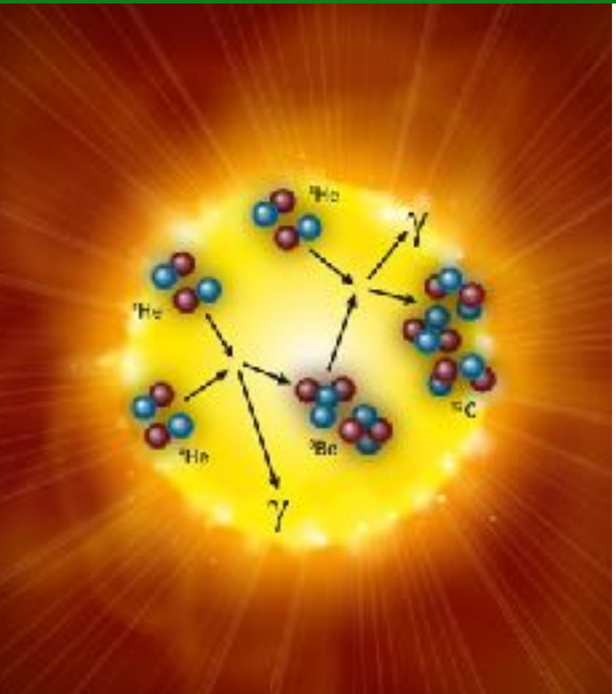
Евгений Эпельбаум, Ruhr University Bochum

Семинар по ядерной физике НИИЯФ МГУ
19 января 2021

Ядерные взаимодействия в киральной эффективной теории поля: достижения и вызовы



Введение

Киральная теория ядерных взаимодействий

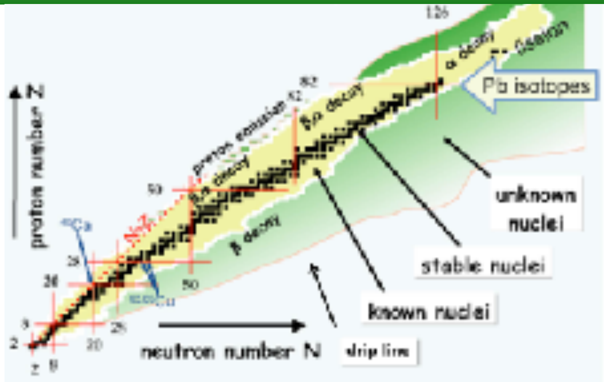
Избранные применения

Нерешенные вопросы

Заключение

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

Why (precision) nuclear physics?



After the discovery of Higgs boson,

the strong sector remains the only poorly understood part of the SM!

Interesting topic on its own. Some current frontiers:

— the nuclear chart and limits of stability FAIR, GANIL, ISOLDE,…

— EoS for nuclear matter (gravitational waves from n-star mergers) LIGO/Virgo,…

— hypernuclei (neutron stars) JLab, JSI/FAIR, J-PARC, MAMI,…

But also highly relevant for searches for BSM physics, e.g.:

— direct Dark Matter searches (WIMP-nucleus scattering)

— searches for $0\nu\beta\beta$ decays

— searches for nucleon/nuclear EDMs

— proton/deuteron radius puzzle (complementary experiments with light nuclei…)

→ need a reliable approach to nuclear structure with quantified uncertainties:

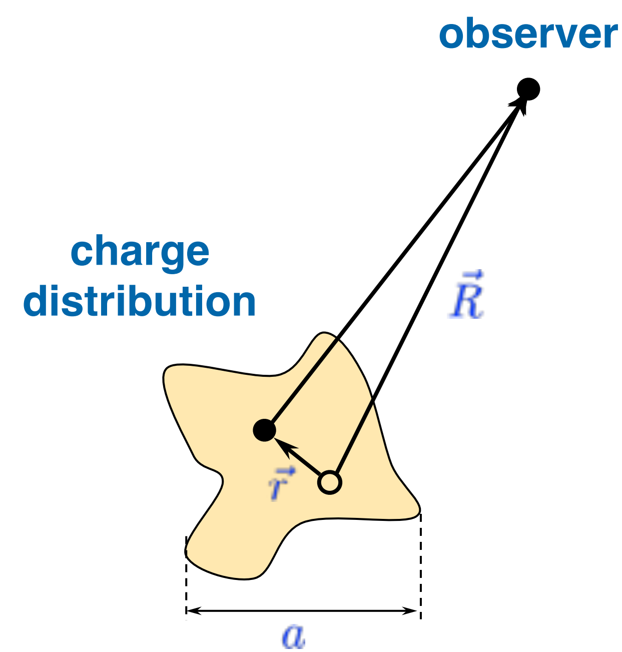
Effective Field Theory

What is an effective theory?

Example from electrostatics

The goal: compute electric potential generated by a localized charge distribution $\rho(\vec{r})$

- The correct answer: $V(\vec{R}) \propto \int d^3r \, \frac{\rho(\vec{r})}{|\vec{R} - \vec{r}|}$



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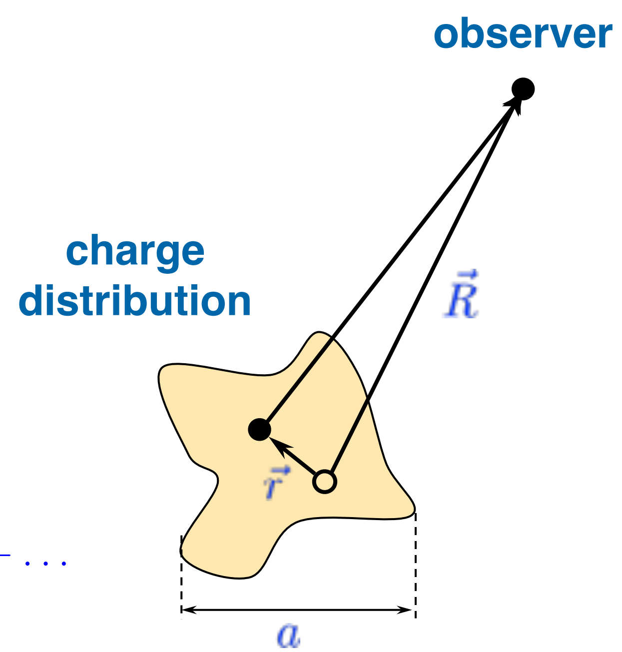

- The correct answer: $V(\vec{R}) \propto \int d^3r \, \frac{\rho(\vec{r})}{|\vec{R} - \vec{r}|}$

- For $R \gg a$, only moments of $\rho(\vec{r})$ are needed:

$$V(\vec{R}) = \frac{\boldsymbol{q}}{R} + \frac{1}{R^3} \sum_i R_i \boldsymbol{P}_i + \frac{1}{6R^5} \sum_{ij} (3R_iR_j - \delta_{ij}R^2) \boldsymbol{Q}_{ij} + \ldots$$

with multipole moments („low-energy constants“):

$$\boldsymbol{q} = \int d^3r \, \rho(\vec{r}), \qquad \boldsymbol{P}_i = \int d^3r \, \rho(\vec{r}) \, r_i, \qquad \boldsymbol{Q}_{ij} = \int d^3r \, \rho(\vec{r})(3r_ir_j - \delta_{ij}r^2)$$

Remember: multipole expansion just follows from:

$$\frac{1}{|\vec{R} - \vec{r}|} = \frac{1}{R} \sum_{l=0}^{\infty} \left(\frac{r}{R}\right)^l P_l(\cos\alpha) = \frac{1}{R} \sum_{l=0}^{\infty} \left(\frac{r}{R}\right)^l \frac{4\pi}{2l+1} \sum_m Y_{lm}(\hat{R}) Y_{lm}^{\star}(\hat{r})$$

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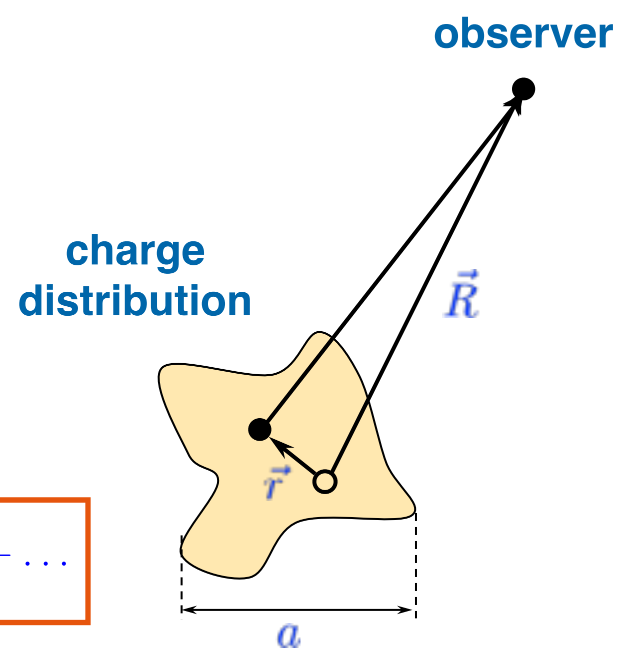
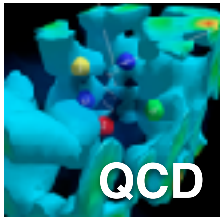
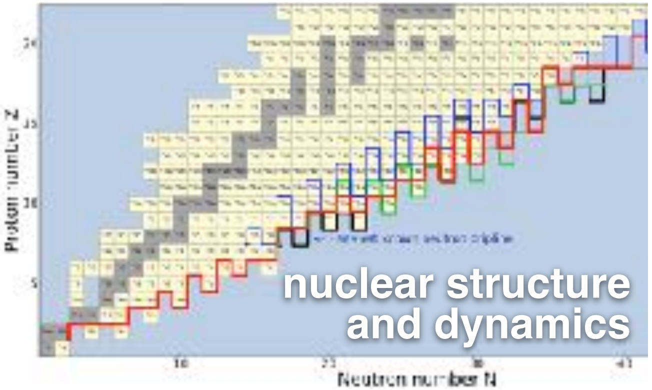


- Getting the right answer without making calculations (and even without knowing $\rho(\vec{r})$)
  - — write down the most general rotationally invariant (symmetry!) expression for $V(\vec{R})$
  - — expected natural size of the LECs (dimensional analysis): $q \sim a^0, \;\; P_i \sim a, \;\; Q_{ij} \sim a^2, \;\; \ldots$
  - — measure LECs & compute $V(\vec{R})$ via expansion in $\frac{a}{R}$

From QCD to nuclei: The framework in a nutshell





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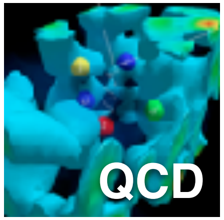


lattice

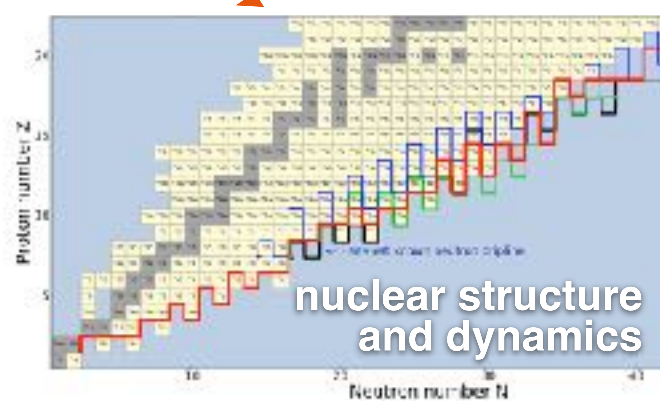
symmetries (especially the chiral symmetry);
lost of information (LECs)

effective chiral Lagrangian $\mathcal{L}_{\rm eff}(\pi, N)$

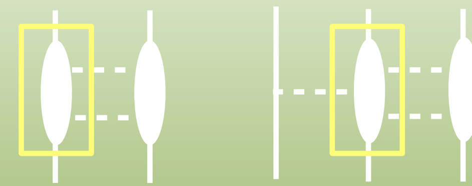
integrate out pions (valid for $|\vec{p}| \sim M_\pi \ll \sqrt{M_\pi m_N}$):
Chiral Perturbation Theory

nuclear forces and currents

ab initio many-body methods:
FY, NCSM, lattice,…



Ultimate goal: predictive and systematically improvable QCD-based approach to nuclei, nuclear reactions and nuclear matter with quantified uncertainties

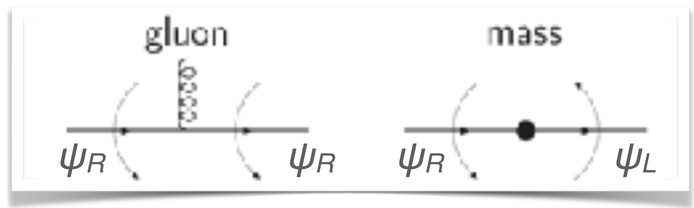


GB and 1N-sectors:

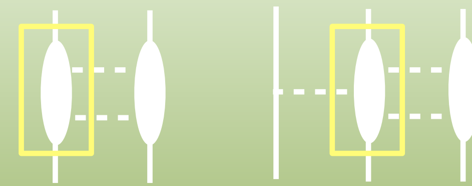
Chiral Perturbation Theory

Weinberg, Gasser, Leutwyler, Bernard, Kaiser, Meißner, …

Observations: QCD is approximately chiral invariant; $SU(2)_L$ x $SU(2)_R$ is spontaneously broken down to $SU(2)_{isospin}$



Idealized world [$m_u = m_d = 0$], zero-energy limit: non-interacting massless GBs
(+ strongly interacting massive hadrons)



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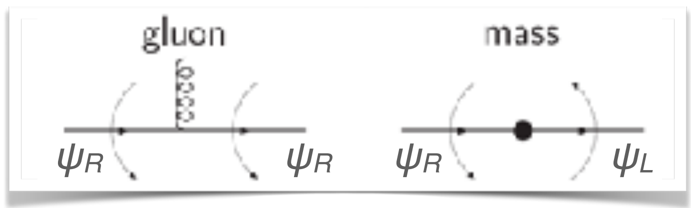

Idealized world $[m_u = m_d = 0]$, zero-energy limit: non-interacting massless GBs (+ strongly interacting massive hadrons)

Real world $[m_u,\ m_d \ll \Lambda_{QCD}]$, low energy: weakly interacting light GBs (pions) (+ strongly interacting massive hadrons)

Chiral perturbation theory: expansion of observables in $Q = \dfrac{\text{momenta of particles or } M_\pi \sim 140\text{ MeV}}{\text{breakdown scale } \Lambda_b}$

Pion-nucleon scattering

Effective chiral Lagrangian:

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\Big(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\Big)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \boldsymbol{e}_i \bar{N}\hat{O}_i^{(4)}[\pi]N}_{\mathcal{L}_{\pi N}^{(4)}} + \ldots$$

Pion-nucleon scattering amplitude for $\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$:

$$T_{\pi N}^{ba} = \frac{E+m}{2m}\Big(\delta^{ba}\Big[g^+(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\, h^+(\omega,t)\Big] + i\epsilon^{bac}\tau^c\Big[g^-(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\, h^-(\omega,t)\Big]\Big)$$

calculated within the chiral expansion

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Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

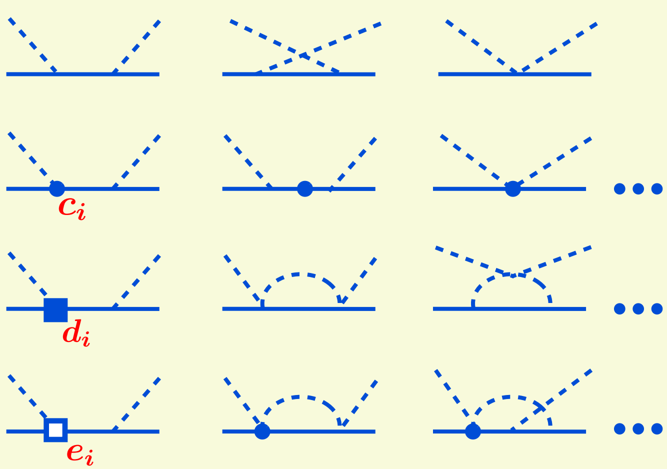
Fettes, Meißner ’00; Krebs, Gasparyan, EE ’12

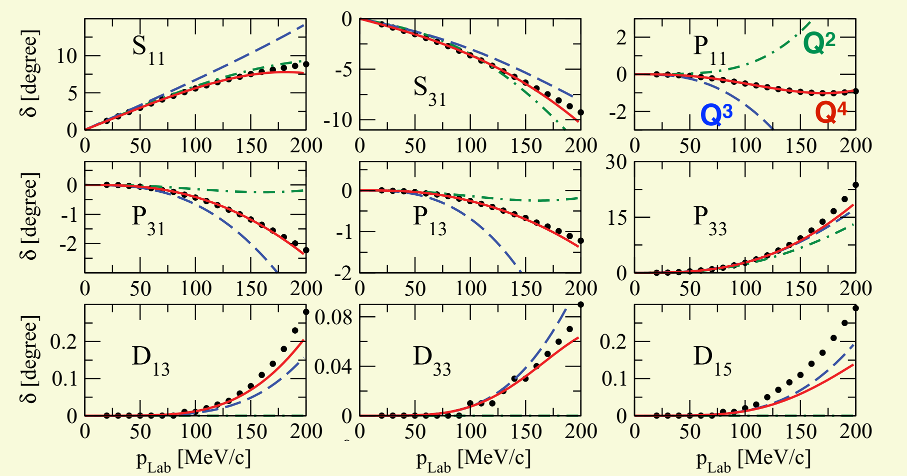
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:





Pion-nucleon scattering

Why does a perturbation theory work?

