Skyrme-RPA analysis of GDR in heavy and superheavy nuclei

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НИИЯФ МГУ, 06.01.2014

Content

☆ GDR in deformed (rare-earth, actinides) and superheavy nuclei

W. Kleinig, VON, J. Kvasil, P.-G. Reinhard and P. Vesely, Phys. Rev. C, 78, 044313 (2008)

- **high accuracy of the model**
- **GDR width: mainly Landau damping and deformation**
- **similar GDR in stable and superheavy nuclei**

☆ Origin of pygmy resonance, nuclear vorticity

- **PDR as a peripheral part of toroidal and compression E1 modes**

↓

- **PDR region as a main location of the dipole nuclear vorticity**

A. Repko, P.-G. Reinhard, VON, and J. Kvasil, PRC 87, 024305 (2013)

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, and P. Vesely, PRC, 84, 034303 (2011)

P.-G. Reinhard, VON, A. Repko, and J. Kvasil, to be published in Phys. Rev. C, arXiv:1312.7216[nucl-th]

Model

VON, J. Kvasil, and P.-G. Reinhard, PRC, 66, 044307 (2002).

VON, W. Kleinig, J. Kvasil, P. Vesely, P.-G. Reinhard, and D.S. Dolci, PRC, 74, 064306 (2006).

Skyrme separable random-phase approximation (SRPA):

- fully self-consistent:
  - both mean field and separable residual interaction are derived from the initial Skyrme functional
  - all terms of the functional are taken into account + Coulomb
- surface and volume pairing
- self-consistent factorization of the resid. Inter. → low rank of RPA matrix (~8-16)
- spherical and axial deformed nuclei, from light to superheavy nuclei
- small computational effort + high accuracy

- already used for GDR, E1(toroidal compression), E2,E3, E0, spin-flip M1 GR

Various codes:
- 1d-SRPA
- 2d-SRPA
- **1d full RPA**

Skyrme parameterization:
SkT6,SkM*,SLy6, SkI3, Sv-bas

can be energy-dependent

Lorentz weight

$$\xi(\omega-\omega_\nu)=\frac{1}{2\pi}\frac{\Delta}{(\omega-\omega_\nu)^2+(\Delta/2)^2}$$

L=0,1,3

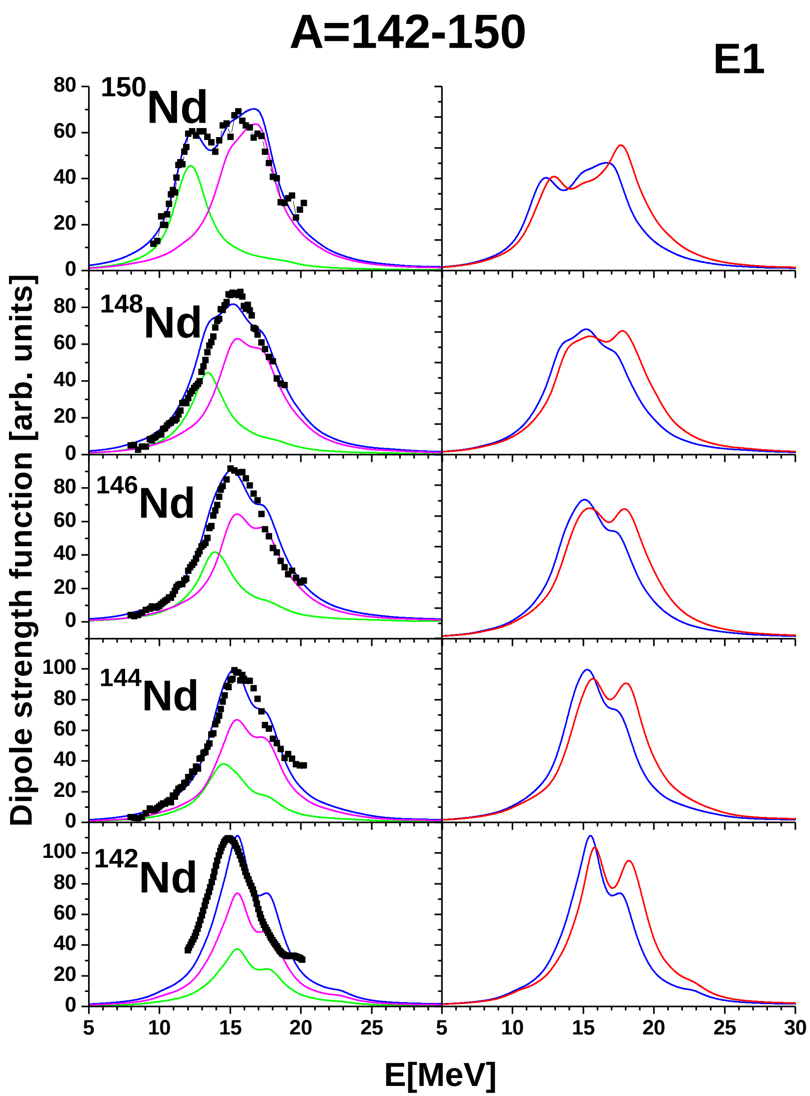
Strength function:

$$S_L(D_{X\lambda\mu})=\sum_\nu \omega_\nu^L <\nu|\hat{D}_{X\lambda\mu}|0>^2\ \xi(\omega-\omega_\nu)=$$

$$=\frac{1}{\pi}\Im\Big[\frac{z^L\sum_{\beta\beta'}F_{\beta\beta'}(z)A_\beta(z)A_{\beta'}(z)}{F(z)}\Big]_{z=\omega+i\Delta/2}+\sum_{ph}\varepsilon_{ph}^L<ph|\hat{D}_{X\lambda\mu}|0>^2\ \xi(\omega-\omega_\nu)$$

Nd chain:
effect of current densit

VON, W. Kleinig, J. Kvasil, P. Vesely, P.-G. Reinhard, J. Mod. Phys. (E), 17, 89 (2008).



— with current — $\mu = 0$

— no current — $\mu = 1$

SLy6

$\Delta = 1\text{MeV}$

- **Good agreement with experiment for all isotopes except of semi- magic A=142**

- **Noticeable effect of time-odd current density**

$$\delta \vec{j}_\nu(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}} \,|\, 0 \right\rangle$$

$$\hat{\vec{j}} = \frac{1}{2} \sum_{i=1}^{A} [\vec{\nabla}_i, \delta(\vec{r} - \vec{r}_i)]$$

Experiment:

A.V. Varlamov, V.V. Varlamov, D.S. Rudenko and M.E. Stepanov, ***Atlas of Giant Resonances, (INDC(NDS)-394, 1999),***
JANIS database.

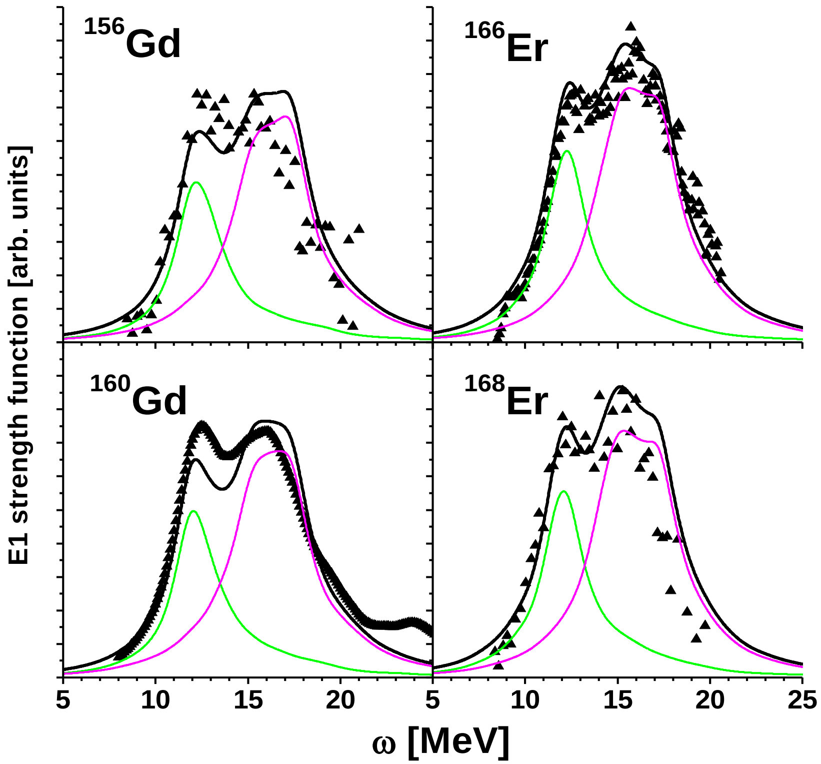
P. Carlos et al, *NPA, **172**, 437* (1971).

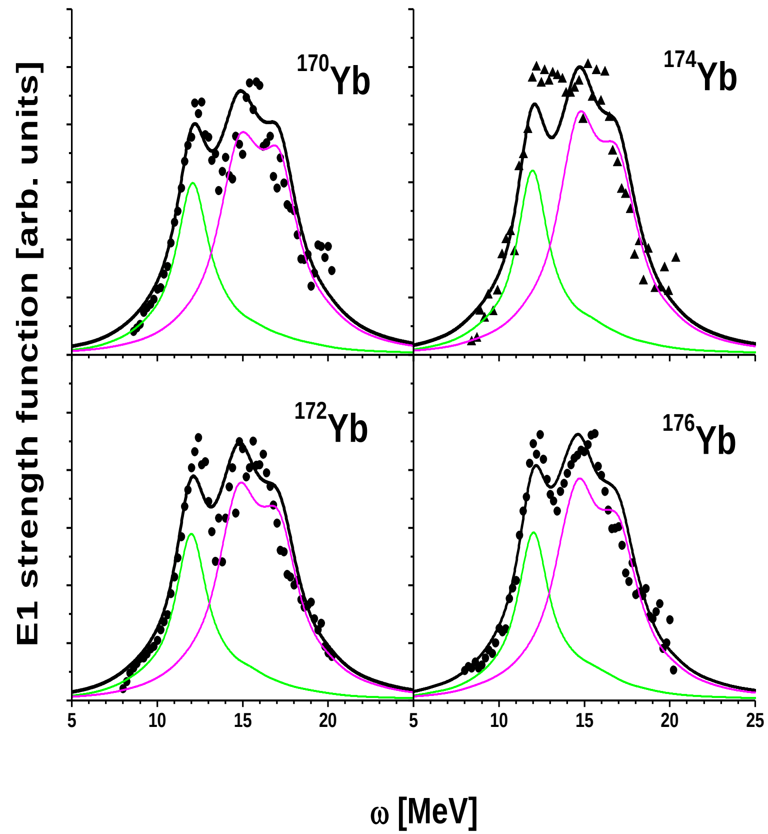
Systematic description of E1(T=1) GR in rare-earth and actinide nuclei

— $\mu = 0$ (green)
— $\mu = 1$ (magenta)

-SLy6
- Lorentz averaging $\Delta = 2\,\text{MeV}$

W. Kleinig, VON, J. Kvasil, P.-G. Reinhard and P. Vesely,
PRC, 78, 044313 (2008)





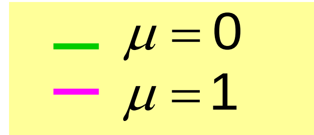
^{156,160}Gd, ^{166,168}Er : **(G,ABS)**

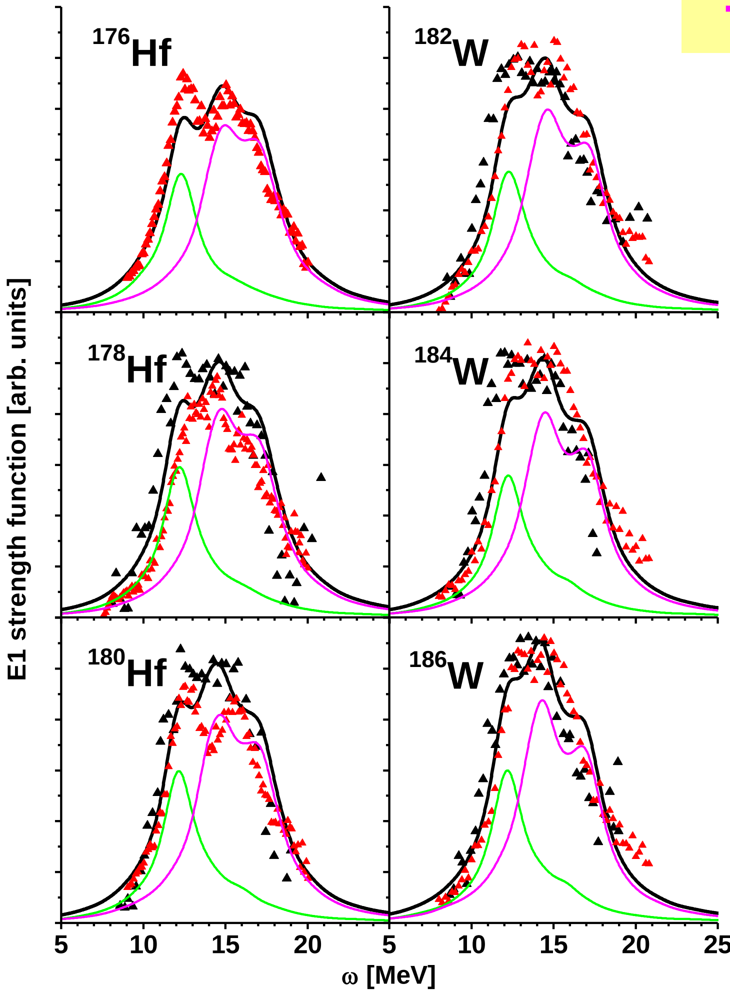
Experiment:
G. M. Gurevich et al, NPA, **351,** 257 (1981)
V. V. Varlamov et al, Bull.Russ. Acad. Sci. Phys. Ser. **67,** 724 (2003).
B. I. Goryachev et al, Yad. Fiz. **23,** 1145 (1976)..

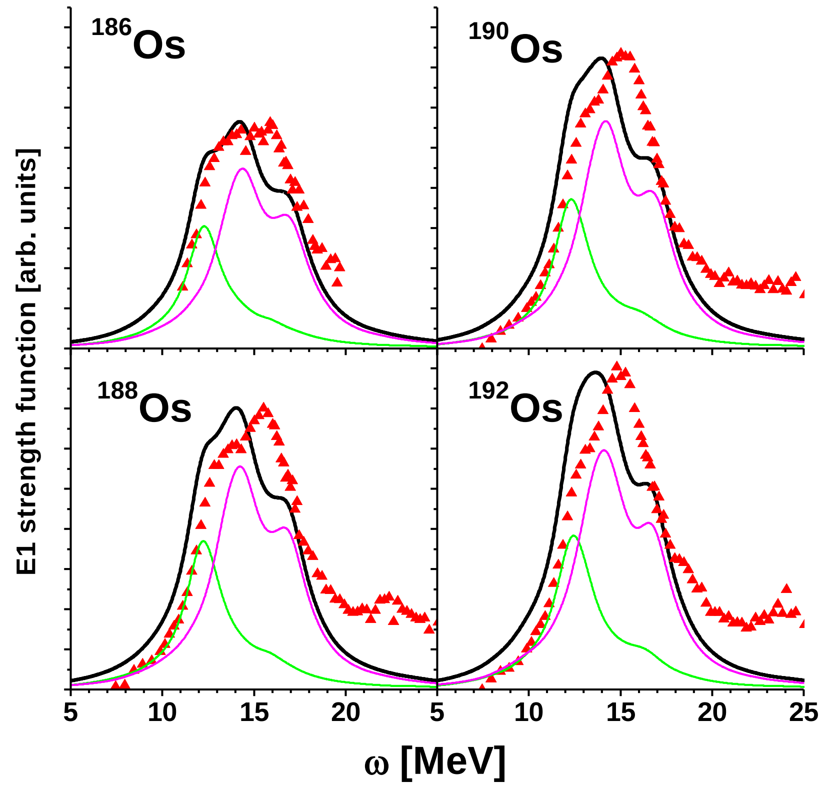
$^{170\text{-}176}$Yb : **(G,XN)**

Experiment:
G. M. Gurevich et al, NPA, **351,** 257 (1981)
A. M. Goryachev and G. N. Zalesnyy, Vopr. Teor. Yad. Fiz. **5,** 42 (1976).

