QED with heavy ions: on the way to supercritical fields

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Outline of the talk

- Introduction
- QED at strong electromagnetic fields
- QED at supercritical fields
  - Schwinger mechanism
  - Pair creation in the U^{92+}—U^{92+} collision
  - How to observe the vacuum decay
- Conclusion

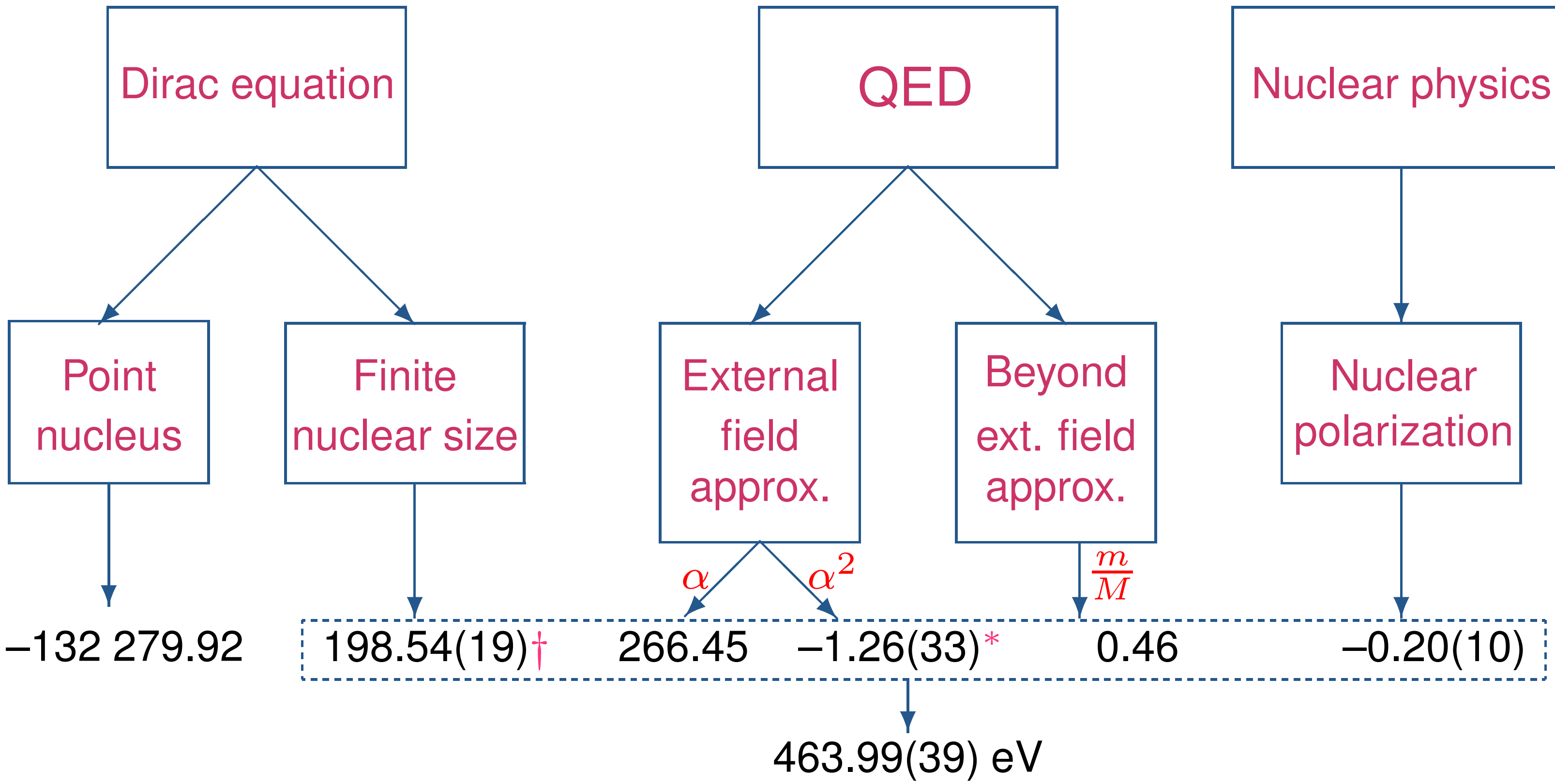
Introduction: tests of QED with atomic systems

Light atoms ($\alpha Z \ll 1$, weak fields):
Tests of QED to lowest orders in α and αZ.

Heavy few-electron ions ($\alpha Z \sim 1$, strong fields):
Tests of QED in nonperturbative in αZ regime.

Low-energy heavy-ion collisions at $Z_1 + Z_2 > 173$ (supercritical fields):
Tests of QED in supercritical regime.

1s Lamb shift in H-like uranium, in eV



Experiment: 460.2(4.6) eV

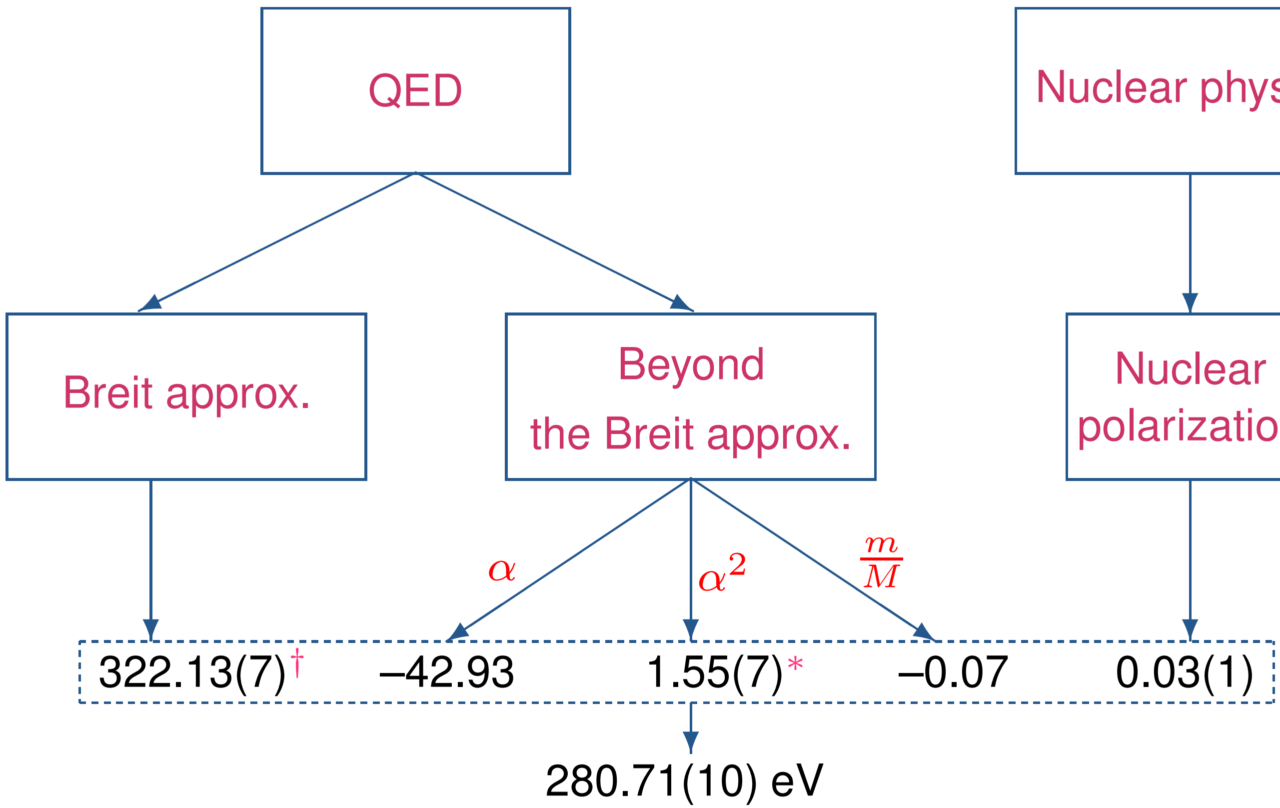
(A. Gumberidze, T. Stöhlker, D. Banas et al., PRL, 2005)

Test of QED: $\sim$ 2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

$2p_{1/2}$-$2s$ *transition energy in Li-like uranium, in eV*



Experiment: 280.59(10) eV (J. Schweppe et al., PRL, 1991)
280.52(10) eV (C. Brandau et al., PRL, 2003)
280.645(15) eV (P. Beiersdorfer et al., PRL, 2005)

Test of QED: $\sim$ 0.2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

Current value for the specific HFS difference in Bi

Theoretical contributions to $\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}$ (in meV) for $\mu/\mu_N = 4.1106(2)$ *(A.V. Volotka et al., PRL, 2012; O.V. Andreev et al., PRA, 2012)*

Dirac value	-31.809
Interel. inter., $\sim 1/Z$	-29.995
Interel. inter., $\sim 1/Z^2$ and h.o.	0.255(3)
One-electron QED	0.036
Screened QED	0.193(2)
Total	-61.320(6)
Experiment *[1]*	-61.012 (5)(21)

[1] J. Ullmann et al., Nature Communications, 2017.

