Microscopic description of dipole pygmy resonances in spherical nuclei

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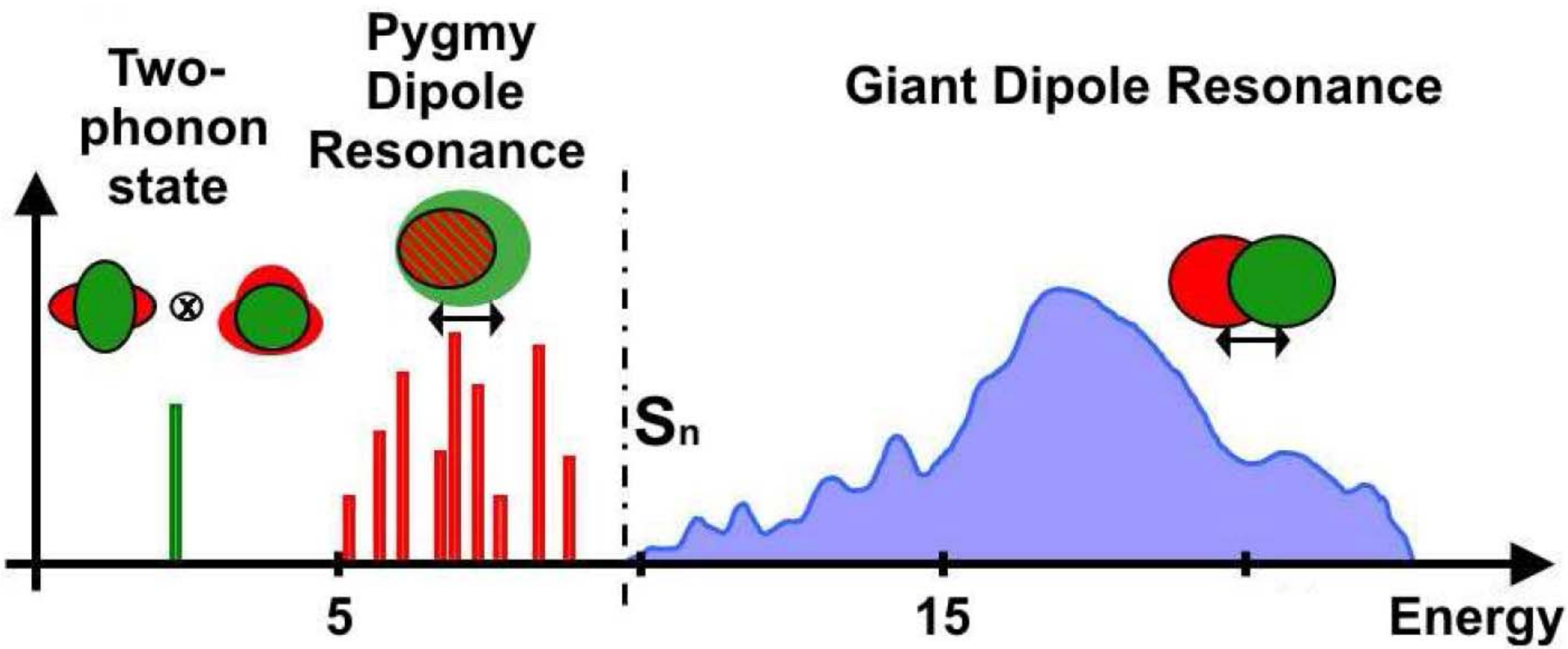
Outline

- Introduction
- The method
- Properties of GDR in $^{124-132}Sn$
- Low-energy spectrum of 1^- states of $^{124-132}Sn$
- Summary

$E1$ strength in (spherical) atomic nuclei



Courtesy: N. Pietralla

1. The properties of the low-lying electric dipole strength in stable and radioactive atomic nuclei have been extensively investigated in many experiments:

– *D. Savran et al.*, Prog. Part. Nucl. Phys. **70**, 210 (2013).

2. The experimental efforts have stimulated theoretical analysis based on the quasiparticle random phase approximation (QRPA)

– *J. Terasaki and J. Engel*, Phys. Rev. **C74**, 044301 (2006);

on the quasiparticle phonon model (QPM) including complex configurations

– *V.Yu. Ponomarev et al.*, Phys. Rev. **C57**, 2229 (1998), Phys. Rev. **C84**, 024326 (2011).

– *N. Tsoneva and H. Lenske*, Phys. Rev. **C77**, 024321 (2008);

on the relativistic RPA and QRPA

– *N. Paar et al.*, Phys. Lett. **B606**, 288 (2005);

on the extended theory of finite Fermi systems (ETFFS)

– *S. Kamerdzhiev et al.*, Phys. Rep. **393**, 1 (2004).

3. The PDR might play an important role in nuclear astrophysics. For example, the occurrence of the PDR could have a pronounced effect on neutron-capture rates in the r-process nucleosynthesis, and consequently on the calculated elemental abundance distribution.

– *S. Goriely*, Phys. Lett. **B436**, 10 (1998).

METHOD OF CALCULATIONS

1. We employ the effective Skyrme interaction in the particle-hole channel

$$\mathbf{V_{12}} = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) +$$

$$(1/2)t_1(1 + x_1 P_\sigma)[\mathbf{k}'^2\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{k}^2] + t_2(1 + x_2 P_\sigma)\mathbf{k}'\delta(\mathbf{r})\mathbf{k} +$$

$$(1/6)t_3(1 + x_3 P_\sigma)\rho^\alpha(\mathbf{R})\delta(\mathbf{r}) + iW_0[\mathbf{k}' \times \delta(\mathbf{r})\mathbf{k}](\sigma_i + \sigma_j),$$

For the pairing interaction we take a density-dependent zero-range force given by

$$\mathbf{V_{12}} = V_0 \left[1 - \eta \left(\frac{\rho(\mathbf{R})}{\rho_c}\right)^\gamma\right] \delta\left(\mathbf{r}\right)$$

where

$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $P_\sigma = (1 + \sigma_1\sigma_2)/2$,

$\mathbf{k} = (\overrightarrow{\nabla}_1 - \overrightarrow{\nabla}_2)/2i$, $\mathbf{k}' = -(\overleftarrow{\nabla}_1 - \overleftarrow{\nabla}_2)/2i$.