W. Kleinig, VON, J. Kvasil,
P.-G. Reinhard and P. Vesely,
PRC, <u>78</u>, 044313 (2008)







(G,ABS) , (G,XN)

Experiment:
G. M. Gurevich et al, NPA, **351,** 257 (1981)
A.M. Goryachev and G. N. Zalesnyy, Yad. Fiz. **26,** 465 (1977).

Experiment:
B. L. Berman et al, PRC **19**, 1205 (1979).

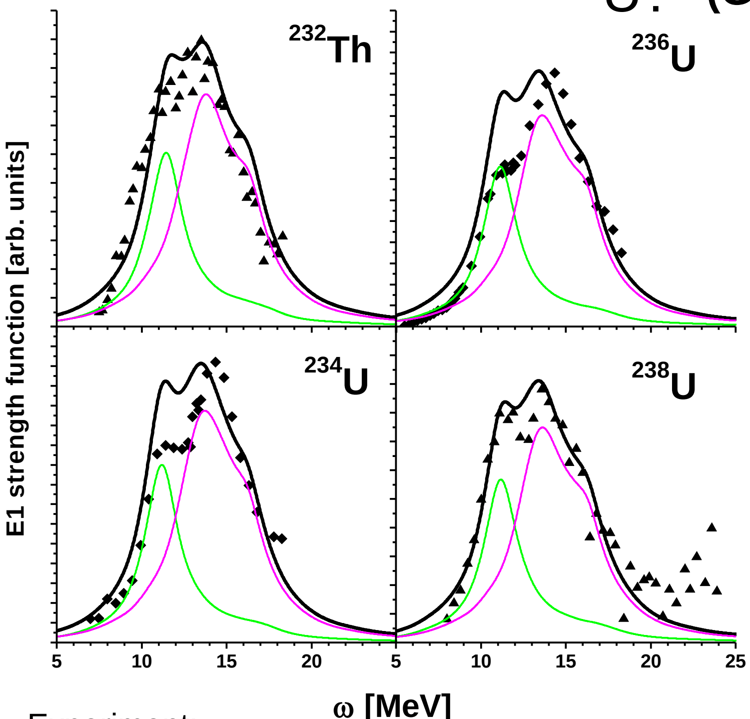
— $\mu = 0$
— $\mu = 1$

^{232}Th, ^{238}U: **(G,ABS)**

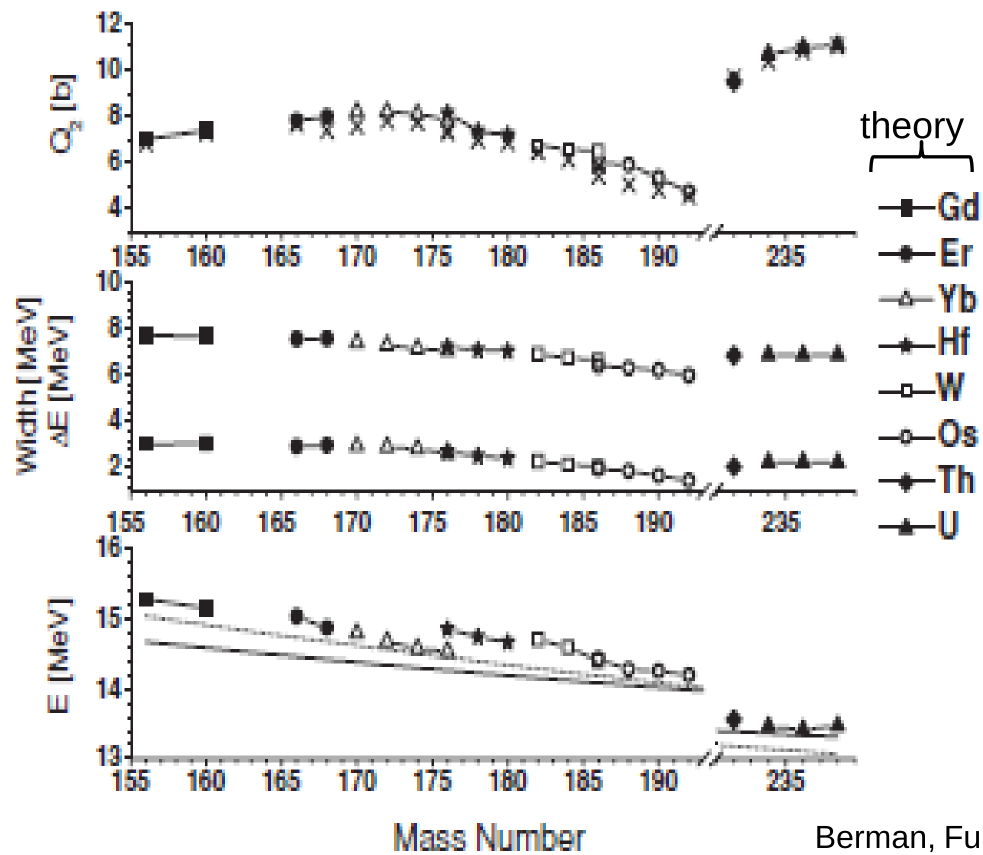
^{234,236}U: **(G,XN)**

**W. Kleinig, VON, J. Kvasil,
P.-G. Reinhard and P. Vesely,**
PRC, 78, 044313 (2008)

- Quadrupole moments,
- GDR width and deformation splitting
- GDR energy centroids



x - experiment



Experiment:
G. M. Gurevich et al, NPA **273,** 326 (1976).
B. L. Berman et al, PRC **34,** 2201 (1986).
J. T. Caldwell et al, PRC **21,** 1215 (1980).

Good agreement for:
- strength functions,
- quadrupole moments
- trends

Deformation splitting: ~ 40-50% of GDR width.

- - - - SJ: $E = 81 A^{-1/3}$ MeV

------ SJ+GT: $(31.2\, A^{-1/3} + 20.6\, A^{-1/6})$ MeV

Berman, Fultz, RMP'75

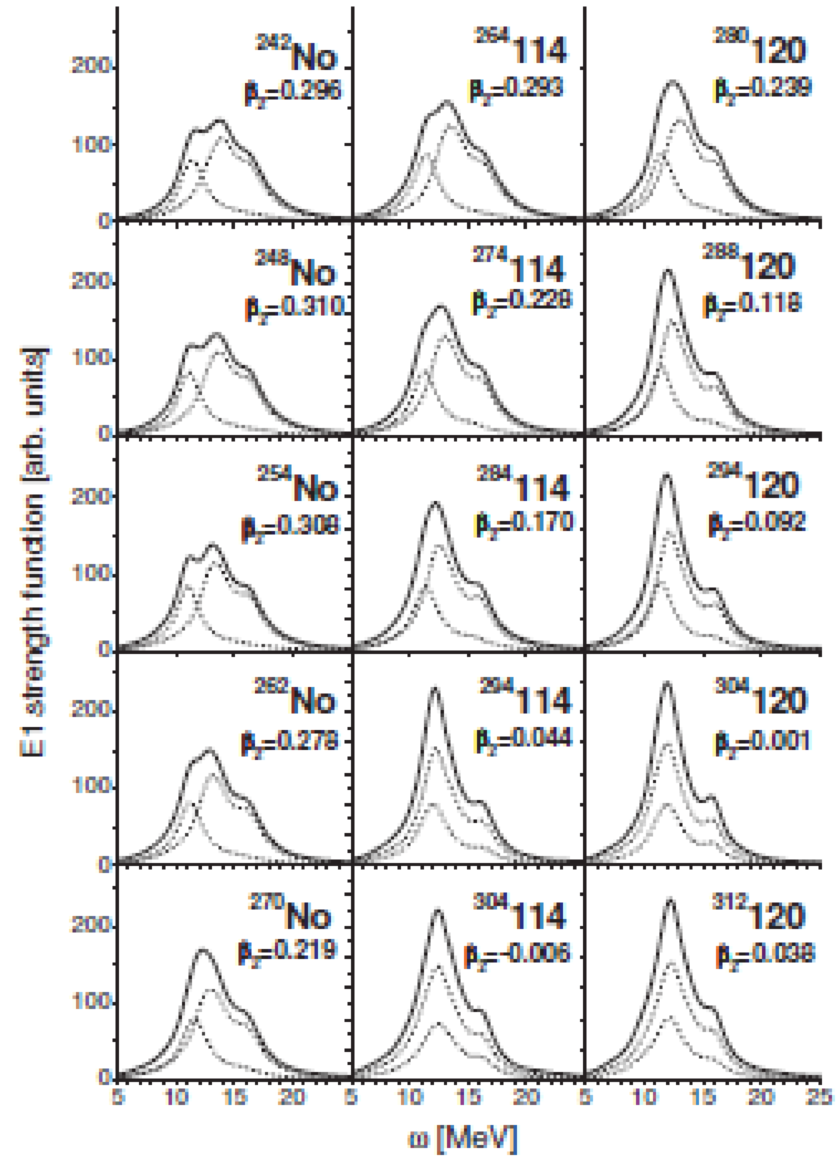
GDR in superheavy nuclei

Isotopic chains:

Z=102, A=242-270
Z=114, A=264-304
Z=120, A=280-312

- **from neutron-deficite to neutron-excess**
- **onset, mid and over of superheavy region**
- **Z=102: all isotopes are well deformed**
- **Z=114, 120: tentatively magic, strong vary of deformation**

GDR in superheavy nuclei is basically the same as in rare-earth and actinide nuclei
(within RPA without continuum, …)

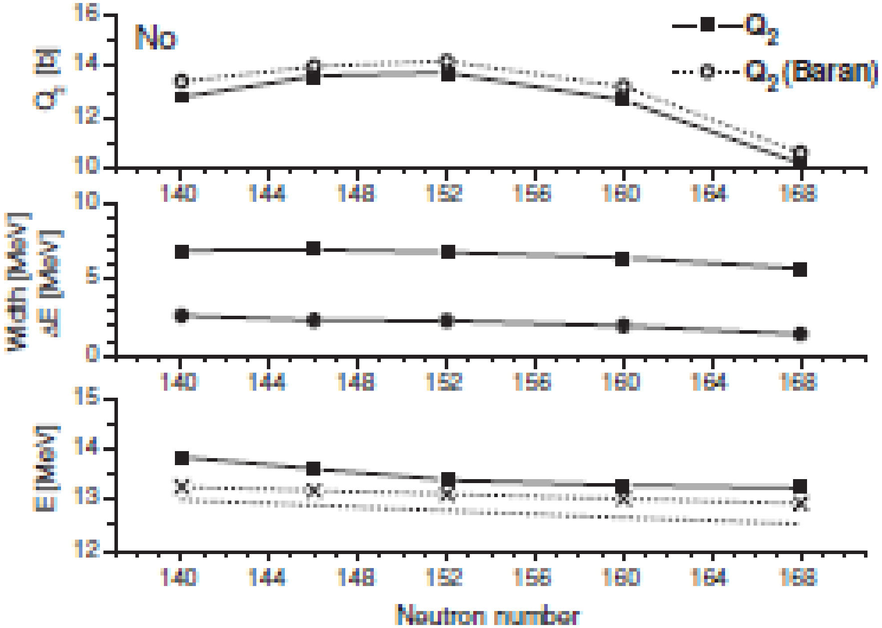


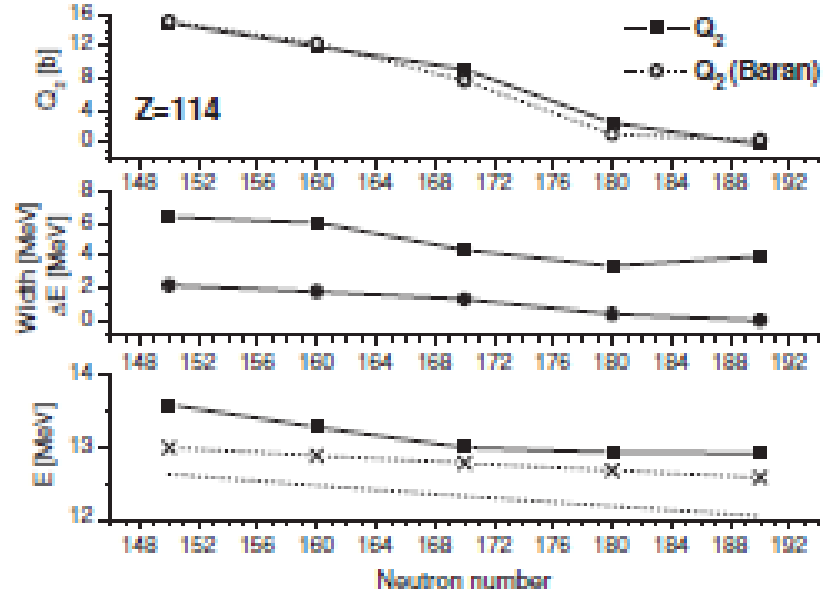
W. Kleinig, VON, J. Kvasil,
P.-G. Reinhard and P. Vesely,
PRC, 78, 044313 (2008)

Superheavy nuclei

---- **SJ** $E = 81\,A^{-1/3}\,\mathrm{MeV}$

--x- **SJ+GT** $E = (31.2\,A^{-1/3} + 20.6\,A^{-1/6})\,\mathrm{MeV}$

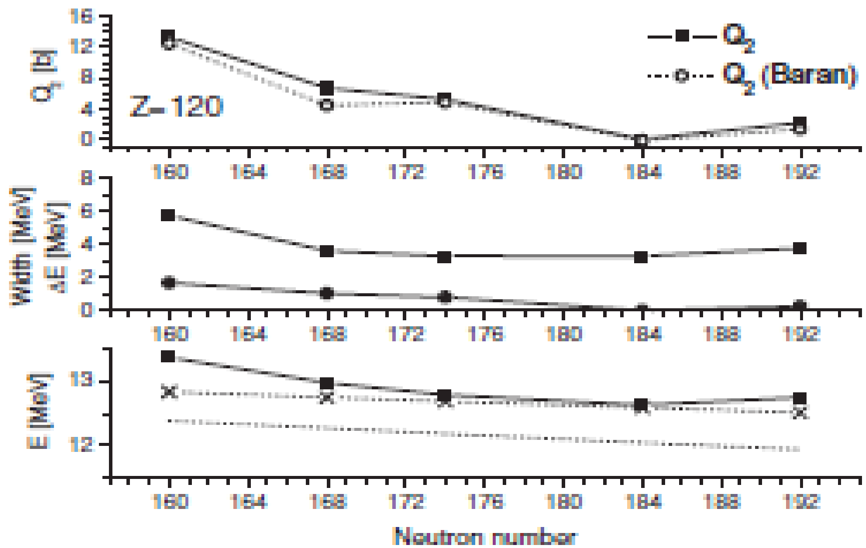




- **deformations: good agreement with macro-micro model of Baran**

A.Baran et al, PRC72, 044310 (2005)

- **energy trend:**
  - **favors SJ+GT,**
  - **rise near drip line (symmetry energy)**
- **generally similar to GDR in rare-earth and actinide nuclei**



