История и современный статус физики нейтрино

Александр Студеникин

25/04/2006

НИИЯФ
МГУ

кафедра
теоретической
физики

- Вступление
- История нейтрино
- Экспериментальный статус осцилляций нейтрино
- Электромагнитные свойства нейтрино
- Новые эффекты (1, 2, 3, 4) в нейтринных осцилляциях
- Квантовый подход к описанию нейтрино в веществе

Faculty of Physics

Moscow State University

Dedicated to the 250th Anniversary of Moscow State University

TWELFTH LOMONOSOV CONFERENCE ON ELEMENTARY PARTICLE PHYSICS

Mikhail Lomonosov
1711-1765

Moscow, August 25-31, 2005

www.icas.ru

Electroweak Theory
Tests of Standard Model and Beyond
Developments in QCD (Perturbative and Non-Perturbative Effects)
Heavy Quark Physics
Neutrino Physics
Astroparticle Physics
Gravitation and Cosmology
Physics at the Future Accelerators

SIXTH INTERNATIONAL MEETING ON PROBLEMS OF INTELLIGENTSIA

August 31, 2005

The Intelligentsia and Violence: Responses to Repression and Terrorism

V.Sadovnichy (Rector of MSU) - Chairman

International Advisory Committee

E.Akhmedov (ICTP, Trieste & Kurchatov Inst., Moscow)
S.Belayev (Kurchatov Inst., Moscow)
V.Berezinsky (LNGS, Gran Sasso)
S.Bilenky (JINR, Dubna)
J.Bleimaier (Princeton)
M.Danilov (ITEP, Moscow)
G.Diambrini-Palazzi (Univ. of Rome)
A.Dolgov (INFN, Ferrara & ITEP, Moscow)
V.Kadyshevsky (JINR, Dubna)
S.Kapitza (EAPS, Moscow)
A.Logunov (IHEP, Protvino)
V.Matveev (INR, Moscow)
P.Nowosad (Univ.of Sao Paulo)
L.Okun (ITEP, Moscow)
V.Rubakov (INR, Moscow)
D.Shirkov (JINR,Dubna)
J.Silk (Univ. of Oxford)
A.Skrinsky (INP, Novosibirsk)
A.Slavnov (MSU & Steklov Math. Inst.)
A.Smirnov (ICTP, Trieste & INR, Moscow)
P.Spillantini (INFN, Florence)

Organizing Committee

V.Bagrov (Tomsk Univ.)
V.Belokurov (MSU)
V.Braginsky (MSU)
A.Egorov (ICAS)
D.Galtsov (MSU)
A.Grigoriev (MSU)
P.Kashkarov (MSU)
A.Kataev (INR, Moscow)
O.Khrustalev (MSU)
V.Mikhailin (MSU & ICAS)
A.Mourao (IST/CENTRA, Lisbon)
N.Narozhny (MEPHI, Moscow)
A.Nikishov (Lebedev Phys. Inst., Moscow)
N.Nikiforova (MSU)
V.Ritus (Lebedev Phys. Inst., Moscow)
Yu.Popov (MSU)
V.Savrin (MSU)
Yu.Simonov (ITEP, Moscow)
A.Sissakian (JINR, Dubna)
A.Studenikin (MSU & ICAS)
V.Trukhin (MSU)

Organizers and sponsors

Faculty of Physics, Moscow State University
Interregional Centre for Advanced Studies
Joint Institute for Nuclear Research (Dubna)
Institute of Theoretical and Experimental Physics (Moscow)
Institute for Nuclear Research (Moscow)
Institute for High Energy Physics (Protvino)
Russian Foundation for Basic Research
Centre for International Projects of Russia
Ministry for Education and Science

INTERREGIONAL CENTRE
FOR ADVANCED STUDIES

For contact: Alexander Studenikin, Chairman, studenik@srd.sinp.msu.ru; Alexander Grigoriev, Scientific Secretary, ane@srd.sinp.msu.ru

Department of Theoretical Physics, Moscow State University, 119992 Moscow, Russia
Phone (007-095) 939-50-33 Fax (007-095) 932-88-20 http://www.icas.ru


TWELFTH
CONFERENCE

XII Lomonosov Conference on Elementary Particle Physics
Moscow State University, August 25-31, 2005

Round Table discussion on

"Neutrino and Astroparticle Physics"

Memorandum

www.icas.ru

XII Lomonosov Conference on Elementary Particle Physics
Moscow State University, August 25-31, 2005

Memorandum

of the Round Table discussion on "Neutrino and Astroparticle Physics"

The progress in particle physics comes from both the high energy frontier and precision experiments. This applies to accelerator and non-accelerator physics. In the last years, field like neutrino physics, astroparticle physics and cosmology have had an spectacular development.

One may anticipate that these clues to the knowledge of nature will develop more along the XXI century, particular when taking into account the longer time periods involved in the construction of higher energy facilities.

There has been important progress in neutrino and astroparticle physics achieved during the last several years:

- The non-vanishing neutrino mass and flavour violation has been observed in neutrino oscillation experiments.

- The validity of the Standard Solar Model has also been proven.

- The non-zero neutrino mass can have an important impact on cosmology, in particular, for our understanding of the baryon asymmetry of the universe. On the other hand, the upper boundary of the sum of three neutrino masses can now be constrained on the level of the order of 1 eV from cosmology.

- Observations of tritium beta-decay have lowered the neutrino upper mass limit to the level of 2.1 eV.

- Double beta decay experiments have reached a sensitivity ~ (0.5 – 1) eV for effective Majorana mass of the neutrino.

World-wide recognition of the obtained results has been evidenced by two Nobel Prizes which have been recently awarded for research in neutrino and astroparticle physics.

Further progress in the study of the fundamental properties of neutrinos will open the window to a new physics. Application of these studies could also play a very important role in our understanding of the inner structure of stellar cores as well as of the early stages of evolution of the universe. Studies of geo neutrinos also open promising possibilities for the future.

More accurate measurements of neutrino characteristics will make further progress in the field possible. Our conference also focused on the need to train specialized manpower in this field for the future.

Problem of $\nu \Longleftrightarrow$ fundamentals of particle physics

- ✓ * ν (W. Pauli, 1930) $\longleftrightarrow$ energy conservation
- ✓ * theory of β-decay (E. Fermi, 1932-1934)
- ✓ * parity violation (T. Lee, C. Yang, 1956)
- ✓ * (V-A) model of weak interaction (E. Sudarshan, R. Marshak, 1956; R. Feynman, M. Gell-Mann, 1958)
- ✓ * Standard Model (S. Glashow, 1961; S. Weinberg, 1967; A. Salam, 1968)
- ✓ * New Physics ? (• Kamiokande, 1998; • LSND; ...; SNO, 2001) • KamLand 2002 ! !

Open questions:

- $m_\nu = ?$ (why $m_\nu \ll m_f$)
- N of flavours ($\nu_e, \nu_\mu, \nu_\tau, \nu_4$)
- mixing between the ν and $\nu_1 \leftrightarrow \nu_2$
- Dirac $\leftrightarrow$ Majorana
- electromagnetic properties
- ν in astrophysics ($\nu_\odot$, SN 1987A, relic ν, dark matter, stellar nucleosynthesis and cooling)

Crucial role of neutrino

ν **is a “tiny” particle :**

- **very light** $m_{\nu_f} \ll m_f, \quad f = e, \mu, \tau$
- **electrically neutral** $q_\nu = 0$ $q_\nu < 4 \times 10^{-17} e$
- **with very small magnetic moment** μ_ν **?**
- **weak interactions are indeed weak**

$\sigma_{\nu_e N} \sim 10^{-39}\ cm^2$ ν-N scattering

$\sigma_{\bar{\nu}_e p} \sim 10^{-40}\ cm^2$ inverse β-decay

$\sigma_{\nu_e e} \sim 10^{-43}\ cm^2$ ν-e scattering

at the final stages of development of particular elementary particle physics framework

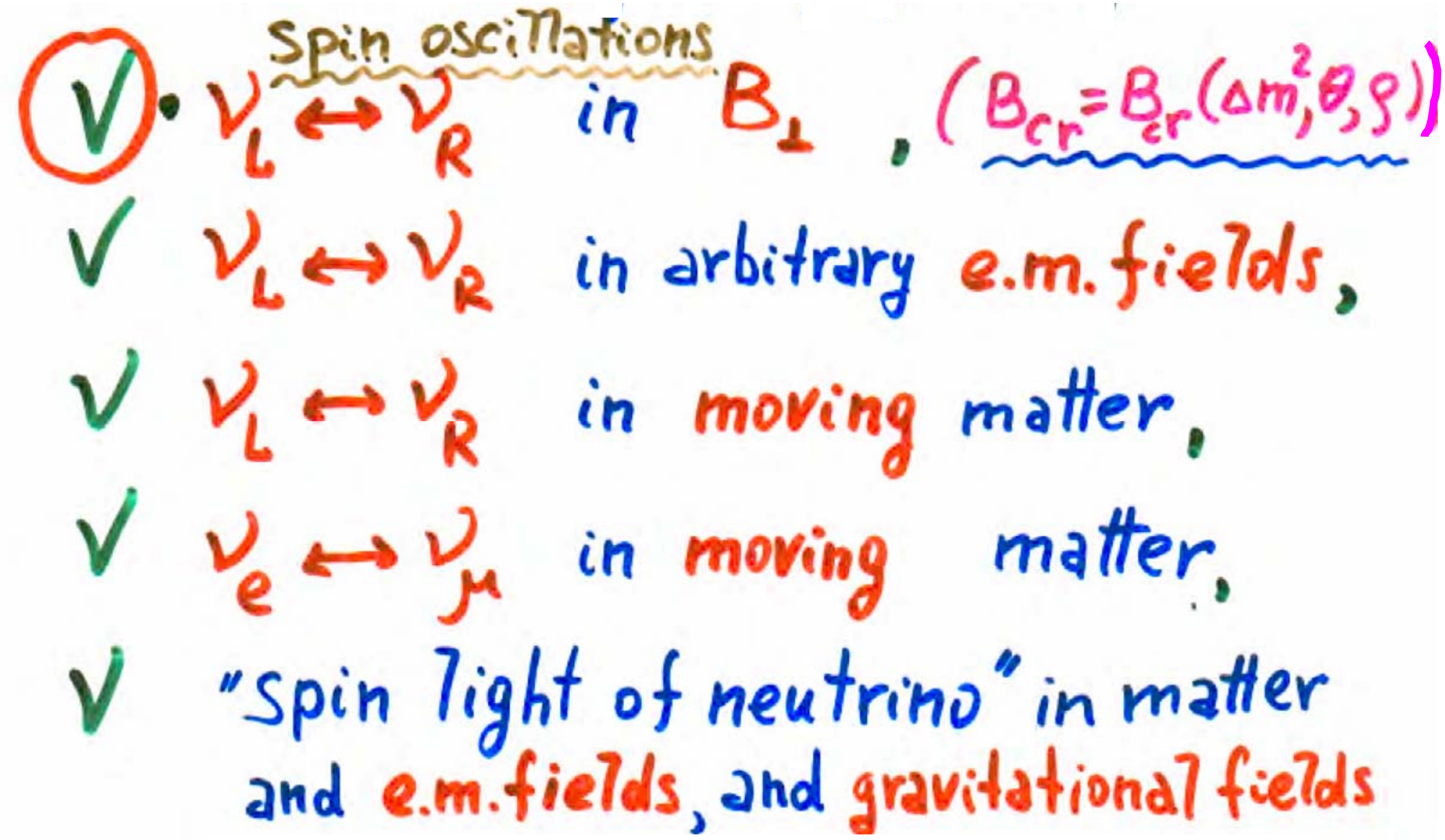
ν manifests itself most vividly under the influence of external conditions:

- **bakground matter**

and

- **external (electromagnetic etc) fields**

Main results of our research group



Spin oscillations

1994-1997 ✓ • $\nu_L \leftrightarrow \nu_R$ in $B_\perp$, $(B_{cr} = B_{cr}(\Delta m^2, \theta, \rho))$

1998-2000 ✓ $\nu_L \leftrightarrow \nu_R$ in arbitrary e.m. fields,

2000-2002 ✓ $\nu_L \leftrightarrow \nu_R$ in moving matter,

1995-2002 ✓ $\nu_e \leftrightarrow \nu_\mu$ in moving matter,

2003-2005 ✓ "spin light of neutrino" in matter and e.m. fields, and gravitational fields

2004-2006… ✓ **quantum theory of neutrino motion in background matter**

A.Studenikin, *J.Phys.A: Math. Gen.* **39** (2006)

A.Studenikin, A.Ternov, *Phys.Lett.**B*** **608** (2005) 107

A.Studenikin, *Nucl.Phys.**B*** (Proc.Suppl.) **143** (2005) 570

A.Grigoriev, A.Studenikin, A.Ternov, *Phys.Lett.**B*** **622** (2005) 199

Grav. & Cosm. **11** (2005) 132

K.Kouzakov, A.Studenikin, *Phys.Rev.**C*** 72 (2005) 015502

M.Dvornikov, A.Grigoriev, A.Studenikin, *Int.J Mod.Phys.**D*** **14** (2005) 309

S.Shinkevich, A.Studenikin, *Pramana* **64** (2005) 124

A.Studenikin, *Phys.Atom.Nucl.* **67** (2004) 1014

M.Dvornikov, A.Studenikin, *Phys.Rev.**D*** **69** (2004) 073001

Phys.Atom.Nucl. **67** (2004) 719

JETP **99** (2004) 254

JHEP **09** (2002) 016

A.Lobanov, A.Studenikin, *Phys.Lett.**B*** **601** (2004) 171

*Phys.Lett.**B*** **564** (2003) 27

*Phys.Lett.**B*** **515** (2001) 94

A.Grigoriev, A.Lobanov, A.Studenikin, *Phys.Lett.**B*** **535** (2002) 187

A.Egorov, A.Lobanov, A.Studenikin, *Phys.Lett.**B*** **491** (2000) 137

⊛ New effects: #1, #2, #3, #4

hep-ph/0407010,

A. Studenikin: Neutrino in electromagnetic fields and moving matter,

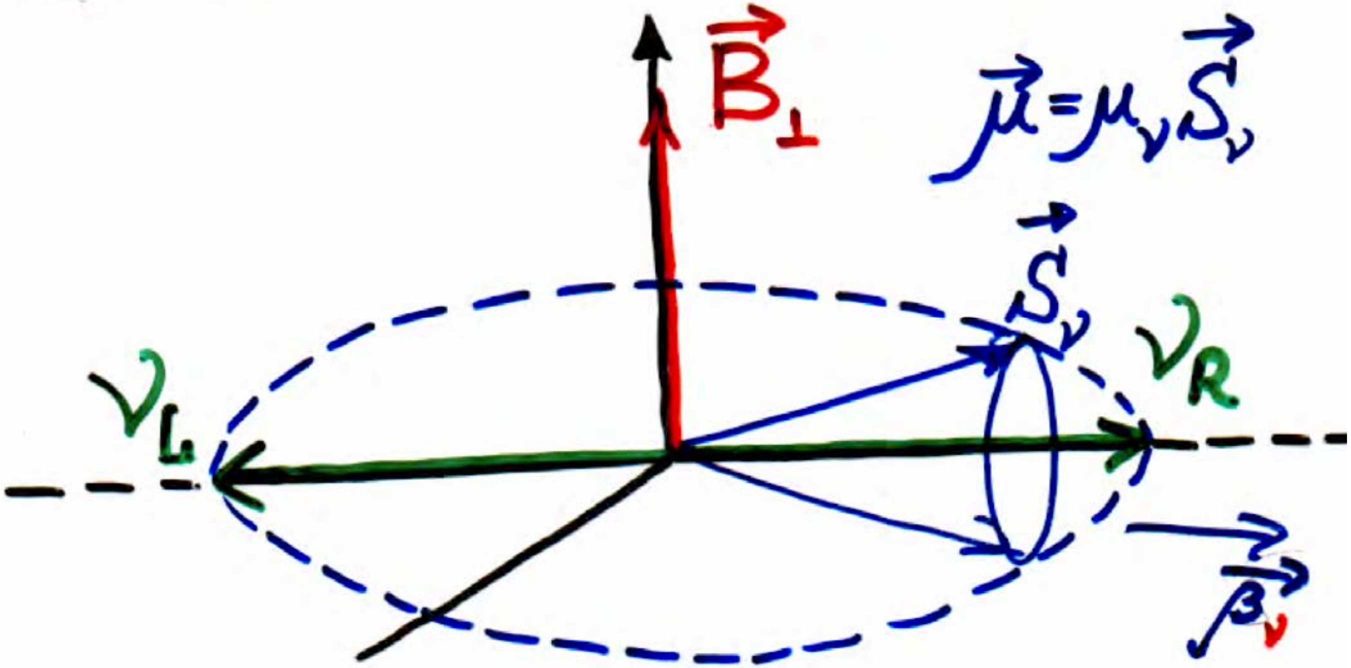
Phys. Atom. Nucl. 67 (N5) 1024, 2004.

#1 Lorentz invariant approach to ν spin evolution in arbitrary e.m. field $F_{\mu\nu}$ (only $B_\perp$ ⊛ was considered before)

⇨ predictions for new resonances in $\nu_L \leftrightarrow \nu_R$ in various configurations of e.m. fields (e.m. wave etc ...)

#2 ... matter effect included ...

ν spin precession can be stimulated not only by e.m. interactions with e.m. field $F_{\mu\nu}$ but also by ν weak interactions with matter!



$$\frac{d\vec{S}_\nu}{dt} = 2\mu_\nu[\vec{S}_\nu \times \underline{\underline{\vec{B}}}\,] + \boxed{2\mu_\nu[\vec{S}_\nu \times \vec{G}\,]}$$

electromagnetic interacton with e.m. field

weak interaction with matter

.

③ $\nu_L \leftrightarrow \nu_R$ and $\nu_\ell \leftrightarrow \nu_{\ell'}$, $\ell \neq \ell'$

(neutrino spin and flavour oscillations)

in moving and polarized matter

$\Rightarrow$

|| matter motion can significantly
|| change the neutrino oscillation pattern

#4

New mechanism of e.m. radiation by ν in matter and e.m. fields, and gravitational fields

$$\boxed{\nu \xrightarrow[\text{environment}]{\text{background}} \nu + \gamma,}$$

‖ "Spin Light of Neutrino": "$SL\nu$"

В. Паули (1930г.) – открытое письмо

Тюбенгенскому физическому обществу:

ν – „нейтрон"

$m_\nu < m_e$ или 0

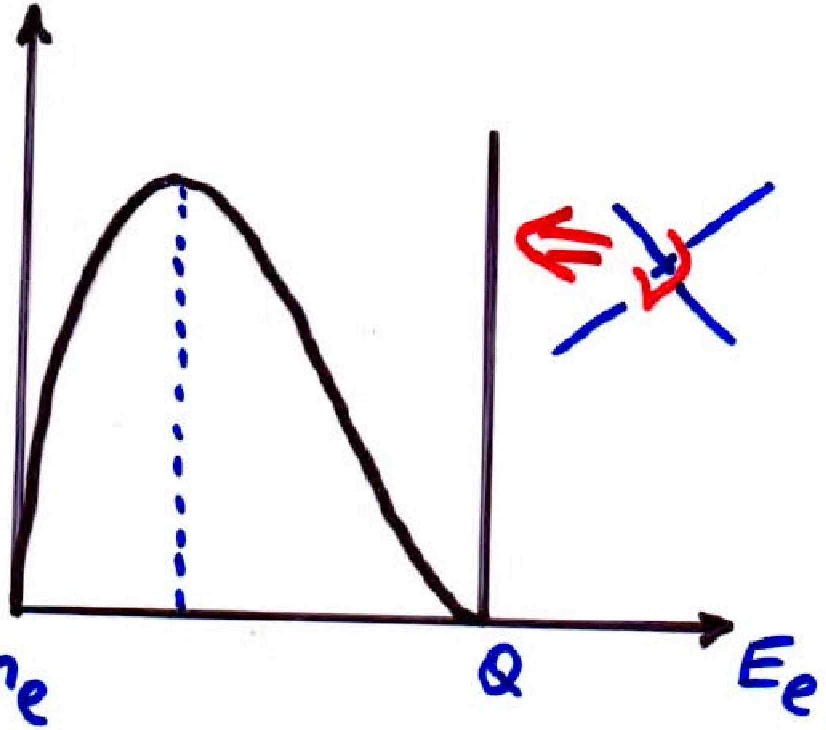
$s = 1/2$

Существование ν „спасает" закон сохранения энергии в β-распаде ядер

Н. Бор ...

$$\frac{d\Gamma}{dE_e}$$

дифференц. вероятность распада



энергия электронов распада

спектр e: ✓

* ∃ ✓

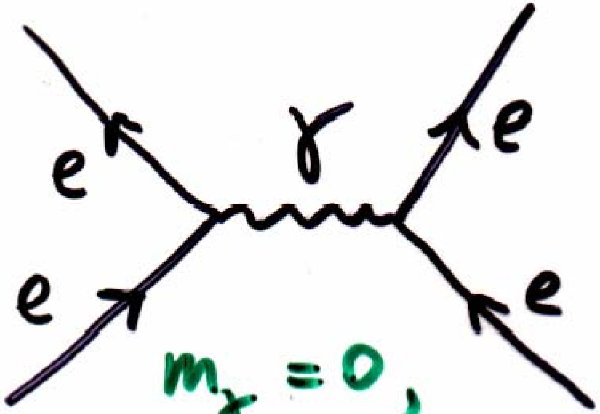
* ограничение на m_{ν_e}

* Теория слабых взаимодействий

Э. Ферми (1933) – аналогия с моделью Дирака электромагнитных взаимодействий

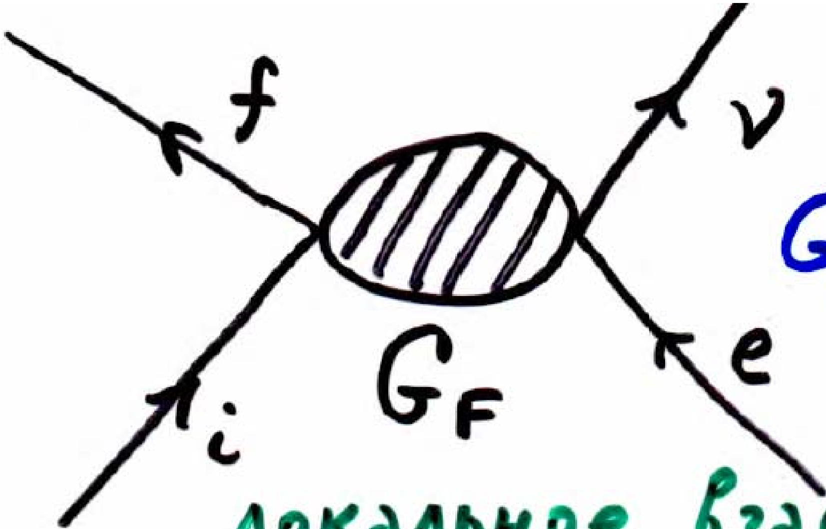
„ток" × „ток"

⇙



$m_\gamma = 0$,

дальнодействие



$$G_F \approx \frac{1}{(1\,\text{GeV})^2} \approx 10^{-5}$$

локальное взаимодействие

$$H = \frac{G_F}{\sqrt{2}} \Psi_p^+ O_\mu \Psi_n \Psi_e^+ O^\mu \Psi_\nu .$$

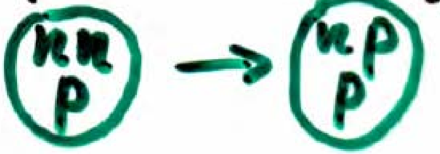
Э. Ферми, Ф. Перрен (1934) — метод прямого измерения массы ν по β-спектру вблизи его конца.

$$(A, Z) \to (A, Z \pm 1) + e^{\mp} + \nu(\bar{\nu})$$

$$n \to p + e + \underset{\sim}{\bar{\nu}_e}$$

Тритий :

$$^{3}_{1}H \to {}^{3}_{2}He + e + \boxed{\bar{\nu}} \;, \qquad T_{1/2} = 12,3 \text{ лет}$$



гелий

$$Q = M(A, Z) - M(A, Z+1) = 18.6 \text{ keV}$$

разность масс родительского и дочернего ядер

Диаграмма Кюри

(спектр электронов распада)

функция Ферми

$$N(p_e)dp_e \sim |\langle f|H_\beta|i\rangle|^2 F(z,E_e)\times$$

$$\times p_e(Q-E_e)^2\sqrt{1-\left(\frac{m_\nu c^2}{Q-E_e}\right)^2}\,dp_e$$

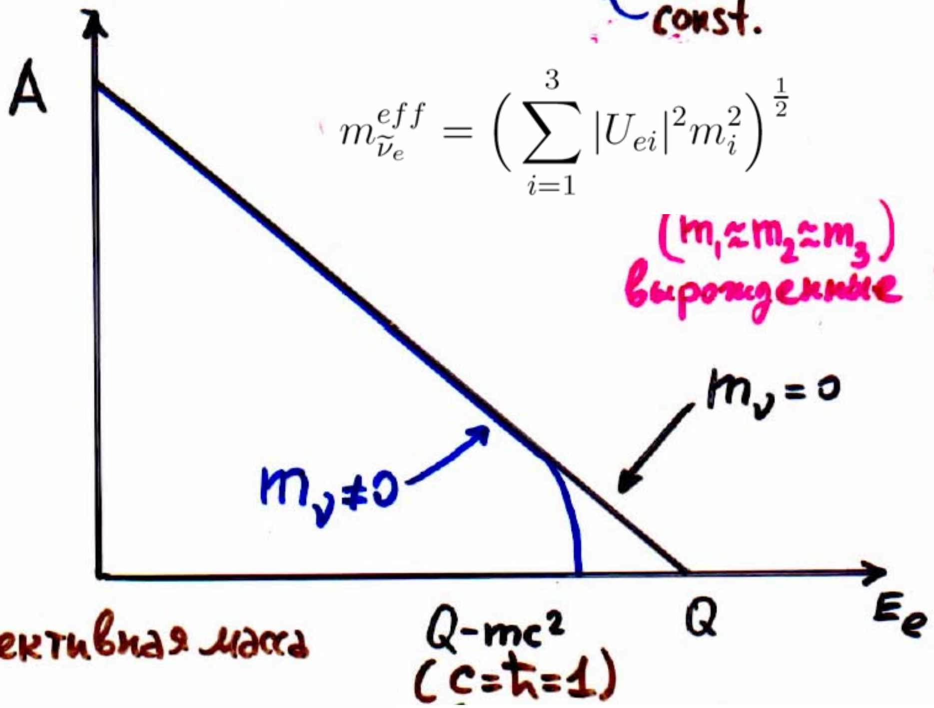
$$\Downarrow$$

энерговыделение

кинетическая энергия электрона

$$A=\sqrt{N(p_e)/p_e^2 F(z,E_e)} = B\cdot(Q-E_e)$$

const.



$$m^{eff}_{\tilde{\nu}_e} = \left(\sum_{i=1}^{3}|U_{ei}|^2 m_i^2\right)^{\frac{1}{2}}$$

$(m_1 \simeq m_2 \simeq m_3)$

вырожденные ν

эффективная масса

эффективная масса

$$\boxed{m_{\nu_e} < 2.2 \text{ eV}}$$

(В. М. Лобашев, Троицк 2003

(Ch. Weinheimer, Mainz 2003)

⊛ $m_{\nu_\mu} < 170$ keV $(\pi^+ \to \mu^+ + \nu_\mu)$

⊛ $m_{\nu_\tau} < 18.2$ MeV $(\tau^\pm \to \nu_\tau + \pi^\pm + \pi^+ + \pi^-)$

Стандартная модель электрослабых взаимодействий Вайнберга-Салама:

✓ $m_\nu = 0$.