$\Downarrow$

2. **We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:**

$$a_{jm}^{+} = u_j \alpha_{jm}^{+} + (-1)^{j-m} v_j \alpha_{j-m}.$$

The single-quasiparticle spectrum is generated by the HF-BCS calculations , where spherical symmetry is assumed for the ground state. ***The parameters of the particle-particle interaction are fixed to reproduce the odd-even mass difference of neighboring nuclei.***

$\Downarrow$

3. **The residual interaction V_{res} can be obtained as the second derivative of the energy density functional $\mathcal{H}$ with respect to the particle density ρ and the pairing density $\tilde{\rho}$**

$$V_{res}^{p-h} \sim \frac{\delta^2 \mathcal{H}}{\delta \rho_1 \delta \rho_2}$$

$$V_{res}^{p-p} \sim \frac{\delta^2 \mathcal{H}}{\delta \tilde{\rho}_1 \delta \tilde{\rho}_2}$$

$\Downarrow$

4. We simplify the p-h interaction by approximating it by its Landau-Migdal form. We keep only Landau parameters F_0, G_0, F_0', G_0'

$$V_{res}^{p-h}(\mathbf{r}_1, \mathbf{r}_2) = N_0^{-1}\left[F_0(r_1) + G_0(r_1)\sigma_1\sigma_2 + (F_0'(r_1) + G_0'(r_1)\sigma_1\sigma_2)\tau_1\tau_2\right]\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

The corresponding Landau parameters can be expressed via the Skyrme force parameters

$$F_0 = N_0\{\tfrac{3}{4}t_0 + \tfrac{1}{16}t_3\rho^\alpha(\alpha+1)(\alpha+2) + \tfrac{1}{8}k_F^2[3t_1 + (5+4x_2)t_2]\},$$

$$F_0' = -N_0\{\tfrac{1}{4}t_0(1+2x_0) + \tfrac{1}{24}t_3\rho^\alpha(1+2x_3) + \tfrac{1}{8}k_F^2[t_1(1+2x_1) - t_2(1+2x_2)]\},$$

$$G_0 = -N_0\{\tfrac{1}{4}t_0(1-2x_0) + \tfrac{1}{24}t_3\rho^\alpha(1-2x_3) + \tfrac{1}{8}k_F^2[t_1(1-2x_1) - t_2(1+2x_2)]\},$$

$$G_0' = -N_0[\tfrac{1}{4}t_0 + \tfrac{1}{24}t_3\rho^\alpha + \tfrac{1}{8}k_F^2(t_1 - t_2)],$$

where $N_0 = 2k_F m^*/\pi^2\hbar^2$. Nguyen Van Giai, H. Sagawa, Phys. Lett. B 106, 379 (1981).

5. The residual interaction is presented as a sum of N separable terms.

$$\hat{V}_{res} = \frac{1}{2}\sum_{1234} V_{1234} : a_1^+ a_2^+ a_4 a_3 :$$

$$V_{1234} = \int \phi_1^*(\mathbf{r}_1)\phi_2^*(\mathbf{r}_2)V_{res}(\mathbf{r}_1,\mathbf{r}_2)\phi_3(\mathbf{r}_1)\phi_4(\mathbf{r}_2)\mathbf{dr}_1\mathbf{dr}_2$$

Let us explain this procedure for making the finite rank approximation by examining only the contribution of the term F_0

$$V_{1234}^{p-h(J)} \sim \langle j_1||i^J Y_J||j_3\rangle\langle j_2||i^J Y_J||j_4\rangle I^{p-h}(j_1 j_2 j_3 j_4)$$

$$V_{1234}^{p-p(J)} \sim \sum_{\lambda}(-1)^{\lambda}\begin{Bmatrix} j_4 & j_3 & J \\ j_1 & j_2 & \lambda \end{Bmatrix} \times \langle j_1||i^{\lambda} Y_{\lambda}||j_3\rangle\langle j_2||i^{\lambda} Y_{\lambda}||j_4\rangle I^{p-p}(j_1 j_2 j_3 j_4)$$

$$V^{pp}_{1234} - V^{pp}_{1243} = \sum_{JM} C^{JM}_{j_1 m_1 j_2 m_2} C^{JM}_{j_3 m_3 j_4 m_4} \frac{\langle j_1 \| i^J Y_J \| j_2 \rangle \langle j_3 \| i^J Y_J \| j_4 \rangle}{2J+1} I^{pp}(j_1 j_2 j_3 j_4)$$

A.P.S., V.V. Voronov, and Nguyen Van Giai, Phys. Rev. C 77, 024322 (2008).

$$I(j_1j_2j_3j_4) \;=\; N_0^{-1}\int_0^{\infty\to R} \frac{F_0(r)}{r^2}u_{j_1}(r)u_{j_2}(r)u_{j_3}(r)u_{j_4}(r)dr$$

This radial integral can be calculated by using a *N-point integration Gauss formula* with abscissas and weights $r_k, \mathrm{w_k}$

$$I(j_1j_2j_3j_4) \simeq N_0^{-1}\frac{R}{2}\sum_{k=1}^{N}\frac{\mathrm{w_k F_0(r_k)}}{r_k^2}u_{j_1}(r_k)u_{j_2}(r_k)u_{j_3}(r_k)u_{j_4}(r_k)$$

$$\Downarrow$$

$$\hat{V}_{res}^{p-h} \sim \sum_{k=1}^{N}\sum_{\lambda m}\frac{\mathrm{w_k F_0^{p-h}(r_k)}}{r_k^2}\hat{M}^+_{\lambda m,k}\hat{M}_{\lambda m,k}$$

$$\hat{V}_{res}^{p-p} \sim \sum_{k=1}^{N}\sum_{\lambda m}\frac{\mathrm{w_k F_0^{p-p}(r_k)}}{r_k^2}\hat{P}^+_{\lambda m,k}\hat{P}_{\lambda m,k}$$

$$\Downarrow$$

Thus, the QRPA solutions would be obtained with the finite rank approximation to the residual interaction.

We introduce the phonon creation operators

$$Q^{+}_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\left(X^{\lambda i}_{jj'}\,A^{+}(jj';\lambda\mu) - (-1)^{\lambda-\mu}Y^{\lambda i}_{jj'}\,A(jj';\lambda-\mu)\right)$$

$$A^{+}(jj';\lambda\mu) = \sum_{mm'}\langle jmj'm' \mid \lambda\mu\rangle \alpha^{+}_{jm}\alpha^{+}_{j'm'}$$

One assumes that the QRPA ground state is the phonon vacuum $\mid 0\rangle$ $\quad Q_{\lambda\mu i}\mid 0\rangle = 0$

We define the excited states by $\quad Q^{+}_{\lambda\mu i}\mid 0\rangle$

QRPA equations

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \omega\begin{pmatrix} X \\ Y \end{pmatrix}$$

Solutions of this set of linear equations yield the eigen-energies and the amplitudes X, Y of the excited states. A dimension of the matrices $\mathcal{A}, \mathcal{B}$ is a space size of the two-quasiparticle configurations.