W. Kleinig, VON, J. Kvasil, P.-G. Reinhard and P. Vesely, PRC, 78, 044313 (2008)

E1(T=1) widths

W. Kleinig, VON, J. Kvasil,
P.-G. Reinhard and P. Vesely,
PRC, 78, 044313 (2008)

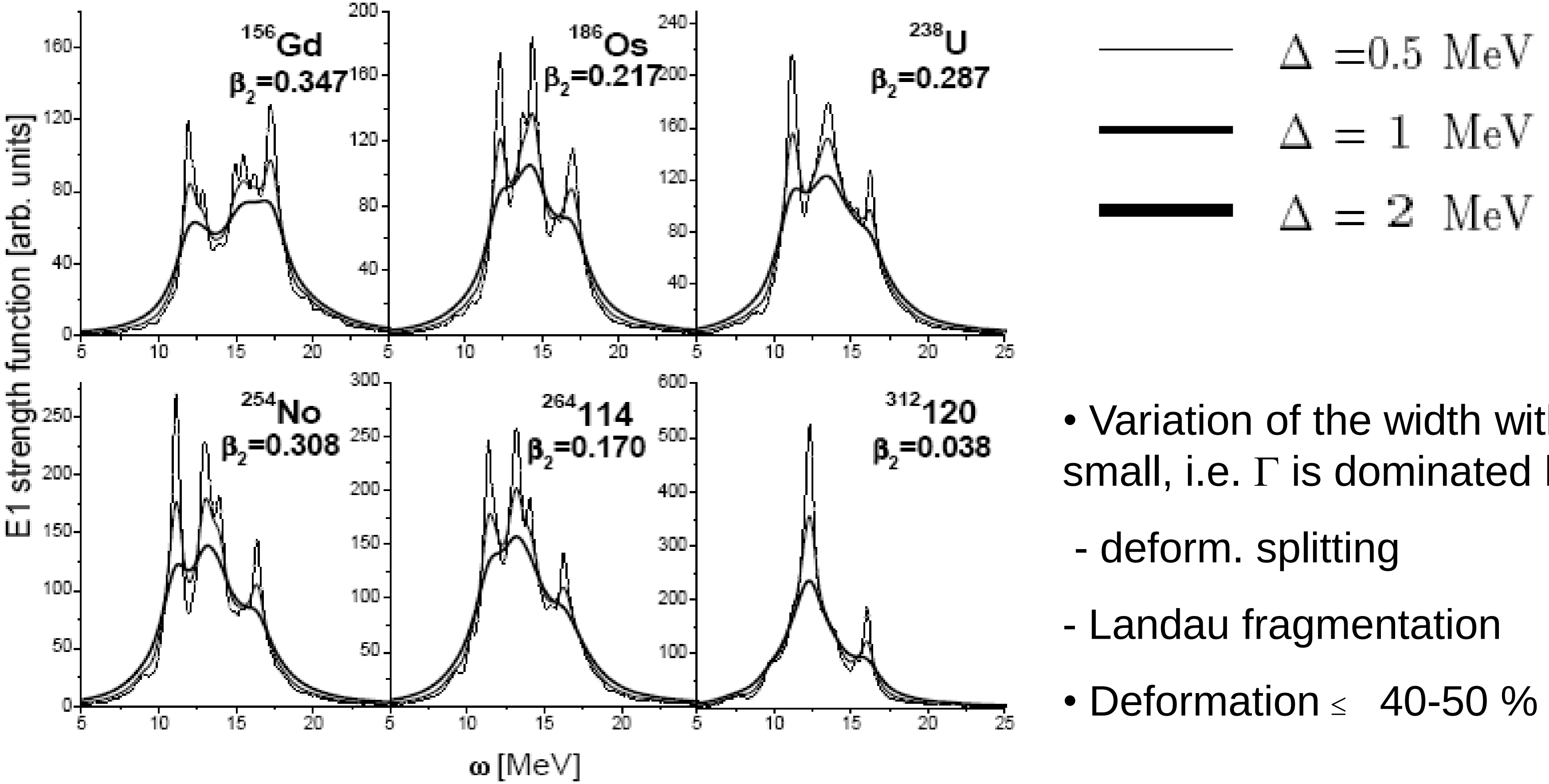
Main width mechanisms:

- **Landau fragmentation**
- **deformation splitting**

directly included

- **escape width**
- **coupling with complex configurations**

simlated by smoothing



- Variation of the width with Δ is small, i.e. Γ is dominated by:
  - deform. splitting
  - Landau fragmentation
- Deformation $\leq$ 40-50 % of Γ

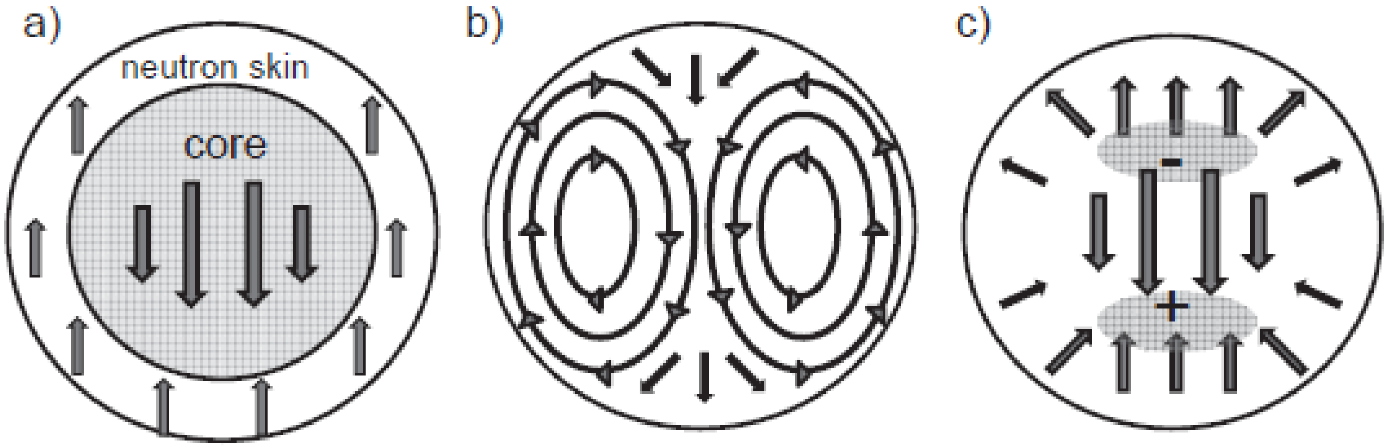
Does the PDR E1 strength indeed correspond

to naive PDR view?

Another physical origin?

Coexistence of different E1 modes in PDR region

R. Mohan et al (1971), | V.M. Dubovik (1975) S.F. Semenko (1981) | M.N. Harakeh (1977) S. Stringari (1982)



E1 pygmy | E1 toroidal | E1 compression

Does the low-energy E1 strength indeed correspond by naive PDR picture?

Dominate in E1(T=0) channel
(after exclusion of spurious E1(T=0) c.m. motion)

$$E = 68\, A^{-1/3} MeV \qquad E = 132\, A^{-1/3} MeV$$

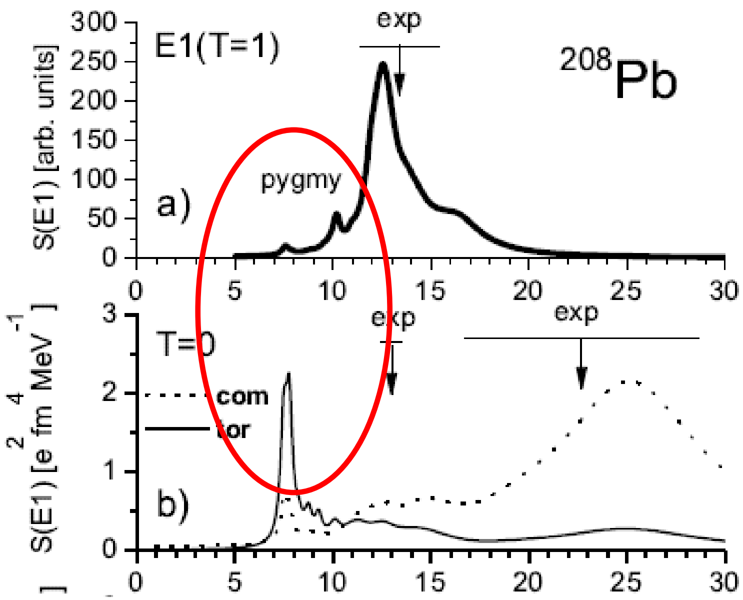
irrotationalal | vortical | irrotational

HD | Elastic,no restoring force in HD | HD

Strength functions SLy6

A. Repko, P.-G. Reinhard, VON, and J. Kvasil, PRC 87, 024305 (2013)

P.-G. Reinhard, VON, A. Repko, and J. Kvasil, to be published in Phys. Rev. C, arXiv:1312.7216[nucl-th]



Two peaks at 7.5 and 10.3 MeV are obtained in agreement to RMF calculations

(D. Vretenar, N. Paar, P. Ring, PRC**, 63**, 047301 (2001))

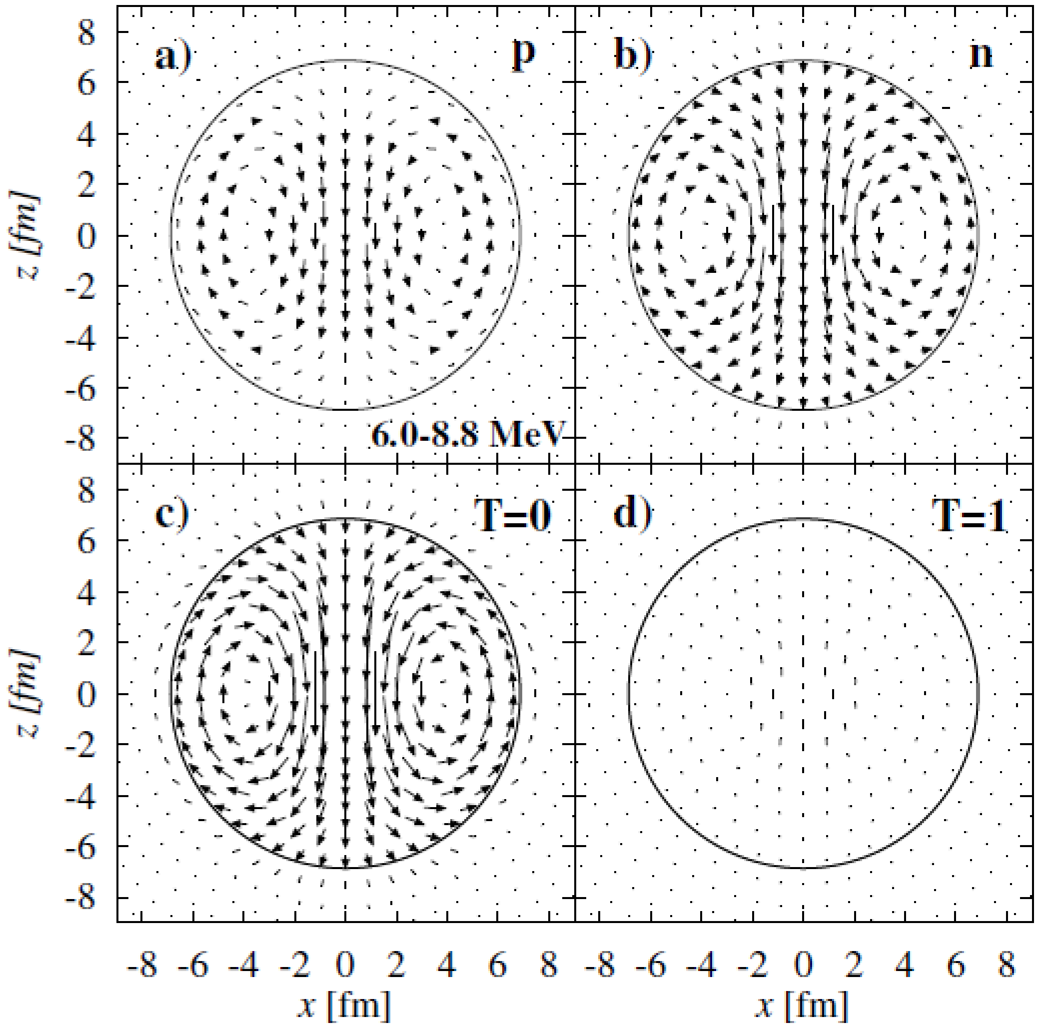
(α, α') experiment of Uchida et al (2003)

PDR region may host toroidal and compression strength

Further analysis of:

- transition densities,
- current fields (!)

E1 flow patterns (current fields): 6.0-8.8 MeV



- **Toroidal flow:**
  - **strong for neutrons,**
  - **weaker for protons**
  - **strong for T=0**
- **faint T=1 flow**

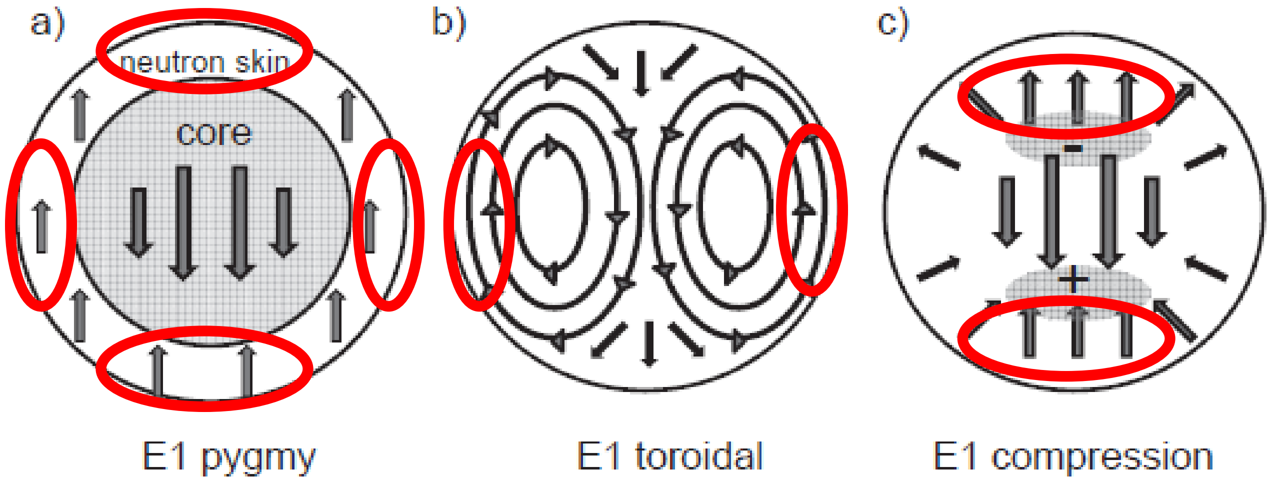
The E1 flow at 6-8.8 MeV looks like pure T=0 toroidal resonance!

No evidence for PDR flow!

-T=0 strictly dominates over T=1

in accordance to:

- N. Ryezaeva et al, PRL, 2002



- PDR originates from TM/CM and represents their peripheral part.