⊛ Однако ещё в 1957 г.
Б. Понтекорво рассматривал
возможность $m_\nu \neq 0$ и

✓ $\nu \leftrightarrow \bar{\nu}$. (Н.п. ?)

Осцилляции нейтрино !

Экспериментальное
открытие $\bar{\nu}$: $\bar{\nu} + p \rightarrow n + e^+$ (1956)

Ф. Райнес – $\boxed{\text{Н.п.}}$ 1996
С. Кован

A.I.Leipunsky (1936),
J.Allen (1942) –
recoil of nucleus (indirect evidence)

Экспер. открытие

$\nu_\mu \neq \nu_e$:

$$\pi^+ \to \mu^+ + \underbrace{\nu \Leftarrow n}_{\downarrow\ \mu,\ \cancel{e}} \quad (1962)$$

Р. Дэвис
М. Кошиба } — $\boxed{\text{Н.п. 2002}}$

Л. Ледерман
М. Шварц
Дж. Стейнберг } — $\boxed{\text{Н.п.}}$ 1988

⊛ З. Маки, М. Хакагава, С. Саката (1962)
смешивание ν: $\nu_e \leftrightarrow \nu_\mu$.

J. Exptl. Theoret. Phys.
<u>34</u> (1957), p. 549.

Bruno Pontecorvo : (1957)

neutrino oscillations

$$\nu \leftrightarrow \bar{\nu}$$

В. Грибов, Б. Понтекорво (1969)
С. Биленкий, Б. Понтекорво (1976)

✱ Осцилляции ν в вакууме

$$\underbrace{\nu^f = \begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}}_{\text{состояния взаимодействия}} \Longleftrightarrow \underbrace{\nu^p = \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}}_{\text{массовые состояния}}$$

(m_1, m_2)

$$\begin{pmatrix}\pi^+ \to \mu^+ + \nu_\mu\\ n \to p + e + \bar{\nu}_e\end{pmatrix}$$

$$\nu_e = \nu_1 \cos\theta_v + \nu_2 \sin\theta_v$$

$$\nu_\mu = -\nu_1 \sin\theta_v + \nu_2 \cos\theta_v$$

← угол смешиван
нейтрино в
вакууме

$$\nu_e = \nu_1 \cos\theta_\nu + \nu_2 \sin\theta_\nu$$

угол смешивани
нейтрино в
вакууме

$$\nu_\mu = -\nu_1 \sin\theta_\nu + \nu_2 \cos\theta_\nu$$

Эволюция пучка ν во времени (пространств)

$$i\frac{d}{dt}\nu^p(t) = H\nu^p(t), \quad H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \quad E_i \approx |\vec{p}| + \frac{m_i^2}{2|\vec{p}|}$$

ν

$$\boxed{P_{\nu_e \to \nu_\mu}(x) = \sin^2 2\theta_\nu \sin^2 \frac{\pi x}{L}}$$

путь, пройденный нейтрино

энергия нейтрино

амплитуда осцилляций

Осцилляции нейтрино

$$L = \frac{4\pi E}{\Delta m^2}, \quad \Delta m^2 = m_2^2 - m_1^2$$

длина осцилляций

1

Matter effect in ν flavour oscillations

Осцилляции нейтрино в веществе

Распространение ν в веществе ($\nu_\odot$, ν_{NS}...

... аналогия с фотоном:

① vacuum $\longrightarrow$ закон дисперсии $E_\gamma = |\vec{p}|$, $v_\gamma = c$,

② matter $\longrightarrow$ $m_\gamma^{eff} \neq 0$, $v_\gamma < c$.

=> Взаимодействие с веществом изменяет закон дисперсии => изменяется гамильтониан системы => изменяется временная эволюция нейтринного пучка

L. Wolfensein, 1978

Обычное вещество (Солнце, NS, …):

e, p, n (нейтральность: $n_e = n_p$)

Рассмотрим распространение $\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$

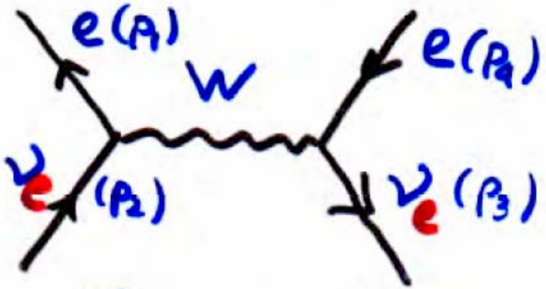
с учетом рассеяния на e, p и n $\Rightarrow$

электрослабые взаимодействия за счет

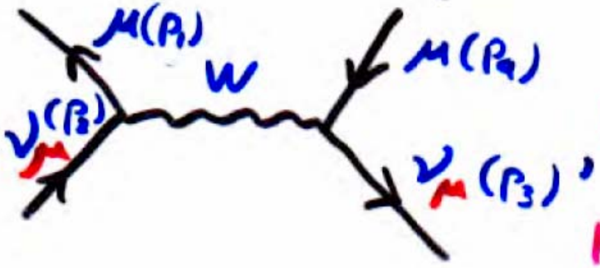
CC-заряженных, **NC**-нейтральных токов:

Ⓘ (CC) Вклад заряженных токов

✓ ν_e:



✓ ν_μ:



, НО!

$n_\mu = 0$

$(T \sim 1\,эВ \sim 10^4\,K)$

⊛ $\boxed{\Delta \mathcal{L}^{(\mu)}_{eff} = 0,}$

⊛ $\Delta \mathcal{L}^{(e)}_{eff} = \left(\frac{ig}{\sqrt{2}}\right)^2 [\bar{e}_L \gamma^\lambda \nu_{e_L}] \frac{-ig_{\lambda\rho}}{(p_2-p_1)^2 - M_W^2} [\bar{\nu}_{e_L} \gamma^\rho e_L] \approx$

$$\approx -\frac{4G_F}{\sqrt{2}} [\bar{e}_L \gamma^\lambda e_L][\bar{\nu}_{e_L} \gamma_\lambda \nu_{e_L}],$$

$$\boxed{\Delta \mathcal{L}^{(e)}_{CC} = -\sqrt{2}\, G_F\, n_e\, \bar{\nu}_{e_L} \gamma_0 \nu_{e_L}}$$

⊙ неподвижная ⊙ неполяризованная среда

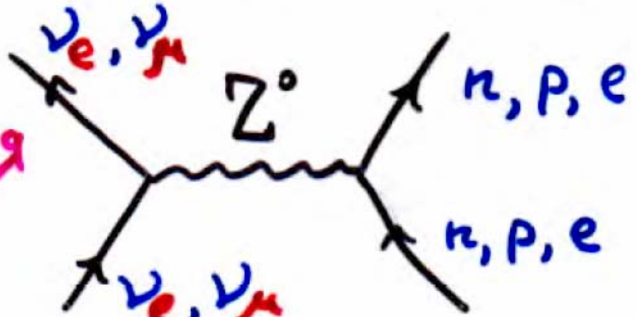
$\langle \bar{e} \gamma_\lambda \gamma_5 e \rangle = \left\langle \frac{\vec{\sigma}_e \vec{p}_e}{E_e} \right\rangle$

$\langle \bar{e} \vec{\gamma} e \rangle = \vec{v}_e \approx 0$

$\langle \bar{e} \gamma_0 e \rangle = n_e$

плотность электронов в $см^{-3}$

Ⅱ NC Вклад нейтральных токов



Взаимодействия ν_e и ν_μ с
$f = n, p, e$
одинаковые:

$$\Delta \mathcal{L}^{(e,\mu)}_{eff} = -\frac{4G_F}{\sqrt{2}}\left[\bar f \gamma^\lambda \left(\underset{\sim}{I_{3L}} \frac{1-\gamma_5}{2} - \underset{\sim}{Q} \sin^2\theta_W\right) f\right]\left[\bar\nu_L \gamma_\lambda \nu_L\right]$$

		e	p	n
компонента слабого изоспина	I_{3L}	$-1/2$	$1/2$	$-1/2$
заряд	Q	-1	$+1$	0

Нейтральная среда: $n_e = n_p \Rightarrow$

$$\boxed{\Delta \mathcal{L}^{(e,\mu)}_{NC} = \frac{1}{\sqrt{2}} G_F n_n \left(\bar\nu_{e_L} \gamma_0 \nu_{e_L} + \bar\nu_{\mu_L} \gamma_0 \nu_{\mu_L}\right)}$$

Итого: $\Delta\mathcal{L}_{eff} = \Delta\mathcal{L}^{(e)}_{CC} + \Delta\mathcal{L}^{(\mu)}_{CC} + \Delta\mathcal{L}^{(e,\mu)}_{NC} =$!!! ✓

$$= -\sum_{\ell=e,\mu} \bar{\nu}_{\ell_L}\gamma_0 V_{\nu_\ell}\nu_{\ell_L},$$

…потенциальная энергия ν_μ и ν_e в среде…

$$V_{\nu_e} = \sqrt{2}\,G_F\,(\underset{CC}{\underline{n_e}} - \underset{NC}{\tfrac{1}{2} n_n})$$ ✓

$$V_{\nu_\mu} = -\frac{1}{\sqrt{2}}\,G_F\,\underset{NC}{n_n}.$$ ✓

С учётом $\Delta\mathcal{L}_{eff}$ вместо ур. Дирака

$$(\hat{p} - m)\Psi_{\nu_e,\nu_\mu} = 0$$

имеем:

$$\boxed{(\gamma_0 E - \vec{\gamma}\vec{p} - m_\nu)\Psi_{\nu_e,\nu_\mu} = \gamma_0 V_{e,\mu}\Psi_{\nu_e,\nu_\mu}} \Rightarrow$$

$$E_{\nu_e,\nu_\mu} = \sqrt{p^2 + m_\nu^2} + \underline{V_{e,\mu}}.$$

$$\boxed{(\gamma_0 E - \vec{\gamma}\,\vec{p} - m_\nu)\,\Psi_{\nu_e,\,\nu_\mu} = \gamma_0\, V_{e,\mu}\,\Psi_{\nu_e,\,\nu_\mu}} \Rightarrow$$

$$E_{\nu_e,\,\nu_\mu} = \sqrt{p^2 + m_\nu^2} + \underset{\sim}{V_{e,\mu}}.$$

Уравнение эволюции $\nu^f_{(x)}$ в веществе:

$$i\frac{d}{dx}\,\nu^f = \tilde{H}\,\nu^f,$$

$$\tilde{H} = \left(E + \frac{m_1^2 + m_2^2}{4E} - \frac{1}{\sqrt{2}}G_F n_n\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{1}{2E}\tilde{M}^2,$$

$$\tilde{M}^2 = \frac{1}{2}\begin{pmatrix}-\Delta m^2\cos 2\theta + 2\underset{\sim}{A} & \Delta m^2\sin 2\theta\\ \Delta m^2\sin 2\theta & \Delta m^2\cos 2\theta\end{pmatrix}.$$

$$\uparrow$$

$$A = 2\sqrt{2}\,G_F n_e E$$

Neutrino flavour oscillations in matter

travelled distance

✓

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\theta_{eff}\,\sin^2\frac{\pi x}{L_{eff}}\,,}$$

$L_{eff}=L_{eff}(\theta, E_\nu, \delta m_\nu^2)$

amplitude of oscillations:

✓

$$\boxed{\sin^2 2\theta_{eff} = \frac{\Delta^2 \sin^2 2\theta_{vac}}{(\Delta \cos 2\theta_{vac} - \tilde{A})^2 + \Delta^2 \sin^2 2\theta_{vac}}\,,}$$

$$L_{eff} = \frac{4\pi}{((\Delta \cos 2\theta - \tilde{A})^2 + (\Delta \sin 2\theta)^2)^{1/2}} \qquad \Delta = \frac{\delta m_\nu^2}{2E_\nu} = \frac{m_2^2 - m_1^2}{2E_\nu}\,,$$

✓

For unpolarized matter at rest, ⊛

$$\tilde{A} = \sqrt{2}\, G_F\, n \leftarrow \text{particle number density}$$

! $\theta_{vac} \to \theta_{eff}$

Mikheev-Smirnov-Wolfenstein resonance:

✓

$$\boxed{\frac{\delta m_\nu^2}{2E_\nu}\cos 2\theta_{vac} = \sqrt{2}\, G_F\, n}$$

1978, 1985 !

$$\Downarrow$$

n_{res}

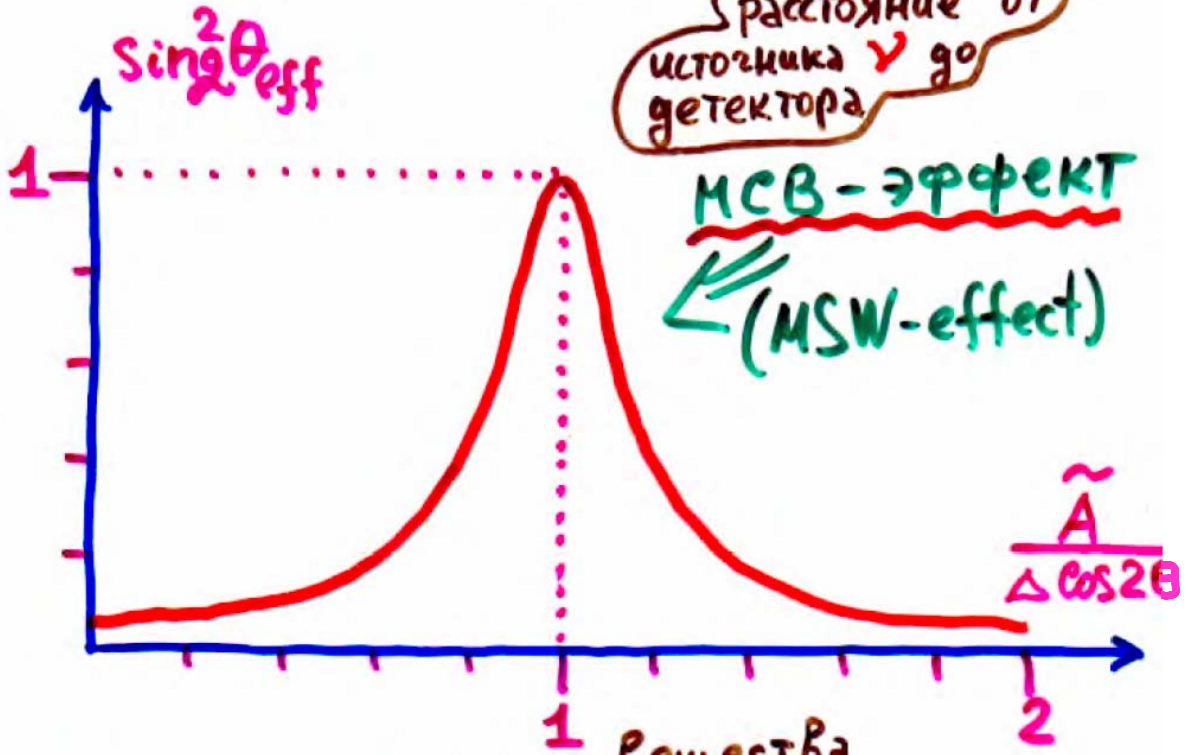
$$P_{\nu_e\to\nu_\mu}(x) = \underbrace{\sin^2 2\theta_{eff}}\cdot \sin^2 \frac{\pi x}{L_{eff}}$$

амплитуда осцилляций

расстояние от источника ν до детектора

МСВ-эффект

(MSW-effect)



плотность вещества

$\tilde{A} = \sqrt{2}\, G_F\, n$, $\Delta = \frac{\Delta m^2}{2E}$,

θ - угол смешивания в вакууме

θ_{eff} - угол смешивания в веществе

Экспериментальный статус смешивания и осцилляций нейтрино

R.N.Mohapatra, A.Y.Smirnov,
"Neutrino mass and New Physics", hep-ph/0603118 :

"Recent discovery of
flavour conversion of
solar, atmospheric, reactor and accelerator
neutrinos
have conclusively established that neutrinos have
nonzero mass
and they
mix among themselves
much like quarks, providing the first evidence of
new physics
beyond the standard model"

Стратегия поиска осцилляций ν

(некоторые осцилляторные эксперименты)

Источники ν (схемы экспериментов):

- солнце,
- атмосфера Земли (космические лучи),
- реакторы,
- ускорители.

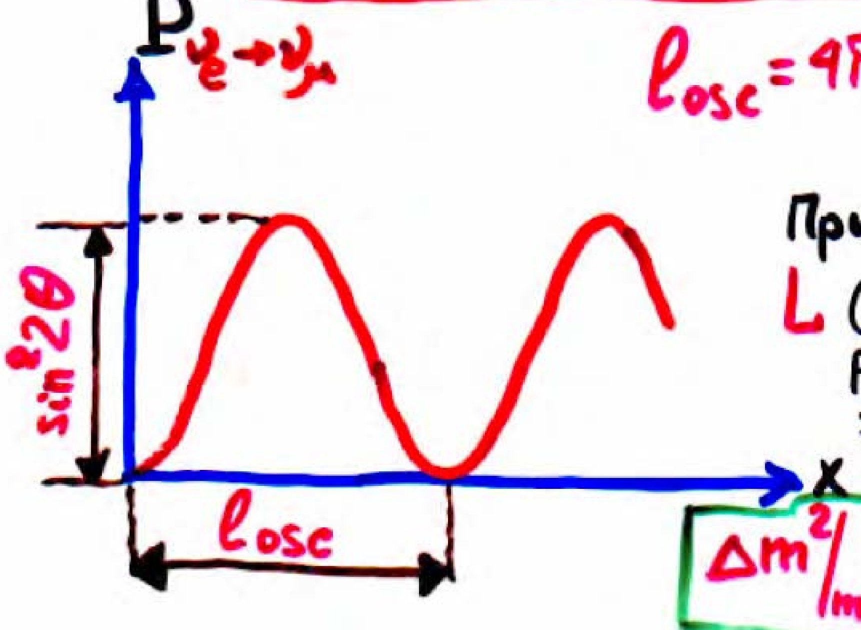
Типы экспериментов – детектирование осцилляций по:

- ⊙ возникновению ν „нового" флейвора ($\nu_e \to \underline{\nu_\mu}$)
- ⊙ исчезновению ν „начального" флейвора ($\underline{\nu_e} \to \nu_\mu$).

Вероятность флейворных осцилляций:

$$\boxed{P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \cdot \sin^2 \frac{x\,\Delta m^2}{4E}}$$

$$\sin^2 \pi \frac{x}{l_{osc}}$$

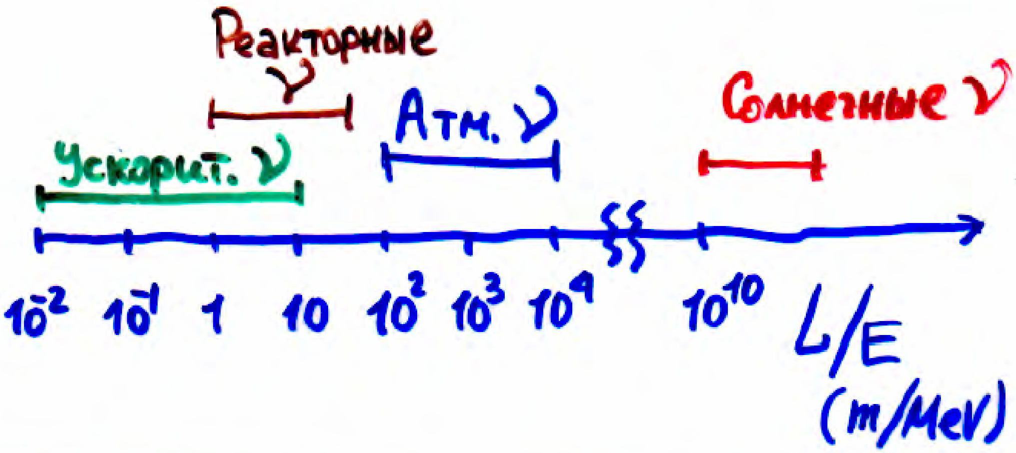


$$l_{osc} = 4\pi \frac{E}{\Delta m^2} = 2.5\,м\,\frac{E}{1\,MeV} \cdot \frac{1\,eV^2}{\Delta m^2}$$

При заданных E и L (расстояние от точки рождения до детектора) эксперимент чувствителен к:

$$\boxed{\Delta m^2\big|_{min} \sim \frac{E}{L}}$$

Области значений L/E, доступные в различных эксперимента



	Флайвор	E(GeV)	L(km)	$\Delta m^2/(eV^2)_{min}$
Атм.	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 10^{2}$	$10 \ldots 10^{4}$	10^{-6}
Солн.	ν_e	$10^{-3} \ldots 10^{-2}$	10^{8}	10^{-11}
Реакт.	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{-1}	10^{-3}
Ускор.	$\nu_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^{2}	10^{-1}

„Новая физика" (экспериментальные указания на $m_\nu \neq 0$ и смешивание ν)

т.е., осцилляции нейтрино существуют

✓ ① Атмосферные ν – „исчезновение" ν_μ

✓ ($\boxed{\Delta m^2_{atm} = 2.5 \cdot 10^{-3} eV^2}$, $\sin^2 2\theta_{atm} = 1$)

(SK, Soudan, MACRO)

✓ ② Ускорительные длинно-базовые ("long-base") эксперименты K2K (KEK to Kamioka, Japan)

✓ – „появление" ν_μ (Δm^2_{atm}, θ_{atm}), 2003

③ ✓ Солнечные $\nu_\odot$ —

$\nu_e \to \nu_\mu$ и ν_τ (Homestake, GALLEX-GNO, ✓SNO, SAGE, Superkamiokande)

LMA ("large-mixing-angle")-решение:

$$\boxed{\Delta m^2_\odot \approx 5\cdot 10^{-5} eV^2}$$

④ ✓ Реакторные эксперименты KamLAND (Japan)

(подтверждение LMA-решения для $\nu_\odot$)

⑤ Ускорительные эксперименты (MiniBooNE 2002, FermiLab)

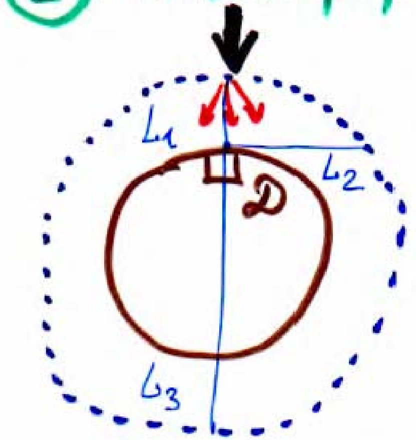
LSND (Liquid Seintillating ν Detector, LAMPF - мезонная фабрика, Лос-Аламос, США)

✓ $\boxed{\Delta m^2_{LSND} \lesssim 1 eV^2}$, $\sin^2 2\theta|_{LSND} \sim 10^{-2} \div 10^{-3}$

{Новая эра в физики элементарных частиц}

ν

① Атмосферные ν (Супер-Камиоканде) 1998



космические лучи

ядра

$(p+N \to n+\pi^{\pm}(K^{\pm})+$

$$P(\alpha\ldots)+Air \to \pi^{\pm}(K^{\pm})+X$$

$$\mu^{\pm}+\nu_\mu(\bar{\nu}_\mu) \quad \textcircled{\mu}_1$$

$$\hookrightarrow e^{\pm}+\nu_e(\bar{\nu}_e)+\bar{\nu}_\mu(\nu_\mu) \quad \textcircled{e}_1 \quad \textcircled{\mu}_2$$

$L_1 = 15 km \; (\theta_\nu = 0)$

$L_2 = 500 km \; (\theta_\nu = 90°)$

$L_3 = 13\,000 km \; (\theta_\nu = 180°)$

Теория (расчеты):

$$N(\nu_\mu/\nu_e) = \frac{N(\nu_\mu,\bar{\nu}_\mu)}{N(\nu_e,\bar{\nu}_e)} = 2$$

(есть зависимость от $\bar{E}_\nu$ и угла падение на детектор)

Неопределенности:

- геомагнитные эффекты
- солнечная активность
- хим. состав космических лучей и т.д.

Сравнение „теории" и „эксперимента":

$$R(\mu/e)=\frac{(\nu_\mu+\bar{\nu}_\mu)/(\nu_e+\bar{\nu}_e)_{exp}}{(\nu_\mu+\bar{\nu}_\mu)/(\nu_e+\bar{\nu}_e)_{\underset{\text{MC}}{th}}},$$

„двойное отношение"

должно быть:

✓ $$\boxed{R(\mu/e)=1}.$$

{Черенковска
{излучение

(n и p мишени
50kt H_2O)

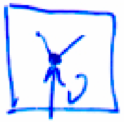
$\nu_e(\bar{\nu}_e)+A\to e^-(e^+)+X$

$\nu_\mu(\bar{\nu}_\mu)+A\to\mu^-(\mu^+)+X$

Однако:

✓ $$\boxed{R(\mu/e)\approx 0.7}\Rightarrow$$

fully contained
events



$E<1.33$ GeV
(Sub-GeV)

$E>1.33$ GeV
(multi-GeV)

✓ ! $\Rightarrow$ дефицит ν_μ^{atm} ($E\sim 1$ GeV).