New calculations of the shielding constant and new NMR measurements in $Bi(NO_3)_3$ and BiF_6^- yielded $\mu/\mu_N = 4.092(2)$ *(L. Skripnikov et al., PRL, 2018)*, which gave $\Delta' E = -61.043(5)(30)$ meV.

g factor of H-like ions

High-precision measurement of the g-factor of $^{12}\mathrm{C}^{5+}$ using a single ion confined in a Penning ion trap *(H. Häffner et al., PRL, 2000)*:

$$g_{\rm exp} = 2(\omega_L/\omega_c)(m_e/M)(q/|e|) = 2.001\,041\,596\,3\,(10)(44)\,.$$

Here $\omega_c = (q/M)B$ is the cyclotron frequency, $\omega_L = \Delta E/\hbar$, M is the ion mass, and q is the ion charge. The second uncertainty (44) was due to the uncertainty of the (m_e/M) ratio. Combined with the related theory *(V.M. Shabaev and V.A. Yerokhin, PRL, 2002; V.A. Yerokhin et al., PRL, 2002)*, this resulted in four-times improvement of the accuracy of the electron mass.

In *(S. Sturm et al., Nature, 2014)*, the precision of the atomic mass of the electron was improved by a factor of 13 (compared to the current CODATA value) : $m_e = 0.000548579909067(14)(9)(2)$.

Future prospects for the g-factor investigations

1) Tests of bound-state QED at strong fields

For stringent tests of QED in the g-factor experiments, one should study specific differences of the g factors of H-, Li- and B-like ions.

2) Tests of QED beyond the Furry picture *(A.V. Malyshev, V.M. Shabaev, D.A. Glazov, and I.I. Tupitsyn, JETP Letters, 2017).*

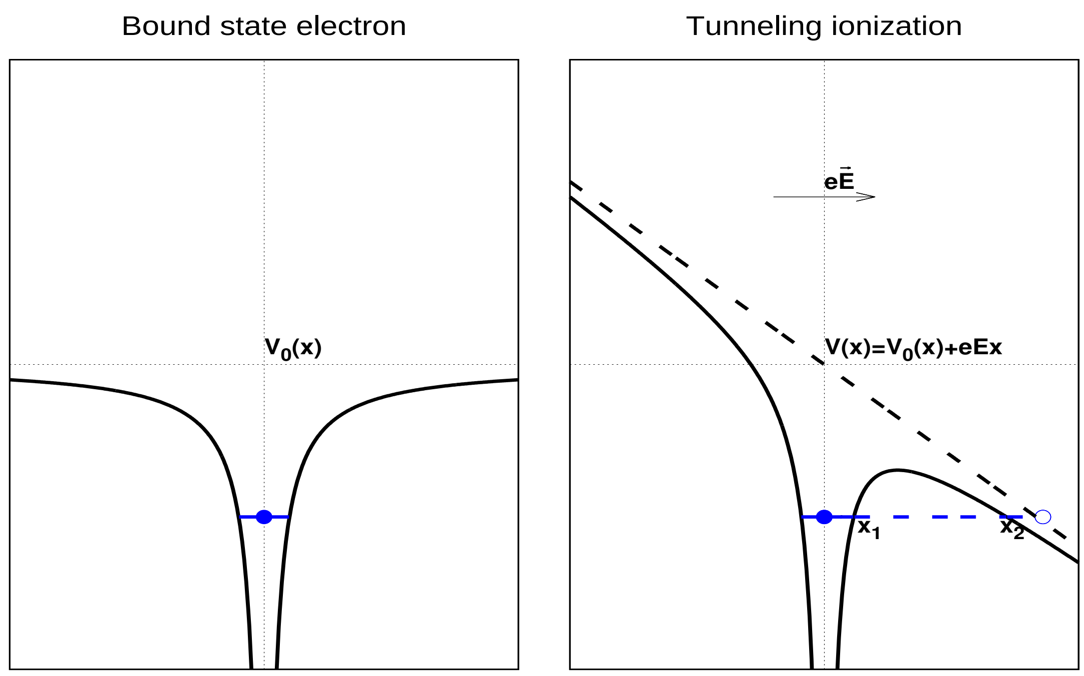
3) Determination of the nuclear magnetic moments

$$g_{\rm atom} = g^{(e)} \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} - \frac{m_e}{m_p} g^{(N)} \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} .$$

4) Determination of the fine structure constant by studying the g factors of H-, Li-, and B-like ions *(V.M. Shabaev et al., PRL, 2006; V.A. Yerokhin et al., PRL, 2016).*

QED at supercritical fields

Tunneling ionization in quantum mechanics



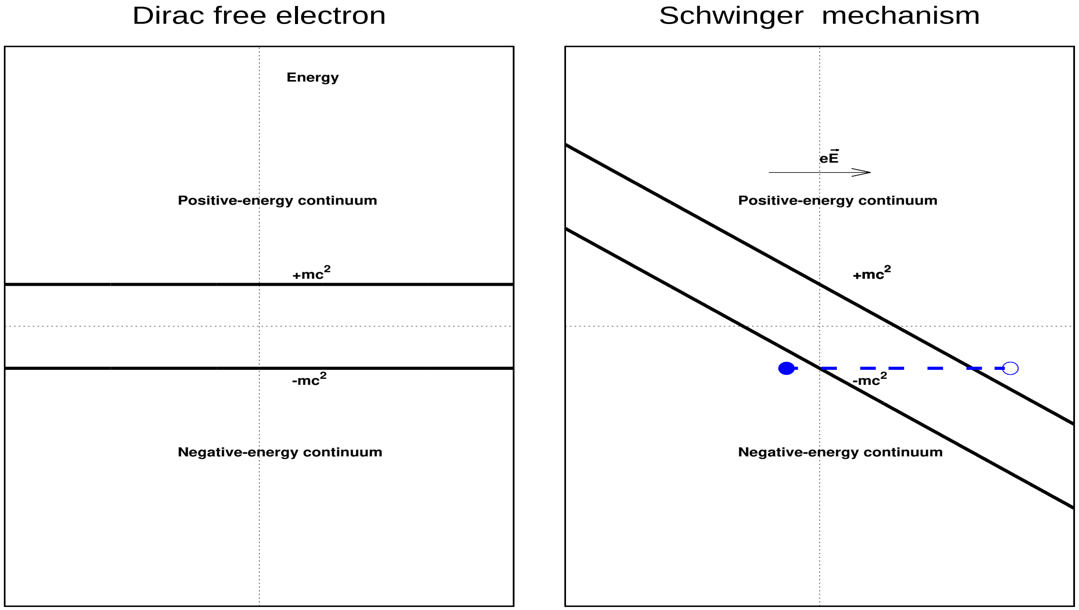
The tunneling probability for a static uniform electric field E:

$$W \sim \exp\Big\{-\frac{4\pi}{\hbar}\int_{x_1}^{x_2} dx\sqrt{2m(V(x)-\mathcal{E})}\Big\}$$

where $V(x) = V_0(x) + eEx$ and $\mathcal{E}$ is the electron energy.

QED at supercritical fields

Electron-positron pair creation by a static uniform electric field



The rate of pair production for a static uniform electric field E:

$$\frac{d^4 n_{e^+e^-}}{d^3x\,dt} \sim \frac{c}{4\pi^3 {\lambda_C}^4} \exp\left(-\pi \frac{E_c}{E}\right)$$

where $\lambda_C = \hbar/(mc)$ and $E_c = m^2c^3/(e\hbar) \approx 1.3 \times 10^{16}\,\mathrm{V/cm}$.

QED at supercritical fields

Electron-positron pair creation by a static electric field

The Schwinger effect has never been observed experimentally as the required field strength, $E_c \approx 1.3 \times 10^{16}\mathrm{V/cm}$, is extremely large. The recent developments of the laser technologies have triggered a great interest to theoretical calculations of this effect for various scenarios *[A. Di Piazza, C. Müller, K.Z. Hatsagortsyan, and C.H. Keitel, RMP, 2012]*.

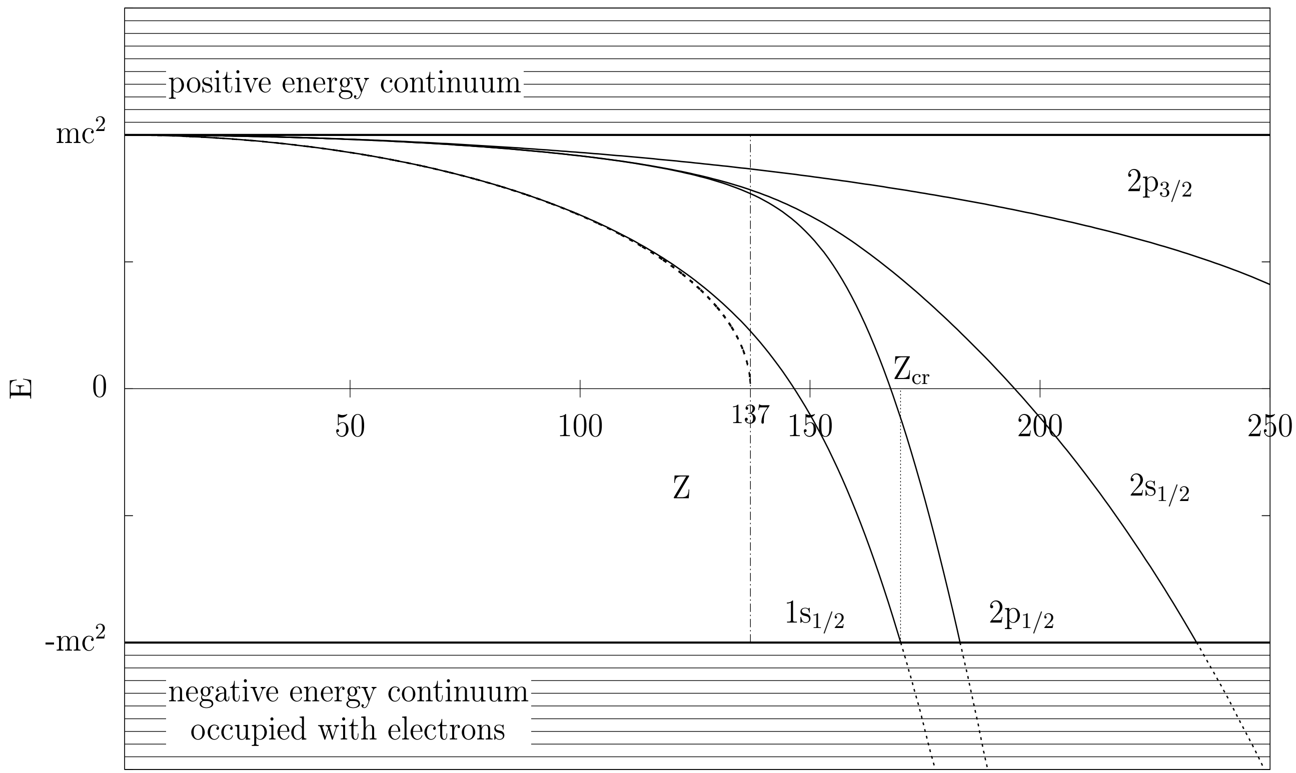
The scenario with two counter-propagating laser pulses is considered as one of most favorable. For the recent progress on the calculations for this scenario we refer to *[I.A. Aleksandrov, G. Plunien, and V.M. Shabaev, PRD, 2018].*