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} X^{\lambda i} \\ Y^{\lambda i} \end{pmatrix} = \omega_{\lambda i}\begin{pmatrix} X^{\lambda i} \\ Y^{\lambda i} \end{pmatrix}$$

$$\Downarrow$$

the finite rank approximation

$$V_{1234}^{p-h(J)} \sim \sum_{k=1}^{N} \frac{\mathrm{w}_k F_0^{p-h}(r_k)}{r_k^{\;2}} M_{13}^{(J,k)} M_{24}^{(J,k)}$$

$$\Downarrow$$

the secular equation $(N \times N)$

$$\det\left|\frac{\omega_k F_0^{p-h}(r_k)}{r_k^{\;2}} T^{kk'} - \delta_{kk'}\right| = 0 \quad \Rightarrow \quad \omega_{\lambda i}$$

where

$$T^{kk'} = \sum_{jj'} \frac{M_{jj'}^{(J,k)} M_{jj'}^{(J,k')} (\varepsilon_j + \varepsilon_{j'})(u_j v_{j'} + u_{j'} v_j)^2}{(\varepsilon_j + \varepsilon_{j'})^2 - \omega_{\lambda i}^2}$$

Using the finite rank approximation we need to invert a matrix having a dimension independently of the configuration space size.

$$\Downarrow$$

Our calculations show that, for the normal parity states one can neglect the spin-multipole interactions. **In this case the matrix dimension is** $6N \times 6N$**.**

If we omit terms of the residual interaction in the particle-particle channel then the matrix dimension is reduced by a factor 3 $(2N \times 2N)$ **. Our investigations enable us to conclude that** N**=45** **is enough for multipolarities** $\lambda \leq 6$ **in nuclei with** $A \leq 208$**.**

6. The effect of the phonon-phonon coupling.

In the simplest case we can write the wave functions of excited states as:

$$|\Psi_{JM\nu}\rangle = \{\sum_{i} R_i(J\nu) Q^{+}_{JMi} +$$

$$+ \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^{+}_{\lambda_1 \mu_1 i_1} Q^{+}_{\lambda_2 \mu_2 i_2}\right]_{JM}\}|0\rangle$$

with the normalization condition

$$\langle \Psi_{JM\nu} | \Psi_{JM\nu} \rangle = \sum_{i} R_i^2(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} (P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 = 1.$$

Using the variational principle in the form:

$$\delta \left(\langle \Psi_{JM\nu} | H | \Psi_{JM\nu} \rangle - E_\nu (\langle \Psi_{JM\nu} | \Psi_{JM\nu} \rangle - 1) \right) = 0$$

one obtains a set of linear equations for the unknown amplitudes $\mathrm{R_i(J\nu)}$ and $\mathrm{P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)}$:

$$(\omega_{\mathbf{Ji}} - \mathbf{E}_\nu)\ \mathbf{R_i}(\mathbf{J}\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} \mathbf{U}^{\lambda_1 i_1}_{\lambda_2 i_2}(\mathbf{J}\nu)\mathbf{P}^{\lambda_1 i_1}_{\lambda_2 i_2}(\mathbf{J}\nu) = \mathbf{0}$$

$$\sum_i \mathbf{U}^{\lambda_1 i_1}_{\lambda_2 i_2}(\mathbf{Ji})\,\mathbf{R_i}(\mathbf{J}\nu) + \mathbf{2}(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \mathbf{E}_\nu)\mathbf{P}^{\lambda_1 i_1}_{\lambda_2 i_2}(\mathbf{J}\nu) = \mathbf{0}$$

$\mathbf{U}^{\lambda_1 i_1}_{\lambda_2 i_2}(\mathbf{Ji},\tau)$ denote matrix elements coupling one- and two-phonon configurations

$$\mathbf{U}^{\lambda_1 i_1}_{\lambda_2 i_2}(\mathbf{Ji},\tau) = \langle \mathbf{0}|\mathbf{Q_{Ji}H}\left[\mathbf{Q}^+_{\lambda_1 i_1}\mathbf{Q}^+_{\lambda_2 i_2}\right]_{\mathbf{J}}|\mathbf{0}\rangle$$

These equations have the same form as basic equations of the quasiparticle-phonon model (QPM). However, in contrast to the QPM all parameters of the model hamiltonian are expressed through parameters of the Skyrme forces.

A.P.S., V.V. Voronov, and Nguyen Van Giai, Eur. Phys. J. A22, 397 (2004).

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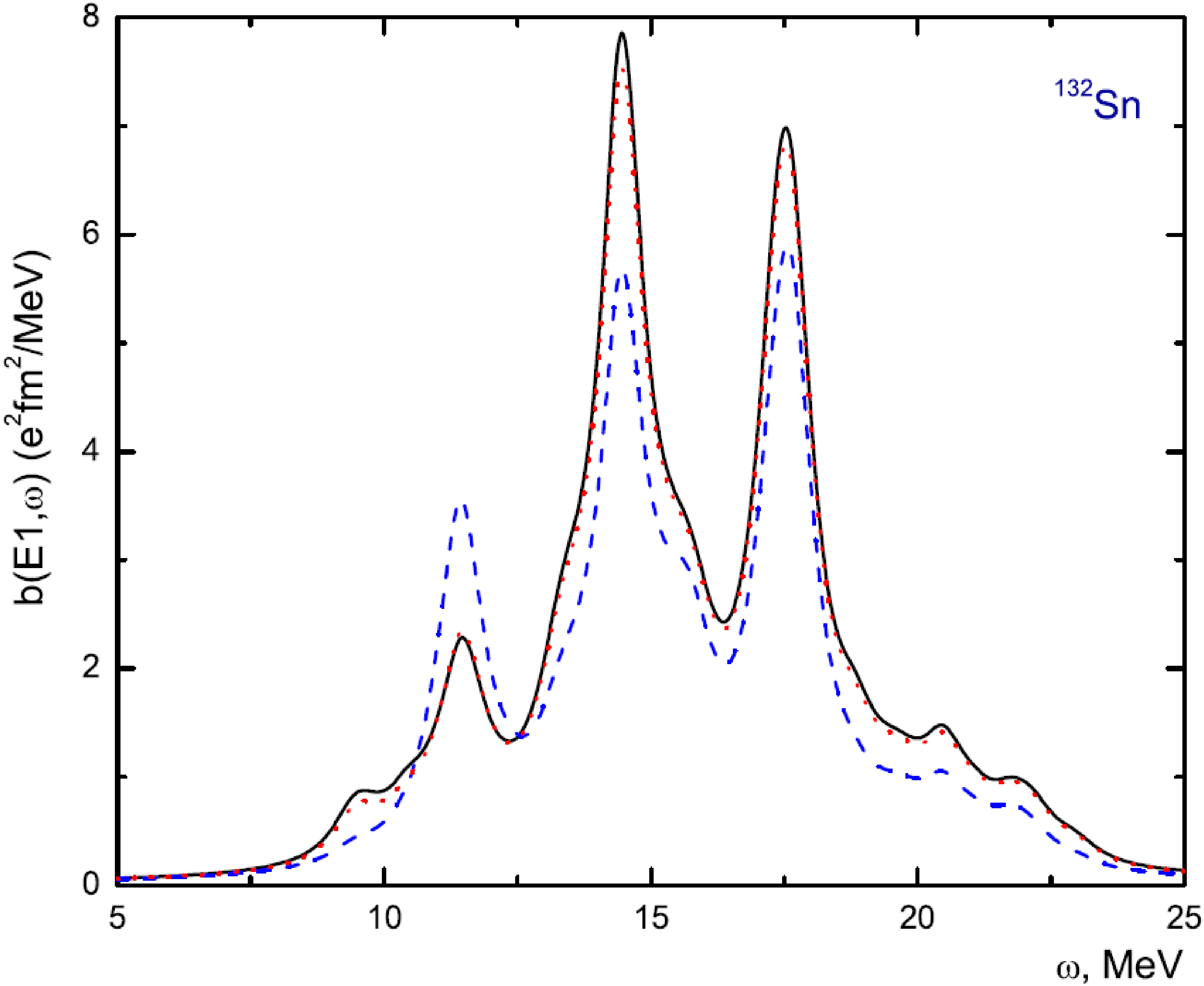