✓ Решение проблемы: вакуумные

осцилляции $\nu_\mu \longleftrightarrow \nu_\tau$,

M. Gonzalez-Garcia, hep-ph/0410030

$$\boxed{\begin{cases} \Delta m^2 = 2.2 \cdot 10^{-3} eV^2 \\ \sin^2 2\theta_\nu = 1 \end{cases}}$$

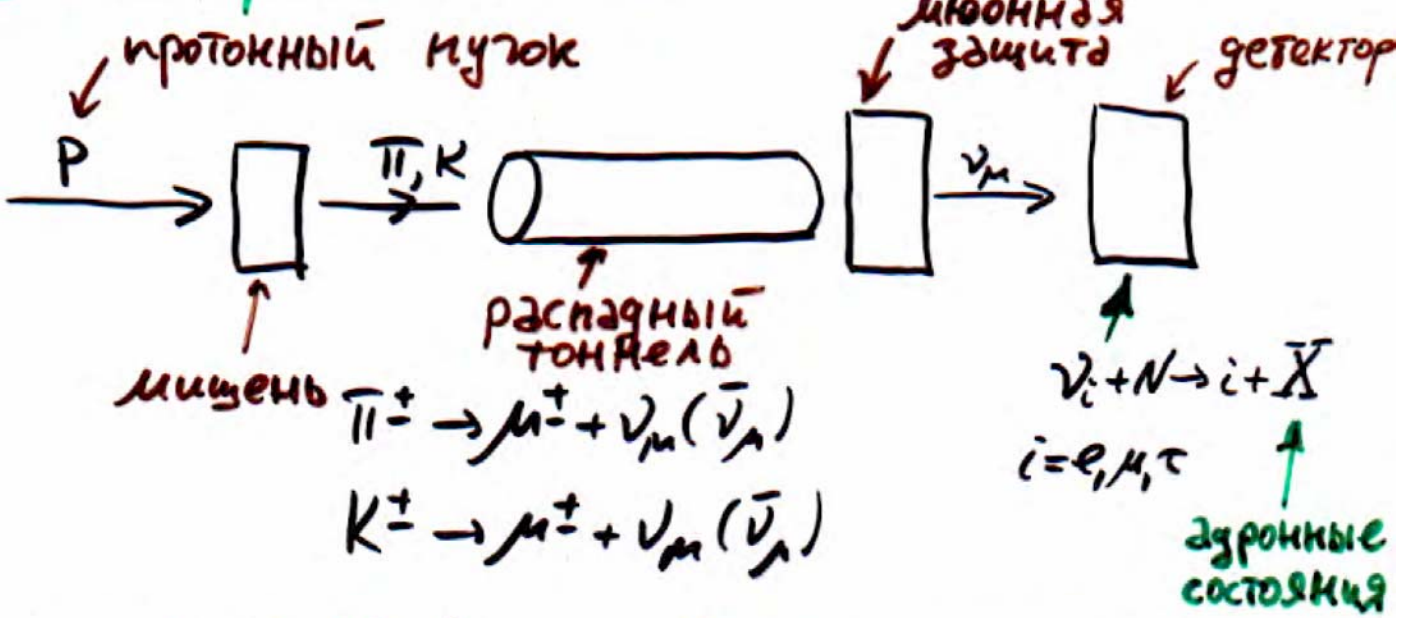
✓

⊛ **NB!** (1998...)

(распределение ν по углу θ_ν; „up-down" асимметрия)

$\left\{\text{Решение проблемы атмосферных нейтрино}\right\}$ Зависимость N_μ от расстояния до точки рождения

② Ускорительные ν



$$\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$$

$$K^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$$

$$\nu_i + N \to i + X$$

$$i = e, \mu, \tau$$

LSND Coll. (Liquid Scintillator ν Detector at LAMPF), Лос-Аламос (США)

$$\left\{ \begin{array}{l} \nu_\mu \to \nu_e \\ \bar{\nu}_\mu \to \bar{\nu}_e \end{array} \right. , \quad \boxed{\begin{array}{l} \Delta m^2_{LSND} \sim 1\,eV^2 \\ \sin^2 2\theta_{LSND} \sim 10^{-3} \div 10^{-2} \end{array}}$$

$E_\nu \sim 30 \div 50\,Mev$

↑средняя энергия ν

②' Реакторные ν $(\bar{\nu}_e \to \bar{\nu}_x\ ?)$

$$\left.\begin{array}{r}{}^{235}U\\ {}^{238}U\\ {}^{239}Pu\\ {}^{241}Pu\end{array}\right) \longrightarrow \bar{\nu}_e + p \to \boxed{e^+} + n$$

- * измерение спектра e^+
- * зависимость спектра e^+ от расстояния

Длинно-базовые эксперименты:

✓ ① K2K (июнь 1999 г.) ✓

$\sin^2 2\theta = 1.0$
$\Delta m^2 = 2.8 \cdot 10^{-3} eV^2$

High Energy Accelerator Research Organization

ускоритель
KEK

$\nu_\mu \to \nu_\tau$

$E = 1.5$ Gev

детектор ~1-0.7%
Супер-Камиоканде

$P + Al \to \pi^+ \to \mu^+ + \nu_\mu$
(как и ν_{atm}...)

$L = 250$ km

$\nu_\mu + n \to \mu + p$
(исчезновение)

Измеряется полное число событий и спектр энергий ν_μ

✓ ② CERN-GRAN-SASSO (~2006)

OPERA, $L = 750$ km

③ Fermilab (MINOS) → Soudan Mine

$L = 730$ km

Проблема солнечных ν

Основной источник солнечной энергии

$$4p \rightarrow {}^{4}He + 2e^{+} + \underline{\underline{2}}\,\nu_e + 2\underline{\underline{7}}\,MeV$$

97% тепловая энергия

3% E_ν

Светимость : $L_\odot = 4\cdot 10^{33}$ эрг·с$^{-1}$,

число ν/с : $\frac{N_\nu}{1c} = 2L_\odot / 27\,MeV$ =>

плотность потока ν

$$\Phi_\nu = \frac{2L_\odot}{27\,MeV}\,\frac{1}{4\pi D^2}\ ,\quad D = 1.5\cdot 10^{13}\ см$$

$$\boxed{\Phi_\nu = 6\cdot 10^{10}\ см^{-2}\,с^{-1}}$$

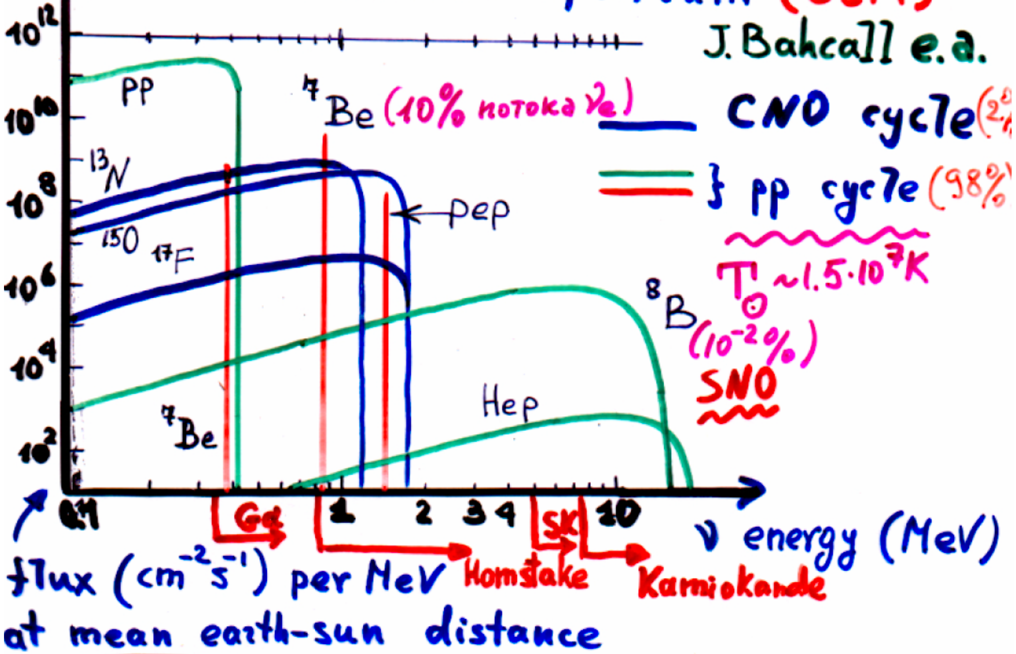
Процесс термоядерного синтеза и эволюция звезд ⟶ Standard Solar Model (ССМ, SSM).

J. Bahcall e.a., 2002

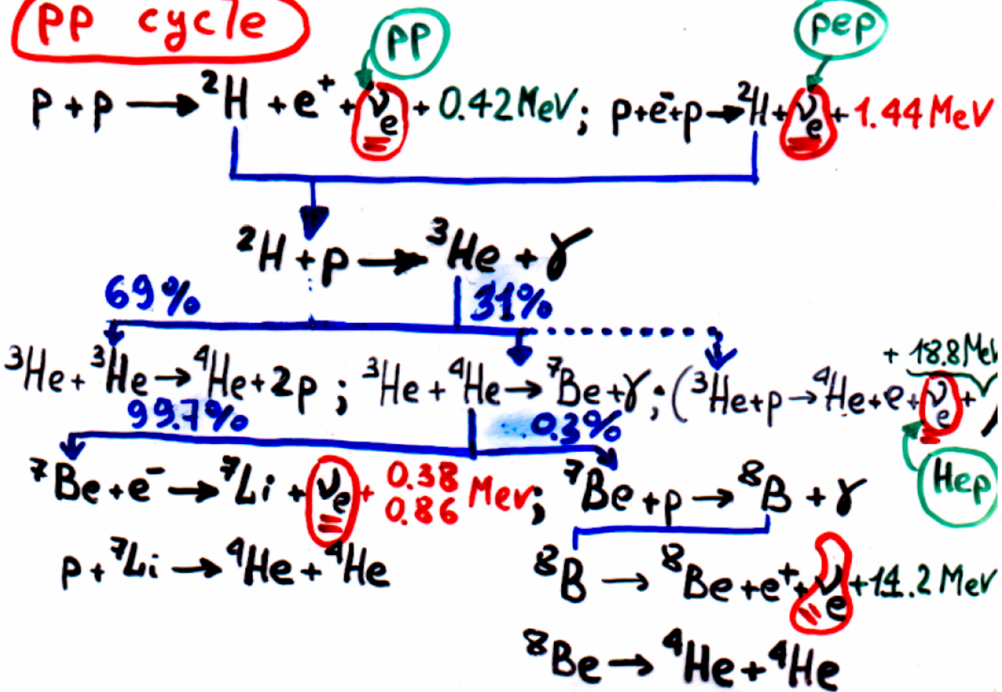
⇅

эксперименты по регистрации $\nu_\odot$

Solar neutrino spectrum (SSM)



pp cycle



pp: $p + p \to {}^2H + e^+ + \nu_e + 0.42\,MeV$; pep: $p + e^- + p \to {}^2H + \nu_e + 1.44\,MeV$

$^2H + p \to {}^3He + \gamma$

69% | 31%

$^3He + {}^3He \to {}^4He + 2p$; $^3He + {}^4He \to {}^7Be + \gamma$; ($^3He + p \to {}^4He + e + \nu_e + \underline{18.8\,MeV}$ Hep

99.7% | 0.3%

$^7Be + e^- \to {}^7Li + \nu_e + \begin{matrix}0.38\\0.86\end{matrix}\,MeV$; $^7Be + p \to {}^8B + \gamma$

$p + {}^7Li \to {}^4He + {}^4He$

$^8B \to {}^8Be + e^+ + \nu_e + 14.2\,MeV$

$^8Be \to {}^4He + {}^4He$

Различные методы регистрации $\nu_\odot$

① Хлорный детектор

$$\nu_e + {}^{37}Cl \to e^- + {}^{37}Ar$$ (Б. Понтекорво, 1946)

порог реакции $E_{th} = 0.81 MeV$

Эксперименты Homestake (BNL, USA)
(Р. Дэвис, 1965)

② Черенковский детектор

$$\nu + e \to \nu + e$$ ← регистрация вспышки черенковского излучения

Эксперименты Kamiokande ($E_{th} \approx 7.5 MeV$)
Super-Kamiokande ($E_{th} \approx 5 MeV$)

③ Галлиевый детектор

$$\nu_e + {}^{71}Ga \to e^- + {}^{71}Ge$$ ($E_{th} = 0.233 MeV$)

Эксперименты SAGE (Баксан, Россия)
GALLEX-GNO (Гран Сассо, Италия)

③ Солнечные $\nu_\odot$

$N^{-1} = \frac{\Phi_{exp}}{\Phi_{th}^{SSM}}$ $\left(\frac{\text{„эксперимент"}}{\text{„теория"}}\right)$

	Homestake	GALLEX +GNO	SAGE	Kam.	S-Kam.
N	3	1.7	1.7	1.8	2.2

✓ => {существенное подавление ν_e от Солнца:}

Условие МСВ-резонанса $\Delta m^2 \cos 2\theta = 2\sqrt{2}\, G_F n_e E$ или

$$\boxed{\Delta m^2 \cos 2\theta \approx 0.7\cdot 10^{-7} \frac{E}{1\,Mev}\,\frac{\rho}{g\cdot cm^{-3}}\; eV^2},$$

при $E \sim 1\,MeV$, $\rho \sim 10^2\, g\cdot cm^{-3}$ => $\{\Delta m^2 \sim 10^{-5}\, eV^2\}$ ✓

большой угол смешивания {MSW („LMA-решение")}

Решение проблемы солнечных ν

$\boxed{\nu_e \to \nu_\mu, \nu_\tau}$

① LMA (MSW): $\sin^2 2\theta \sim 1$

$\Delta m^2 \sim 3 \cdot 10^{-5} eV^2$!

✓

② SMA (MSW):

$\sin^2 2\theta \sim 5 \cdot 10^{-3}$ $\Delta m^2 \sim 7 \cdot 10^{-6} eV^2$

③ LOW (MSW):

$\sin^2 2\theta \sim 1$ $\Delta m^2 \sim 10^{-7} eV^2$

④ „вакуумное решение" ("just-so") $\sin^2 2\theta \sim 1$ $\underline{\underline{\Delta m^2}} \sim 10^{-2} eV^2$

малые Δm^2: $L = \frac{4\pi E_\nu}{\Delta m^2} \sim L_{\odot\text{-земля}}$

Реакторный эксперимент KamLAND (2002г.)

(исчезновение $\bar{\nu}_e$ от 26 реакторов Японии и Ю.Кореи) $L = 138 \div 214$ km

$$\bar{\nu}_e + p \to e^+ + n \quad (E_{th} = 1.8 MeV)$$

Kamioka (Япония) - 1 kt сцинтилляционный детектор на глубине 1 km под землей

За 145 дней 54 $\bar{\nu}_e$ события (без осцилляций должно быть 86.8):

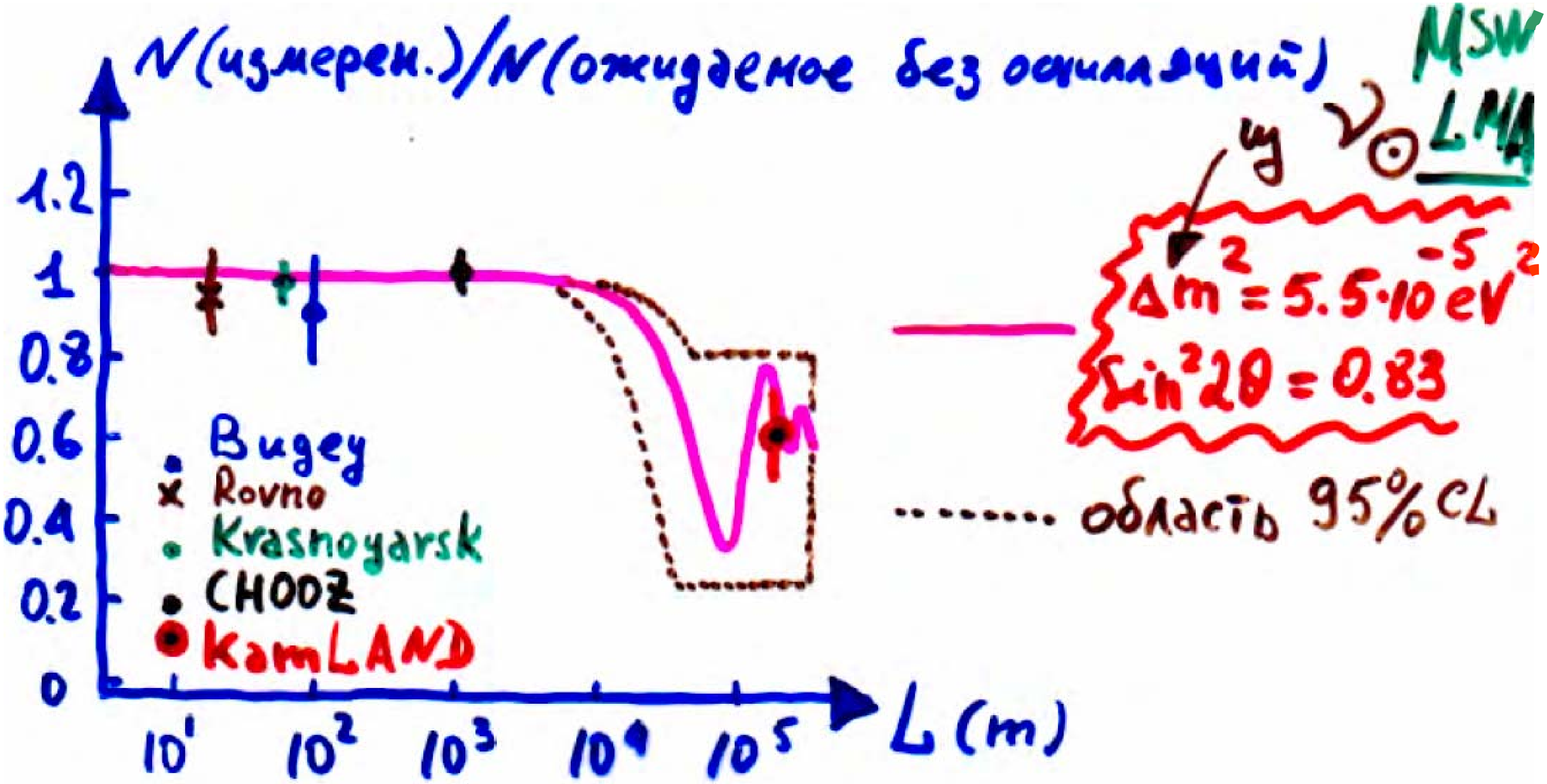
$$\frac{N(\text{изм.}) - N(\text{фон.})}{N(\text{ожидаемое})} \approx 0.6,$$

KamLAND

$$\boxed{\begin{array}{l} \Delta m^2 = 6.9 \cdot 10^{-5} eV^2 \\ \sin^2 2\theta = 1. \end{array}}$$

V

...вакуумные осцилляции


N(измерен.)/N(ожидаемое без осцилляций)
1.2
1
0.8
0.6
0.4
0.2
0
10¹
10²
10³
10⁴
10⁵
L (m)
Bugey
Rovno
Krasnoyarsk
CHOOZ
KamLAND
MSW
LMA
у ν⊙
Δm² = 5.5·10⁻⁵ eV²
sin²2θ = 0.83
область 95% CL

(2001) SNO - эксперимент (поиск $\nu_{e_\odot}$, $\nu_{x_\odot}$, $x=\mu,\tau$)

Sudbury Neutrino Observatory (Ontario, Canada)

Черенковский детектор (тяжёлая вода D_2O, ⌀12м, 1 kt):

CC: $\nu_e + d \to e^- + p + p$, только ν_e ✓

дейтрон $E_e^{th} = 5\,MeV$

NC: $\nu_x + d \to \nu_x + n + p$, ν_e, ν_μ и ν_τ ✓

$E_{th} = 2.2\,Mev$

ES: $\nu_x + e \to \nu_x + e$, ν_e, $\underbrace{\nu_\mu \text{ и } \nu_\tau}_{\text{подавлены}}$ ✓

=> только $^8B \to {^8Be} + e^+ + \nu_e$ от Солнца детектируются.

Измерение потока ν_e и $\nu_{\mu,\tau}$ показало, что:

① $\Phi^{exp}_{\nu_e+\underbrace{\nu_\mu+\nu_\tau}_{}} = \Phi^{SSM}_{\nu_e}$ (Стандартная модель Солнца, Дж. Бакола е. а. 2002)

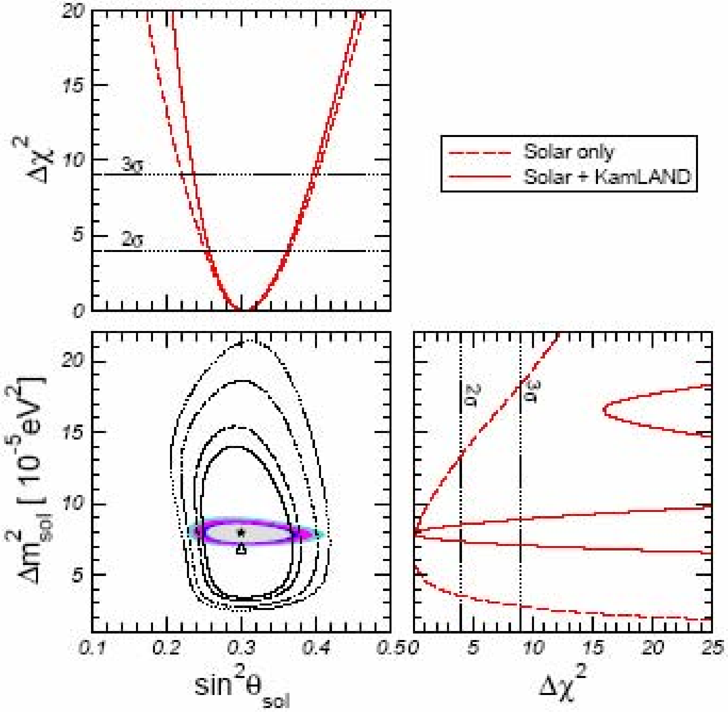
② решение проблемы $\nu_\odot$

$$\boxed{\begin{aligned}\Delta m^2_{\nu_\odot} &= 5\cdot10^{-5}\,eV^2\\ \tan^2\theta &= 0.34\end{aligned}}$$

LMA (MSW) - решение.

...большой угол смешивания...

- LMA–oscillation solution confirmed $\rightarrow \Delta m^2_{\text{SOL}}, \theta_{\text{SOL}}$.
- Combined analysis solar + KamLAND data:



Maltoni et al., NJP, 2004.

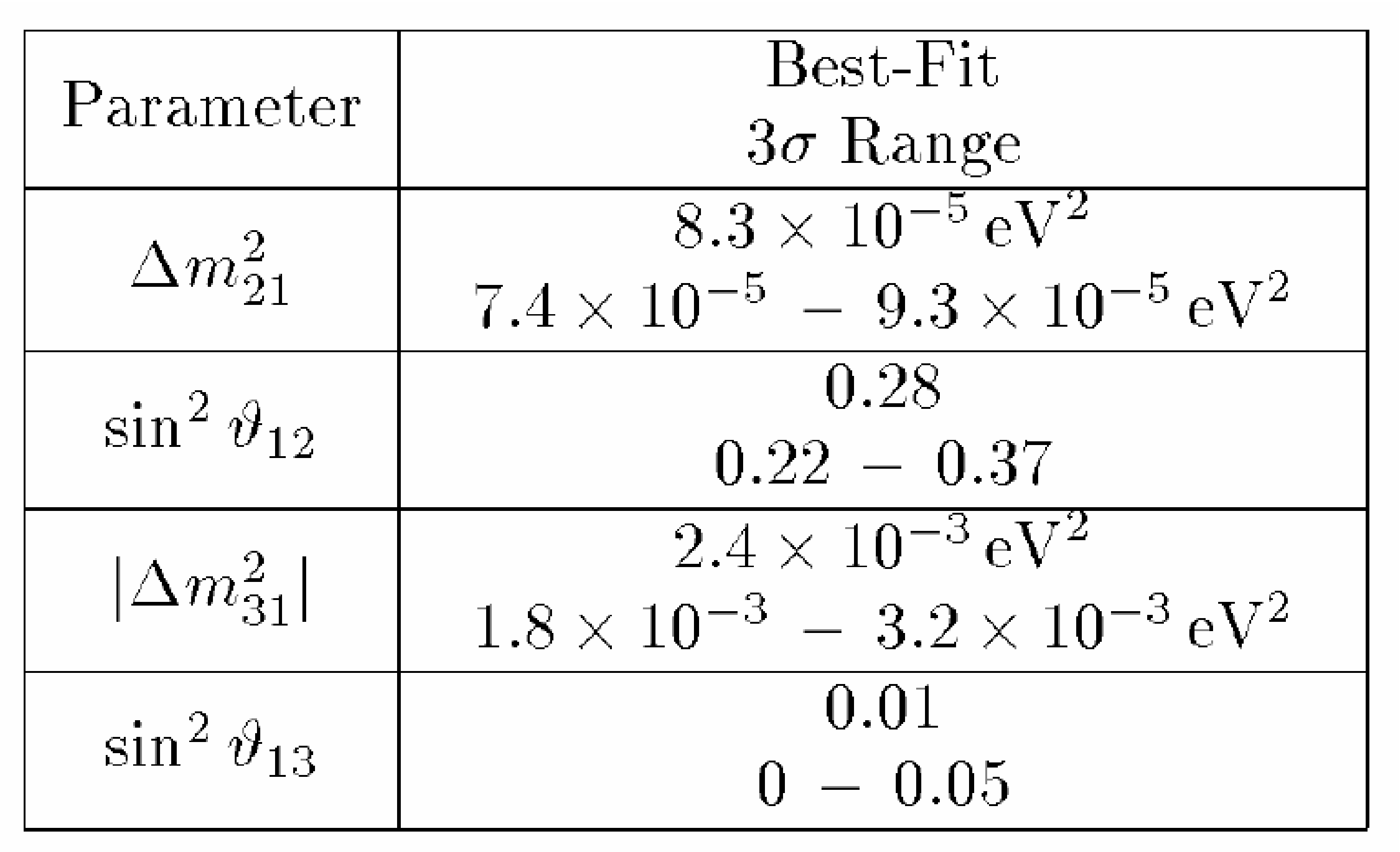
- maximal mixing excluded at 5.3σ.
- Best fit point:

$$\left[\begin{array}{l} \sin^2\theta_{\text{SOL}} = 0.30 \\ \Delta m^2_{\text{SOL}} = 7.9 \times 10^{-5}\ \text{eV}^2 \end{array}\right]$$

- Allowed 3σ region:

$$\left[\begin{array}{l} 0.24 < \sin^2\theta_{\text{SOL}} < 0.40 \\ 7.1 < \Delta m^2_{\text{SOL}} < 8.9 \times 10^{-5}\ \text{eV}^2 \end{array}\right]$$

Table 1. Best-fit and 3σ range for the three-neutrino oscillation parameters obtained in the global fit of Ref. [19]. **G.Fogli et al, 2004**



Parameter	Best-Fit 3σ Range
Δm^2_{21}	$8.3 \times 10^{-5}\,\mathrm{eV}^2$ $7.4 \times 10^{-5} - 9.3 \times 10^{-5}\,\mathrm{eV}^2$
$\sin^2 \vartheta_{12}$	0.28 $0.22 - 0.37$
$\lvert\Delta m^2_{31}\rvert$	$2.4 \times 10^{-3}\,\mathrm{eV}^2$ $1.8 \times 10^{-3} - 3.2 \times 10^{-3}\,\mathrm{eV}^2$
$\sin^2 \vartheta_{13}$	0.01 $0 - 0.05$

Теория
Осцилляций нейтрино

... если $m_\nu \neq 0$, то:

① $$\nu_e \xleftrightarrow{\text{Вакуум}} \bar{\nu}_e$$ Понтекорво, 1957

② $$\nu_e \xleftrightarrow{\text{Вакуум}} \nu_\mu$$ Маки и др., 1962

③ Вещество, $\rho = const$, $v \ll 1$
$$\nu_e \longleftrightarrow \nu_\mu$$ Вольфенштейн, 1978

скорость движения вещества

!!!

④ Вещество, $\rho \neq const$, $v \ll 1$
$$\nu_e \longleftrightarrow \nu_\mu$$ Михеев–Смирнов, 1985

!