- PDR flow is locally irrotational part of vortical TM flow

- There is no contradiction between PDR, TM and CM if to take into account that:
  - E1(T=0) is measured mainly in peripheral reaction (α, α')
  - PDR, TM and CM have similar peripheral parts

The oversimplified PDR view still persists but has another origin

Conclusions

☆ GDR in deformed (rare-earth, actinides) and superheavy nuclei

- **High SRPA accuracy**
- **Noticeable impact of the current transition density**
- **GDR width: Landau damping and deformation,**
- **Similar GDR in stable and superheavy nuclei**

☆ Complicated origin of pygmy resonance: interplay with toroidal E1 mode

- **PDR as a peripheral part of TM and CM.**
- **Naïve PDR picture persists but has another origin.**

⇓

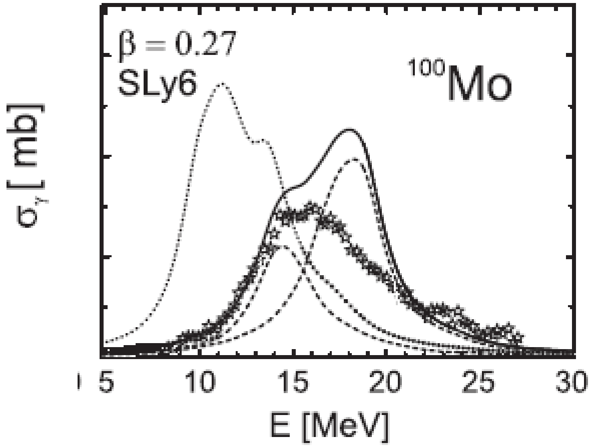
- **PDR region as a main location of the dipole vorticity**

Thank you
for the attention

Low-energy E1 strength near particle emission threshold

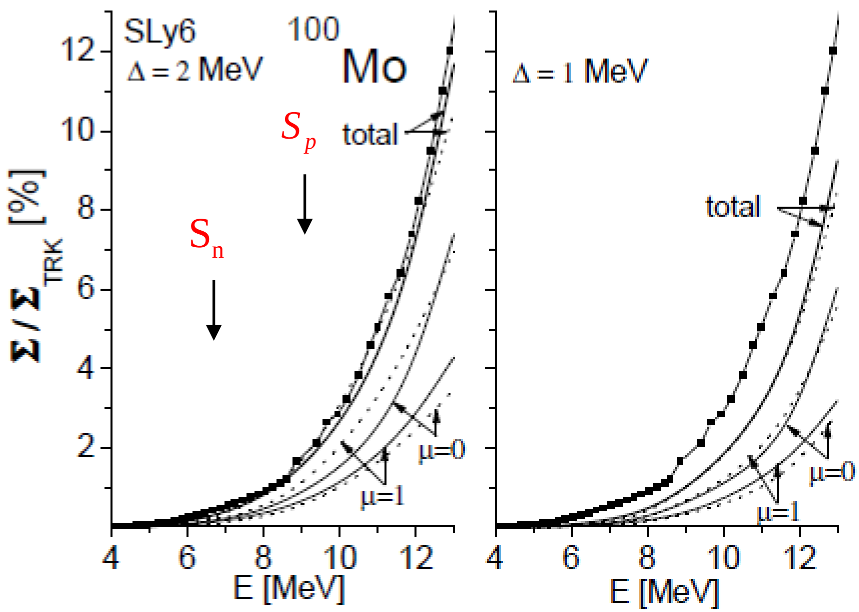
-**important for astrophysical problems**

F. Donau et al, PRC, 76, 014317 (2007):
strong deformation impact on low-energy E1 strength

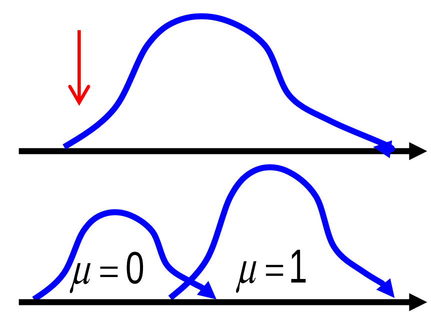


J. Kvasil, P. Vesely, VON W. Kleinig, P.-G. Reinhard, and S. Frauendorf IJMP (E)), 18, n.4, 975 (2009)

spherical
deformed



Deformation has a modest impact due to the compensation effect between two GDR branches!



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- high accuracy of the model
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- similar GDR in stable and superheavy nuclei

☆ Deformation effect on E1 strength near the particle threshold

- Minor deformation effect in contrast to previous predictions

J. Kvasil, P. Vesely, VON W. Kleinig, P.-G. Reinhard, and S. Frauendorf, IJMP (E), 18, n.4, 975 (2009)

☆ Origin of pygmy resonance

- PDR as a peripheral part of toroidal and compression E1 modes

A. Repko, P.-G. Reinhard, VON, and J. Kvasil, PRC 87, 024305 (2013)

☆ Nuclear vorticity

- PDR region as a main location of the dipole nuclear vorticity

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P.-G. Reinhard, VON, A. Repko, and J. Kvasil, to be published in Phys. Rev. C, arXiv:1312.7216[nucl-th]

Conclusions

☆ GDR in deformed (rare-earth, actinides) and superdeformed nuclei

-**High SRPA accuracy**
- **Noticeable impact of the current transition density**
-**GDR width: Landau damping and deformation, no need in complex config.**
-Similar GDR in stable and superheavy nuclei

☆ Deformation effect on E1 strength near the particle threshold

- Minor deformation effect in contrast to previous predictions

☆ Complicated origin of pygmy resonance: interplay with toroidal E1 mode

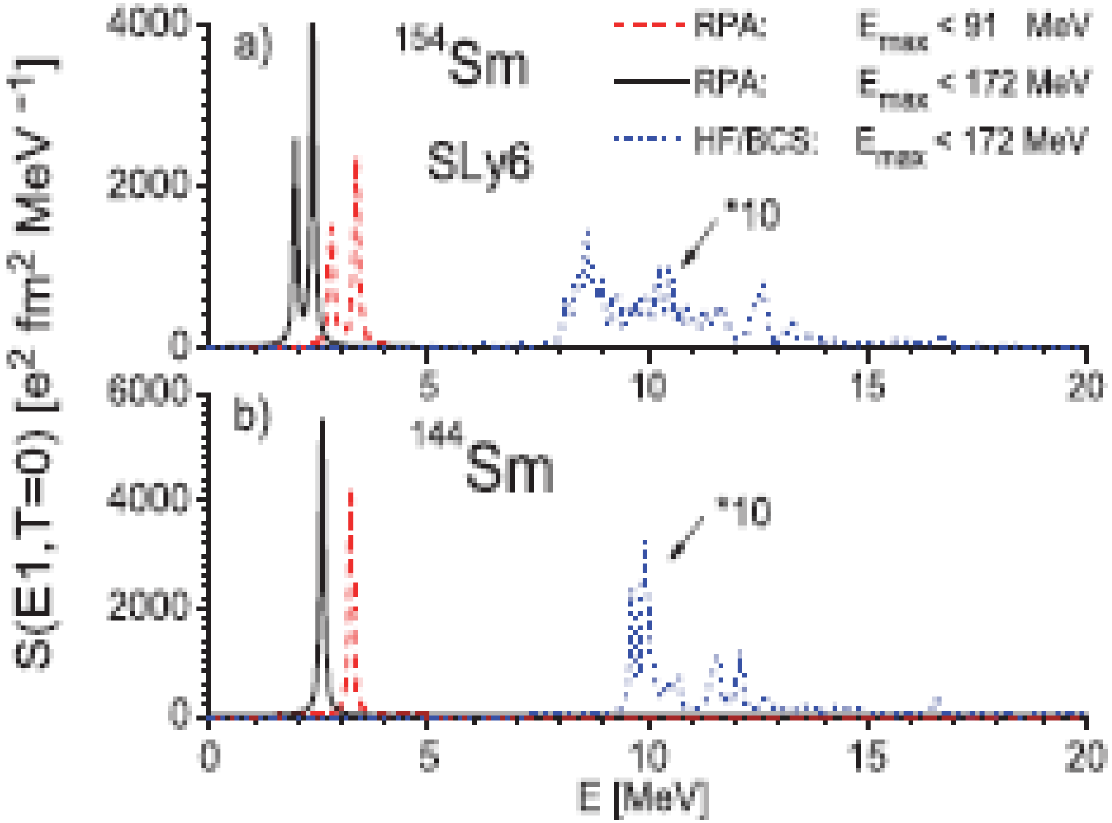
- PDR as a peripheral part of TM and CM

☆ Nuclear vorticity

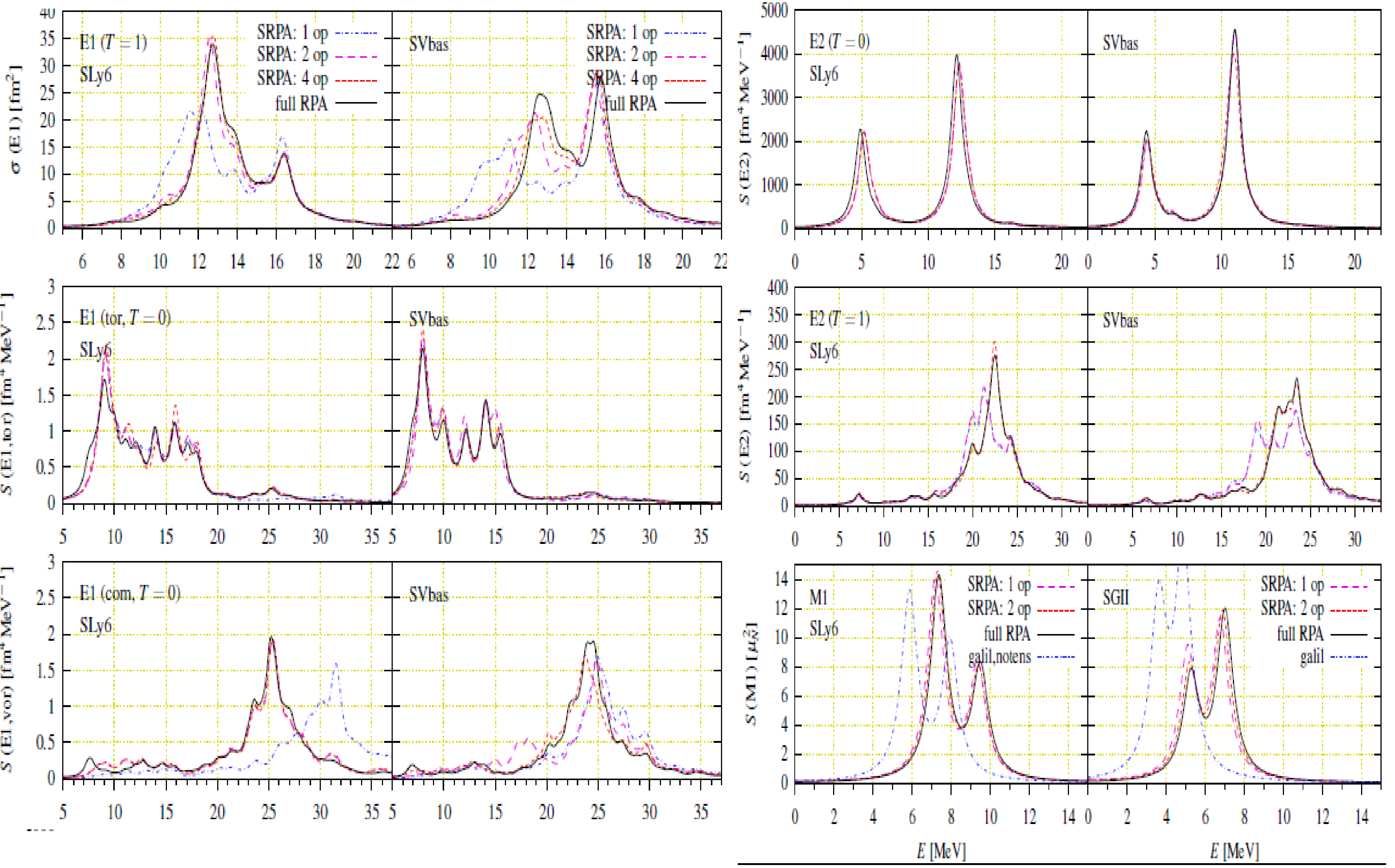
- Toroidal current as a measure of nuclear vorticity
- PDR region as a main location of the dipole vorticity

Spurious E1(T=0,m=0) strength in HF/BSC and SRPA

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard
and N. Lo Iudice,
EPJA, v.49, 119 (2013)



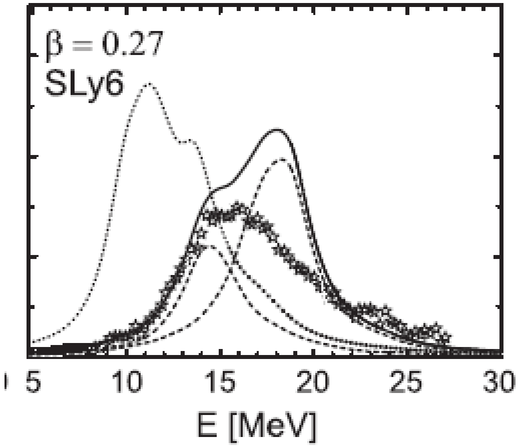
SRPA vs full RPA: GR in 208Pb



GDR description within Skyrme RPA: some news and trends

1) The simultaneous description of E1(T=1) and E2(T=0) can be now obtained within the recent Skyrme parameterization SV-bas

P. Kluepfel, P.-G. Reinhard, T.J. Buervenich, J.A. Maruhn, PRC 79, 034310 (2009).



Details of the calculations

Effective charges: $\lambda = 1, T = 1 \rightarrow e_p^{eff} = N/A, \quad e_n^{eff} = -Z/A$

Pairing: BCS, DF, no blocking

G_p, G_n from odd-even mass differences, $\Delta^{(5)}$ (Reinhard)

2d-grid in cylindrical coordinates (ρ, z)

calculation box: $dz = d\rho = 0.7\,\text{fm}\ z_{\text{max}} = 24.5\,\text{fm}, \quad \rho_{\text{max}} = 18.9\,\text{fm}$

(about 3R)

Single-particle basis:

^{154}Sm **SkM*, p: [- 43, +20] MeV, 248 levels,**
n: [- 49, +15] MeV, 307 levels

^{238}U **SkM*, p: [- 40, +19] MeV, 307 levels,**
n: [- 49, +15] MeV, 400 levels

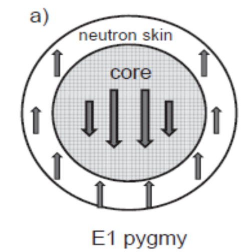
Number of 2qp pairs:

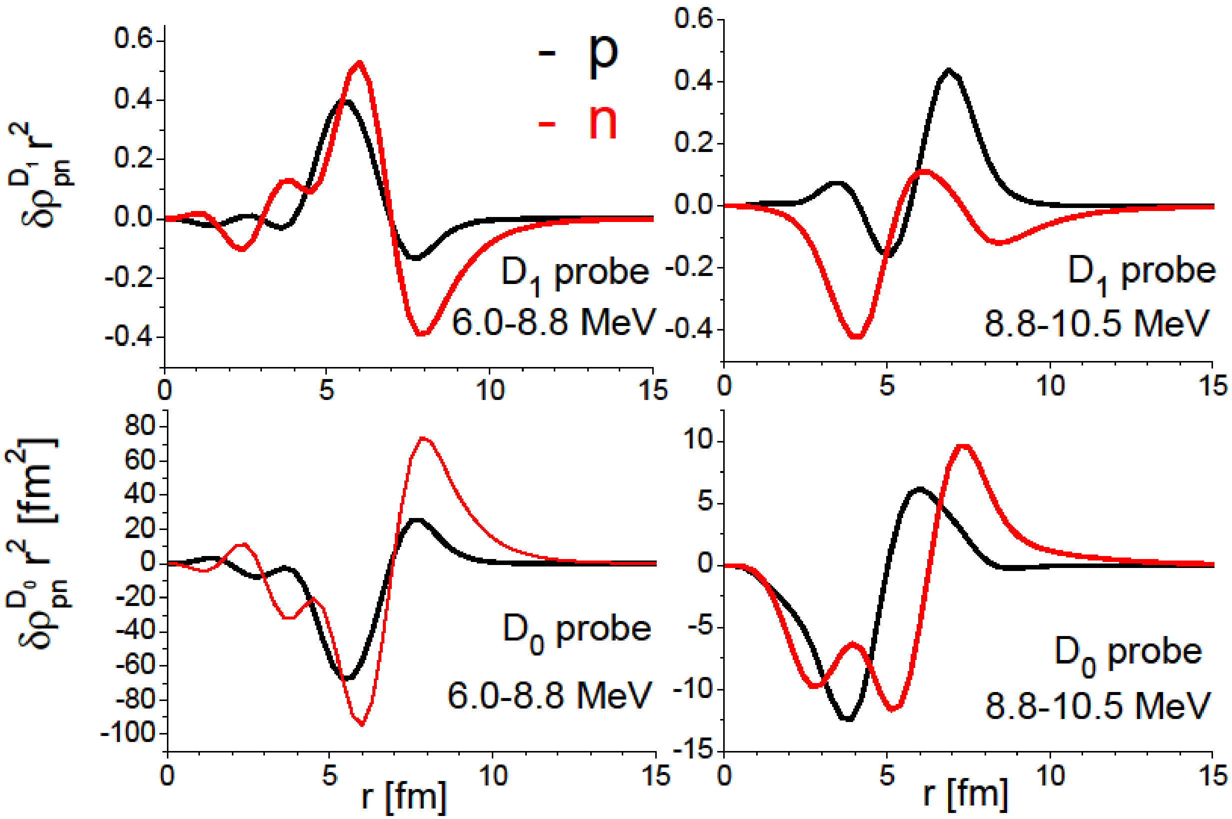
$u_{12}^{(+)} = u_1 v_2 + u_2 v_1 < 10^{-8}$

^{154}Sm **E1: 3900(p) + 9400(n)**
E2: 6200(p) + 15300(n)
p[0, 26-63], n[0, 23-63] MeV

^{238}U **E1: 6200(p) + 13900(n)**
E2:10000(p) + 22700(n)
p[0, 25-59], n[0, 21-64] MeV

Summed r^2 -weighted transition densities (TD)
for two parts of PDR region: 6-8.8 MeV and 8.8-10.5 MeV





Bin 6-8.8 MeV:

- **typical TD structure used to justify the PDR picture: neutron excess (7-10 fm) oscillates against the nuclear core (4-7 fm)**

The flow in nuclear interior (r< 4 fm) is damped though It may be important for disclosing the true PDR origin.

TD lose angular dependence of the flow are so are too rough in general.

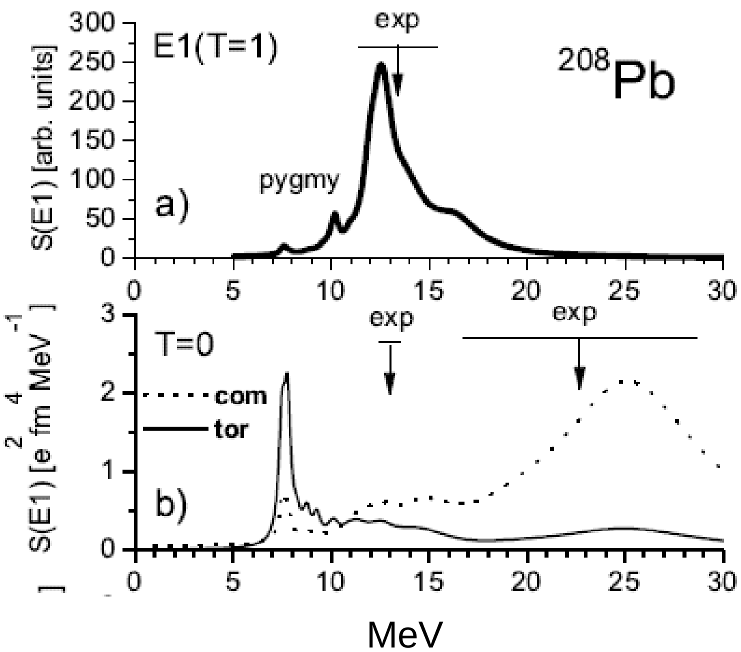
More detailed characteristics (velocity fields) are necessary.

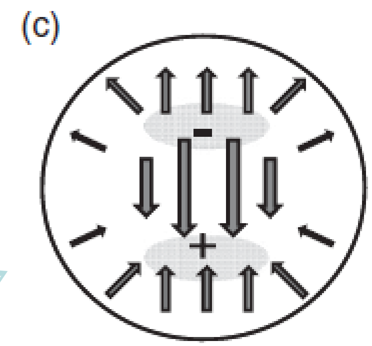
Bins 6-8.8 MeV and 8.8-10.5 MeV :

- **different scales of IS DT ⟶ the bin 6-8.8 MeV id more IS than 8.8-10.5 MeV**

Bin 8.8-10.5 MeV: mixed IS/IV structure

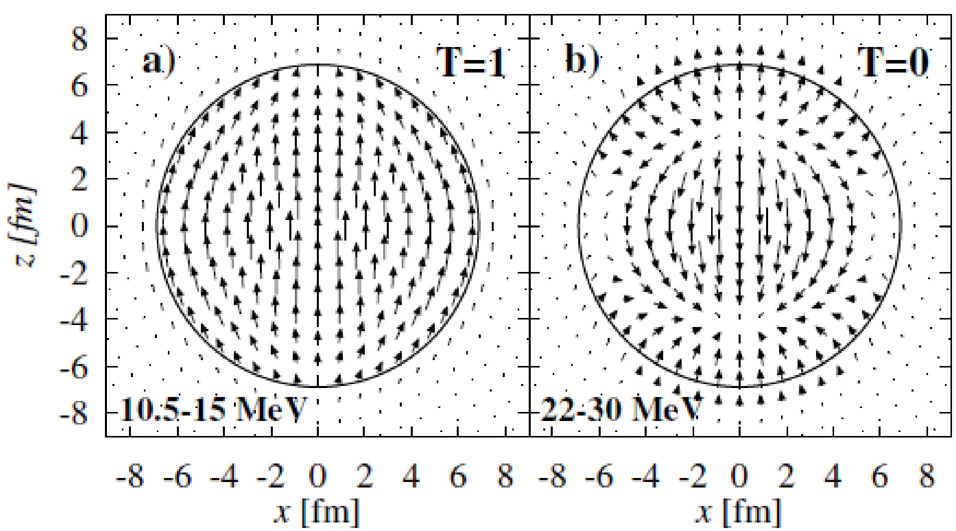
Benchmark examples





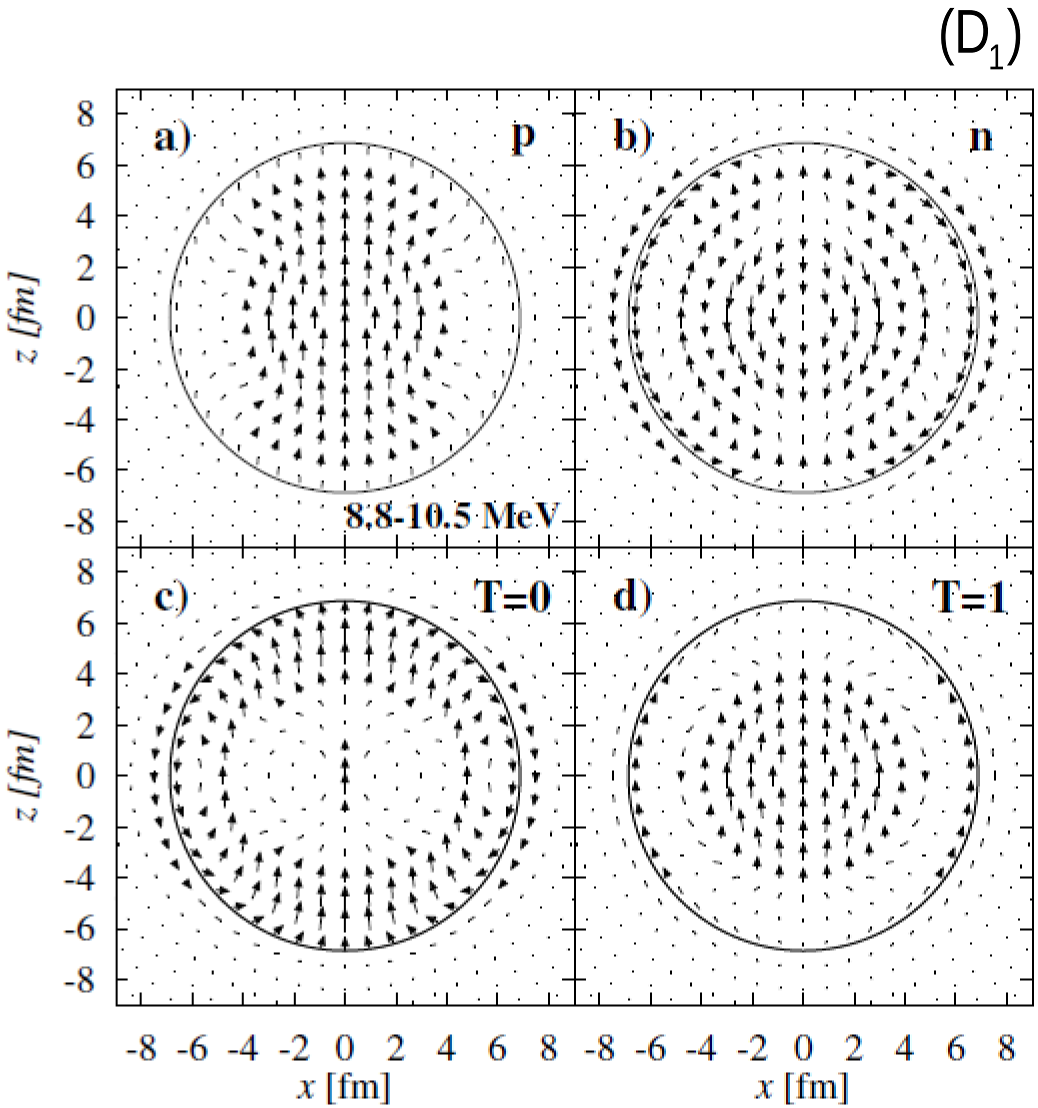
E1 compression

GDR (D_1) compression (D_0)



- good reproduction of well known fields,
- justifies accuracy of our model

Flow patterns : 8.8-10.5 MeV



- mainly T=1 in interior and T=0 at the surface,

-TR: (n, T=0)
CR: (n)
linear dipole: (p, T=1)

- complex structure with mixed is/iv, TR/CR/dipole

- More significant T=1 contribution than at 6-8.8 MeV

in accordance to:
- experiment for 124Sn, $(\alpha,\alpha'\gamma')$
(Enders et al, PRL, 2010)

References:

1) V.O. Nesterenko, J. Kvasil, and P.-G. Reinhard,
"Separable random-phase-approximation for self-consistent nuclear models",
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Phys.Rev C74, 064306 (2006).

4) V.O. Nesterenko, W. Kleinig, J. Kvasil, P. Vesely, P.-G. Reinhard,
"Giant dipole resonance in deformed nuclei: dependence on Skyrme forces",
Int. J. Mod. Phys. (E), 16, 624 (2007).

5) V.O. Nesterenko, W. Kleinig, J. Kvasil, P. Vesely, P.-G. Reinhard,
“TDDFT with Skyrme forces: effect of time-odd densities on electric giant resonances"
J. Mod. Phys. (E), 17, 89 (2008).

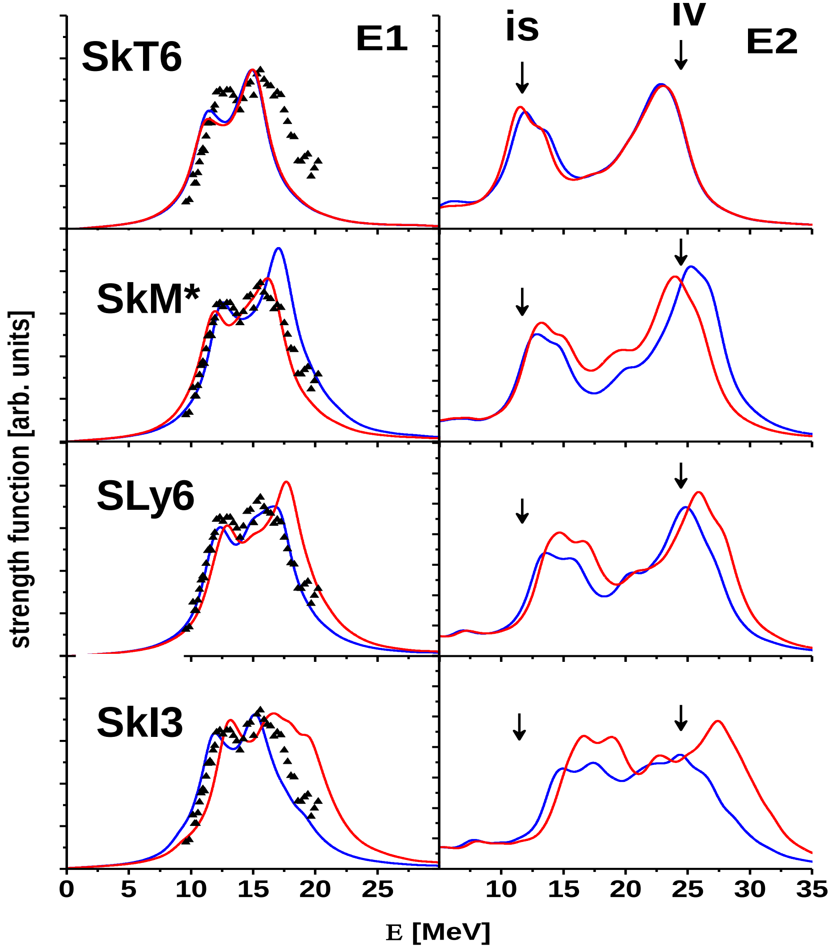
6) W. Kleinig, V.O. Nesterenko, J. Kvasil, P.-G. Reinhard and P. Vesely,
"Description of dipole giant resonance in heavy and superheavy nuclei within Skyrme random-phase-approximation",
Phys. Rev. C, 78, 044313 (2008)

^{150}Nd

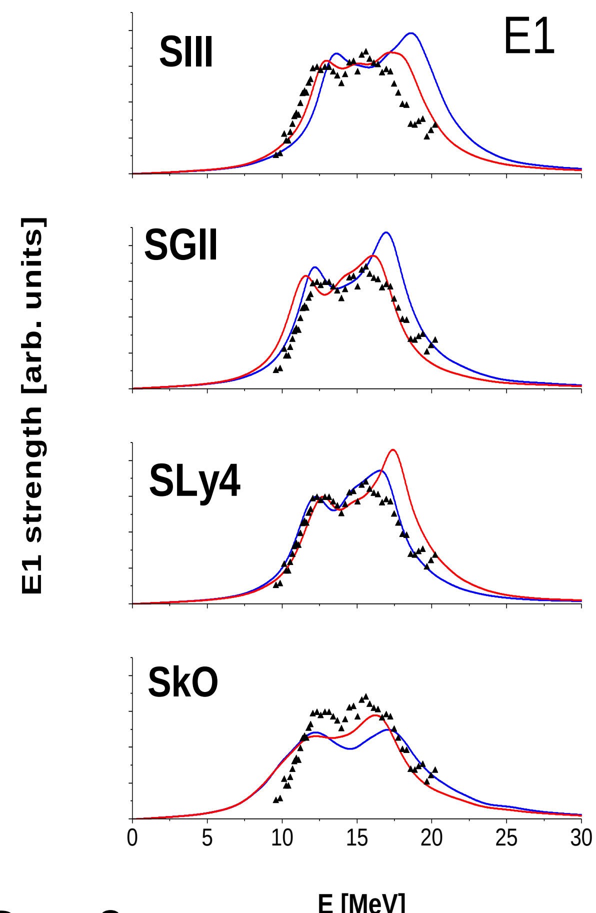
$$E2(T=0) \approx 62\,A^{-1/3}\ MeV$$

$$E2(T=1) \approx 130\,A^{-1/3}\ MeV$$

— with current
— no current



B_1	B_0
0	0
34	35
-26	58
-64	96



B_1
31
29
-25
8

Regular t-odd impact for E2(T=0) because always $B_0 > 0$

Irregular t-odd impact for E1(T=1):