The spontaneously broken chiral symmetry only allows for **derivative** pion couplings!

E.g., for the pseudoscalar (Yukawa) coupling:

$\sim Q^{-1}$

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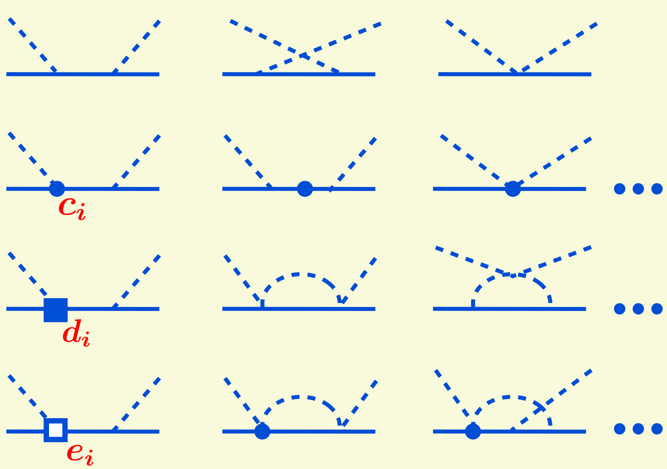
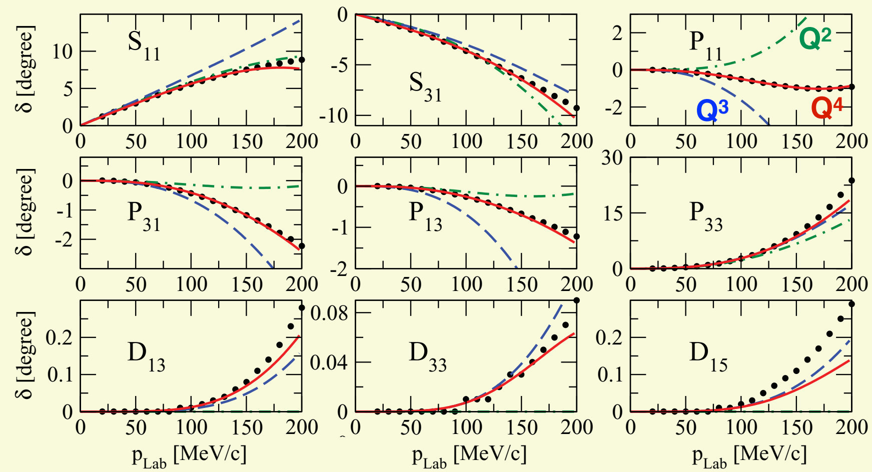


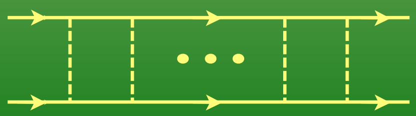


Chiral EFT for nuclear systems

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

— contrary to pions, the interaction between nucleons is **NOT** suppressed at low energy

— certain Feynman diagrams (ladder) are enhanced and **need to be resummed**



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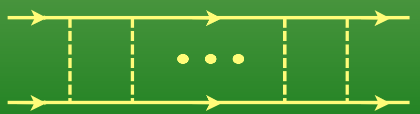
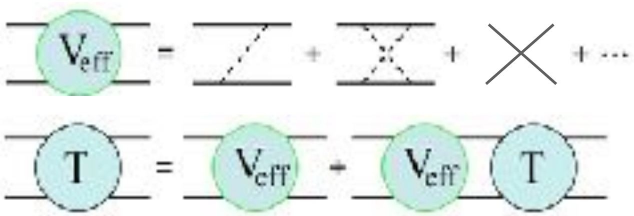


The approach proposed by Weinberg:

(1) Use chiral EFT to compute nuclear forces

(2) Solve the A-body Schrödinger equation to calculate nuclear observables



$$\boxed{\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle}$$

A **systematically improvable**, unified approach for $\pi\pi$, πN, NN that allows one to derive **consistent** many-body forces and currents

Framework in a nutshell

EE, Krebs, Reinert, Front. in Phys. 8 (2020) 98

- Write down the **effective chiral Lagrangian** for π, N (if needed, + Δ and external fields…)

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- Compute the **irreducible part of the few-N amplitude** (i.e. nuclear forces & currents) by combining ChPT with TOPT, method of UT, unitary clothing or S-matrix matching
  - — unitary ambiguities (consistency!)
  - — pion loops usually computed in DR
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  - — long range: $\dfrac{1}{\vec{q}^2 + M_\pi^2} \longrightarrow \dfrac{1}{\vec{q}^2 + M_\pi^2} e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} \simeq \dfrac{1}{\vec{q}^2 + M_\pi^2} (1 + \text{short-range terms})$
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still consistent beyond the NN system?

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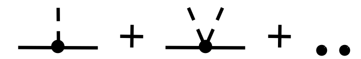
Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

- **Canonical transformation & quantization:** $\mathcal{L}_{\pi N} \longrightarrow \mathcal{H}_{\pi N} =$ 

nucleonic states $|N\rangle,\ |NN\rangle,\ \ldots$

EOM: $$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

projectors

states with mesons $|N\pi\rangle,\ |N\pi\pi\rangle,\ \ldots$

can not solve
(infinite-dimensional eq.)

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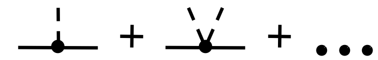
- **Canonical transformation & quantization:** $\mathcal{L}_{\pi N} \longrightarrow \mathcal{H}_{\pi N} =$ 

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projectors

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can not solve (infinite-dimensional eq.)

- **Decouple pions via a suitable UT:** $\tilde{H} \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$

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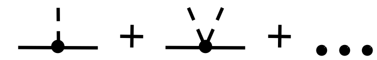
- **Canonical transformation & quantization:** $\mathcal{L}_{\pi N} \longrightarrow \mathcal{H}_{\pi N} =$ 

nucleonic states $|N\rangle, |NN\rangle, \ldots$

EOM: $$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$ ⟵ **can not solve (infinite-dimensional eq.)**

projectors

states with mesons $|N\pi\rangle, |N\pi\pi\rangle, \ldots$

- **Decouple pions via a suitable UT:** $$\tilde{H} \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$$

(Minimal) ansatz: $$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}, \quad A = \lambda A \eta$$

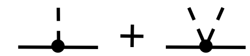
Okubo '54

Require: $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0 \longrightarrow \boxed{\lambda \left(H - [A, H] - AHA \right) \eta = 0}$

The decoupling equation is solved perturbatively (chiral expansion)

Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

- **Canonical transformation & quantization:** $\mathcal{L}_{\pi N} \longrightarrow \mathcal{H}_{\pi N} =$  $+ \ldots$

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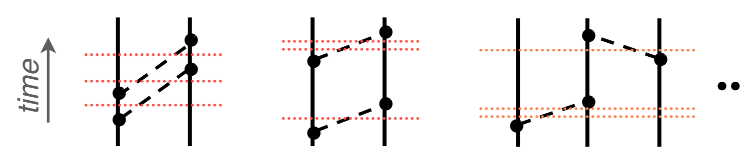
E.g., for the 2-pion exchange $\propto g_A^4$ one finds:

$$V^{(2)} = \eta\Big[- H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} + \frac{1}{2} H^{(1)} \frac{\lambda}{E_\pi^2} H^{(1)} \eta H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} + \frac{1}{2} H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \eta H^{(1)} \frac{\lambda}{E_\pi^2} H^{(1)} \Big] \eta$$



EE, e-Print: 1001.3229 [nucl-th]

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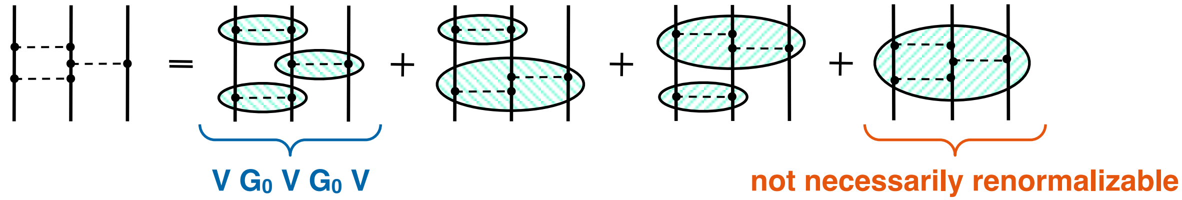

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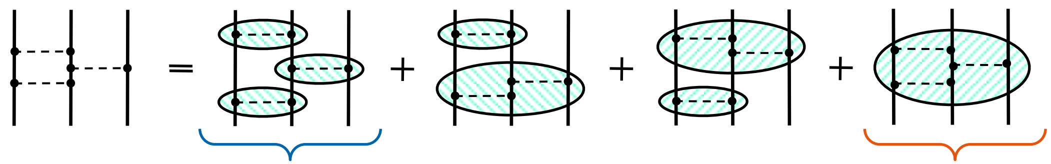
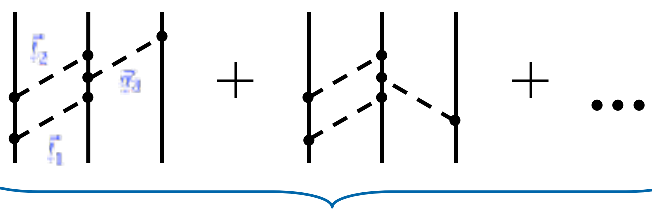


$V\, G_0\, V\, G_0\, V$

not necessarily renormalizable

Indeed, explicit calculations of e.g. the 3NF $\propto g_A^6$ yield:



time-ordered-like diagrams from the method of UT

$$V = \ldots = \int d^3 l_1\, d^3 l_2\, \delta(\vec{l}_1 - \vec{l}_2 - \vec{q}_1) \left[\ldots \right]$$

$$\times \left[2\frac{\omega_1^2 + \omega_2^2}{\omega_1^4\, \omega_2^4\, \omega_3^2} + \frac{8}{\omega_1^2\, \omega_2^2\, \omega_3^4} - \frac{\omega_1 + \omega_2}{\omega_1^3\, \omega_2^3\, \omega_3^3} - \frac{2}{\omega_1^4\, \omega_2^2\, \omega_3\, (\omega_1 + \omega_3)} - \frac{2}{\omega_1^2\, \omega_2^4\, \omega_3\, (\omega_2 + \omega_3)} \right]$$

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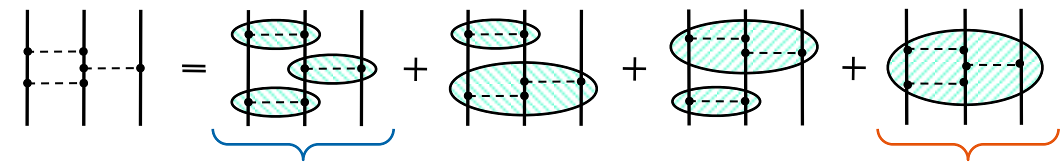

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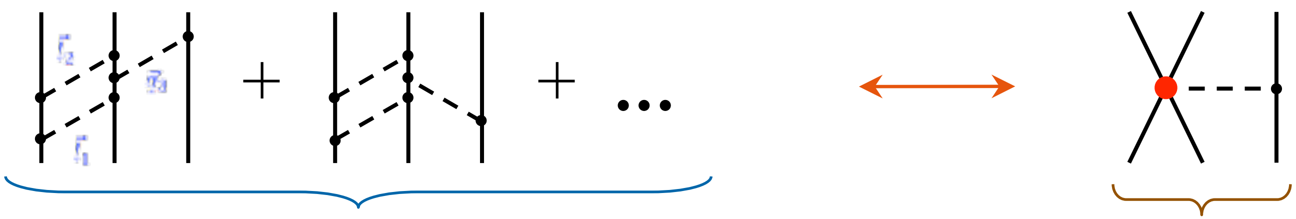

time-ordered-like diagrams from the method of UT

$\propto 1/\omega_3^2$

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$\longrightarrow$ cannot renormalize the potential !

Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, ...

Solution [EE, PLB 639 (2006) 654]: Nuclear potentials are not uniquely defined. Starting from N^{3}LO, one can construct additional UTs in Fock space beyond the (minimal) Okubo Ansatz.

The UTs relevant for the N^{3}LO terms $\propto g_A^6$ are $U = e^{\alpha_1 S_1 + \alpha_2 S_2}$, with the generators

$$S_1 = \eta \left[H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \eta H^{(1)} \frac{\lambda}{E_\pi^3} H^{(1)} - \mathrm{h.\,c.} \right] \eta, \qquad S_2 = \eta \left[H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \frac{\lambda}{E_\pi^2} H^{(1)} - \mathrm{h.\,c.} \right] \eta$$

They induce additional contributions in the Hamiltonian starting from N^{3}LO

$$\delta V^{(4)} = [(H_{\mathrm{kin}} + V^{(0)}),\ \alpha_1 S_1 + \alpha_2 S_2] = -\alpha_1 H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \eta H^{(1)} \frac{\lambda}{E_\pi} H^{(1)} \eta H^{(1)} \frac{\lambda}{E_\pi^3} H^{(1)} + \ldots$$

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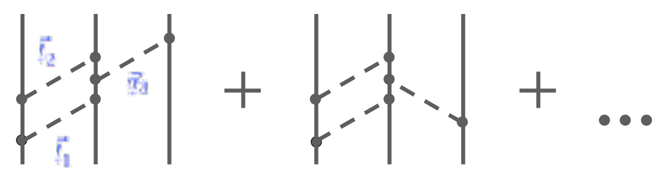
Choose α_1, α_2 such that the 1π-exchange factorizes out (i.e. the 3NF is renormalizable):



$\propto \dfrac{1}{\omega_3^2}$

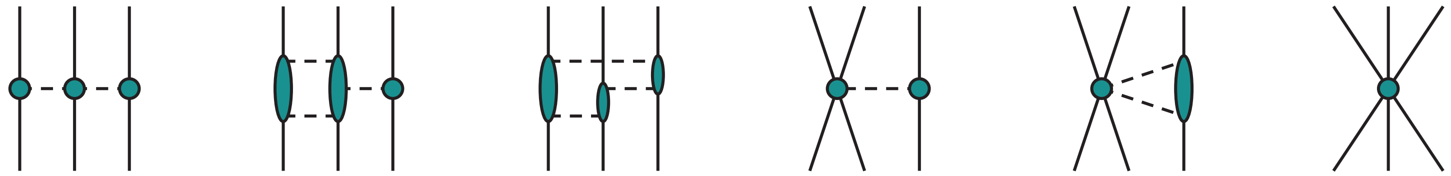
time-ordered-like diagrams from the method of UT

So far, it was **always** possible to tune the phases of the additional UTs in such a way that nuclear potentials and currents remain finite (using DR).