N. N. Arsenyev and A. P. Severyukhin, Phys. Part. Nucl. Lett. 7, 112 (2010).

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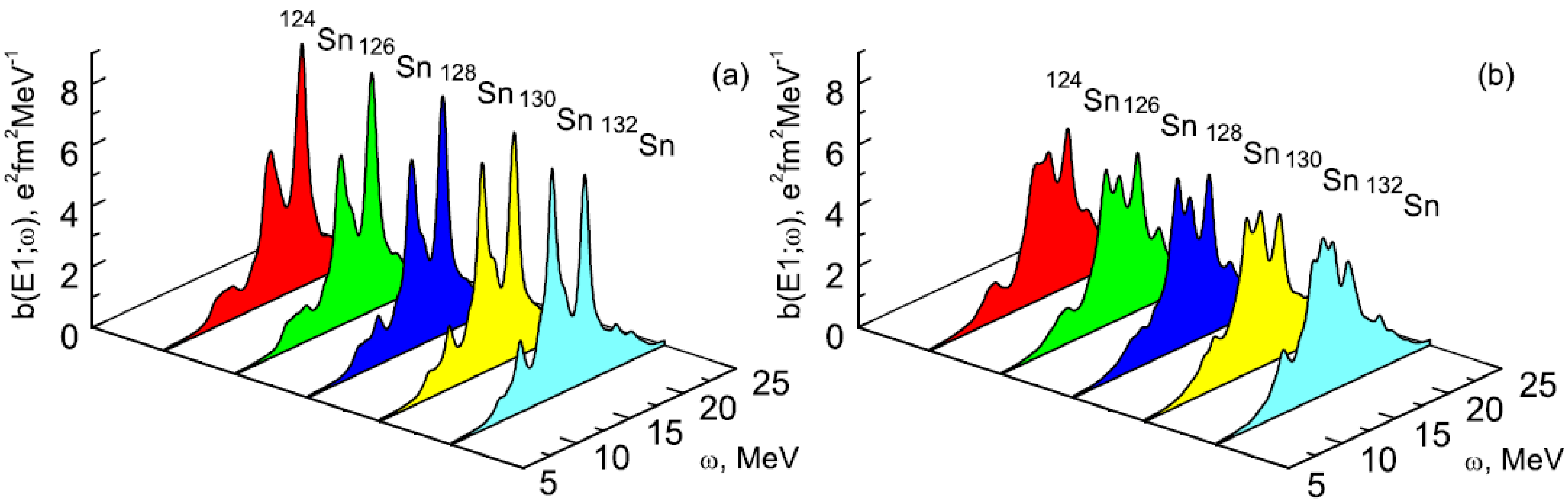

N. N. Arsenyev, A. P. S., V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC **38**, 17002 (2012).

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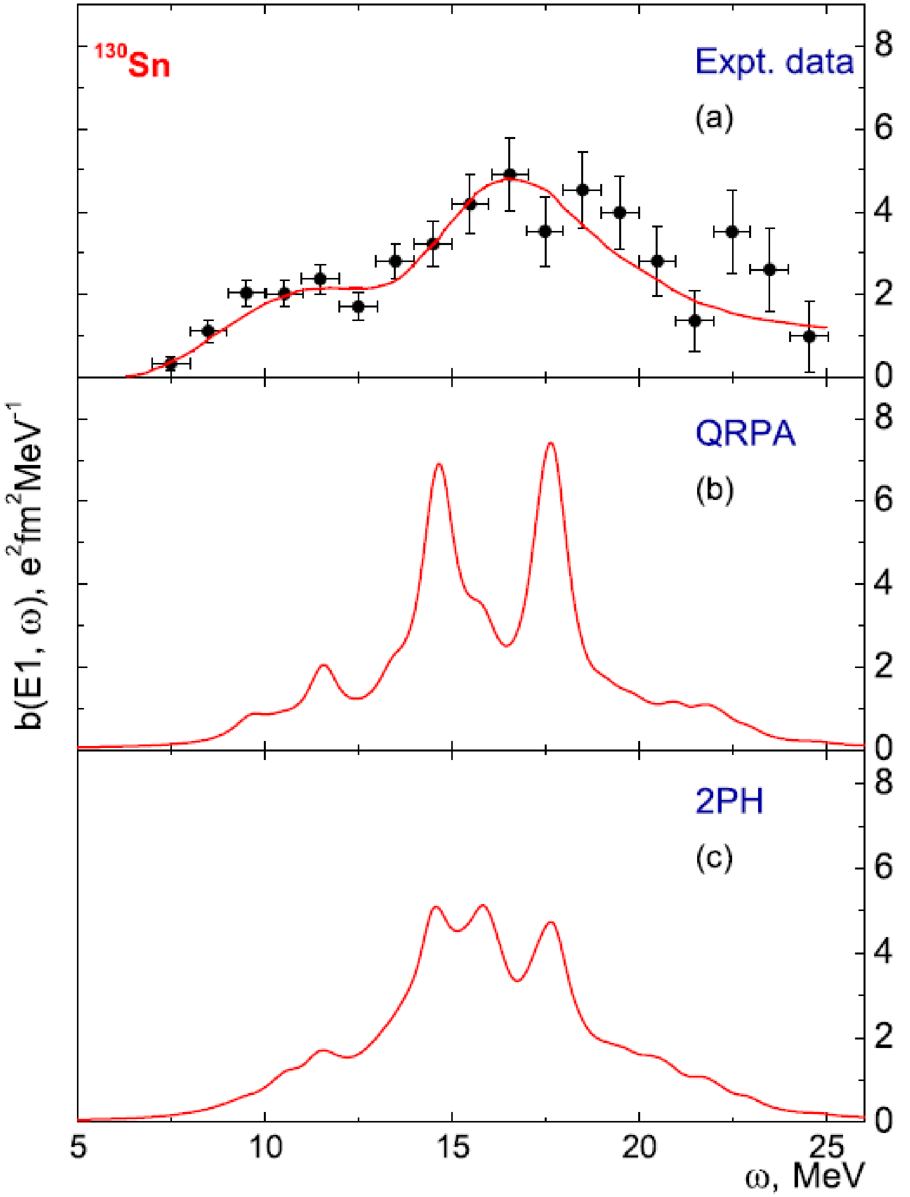

N. N. Arsenyev, A. P. S., V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC **38**, 17002 (2012).

P. Adrich et al., Phys. Rev. Lett. **95**, 132501 (2005).

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Nucleus	$\bar{E}$, MeV			Γ, MeV		
	QRPA	2PH	Expt.	QRPA	2PH	Expt.
^{124}Sn	16.4	16.3	15.3	4.4	4.7	4.8
^{126}Sn	16.2	16.2		4.4	4.7	
^{128}Sn	16.1	16.0		4.7	4.7	
^{130}Sn	15.8	15.7	15.9±0.5	4.8	4.8	4.8±1.7
^{132}Sn	15.5	15.4	16.1±0.7	4.9	5.0	4.7±2.1