МСВ-эффект

Вещество, ν – любое !

$$\begin{tikzcd} \nu_e \arrow[r, leftrightarrow] & \nu_\mu \\ \nu_L \arrow[r, leftrightarrow, "\text{магнитное поле}", "\text{гравитационное поле}"'] & \nu_R \end{tikzcd}$$

1995

1994

2004

„Исследования осцилляций нейтрино в МГУ":

> 10 лет с 1994 г.

Neutrino flavour oscillations $\nu_{l=e} \leftrightarrow \nu_{\substack{l'\neq e \\ {}^{\prime\prime}\mu}}$

in **moving** and **polarized** matter

G. Likhachev,
A.S., 1995;

A. Grigoriev,
A. Lobanov,
A.S., Phys. Lett. B353, 187,
2002.

For each of matter components

$$f = e, n, p, \mu, \dots, \nu_e, \nu_\mu, \dots \qquad :$$

Matter ($f = e, n, p, \dots$) motion and polarization in laboratory frame:

the frame of oscillation experiment

$$j_f^{\mu} = (n_f, n_f \vec{v}_f),$$

currents

fermions

polarizations

$$\lambda_f^{\mu} = \Big(n_f \vec{\zeta}_f \vec{v}_f ,\; n_f \vec{\zeta} \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}} \Big).$$

- $\vec{v}_f$ ⤳ speed of reference frame in which mean momentum of fermions f is zero (rest frame of background fermions f)

  total speed of matter

- $n_f^{(0)}$ ⤳ number density of f in r.f. of

  invariant number density

$$\circledast \quad n_f = \frac{n_f^{(0)}}{\sqrt{1-v_f^2}}$$

in the laboratory frame
(ν oscillation experimentf.)

- $\vec{\zeta}_f$ ⤳ mean value of polarization vectors of f in r.f.

delicate procedure...

Two-step averaging procedure

→

(Fermi-Dirac)

Ⓘ $$\langle \vec{O} \rangle = \int \Psi^{+}_{f}(x)\, \vec{O}\, \Psi_{f}(x)\, dx ,$$

fermion quantum states in e.m. field

$$\vec{O} = \gamma_0 \vec{\Sigma} - \gamma_5 \frac{\vec{p}}{E} - \gamma_0 \frac{\vec{p}(\vec{p}\vec{\Sigma})}{E(E+m)} ,$$

fermion momentum energy mass

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} .$$

relativistic spin operator of fermion f

In the reference frame $\vec{v}_f = 0 \Rightarrow$

$$\lambda_f^{\mu} = (0, n_f \vec{\zeta}_f),$$

$$(0 \leqslant |\vec{\zeta}_f|^2 \leqslant 1).$$

A.Lobanov,
A.S., PLB, 2001

Mean value of polarization vectors of **f** :

(II) $$\vec{\zeta}_f = \sum_{\{n\}} \langle \vec{O} \rangle \underbrace{\rho_f(\{n\})} \Big/ \sum_{\{n\}} \underbrace{\rho_f(\{n\})},$$

fermion distribution
(Fermi-Dirac)

For slowly moving matter component f
($v_f \approx 0$ in lab. frame):

$$j_f^{\mu} = (n_f^{(0)} , 0, 0, 0) ,$$

$$\lambda_f^{\mu} = (0 , \; n_f^{(0)} \vec{\xi}_f) .$$

Let us suppose that at least one of matter components is moving with

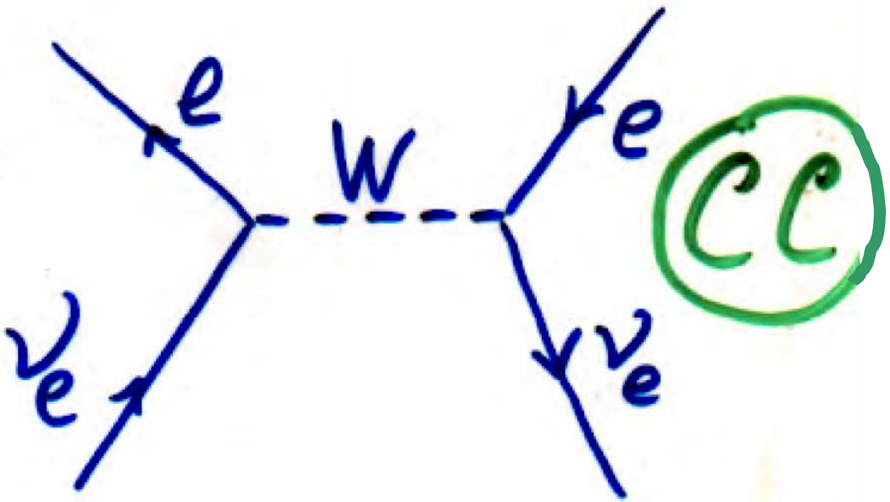
$$v_f \approx 1 \quad .$$

Two-flavour oscillations $\nu_e \leftrightarrow \nu_\mu$ in matter $f = e$ (gas of electrons with $v_e \sim 1$).

Matter effect in $\nu_e \leftrightarrow \nu_\mu$ (elastic forward scattering of ν's on background electrons):

$$\Delta V = V_{\nu_e} - V_{\nu_\mu}$$

effective neutrino potentials



! NC contributes in case of $\nu_{\text{active}} \longleftrightarrow \nu_{\text{sterile}}$

Total Lagrangian density for ν_e

(CC interaction with $f=e$ is included)

$$\mathcal{L} = \bar{\Psi}_{\nu_e}(i\not{\partial} - m_{\nu_e})\Psi_{\nu_e} + \bar{\Psi}_e(i\not{\partial} - m_e)\Psi_e -$$

$$-\frac{G_F}{\sqrt{2}}\left(\bar{\Psi}_{\nu_e}\gamma_\mu(1+\gamma_5)\Psi_{\nu_e}\right)\underbrace{\left(\bar{\Psi}_e\gamma^\mu(1+\gamma_5)\Psi_e\right)} \quad \left(\text{Fierz transformed}\right)$$

$\Rightarrow$

⇒ additional matter term

$$\Delta \mathcal{L}_{eff} = - f_\mu \left(\bar{\nu} \gamma_\mu \frac{1+\gamma_5}{2} \nu\right),$$

$$f_\mu = \sqrt{2}\, G_F \left(j_e^\mu - \lambda_e^\mu\right)$$

electrons current and polarization

modifies the Dirac equation

$$(\gamma_0 E_{\nu_e} - \vec{\gamma}\vec{p}_{\nu_e} - m_{\nu_e})\Psi_{\nu_e} = (\gamma_\mu f^\mu)\Psi_{\nu_e} \quad .$$

$$\Downarrow$$

$$\boxed{E_{\nu_e} = \sqrt{(\vec{p}_{\nu_e} - \vec{f})^2 + m_{\nu_e}^2} + f^0}$$

neutrino dispersion relation in matter

in the limit of weak potential

$$|\vec{f}| \ll P_0 = \sqrt{\vec{p}_\nu^{\,2} + m_\nu^2} \quad \Longrightarrow$$

Effective energy of ν_e in moving and polarized matter $(f = e)$:

$$\gamma_\nu = \frac{1}{\sqrt{1-\beta_\nu^2}}$$

$$\boxed{E_{\nu_e} = P_0 + \sqrt{2}\, G_F \underline{\underline{n_e}} \Big\{ (1 - \underline{\underline{\vec{\xi}_e}} \underline{\underline{\vec{v}_e}})(1 - \vec{\beta}_\nu \underline{\underline{\vec{v}_e}}) + \sqrt{1-v_e^2}\Big[\vec{\xi}_e \vec{\beta}_\nu - \frac{(\vec{\beta}_\nu \vec{v}_e)(\vec{\xi}_e \vec{v}_e)}{1 + \sqrt{1-v_e^2}} \Big] \Big\} + O(\gamma_\nu^{-1}),}$$

$\vec{\beta}_\nu$ – neutrino speed

$\vec{v}_e$ – speed of matter

$\vec{\xi}_e$ – mean polarization vector of electrons in r.f. of matter

$$n_e = \frac{n_e^{(0)}}{\sqrt{1-v_e^2}} \longleftarrow \text{invariant number density (in r.f. of matter)}$$

non-moving and unpolarized matter

If $v_e \approx 0$ (and also $\zeta_e \approx 0$)

$E_{\nu_e} = p_0 + \underline{\sqrt{2}\, G_F\, n_e^{(0)}}$ ← Wolfenstein matter term,

No matter term for ν_μ $(f = e, \cancel{\mu}, \cancel{\ldots})$.

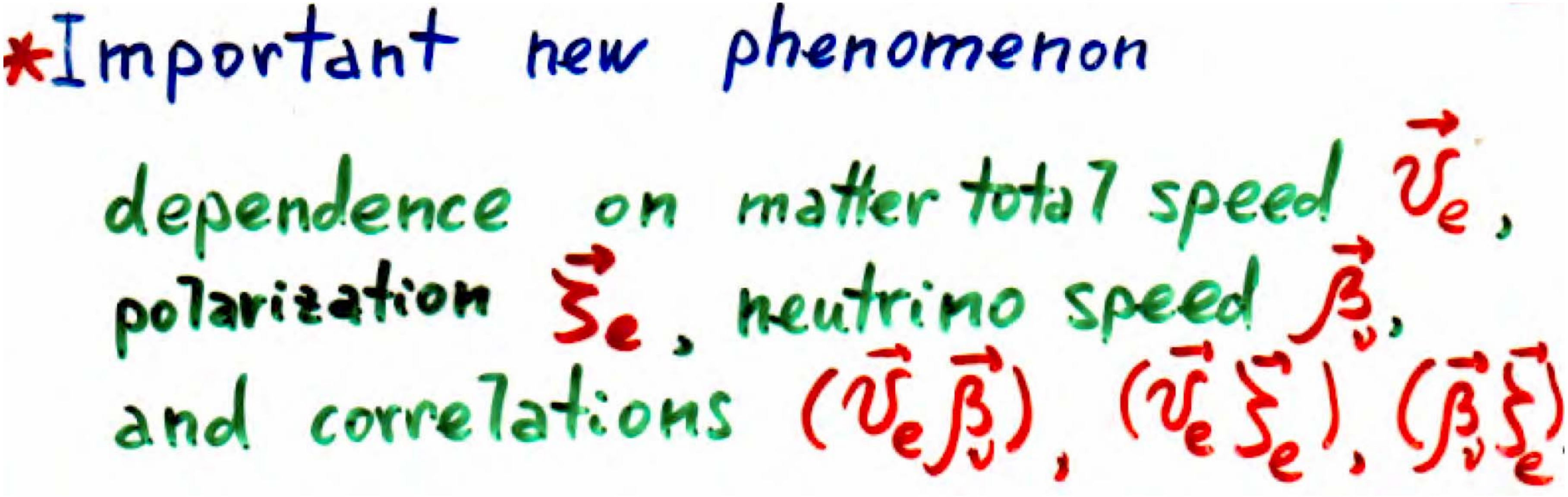
Probability of oscillations (in adiabatic limit)

$$\boxed{\begin{aligned}
&P_{\nu_e \leftrightarrow \nu_\mu}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}} \,, \\
&\sin^2 2\theta_{eff} = \frac{\underline{\underline{\Delta}}^2 \sin^2 2\theta_{vac}}{(\Delta \cos 2\theta_{vac} - \underline{\underline{A}})^2 + \Delta^2 \sin^2 2\theta_{vac}} \,, \\
&L_{eff} = 2\pi \left((\Delta \cos 2\theta_{vac} - \underline{\underline{A}})^2 + \Delta^2 \sin^2 2\theta_{vac} \right)^{-1/2} \,,
\end{aligned}}$$

$$\Delta = \frac{\delta m_\nu^2}{2|\vec{p}|} \,, \quad \delta m_\nu^2 = m_2^2 - m_1^2$$

$$A = \sqrt{2}\, G_F \frac{n_e^{(0)}}{\sqrt{1 - \underline{\underline{v_e}}^2}} \Big\{ (1 - \underline{\underline{\vec{\beta}_\nu}} \underline{\underline{\vec{v}_e}})(1 - \underline{\underline{\vec{\zeta}_e}} \underline{\underline{\vec{v}_e}}) +$$

$$+ \sqrt{1 - \underline{\underline{v_e}}^2} \Big[\underline{\underline{\vec{\zeta}_e}} \underline{\underline{\vec{\beta}_\nu}} - \frac{(\underline{\underline{\vec{\beta}_\nu}} \underline{\underline{\vec{v}_e}})(\underline{\underline{\vec{\zeta}_e}} \underline{\underline{\vec{v}_e}})}{1 + \sqrt{1 - \underline{\underline{v_e}}^2}} \Big] \Big\}$$



*Important new phenomenon

dependence on matter total speed $\vec{v}_e$, polarization $\vec{\xi}_e$, neutrino speed $\vec{\beta}_\nu$, and correlations $(\vec{v}_e\vec{\beta}_\nu)$, $(\vec{v}_e\vec{\xi}_e)$, $(\vec{\beta}_\nu\vec{\xi}_e)$

G.Likhachev, A.Studenikin, 1995 (unpublished)

A.Grigoriev, A.Lobanov, A.Studenikin,
Phys.Lett.B 535 (2002) 187

The resonance condition in moving and polarized matter

Mikheyev, Smirnov,
Wolfenstein
1978, 1985

$$\boxed{\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta = A,}$$

$$\boxed{|\vec{p}| = m_\nu \gamma_\nu}$$

$$A = \sqrt{2}\, G_F \boxed{\frac{h_e^{(0)}}{\sqrt{1-\underline{\underline{v_e}}^2}}} \Big\{ (1-\underline{\underline{\vec{\beta}_\nu}}\,\underline{\underline{\vec{v}_e}})(1-\underline{\underline{\vec{\xi}_e}}\,\underline{\underline{\vec{v}_e}}) +$$

$$+\sqrt{1-\underline{\underline{v_e}}^2}\Big[\underline{\underline{\vec{\xi}_e}}\,\underline{\underline{\vec{\beta}_\nu}} - \frac{(\underline{\underline{\vec{\beta}_\nu}}\,\underline{\underline{\vec{v}_e}})(\underline{\underline{\vec{\xi}_e}}\,\underline{\underline{\vec{v}_e}})}{1+\sqrt{1-\underline{\underline{v_e}}^2}}\Big].$$

A.Grigoriev, A.Lobanov, A.Studenikin,
Phys.Lett.B 535 (2002) 187

The resonance in $\nu_e \leftrightarrow \nu_\mu$ can occur for moving matter even if for given δm_ν^2, $|\vec{p}|$, θ, and $n_e^{(0)}$ the resonance in non-moving matter is impossible

$$\boxed{\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta \cancel{=} \sqrt{2}\, G_F\, n_e^{(0)}.}$$

$$A\Big|_{v_e \to 0} \approx \sqrt{2}\, G_F\, n_e^{(0)}$$

Unpolarized but moving matter

$(\vec\zeta_e = 0,\ v_e \neq 0)$

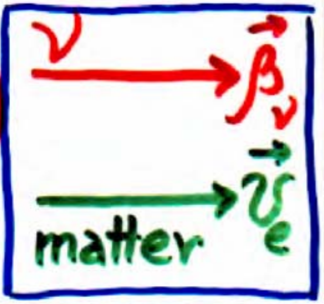
Resonance condition:

- $$\boxed{\frac{\delta m_\nu^2}{2|\vec p|}\cos 2\theta = \sqrt{2}\, G_F\, n_e^{(0)} \frac{1-\vec\beta_\nu \vec v_e}{\sqrt{1-v_e^2}}}$$

$n_e^{(0)}$ ← invariant matter density in r.f.

If $\longleftrightarrow \vec\beta_\nu$, $\longleftrightarrow \vec v_e$:

- $$\left.\frac{1-\vec\beta_\nu \vec v_e}{\sqrt{1-v_e^2}}\right|_{\beta_\nu\approx 1} = \begin{cases} \left.\sqrt{\dfrac{1-v_e}{1+v_e}}\right|_{v_e\approx 1} \approx \dfrac{\sqrt{1-v_e}}{\sqrt{2}} \\ \left.\sqrt{\dfrac{1+v_e}{1-v_e}}\right|_{v_e\approx 1} \approx \dfrac{\sqrt{2}}{\sqrt{1-v_e}} \end{cases}$$



Relativistic motion of matter along (against) neutrino propagation could provide resonance in $\nu \leftrightarrow \nu$ if matter density $n_e^{(0)}$ is too high (low) for resonance appearance in non-moving matter.

in restframe of matter

2

Electromagnetic properties of ν

Neutrino mass

$$m_\nu \neq 0 \; !$$

$$\Longrightarrow$$

Neutrino magnetic moment ν ⊙

$$\mu_\nu \neq 0 \; ? \; (!)$$

⊛ Lee, Shrock 1977

Shrock, Fujikawa 1980

... Massive neutrino electromagnetic properties ...

Bounds on ν magnetic moments

Laboratory bounds

$\mu_B = \frac{e}{2m_e}$

$\nu_\odot$

$$\mu_{\nu_e} \ll 1.5 \cdot 10^{-10} \mu_B \quad (\bar{\nu}_e - e)$$

$$\mu_{\nu_\mu} \ll 6.8 \cdot 10^{-10} \mu_B \quad (\nu_\mu, \bar{\nu}_\mu - e)$$

$$\mu_{\nu_\tau} \ll 3.9 \cdot 10^{-7} \mu_B \quad (\nu_\tau - e)$$

$$\boxed{\mu_{\nu_e} \leq 0.9 \times 10^{-10} \mu_B}$$

MUNU Coll. 2005

Astrophysical bounds

$$\mu_\nu \ll 3 \cdot 10^{-12} \mu_B$$

(Red Giant lumin.)
etc.

G. Raffelt, D. Dearborn,
J. Silk, 1989.

Theory (Standard Model with ν_R)

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1eV}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee
Shrock, 1977; Fujikawa
Shrok, 1980

ν large magnetic moment

- In the Standard Model: $m_\nu = 0$, there is no ν_R $\Rightarrow$ ν magnetic moment $\mu_\nu = 0$.
- Thus, $\mu_\nu \neq 0 \longleftarrow$ beyond the SM.

$$\mu_\nu = \mu_\nu(m_\nu, m_{B^+}; m_{e^-})$$

$$\Uparrow$$

- In the L-R symmetric models $\big(SU(2)_L \times SU(2)_R \times U(1)\big)$

Kim, 1976

Beg, Marciano, Ruderman, 1978

✓

- **M.Voloshin (ITEP),**

"On compatibility of small m_ν with large μ_ν of neutrino", Sov.J.Nucl.Phys. 48 (1988) 512

… there may be $SU(2)_\nu$ symmetry that forbids m_ν but not μ_ν

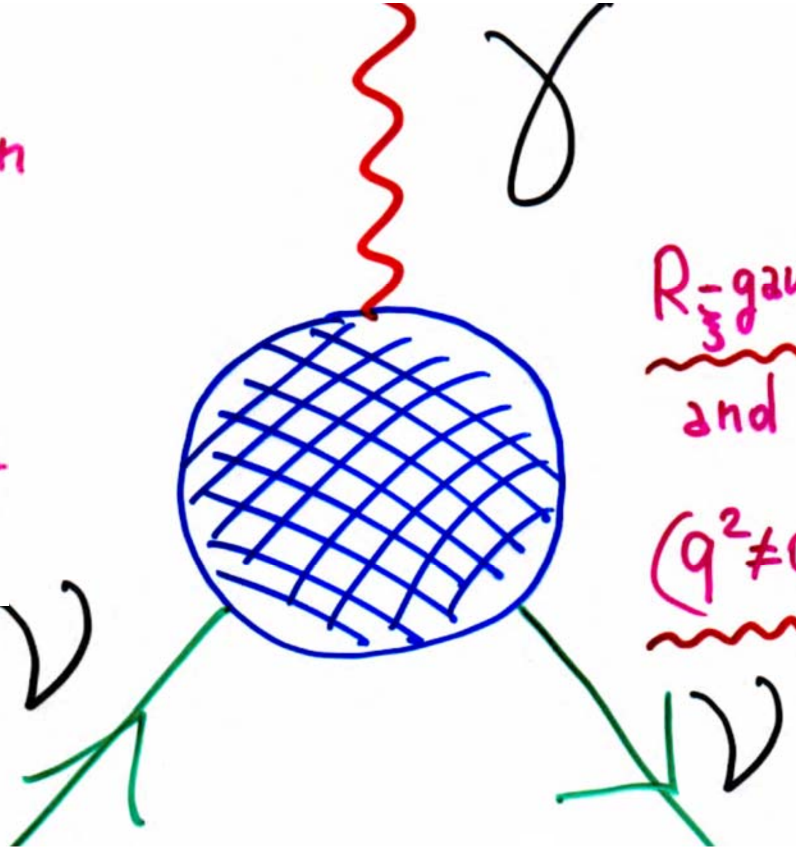
The most general study of the **massive neutrino** vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R accounting for masses of particles in polarization loops

M. Dvornikov, A. Studenikin

⊛ Phys. Rev. D 63, 073001, 2004,

"Electric charge and magnetic moment of massive neutrino";
JETP 126 (2004), N8, 1

⊛ "Electromagnetic form factors of a massiv neutrino."



R_ξ-gauge
and
$(q^2 \neq 0)$

charge　　　magnetic moment

$$\Delta_\mu(q) = \underline{f_Q}(q^2)\gamma_\mu + \underline{f_M}(q^2)\, i\sigma_{\mu\nu}q^\nu -$$
$$- \underline{f_E}(q^2)\, i\sigma_{\mu\nu}q^\nu\gamma_5 - \underline{f_A}(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

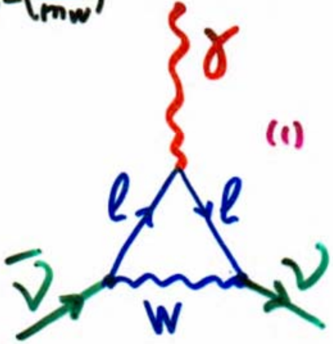
electric moment　　　anapole moment

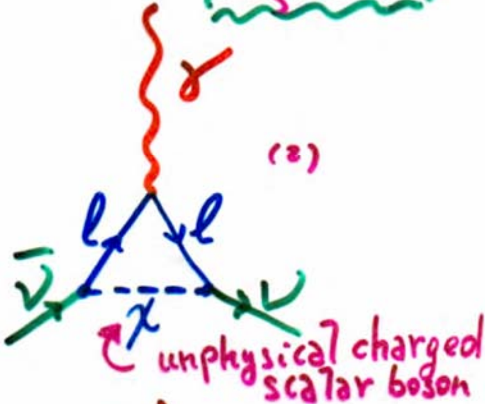
$a = \left(\frac{m_\ell}{m_W}\right)^2,$

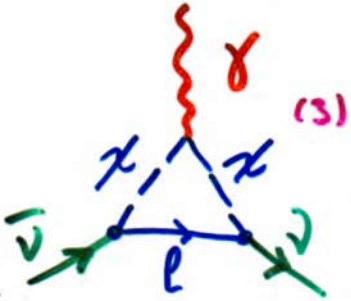
$b = \left(\frac{m_\nu}{m_W}\right)^2$

Proper vertices

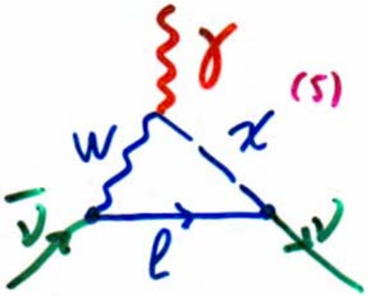
R_ξ-gauge

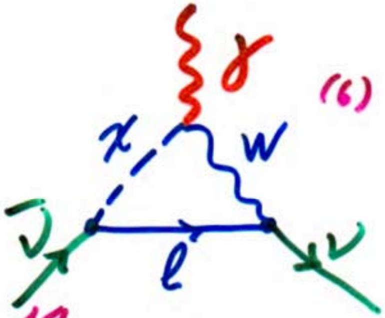

(1)


(2)


(3)


(4)


(5)

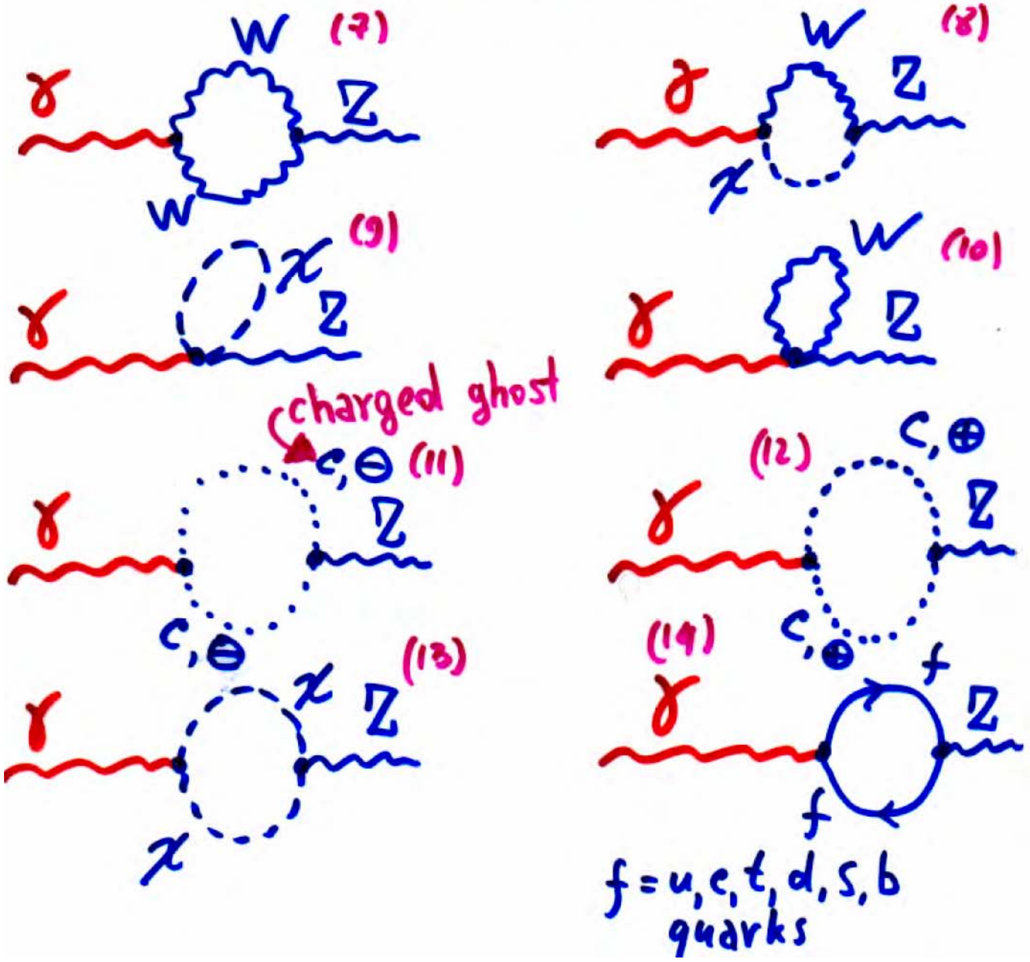

(6)

$$\Lambda_\mu(q) = \sum_{i=1}^{14} \Lambda_\mu^i(q)$$

$$\Lambda^{j}_{\mu}(q) = \frac{g}{2\cos\theta_W}\,\Pi^{(j)}_{\mu\nu}(q)\,\frac{1}{q^2 - M_Z^2}$$

$$\times \left\{ g^{\nu\alpha} - (1-\alpha_Z)\frac{q^\nu q^\alpha}{q^2 - \alpha_Z M_Z^2} \right\} \gamma^{L}_{\alpha}, \; j=7,\ldots,14$$