However, even in the most promising scenarios the electric field strength reached with new laser technologies in the not too distant future is expected to be two orders of magnitude smaller than the critical value.

Low-energy heavy-ion collisions

Access to supercritical fields

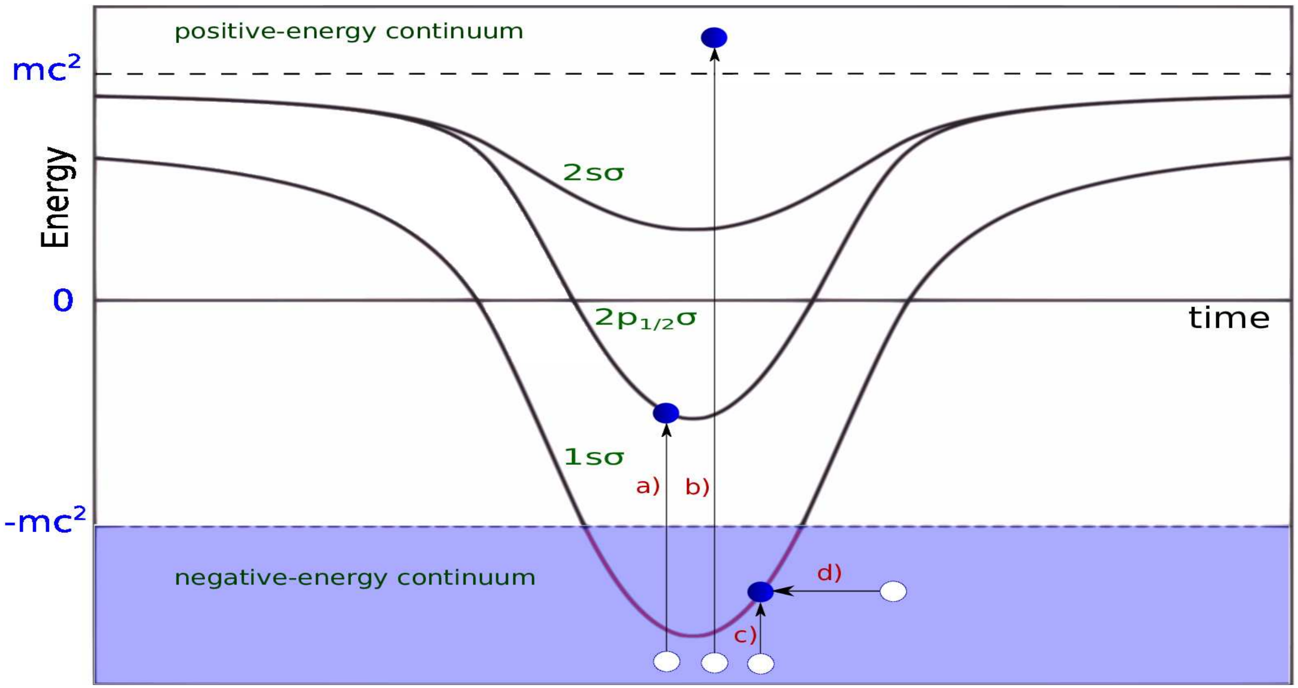
S.S. Gershtein, Ya.B. Zel'dovich, 1969; W. Pieper, W. Greiner, 1969



The $1s$ level dives into the negative-energy continuum at $Z_{\mathrm{crit}} \approx 173$.

Low-energy heavy-ion collisions

Creation of electron-positron pairs in low-energy heavy-ion collisions, with $Z_1 + Z_2 > 173$

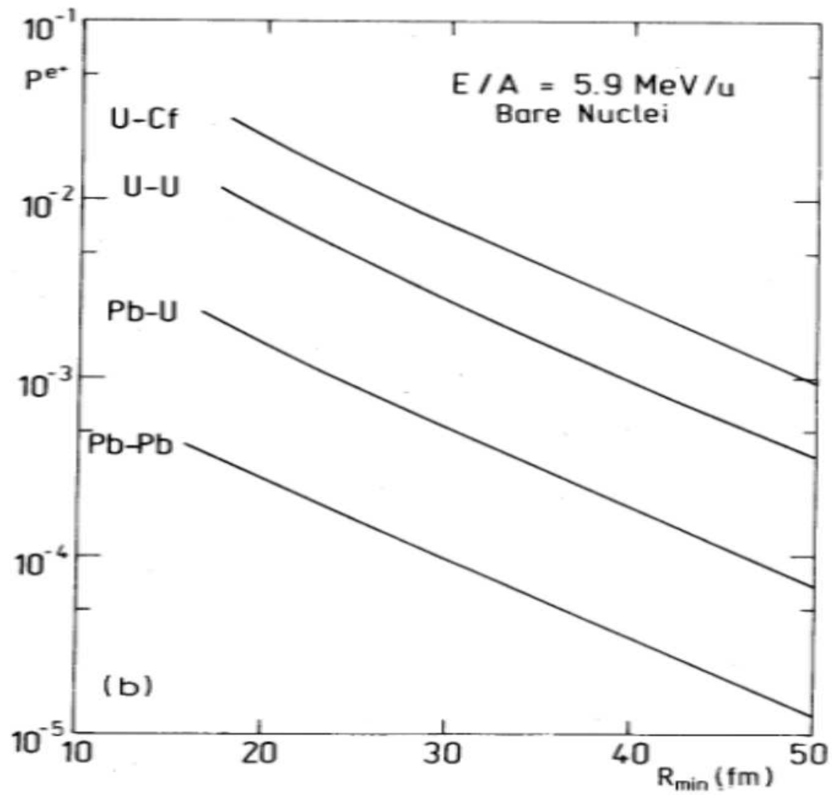


Dynamical mechanism: a),b),c). Spontaneous mechanism (vacuum decay): d). The $1s$ state dives into the negative-energy continuum for about 10^{-21} sec.

Low-energy heavy-ion collisions

Positron production probability in 5.9 MeV/u collisions of bare nuclei as a function of distance of closest approach $R_{\min}$

(J. Reinhardt, B. Müller, and W. Greiner, Phys. Rev. A, 1981).



Conclusion by Frankfurt's group (2005):The vacuum decay could only be observed in collisions with nuclear sticking, in which the nuclei are bound to each other for some period of time by nuclear forces.

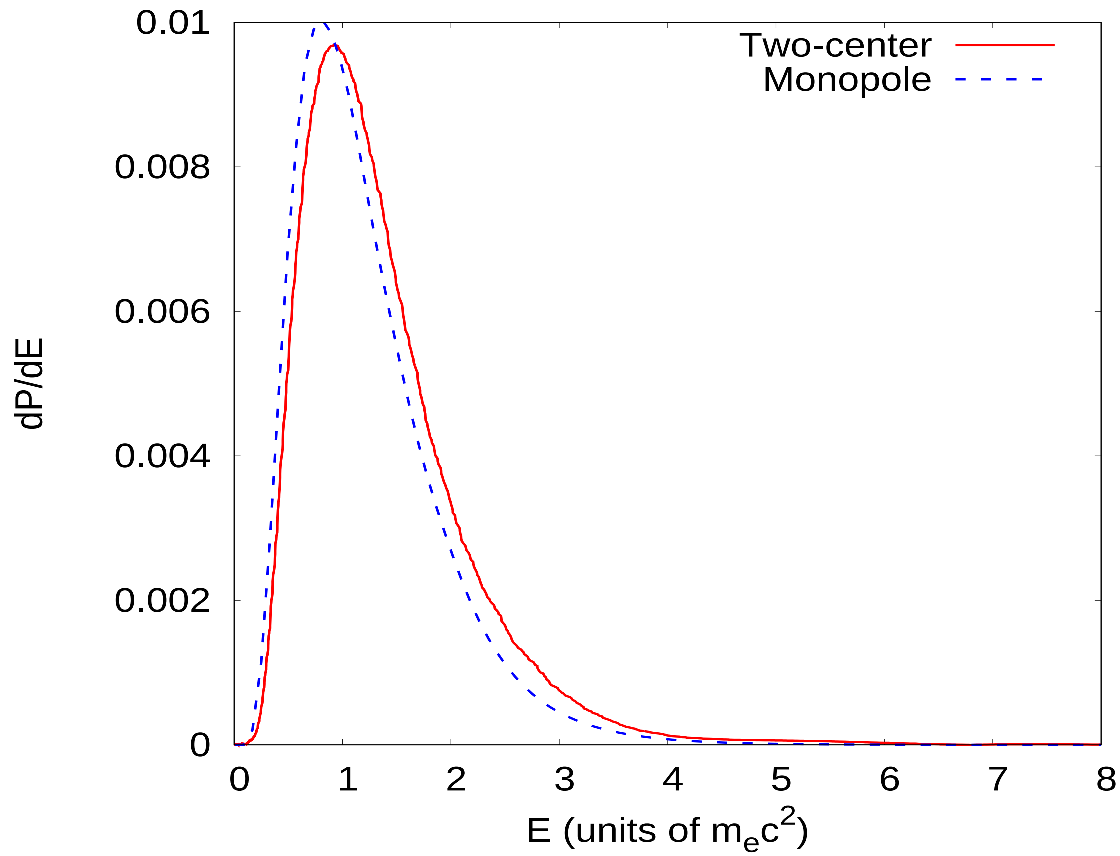
Low-energy heavy-ion collisions

New methods for calculations of quantum dynamics of electron-positron field in low-energy heavy-ion collisions at subcritical and supercritical regimes have been developed:

- *I.I. Tupitsyn, Y.S. Kozhedub, V.M. Shabaev et al., Phys. Rev. A 82, 042701 (2010).*
- *I. I. Tupitsyn, Y. S. Kozhedub, V. M. Shabaev et al., Phys. Rev. A 85, 032712 (2012).*
- *G. B. Deyneka, I. A. Maltsev, I. I. Tupitsyn et al., Russ. J. of Phys. Chem. B 6, 224 (2012).*
- *G. B. Deyneka, I. A. Maltsev, I. I. Tupitsyn et al., Eur. Phys. J. D 67, 258 (2013).*
- *Y.S. Kozhedub, V.M. Shabaev, I.I. Tupitsyn et al., Phys. Rev. A 90, 042709 (2014).*
- *I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn et al., NIMB, 408, 97 (2017).*
- *R.V. Popov, A.I. Bondarev, Y.S. Kozhedub et al., Eur. Phys. J. D 72, 115 (2018).*
- *I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., Phys. Rev. A 98, 062709 (2018).*

Pair creation beyond the monopole approximation

Positron energy spectrum for the U−U head-on collision at energy $E_{\rm cm}$ = 740 MeV *(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRA, 2018)*.



Low-energy heavy-ion collisions

Pair creation beyond the monopole approximation

U-U, $E_{\rm cm}$ = 740 MeV

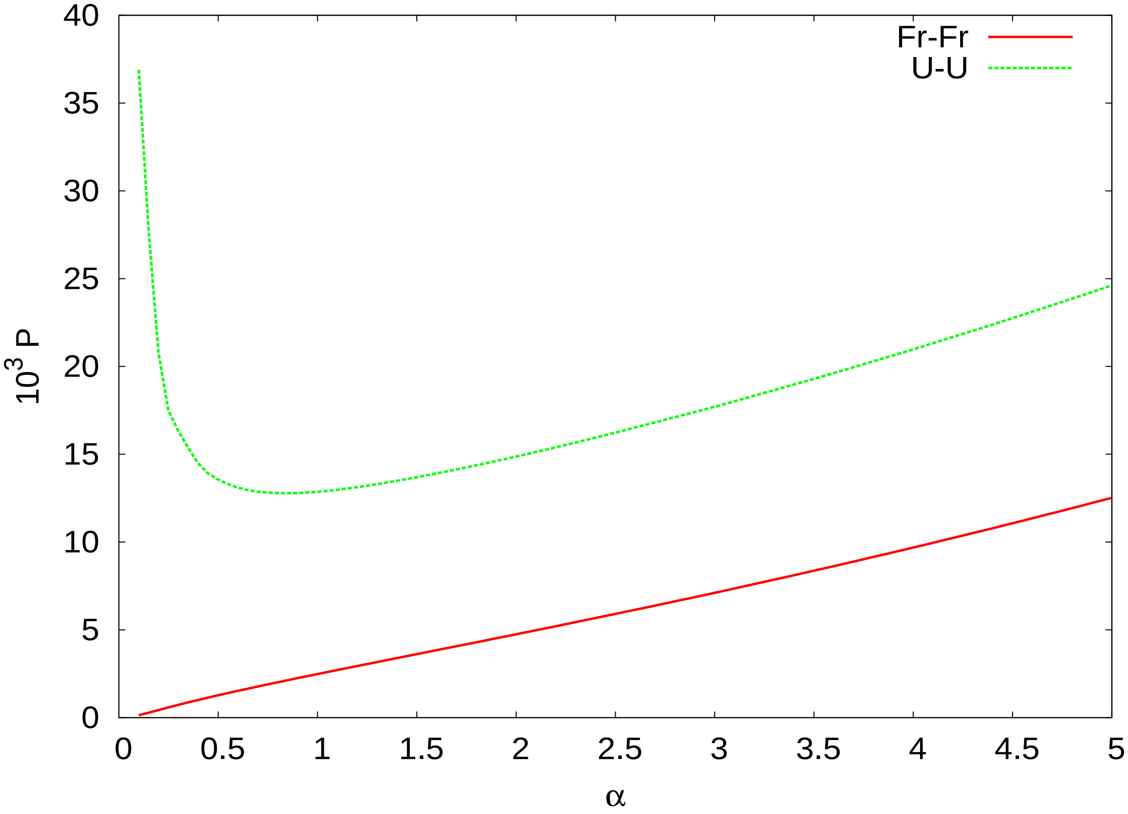
Expected number of created pairs as a function of the impact parameter b

(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRA, 2018) .

b (fm)	Monopole approximation	Two-center approach
0	1.29 $\times 10^{-2}$	1.38 $\times 10^{-2}$
10	7.26 $\times 10^{-3}$	8.01 $\times 10^{-3}$
20	2.75 $\times 10^{-3}$	3.46 $\times 10^{-3}$
30	1.04 $\times 10^{-3}$	1.42 $\times 10^{-3}$
40	4.12 $\times 10^{-4}$	7.04 $\times 10^{-4}$

The two-center result for $b = 0$ has been confirmed by a different method *(R.V. Popov, A.I. Bondarev, Y.S. Kozhedub et al., EPJD, 2018)* .

Low-energy heavy-ion collisions

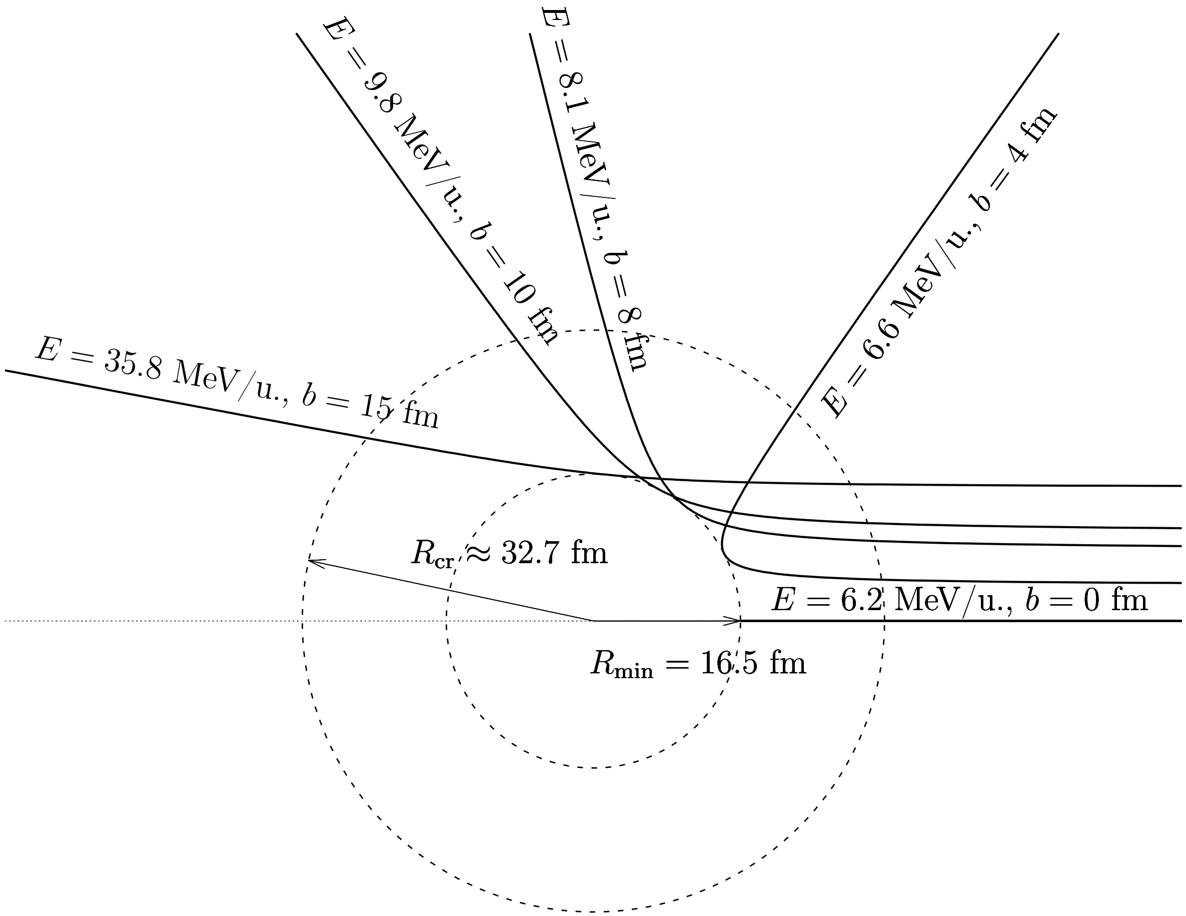


Pair creation with artificial trajectories for the supercritical U−U and subcritical Fr−Fr head-on collisions at $E_{\rm cm} = 674.5$ and $E_{\rm cm} = 740$ MeV, respectively. The trajectory $R_\alpha(t)$ is defined by $\dot{R}_\alpha(t) = \alpha \dot{R}(t)$, where $R(t)$ is the classical Rutherford trajectory *(I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn et al., PRA, 2015)*.