An example:
chiral expansion of the 3NF

Up to N^{4}LO (Q^5), the 3NF receives contributions from 6 topologies:

(tree-level diagrams start contributing from N^{2}LO, one-loop graphs from N^{3}LO)



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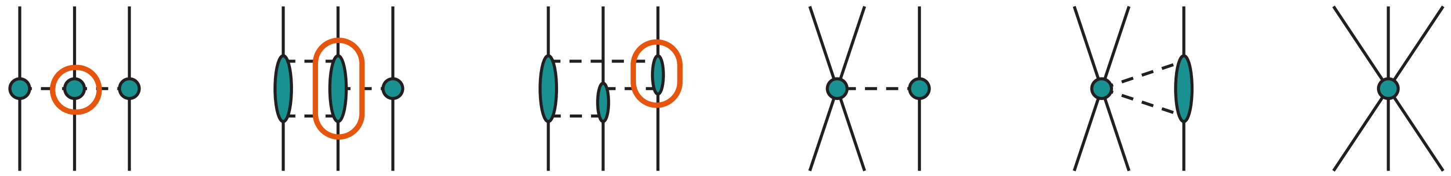
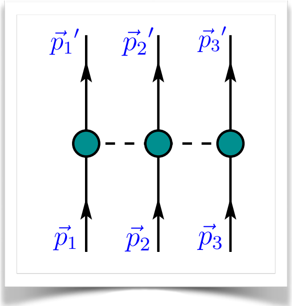


The large-distance behavior is completely predicted in a parameter-free way by the chiral symmetry of QCD + exp. information on πN system

The longest-range 3NF

The TPE 3NF has the form (modulo 1/m-terms):

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$



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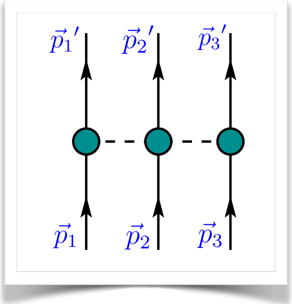

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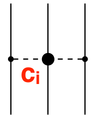
- N^{2}LO [Q^3]:
  van Kolck '94



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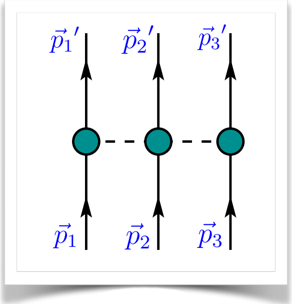

- N^2LO $[Q^3]$: $$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4} \Big((2c_3 - 4c_1) M_\pi^2 + c_3 q_2^2 \Big) , \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

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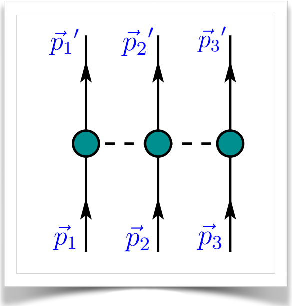

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  van Kolck '94

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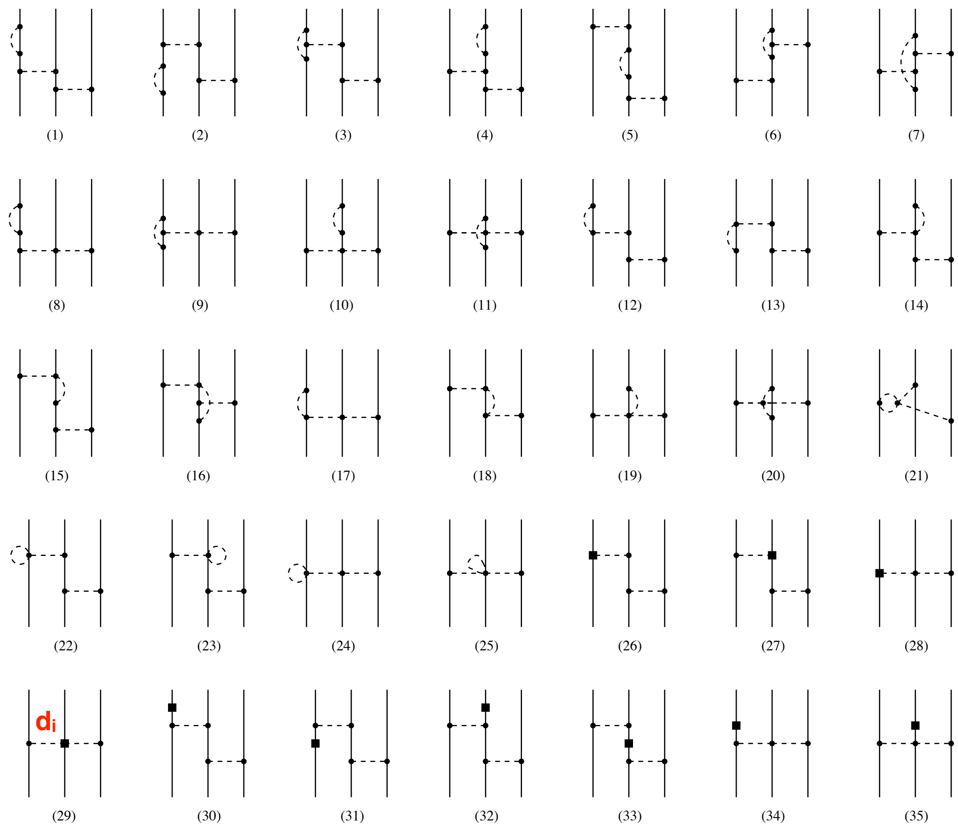

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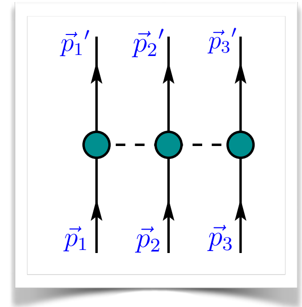

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$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2 + 1\right) M_\pi^3 + 2\left(g_A^2 + 1\right) M_\pi q_2^2\Big],$$

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Ishikawa, Robilotta '07
Bernard, EE, Krebs, Meißner '08

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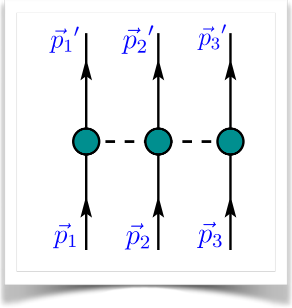

- N^{2}LO [Q^3]:
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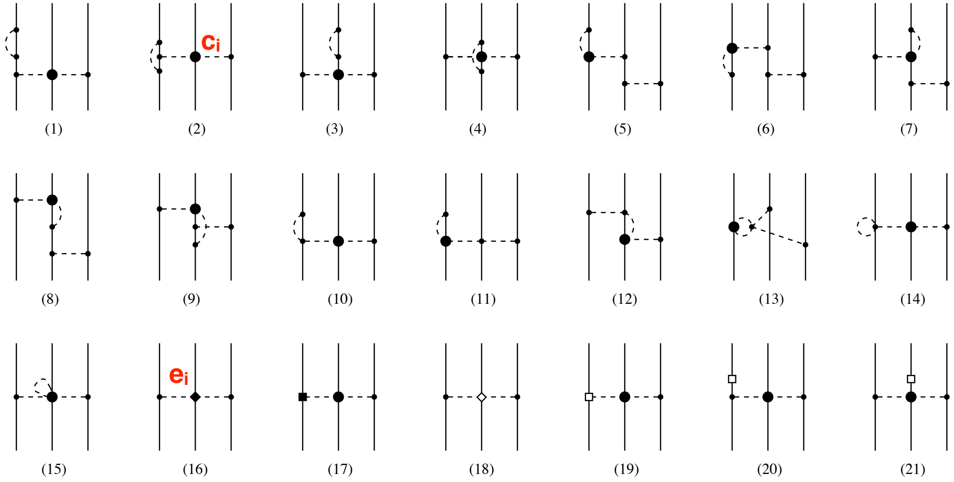
- N^{4}LO [Q^5]:
  Krebs, Gasparyan, EE '12



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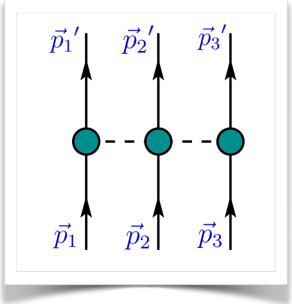

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2] \, [q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

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Ishikawa, Robilotta '07
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  Krebs, Gasparyan, EE '12

$$\begin{aligned}
\mathcal{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2 (F_\pi^2 \left(2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3\right) \\
&+ g_A(144c_1 - 53c_2 - 90c_3)) + M_\pi^4\left(F_\pi^2\left(4608\pi^2 \bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A(2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\right)\right. \\
&+ \left. g_A\left(72\left(64\pi^2 \bar{l}_3 + 1\right)c_1 - 24c_2 - 36c_3\right)\right) + q_2^4\left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A(5c_2 + 18c_3)\right)\Big] \\
&- \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2)\left(M_\pi^2 + 2q_2^2\right)\left(4M_\pi^2(6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\right)\,, \\
\mathcal{B}^{(5)}(q_2) &= -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2\left(F_\pi^2\left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A(2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})\right) + 108 g_A^3 c_4 + 24 g_A c_4\right) \\
&+ q_2^2\left(5g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A\right)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2)\left(4M_\pi^2 + q_2^2\right)
\end{aligned}$$

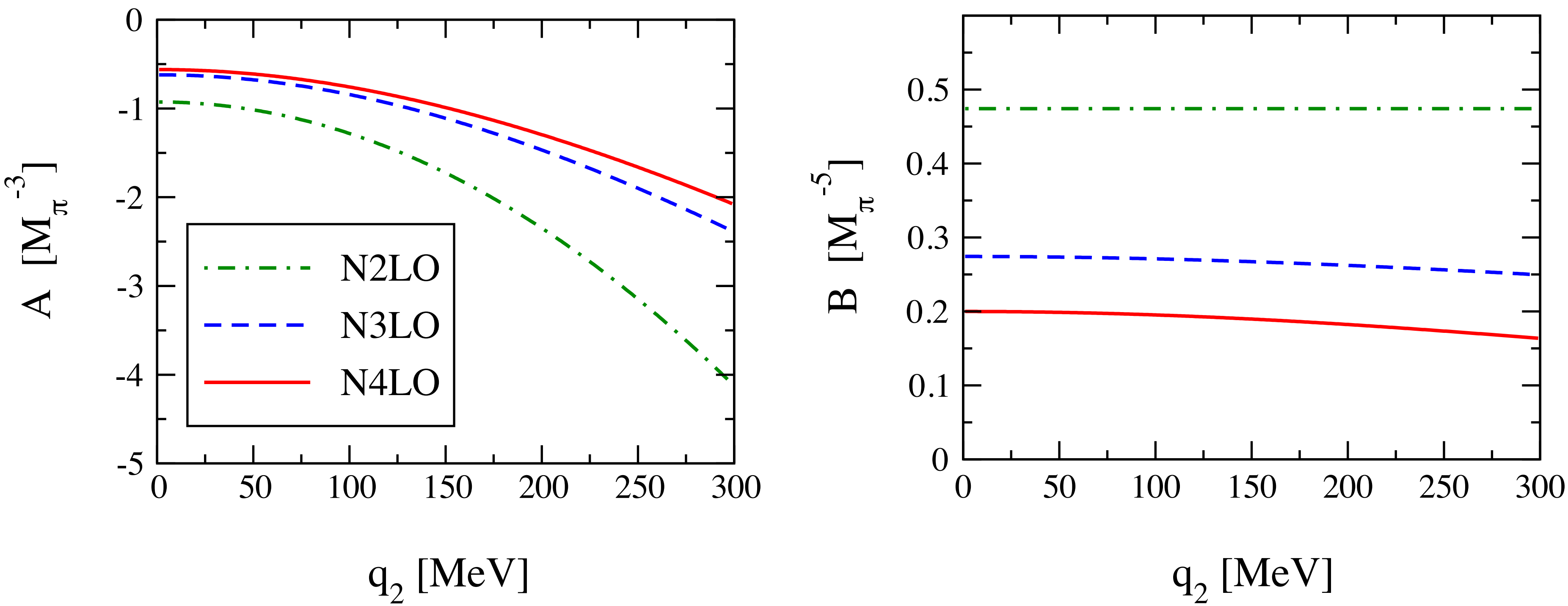
$$L(q) = \frac{\sqrt{4M_\pi^2 + q^2}}{q} \ln \frac{\sqrt{4M_\pi^2 + q^2} + q}{2M_\pi}$$

All LECs are known from pion-nucleon scattering!

The longest-range 3NF

Krebs, Gasparyan, EE ’12

Chiral expansion of the structure functions $A(q_2)$, $B(q_2)$

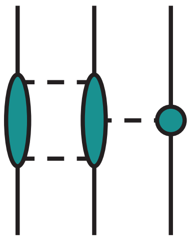


— good convergence for the longest-range 3NF

— higher-order corrections tend to weaken the long-range 3NF

Intermediate range: 2π-1π exchange

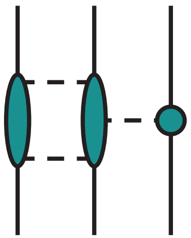
The chiral expansion starts at N3LO



$$
\begin{aligned}
V_{2\pi\text{-}1\pi} = \frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\Big[& \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\ [\vec{\sigma}_2\cdot\vec{q}_1\ \vec{q}_1\cdot\vec{q}_3\ F_1(q_1)+\vec{\sigma}_2\cdot\vec{q}_1\ F_2(q_1)+\vec{\sigma}_2\cdot\vec{q}_3\ F_3(q_1)] \\
& +\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\ [\vec{\sigma}_1\cdot\vec{q}_1\ \vec{q}_1\cdot\vec{q}_3\ F_4(q_1)+\vec{\sigma}_1\cdot\vec{q}_3\ F_5(q_1)+\vec{\sigma}_2\cdot\vec{q}_1\ \vec{q}_1\cdot\vec{q}_3\ F_6(q_1) \\
& +\vec{\sigma}_2\cdot\vec{q}_1\ F_7(q_1)+\vec{\sigma}_2\cdot\vec{q}_3\ \vec{q}_1\cdot\vec{q}_3\ F_8(q_1)+\vec{\sigma}_2\cdot\vec{q}_3\ F_9(q_1)] \\
& +\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,[\vec{\sigma}_1\times\vec{\sigma}_2\cdot\vec{q}_1\ (\vec{q}_1\cdot\vec{q}_3\ F_{10}(q_1)+F_{11}(q_1))+\vec{q}_1\times\vec{q}_3\cdot\vec{\sigma}_1\ \vec{q}_1\cdot\vec{\sigma}_2\ F_{12}(q_1)]\Big]
\end{aligned}
$$

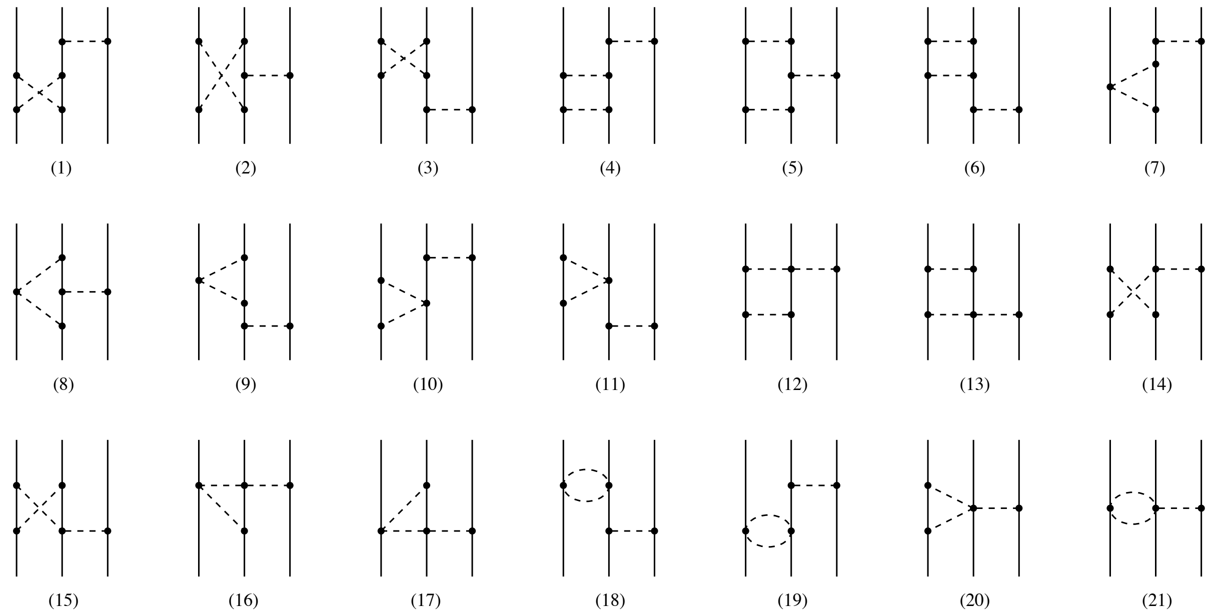
Intermediate range: 2π-1π exchange

The chiral expansion starts at N3LO



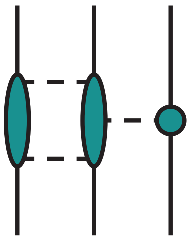
$$
\begin{aligned}
V_{2\pi\text{-}1\pi} \;=\; \frac{\vec\sigma_3\cdot\vec q_3}{q_3^2+M_\pi^2}\Big[& \boldsymbol\tau_1\cdot\boldsymbol\tau_3\;[\vec\sigma_2\cdot\vec q_1\;\vec q_1\cdot\vec q_3\;F_1(q_1)+\vec\sigma_2\cdot\vec q_1\;F_2(q_1)+\vec\sigma_2\cdot\vec q_3\;F_3(q_1)] \\
& +\boldsymbol\tau_2\cdot\boldsymbol\tau_3\;[\vec\sigma_1\cdot\vec q_1\;\vec q_1\cdot\vec q_3\;F_4(q_1)+\vec\sigma_1\cdot\vec q_3\;F_5(q_1)+\vec\sigma_2\cdot\vec q_1\;\vec q_1\cdot\vec q_3\;F_6(q_1) \\
& +\vec\sigma_2\cdot\vec q_1\;F_7(q_1)+\vec\sigma_2\cdot\vec q_3\;\vec q_1\cdot\vec q_3\;F_8(q_1)+\vec\sigma_2\cdot\vec q_3\;F_9(q_1)] \\
& +\boldsymbol\tau_1\times\boldsymbol\tau_2\cdot\boldsymbol\tau_3\,[\vec\sigma_1\times\vec\sigma_2\cdot\vec q_1\;(\vec q_1\cdot\vec q_3\;F_{10}(q_1)+F_{11}(q_1))+\vec q_1\times\vec q_3\cdot\vec\sigma_1\;\vec q_1\cdot\vec\sigma_2\;F_{12}(q_1)]\Big]
\end{aligned}
$$