γ-Z self-energy diagrams



ν magnetic moment

(heavy massive neutrino)

- **LEP data** →

only 3 light ν s coupled to Z° ,

for any additional neutrino

$$\boxed{m_{\nu} \geq 80Gev}$$

- ● $m_\nu \ll m_e \ll M_W$ **light** ν

$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu$

$$\boxed{\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}\left(2 - 7a + 6a^2 - 2a^2 \ln a - a^3\right)}\ , \quad a = \left(\frac{m_e}{M_W}\right)^2$$

- ● $m_e \ll m_\nu \ll M_W$ **intermediate** ν

$$\boxed{\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1 + \frac{5}{18} b\right\}}\ , \quad b = \left(\frac{m_\nu}{M_W}\right)^2$$

M.Dvornikov,
A.Studenikin,
*Phys.Rev.**D*** **69** (2004)
073001

- ● $m_e \ll M_W \ll m_\nu$ **heavy** ν

$$\boxed{\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu}$$

ν e.m. form factors are affected by matter and B

⊛ magnetic moment $\mu_\nu = \mu_\nu(B)$

Egorov, Studenikin 1993

Borisov, Zhukovsky, Kurilin, Ternov, 1985

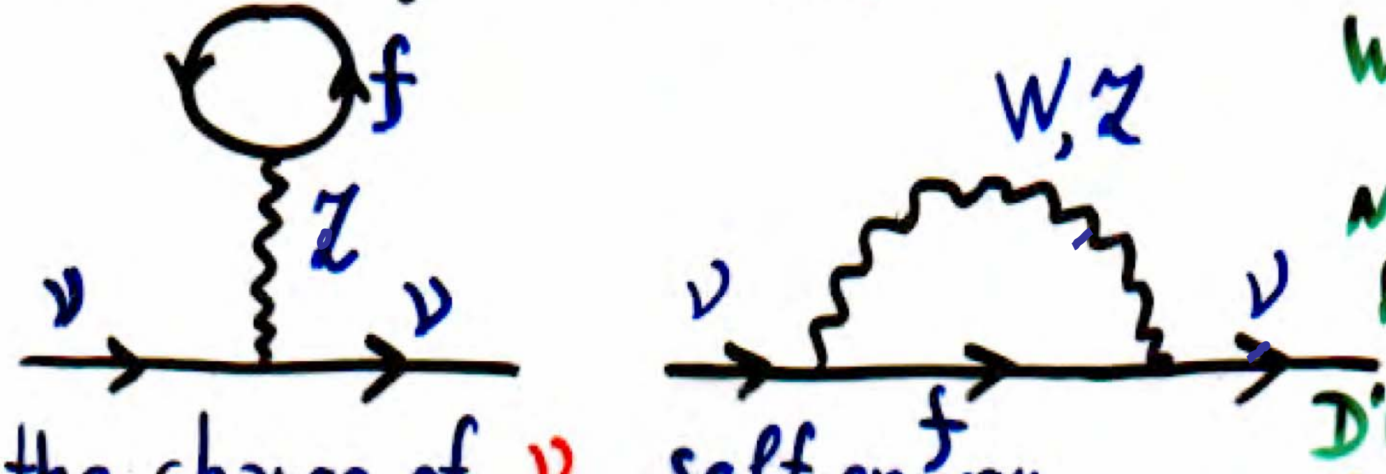
⊛ induced electric charge of ν in magnetized matter

⊛ Oraevsky, Semikoz, Smorodinsky, 1986

Bhattacharaya, Ganguly, Konar, 2002

Nieves, 2003

• Electromagnetic ν properties in medium



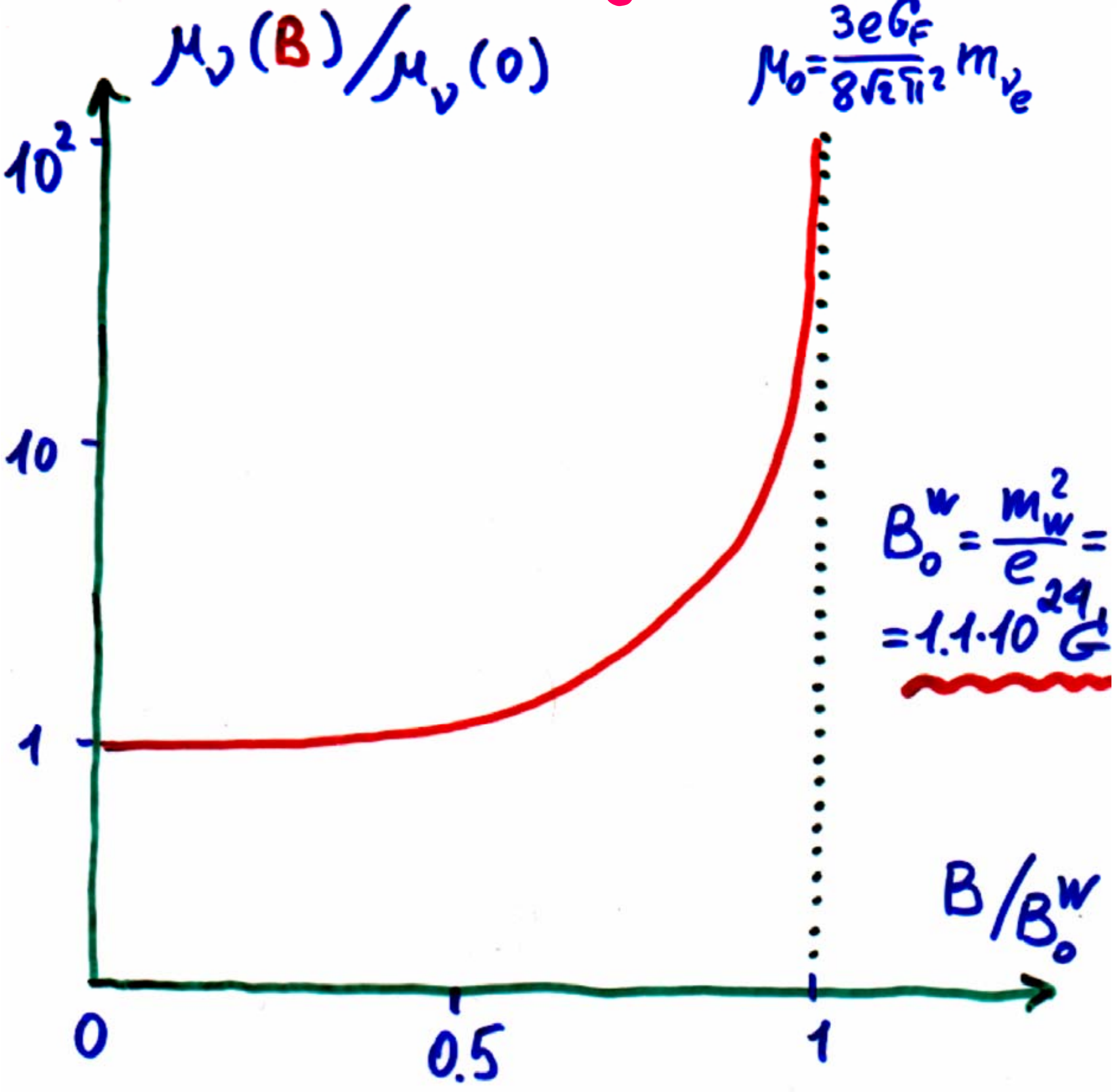
Wolfenstein 1978

Nötzold Raffelt, 1988

D'Olivo, Nieves, Pal, 1989

⇒ the change of ν self-energy (even a Majorana ν can have μ, $d \neq 0$).

Neutrino magnetic moment



$$\mu_0 = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$

$$B_0^w = \frac{m_w^2}{e} = 1.1 \cdot 10^{24} G$$

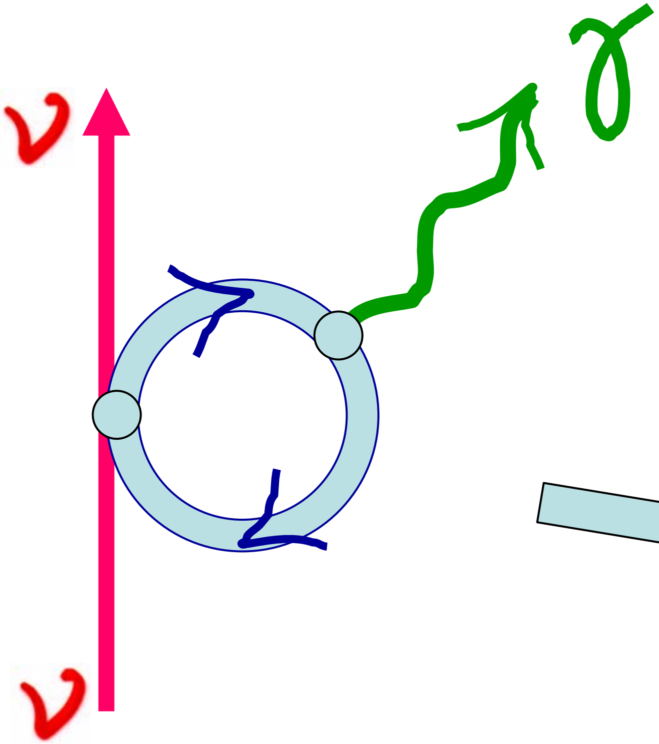
Borisov,
Zhukovs'ky,
Kurilin,
Ternov, 1985;

Masood,
Perez Rojas,
Gaitan,
Rodrigues-Romo,
1999

ν "effective electric charge"
in magnetized plasma

- ν's do not couple with γ's in vacuum,

  however, when
- ν in thermal medium (e^- and e^+)



ν V.Oraevsky, V.Semikoz, Ya.Smorodinsky,
JETP Lett. 43 (1986) 709;
J.Nieves, P.Pal, Phys.Rev.D 49 (1994) 1398;
T.Altherr, P.Salati, Nucl.Phys.B421 (1994) 662;
K.Bhattacharya, A.Ganguly, 2002

…different νγ interactions in astrophysical and cosmological media

⊙ Self-energy of ν in magnetized medium has been also studied

Esposito, Capone } D'Olivo, Nieves } Elmfors, Grasso, Rafelt } 1996

⊙ Magnetic moment of ν in $\vec{B}$ at high temperature and density

Masood, 1992; Giunti, Kim, Lam; 1991
Zhukovskii, Shonia, Eminov, 1993;
Akhter, Skalozub, Vilensky, 1994.

3

Matter effect in ν spin (spin-flavour) oscillations

Chirality and helicity

Ст. предст. $\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$, $i = 1, 2, 3$, $\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$

⊛ ν chirality states:

chirality projector operators

$$\Psi_{L,R} = \frac{1 \mp \gamma_5}{2} \Psi_\nu , \quad P_{L,R} = \frac{1 \mp \gamma_5}{2} ;$$

⊛ ν helicity states:

helicity projector operators

$$\Psi_{+,-} = \frac{1}{2}\left(1 \mp \frac{\vec{\sigma}\vec{p}_\nu}{|\vec{p}_\nu|}\right) \Psi_\nu , \quad P_{+,-} = \frac{1}{2}\left(1 \mp \frac{\vec{\sigma}\vec{p}_\nu}{|\vec{p}_\nu|}\right)$$

⊛ For massive ν chirality is not concerved (it is concerved if $m_\nu = 0$):

$$(i\gamma^\mu \partial_\mu - m_\nu)\Psi_\nu = 0, \quad \Psi_\nu = \Psi_L + \Psi_R \Rightarrow$$

$$\begin{cases} i\gamma^\mu \partial_\mu \Psi_L = m_\nu \Psi_R \\ i\gamma^\mu \partial_\mu \Psi_R = m_\nu \Psi_L \end{cases}, \quad \text{if } m_\nu = 0 \Rightarrow$$

Ψ_L and Ψ_R are not coupled with one another.

$$\boxed{\begin{array}{l} \Psi_L^{(e)} \approx \Psi_-^{(e)}, \\ \Psi_R^{(\mu)} \approx \Psi_+^{(\mu)}, \ \frac{E_\nu}{m_\nu} \gg 1 \end{array}}$$

Спиновые осцилляции ν в магнитном поле (и проблема $\nu_\odot$)

магнитный момент ν

Если $m_\nu \neq 0$, то $\mu_\nu \neq 0$! $\Rightarrow$

ν взаимодействует с э.м. полем:

$$\Delta\mathcal{L} = \mu_\nu \bar{\Psi}_\nu \sigma_{\lambda\rho} \Psi'_\nu F^{\lambda\rho}, \quad \sigma_{\lambda\rho} = \frac{1}{2}(\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda),$$

тензор э.м. поля,

$$\bar{\Psi}\sigma_{\lambda\rho}\Psi' = \ldots = \bar{\Psi}_L \sigma_{\lambda\rho}\ {}_{R}\ {}_{R}\ {}_{\lambda\rho}\ {}_{L},$$

$$\Psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\Psi .$$

ν magnetic moment interaction

electromagnetic field

$$\Delta\mathcal{L} = \mu_\nu \bar{\Psi}_\nu \sigma_{\lambda\rho} \Psi'_\nu F^{\lambda\rho}, \quad \sigma_{\lambda\rho} = \frac{1}{2}(\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda),$$

$$\bar{\Psi}\sigma_{\lambda\rho}\Psi' = \dots = \bar{\Psi}_L \sigma_{\lambda\rho} \Psi'_R + \bar{\Psi}_R \sigma_{\lambda\rho} \Psi'_L,$$

$$\Psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\Psi .$$

{Neutrino spin $\nu_L \xleftrightarrow{B_\perp} \nu_R$ oscillations

(mixing due to $\frac{\Delta m_\nu^2}{2E_\nu}\sin 2\theta_{vac} \rightarrow \boxed{2\mu B_\perp}$)

$$\boxed{\begin{aligned} P(\nu_L \leftrightarrow \nu_R) &= \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}} , \\ \sin^2 2\theta_{eff} &= \frac{(2\mu B_\perp)^2}{(2\mu B_\perp)^2 + \underline{\Omega^2}} , \end{aligned}}$$

⊛

$$* \quad \Omega = \frac{\Delta m_\nu^2}{2E_\nu} A(\theta_{vac}) - \sqrt{2}\, G_F\, n_{eff} ,$$

particle number density

$$L_{eff} = \frac{2\pi}{\sqrt{\underline{\Omega^2} + (2\mu B_\perp)^2}}$$

E. Akhmedov
C.-S. Lim, W. Marciano

(1988)

$\Omega^2 \rightarrow 0$: resonance in $\nu_L \leftrightarrow \nu_R$ neutrino spin oscillations

$\Downarrow$

Spin and spin-flavour oscillations for $\nu_\odot$ and ν_{SN}

Main steps in ν oscillations

① $\nu_e \xleftrightarrow{vac} \bar{\nu}_e$, B. Pontecorvo, 1957

② $\nu_e \xleftrightarrow{vac} \nu_\mu$, Z. Maki, M. Nakagava, S. Sakata, 1962

③ $\nu_e \xleftrightarrow{matter, \rho = const} \nu_\mu$, L. Wolfenstein, 1978

④ $\nu_e \xleftrightarrow{matter, \rho \neq const} \nu_\mu$, S. Mikheev, A. Smirnov, 1985

- resonances in ν flavour oscillations ⇒ MSW-effect, solution for $\nu_\odot$-problem

⑤ $\nu_{e_L} \xleftrightarrow{B_\perp} \nu_{e_R}$, A. Cisneros, 1971, M. Voloshin, M. Vysotsky, L. Okun, 1986, $\nu_\odot$

$\nu_\odot$

⑥ $\nu_{e_L} \xleftrightarrow{B_\perp} \nu_{e_R}, \nu_{\mu_R}$, E. Akhmedov, 1988, C.-S. Lim & W. Marciano, 1988

- resonances in ν spin (spin-flavour) oscillations in matter

Neutrino conversions and oscillations in magnetic field

⊛ $\nu_\odot$ problem

$\nu_L \overset{B}{\leftrightarrow} \nu_R$

Cisneros, 1971

* { Voloshin, Vysotsky, Okun, 1986
Barbieri, Fiorentini, 1988

⊙ twisting B { Smirnov, 1991
Akhmedov, Petcov, Smirnov, 1993

⊛ Supernova $\nu_L \overset{B}{\leftrightarrow} \nu_R$

Dar, 1987

Fujikawa, Shrock, 1988

Voloshin, 1988

$\nu_\odot$

SPIN FLAVOUR PRECESSION AND LMA

João M. Pulido

CFTP - Instituto Superior Técnico, Lisbon

12th Lomonosov Conference, Moscow

26th August, 2005

Long term periodicity may have been observed by the Gallium experiments. In fact

Period	1991-97	1998-03
SAGE+Ga/GNO	77.8 ± 5.0	63.3 ± 3.6
Ga/GNO only	77.5 ± 7.7	62.9 ± 6.0
no. of suspots	52	100

Notice a 2.4σ discrepancy in the combined results over the two periods. This is suggestive of an anticorrelation of Ga event rate with the 11-year solar sunspot cycle.

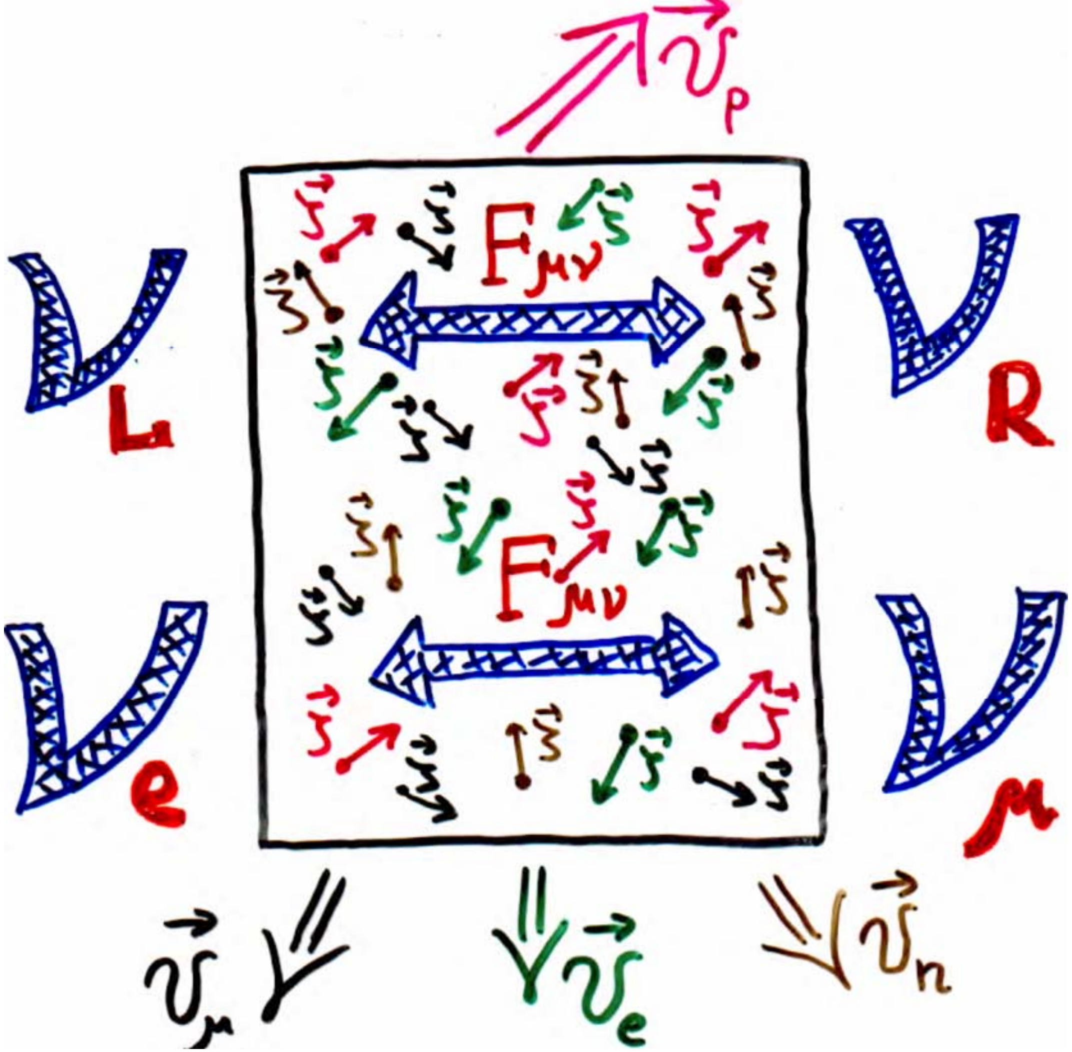
Periodicity of the active solar neutrino flux is probably the most important issue to be investigated after LMA has been ascertained as the dominant solution to the $\odot\ \nu$ problem. If confirmed it will imply the existence of a sizable neutrino magnetic moment μ_ν and hence a wealth of new physics.

Idea was introduced in 1986 by Russian physicists Voloshin, Vysotsky and Okun

↓

$$Strong\ B_\odot \rightarrow large\ \mu_\nu B_\odot \rightarrow large\ conversion$$

…from J.Pulido



moving matter components

$f = e, n, p, \mu,$ etc

with polarizations

$\vec{\xi}, \vec{\xi}, \vec{\xi}, \vec{\xi},$ etc

G.Likhachev,
A.Studenikin,
1995

A.Egorov,
A.Lobanov,
A.Studenikin,
PLB 2000

A.Lobanov,
A.Studenikin,
PLB 2001

A.Lobanov,
A.Grigoriev,
A.Studenikin,
PLB 2002

$$\nu_L \overset{F_{\mu\nu}}{\Longleftrightarrow} \nu_R$$

Two steps:

① arbitrary $F_{\mu\nu}$ (1999-2000)

② moving and polarized
✓ matter (2000-2001),

we start with ②

Neutrino spin evolution in arbitrary electromagnetic field $F_{\mu\nu}$ and moving and polarized matter

Bargmann-Michel-Telegdi equation for spin vector S_μ of neutral particle:

$$\frac{dS^\mu}{d\tau} = 2\underset{\text{magnetic}}{\mu}\left[F^{\mu\nu}S_\nu - u^\mu(u_\nu F^{\nu\lambda}S_\lambda)\right] + \underbrace{2\underset{\text{electric}}{\epsilon}\left[\tilde{F}^{\mu\nu}S_\nu - u^\mu(u_\nu \tilde{F}^{\nu\lambda}S_\lambda)\right]}_{\cancel{\text{T-invariance}}}$$

dipole moments

- direct interaction of ~~ν~~ spin with $F_{\mu\nu}$
- P invariant theory

$\boxed{\text{arbitrary e.m. field}} \nearrow F_{\mu\nu}$

Neutrino spin evolution equation

for ν general interactions (e.g., ~~P-in~~variant weak interactions) with moving and polarized matter

neutrino speed

$u_\mu = (\gamma, \gamma\vec{\beta})$, $\gamma = (1-\beta^2)^{-1/2}$, $\vec{\beta} =$ const., $S^2 = -1$, $u_\mu S^\mu = 0$

Lorentz invariant generalization of BMT eq. :

$$F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}$$

ν interactions with moving and polarized matter

⊛ Evaluation of $G_{\mu\nu}$:

- ν evolution eq. has to be linear over S_μ , $F_{\mu\nu}$ and characteristics of matter

$$j_f^\mu = (n_f , n_f \vec{v}_f), \quad \boxed{f = e, n, p, \mu,} \dots$$

fermions currents

$$\lambda_f^\mu = \left(n_f \vec{\xi}_f \vec{v}_f , n_f \vec{\xi}_f \sqrt{1-v_f^2} + \frac{n_f \vec{v}_f (\vec{\xi}_f \vec{v}_f)}{1+\sqrt{1-v_f^2}} \right)$$

polarizations

in the laboratory frame of reference

n_f → number density of background f

$\vec{v}_f$ → speed of reference frame in which mean momentum of fermions f is zero

$\vec{\xi}_f$ → mean value of polarization vectors of f in above mentioned ref. frame

For each of fermions there are only $\overset{\text{speed}}{u_\mu^f}$, $\overset{\text{current}}{j_\mu^f}$, $\overset{\text{polaritation}}{\lambda_\mu^f}$ to constract $G_{\mu\nu}$.

If j_μ^f, λ_μ^f → slowly varying functions in space and time (similar to $F_{\mu\nu}$ in BMT eq.)

⇒ only four tensors (for each of f) linear in respect to matter charact.:

$$G_1^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda}\, u_\lambda\, \underline{\underline{j_\rho}} \,,$$

$$\Updownarrow$$

$$G_2^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda}\, u_\lambda\, \underline{\underline{\lambda_\rho}} \,,$$

$$\boxed{\begin{aligned} & u, j, \lambda \equiv \\ & \equiv u^f, j^f, \lambda^f \end{aligned}}$$

$$G_3^{\mu\nu} = u^\mu \underline{\underline{j^\nu}} - \underline{\underline{j^\mu}} u^\nu \,,$$

$$\Updownarrow$$

$$G_4^{\mu\nu} = u^\mu \underline{\underline{\lambda^\nu}} - \underline{\underline{\lambda^\mu}} u^\nu \,.$$

⚡ for each of background fermions

$$G_1(j_\rho) = G_2(\lambda_e \to j_\rho);\; G_3(j^\nu) = G_4(\lambda^\nu \to j^\nu)$$

Thus, in general case of ν interaction with different background fermions f matter effects are described by antisymmetric tensor

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g^{(1)}_{\rho} \underset{\uparrow\ \nu\text{ speed}}{u_{\lambda}} - \left(g^{(2)\mu} u^{\nu} - u^{\mu} g^{(2)\nu}\right),$$

where

$$g^{(1)\mu} = \sum_{f} \rho^{(1)}_{f} \overset{\text{matter current}}{j^{\mu}_{f}} + \rho^{(2)}_{f} \overset{\text{and polarization}}{\lambda^{\mu}_{f}},$$

$$g^{(2)\mu} = \sum_{f} \xi^{(1)}_{f} j^{\mu}_{f} + \xi^{(2)}_{f} \lambda^{\mu}_{f},$$

- summation is performed over fermions f,
- coefficients $\rho^{(1),(2)}_{f}$, $\xi^{(1),(2)}_{f}$ are determined by ν interaction model.

