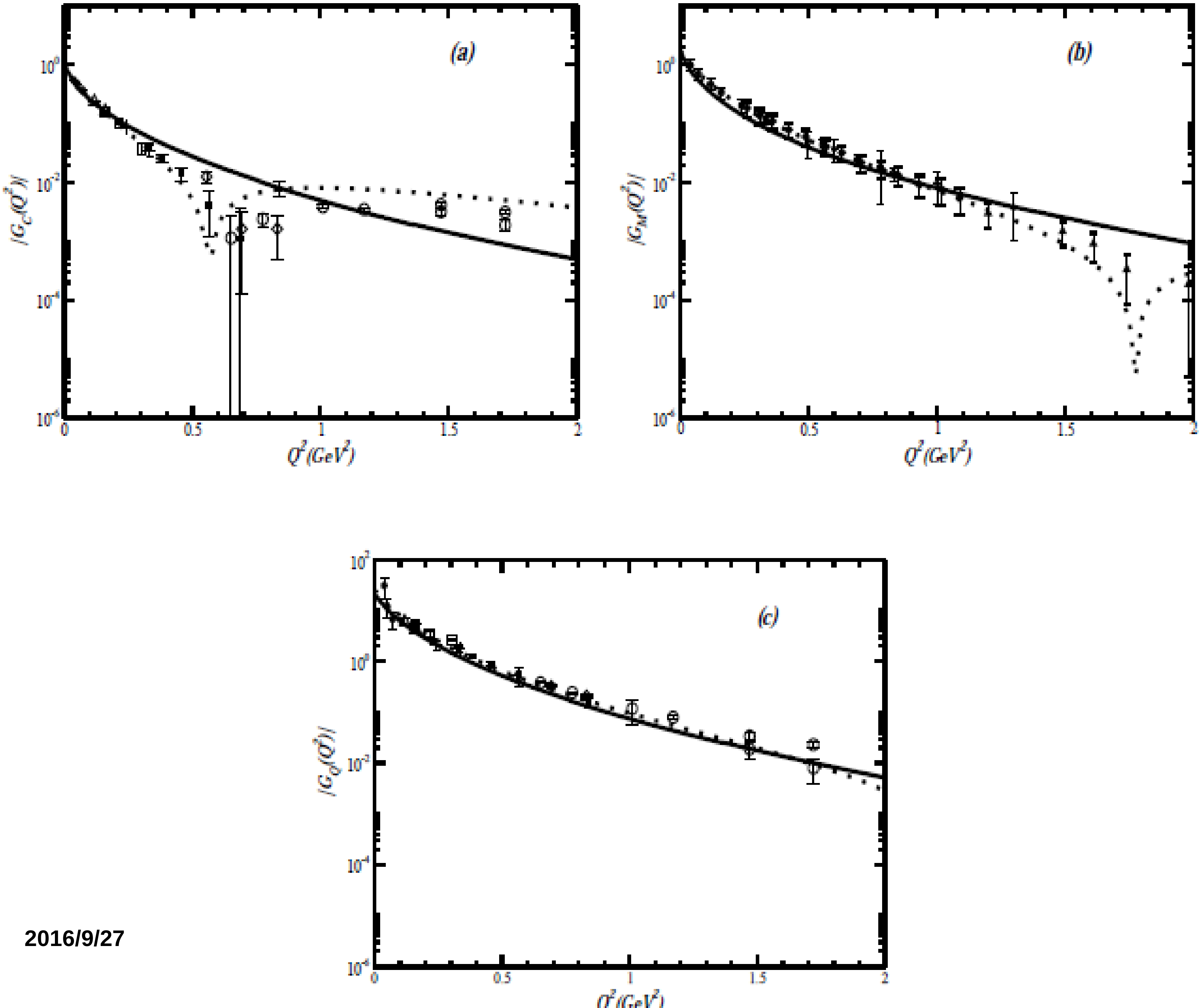
$$\mathcal{L}_D(x) = g_D D^\dagger_\mu(x) \int dy \bar{p}^c(x+y/2) \boxed{\Phi_D(y^2)\Gamma^\mu} n(x-y/2) + H.c.,$$