Low-energy heavy-ion collisions

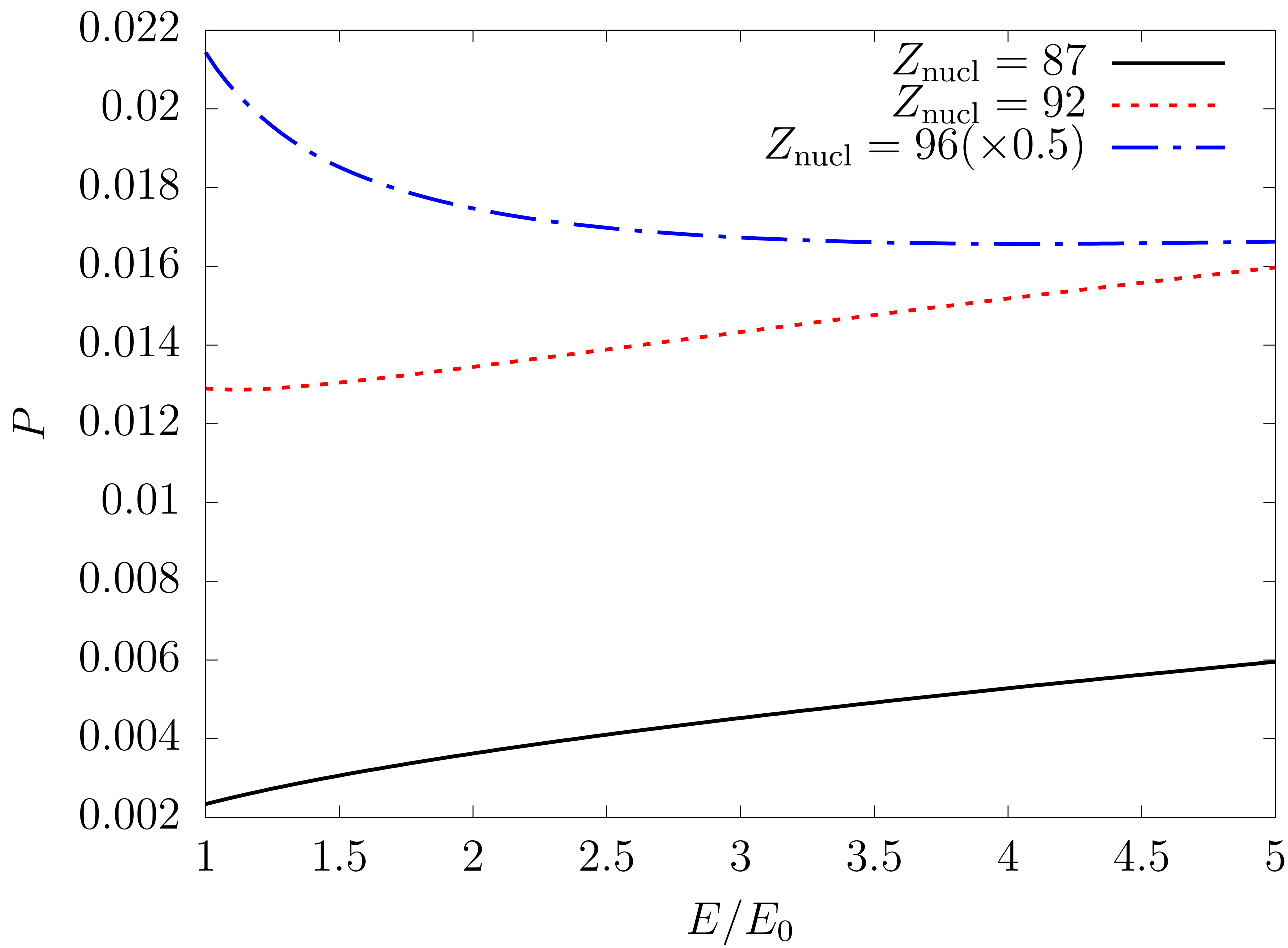
How to observe the vacuum decay

(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRL, 2019.)



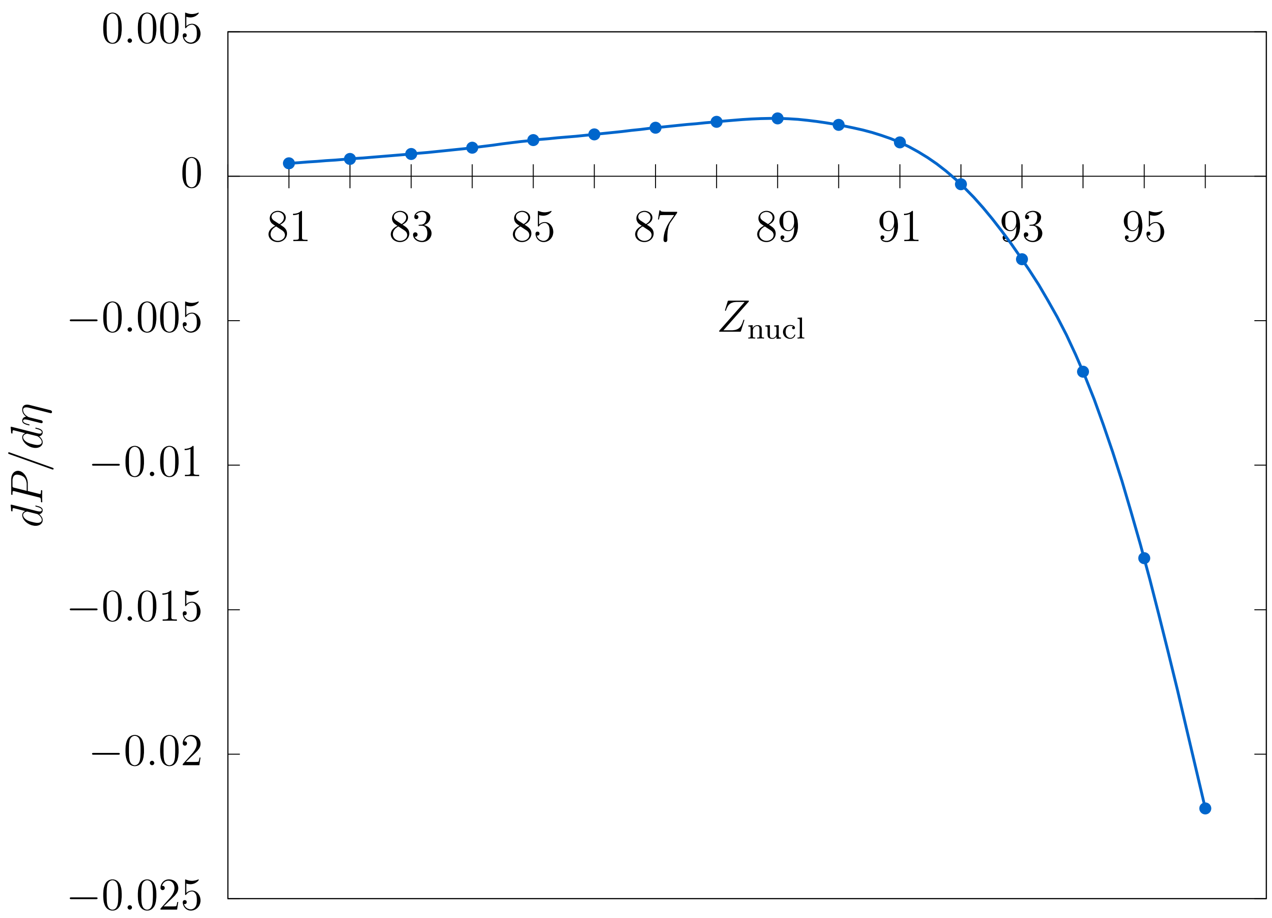
We consider only the trajectories for which the minimal internuclear distance is the same: $R_{\min} = 16.5$ fm.