In the usual notations

$$\underset{\substack{\uparrow \\ \text{e.m. field}}}{F_{\mu\nu}} = (\vec{E}, \vec{B}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\boxed{G_{\mu\nu} = (-\vec{P}, \vec{M})},$$

where

$$g_\mu^{(1,2)} \equiv (g_0^{(1,2)}, \vec{g}^{(1,2)})$$

$$\vec{M} = \gamma\left\{ g_0^{(1)}\vec{\beta} - \vec{g}^{(1)} - [\vec{\beta} \times \vec{g}^{(2)}] \right\},$$

$$\vec{P} = -\gamma\left\{ g_0^{(2)}\vec{\beta} - \vec{g}^{(2)} + [\vec{\beta} \times \vec{g}^{(1)}] \right\}.$$

Substitution $\boxed{F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}}$

implies:

✓ $$\boxed{\begin{array}{l} \vec{B} \longrightarrow \vec{B} + \vec{M} \\ \\ \vec{E} \longrightarrow \vec{E} - \vec{P} \end{array}}$$

effects of ν interaction with moving and polarized matter

$\boxed{\vec{B}_0, \vec{E}_0, \vec{M}_0, \vec{P}_0}$

in the rest frame of ν are expressed in terms of quantities determined in laboratory frame

weak interaction of neutrino with matter

Finally:

three-dimentional ν spin vector

$$\boxed{\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma_\nu}\left[\vec{S}\times(\vec{B}_0+\vec{M}_0)\right] + \frac{2\epsilon}{\gamma}\left[\vec{S}\times(\vec{E}_0-\vec{P}_0)\right],}$$

effects of matter

laboratory frame

$$\vec{B}_0 = \gamma_\nu\left(\vec{B}_\perp + \frac{1}{\gamma_\nu}\vec{B}_\parallel + \sqrt{1-\frac{1}{\gamma_\nu^2}}\,[\vec{E}_\perp\times\vec{n}]\right), \quad \gamma_\nu = \frac{E_\nu}{m_\nu}$$

energy

mass of ν

$$\vec{E}_0 = \gamma_\nu\left(\vec{E}_\perp + \frac{1}{\gamma_\nu}\vec{E}_\parallel - \sqrt{1-\frac{1}{\gamma_\nu^2}}\,[\vec{B}_\perp\times\vec{n}]\right),$$

in rest frame of ν

$$\boxed{\vec{n} = \vec{\beta}/\beta}$$

A. Egorov,
A. Lobanov,
A.S.
PLB491(200
p.137

$$\vec{M}_0 = \gamma_\nu\vec{\beta}\left(g_0^{(1)} - \frac{\vec{\beta}\vec{g}^{(1)}}{1+\gamma_\nu^{-1}}\right) - \vec{g}^{(1)},$$

$$\vec{P}_0 = -\gamma_\nu\vec{\beta}\left(g_0^{(2)} - \frac{\vec{\beta}\vec{g}^{(2)}}{1+\gamma_\nu^{-1}}\right) + \vec{g}^{(2)}.$$

$$g = g(j_f, \lambda_f)$$

For SM+SU(2)-singlet ν_R and matter $f=e$

$$\boxed{\frac{d\vec{S}_\nu}{dt} = \frac{2\mu_\nu}{\gamma_\nu}\left[\vec{S}_\nu \times (\vec{B}_0 + \vec{M}_0)\right]},$$

in rest frame of neutrino

$$\vec{B}_0 = \gamma_\nu\left(\underline{\underline{\vec{B}_\perp}} + \frac{1}{\gamma_\nu}\underline{\underline{\vec{B}_\parallel}} + \sqrt{1-\frac{1}{\gamma_\nu^2}}\left[\underline{\underline{\vec{E}_\perp}} \times \vec{n}\right]\right),$$

interaction of neutrino with matter

$$\vec{M}_0 = \gamma_\nu \rho n_e \left(\underline{\underline{\vec{\beta}_\nu}}\left(1-\underline{\underline{\vec{\beta}_\nu}}\,\underline{\underline{\vec{v}_e}}\right) - \frac{1}{\gamma_\nu}\underline{\underline{\vec{v}_{e\perp}}}\right),$$

matter density

$\parallel$

$\perp$

$$\gamma_\nu = \frac{E_\nu}{m_\nu},$$

ν *spin procession in matter !!!*
with out any electromagnetic field

How $\rho_f^{(i)}$ and $\xi_f^{(i)}$ are determined?

To fix ν interactions: SM + SU(2)-singlet ν_R =>

for ν propagating in moving and polarized gas of **e**

$$L_{eff} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma_5}{2}\nu\right),$$ where

$$f^{\mu} = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j_e^{\mu} - \lambda_e^{\mu}\right)$$

In this case: $\epsilon = 0$ => $\boxed{\xi_e^{(i)} = 0,}$

ν electric dipole moment

and $(f_{\mu} = 2\mu g_{\mu}^{(1)})$ =>

$$\boxed{\begin{array}{l} \rho_e^{(1)} = \dfrac{G_F}{2\mu_{\nu}\sqrt{2}}(1+4\sin^2\theta_W), \\ \rho_e^{(2)} = -\dfrac{G_F}{2\mu_{\nu}\sqrt{2}}. \end{array}}$$

If :

$$\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu ,$$

B. Lee, R. Shrock, 1977
K. Fujikawa, R. Shrock, 1980

$$\boxed{\rho_e^{(1)} = \frac{4\pi^2}{3 e m_\nu}\left(1 + 4\sin^2\theta_W\right), \quad \rho_e^{(2)} = -\frac{4\pi^2}{3 e m_\nu} \quad .}$$

New effects of relativistic matter in neutrino oscillations

⇓

(NO)

NEW NEUTRINO RESONANCES IN MOVING AND POLARIZED MATTER

Neutrino ν_e spin evolution in relativistic flux of electrons ($f \equiv e$)

Effects of moving and polarized matter

$$\vec{M}_0 = n_e \gamma \vec{\beta}_{\nu} \Big\{ \big[\rho^{(1)} + \rho^{(2)} \vec{\zeta}\vec{v}_e \big] (1 - \vec{\beta}\vec{v}_e) +$$

$$+ \rho^{(2)} \sqrt{1 - v_e^2} \Big[\frac{\vec{\zeta}\vec{v}_e \, \vec{\beta}\vec{v}_e}{1 + \sqrt{1 - v_e^2}} - \vec{\zeta}\vec{\beta} \Big] + O(\gamma^{-1}) \Big\}$$

- slowly moving matter, $v_e \ll 1$:

$$\vec{M}_0 = n_e \gamma \vec{\beta} \left(\rho^{(1)} - \rho^{(2)} \vec{\zeta} \vec{\beta} \right). \qquad \gamma = \frac{E_\nu}{m_\nu}$$

Wolfenstein term (1978)

H. Nunokawa,
V. Semikoz,
A. Smirnov,
J.W.F. Valle (1997)

$$\boxed{\vec{\zeta} \equiv \vec{\zeta}_e}$$

mean value of polarization vector of electrons

$$\rho^{(1)} = \frac{G_F}{2\mu\sqrt{2}} (1 + 4\sin^2\theta_W), \quad \rho^{(2)} = -\frac{G_F}{2\mu\sqrt{2}}$$

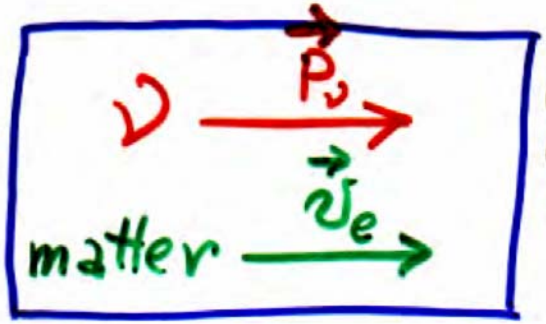
for SM + SU(2)-singlet ν_R

- relativistic flux of e, $v_e \sim 1$:

$$\vec{M}_0 = n_e \gamma \vec{\beta} \left(\rho^{(1)} + \rho^{(2)} \vec{\zeta}_e \vec{v}_e\right) \boxed{\left(1 - \vec{\beta}_\nu \vec{v}_e\right)}$$

!

In case of



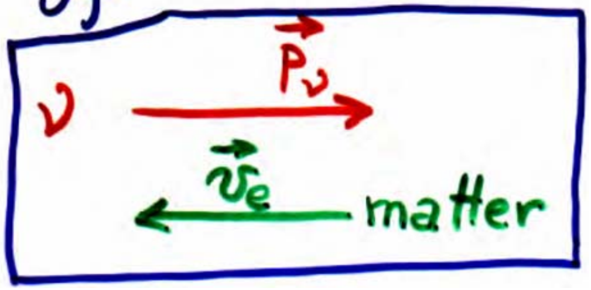
c=1 speed of light

matter effect contribution to ν spin evolution equation is suppressed !

invariant electron number density

$$\boxed{n_e = \frac{n_0}{\sqrt{1 - v_e^2}}}$$

In case of



ν and relativistic matter ($v_e \sim 1$)
motion in opposite directions
matter term gets its maximum

$$\vec{M}_0^{max} = \boxed{\frac{2}{\sqrt{1-v_e^2}}} \vec{M}_0^{(v_e \ll 1)} \gg 1 \text{ if } v_e \sim 1$$

$\Rightarrow$ substantial increase
of matter effects in
ν oscillations !

$$\nu_L \overset{B_\perp}{\Longleftrightarrow} \nu_R$$

* $B_{\text{кр}}(\theta_{vac}, \Delta m^2_\nu, E_\nu, \mu_\nu, n_{\text{эфф}}, \dot{\varphi})$

Neutrino conversions and oscillations in magnetic field

⊛ $\nu_{\odot}$ problem

$$\boxed{\nu_L \overset{B}{\longleftrightarrow} \nu_R}$$

Cisneros, 1971

* { Voloshin, Vysotsky, Okun, 1986
Barbieri, Fiorentini, 1988

⊙ twisting B { Smirnov, 1991
Akhmedov, Petcov, Smirnov 1993

⊛ Supernova $\nu_L \overset{B}{\leftrightarrow} \nu_R$

Dar, 1987

Fujikawa, Shrock, 1988

Voloshin, 1988

Осцилляции нейтрино

$$\nu_L \overset{B_\perp}{\longleftrightarrow} \nu_R$$ (спиновые осцил-ляции)

в сильных магнитных полях

А. Егоров,
Г. Лихачев,
А. Студеникин,
1995, 1997, **ЖЭТФ**

$$\nu_L \overset{B_\perp}{\Longleftrightarrow} \nu_R$$

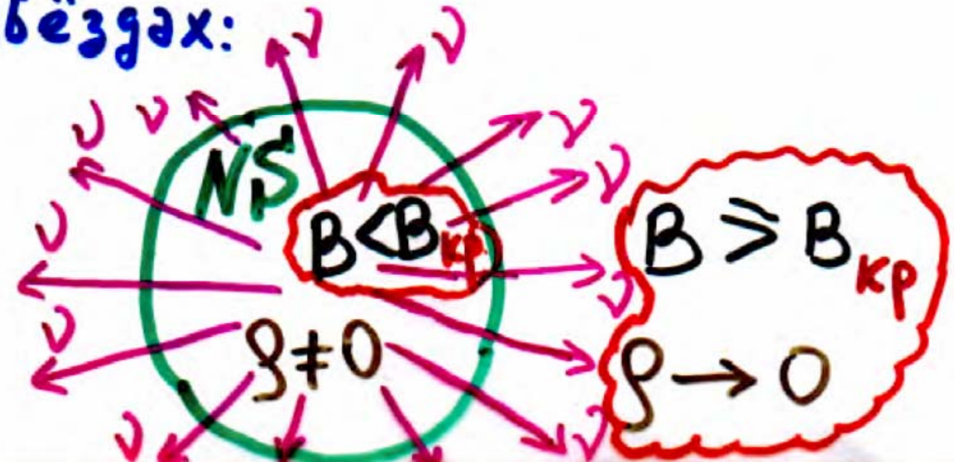
* $B_{кр}(\theta_{vac}, \Delta m^2_\nu, E_\nu, \mu_\nu, n_{эфф}, \dot{\varphi})$

⊛ Критическая напряжённость поля

$$B_{кр} = B(\Delta m^2, \theta_\nu, E_\nu, \mu_\nu, \underline{\underline{\rho}})$$

⊛ $B_{кр} \sim \underline{\underline{\rho}} \leftarrow$ плотность вещества

⊛ „Пограничный эффект" при осцилляциях ν в нейтронных звёздах:

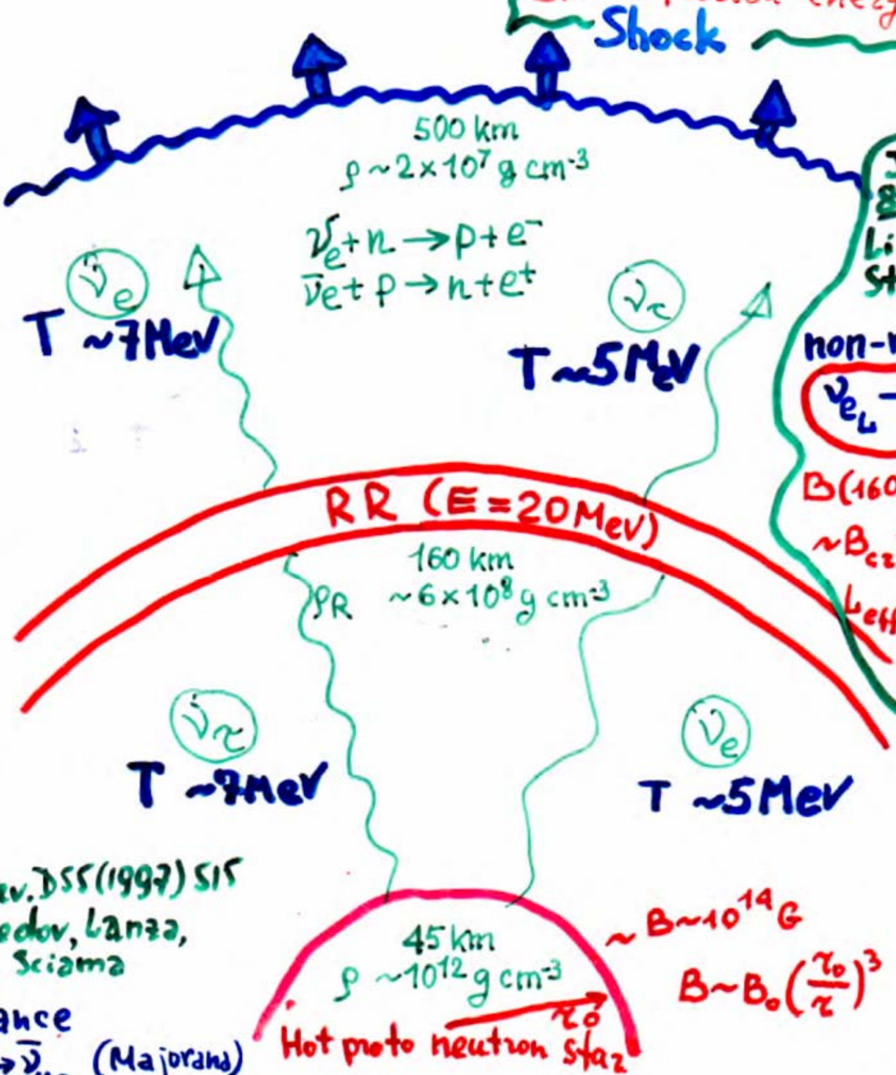


* $\nu_i \overset{B}{\leftrightarrow} \nu_j$ => sufficient consequences on reheating phase of a Type II supernova

G. Fuller
R. Mayle
B. Meyer
J. Wilson

ApJ. (1991) 389, 517 ⇒

• 60% increase of SN explosion energy



• JETP, 1995
81, 419
Likhachev
Studenikin

non-resonance

$\nu_{e_L} \to \nu_{e_R}$

B(160km) ~
~$B_{cr} = 0.6\cdot 10^{13}$ G

$L_{eff} \sim 10$ cm

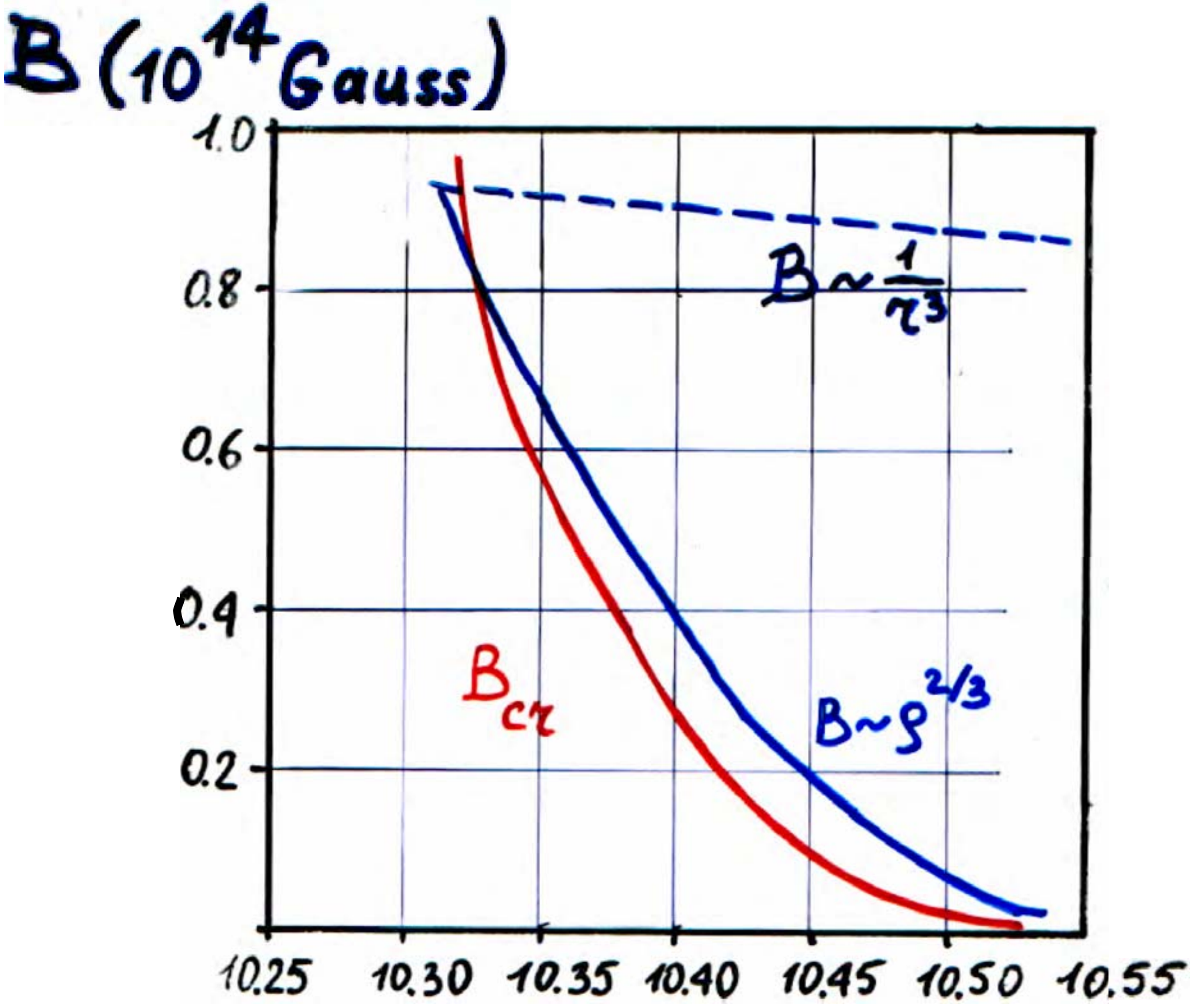
$\mu \sim 10^{-11}\mu_B$

• Phys.Rev. D55 (1997) 515
Akhmedov, Lanza,
Petkov, Sciama

resonance

$\nu_{e_L} \to \bar{\nu}_{\mu_R}$ (Majorana)

$B \sim 5\cdot 10^{14}$ G => $\mu \sim 10^{-13}\mu_B$



A.Egorov,
G.Likhachev,
A.Studenikin,
1995,1997

$R_{NS} \sim 10 km$

$\nu_L \rightarrow \nu_R$

active neutrino

sterile neutrino

Now we know:

①#1 how to treat ν spin oscillations in arbitrary e.m. fields within Lorentz invariant approach $\Rightarrow$ new resonances in $\nu_L \leftrightarrow \nu_R$ in various e.m. fields (e.m. wave etc...)

Now we know:

#2 that ν spin precession can be stimulated by weak interactions with matter and always occurs if initial ν state is not longitudinally polarized.

Now we know:

#3 that account for matter motion can drastically change probabilities of ν spin and flavour oscillations.

These effects are of particular importance in cases when ν propagates in relativistic jets of matter (quasars, gamma-ray bursts...)

How can ν be affected by $\vec{B}$?

non-trivial electromagnetic properties

- $\mu_\nu \neq 0 \, (m_\nu \neq 0)$
- spin and
- spin-flavour oscillations in B

indirect influence of B on interacting with ν particles

- $n \xrightarrow{B} p^+ + e^- + \tilde{\nu}_e$
- $\nu + n \xrightarrow{B} p^+ + e^-$
- flavour and spin ν oscillations in polarized (by B) matter $(e, n, p, \ldots)$

Direct influence of external $F_{\mu\nu}$ on oscillations $\nu_L \leftrightarrow \nu_R$

$\Nearrow$

direct interaction:

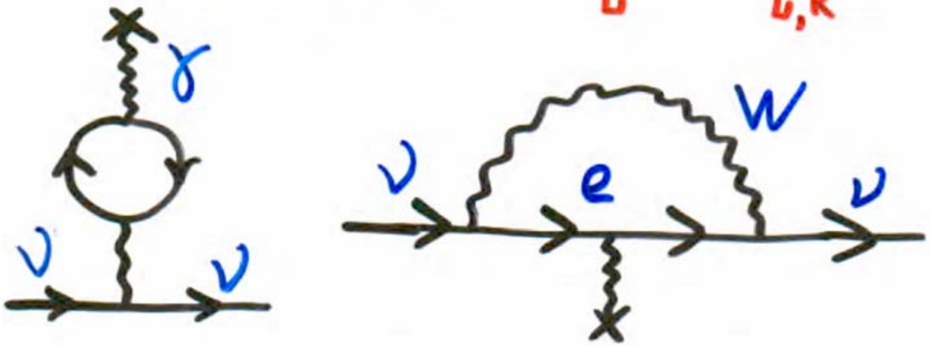
$$\left.\begin{array}{ll}\mu_{\alpha\beta} & \text{magnetic}\\ \epsilon_{\alpha\beta} & \text{electric}\end{array}\right\}\text{(transition, }\alpha\neq\beta\text{) momen}$$

$\alpha, \beta = e, \mu, \tau$ (ν's families)

$\Rightarrow$ non-trivial (beyond SM) electromagnetic properties of ν ($\mu \neq 0$, $\epsilon \neq 0$)

Indirect influence of external $F_{\mu\nu}$ on oscillations $\nu_L \leftrightarrow \nu_{L,R}$ in matter

J. D'Olivo, J. Nieves, P. Pal, 1989,
S. Esposito, G. Capone, 1996,
Elmfors, D. Grasso, G. Raffelt, 1996,
V. Semikoz, J. Valle, 1994; 1997,



J. D'Olivo, J. Nieves, 1996,
H. Nunokawa, V. Semikoz, A. Smirnov, J. Valle
1997

one-loop finite-density contribution to energy of ν in magnetized matter

⇔ matter polarization effects in $\vec{B}$

⇓

Extra term in ν effective potential

✓ $$V_{\nu_e} = \sqrt{2}\, G_F N_e - \frac{e G_F}{\sqrt{2}} \left(\frac{3N_e}{\pi^4}\right)^{1/3} \boxed{\frac{\vec{p}_\nu \vec{B}}{E_\nu}} \sim B_{\parallel}$$

(degenerate electron gas)

$N_e = n_e - n_{\bar{e}}$

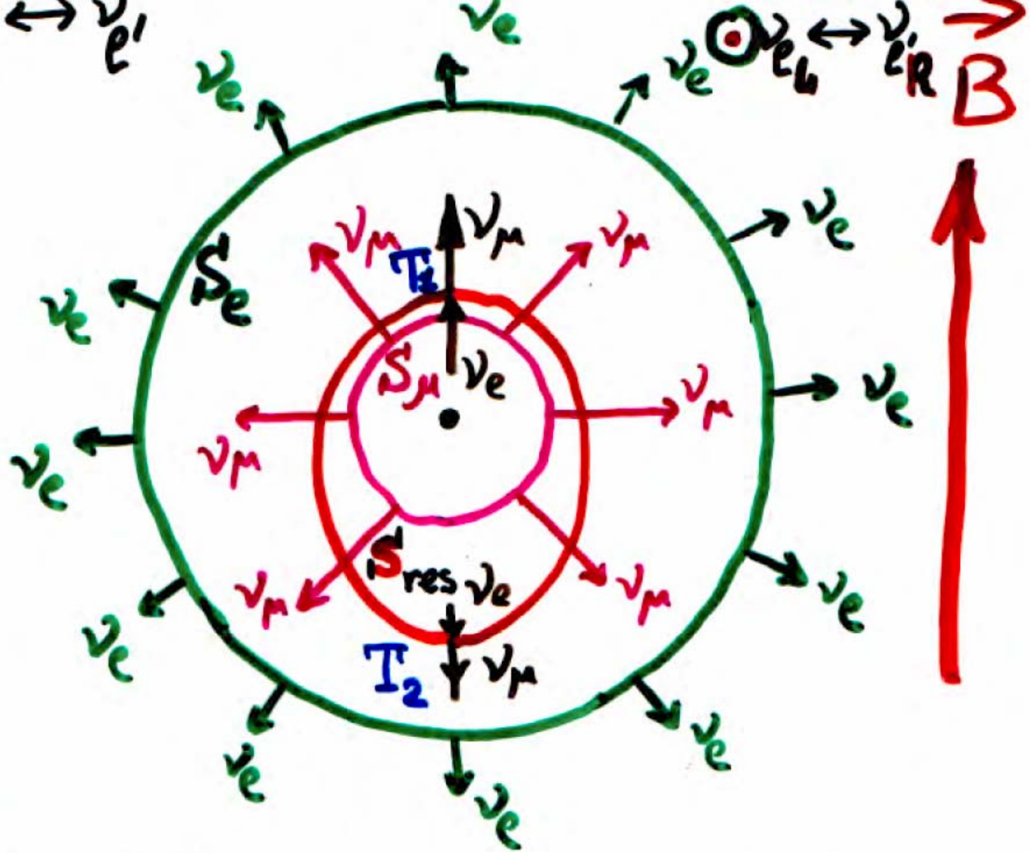
Pulsar kick and ν oscillations in B

{Kusenko, Serge, 1996

$\odot\ \nu_\ell \leftrightarrow \nu_{\ell'}$

{Akhmedov, Lanza, Sciama, 1997

$\odot\ \nu_{\ell_L} \leftrightarrow \nu_{\ell_R}$



S_μ, S_e, S_{res}

are muon, electron neutrino-spheres

and $\nu_e \leftrightarrow \nu_\mu$ resonance surface (ellipsoid)

{In directions where RS is close (far) to center ⟶ larger (smaller) ν momentum, since $T_1 > T_2$.