Correlation function (Cut-off) *PRC78, 035205*

$$\tilde{\Phi}'_D(\vec{k}^{\,2})\tilde{\Gamma}'^{\mu}(k,P) = \left\{ \left[\gamma^\mu + \frac{3k^\mu}{4M_N}\right]\tilde{f}_1(\vec{k}^{\,2}) + \left[\frac{3k^\mu}{4M_N}\right]\tilde{a}_D\tilde{f}_2(\vec{k}^{\,2}) \right\},$$

$$\tilde{\Phi}_D(k^2)\Gamma^\mu(k,P) = \sum_{i=1}^{2} b_i \Gamma^\mu_i f_i(k^2;\Lambda_i),$$

$$b_1 = 1, \quad b_2 = a_D; \quad \Gamma^\mu_1 = \gamma^\mu + \frac{3k^\mu}{4M_N}, \quad \Gamma^\mu_2 = \frac{3k^\mu}{4M_N}; \quad f_i(k^2;\Lambda_i) = exp\left(-\frac{k_E^2}{\Lambda_i^2}\right),$$



2016/9/27

Measurements of proton form factors

The results from the Jlab by using the new polarization transfer method were surprising, as they disagree with the ratios of G_p^E / G_p^M obtained by using the Rothenbluth method (cross section). The later appears to be near unity up to 6 GeV^2 , whereas the the polarization results show the ratio value around 0.3 at Q^2 of 5.6 GeV^2.

Comparing the measurements of Rosenbluth separation and polarization transfer, it is shown an unexpected and significant different dependence on Q^2 for G_p^E than on G_p^M .

This has been interpreted as indicating a difference between the spatial distributions of the charge and magnetization at short distances.

Polarization: P_y

$$P_y = \frac{1}{Dq^4} L_{\mu\nu} H_{\mu\nu}(s_{1y}) = \frac{1}{Dq^4} \left[L_{\mu\nu}(0) H_{\mu\nu}(s_{1y}) + \boxed{L_{\mu\nu}(s) H_{\mu\nu}(s_{1y})} \right]$$

$$P_y = \frac{2\sin\theta}{D\sqrt{\tau}} \left[Im[G_M G_E^* + G_M \Delta G_E^* + \Delta G_M G_E^*] \cos\theta - \sqrt{\tau(\tau-1)}\left(Im\boxed{[G_E \widetilde{F}_3^*]} \sin^2\theta + Im[G_M \widetilde{F}_3^*] \cos^2\theta\right)\right].$$

$$\boxed{P_y(\pi/2) = -2\frac{\sqrt{\tau-1}}{D} Im[G_E \tilde{F}_3^*].}$$

红外发散问题

1. 红外发散只在中间态为核子的双光子交换过程;

2. 软光子近似中，红外发散部分是准确的;

3. 我们最终的结果是去掉了软光子近似中的红外发散部分，是与光子的质量 λ无关的。

The discrepancy is due to the missing physics in the extraction of the ratio from the data, rather than systematic problems. A likely explanation is the two-photon exchange process, which affects both cross section and polarization transfer components. However, because the Rosenbluth method is very sensitive to small variations in the angular dependence of the cross section, the two-photon effects have a much more dramatic impact on the results from the Rosenbluth separation, while modifying the ratio obtained with the polarization method by a few percent only.

Some approaches:

- **M.T. Corrections:**
  - **intermediate state: nucleon;**
  - **finite part is ignored;**
  - **the momentums of one of photons in the denominator and numerator is set to 0(soft)**

- **L. C. Maximon et al.:**

  One of the photon momentum in the denominator is set 0

- **P.G. Blunden et al.:**

  Including some finite contributions of TPE

P.A.M.Guichon and M.Vanderhaeghen, PRL 91, 142303, 03

General expression of the matrix element: including TPE

$$\mathcal{M} = -\frac{ie^2}{q^2}\Big\{\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\left[\tilde{F}_1\gamma^\mu + \tilde{F}_2\frac{i\sigma^{\mu\nu}q_\nu}{2M}\right]u(p_2)$$
$$\boxed{+\bar{u}(p_3)\gamma_\mu\gamma_5 u(p_1)\bar{u}(p_4)\gamma^\mu\gamma^5\tilde{G}_A u(p_2)}\Big\}.$$

$$\bar{u}(p_3)\gamma\cdot P u(p_1)\bar{u}(p_4)\gamma\cdot K u(p_2)$$
$$= \frac{s-u}{4}\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\gamma^\mu u(p_2) + \frac{t}{4}\bar{u}(p_3)\gamma_\mu\gamma_5 u(p_1)\bar{u}(p_4)\gamma^\mu\gamma^5 u(p_2).$$