*

N. N. Arsenyev, A. P. S., V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC **38**, 17002 (2012).

P. Adrich et al., Phys. Rev. Lett. **95**, 132501 (2005).

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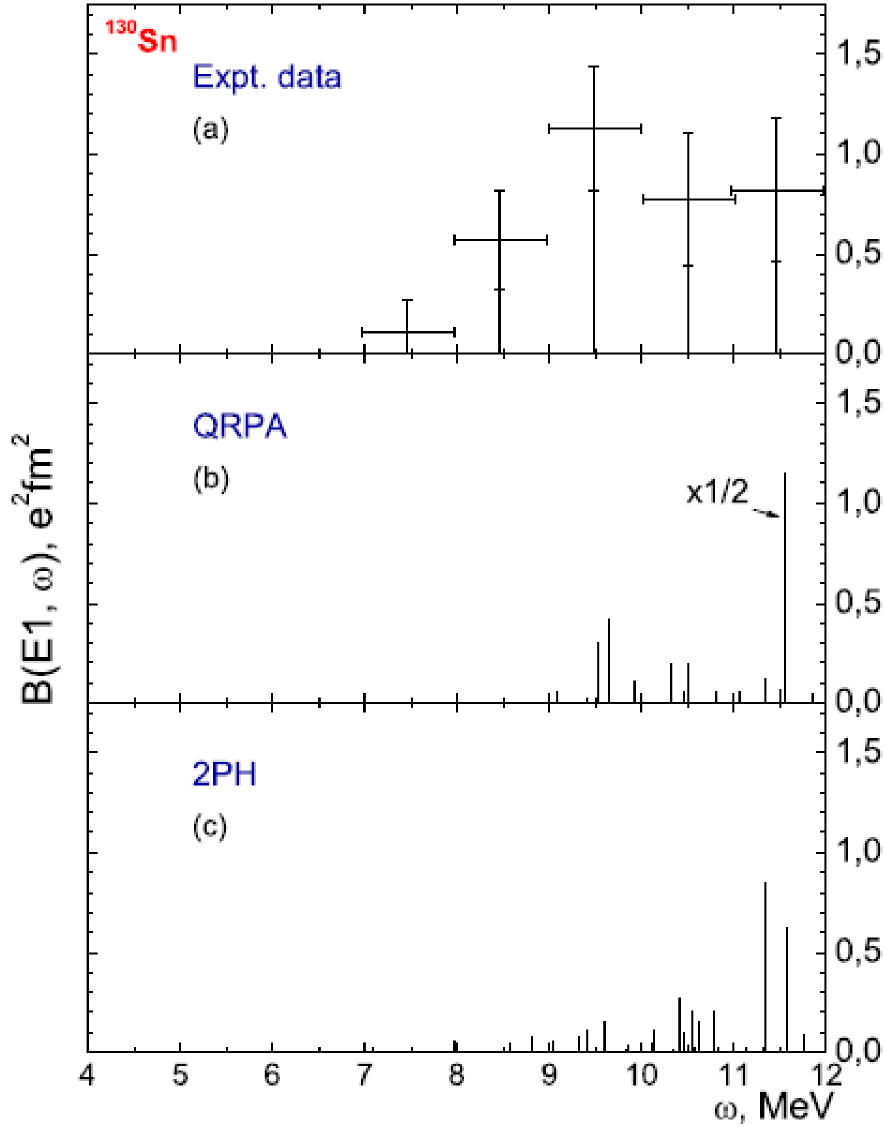

PDR width:
1.0 MeV – QRPA
1.8 MeV – 2PH
< 3.4 MeV – an upper limit

N. N. Arsenyev, A. P. S., V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC 38, 17002 (2012).

A. Klimkiewicz et al., Phys. Rev. C76, 051603R (2007).

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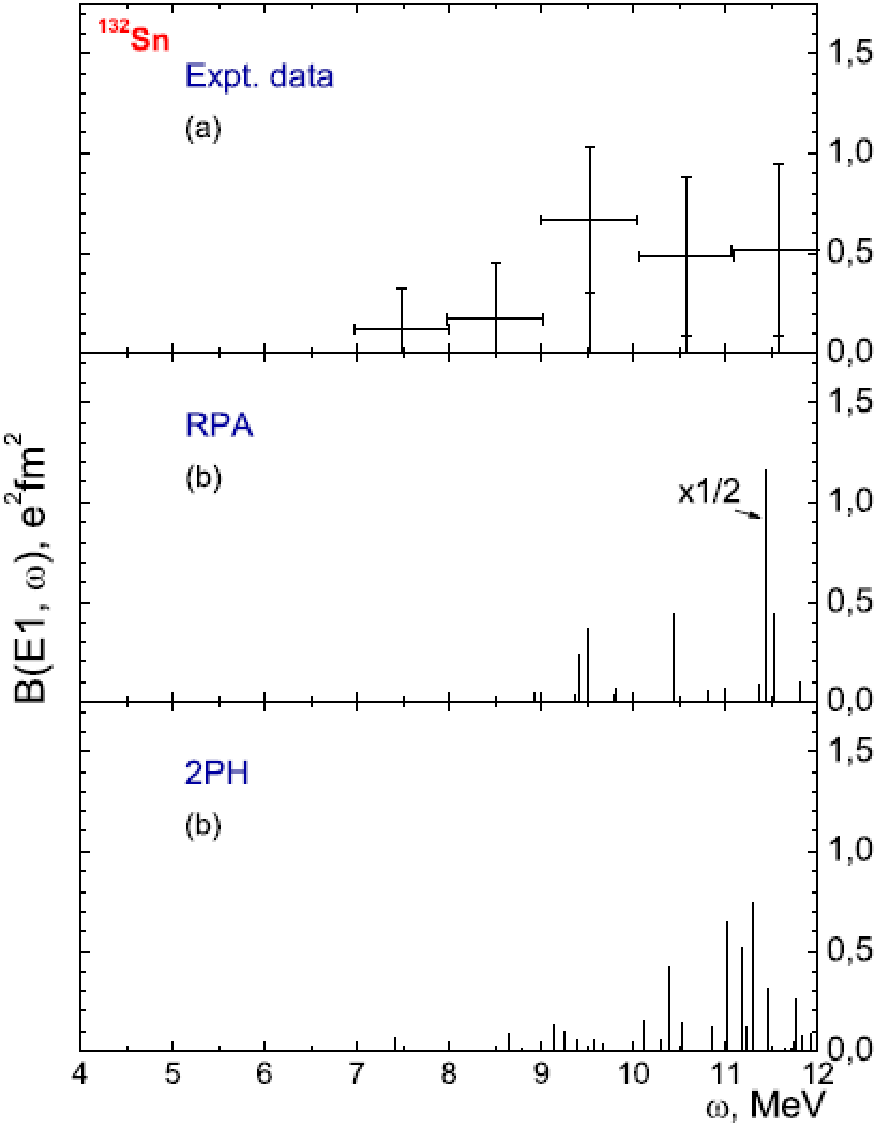