…more about

Indirect influence of external $F_{\mu\nu}$

- $\nu\gamma$ interactions

$$\nu \rightarrow \nu + \gamma$$

$$\gamma \rightarrow \nu + \nu$$

$$\gamma\gamma \rightarrow \nu\bar{\nu} \ldots$$

DeRaad, Milton, Hari Dass
Galtsov, Nikitina, Skobelev
Chistakov, Gvozdev Mikheev, Vasilevskaya
Ionnisian, Raffelt
Dicus, Repko, Shaisultanov
Borisov, Zhukovsky, A.Ternov, Eminov
Radomski, Grimus, Sakuda
Mohanty, Samal
Nieves, Pal . . .

- νe interactions

$$e \rightarrow e\nu\nu$$

$$\nu e \rightarrow \nu e \ldots$$

Landstreet, Baier, Katkov, Strakhovenko
Loskutov, Zakhartsov
Ritus, Nikishov
I.Ternov, Rodionov, Studenikin
Borisov, Kurilin
Narynskaya . . .

β-decay of neutron in magnetic field

{Birth of ν astrophysics in B} $n \xrightarrow{B} p+e+\tilde{\nu}_e$

- L. Korovina, "β-decay of polarized neutron in magnetic field", Izv.Vuz.Phys., # 6, 1964, 86
- I.Ternov, B.Lysov, L.Korovina, Mosc.Univ.Bull.,Phys.,Astron., #5, 1965, 58 "On the theory of neutron β-decay in external magnetic field",
- J.Matese, R.O'Connell, "Neutron beta decay in a uniform magnetic field", Phys.Rev.180, 1969, 1289
- L.Fassio-Canuto, "Neutron beta decay in a strong magnetic field" Phys.Rev.187 1969, 2141
- G.Greenstein, Nature, 223, 1969, 938

⊛ Asymmetry in $\tilde{\nu}$ emission

$$\frac{W(B)}{W_0} = \frac{1}{2}\int \sin\theta_\nu \, d\theta_\nu \left\{ 1 + \frac{2(\alpha^2+\alpha)}{1+3\alpha^2} S_n \underline{\underline{\cos\theta_\nu}} - 4.9\,\frac{eB}{\Delta^2}\left(\frac{\alpha^2-1}{1+3\alpha^2} \underline{\underline{\cos\theta_\nu}} + \frac{2(\alpha^2-\alpha)}{1+3\alpha^2} S_n \right) \right\}$$

astrophysical applications

Weak reaction rates in B

Inter-conversion between n and p through

inverse β-decay

$$* \; n + \nu \overset{B}{\longleftrightarrow} p + e^- ,$$

$$* \; n + e^+ \overset{B}{\longleftrightarrow} p + \bar{\nu} ,$$

$$* \; n \overset{B}{\longleftrightarrow} p + e^- + \bar{\nu} ,$$

n/p ratio in various astrophysical processes such as Greenstein, 1969; Matese, O'Connell, 1970; 1969;

I Big-Bang Nucleosynthesis Cheng, Schramm, Truran, 1993;

* "Magnetic Fields in the Early Universe", Phys. Rep. 348, 2001, 163. } ← Grasso, Rubinstein, 1995, 1996.

II Neutron Star Cooling and kick velocities

* $n + \nu \xleftrightarrow{B} p + e^-$,

* $n + e^+ \xleftrightarrow{B} p + \bar{\nu}$,

* $n \xleftrightarrow{B} p + e^- + \bar{\nu}$,

→ $\boxed{v_{NS} \sim X00 \, \frac{km}{s}}$

* Chugai, 1984;

* Dorofeev, Rodionov, Ternov, 1984;

* Loskutov, Zakhartsov, 1985;

* Studeni'kin, 1988.

Recent studies of β-processes in B

(neutron star cooling and kick velocities)

increasing interest

Vilenkin, 1995
Goyl, 1997;
Roulet, 1997;
Leinson, Perez, 1998;
Lai, Qian, 1998;
Arras, Lai, 1999;
Bhattacharya, Pal, 2003;
Duan, Qian, 2004;
Kauts, Savochkin, Studenikin, 2004.

Gvozdev, Ognev, 1999;
Bisnovatyi-Kogan, 1993.

H. Duan, Y.-Z. Qian,

Phys.Rev.D69 (2004) 123004

astro-ph/0401634,
30 Jan 2004

Neutrino processes

$\nu_e + n \rightleftarrows p + e^-$, $\bar{\nu}_e + p \rightleftarrows n + e^-$

in magnetic field $B \lesssim 10^{16} G$

(non-relativistic approach)

to n and p

$\Rightarrow \Big\{$ ν asymmetry and reduction of supernova cooling rate.

⊙ S. Shinkevich, A.S., **Pramana, 65 (2005) 215-244**

Relativistic theory of inverse β-decay of polarized neutron

$\boxed{\nu_e + n \rightarrow p + e^-}$

in strong B

⇒ effects of proton momentum quantization and proton recoil motion are included.

⊛ neutron matter is transparent for ν if

$\bullet\!\!\longrightarrow \nu \quad S_n \longleftarrow\!\!\bullet\, n$ in strong and super-strong B.

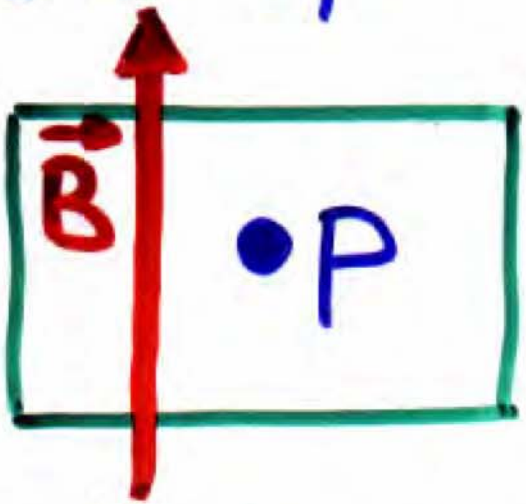
Motivation of the study of $\nu + n \xrightarrow{B} p + e$

* In "some-many-nearly all" of published papers on β-decay the **wrong** statement is made:

The influence of B on proton in β-decay (inverse β-decay and other relative processes) can be neglected if

$$B \ll \frac{m_p^2}{e} = B_0 \frac{m_p^2}{m_e^2} \approx \boxed{2 \cdot 10^{20}\, G}.$$

It is OK if



, But not for

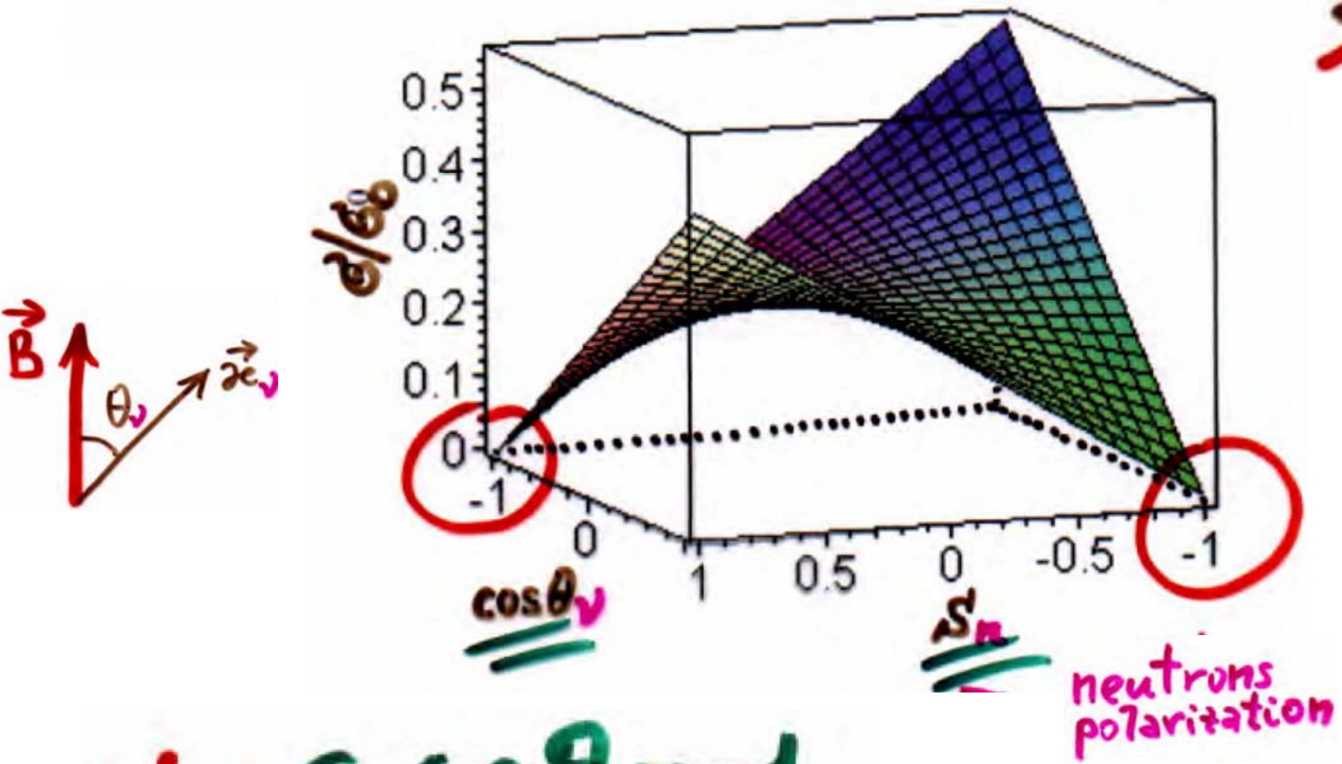
$n \xrightarrow{B} p+e+\tilde{\nu}$ or $\nu+n \xrightarrow{B} p+e$ etc...

(Studenikin, 1989)

⊛ ordinary mistakes in calculations ...

Cross section σ in strong $B = B_{cr} = 1.1 \cdot 10^{16}\,G$

$\varkappa = 10\,MeV$



S.Shinkevich,
A.Studenikin,
Pramana, 65 (2005)
215-244

$\alpha = \frac{g_A}{g_V}$

$\alpha = 1.26$

if $S_n \cos\theta_\nu = -1$,

$$\sigma\Big|_{\cos\theta_\nu = -1,\ S_n = 1} = 0, \quad \sigma\Big|_{\cos\theta_\nu = +1,\ S_n = -1} \sim (1-\alpha)^2 < 0.1$$

neutron matter is transparent for ν...

⊙ **K.Kouzakov, A.S.**

Phys.Rev.C 72 (2005) 015502

$n \xrightarrow{B} pe + \tilde{\nu}_e$

"Bound-state beta-decay of neutron in strong magnetic field"

Usual (continuum-state) β decay $\quad n \to p + e^- + \bar{\nu}_e$

"Rare" (bound-state) β decay $\quad n \to (pe^-) + \bar{\nu}_e$

R. Daudel, M. Jean, and M. Lecoin, J. Phys. Radium **8**, 238 (1947)

$$\boxed{\frac{w_b}{w_c} \cong 4.2 \times 10^{-6}}$$

$$\tau_c \sim 15 \text{ min}$$

$$\tau_b \sim 7 \text{ years}$$

J.N. Bahcall, Phys. Rev. **124**, 495 (1961) [Dirac equation]

L.L. Nemenov, Sov. J. Nucl. Phys. **15**, 582 (1972) [Schrödinger equation]

X. Song, J. Phys. G: Nucl. Phys. **13**, 1023 (1987) [Bethe-Salpeter equation]

K.A. Kouzakov and A.I. Studenikin, Phys. Rev. C **72**, 015502 (2005)

http://arxiv.org/hep-ph/0412134

Summary

First analysis of bound-state β decay in a strong magnetic field (B~10^{13}-10^{18} G)

✓ $w_b/w_c \sim 0.1-0.4$ in contrast to the field-free case, where $w_b/w_c \sim 10^{-6}$

✓ A logarithmic like behavior

$$w_b/w_c \propto \log_{10}(B/B_e)+b \ (b>0)$$

K.A. Kouzakov and A.I. Studenikin, Phys. Rev. C **72**, 015502 (2005)

http://arxiv.org/hep-ph/0412134

Summary

First analysis of bound-state β decay in a strong magnetic field (B~10^{13}-10^{18} G)

✓ $w_b/w_c \sim 0.1\text{-}0.4$ in contrast to the field-free case, where $w_b/w_c \sim 10^{-6}$

✓ A logarithmiclike behavior

$$w_b/w_c \propto \log_{10}(B/B_e)+b \ (b>0)$$

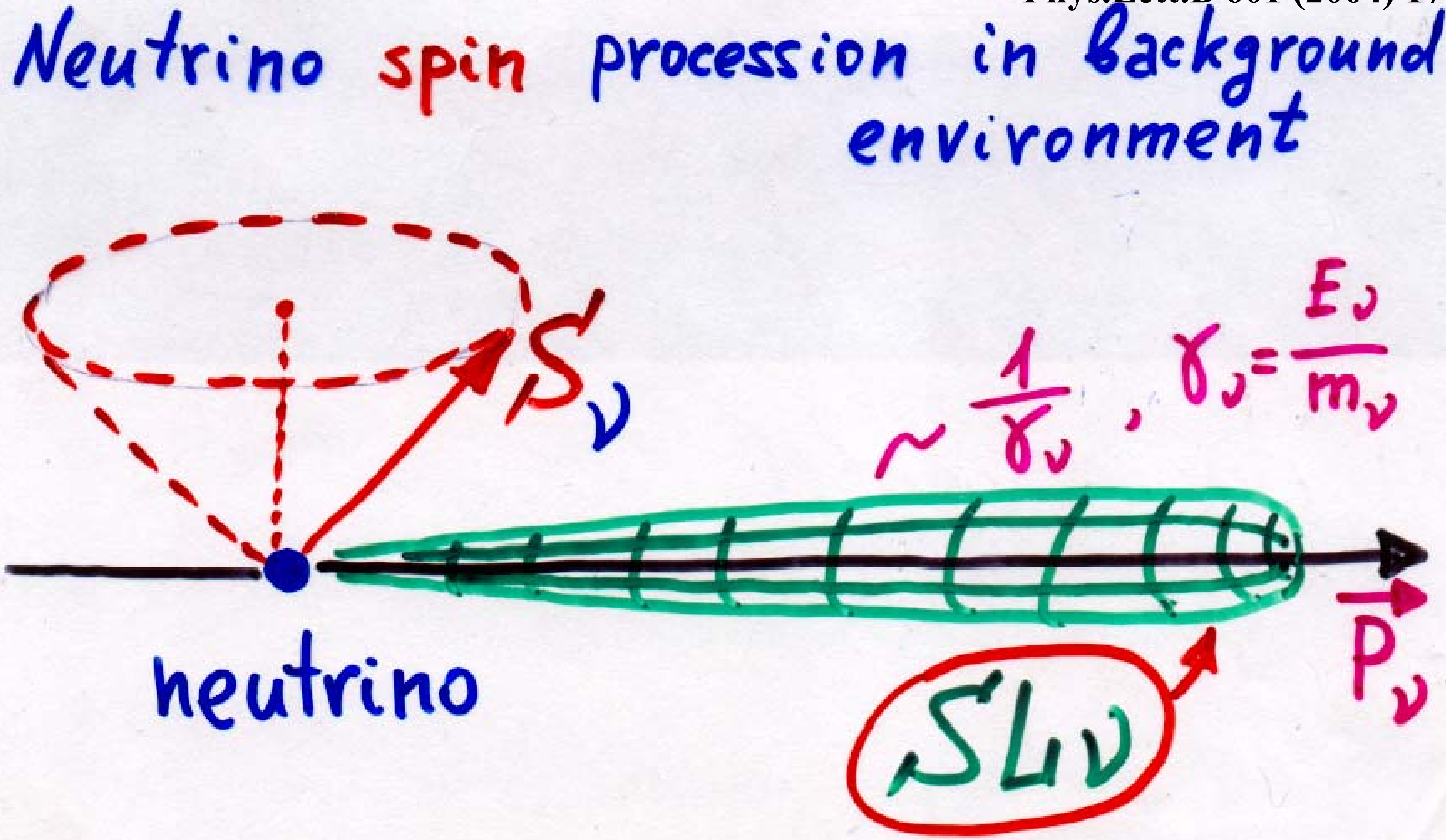
Outlook: Astrophysical applications?

"Spin light of neutrino" in matter and electromagnetic fields

$$\boxed{\nu \xrightarrow{F_{\mu\nu},\ G_{\mu\nu}} \nu + \gamma}$$

Quasi-classical theory of spin light of neutrino in matter

A.Lobanov, A.Studenikin, **Phys.Lett.B 564 (2003) 27,**
Phys.Lett.B 601 (2004) 171



Now we know:

#4 new mechanism of e.m. radiation by ν in matter (with or without e.m. field being superimposed)

– **spin light of neutrino** –

that must be important for

dense astrophysical (SN,... gamma-ray bursts)

cosmological (the early Universe)

environments.

...however !!!

Quantum treatment of neutrino in matter

A.Studenikin, *J.Phys.A: Math.Gen* **39** (2006)

A.Studenikin, A.Ternov, *Phys.Lett.**B*** **608** (2005) 107

A.Grigoriev, A.Studenikin, A.Ternov, *Phys.Lett.**B*** **622** (2005) 199
Grav. & Cosm. **11** (2005) 132

I.Pivovarov, A.Studenikin, *PoS(HEP2005)*191

References

A.Studenikin, *J.Phys.**A:** Math.Gen* **39** (2006) 6761

A.Studenikin, A.Ternov, *Phys.Lett.**B*** **608** (2005) 107

A.Studenikin, *Nucl.Phys.**B*** (Proc.Suppl.) **143** (2005) 570

A.Grigoriev, A.Studenikin, A.Ternov, *Phys.Lett.**B*** **622** (2005) 199
Grav. & Cosm. **11** (2005) 132

K.Kouzakov, A.Studenikin, *Phys.Rev.**C*** 72 (2005) 015502

M.Dvornikov, A.Grigoriev, A.Studenikin, *Int.J Mod.Phys.**D*** **14** (2005) 309

S.Shinkevich, A.Studenikin, *Pramana* **64** (2005) 124

A.Studenikin, *Phys.Atom.Nucl.* **67** (2004) 1014

M.Dvornikov, A.Studenikin, *Phys.Rev.**D*** **69** (2004) 073001
Phys.Atom.Nucl. **67** (2004) 719
JETP **99** (2004) 254
JHEP **09** (2002) 016

A.Lobanov, A.Studenikin, *Phys.Lett.**B*** **601** (2004) 171
*Phys.Lett.**B*** **564** (2003) 27
*Phys.Lett.**B*** **515** (2001) 94

A.Grigoriev, A.Lobanov, A.Studenikin, *Phys.Lett.**B*** **535** (2002) 187

A.Egorov, A.Lobanov, A.Studenikin, *Phys.Lett.**B*** **491** (2000) 137

Matter effect in neutrino flavour oscillations

- L.Wolfenstein,
  Neutrino oscillations in **matter**, Phys.Rev.D 17 (1978) 2369;
- S.Mikheyev, A.Smirnov,
  Resonance amplification of neutrino oscillations in matter and the spectroscopy of the solar neutrino, Sov.J.Nucl.Phys.42 (1985) 913.

$$\boxed{P_{\nu_e \to \nu_\mu}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}},}$$

effective mixing angle, θ_{eff}, and the effective oscillation length, L_{eff}, are given by

$$\sin^2 2\theta_{eff} = \frac{\Delta^2 \sin^2 2\theta}{\left(\Delta \cos 2\theta - A\right)^2 + \Delta^2 \sin^2 2\theta}, \quad L_{eff} = \frac{2\pi}{\sqrt{\left(\Delta \cos 2\theta - A\right)^2 + \Delta^2 \sin^2 2\theta}}, \quad \Delta = \delta m_\nu^2 / 2|\vec{p}\,|$$

θ is the vacuum mixing angle , $\boxed{A = \sqrt{2} G_F n_e}$, n_e ← the particle number density

MSW effect

Matter effect in neutrino spin (spin-flavour) oscillations

- M.Voloshin, M.Vysotsky, L.Okun,
  Electrodynamics of the neutrino and possible effects for solar neutrinos, JETP 64 (1986) 446;

neutrino **spin procession** in **magnetic field** and solar **matter** is considered

- C.-S.Lim, W.Marciano,
  Resonant **spin-flavour precession** of solar and supernova neutrinos, Phys.Rev.D37 (1988) 1368;
- E.Akhmedov,
  Resonant amplification of neutrino **spin rotation** in matter and the solar-neutrino problem, Phys.Lett.B213 (1988) 64.

$$\boxed{P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}}$$

neutrino magnetic moment

$$\sin^2 2\theta_{eff} = \frac{(2\mu B_\perp)^2}{(2\mu B_\perp)^2 + \Omega^2}, \quad L_{eff} = \frac{2\pi}{\sqrt{\Omega^2 + (2\mu B_\perp)^2}},$$

magnetic field

$$\boxed{\Omega = \frac{\delta m_\nu^2}{2|\vec{p}|} A(\theta_{vac}) - \sqrt{2} G_F n_{eff},}$$

$$\boxed{\Omega = 0}$$

resonance in neutrino spin-flavour oscillations

Neutrino decay in matter

- Z.Berezhiani, M.Vysotsky,
  Neutrino **decay** in matter, Phys.Lett.B 199 (1987) 281;
- Z.Berezhian, A.Smirnov,
  Matter-induced neutrino **decay** and supernova 1987A,
  Phys.Lett.B 220 (1989) 279;
- C.Giunti, C.W.Kim, U.W.Lee, W.P.Lam,
  Majoron **decay** of neutrinos in matter,
  Phys.Rev.D 45 (1992) 1557.

Matter can induce the neutrino **decay** into antineutrino and a light scalar particle (**majoron**):

$$\boxed{\nu \to \tilde{\nu} + \chi} \quad , \quad \text{or vice versa} \quad , \boxed{\tilde{\nu} \to \nu + \chi.}$$

- Z.Berezhiani, A.Rossi,
  Majoron **decay** in matter, Phys.Lett.B 336 (1994) 439:

$$\boxed{\chi \to \nu + \nu, \qquad \chi \to \tilde{\nu} + \tilde{\nu}.}$$

● … beyond the Standard Model …

Outline

“**Quantum approach**” to description of neutrino motion in the **background matter**

- **Modified Dirac equation** for neutrino in **background matter**
  - **Modified Dirac-Pauli equation** for neutrino in **background matter** and **magnetic field**

- Exact solutions of **modified Dirac (**and **Dirac-Pauli) equations** in matter
  - Neutrino **wave function** and **energy spectrum** in matter
  - Neutrino **oscillations** in **matter** and **magnetic field**

- Quantum theory of ***neutrino spin light*** in matter
  - Transition **rate,** radiation **power,** photon **energy**
  - Spatial angular **distribution** and **polarization**

Applications to different neutrino processes in astrophysical and cosmological environments

Standard model electroweak interaction of a flavour neutrino in matter (*f* = *e*)

Interaction Lagrangian (it is supposed that **matter contains only electrons**)

$$L_{int} = -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1+\gamma_5)\nu_e - \bar{e}\gamma^\mu(1-4\sin^2\theta_W+\gamma_5)e\right]Z_\mu$$

$$-\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu(1+\gamma_5)eW_\mu^+ - \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu_e W_\mu^-$$

Charged current interactions contribution to neutrino potential in matter

$$\Delta L_{eff}^{CC} = \sqrt{2}\,G_F\left\langle\bar{e}\gamma^\mu(1+\gamma_5)e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

Neutral current interactions contribution to neutrino potential in matter

$$\Delta L_{eff}^{NC} = -\frac{G_F}{\sqrt{2}}\left\langle\bar{e}\gamma^\mu[(1-4\sin\theta_W^2)+\gamma_5]e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

Matter current and polarization

When the **electron field bilinear**

$$\left\langle \bar{e}\gamma^{\mu}(1+\gamma_5)e \right\rangle$$

is **averaged** over the background

$$\langle \bar{e}\gamma_0 e\rangle \sim density\,,$$

$$\langle \bar{e}\gamma_i e\rangle \sim velocity\,, \qquad \boldsymbol{i = 1, 2, 3}$$

$$\langle \bar{e}\gamma_\mu\gamma_5 e\rangle \sim spin\,,$$

it can be replaced by the **matter** (electrons) **current**

☆ $$\boxed{j^{\mu} = (n, n\mathbf{v}),}$$

invariant number density

speed of matter

and **polarization**

☆ $$\boxed{\lambda^{\mu} = \left(n(\boldsymbol{\zeta}\mathbf{v}), n\boldsymbol{\zeta}\sqrt{1-v^2} + \frac{n\mathbf{v}(\boldsymbol{\zeta}\mathbf{v})}{1+\sqrt{1-v^2}} \right)}$$

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L^{CC}_{eff} + \Delta L^{NC}_{eff} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\right)$$

matter current

where $$f^{\mu} = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j^{\mu} - \lambda^{\mu}\right)$$

matter polarization

$$\boxed{\left\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\right\}\Psi(x) = 0}$$

It is suppose that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,’88; J.Panteleone,’91; K.Kiers, N.Weiss, M.Tytgat,’97-’98; P.Manheim,’88; D.Nötzold, G.Raffelt,’88; J.Nieves,’89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang’91; M.Kachelriess,’98; A.Kusenko, M.Postma,’02.

A.Studenikin, A.Ternov, hep-ph/0410297;
*Phys.Lett.**B*** **608** (2005) 107

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Neutrino wave function and energy spectrum in matter (I)

In the **rest frame** of **unpolarized matter**

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

$$f^\mu = \frac{1}{2\sqrt{2}} \tilde{G}_F (n, 0, 0, 0), \quad \tilde{G}_F = G_F (1 + 4 \sin^2 \theta_W)$$

The Hamiltonian form of the equation:

$$i \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H}_{matt} \Psi(\mathbf{r}, t),$$

where

$$\hat{H}_{matt} = \hat{\boldsymbol{\alpha}} \mathbf{p} + \hat{\beta} m + \hat{V}_{matt},$$

$$\hat{V}_{matt} = \frac{1}{2\sqrt{2}} (1 + \gamma_5) \tilde{G}_F n$$

number density of background matter (electrons)

$$\hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_0 \boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0,$$

$$\tilde{G}_F = G_F (1 + 4 \sin^2 \theta_W)$$

The form of the Hamiltonian implies that the **operators of the momentum,** $\hat{\mathbf{p}}$, and **longitudinal polarization,** $\hat{\Sigma}\mathbf{p}/p$, are the **integrals of motion:**

$$\frac{\hat{\Sigma}\mathbf{p}}{p} \Psi(\mathbf{r}, t) = s \Psi(\mathbf{r}, t), \quad \hat{\Sigma} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix}, \quad s = \pm 1$$

positive-helicity

negative-helicity

In the relativistic limit the **negative-helicity** neutrino state is dominated by the **left-handed chiral** state: $\nu_- \approx \nu_L$, $\nu_+ \approx \nu_R$.