Low-energy heavy-ion collisions



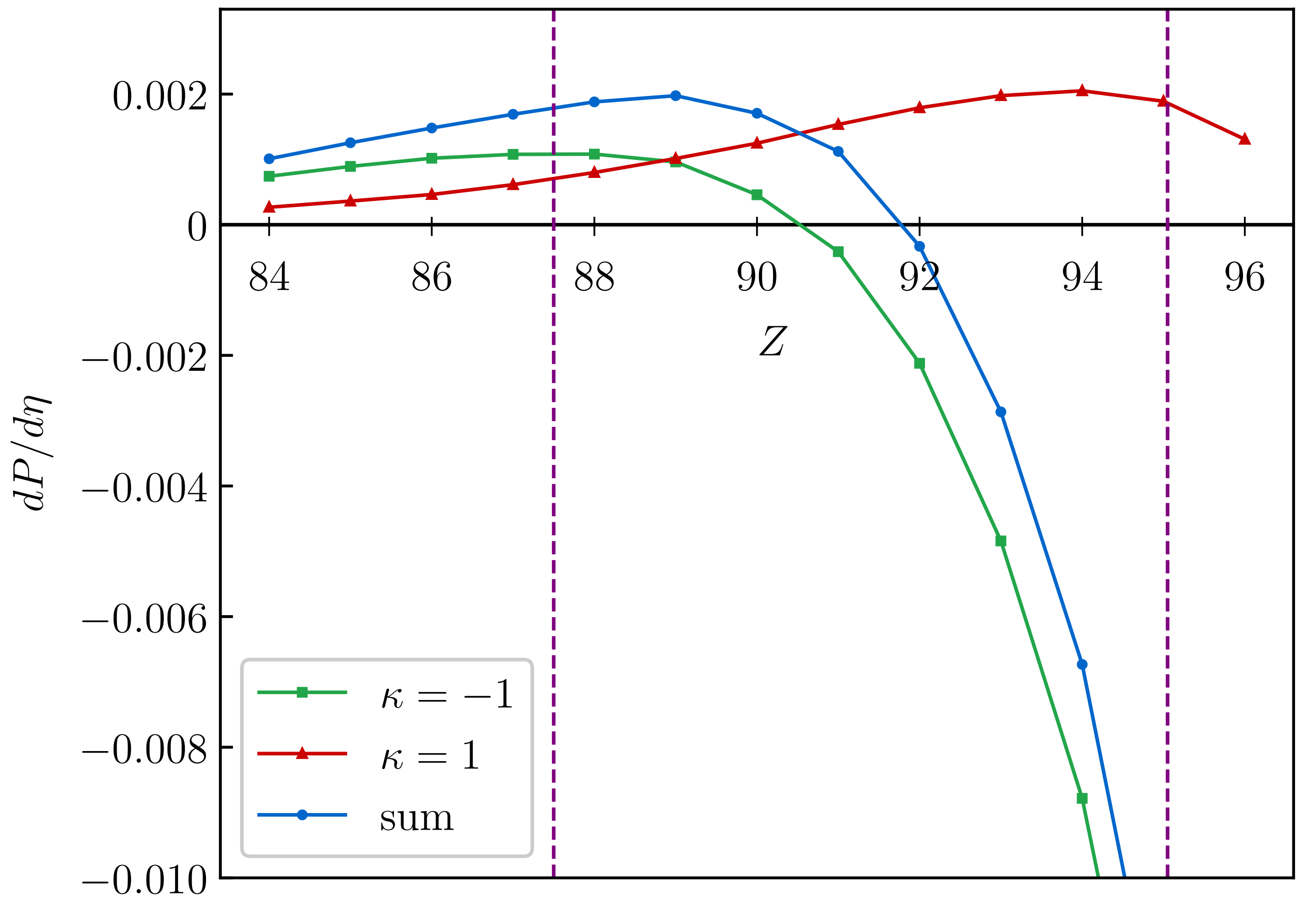
Total pair-production probability for symmetric ($Z_1 = Z_2 = Z_{\rm nucl}$) collisions as a function of the collision energy at $R_{\rm min} = 16.5$ fm.

Low-energy heavy-ion collisions



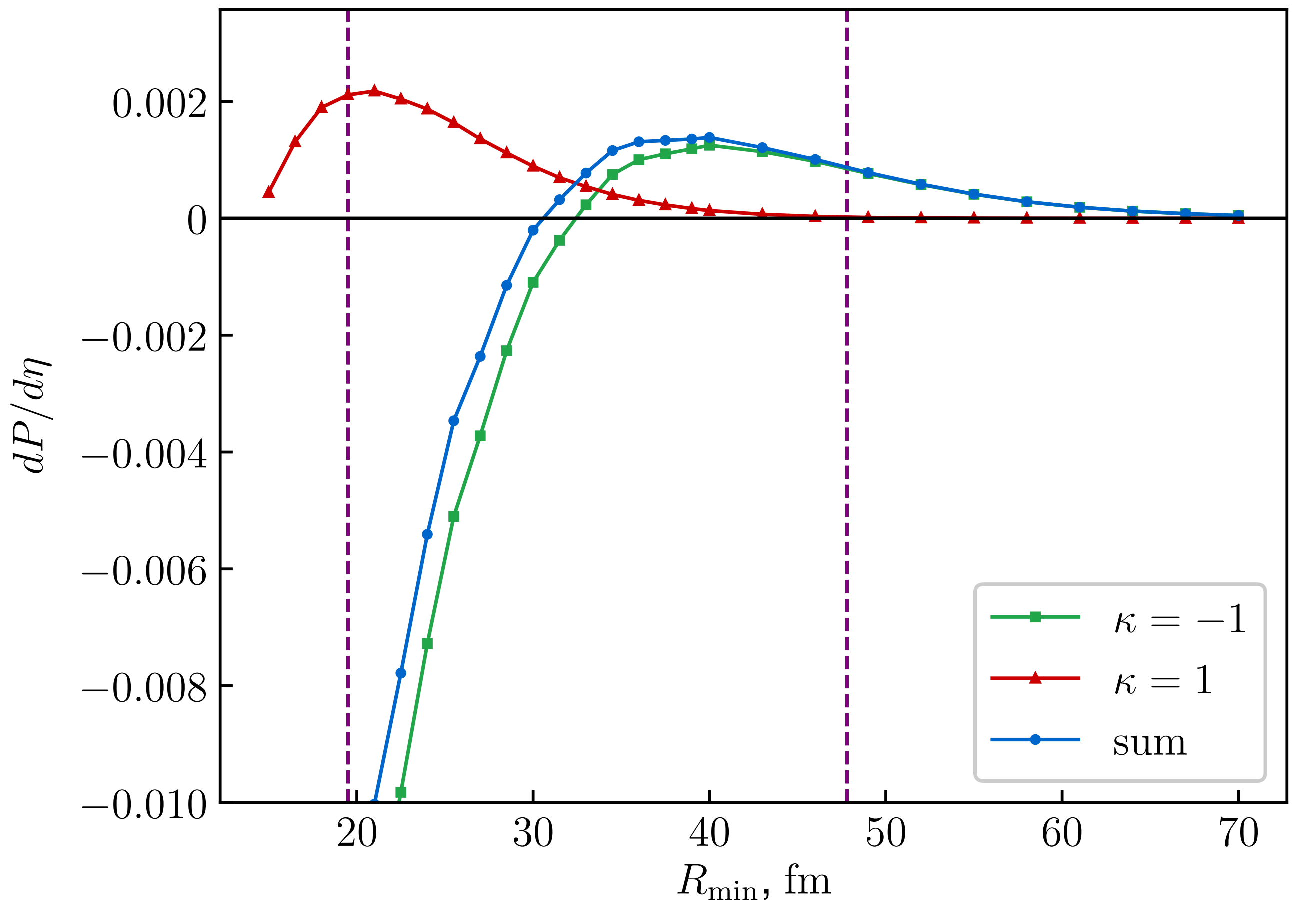
The derivative of the pair-production probability with respect to the energy $dP/d\eta$, where $\eta = E/E_0$, at the point $\eta = 1$ as a function of the nuclear charge number $Z_{\text{nucl}} = Z_1 = Z_2$ at $R_{\text{min}} = 16.5$ fm.

Low-energy heavy-ion collisions



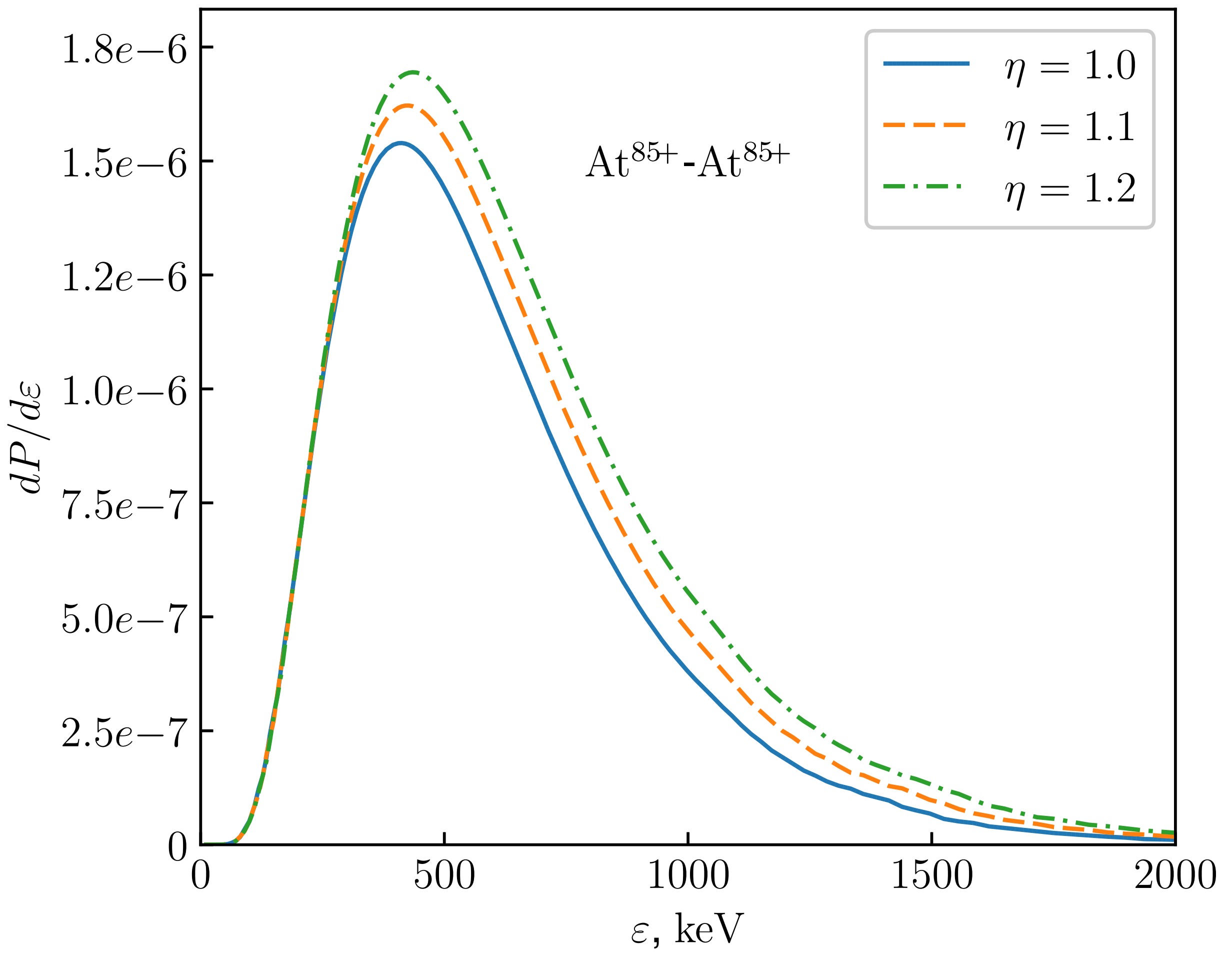
The $\kappa = \pm 1$ contributions to the derivative of the pair-production probability with respect to the energy $dP/d\eta$, where $\eta = E/E_0$, at the point $\eta = 1$ as a function of the nuclear charge number $Z = Z_1 = Z_2$ at $R_{\rm min} = 16.5$ fm.