PDR width:

1.2 MeV – QRPA

2.0 MeV – 2PH

< 2.5 MeV – an upper limit

N. N. Arsenyev, A. P. S., V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC 38, 17002 (2012).

A. Klimkiewicz et al., Phys. Rev. C76, 051603R (2007).

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Nucleus	$\bar{E}$, MeV			$\sum B(E1)$, e^2fm^2		
	QRPA	2PH	Expt.	QRPA	2PH	Expt.
^{124}Sn	9.7	9.1	6.97	0.86	0.59	0.379±0.045
^{126}Sn	10.1	10.0		1.82	1.86	
^{128}Sn	10.0	10.0		1.63	1.78	
^{130}Sn	10.0	10.0	10.1±0.7	1.40	1.80	2.4±0.7
^{132}Sn	9.9	9.9	9.8±0.7	1.27	1.42	1.3±0.8

N. N. Arsenyev, A. P. S., V. V. Voronov, Nguyen Van Giai, Eur. Phys. J. WC **38**, 17002 (2012).

P. Adrich et al., Phys. Rev. Lett. **95**, 132501 (2005).

B. Özel et al., Nucl. Phys. **A788**, 385c (2007).

Summary

- We have presented the finite rank separable approximation for the QRPA calculations with Skyrme interactions. This approximation enables one to reduce remarkably the dimensions of the matrices that must be diagonalized to perform the calculations in very large configuration spaces.

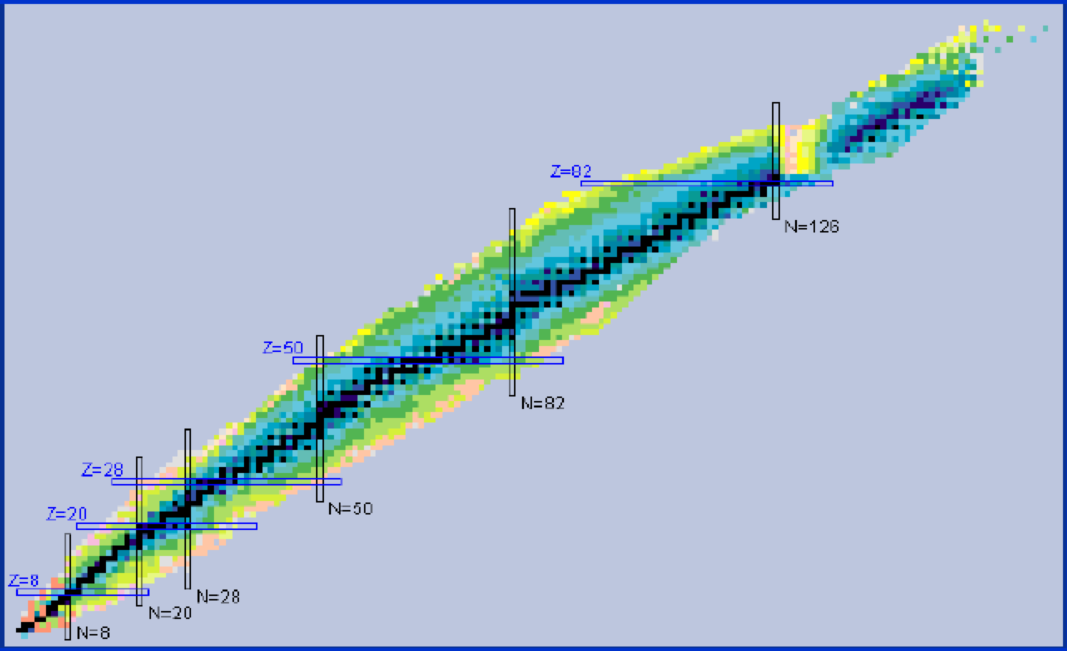
- Using the same set of parameters we have studied the behaviour of the energies, the B(E1) values of the 1^- states in neutron-rich Sn isotopes. It is shown that the coupling between one- and two-phonon terms in the wave functions of excited states is essential for the description of the dipole strength distribution at energy region below 11 MeV. The inclusion of the two-phonon configurations results in a remarkable increase of the width of the pygmy resonance. At the same time there is a minor influence on the width of the GDR which is successfully described within the one-phonon approximation. Our results for ^{124,130,132}Sn are in reasonable agreement with experimental data.

Thanks for collaboration:

N.N. Arsenyev – BLTP, JINR

Nguyen Van Giai – IPN, Orsay

V.V. Voronov – BLTP, JINR



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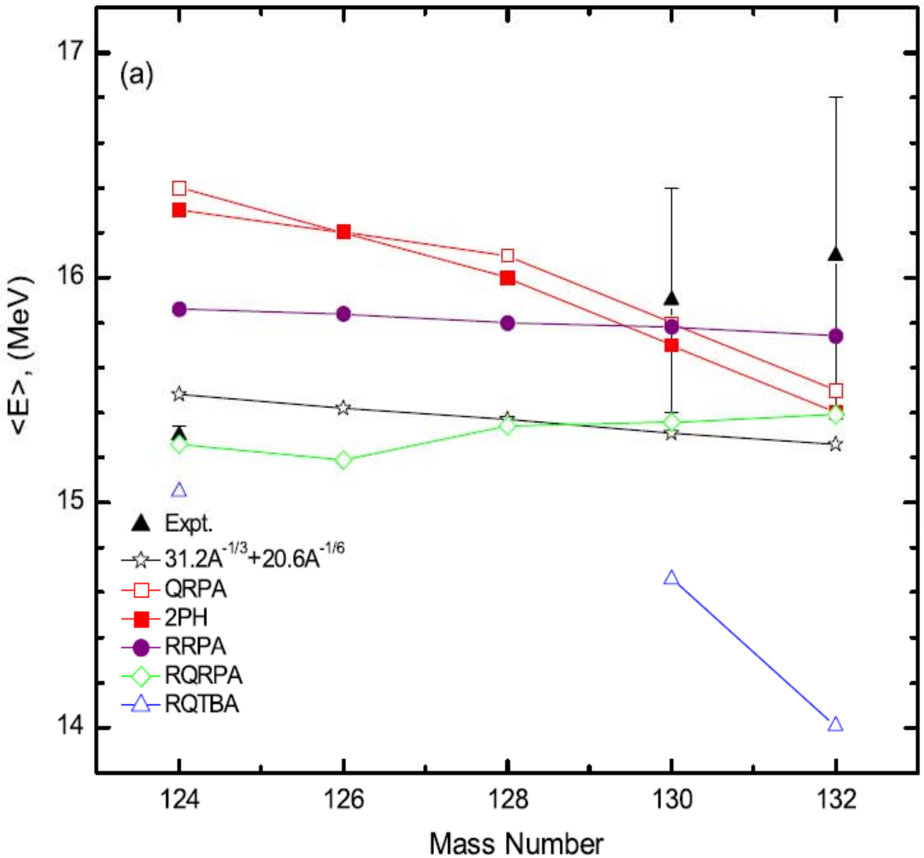