- N3LO [Q^4], **Bernard, EE, Krebs, Meißner ’08**



Intermediate range: 2π-1π exchange

The chiral expansion starts at N³LO



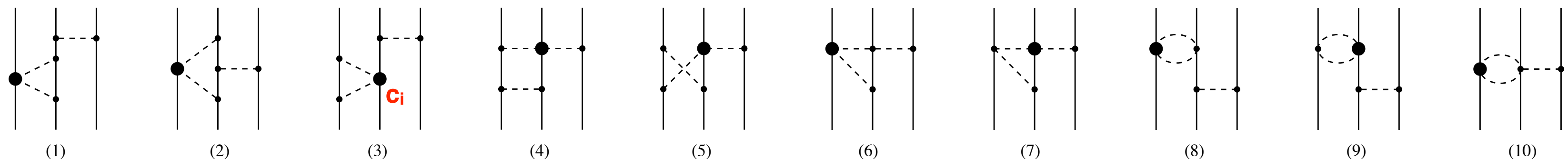
$$
\begin{aligned}
V_{2\pi\text{-}1\pi} = \frac{\vec\sigma_3\cdot\vec q_3}{q_3^2+M_\pi^2}\Big[& \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\;[\vec\sigma_2\cdot\vec q_1\;\vec q_1\cdot\vec q_3\;F_1(q_1)+\vec\sigma_2\cdot\vec q_1\;F_2(q_1)+\vec\sigma_2\cdot\vec q_3\;F_3(q_1)]\\
&+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\;[\vec\sigma_1\cdot\vec q_1\;\vec q_1\cdot\vec q_3\;F_4(q_1)+\vec\sigma_1\cdot\vec q_3\;F_5(q_1)+\vec\sigma_2\cdot\vec q_1\;\vec q_1\cdot\vec q_3\;F_6(q_1)\\
&+\vec\sigma_2\cdot\vec q_1\;F_7(q_1)+\vec\sigma_2\cdot\vec q_3\;\vec q_1\cdot\vec q_3\;F_8(q_1)+\vec\sigma_2\cdot\vec q_3\;F_9(q_1)]\\
&+\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,[\vec\sigma_1\times\vec\sigma_2\cdot\vec q_1\;(\vec q_1\cdot\vec q_3\;F_{10}(q_1)+F_{11}(q_1))+\vec q_1\times\vec q_3\cdot\vec\sigma_1\;\vec q_1\cdot\vec\sigma_2\;F_{12}(q_1)]\Big]
\end{aligned}
$$

- N³LO [Q⁴], **Bernard, EE, Krebs, Meißner '08**

$$
\begin{aligned}
F_1^{(4)}(q_1) &= \frac{g_A^4}{256\pi F_\pi^6 q_1^2}\Big[A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(g_A^2+1\right)q_1^2\right)-\frac{M_\pi}{4M_\pi^2+q_1^2}\left(\left(8g_A^2-4\right)M_\pi^2+\left(3g_A^2-1\right)q_1^2\right)\Big]\\
F_2^{(4)}(q_1) &= \frac{g_A^4}{128\pi F_\pi^6}A(q_1)\left(2M_\pi^2+q_1^2\right)\\
F_3^{(4)}(q_1) &= -\frac{g_A^4}{256\pi F_\pi^6}A(q_1)\left(\left(8g_A^2-4\right)M_\pi^2+\left(3\,g_A^2-1\right)q_1^2\right)\\
F_4^{(4)}(q_1) &= -\frac{F_5^{(4)}(q_1)}{q_1^2}=-\frac{g_A^6}{128\pi F_\pi^6}A(q_1)\\
F_6^{(4)}(q_1) &= F_8^{(4)}(q_1)=F_9^{(4)}(q_1)=F_{10}^{(4)}(q_1)=F_{12}^{(4)}(q_1)=0\\
F_7^{(4)}(q_1) &= \frac{g_A^4}{128\pi F_\pi^6}A(q_1)\left(2M_\pi^2+q_1^2\right)\\
F_{11}^{(4)}(q_1) &= -\frac{g_A^4}{512\pi F_\pi^6}A(q_1)\left(4M_\pi^2+q_1^2\right)
\end{aligned}
$$

Intermediate range: 2π-1π exchange

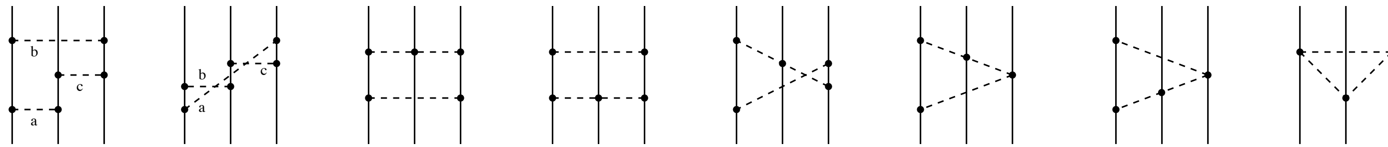
- N[4]LO [Q[5]], **Krebs, EE, Gasparyan ’13**



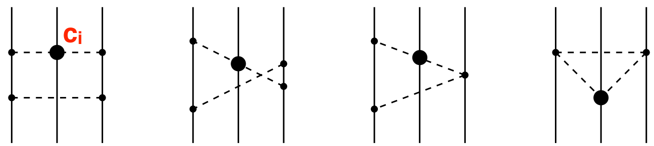
$$
\begin{aligned}
F_1^{(5)} &= -\frac{g_A^2 c_4}{96\pi^2 F_\pi^6 q_1^2 \left(4M_\pi^2+q_1^2\right)} L(q_1) \left(8\left(4g_A^2-1\right) M_\pi^4 + 2\left(5g_A^2+1\right) M_\pi^2 q_1^2 - \left(g_A^2-1\right) q_1^4\right) - \frac{\left(1-4g_A^2\right) g_A^2 c_4 M_\pi^2}{48\pi^2 F_\pi^6 q_1^2} \\
F_2^{(5)} &= F_8^{(5)} = F_{11}^{(5)} = 0 \\
F_3^{(5)} &= -\frac{g_A^2 c_4}{48\pi^2 F_\pi^6 \left(4M_\pi^2+q_1^2\right)} L(q_1) \left(4\left(4g_A^2-1\right) M_\pi^4 + \left(17g_A^2-5\right) M_\pi^2 q_1^2 + \left(4g_A^2-1\right) q_1^4\right) \\
F_4^{(5)} &= -\frac{F_5^{(5)}}{q_1^2} = -\frac{g_A^4 c_4}{16\pi^2 F_\pi^6} L(q_1) \\
F_6^{(5)} &= \frac{g_A^4 M_\pi^2 (6c_1 + c_2 - 3c_3)}{96\pi^2 F_\pi^6 q_1^2} \\
&+ \frac{g_A^4 L(q_1)}{192\pi^2 F_\pi^6 q_1^2 \left(4M_\pi^2+q_1^2\right)} \left(-48 c_1 M_\pi^4 + c_2\left(-8M_\pi^4 + 2M_\pi^2 q_1^2 + q_1^4\right) + 12 c_3 M_\pi^2 \left(2M_\pi^2 + q_1^2\right)\right) \\
F_7^{(5)} &= -\frac{g_A^2}{192\pi^2 F_\pi^6} L(q_1) \left(24 c_1 M_\pi^2 - c_2\left(4M_\pi^2+q_1^2\right) - 6c_3\left(2M_\pi^2+q_1^2\right)\right) \\
F_9^{(5)} &= -\frac{g_A^4 L(q_1)}{128\pi^2 F_\pi^6 \left(4M_\pi^2+q_1^2\right)} \Big[-32 c_1 M_\pi^2 \left(3M_\pi^2+q_1^2\right) + c_2\left(16M_\pi^4 + 16M_\pi^2 q_1^2 + 3q_1^4\right) \\
&+ c_3\left(80M_\pi^4 + 68 M_\pi^2 q_1^2 + 13 q_1^4\right) \Big] \\
F_{10}^{(5)} &= F_{12}^{(5)} = \frac{g_A^4 c_4 L(q_1)}{64\pi^2 F_\pi^6}
\end{aligned}
$$

Intermediate range: ring diagrams

- N^{3}LO [Q^4]:
  Bernard, EE, Krebs, Meißner '08

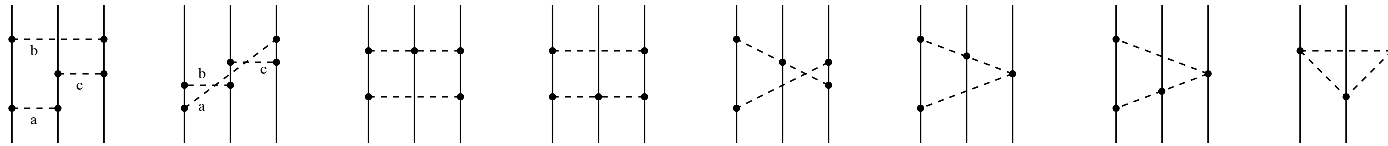


- N^{4}LO [Q^5]:
  Krebs, Gasparyan, EE '13

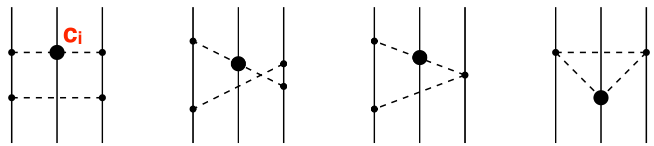


Intermediate range: ring diagrams

- N^{3}LO [Q^4]:
  Bernard, EE, Krebs, Meißner '08



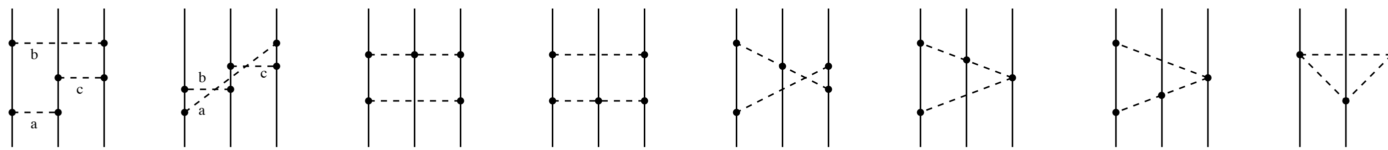
- N^{4}LO [Q^5]:
  Krebs, Gasparyan, EE '13



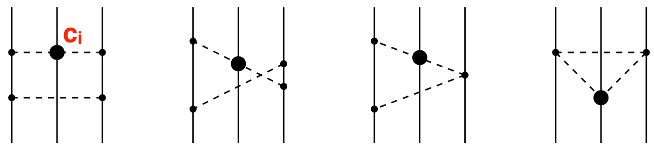
Expressions in momentum space are complicated (3-point function), e.g. at N^{3}LO:

Intermediate range: ring diagrams

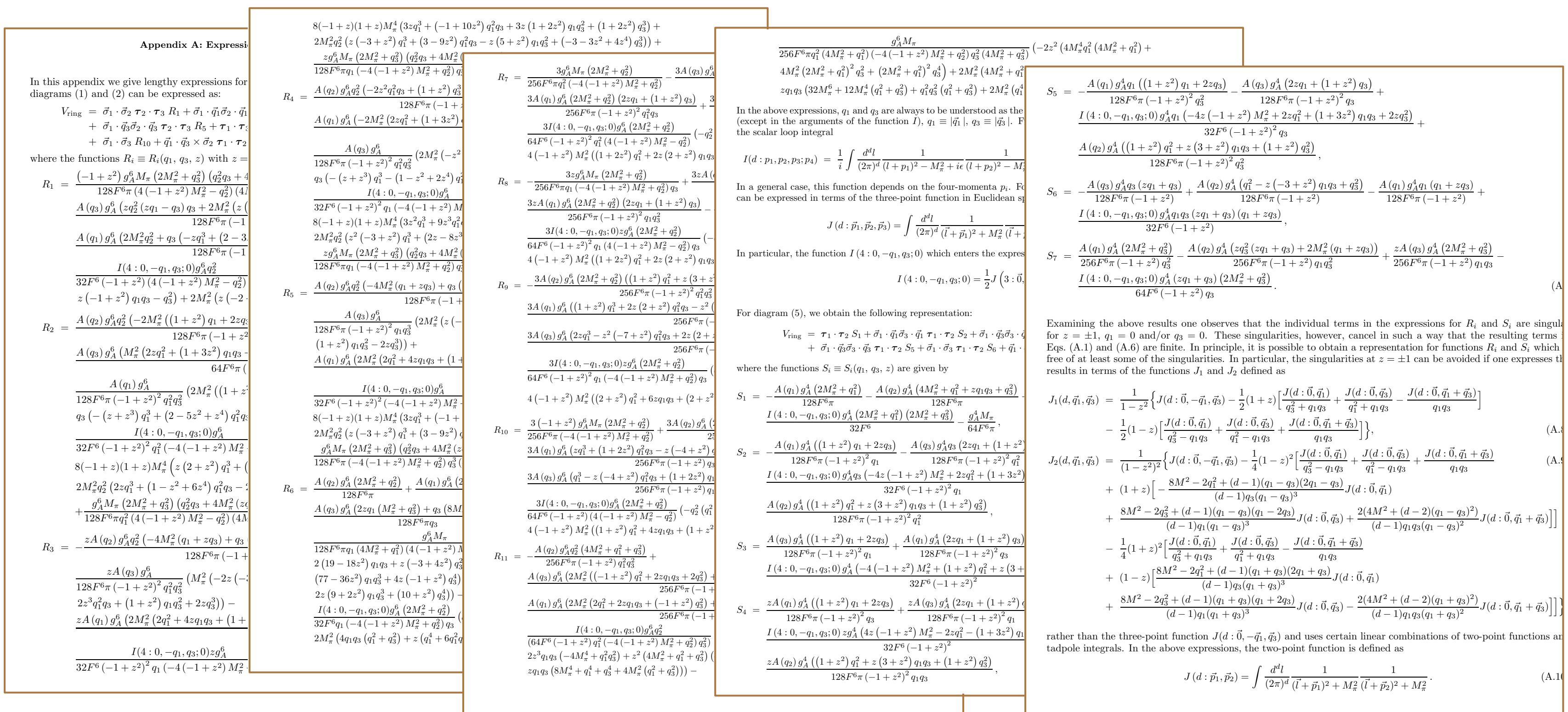
- N³LO [Q⁴]:
  Bernard, EE, Krebs, Meißner '08



- N⁴LO [Q⁵]:
  Krebs, Gasparyan, EE '13

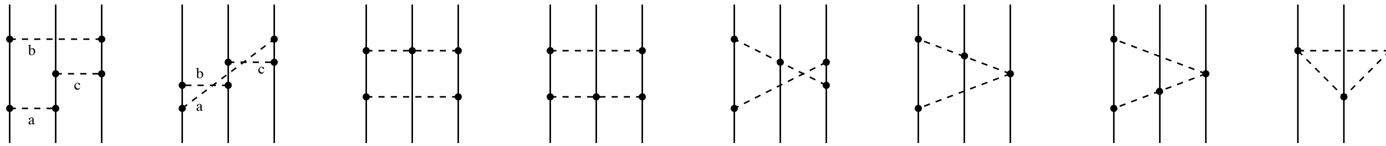


Expressions in momentum space are complicated (3-point function), e.g. at N³LO:

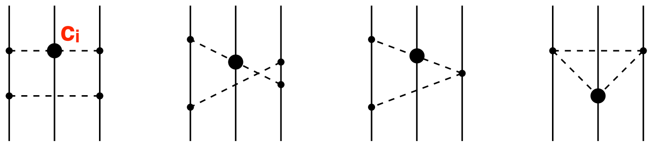


Intermediate range: ring diagrams

- N³LO [Q⁴]:
  Bernard, EE, Krebs, Meißner '08



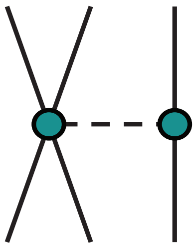
- N⁴LO [Q⁵]:
  Krebs, Gasparyan, EE '13



However, pion exchanges factorize in coordinate space leading to very simple expressions:

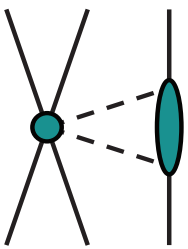
$$
\begin{aligned}
V^{(4)}_{\rm ring}(\vec r_{12}, \vec r_{32}\,) \;=\; & -\frac{g_A^6\, M_\pi^7}{4096\,\pi^3\, F_\pi^6}\Big[-4\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\, \vec\nabla_{23}\times\vec\nabla_{12}\cdot\vec\sigma_2\, \vec\nabla_{23}\times\vec\nabla_{31}\cdot\vec\sigma_3\, \vec\nabla_{31}\cdot\vec\nabla_{12} \\
& \qquad - \; 2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\, \vec\nabla_{23}\cdot\vec\nabla_{31}\, \vec\nabla_{23}\cdot\vec\nabla_{12}\, \vec\nabla_{31}\cdot\vec\nabla_{12} \\
& \qquad + \; \boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\, \vec\nabla_{23}\times\vec\nabla_{12}\cdot\vec\sigma_2\, \vec\nabla_{23}\cdot\vec\nabla_{31}\, \vec\nabla_{31}\cdot\vec\nabla_{12} \\
& \qquad + \; 3\vec\nabla_{31}\times\vec\nabla_{12}\cdot\vec\sigma_1\, \vec\nabla_{23}\times\vec\nabla_{31}\cdot\vec\sigma_3\, \vec\nabla_{23}\cdot\vec\nabla_{12}\Big]\, U_1(x_{23})\, U_2(x_{31})\, U_1(x_{12}) \\
& + \; \frac{g_A^4\, M_\pi^7}{2048\,\pi^3\, F_\pi^6}\Big[2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec\nabla_{23}\cdot\vec\nabla_{31}\, \vec\nabla_{31}\cdot\vec\nabla_{12} - \vec\nabla_{31}\times\vec\nabla_{12}\cdot\vec\sigma_1\, \vec\nabla_{23}\times\vec\nabla_{31}\cdot\vec\sigma_3) \\
& \qquad + \boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\, \vec\nabla_{31}\times\vec\nabla_{12}\cdot\vec\sigma_1\, \vec\nabla_{23}\cdot\vec\nabla_{31}\Big] U_1(x_{23})\, U_1(x_{31})\, U_1(x_{12})
\end{aligned}
$$

Shorter-range contributions



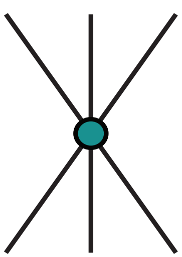
$$V_{1\pi-\mathrm{cont}}^{(3)} = -\frac{g_A D}{8F_\pi^2}\frac{\vec{\sigma}_3\cdot\vec{q}_3}{q_3^2+M_\pi^2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\vec{\sigma}_1\cdot\vec{q}_3 \qquad V_{1\pi-\mathrm{cont}}^{(4)} = 0$$

N[4]LO contribution still to be worked out (several new LEC…)



$$\begin{aligned} V_{2\pi-\mathrm{cont}}^{(4)} &= \frac{g_A^4 C_T}{48\pi F_\pi^4}\Big\{2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec{\sigma}_2\cdot\vec{\sigma}_3)\Big[3M_\pi - \frac{M_\pi^3}{4M_\pi^2+q_1^2} + 2(2M_\pi^2+q_1^2)A(q_1)\Big] \\ &\quad + 9\Big[(\vec{q}_1\cdot\vec{\sigma}_1)(\vec{q}_1\cdot\vec{\sigma}_2) - q_1^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)\Big]A(q_1)\Big\} \\ &\quad - \frac{g_A^2 C_T}{24\pi F_\pi^4}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec{\sigma}_2\cdot\vec{\sigma}_3)\Big[M_\pi + (2M_\pi^2+q_1^2)A(q_1)\Big] \end{aligned}$$

…N[4]LO contribution still to be worked out…



$$V_{\mathrm{cont}}^{(3)} = \frac{1}{2}E\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

9 out of 10 LECs at N[4]LO can be determined in Nd scattering

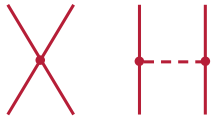
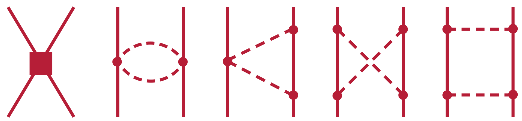
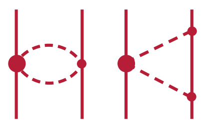
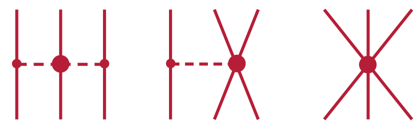
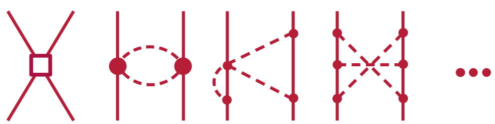
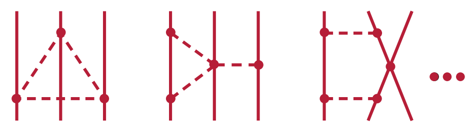
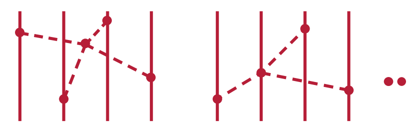
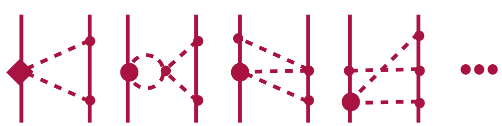
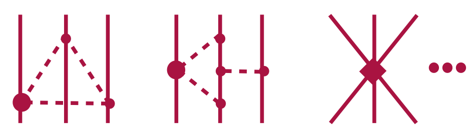
$$\begin{aligned} V_{\mathrm{cont}}^{(5)} &= -E_1 q_1^2 - E_2 q_1^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - E_3 q_1^2\vec{\sigma}_1\cdot\vec{\sigma}_2 - E_4 q_1^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\vec{\sigma}_1\cdot\vec{\sigma}_2 - E_5(3\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2 - q_1^2) \\ &\quad - E_6(3\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2 - q_1^2)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + iE_7\vec{q}_1\times(\vec{k}_1-\vec{k}_2)\cdot(\vec{\sigma}_1+\vec{\sigma}_2) \\ &\quad + iE_8\vec{q}_1\times(\vec{k}_1-\vec{k}_2)\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3 - E_9\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_2\cdot\vec{\sigma}_2 - E_{10}\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_2\cdot\vec{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \end{aligned}$$

Girlanda, Kievsky, Viviani, PRC 84 (2011) 014001

Finally, starting from N[3]LO, one has to account for relativistic corrections (parameter-free).

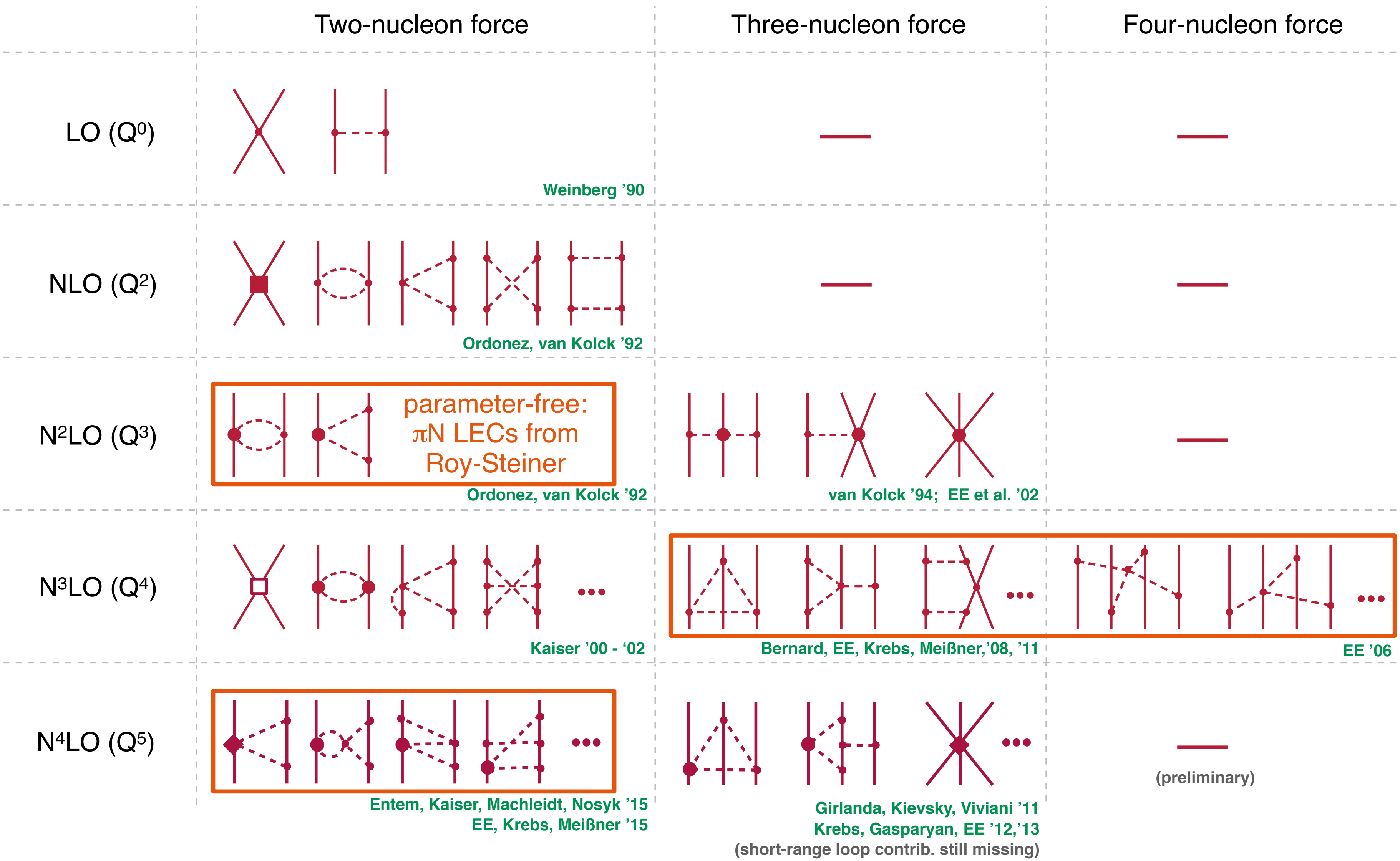
State of the art

Chiral expansion of the nuclear forces [W-counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)	 Ordonez, van Kolck '92	—	—
N^{2}LO (Q^3)	 Ordonez, van Kolck '92	 van Kolck '94; EE et al. '02	—
N^{3}LO (Q^4)	 Kaiser '00 - '02	 Bernard, EE, Krebs, Meißner,'08, '11	 EE '06
N^{4}LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	 Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12,'13 (short-range loop contrib. still missing)	— (preliminary)

— Electroweak and scalar currents worked out to N^{3}LO **Krebs, Kölling, EE, Meißner; Baroni, Pastori, Schiavilla et al.**

Chiral expansion of the nuclear forces [W-counting]



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Chiral EFT: The status

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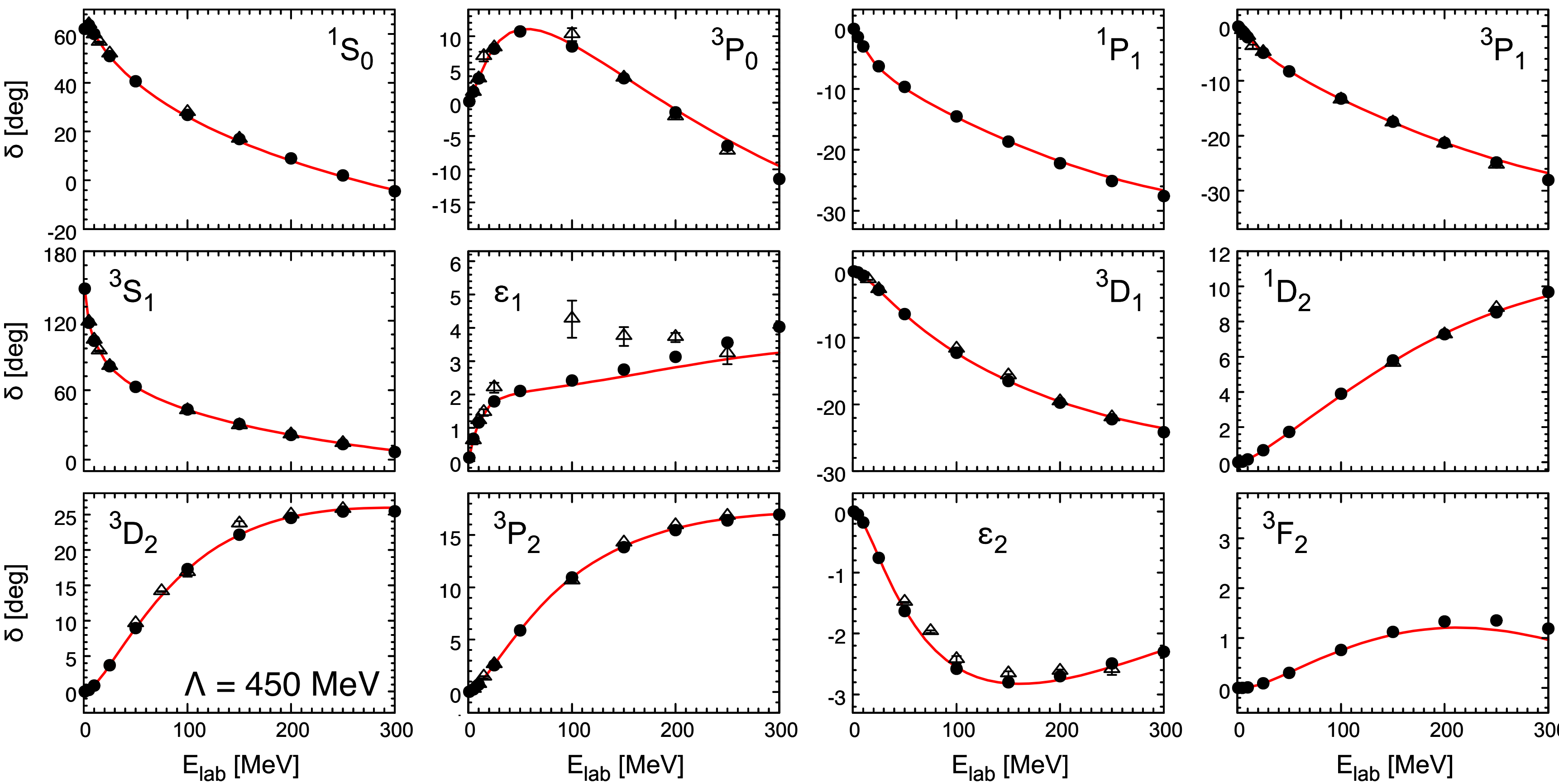
χ^2/datum (pp, $0-300$ MeV)	1380	91	41	3.43	1.00

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88 [Incomplete treatment of IB effects!]