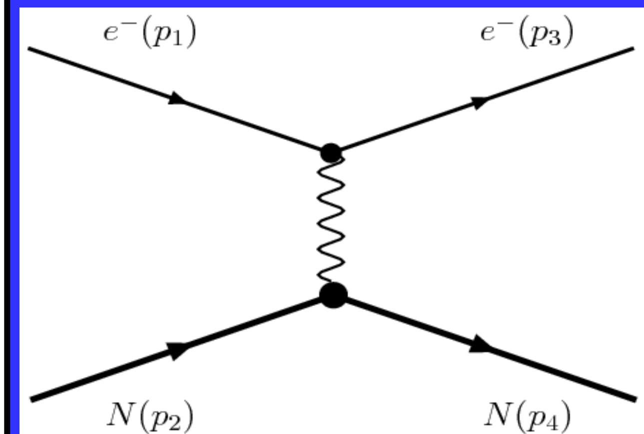
$$-q^2\overline{U}'\gamma_5\gamma_\mu U = 2mq_\mu\overline{U}'\gamma_5 U + 2i\varepsilon_{\mu\nu\sigma\tau}K^\nu q^\sigma\vec{U}'\gamma^\tau U$$

$$\mathcal{M} = -\frac{ie^2}{q^2}\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\left[\tilde{F}_1\gamma^\mu + \tilde{F}_2\frac{i\sigma^{\mu\nu}q_\nu}{2M} + \tilde{F}_3\frac{\gamma\cdot K P^\mu}{M^2}\right]u(p_2).$$

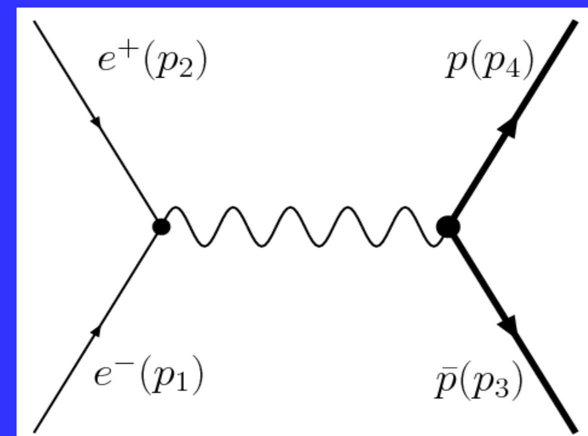
Cross symmetry

$$\overline{|\mathcal{M}(e^-p \to e^-p)|^2} = f(s,t) = \overline{|\mathcal{M}(e^+e^- \to p\bar{p})|^2}.$$

Scattering



Annihilation



$$\begin{aligned} f(s,t) \;=\; & \frac{8e^4}{(4M^2-t)t}\Big\{8|\widetilde{G}_E|^2M^2\left[M^4-2sM^2+s(s+t)\right]- \\ & |\widetilde{G}_M|^2t\left[2M^4-4M(s+t)+2s^2+t^2+2st\right] \\ & -M^{-2}\left[2M^6-M^4(6s+t)+2M^2s(3s+2t)-\right. \\ & \left. s(2s^2+3ts+t^2)\right]Re\left[(4M^2\widetilde{G}_E-t\widetilde{G}_M)^*\widetilde{F}_3\right]\Big\}. \end{aligned}$$

$$\begin{aligned} s &= (p_1+p_2)^2 \\ t &= (p_1-p_3)^2 \end{aligned}$$

$$\begin{aligned} s &= (p_1-p_3)^2 \\ t &= (p_1+p_2)^2 \end{aligned}$$

Δ contribution

$$j_c^{\mu\nu} = \bar{u}(-p_2)\gamma^\mu(\hat{p}_1 - \hat{k})\gamma^\nu u(p_1),$$

$$J_c^{\mu\nu} = \bar{u}(p_4)\boxed{\Gamma^{\mu\alpha}_{\gamma\Delta\to N}(p_1+p_2-k)}(\hat{k}-\hat{p}_3-m_\Delta)\boxed{P^{3/2}_{\alpha\beta}}\boxed{\Gamma^{\nu\beta}_{\gamma\to\bar{N}\Delta}(k)}u(-p_3)$$

$$D_c(k) = [k^2-\lambda^2][(p_1+p_2-k)^2-\lambda^2][(p_1-k)^2-m_e^2][(k-p_3)^2-m_\Delta^2]$$

$$\Gamma^{\mu\alpha}_{\gamma\Delta\to N} = \frac{-F_\Delta(q_1^2)}{M_N^2}[g_1(g^\alpha_\mu \hat{k}\hat{q}_1 - k_\mu\gamma^\alpha\hat{q}_1 - \gamma_\mu\gamma^\alpha k\cdot q_1 + \gamma_\mu\hat{k}q_1^\alpha)$$
$$+g_2(k_\mu q_1^\alpha - k\cdot q_1 g^\alpha_\mu) + g_3/M_N(q_1^2(k_\mu\gamma^\alpha - g^\alpha_\mu\hat{k})$$
$$+q_{1\mu}(q_1^\alpha\hat{k} - \gamma^\alpha k\cdot q_1))]\gamma_5 T_3,$$