- **upshift (SkM*,SkIII, SGII)** → $B_1 > 0$
- **downshift (SLY4,SLy6,SkI3)** → $B_1 < 0$
- **no shift (SkT6, SkO)** → $B_1 \sim 0$

⇒ **Classification of Skyrme forces**

Skyrme functional for atomic nuclei

Y.M. Engel et al, NPA 249, 215 (1975).
J. Dobaczewski and J. Dudek, PRC, 52 1827 (1995).

$$E=\int H(\vec r)\,d\vec r, \qquad H=T+H_{Sk}+H_{Coul}$$

s=p,n

$$H_{Sk}(\rho,\tau,\vec{\Im},\vec j,\vec s,\vec T)=\frac{b_0}{2}\rho^2-\frac{b_0'}{2}\sum_q\rho_q^2+\frac{b_3}{2}\rho^{\alpha+2}-\frac{b_3'}{2}\sum_q\rho_q^2$$
$$+b_1(\rho\tau-\vec j^2)-b_1'\sum_q(\rho_q\tau_q-\vec j_q^2)-\frac{b_2}{2}\rho\Delta\rho+\frac{b_2'}{2}\sum_q\rho_q\Delta\rho_q$$
$$-b_4(\rho\vec\nabla\cdot\vec{\Im}+\vec s\cdot(\vec\nabla\times\vec j))+b_4'\sum_q(\rho_q\vec\nabla\cdot\vec{\Im}_q+\vec s_q\cdot(\vec\nabla\times\vec j_q))+\dots$$

(+) density $\rho(\vec r)=\sum_j|\phi_j(\vec r)|^2$

(-) current $\vec j(\vec r)=-\frac{i}{2}\sum_j(\phi_j^+(\vec r)\vec\nabla\phi_j(\vec r)-\phi_j(\vec r)\vec\nabla\phi_j^+(\vec r))$

(+) kin. en. dens. $\tau(\vec r)=\sum_j|\vec\nabla\phi_j(\vec r)|^2$

(-) spin dens. $\vec s(\vec r)=-i\sum_j\phi_j^+(\vec r)\vec\sigma\phi_j(\vec r)$

(+) spin-orb. dens. $\vec{\Im}(\vec r)=-i\sum_j\phi_j^+(\vec r)\vec\nabla\times\vec\sigma\phi_j(\vec r)$

(-) spin kin. en. dens. $\vec T(\vec r)=\sum_j\vec\nabla\phi_j(\vec r)\vec\sigma\cdot\vec\nabla\phi_j(\vec r)$

t-odd densities:
- follow from initial Skyrme forces
- keep Galilean invariance of the functional
- come only in specific combinations with t-even densities: →
- do not need new parameters

$$\vec s\cdot\vec T-\vec{\Im}^2$$

$$\rho\vec\nabla\cdot\vec{\Im}+\vec s\cdot(\vec\nabla\times\vec j)$$

$$\rho\tau-\vec j^2$$

!!!

Which densities do we need?

General arguments:

Single-particle density matrix:

$$\rho(\vec r\sigma,\vec r'\sigma') = \sum_i \phi_i(\vec r,\sigma)\phi_i^*(\vec r',\sigma')$$

$$= \frac{1}{2}[\rho(\vec r,\vec r')\delta_{\sigma\sigma'} + \sum_\nu \langle\sigma|\hat\sigma_\nu|\sigma'\rangle s_\nu(\vec r,\vec r')]$$

$$\rho(\vec r,\vec r') = \sum_\sigma \rho(\vec r\sigma,\vec r'\sigma),$$

$$s_\nu(\vec r,\vec r') = \sum_{\sigma\sigma'} \rho(\vec r\sigma,\vec r'\sigma')\langle\sigma'|\hat\sigma_\nu|\sigma\rangle$$

Other densities are first and second derivatives of basic densities ρ, $\vec s$:

$$\rho(\vec r) = \rho(\vec r,\vec r) \qquad \vec s(\vec r) = \vec s(\vec r,\vec r)$$

basic densities

$$\vec j(\vec r) = \frac{1}{2i}[(\vec\nabla - \vec\nabla')\rho(\vec r,\vec r')]_{\vec r=\vec r'} \qquad \vec{\Im}_{\mu\nu}(\vec r) = \frac{1}{2i}[(\vec\nabla_\mu - \vec\nabla_\mu')s_\nu(\vec r,\vec r')]_{\vec r=\vec r'}$$

their momenta (first derivatives)

$$\tau(\vec r) = [\vec\nabla\cdot\vec\nabla'\rho(\vec r,\vec r')]_{\vec r=\vec r'} \qquad \vec T(\vec r) = [\vec\nabla\cdot\vec\nabla'\vec s(\vec r,\vec r')]_{\vec r=\vec r'}$$

their kin. energies (second derivatives)

- **Some kind of gradient expansion**

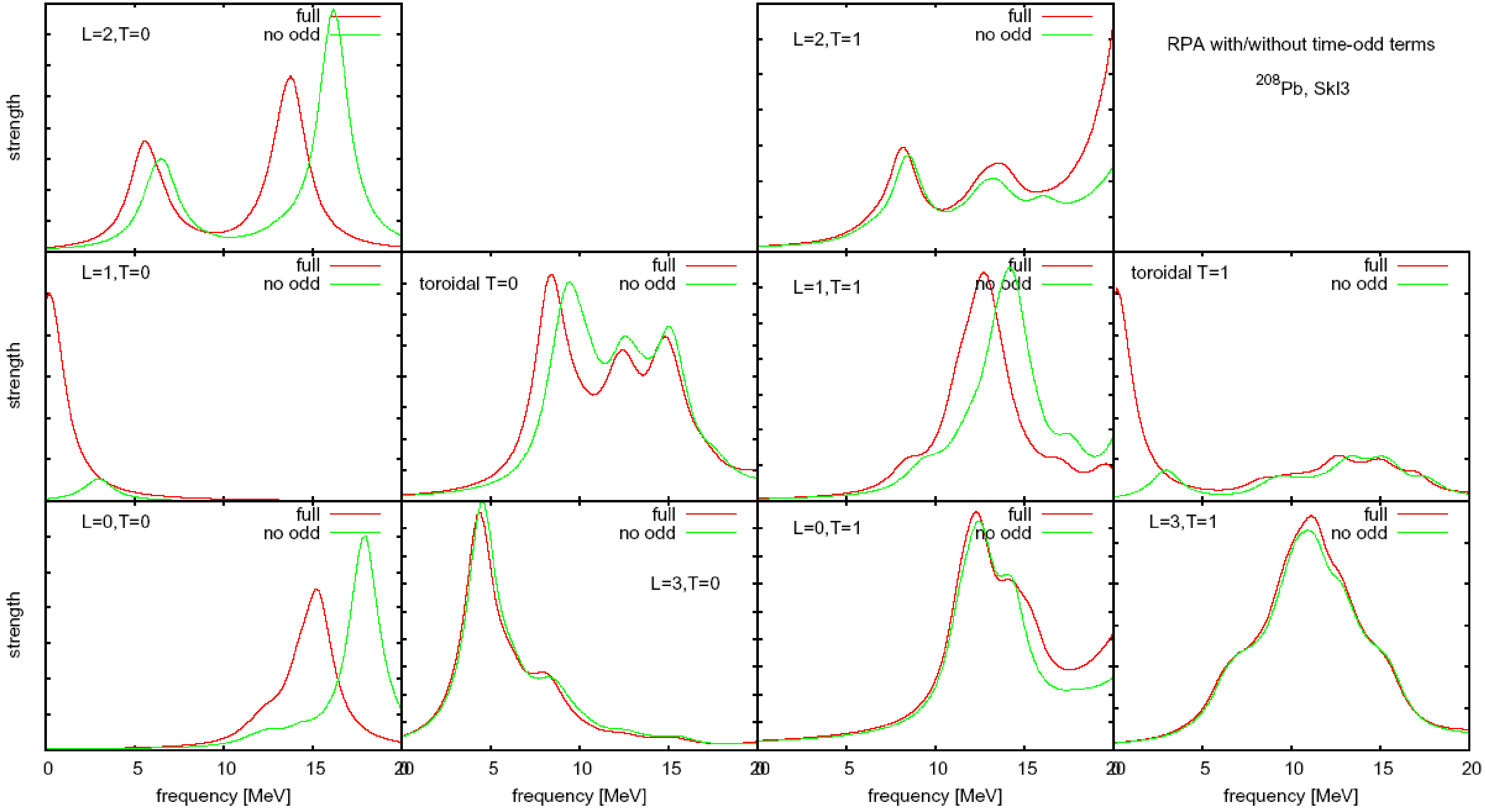
- **Combinations of densities in the functional must:**
  - **a) be time-even,**
  - **b) fulfill local gauge (Galilean) invariance**

Local gauge transformation

$$\Psi' = \exp\{i\sum_{j=1}^{N}\varphi(\vec r_j)\}\Psi$$

$\varphi(\vec r)$ **is real function**

Impact of T-odd densities
(P.-G. Reinhard)



SRPA (1)

Time-dependent formulation:

$$E(J_\alpha(\vec{r},t)) = \langle \Psi | H | \Psi \rangle ,$$

$$J_\alpha(\vec{r},t) \in \{\rho(\vec{r},t), \vec{j}(\vec{r},t), \dots\}$$

$$J_\alpha(\vec{r},t) =< \Psi | \hat{J}_\alpha | \Psi >$$ ← T-even and T-odd densities

$$J_\alpha(\vec{r},t) = \bar{J}_\alpha(\vec{r}) + \delta J_\alpha(\vec{r},t)$$ ← Linear regime: small time-dependent perturbation

$$h(\vec{r},t) = h_0(\vec{r}) + \delta h_{res}(\vec{r},t)$$ ← Mean field hamiltonian: static g.s. + time-dependent response

$$= \sum_\alpha [\frac{\delta E}{\delta J_\alpha}]_{J=\bar{J}} \hat{J}_\alpha(\vec{r}) + \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}]_{J=\bar{J}} \delta J_\alpha(\vec{r},t) \hat{J}_\alpha(\vec{r})$$

$$\delta J_\alpha(t) = \langle \Psi(t) | J_\alpha | \Psi(t) \rangle - \langle 0 | J_\alpha | 0 \rangle$$ ← **The only unknowns**

Now we have to specify the perturbed many-body wave function Ψ

SRPA (2)

$$V_{res} \Rightarrow \sum_{k,k'=1}^{K} \{\kappa_{kk'}\hat{X}_k \hat{X}_{k'} + \eta_{kk'}\hat{Y}_k \hat{Y}_k\}$$

Macroscopic step:

Perturbed w.f. via scaling: $\Psi(t) = \prod_{k=1}^{K} \exp\{-q_k(t)\hat{P}_k\}\exp\{-p_k(t)\hat{Q}_k\}|0\rangle,$

both $\Psi(t), |0\rangle$ **are Slater determinants**

$$\hat{Q}_k = \hat{Q}_k^+, \quad \hat{T}\hat{Q}_k\hat{T}^{-1} = \hat{Q}_k$$

$$\hat{P}_k = i[\hat{H}, \hat{Q}_k]_{ph} = \hat{P}_k^+, \quad \hat{T}\hat{P}_k\hat{T}^{-1} = -\hat{P}_k$$

$$q_k(t) = \bar{q}_k \cos(\omega t)$$

$$p_k(t) = \bar{p}_k \sin(\omega t)$$

$$\hat{h}_{res}(t) = \sum_k \{-q_k(t)\hat{X}_k + p_k(t)\hat{Y}_k\} \overset{kk'}{=} 1/2\sum_{kk'}\{\kappa_{kk'}\delta\hat{X}_k(t)\hat{X}_k + \eta_{kk'}\delta\hat{Y}_k(t)\hat{Y}_k\}$$

Microscopic step:

Perturbed w.f. via Thouless theorem: $\Psi_{Th}(t) = \{1 + \sum_{ph} c_{ph}(t)\hat{A}_{ph}^+)\}|0\rangle$

$$c_{ph}(t) = c_{ph}^+ e^{i\omega t} + c_{ph}^- e^{-i\omega t}$$

Merging step:

Both scaling and Thouless w.f. $\Psi(t)$ **must give equal variations:**

$$\delta\hat{X}_k(t)|_{sc} = \delta\hat{X}_k(t)|_{Th}, \quad \delta\hat{Y}_k(t)|_{sc} = \delta\hat{Y}_k(t)|_{Th}$$

SRPA (3)

Final RPA equations:

$$\sum_k \{\bar{q}_k (d_{kk'}(XX) - \kappa^{-1}_{kk'}) + \bar{p}_k d_{kk'}(XY)\} = 0$$

$$\sum_k \{\bar{q}_k d_{kk'}(YX) + \bar{p}_k (d_{kk'}(YY) - \eta^{-1}_{kk'})\} = 0$$

T-even T-odd

$$H = h_0 + 1/2 \sum_{kk'} \{\kappa_{kk'} \underbrace{\hat{X}_k \hat{X}_{k'}}_{\text{T-even}} + \eta_{kk'} \underbrace{\hat{Y}_k \hat{Y}_k}_{\text{T-odd}}\}$$

$\det[\omega_j] = 0 \Longrightarrow$ **RPA spectrum**

where e.g. $$d_{kk'}(XY) = \sum_{ph} \left[\frac{\langle ph|\hat{X}_k|0\rangle^* \langle ph|\hat{Y}_{k'}|0\rangle}{(\varepsilon_{ph} - \omega)} + \frac{\langle ph|\hat{X}_k|0\rangle \langle ph|\hat{Y}_{k'}|0\rangle^*}{(\varepsilon_{ph} + \omega)} \right]$$

$$\hat{X}_k(\vec{r}) = i \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}] \langle 0|[J_\alpha, \hat{P}_k]0\rangle \hat{J}_{\alpha'}$$

$$\hat{Y}_k(\vec{r}) = i \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}] \langle 0|[J_\alpha, \hat{Q}_k]0\rangle \hat{J}_{\alpha'}$$

$$\kappa^{-1}_{kk'} = i \int d\vec{r} \langle 0|[\hat{X}_k, \hat{P}_{k'}]|0\rangle$$

$$\eta^{-1}_{kk'} = i \int d\vec{r} \langle 0|[\hat{Y}_k, \hat{Q}_{k'}]|0\rangle$$

$$C^+ = \sum_{ph} [c^-_{ph} a^+_p a_h - c^+_{ph} a^+_h a_p]$$ **RPA phonon**

$$c^{\pm}_{ph} = -\frac{1}{2} \frac{\sum_k \{\bar{q}_k \langle ph|\hat{X}_k|0\rangle \mp i \bar{p}_k \langle ph|\hat{Y}_k|0\rangle\}}{\varepsilon_{ph} \pm \omega}$$

- **Rank of RPA matrix is 4K. For giant resonances usually K=2 is enough.**

Very low rank!