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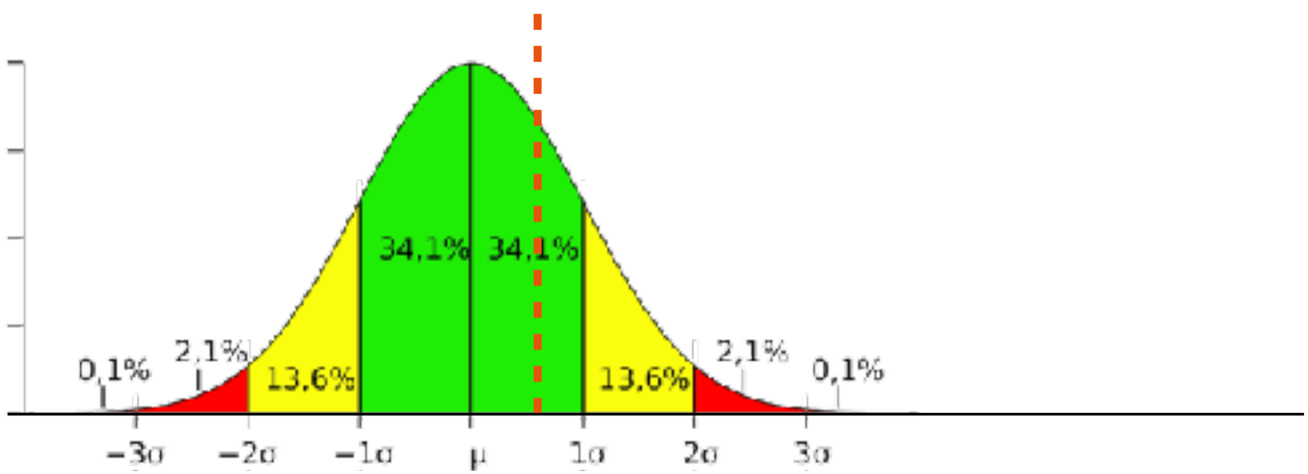

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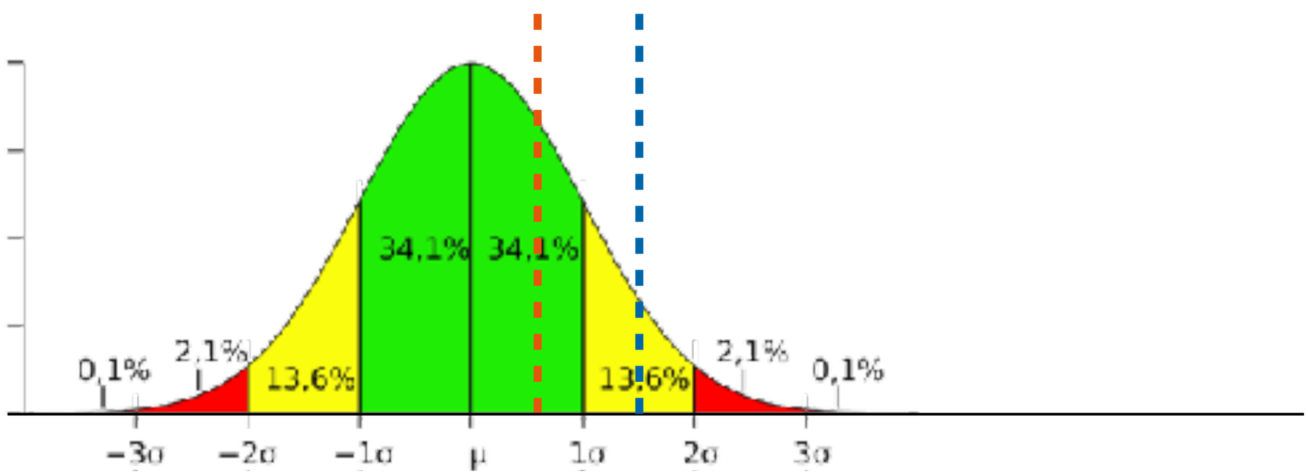

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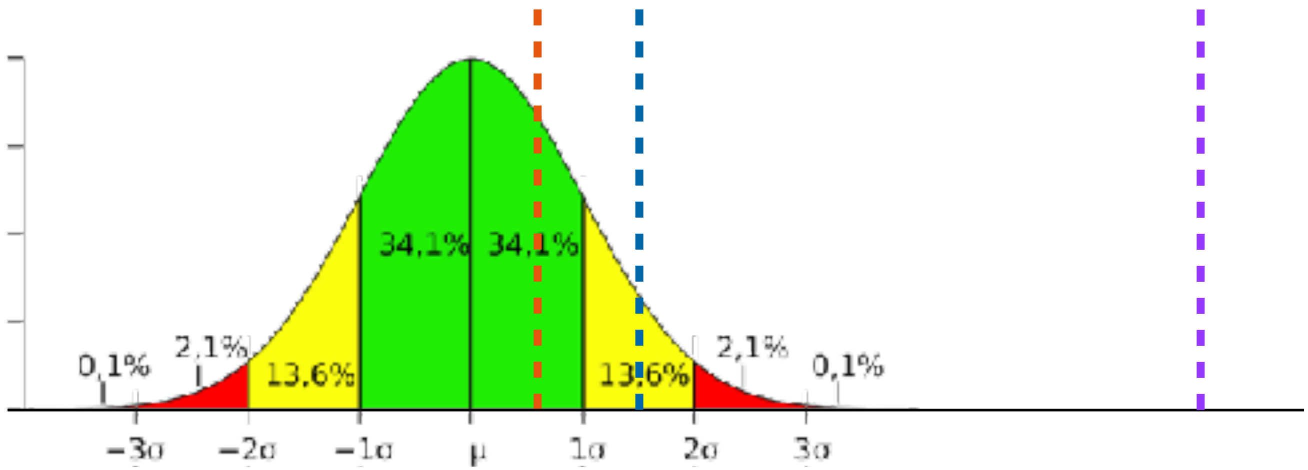
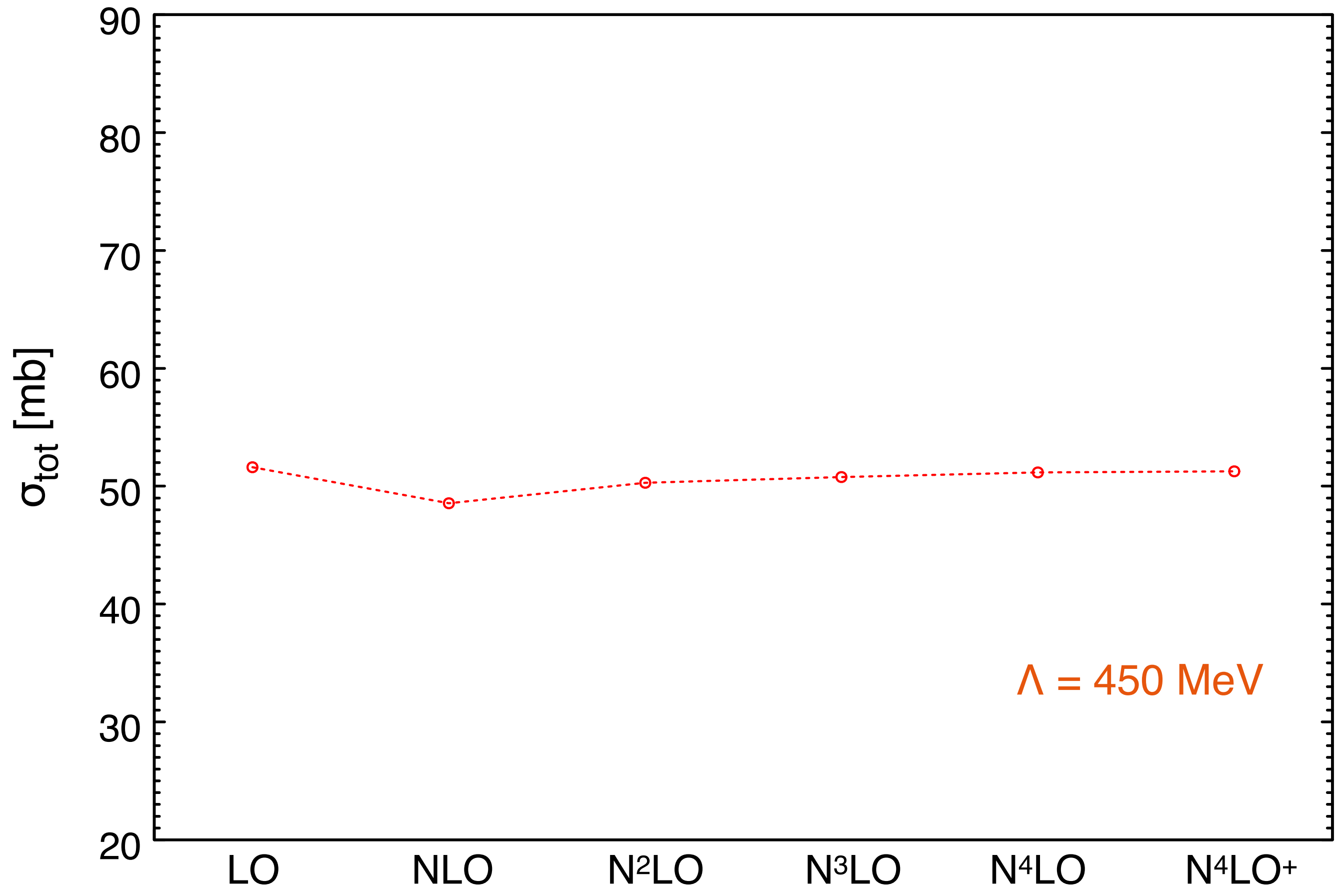


Uncertainty quantification



Neutron-proton total cross section at 150 MeV [Λ = 450 MeV]

$$\sigma_{\rm tot} = 51.4_{\rm LO} - 3.0_{\rm NLO} + 1.7_{\rm N^2LO} + 0.5_{\rm N^3LO} + 0.4_{\rm N^4LO} + 0.1_{\rm N^4LO+}$$

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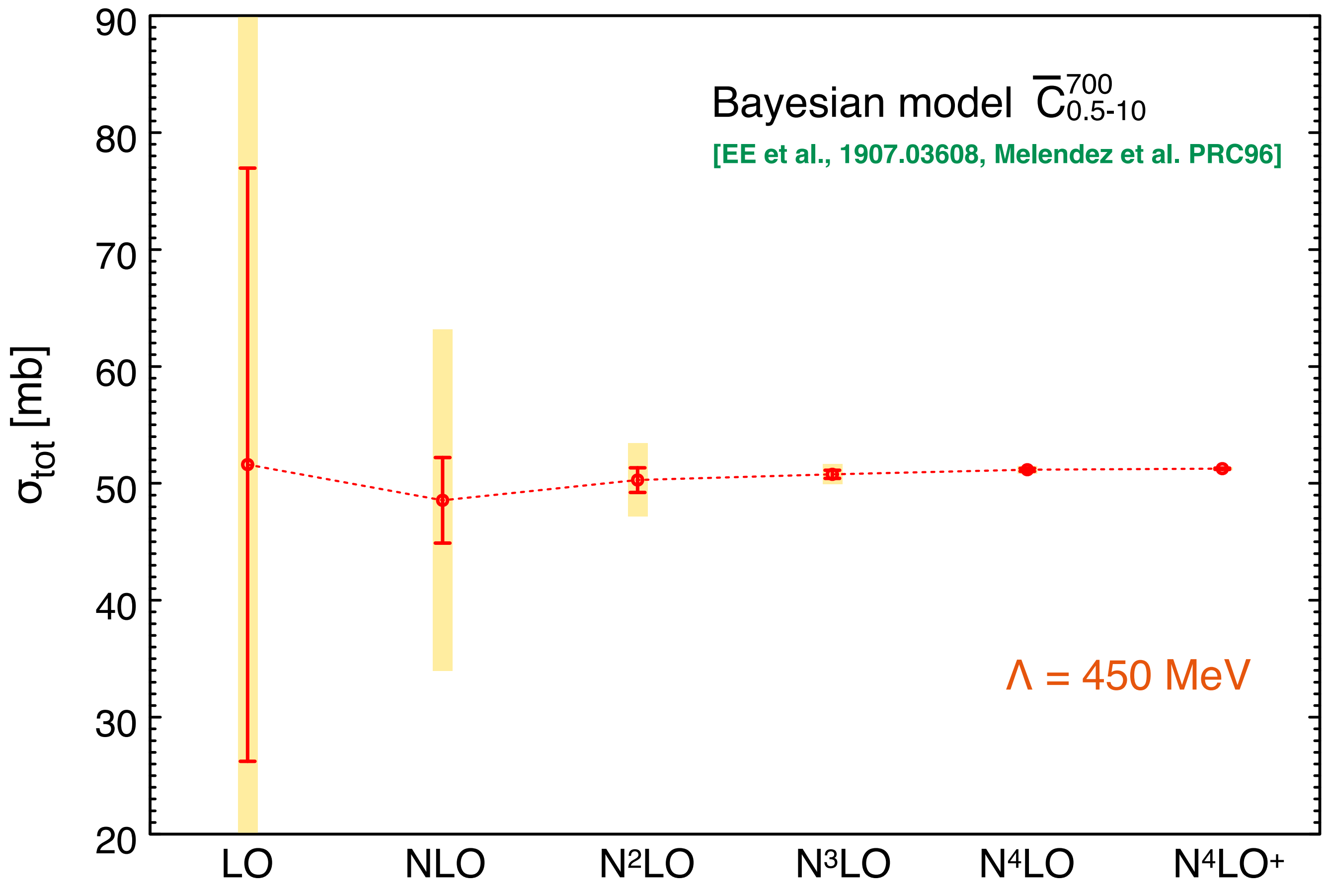

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Lisowski et al. '82

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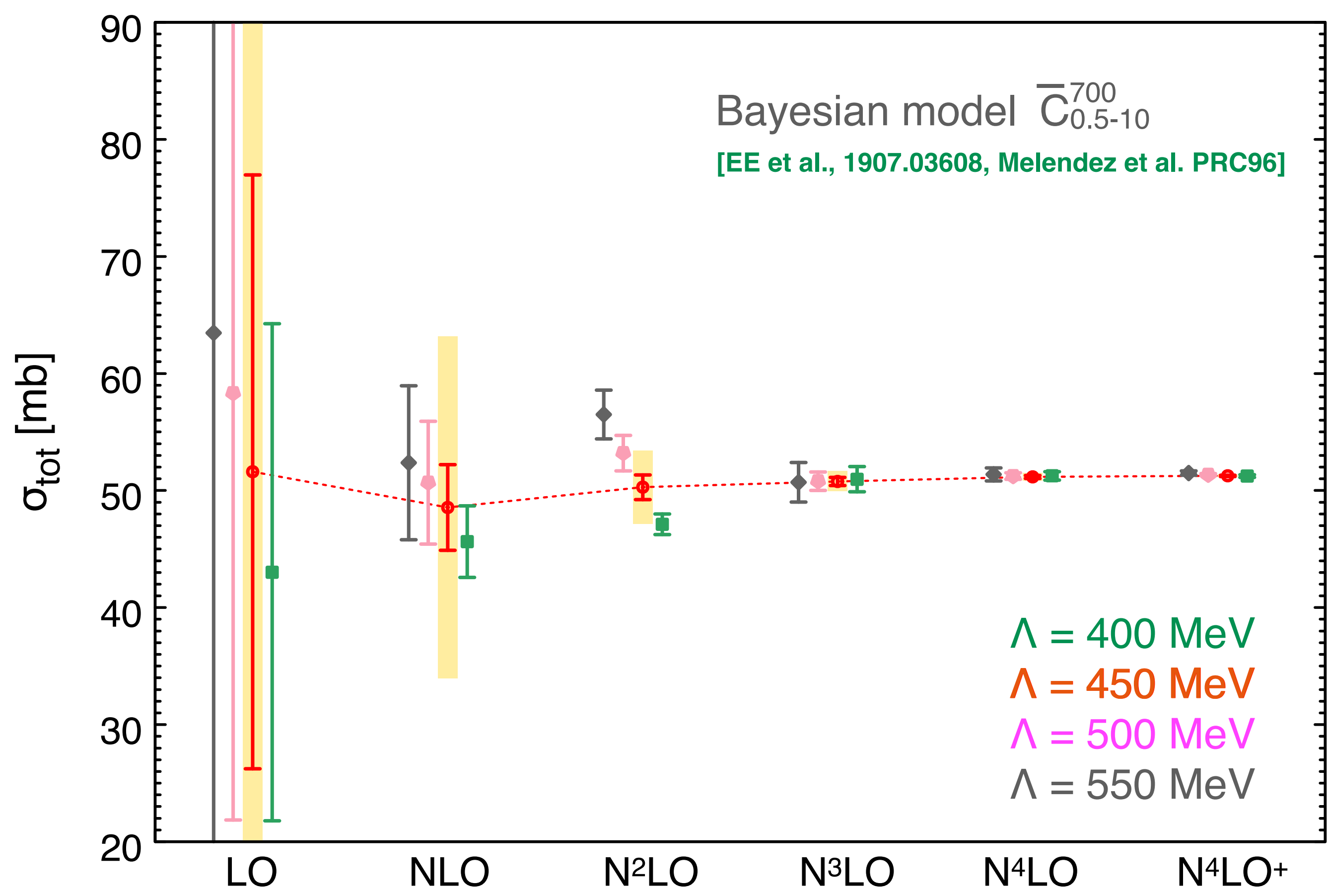