$$\Gamma^{\mu\alpha}_{\gamma\to\bar{N}\Delta} = \frac{-F_\Delta(q_2^2)}{M_N^2}(k)T_3^+\gamma_5[g_1(g^\beta_\nu\hat{q}_2\hat{k} - k_\nu\hat{q}_2\gamma^\beta - \gamma^\beta\gamma_\nu k\cdot q_2 + \hat{k}\gamma_\nu q_2^\beta)$$
$$+g_2(k_\nu q_2^\beta - k\cdot q_2 g^\beta_\nu) - g_3/M_N(q_2^2(k_\nu\gamma^\beta - g^\beta_\nu\hat{k})$$
$$+q_{2\nu}(q_2^\beta\hat{k} - \gamma^\beta k\cdot q_2))].$$

$$S^\Delta_{\alpha\beta}(k) = \frac{-i(\hat{k}+M_\Delta)}{k^2-M_\Delta^2+i\epsilon}P^{3/2}_{\alpha\beta}(k),$$

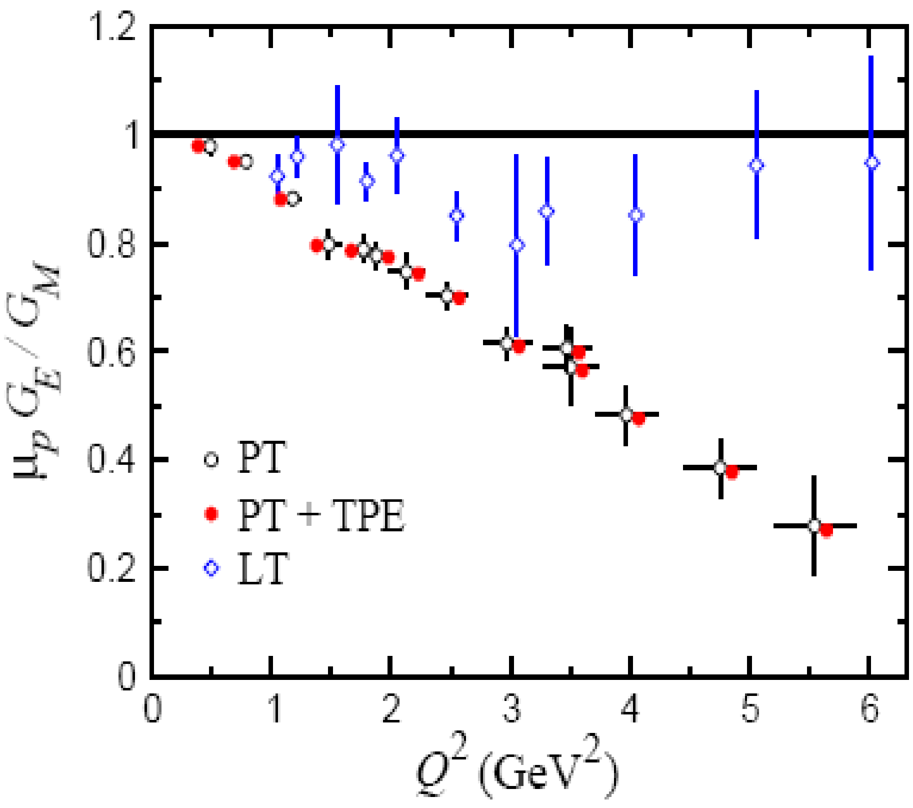
$$P^{3/2}_{\alpha\beta}(k) = g_{\alpha\beta} - \gamma_\alpha\gamma_\beta/3 - (\hat{k}\gamma_\alpha k_\beta + k_\alpha\gamma_\beta\hat{k})/3k^2.$$

2. Polarizations of P_x and P_z

$$P_x = \frac{1}{Dq^4} L_{\mu\nu} H^{\mu\nu}(s_{1x}) = \frac{1}{Dq^4}\left[\boxed{L_{\mu\nu}(0) H^{\mu\nu}(s_{1x})} + L_{\mu\nu}(s) H^{\mu\nu}(s_{1x})\right]$$

$$\boxed{P_z^{1\gamma}(\pi/2) = 0, \quad P_z^{2\gamma} = -\sqrt{\tau(\tau-1)}\,\frac{Re[G_M \widetilde{F}_3^*]}{|G_M|^2}}$$

TPE in SL (To Rosenbluth separation and polarization transfer)



Measuremens of proton form factors

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