SRPA (4)

Accuracy:

- Choice of $\hat{Q}_k(\vec{r})$ is crucial for accuracy and simplicity of the method.

- SRPA itself does not provide the form of the input operators $\hat{Q}_k(\vec{r})$

- The choice is given by physical arguments:

$$\hat{Q}_k(\vec{r}) = f_k(r)\ (Y_{\lambda\mu}(\Omega) + Y^+_{\lambda\mu}(\Omega))$$

$k=1: \quad f_1(r) = r^{\lambda}$

- As external Eλ field in the long-wave limit
- X and Y operators have maxima at the nuclear surface
- dominate contribution

$k>1: \quad f_k(r) = r^{\lambda+2(k-1)}, \quad j_{\lambda}(q_k r)$

- X and Y operators have maxima in the nuclear interior
- minor contribution

Since probe different parts of the system, SRPA provides a high accuracy already for a couple of input operators.

Giant resonances:

E1(T=1) $\longrightarrow$ $f_1(r) = r, \quad f_2(r) = r^3$

E2(T=0) $\longrightarrow$ $f_1(r) = r^2, \quad f_2(r) = r^4$

SRPA accuracy

V.O. Nesterenko, J. Kvasil and P.-G. Reinhard
PRC, 66 044307 (2002)

Comparison of full RPA & SRPA…

Spherical ^{40}Ca, ^{208}Pb

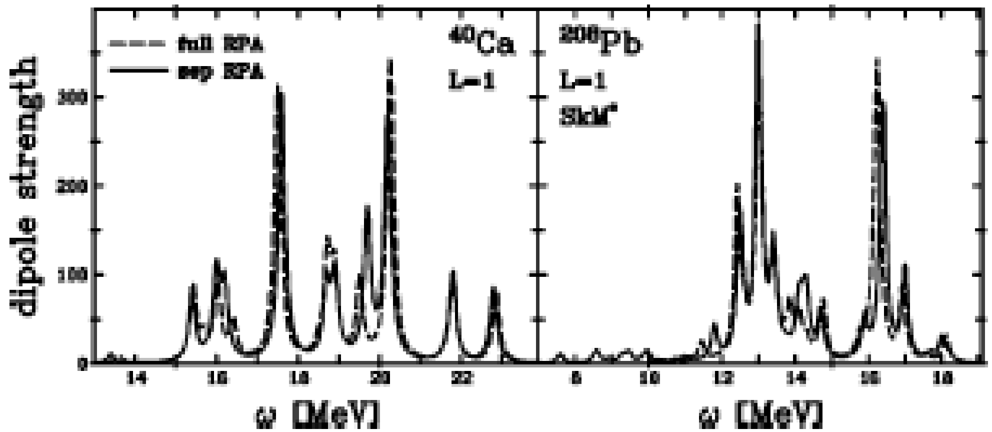
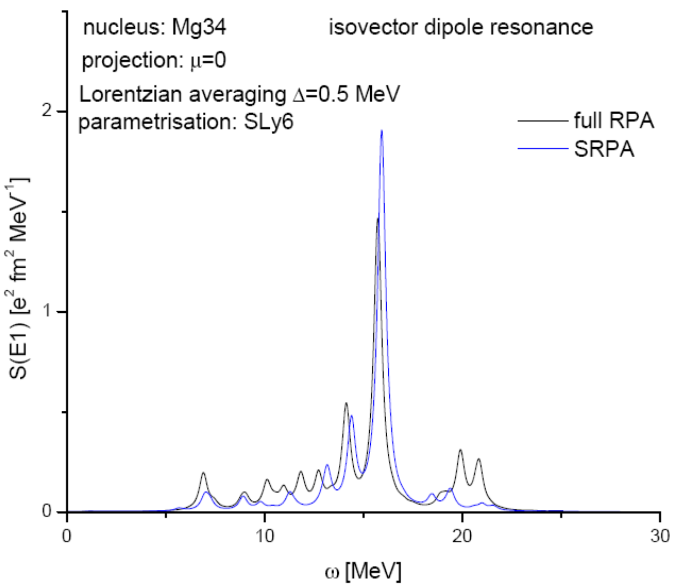
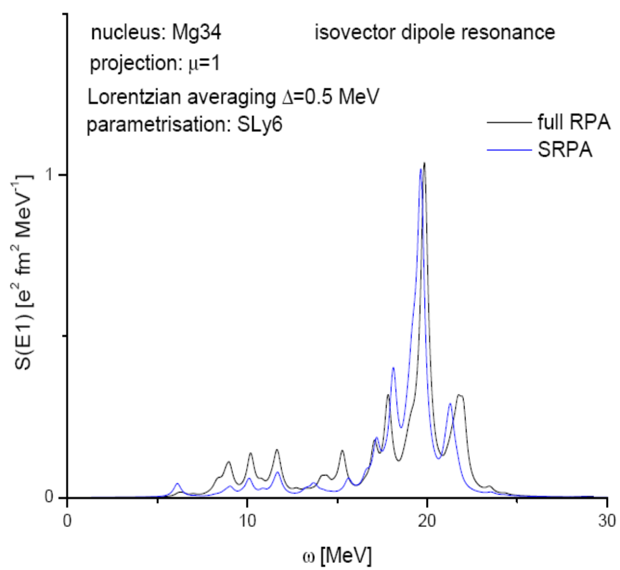


FIG. 2. Isovector $E1$ resonance in ^{40}Ca and ^{208}Pb calculated with SkM* forces in full RPA and SRPA with the complete set $k=1-4$. The responses are weighted by Lorentz function with the small averaging parameter $\Gamma=0.1$ MeV.

configuration space: 1s,1p,2s,1d,1f,2p,$1g_{9/2}$ i.e. first 50 states

Deformed ^{34}Mg





SRPA

Lorentz weight

$$\xi(\omega-\omega_\nu) = \frac{1}{2\pi}\frac{\Delta}{(\omega-\omega_\nu)^2 + (\Delta/2)^2}$$

L=0,1,3

Strength function:

$$S_L(D_{X\lambda\mu}) = \sum_\nu \omega_\nu^L <\nu|\hat{D}_{X\lambda\mu}|0>^2 \; \xi(\omega-\omega_\nu) =$$

$$= \frac{1}{\pi}\Im\Big[\frac{z^L\sum_{\beta\beta'}F_{\beta\beta'}(z)A_\beta(z)A_{\beta'}(z)}{F(z)}\Big]_{z=\omega+i\Delta/2} + \sum_{ph}\varepsilon_{ph}^L <ph|\hat{D}_{X\lambda\mu}|0>^2 \; \xi(\omega-\omega_\nu)$$

Contribution of residual inter. — **Unperturbed 2qp strength**

-30' for one nucleus instead of 2 weeks
- the only 2d Skyrme-RPA code for systematic calculations

Other features: Coulomb, spin-orbital, pairing
Contributions to the residual interaction

Alternative self-consistent separable RPA methods:

- **N.Van Giai, Ch.Stoyanov, V.V.Voronov, Phys.Rev. C57, 1204 (1998)**
- **A.P.Severyuchin, Ch.Stoyanov, V.V.Voronov, N.Van Giai, Phys.Rev. C66,034304 (2002)**

(-)) Larger rank of RPA matrix, K=400
(-) No T-odd densities, Coulomb, spin-orb., ...
(-) No deformed nuclei

Migdal forces with Skyrme parametrizations

Motivation

- Nuclear dynamics:
  - is still not so widely explored within Skyrme DFT as the nuclear ground states,
  - useful to upgrade Skyrme forces.

- Giant resonances:
  - bulk of experimental data,
  - not sensitive to details,
  - still unsolved fundamental problem:

  i) there are no Skyrme forces for simultaneous description of isoscalar E2(T=0) and isovector E1(T=1) GR.

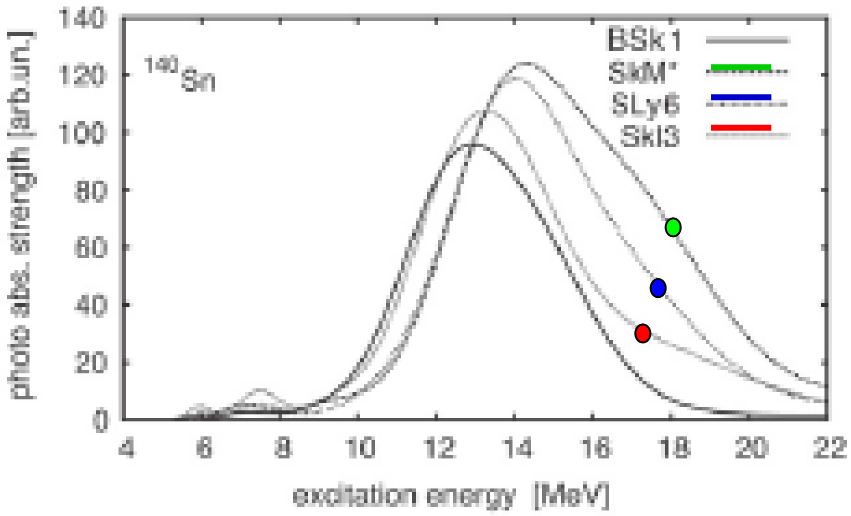
  E2(T=0) ↓ $m^*/m \sim 1$ E1(T=1) ↓ $m^*/m \sim 0.7$

  ii) unexplored magnetic GR (spin M1, …)

- Similar problems for GR in exotic and standard nuclei. But for standard nuclei we have exper. data. So, it is indeed worth to scrutinize carefully standard GR in standard nuclei.

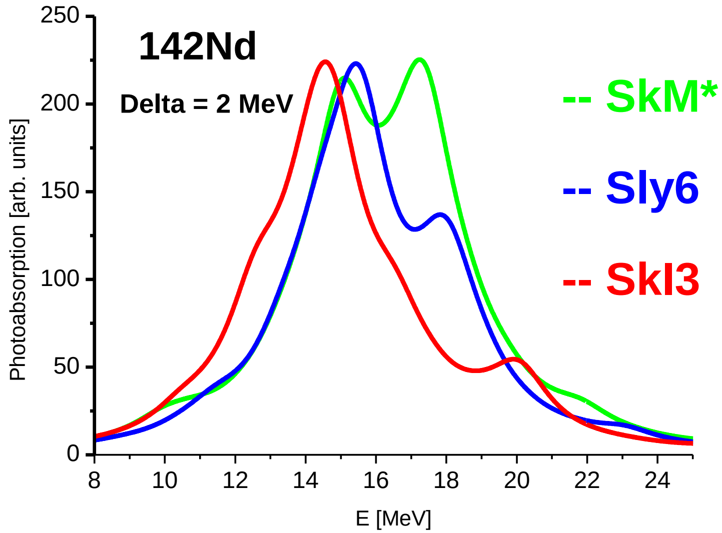
- Need in precise but practicable model for systematic exploration of GR: self-consistent separable Skyrme Random-Phase-Approximation (SRPA) model

Exotic vs Standard



J.R. Stone, P.-G. Reinhard
Prog. Part. Nucl. Phys., 58 (2007) 587-657.

Exotic neutron-excess nucleus 140Sn

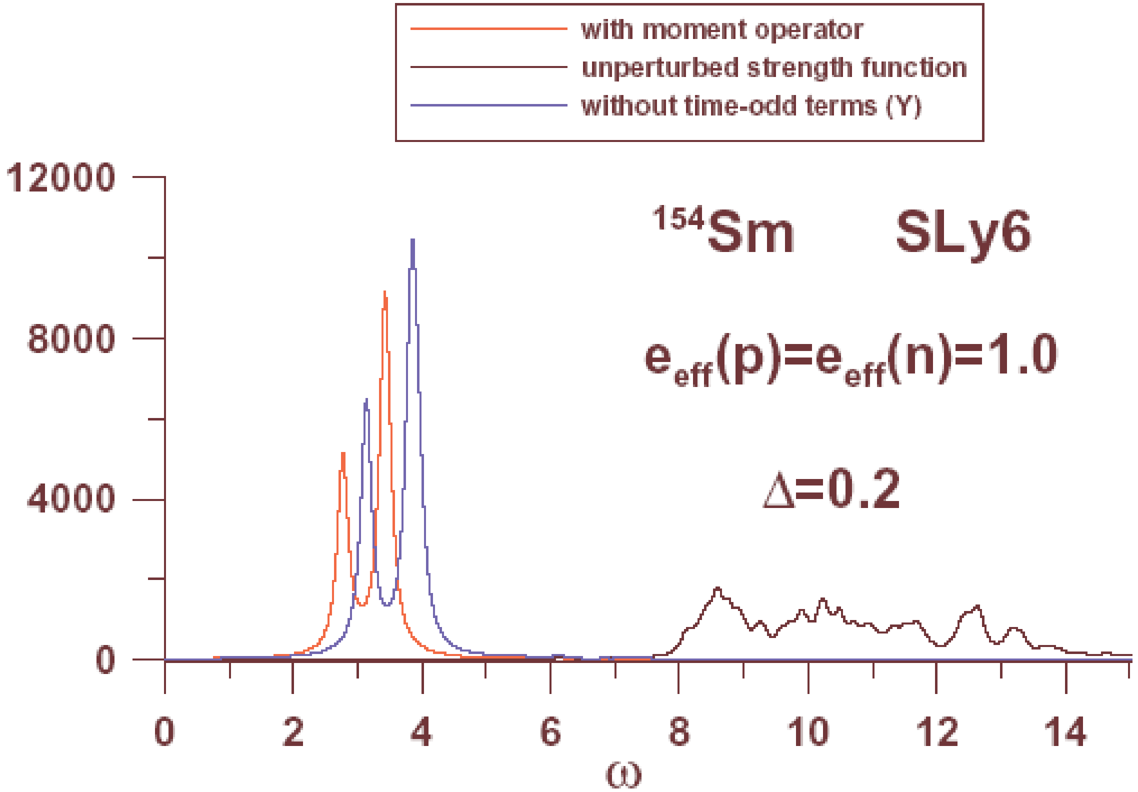


Standard nucleus 142Nd

Spurious admixtures for E1 excitations

$$[H, P_{sym}] = 0 \qquad P_{sym} \in \{P_k\}, \quad Q_{sym} \; [H, P_{sym}] = 0$$

$$[H_{SRPA}, P_{sym}] = 0, \quad [h_0, P_{sym}] = -[V_{res}, P_{sym}] = -X_{sym}$$



- SRPA residual interaction down-shifts the spurious E1(T=0) strength to the region of ~ 3-5 MeV.

- Introduction of P_{sym} does not influence E1(T=0). It seems that the main effect has been already taken into account by familiar operators

$$\hat{Q}_{1\mu k}, \quad \hat{P}_{1\mu k} = [\hat{H}, \hat{Q}_{1\mu k}]$$

- Is not yet clear how to put E1(T=0) precisely to

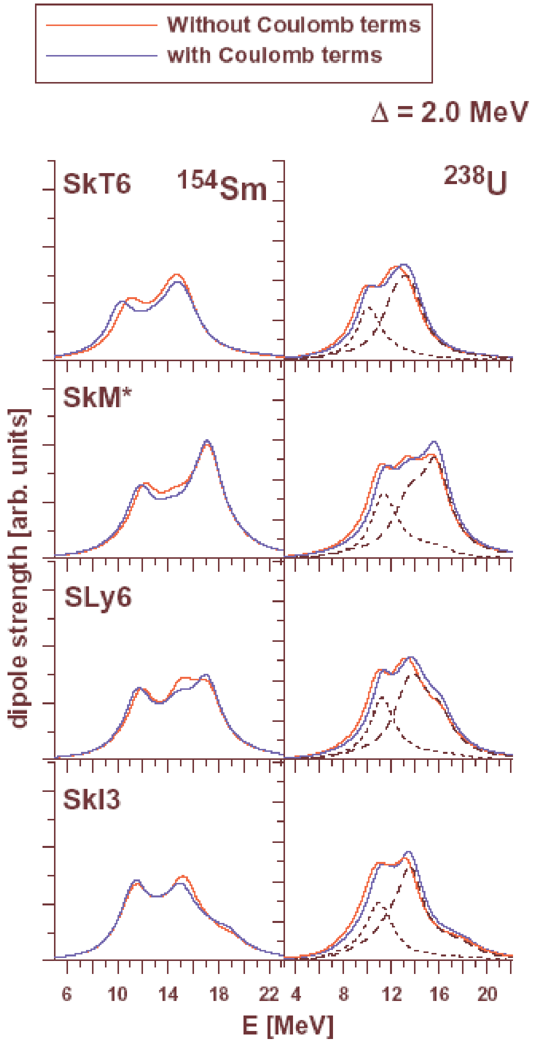
$$\omega = 0$$

Underestimated IS interaction?