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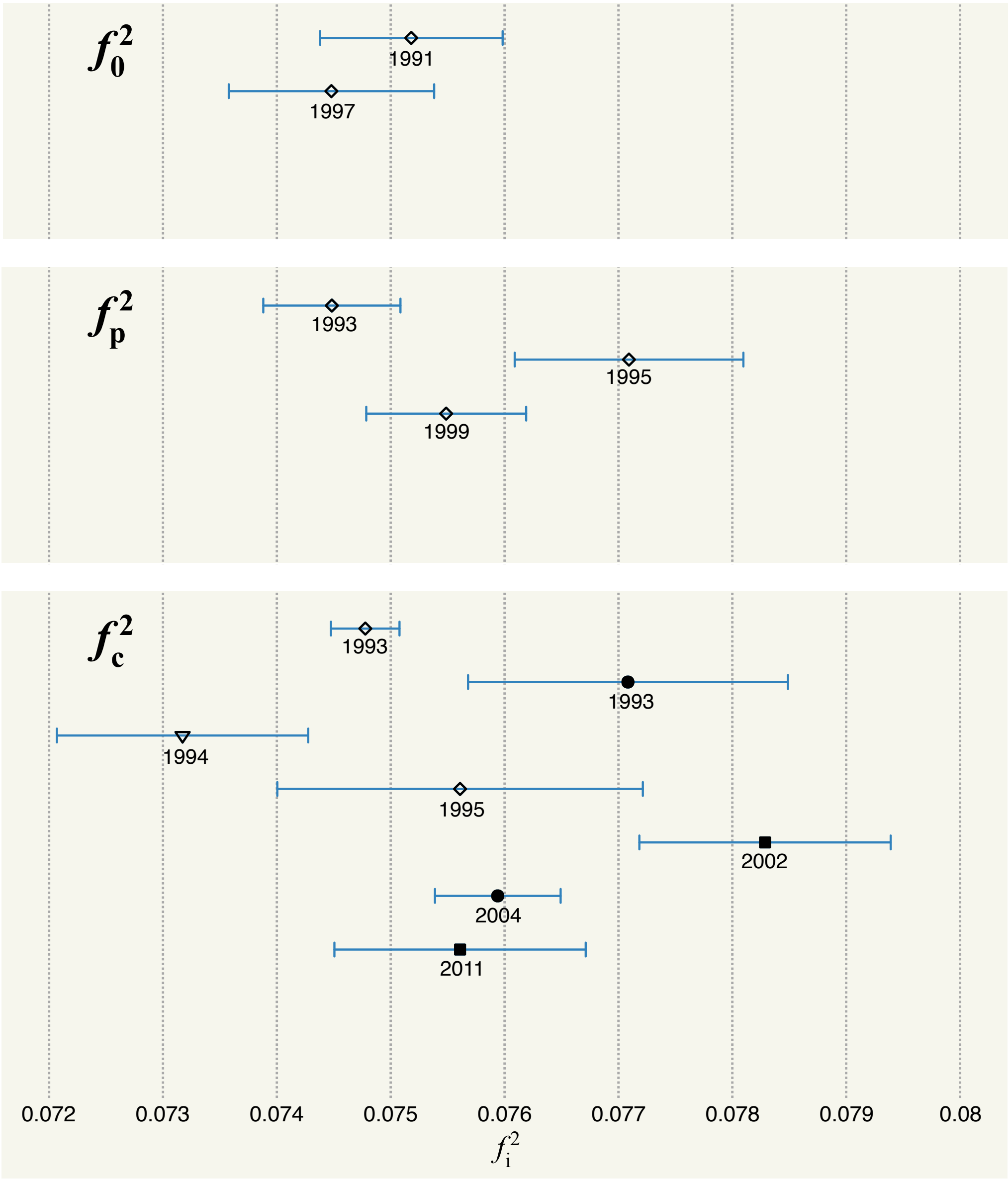
Some recent highlights

Determination of the πN constants

Reinert, Krebs, EE, e-Print: 2006.15360 [nucl-th]



Standard notation:

$$\begin{aligned} f_0^2 &= -f_{\pi^0 nn}\, f_{\pi^0 pp} \\ f_p^2 &= f_{\pi^0 pp}\, f_{\pi^0 pp} \\ 2 f_c^2 &= f_{\pi^\pm pn}\, f_{\pi^\pm pn} \end{aligned}$$

● — fixed-t dispersion relations of πN scattering
Markopoulou-Kalamara, Bugg ’93; Arndt et al. ’04

■ — πN scattering lengths + Goldberger-Miyazawa-Oehme sum rules
Ericson et al. ’02; Baru et al. ’11

▽ — proton-antiproton PWA
Timmermans et al. ’94

◇ — neutron-proton (+ proton-proton) PWA
Klomp et al. ’91; Stoks et al. ’93; Bugg et al. ’95; de Swart et al. ’97; Rentmeester et al. ’99

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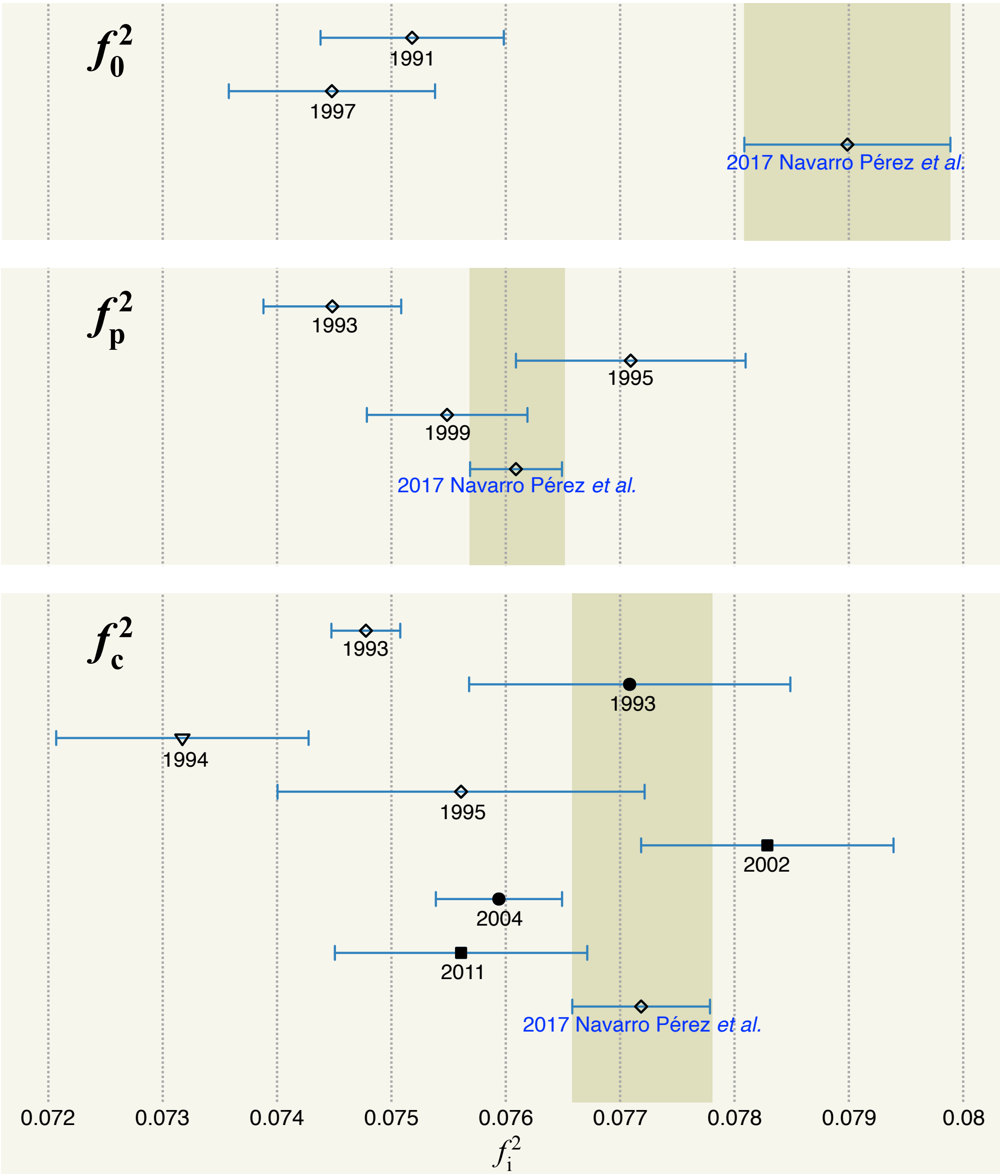

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2017 Granada PWA: evidence for significant charge dependence of the coupling constants:

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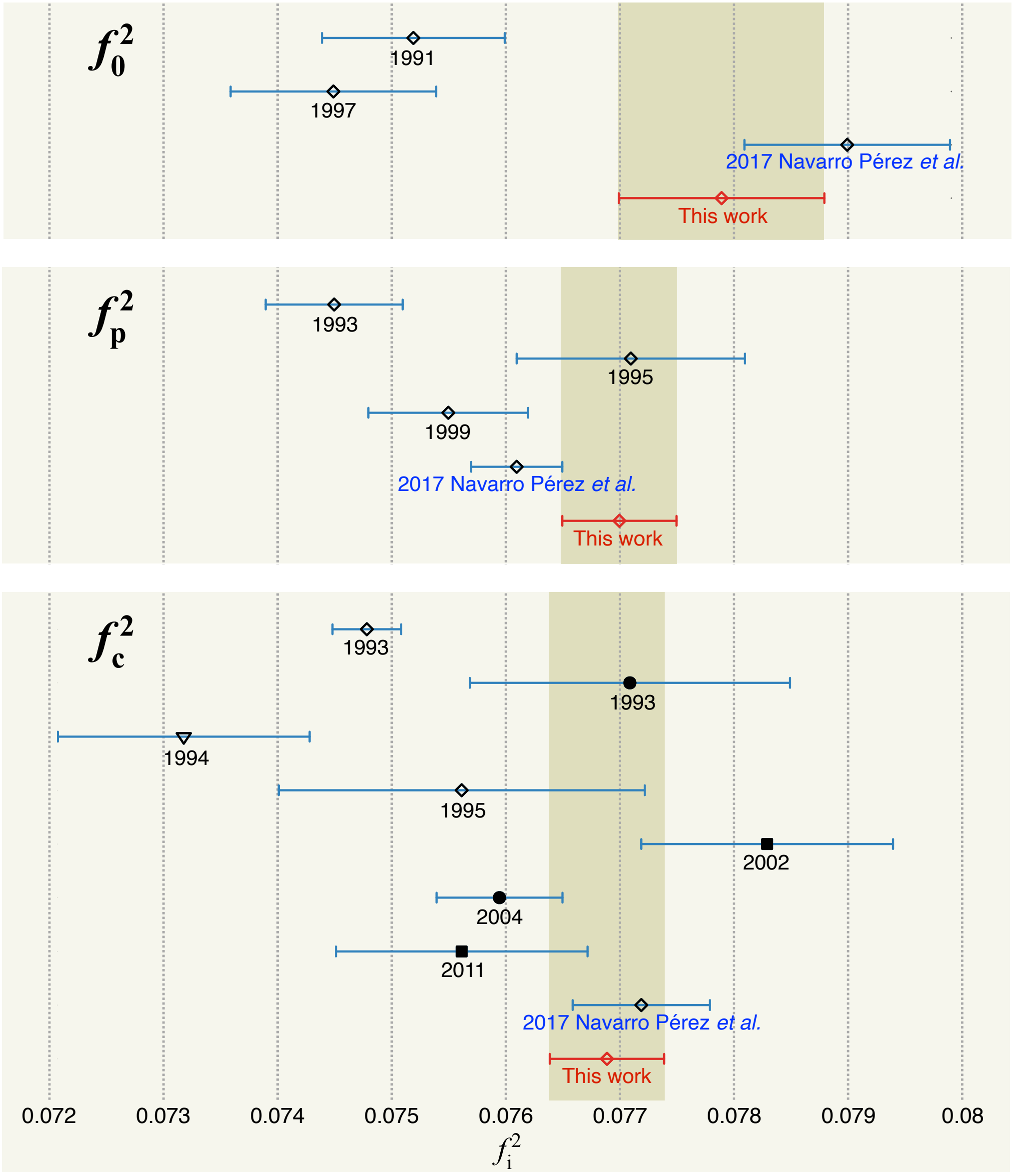

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Our result (χEFT at N⁴LO):

Bayesian determination; statistical **and systematic** uncertainties.

No evidence for charge dependence of the πN coupling constants

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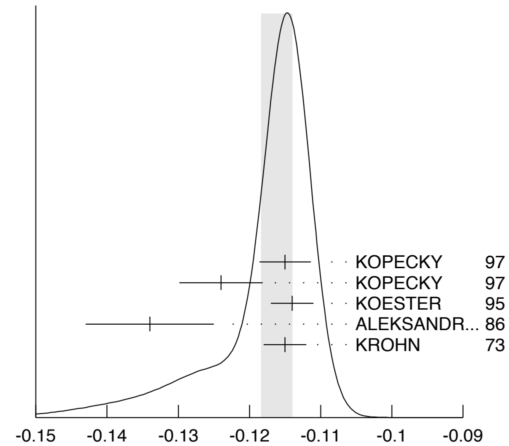
How large is a neutron?

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; e-Print: 2009.08911

While the proton radius puzzle seems settled, what do we know about the neutron radius?

PDG recommended value: $r_n^2 = -0.1161 \pm 0.0022\ \mathrm{fm}^2$

- no neutron targets exist…
- information only from (old) n-scattering experiments on Pb, Bi



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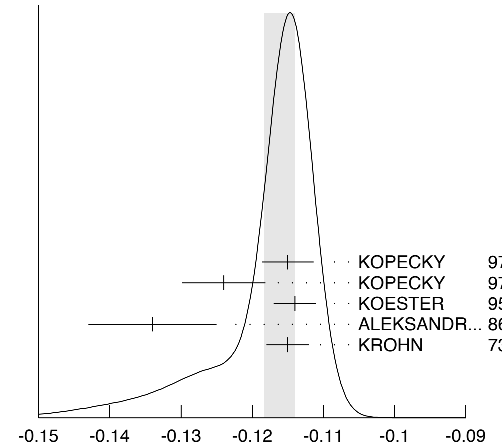

Idea: an accurate calculation of the ^{2}H structure radius

$$r_{\rm str}^2 = r_d^2 - r_p^2 - r_n^2 - \frac{3}{4m_p^2}$$

along with ^{1}H-^{2}H isotope shifts data $r_d^2 - r_p^2 = 3.82070(31) \text{ fm}^2$ can be used to extract r_n^2!