Low-energy heavy-ion collisions



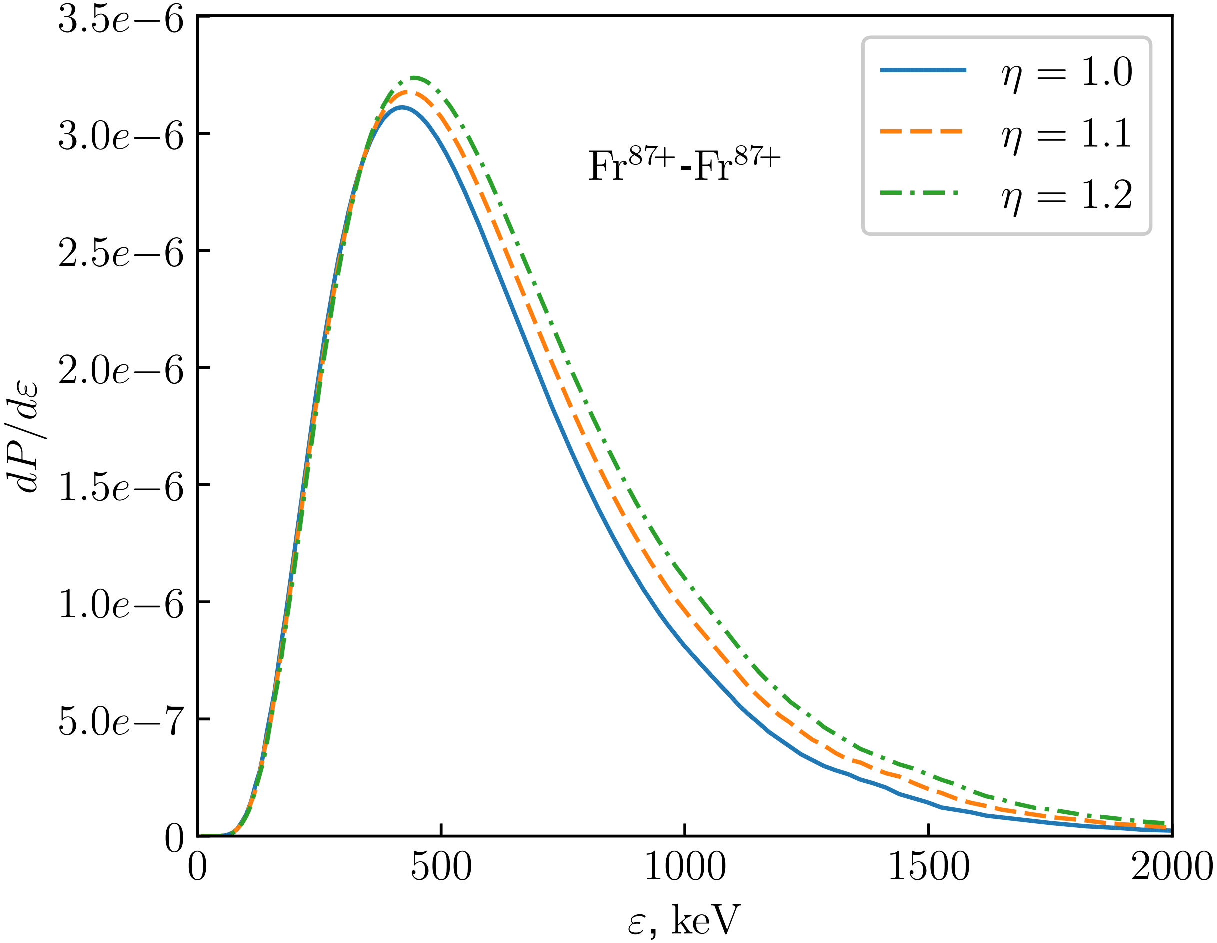
The $\kappa = \pm 1$ contributions to the derivative of the pair-production probability with respect to the energy $dP/d\eta$, where $\eta = E/E_0$, at the point $\eta = 1$ as a function of $R_{\min}$ at the nuclear charge number $Z = Z_1 = Z_2 = 96$.

How to observe the vacuum decay



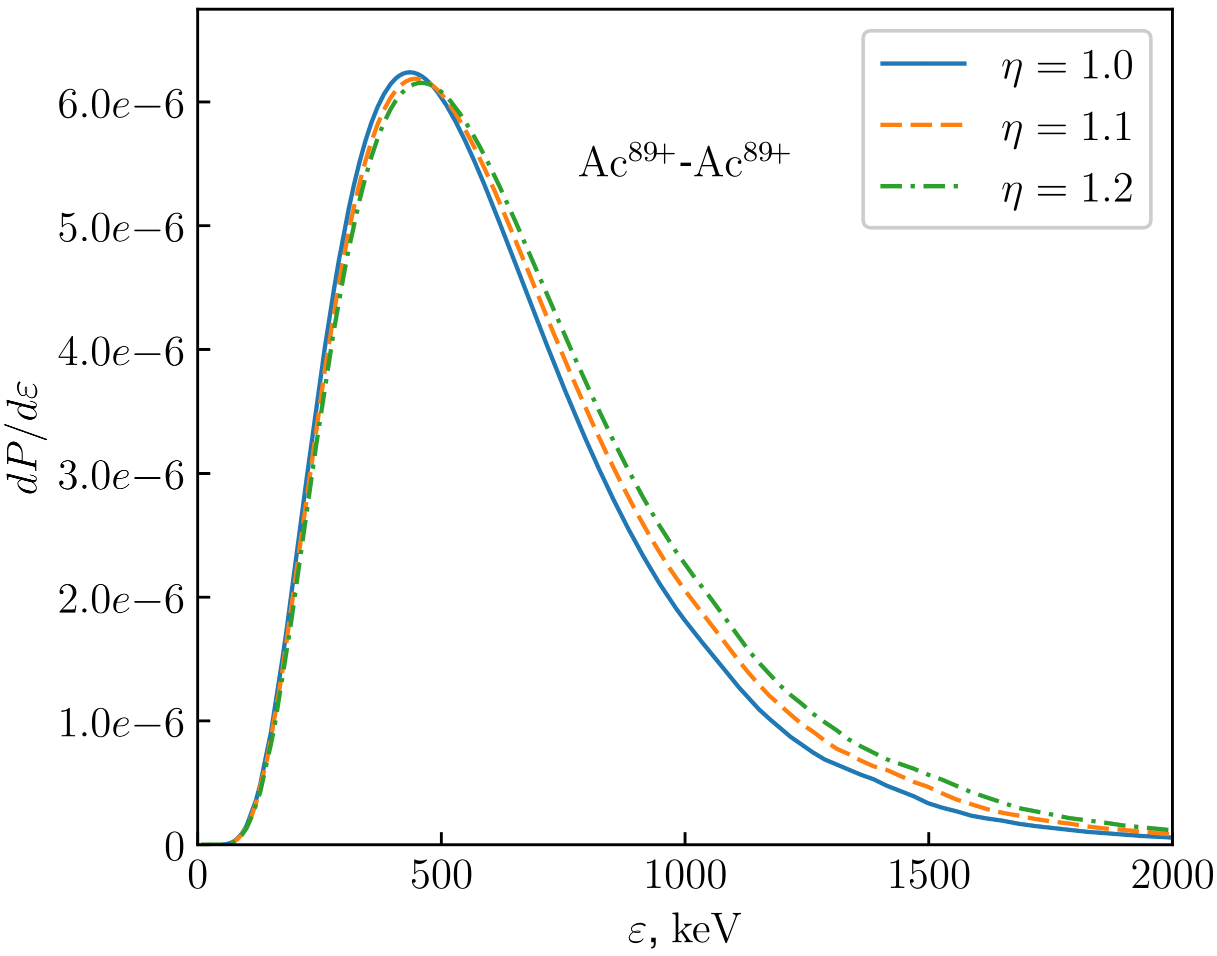
Positron spectra in symmetric ($Z = Z_1 = Z_2 = 85$) collisions for different collision energy $\eta = E/E_0$ at $R_{\rm min} = 17.5$ fm *(R.V. Popov, V.M. Shabaev, D.A. Telnov et al., PRD, 2020)*.