Impact of Coulomb interaction

Impact is negligible for both giant resonances E1(T=1) and E2(T=0)

But Coulomb impact can be essential for low-energy states which are generally quite sensitive to most of ingredients of the residual interaction



Essentially different isoscalar and isovector characteristics

Skyrme forces:

Force	B_0	m_0^*/m	B_1	κ
SkT6	0	1.00	0	0.00
SkM*	35	0.79	34	0.53
SLy6	58	0.69	-26	0.25
SkI3	96	0.58	-64	0.25
SGII	36	0.79	29	0.50
SIII	44	0.76	31	0.52
SLy4	57	0.69	-25	0.25
SkO	15	0.90	8	0.17

$$\frac{1}{8}[t_1(2+x_1)+t_2(2+x_2)](\rho\tau - j^2)$$
$$+\frac{1}{8}[t_2(1+2x_2)-t_1(1+2x_1)](\rho_n\tau_n - j_n^2 + \rho_p\tau_p - j_p^2)]$$

$$\rho_0 = \rho_n + \rho_p$$
$$\rho_1 = \rho_n - \rho_p$$

$$B_0(\rho\tau - j^2) + B_1(\rho_1\tau_1 - j_1^2)$$

One may expect from very beginning correlation

$B_0, B_1 \longleftrightarrow m_0^*/m, \kappa \longleftrightarrow$ j-impact

m_0^*/m - isoscalar effective mass

$m_1^*/m=1/(1+\kappa)$ - isovector effective mass

κ - E1(T=1) sum rule enhancement factor

V.O. Nesterenko, W. Kleinig, J. Kvasil, P. Vesely, and P.-G. Reinhard, IJMP(E), 17, 89-99 (2008)

Energy-weighted sum rules:

E. Lipparini and S. Stringari, Phys. Rep. 175, 103 (1989):

$$EWSR(\lambda>1, T=0) = \frac{(\hbar e)^2}{8\pi m}\lambda(2\lambda+1)^2 A\left\langle r^{2\lambda-2}\right\rangle_A$$

$$EWSR(\lambda>1, T=1) = \frac{(\hbar e)^2}{8\pi m_1^*}\lambda(2\lambda+1)^2 A\left\langle r^{2\lambda-2}\right\rangle_A$$

$$EWSR(\lambda=1, T=1) = \frac{(\hbar e)^2}{8\pi m_1^*}9\frac{NZ}{A} = \frac{(\hbar e)^2}{8\pi m}9\frac{NZ}{A}(1+k)$$

For isoscalar modes the effective mass (τ) and current (j) contributions to EWSR(T=0) fully compensate each other and so EWSR(T=0) acquires the bare mass m.
For isovector modes the compensation is not complete and so EWSR(T=1) has the isovector effective mass m_1^*

	SkM*	Sly6	SkI3
m_0^*/m	0.79	0.69	0.58
m_1^*/m	0.65	0.80	0.80

Sum rules: $\alpha = \frac{EWSR_{SRPA}}{EWSR_{estim}}$

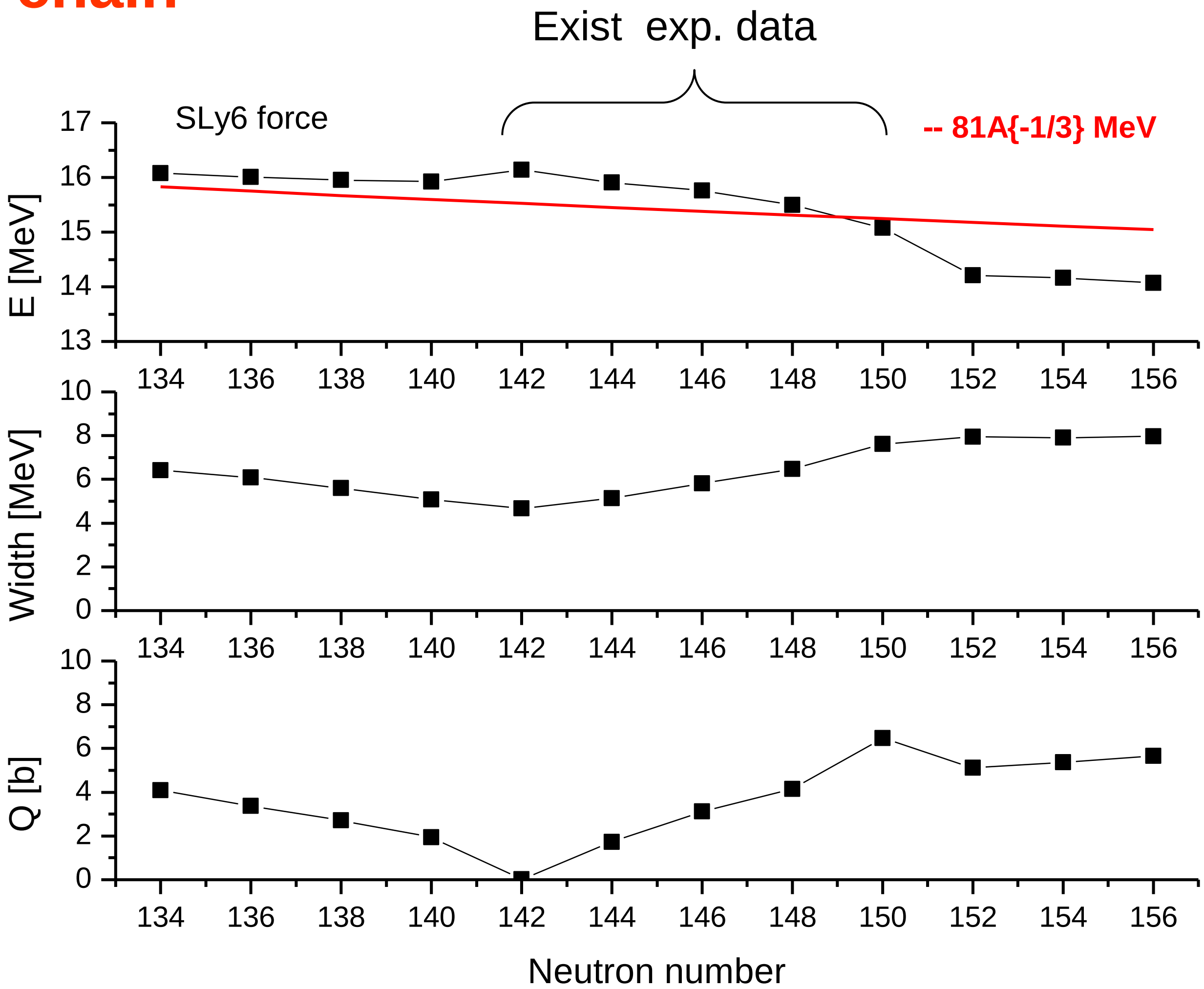
^{150}Nd

	SkT6	SkM*	SLy6	SkI3
$\alpha(E1, T=1)=$	0.99	0.90	0.94	0.93
$\alpha(E2, T=0)=$	0.96	0.97	0.95	0.95
$\alpha(E3, T=0)=$	0.75	0.74	0.73	0.72

Indeed large basis

If the current density j is taken into account, then $EWSR_{2qp} \neq EWSR_{SRPA}$

Nd chain



Exist exp. data
SLy6 force
-- 81A{-1/3} MeV
E [MeV]
17
16
15
14
13
Width [MeV]
10
8
6
4
2
0
Q [b]
10
8
6
4
2
0
134
136
138
140
142
144
146
148
150
152
154
156
Neutron number

Pairing

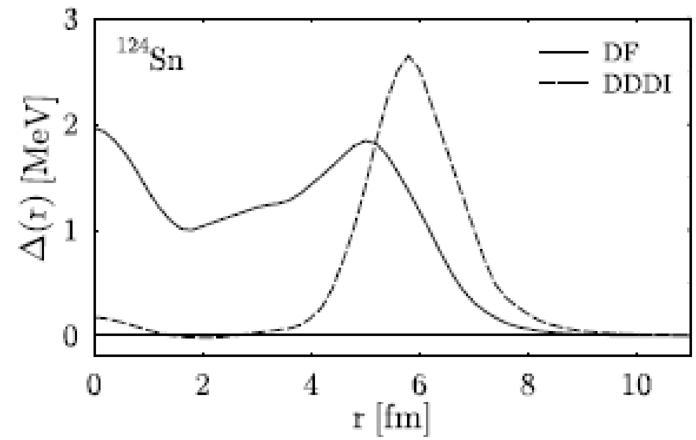
M. Bender et al, EPJA, 8, 59 (2000)

Pairing interaction in delta-force approximation:

$$V_q^{pair}(\vec{r},\sigma,\vec{r}\,',-\sigma') = G_q\,\delta(\vec{r}-\vec{r}\,')F_q(\vec{r})$$

Pairing energy functional:

$$E_{pair} = \frac{1}{4}\sum_{q\in p,n} G_q \int d\vec{r}\,\chi^*_q(\vec{r})\chi_q(\vec{r})F_q(\vec{r})$$



Pairing force:

$$F_q(\vec{r}) = \begin{cases} 1 \\ 1-\left(\dfrac{\rho(\vec{r})}{\rho_0}\right)^{\gamma} \end{cases}$$

- **delta force (DF),**
- **density-dependent delta interaction (DDDI)**

$$\rho(\vec{r}) = \rho_p(\vec{r}) + \rho_n(\vec{r})$$

$\rho_0 \gg \rho(\vec{r}=0)$ → **volume pairing (DF)**

$\rho_0 = \rho(\vec{r}=0) \approx 0.16\ fm^{-3}$, $\gamma \approx 1$ → **surface Pairing (DDDI)**

Pairing density:

$$\chi_q(\vec{r}) = -2\sum_{k>0\in q} u_k v_k\,|\phi_k(\vec{r})|^2$$

$$\chi_q = -2\sum_{k>0\in q} u_k v_k, \quad F_q = 1$$

- constant (schematic) pairing: no radial dependence in both χ_q and F_q

Input operators:

- Choice of $\hat{Q}_k(\vec{r})$ is crucial for accuracy and simplicity of the method.

- SRPA itself does not provide the form of the input operators $\hat{Q}_k(\vec{r})$, k<K

- The choice is given by physical arguments:

$$\hat{Q}_k(\vec{r}) = f_k(r)\ (Y_{\lambda\mu}(\Omega) + Y^{+}_{\lambda\mu}(\Omega))$$

$k=1: \quad f_1(r) = r^{\lambda}$

- As external Eλ field in the long-wave limit
- X and Y operators have maxima at the nucleus surface
- dominate contribution

$k>1: \quad f_k(r) = r^{\lambda+2(k-1)}, \ j_{\lambda}(q_k r)$

- X and Y operators have maxima in the nuclear interior
- minor contribution

Since probe different parts of the system, SRPA provides a high accuracy already for a couple of input operators.

Giant resonances: E1(T=1) $\longrightarrow$ $f_1(r) = r, \quad f_2(r) = r^3$

E2(T=0) $\longrightarrow$ $f_1(r) = r^2, \quad f_2(r) = r^4$

Low-energy states: E2(T=0) $\longrightarrow$ $f_{k=1}(r) = r^2, \quad f_{k=2,3,4}(r) = j(q_k r), \quad Q_{k=5} = r^4(Y_{4\mu} + h.c.)$

E3(T=0) $\longrightarrow$ $f_{k=1}(r) = r^3, \quad f_{k=2,3,4}(r) = j(q_k r), \quad Q_{k=5} = r(Y_{1\mu} + h.c.)$

SRPA (5): detailed expressions with isospin indices s={n,p}

$$h_{res}(\vec{r},t)=\sum_{ss'}\sum_{\alpha\alpha'}[\frac{\delta^2 E}{\delta J_{s\alpha}\delta J_{s'\alpha'}}]_{J=\bar{J}}\,\delta J_{s\alpha}(\vec{r},t)\,\hat{J}_{s'\alpha'}(\vec{r})$$

$$=\sum_{ss'}\sum_{k}\{q_{qs}(t)\hat{X}^{s'}_{sk}+p_{qs}(t)\hat{Y}^{s'}_{sk}\}=\sum_{ss'}\sum_{kk'}\{\kappa^{s'k'}_{sk}\delta\hat{X}_{sk}(t)\hat{X}_{s'k'}+\eta^{s'k'}_{sk}\delta\hat{Y}_{sk}(t)\hat{Y}_{s'k'}\}$$

$$\delta J_{s\alpha}(t)=i\sum_{k}\{q_{s\alpha}(t)\langle[\hat{P}_{sk},J_{s\alpha}]\rangle+p_{s\alpha}(t)\langle[\hat{Q}_{sk},J_{s\alpha}]\rangle\}$$

$$\hat{X}_{sk}=\sum_{s'}\hat{X}^{s'}_{sk}=i\sum_{s'\alpha\alpha'}[\frac{\delta^2 E}{\delta J_{s\alpha}\delta J_{s'\alpha'}}]\ \langle[\hat{P}_{sk},\hat{J}_{s\alpha}]\rangle\ \hat{J}_{s'\alpha'}$$

$$\hat{Y}_{sk}=\sum_{s'}\hat{Y}^{s'}_{sk}=i\sum_{s'\alpha\alpha'}[\frac{\delta^2 E}{\delta J_{s\alpha}\delta J_{s'\alpha'}}]\ \langle[\hat{Q}_{sk},\hat{J}_{s\alpha}]\rangle\ \hat{J}_{s'\alpha'}$$

$$[\kappa^{-1}]^{s'k'}_{sk}=[\kappa^{-1}]^{sk}_{s'k'}=-i\langle[\hat{P}_{s'k'},\hat{X}^{s'}_{sk}]\rangle$$

$$[\eta^{-1}]^{s'k'}_{sk}=[\eta^{-1}]^{sk}_{s'k'}=-i\langle[\hat{Q}_{s'k'},\hat{Y}^{s'}_{sk}]\rangle$$

If $\hat{A}^{\dagger}=\hat{A},\quad \hat{T}^{-1}\hat{A}_{\pm}\hat{T}=\pm\hat{A}_{\pm}$ **then**

$$\langle[\hat{A}_{+},\hat{B}_{+}]\rangle=\langle[\hat{A}_{-},\hat{B}_{-}]\rangle=0$$

$$\langle[\hat{A}_{+},\hat{B}_{-}]\rangle\neq 0$$

Calculation of average commutators via s-p matrix elements →

$$\langle[\hat{A},\hat{B}]\rangle=\sum_{ph}\{\langle|\hat{A}|ph\rangle\langle ph|\hat{B}|\rangle-\langle|\hat{B}|ph\rangle\langle ph|\hat{A}|\rangle\}$$