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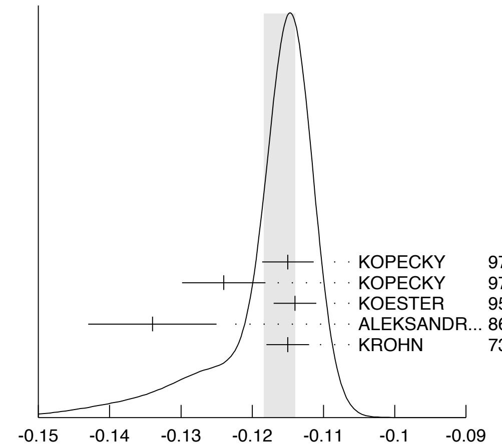

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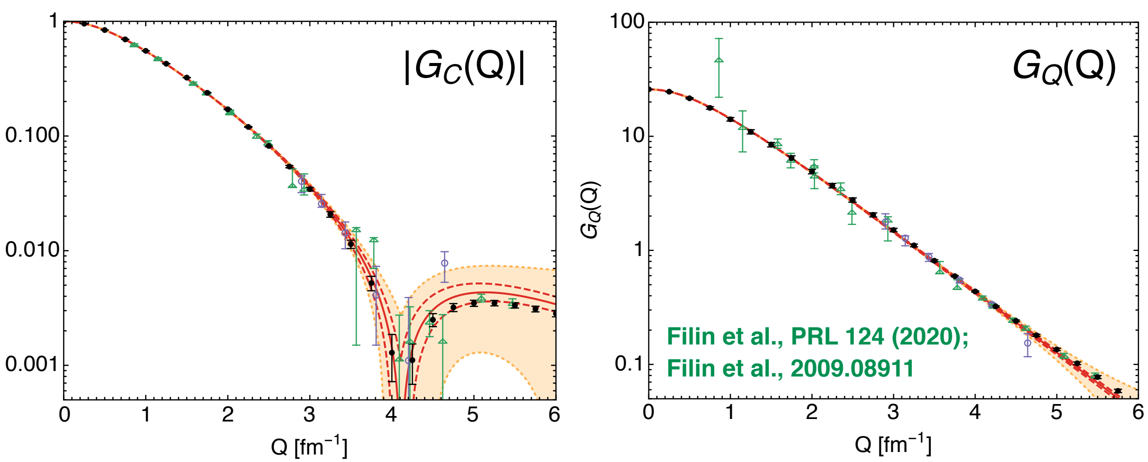
The charge and quadrupole form factors of the deuteron at N⁴LO



The extracted structure radius and quadrupole moment:

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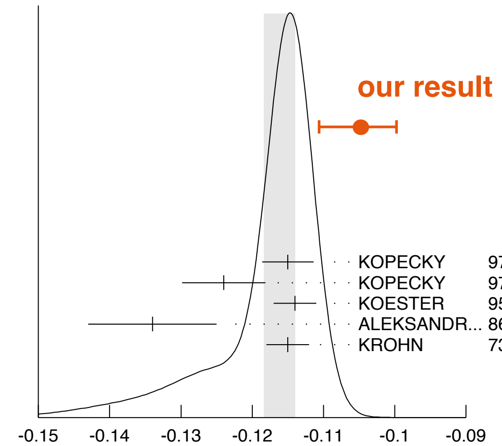

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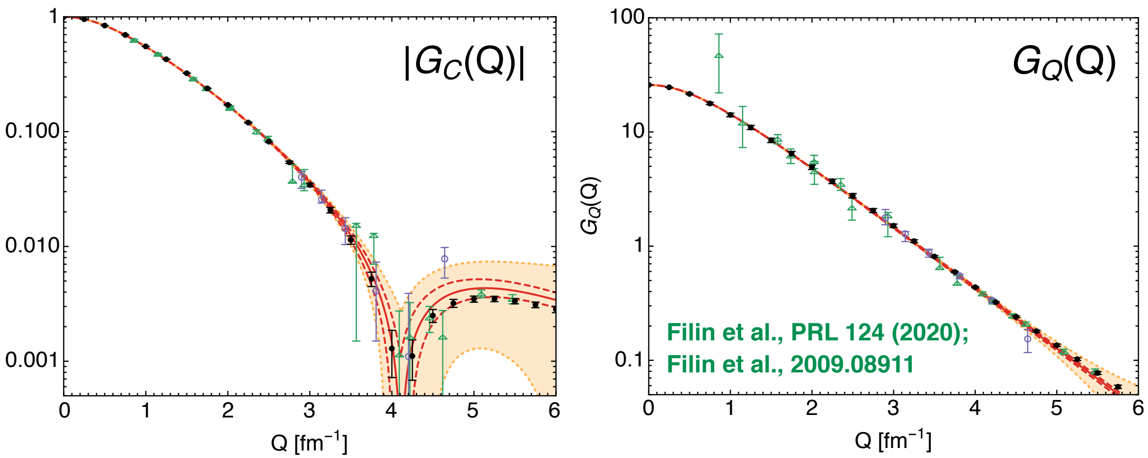

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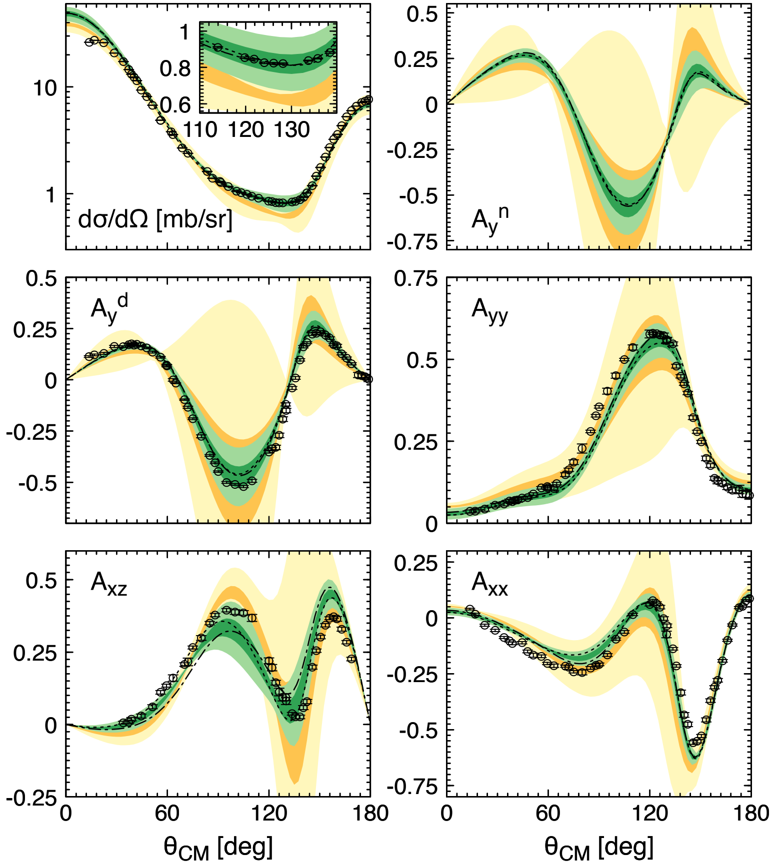
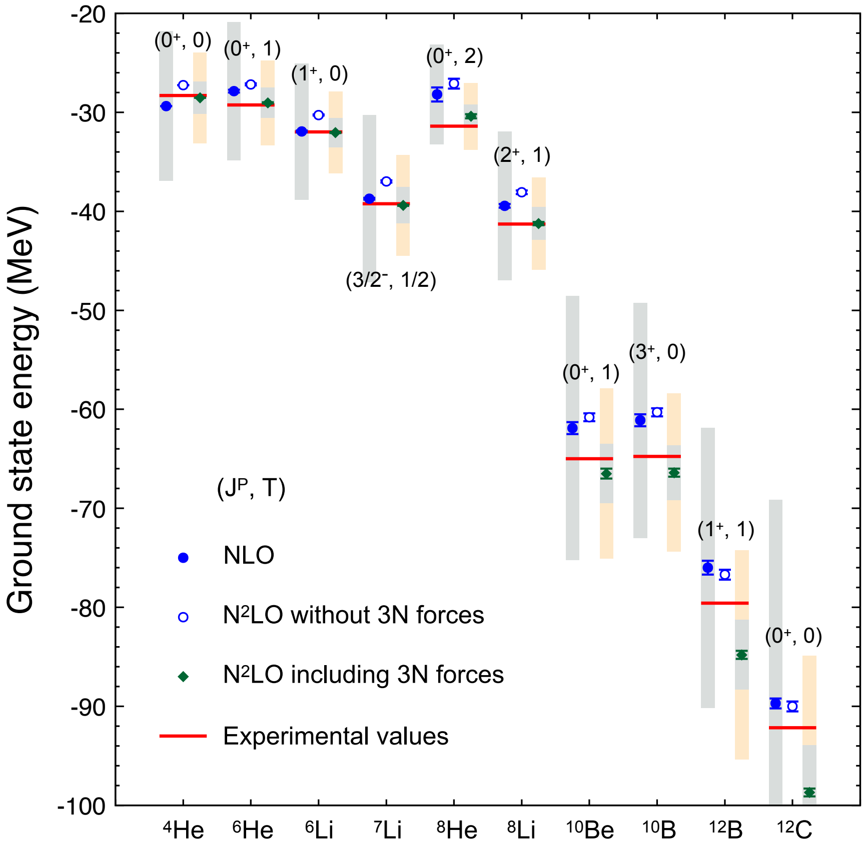
Few-nucleon systems at N²LO

Maris, EE, Furnstahl et al. (LENPIC), e-Print: 2012.12396 [nucl-th]

Nd elastic scattering observables



Ground state energies of p-shell nuclei

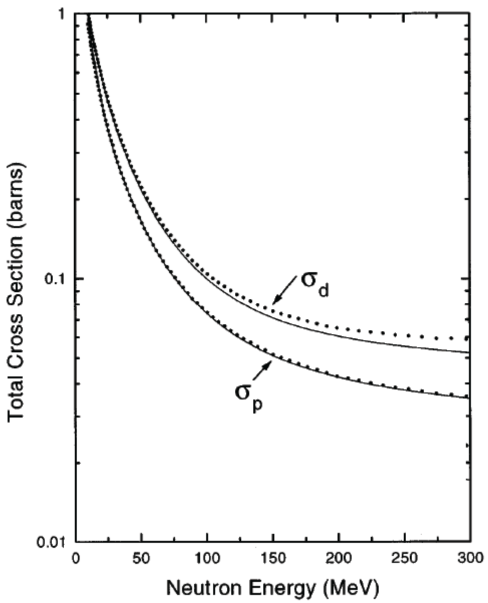


LENPIC: Low Energy Nuclear Physics International Collaboration

Frontiers and challenges

The 3-body force challenge

- Since ~ 25 years, there exist high-precision NN potentials which describe mutually compatible pp+np data below π-production threshold with χ^2/dat ~ 1 (N^{4}LO$^+$, AV18, CD Bonn, Nijm I,II, …)
- On the other hand, the 3N continuum is still not understood; no PWA available…



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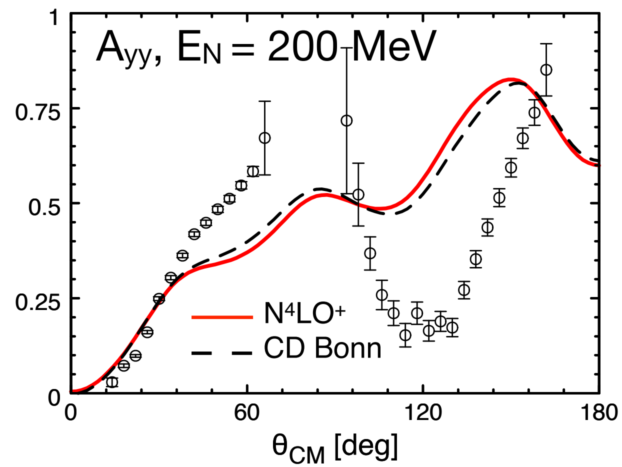

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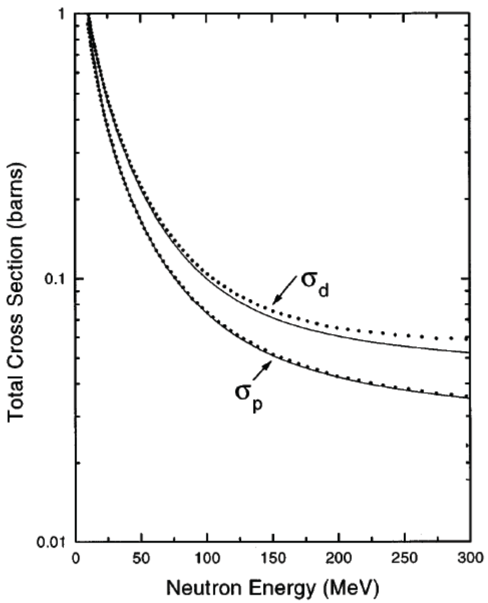

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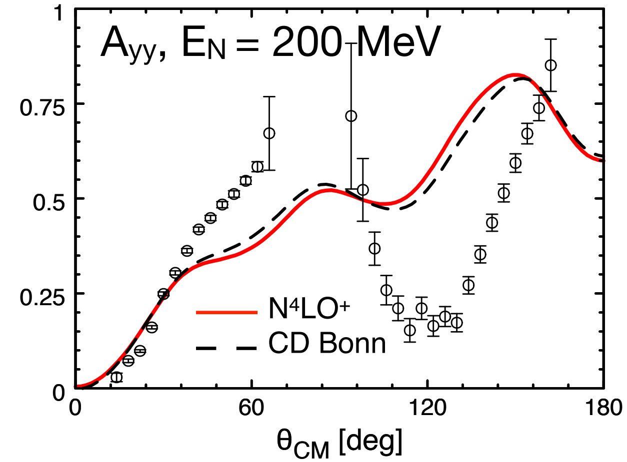

3N forces, in spite of a long history, remain a challenge!

- **Chiral EFT at N^{4}LO achieves a precision sufficient for solving the 3NF problem!**

  Still, both computational and conceptual challenges need to be addressed…

  On the computational side: determination of LECs in the 3NF (~10^7 more CPU time needed to compute the 3N amplitude as compared to the 2N one)

European Research Council

The 3-body force challenge

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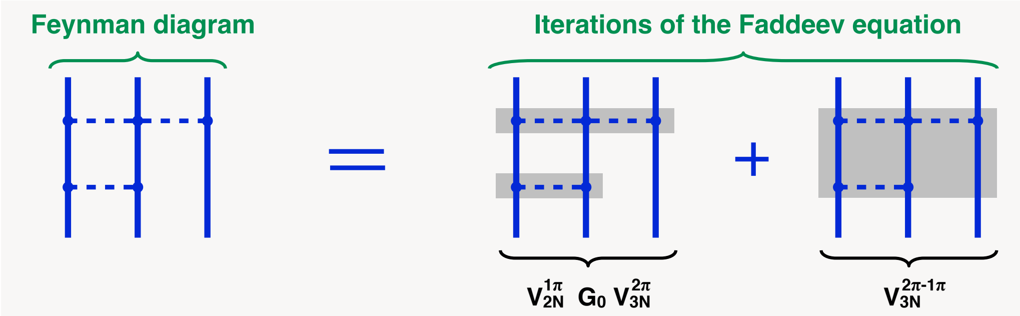

- Using DR to compute the Feynman diagram, 3NF and the iteration of the Faddeev equation leads to the same results (i.e. consistency)
- Using Cutoff Reg. in the iteration of the Faddeev equation and DR in the 3NF, the r.h.s. requires a chiral-symmetry breaking counter term signaling the inconsistency!

[EE, Krebs, Reinert, Front. in Phys. 8 (2020) 98]

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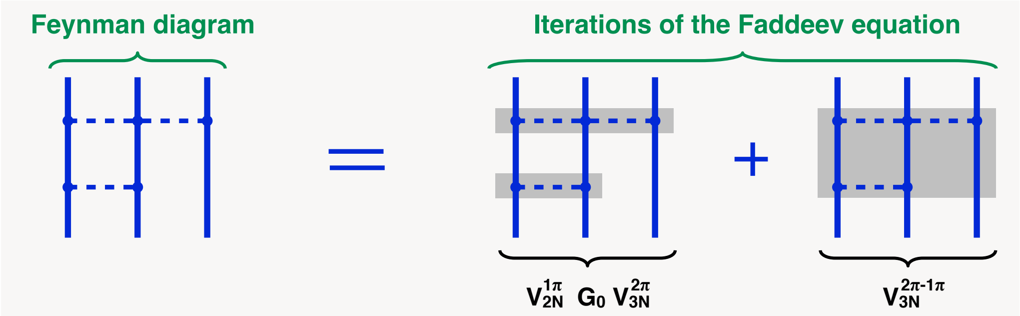

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All 3NF expressions beyond tree level (i.e. starting from N^3LO) as well as exchange currents [Krebs, EPJA 56 (2020) 234] **must be re-derived using cutoff regularization.**

— higher-derivative regularization to maintain the symmetries [Slavnov, NPB 31 (1971) 301]

— a new path-integral approach to derive nuclear forces and currents [Krebs, EE, in preparation]

Summary and outlook

Chiral EFT is becoming a precision tool for nuclear physics!

Some outstanding challenges and unsolved problems:

— consistent regularization of many-body forces and currents (relevant at N^3LO and beyond)

— the three-nucleon force problem

— underpredicted radii for medium-mass and heavy nuclei (and the related issue of the symmetric EoS)

— pushing ab initio frontier to reactions and heavier systems

— quark mass dependence of nuclear forces

…stay tuned for new results in the near future…