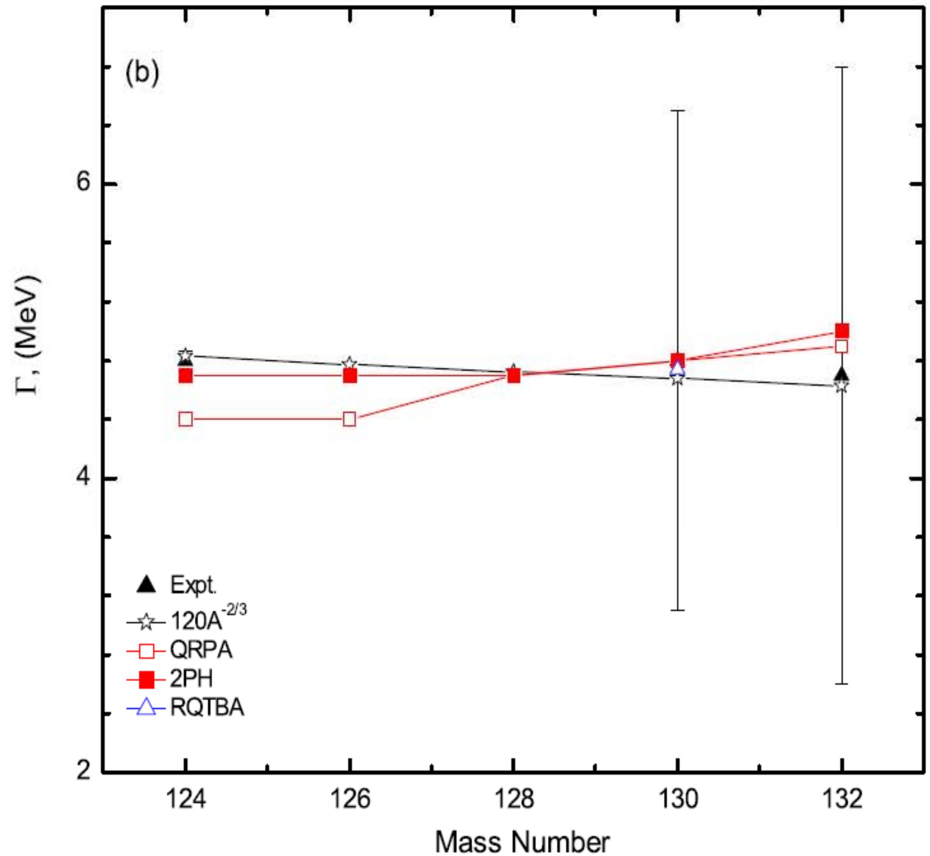

QRPA and 2PH – *N. N. Arsenyev et al.* EPJ WC **38**, 17002 (2012);
RRPA – *J. Piekarewicz*, PRC **73**, 044325 (2006);
RQRPA – *N. Paar et al.*, PLB **606**, 288 (2005);
RQTBA – *E. Litvinova et al.*, PRC **79**, 054312 (2009).

The integral characteristics of $E1$ strength function in these nuclei have been calculated for the energy interval 11-20 MeV.