How to observe the vacuum decay



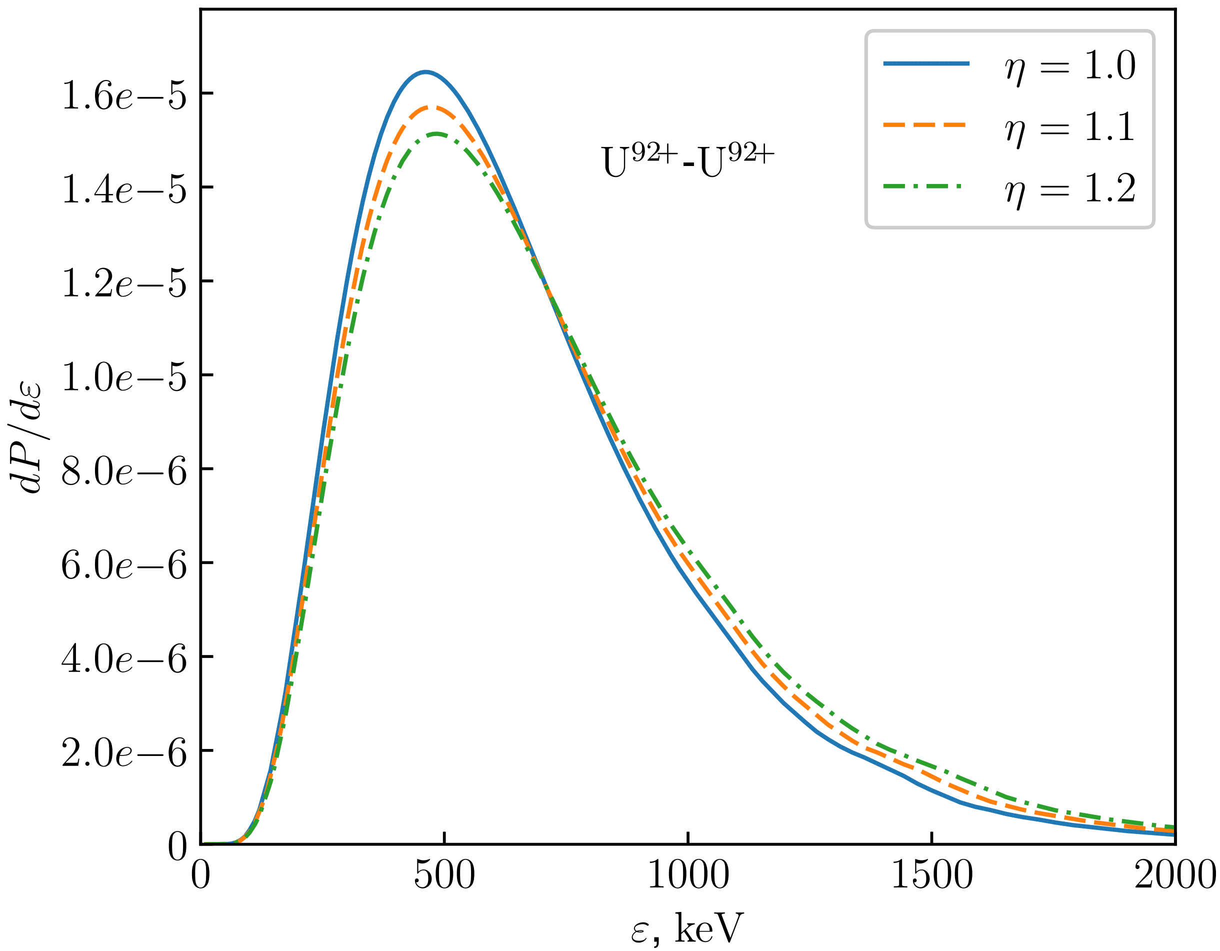
Positron spectra in symmetric ($Z = Z_1 = Z_2 = 87$) collisions for different collision energy $\eta = E/E_0$ at $R_{\rm min} = 17.5$ fm *(R.V. Popov, V.M. Shabaev, D.A. Telnov et al., PRD, 2020)*.

How to observe the vacuum decay



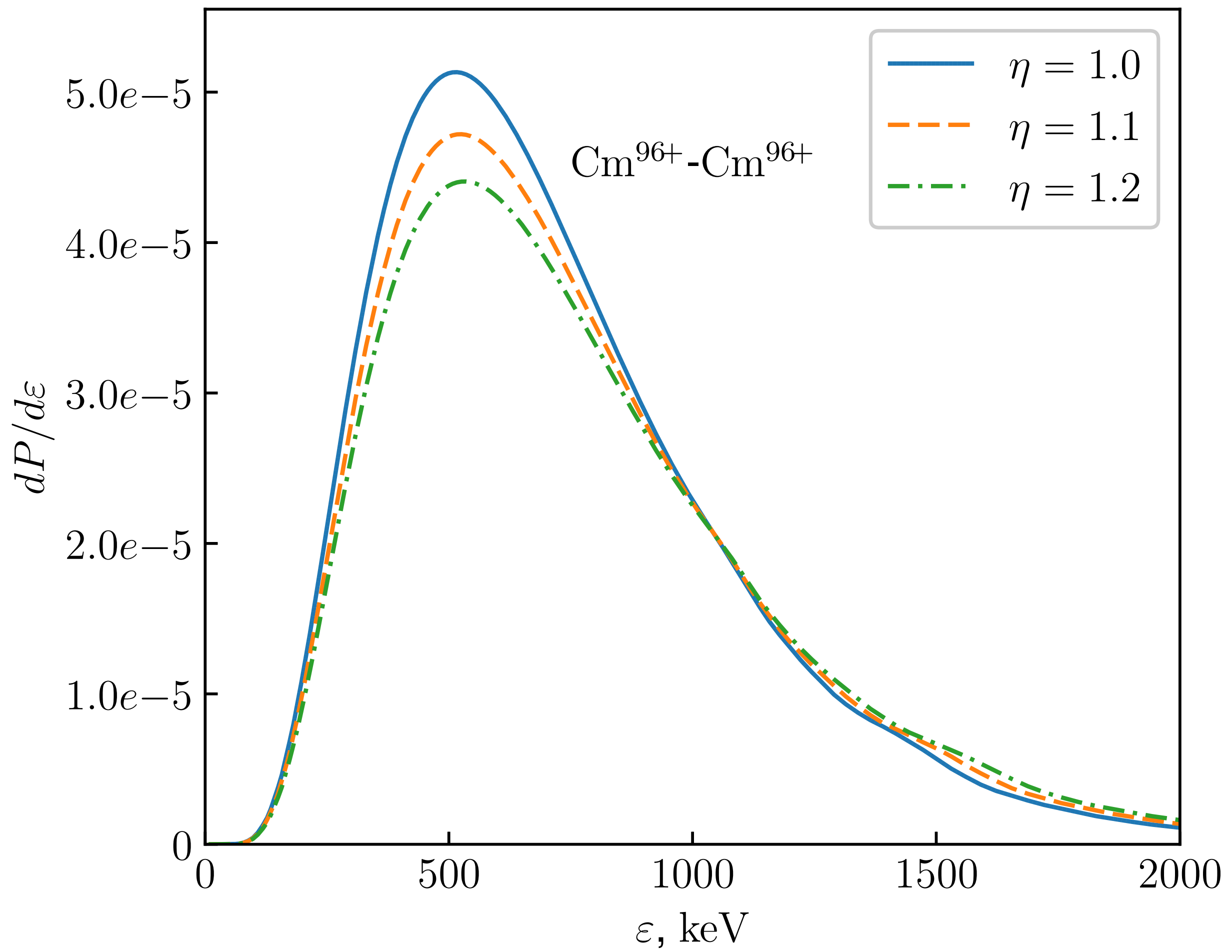
Positron spectra in symmetric ($Z = Z_1 = Z_2 = 89$) collisions for different collision energy $\eta = E/E_0$ at $R_{\min} = 17.5$ fm *(R.V. Popov, V.M. Shabaev, D.A. Telnov et al., PRD, 2020)*.

How to observe the vacuum decay



Positron spectra in symmetric ($Z = Z_1 = Z_2 = 92$) collisions for different collision energy $\eta = E/E_0$ at $R_{\rm min} = 17.5$ fm *(R.V. Popov, V.M. Shabaev, D.A. Telnov et al., PRD, 2020)*.

How to observe the vacuum decay



Positron spectra in symmetric ($Z = Z_1 = Z_2 = 96$) collisions for different collision energy $\eta = E/E_0$ at $R_{\min} = 17.5$ fm *(R.V. Popov, V.M. Shabaev, D.A. Telnov et al., PRD, 2020)*.

Conclusion

Investigations of heavy ions at low-energy regime can provide:

- Tests of QED at strong coupling regime
- Determination of the fundamental constants
- Determinations of the nuclear charge radii and magnetic moments
- Observing the vacuum decay in supercritical fields