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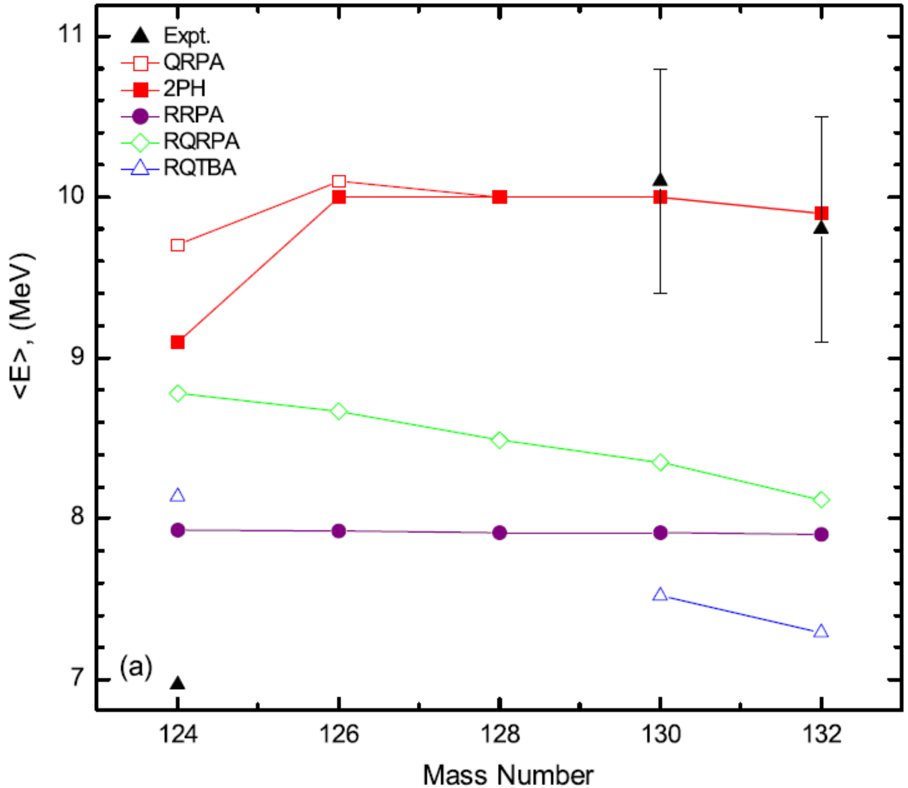
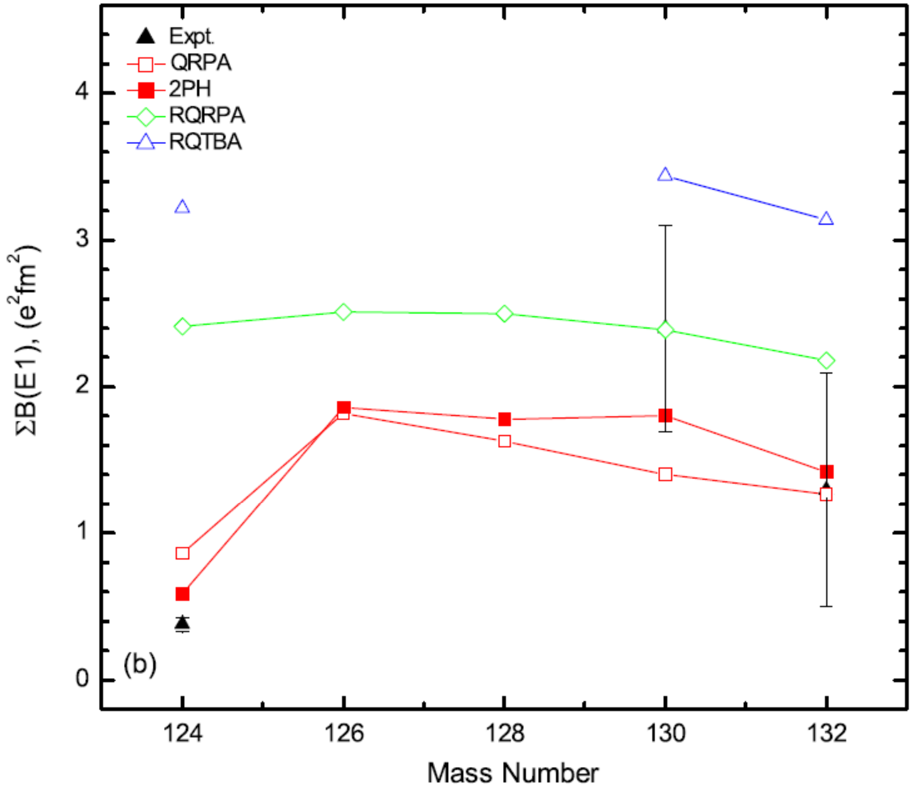
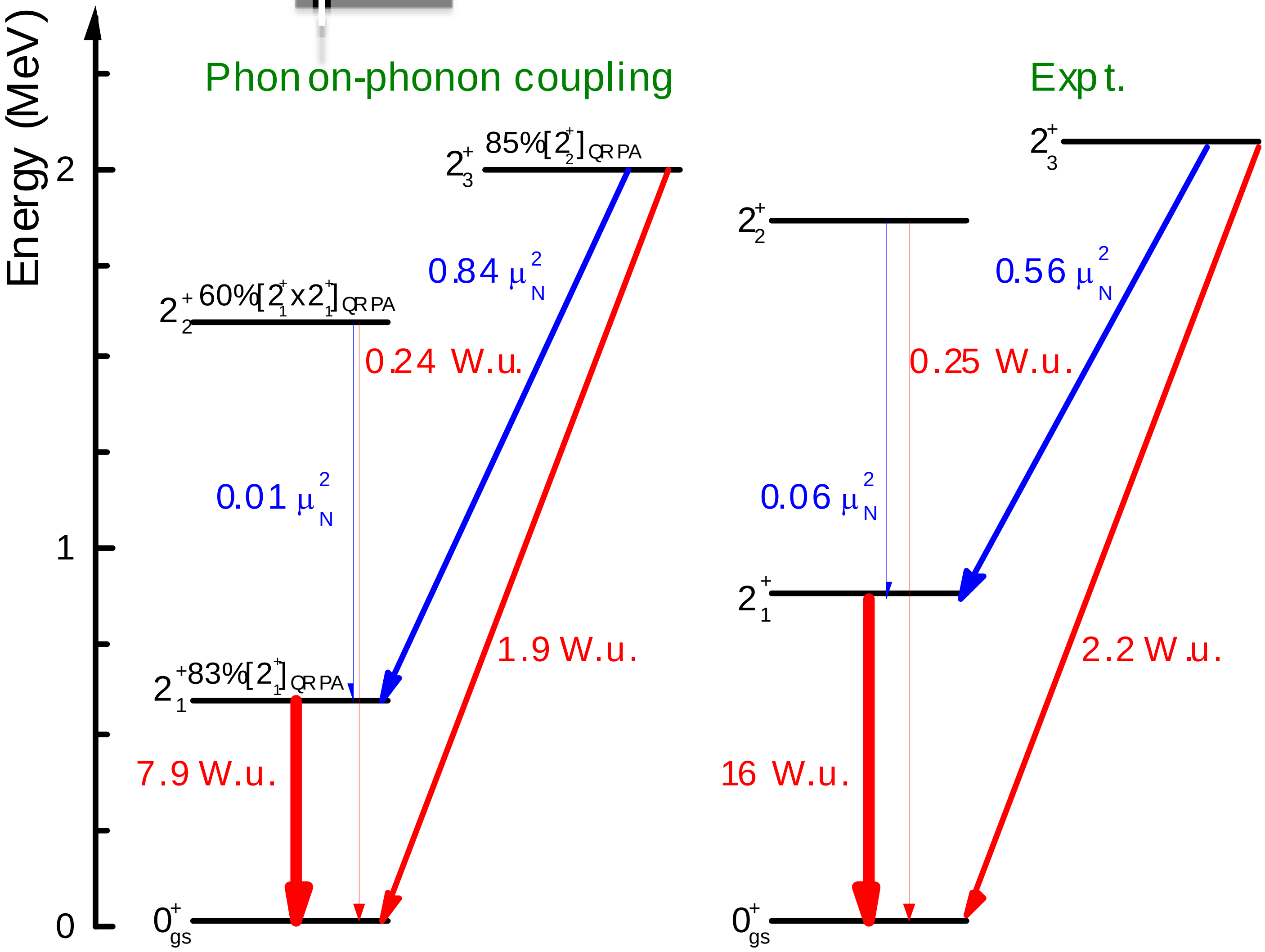




QRPA and 2PH – *N. N. Arsenyev et al.* EPJ WC **38**, 17002 (2012);
RRPA – *J. Piekarewicz*, PRC **73**, 044325 (2006);
RQRPA – *N. Paar et al.*, PLB **606**, 288 (2005);
RQTBA – *E. Litvinova et al.*, PRC **79**, 054312 (2009).

Note, to compare with experimental data in ^{124}Sn we choose the energy interval below 10 MeV.

^{94}Mo



A.P.S., N.N. Arsenyev, N. Pietralla, Phys. Rev. C 86, 024311 (2012).

	$\lambda_i^\pi = 2_i^+$	Energy (MeV)		Structure	$B(E2; 0_{gs}^+ \to 2_i^+)$ (e^2fm^4)		$B(E2; 2_i^+ \to 2_1^+)$ (e^2fm^4)		$B(M1; 2_i^+ \to 2_1^+)$ (μ_N^2)	
		Expt.	Theory		Expt.	Theory	Expt.	Theory	Expt.	Theory
^{90}Zr	2_1^+	2.186	2.6	$93\%[2_1^+]$	643±22	600				
	2_2^+	3.308	3.2	$95\%[2_2^+]$	53±14	1	65±17	1	0.088±0.025	0.00
^{92}Zr	2_1^+	0.934	1.6	$96\%[2_1^+]$	790±62	420				
	2_2^+	1.847	2.7	$87\%[2_2^+]$	419±49	230	10^{+12}_{-7}	4	0.37±0.04	0.41
	2_3^+	2.067	2.6	$45\%[2_4^+] + 37\%[2_1^+ \otimes 2_1^+]$	< 0.62	50	< 395	160	<0.024	0.17
^{92}Mo	2_1^+	1.509	1.9	$99\%[2_1^+]$	1036±62	1160				
	2_2^+	3.091	3.8	$91\%[2_1^+ \otimes 2_1^+]$	254±20	50	96±27	420	0.043±0.007	0.03
^{94}Mo	2_1^+	0.871	0.5	$73\%[2_1^+]$	2031±25	1280				
	2_2^+	1.864	1.8	$53\%[2_1^+ \otimes 2_1^+] + 21\%[2_3^+]$	32±7	170	720±260	190	0.06±0.02	0.07
	2_3^+	2.067	2.3	$87\%[2_2^+]$	279±25	310	124^{+76}_{-58}	10	0.56±0.05	0.68

A.P.S., N.N. Arsenyev, N. Pietralla, Phys. Rev. C 86, 024311 (2012).