Stationary states

$$\boxed{\Psi(\mathbf{r},t) = e^{-i(E_\varepsilon t - \mathbf{pr})} u(\mathbf{p}, E_\varepsilon),}$$

neutrino wave function in matter

and

$$\boxed{E_\varepsilon = \varepsilon\sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m}$$

neutrino energy spectrum in matter

$\boxed{s = \pm 1}$ for two **helicity** states ,

where the **matter density parameter** $\boxed{\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}}$,

J.Panteleone, 1991
(if NC interaction
were left out)

$\boxed{\frac{1}{2\sqrt{2}}\tilde{G}_F n \sim 1\ eV}$ for $\boxed{n = 10^{37}\ cm^{-3}}$

density of matter
in a neutron star

Neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization (helicity), i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies.

Neutrino wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon t-\mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\;\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\;\sqrt{1-s\frac{p_3}{p}}\;\;e^{i\delta}\\ s\varepsilon\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\;\sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\;\sqrt{1-s\frac{p_3}{p}}\;\;e^{i\delta}\end{pmatrix},}$$

$$\delta = \arctan\left(p_2/p_1\right)$$

$$\boxed{E_\varepsilon-\alpha m=\varepsilon\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}}$$

A.Studenikin, A.Ternov,
hep-ph/0410297;
Phys.Lett.B **608** (2005) 107

The quantity $\boxed{\varepsilon = \pm 1}$ splits the solutions into the two branches that

in the limit of **vanishing matter density,** $\boxed{\alpha \longrightarrow 0,}$

reproduce the **positive** and **negative-frequency** solutions, respectively.

Modified Dirac equation for matter composed of electrons, protons and neutrons (I)

The generalizations of the modified Dirac equation for more complicated matter compositions and the other flavour neutrinos are just straightforward.

For matter composed of **electrons** , **protons** and **neutrons** :

$$f^{\mu} = \sqrt{2} G_F \sum_{f=e,p,n} j_f^{\mu} q_f^{(1)} + \lambda_f^{\mu} q_f^{(2)} ,$$

where

$$q_f^{(1)} = (I_{3L}^{(f)} - 2Q^{(f)} \sin^2 \theta_W + \delta_{ef}), \quad q_f^{(2)} = -(I_{3L}^{(f)} + \delta_{ef}), \quad \delta_{ef} = \begin{cases} 1 & \text{for} f = e, \\ 0 & \text{for} f = n, p \end{cases}$$

isospin third component

electric charge of a fermion *f*

current

polarization

$$\boxed{\left\{ i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5) f^{\mu} - m \right\} \Psi(x) = 0}$$

Modified Dirac equation for matter composed of electrons, protons and neutrons (II)

Neutrino **energy spectrum** in matter composed of **electrons** , **protons** and **neutrons** :

$$E_\varepsilon = \varepsilon\sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$\varepsilon = \pm 1$$
$$s = \pm 1 .$$

Density parameter α is determined by particles number densities $n_{e,p,n}$.

★ For **electron neutrino** ν_e :

$$\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(1 + 4\sin^2\theta_W) + n_p(1 - 4\sin^2\theta_W) - n_n\Big) ,$$

✸ in electrically **neutral** and **neutron** reach matter $\alpha_{\nu_e} < 0$,

✸ for **electron antineutrino** $\alpha_{\tilde{\nu}_e} \rightarrow -\alpha_{\nu_e}$.

★ For **muon** and **tau neutrino** ν_μ , ν_τ :

$$\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(4\sin^2\theta_W - 1) + n_p(1 - 4\sin^2\theta_W) - n_n\Big) .$$

An important note (I)

The **modified Dirac equation**
for a neutrino in the background matter
(and the obtained **exact solution** and **energy spectrum**)
establish a basis for an **effective method**
in investigations of different phenomena
that can appear when neutrinos are moving in media.

similar to the **Furry representation**
of quantum electrodynamics

Neutrino and antineutrino energy spectra in matter

For the fixed value of the neutrino momentum **P** there are two values for the **“positive sign”** $\varepsilon = +1$ energies

$$E^{s=+1} = \sqrt{\mathbf{p}^2\left(1-\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m,$$

$$E^{s=-1} = \sqrt{\mathbf{p}^2\left(1+\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

positive-helicity neutrino energy

negative-helicity neutrino energy

particle (neutrino) energies in matter

The two other values of the energy for the **“negative sign”** $\varepsilon = -1$ correspond to the **antiparticle** solutions. By changing the sign of the energy, we obtain

$$\tilde{E}^{s=+1} = \sqrt{\mathbf{p}^2\left(1-\alpha\frac{m}{p}\right)^2 + m^2} - \alpha m,$$

$$\tilde{E}^{s=-1} = \sqrt{\mathbf{p}^2\left(1+\alpha\frac{m}{p}\right)^2 + m^2} - \alpha m$$

positive-helicity antineutrino energy

negative-helicity antineutrino energy

antiparticle (antineutrino) energies in matter

Neutrino processes in matter

- **Neutrino reflection from interface between vacuum and matter**
- **Neutrino trapping in matter**
- **Neutrino-antineutrino pair annihilation at interface between vacuum and matter**
- **Spontaneous neutrino-antineutrino pair creation in matter**

L.Chang, R.Zia,'88
A.Loeb,'90
J.Panteleone,'91
K.Kiers, N.Weiss, M.Tytgat,'97-'98
M.Kachelriess,'98
A.Kusenko, M.Postma,'02 H.Koers,'04
A.Studenikin, A.Ternov,'04
A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, '05
I.Pivovarov, A.Studenikin,'05
A.Ivanov, A.Studenikin, '05

Neutrino reflection from interface between vacuum and matter

$1 < \alpha_1 < 2$

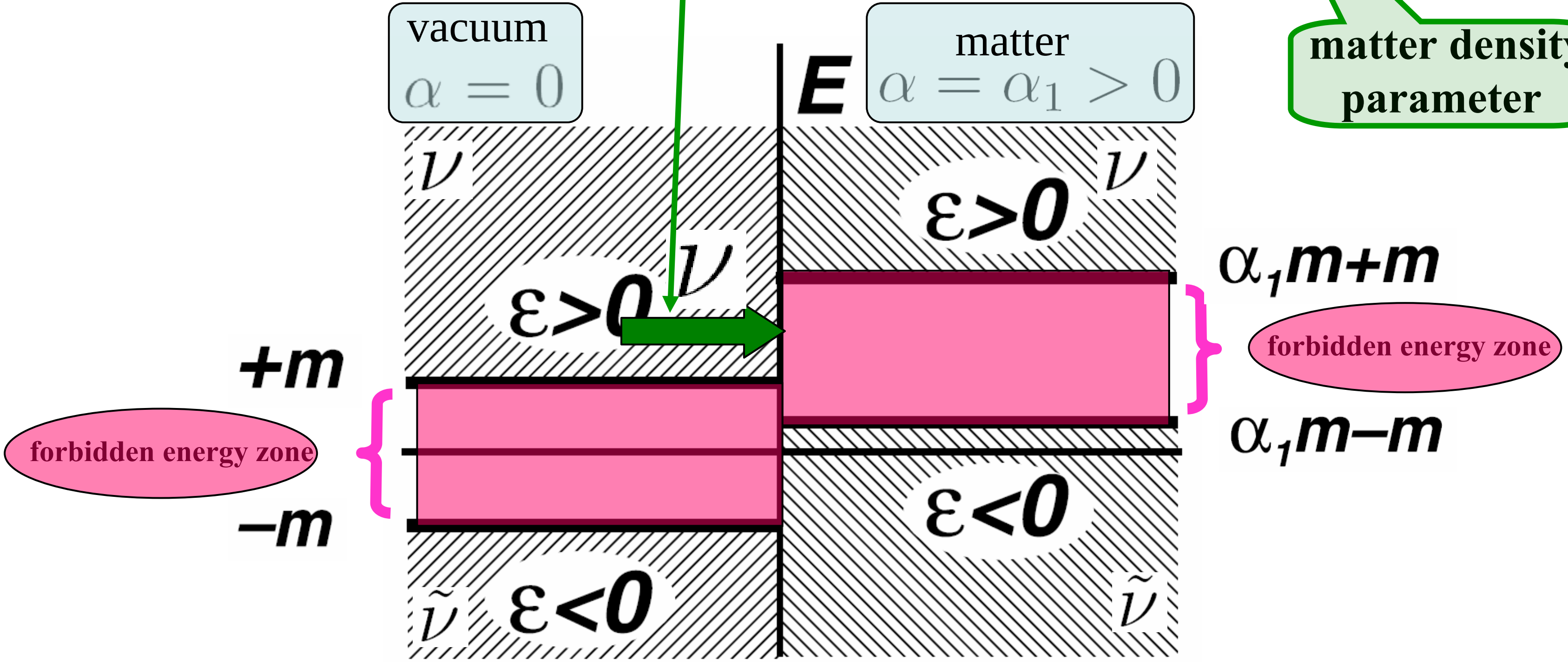
If the neutrino energy in **vacuum** E

is less than the neutrino minimal energy in **medium** $\alpha_1 m + m$

$$\boxed{m \le E < \alpha_1 m + m}$$

$$\boxed{\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}}$$

matter density parameter



then the appropriate energy level inside the medium is **not accessible** for neutrino

neutrino **is reflected** from the interface.

Neutrino trapping in matter

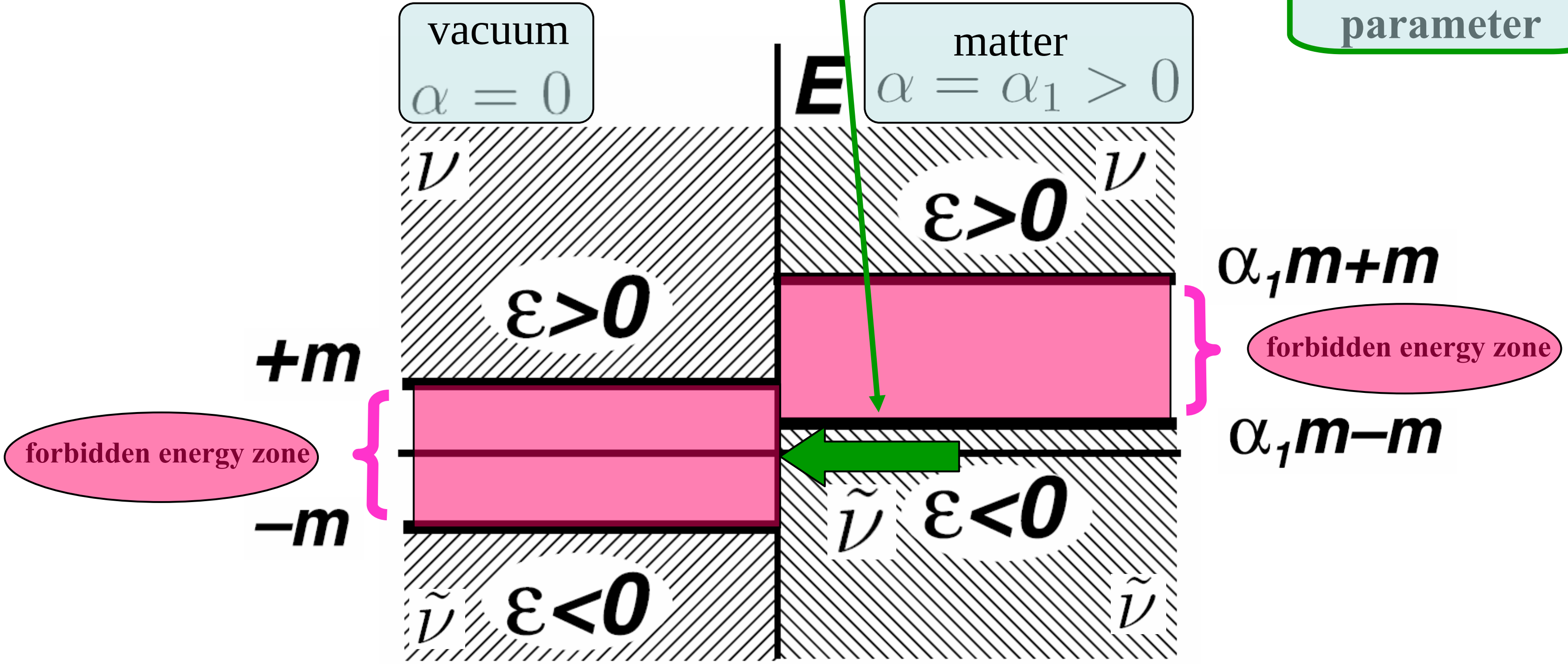
$1 < \alpha_1 < 2$

Antineutrino in **medium** with energy $|\alpha_1 m - m| \leq E < m$

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}$$

matter density parameter

can not escape from the medium because this particular range of energies exactly falls on the forbidden energy zone in **vacuum** :



Antineutrino has not enough energy to survive in **vacuum**

it is **trapped** inside the medium.

Neutrino-antineutrino pair annihilation at interface between vacuum and matter

Consider a **neutrino** with energy $\boxed{m < E \le \alpha_2 m - m}$

$\alpha_2 \ge 2$

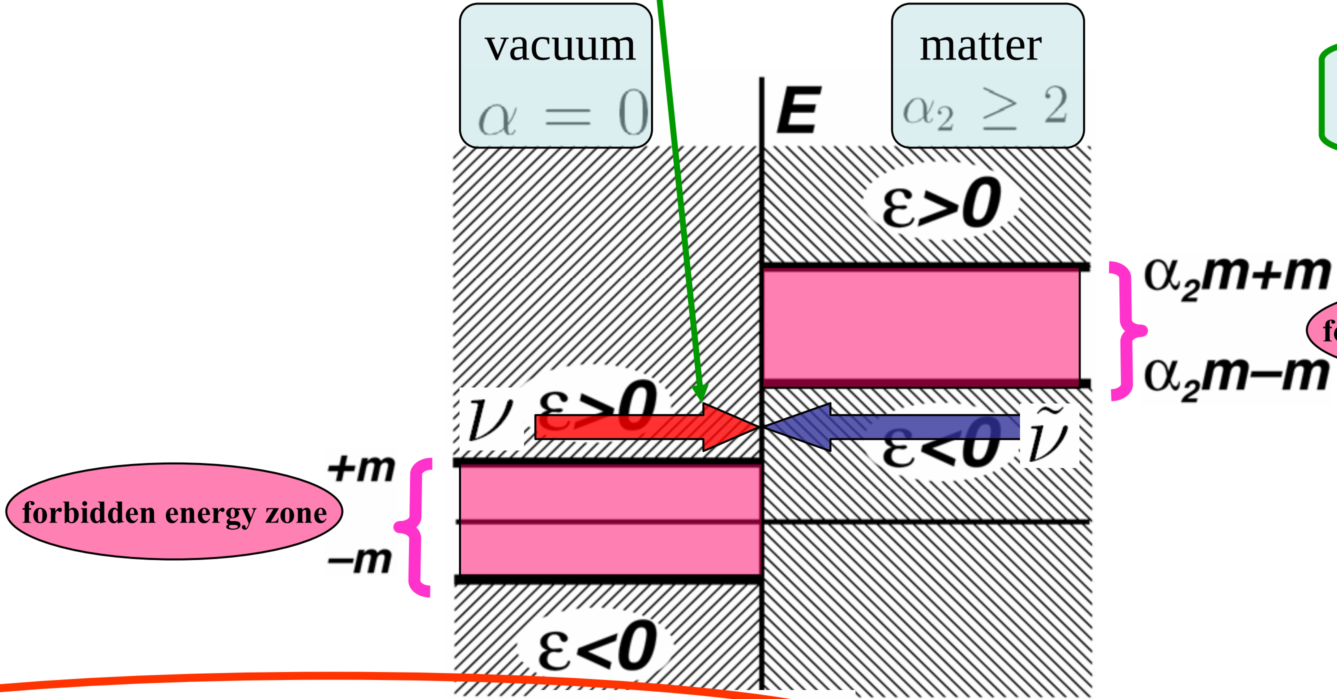
propagating in vacuum towards the interface with matter.

$$\boxed{\alpha_2 = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}}$$

matter density parameter

If not all of "**negative sign**" energy levels are occupied and, in particular, the level with energy exactly equal to E is available

➡ an **antineutrino** exists in matter :



forbidden energy zone

neutrino-antineutrino annihilation $\nu + \tilde{\nu} = \gamma$ at the interface of vacuum and matter.

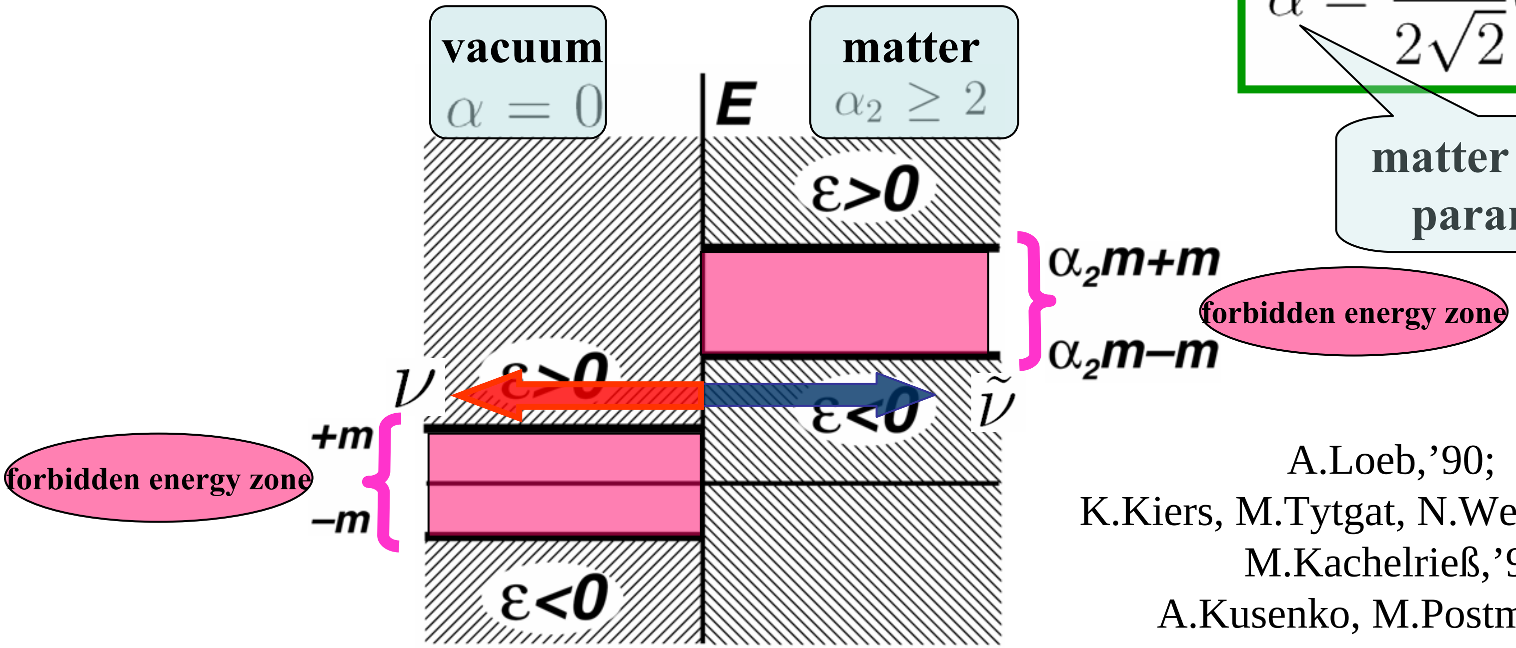
Spontaneous neutrino-antineutrino pair creation in matter

$\alpha_2 \geq 2$

”Negative sign” energy levels in matter have their counterparts in **”positive sign”** energy levels in vacuum:

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}$$

matter density parameter



A.Loeb,’90;
K.Kiers, M.Tytgat, N.Weiss,’97-’98
M.Kachelrieß,’98;
A.Kusenko, M.Postma,’02;
H.Koers,’04;
A.Studenikin, A.Ternov, ‘04

Neutrino-antineutrino pair creation can be interpreted as a process of appearance of a particle state of in the **”positive sign”** energy range accompanied by appearance of the hole state in the **”negative sign”** energy sea.

Spontaneous **electron-positron pair creation** according to Klein’s paradox of electrodynamics.

An important note (II)

The energy spectrum of **active left-handed** and **sterile right-handed** neutrino

$$\boxed{E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m}$$

- in the relativistic case $p \gg m$ and
- for not extremely high densities

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} \ll \frac{p}{m}$$

$$E_0 = \sqrt{p^2 + m^2}$$

the energies of the neutrino **helicity** states

$$\boxed{E^{s=\pm 1} \approx E_0 - \alpha m\left(s\frac{p}{E_0} - 1\right)}$$

correct energies of the neutrino **chiral** states

$$\boxed{E_{\nu_L} \approx E^{s=-1} \approx E_0 + \frac{1}{\sqrt{2}}\tilde{G}_F n}$$

active left-handed neutrino

$$\boxed{E_{\nu_R} \approx E^{s=+1} \approx E_0}$$

sterile right-handed neutrino

Neutrino flavour oscillations in matter

Consider the two flavour neutrinos, ν_e and ν_μ , propagating in electrically neutral matter of **electrons** , **protons** and **neutrons** : $n_e = n_p$.

The matter density parameters are

$$\boxed{\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(1+4\sin^2\theta_W)+n_p(1-4\sin^2\theta_W)-n_n\Big)}$$

and

$$\boxed{\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(4\sin^2\theta_W-1)+n_p(1-4\sin^2\theta_W)-n_n\Big)}$$

, respectively.

The energies of the **relativistic active** neutrinos are

$$\boxed{E^{s=-1}_{\nu_e,\nu_\mu} \approx E_0 + 2\alpha_{\nu_e,\nu_\mu} m_{\nu_e,\nu_\mu}}\ ,$$

and the energy difference

$$\boxed{\Delta E \equiv E^{s=-1}_{\nu_e} - E^{s=-1}_{\nu_\mu} = \sqrt{2}G_F n_e}$$

⇒ **MSW effect**

Modified Dirac-Pauli equation for neutrino in matter

Recently we have developed the quasi-classical approach to a massive **neutrino spin evolution** in the presence of external electromagnetic fields $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$ and **background matter**. The well known **Bargmann-Michel-Telegdi** equation of QED has been generalized for the case of a neutrino moving in matter and external electromagnetic fields by the following substitution of the electromagnetic field tensor **:**

A.Egorov,
A.Lobanov,
A.Studenikin

PLB 491 (2000) 137,
PLB 515 (2001) 94

$$F_{\mu\nu} \to E_{\mu\nu} = F_{\mu\nu} + G_{\mu\nu} \quad , \quad G_{\mu\nu} = (-\mathbf{P}, \mathbf{M})$$

where

$$G^{\mu\nu} = \sum_f \epsilon^{\mu\nu\eta\lambda}\big(\rho_f^{(1)} j_\eta^f + \rho_f^{(2)} \lambda_\eta^f\big) u_\lambda$$

matter current

(for a neutrino with zero dipole electric moment)

neutrino speed

$f = e, n, p, \mu, \tau \ \ldots$ **(matter content)**

matter polarization

$$\rho_e^{(1)} = \frac{G_F}{2\mu\sqrt{2}}(1 + 4\sin^2\theta_W), \quad \rho_e^{(2)} = -\frac{G_F}{2\mu\sqrt{2}}$$

$$\mathbf{B} \to \mathbf{B} + \mathbf{M}, \quad \mathbf{E} \to \mathbf{E} - \mathbf{P} \ ,$$

in particular $\boxed{\mathbf{M} = \gamma\rho^{(1)} n \boldsymbol{\beta}}$ **plays the role of a magnetic field** $\gamma = (1 - \beta^2)^{-1/2}$

Dirac-Pauli equation of electrodynamics

The **Dirac-Schwinger** equation for a massive neutrino in an external electromagnetic field $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$

$$\boxed{(i\gamma^\mu \partial_\mu - m)\Psi(x) = \int M_F(x', x)\Psi(x')dx'} \quad ,$$

neutrino mass operator in electromagnetic field

in the **linear** approximation over the electromagnetic field

the **Dirac-Pauli equation** :

$$\boxed{\left(i\gamma^\mu \partial_\mu - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\Psi(x) = 0} \quad , \quad \sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \; .$$

neutrino magnetic moment

The Hamiltonian form for the case of a magnetic field reads :

$$\boxed{i\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{H}_F \Psi(\mathbf{r}, t)} \quad , \quad \hat{H}_F = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta}m + \hat{V}_F \quad , \quad \hat{V}_F = -\mu\hat{\beta}\hat{\boldsymbol{\Sigma}}\mathbf{B}$$

$$\hat{\boldsymbol{\Sigma}} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix} \quad , \quad \hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_0\boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0, \quad \hat{\boldsymbol{\sigma}} = (\sigma_1, \sigma_2, \sigma_3) \; .$$

Modified Dirac-Pauli equation for neutrino in matter

From

$$\boxed{\left(i\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\Psi(x) = 0}$$

with the substitution

$$F_{\mu\nu} \longrightarrow G_{\mu\nu}$$

$$\boxed{\left(i\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}G_{\mu\nu}\right)\Psi(x) = 0} \quad .$$

neutrino speed

For the electron neutrino moving in unpolarized matter (electrons) at rest :

$$G^{\mu\nu} = \gamma\rho^{(1)}n\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_3 & \beta_2 \\ 0 & \beta_3 & 0 & -\beta_1 \\ 0 & -\beta_2 & \beta_1 & 0 \end{pmatrix}, \quad \rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}, \quad \gamma = (1-\boldsymbol{\beta}^2)^{-1/2}, \quad \boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3) \, .$$

In the Hamiltonian form:

$$\boxed{i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}_G\Psi(\mathbf{r},t)}$$

number density of matter

$$\hat{H}_G = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta}m + \hat{V}_G \, , \quad \hat{V}_G = -\frac{\tilde{G}_F}{2\sqrt{2}}\frac{n}{m}\hat{\beta}\boldsymbol{\Sigma}\mathbf{p} \, , \quad n \, .$$

Stationary states

$$\Psi(\mathbf{r},t)=e^{-i(Et-\mathbf{p}\mathbf{r})}u(\mathbf{p},E)$$

neutrino wave function in matter

$$E=\sqrt{\mathbf{p}^2(1+\alpha^2)+m^2-2\alpha mps}$$

neutrino energy spectrum in matter

and

$$s\;=\;\pm1$$

for two helicity states ,

where the matter density parameter

$$\alpha=\frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m},$$

$$E=\sqrt{\mathbf{p}^2+m^2\Big(1-s\frac{\alpha p}{m}\Big)^2}$$

$$m\to m\Big(1-s\frac{\alpha p}{m}\Big)$$

Neutrino wave function

$$\Psi_{\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(Et-\mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m-s\alpha p}{E}}\;\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m-s\alpha p}{E}}\;\sqrt{1-s\frac{p_3}{p}}\;\;e^{i\delta}\\ s\sqrt{1-\frac{m-s\alpha p}{E}}\;\sqrt{1+s\frac{p_3}{p}}\\ \sqrt{1-\frac{m-s\alpha p}{E}}\;\sqrt{1-s\frac{p_3}{p}}\;\;e^{i\delta}\end{pmatrix}$$

The two energy spectra

Dirac

Dirac-Pauli

$$\left\{i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m\right\}\Psi(x) = 0$$

$$f^\mu = \frac{1}{2\sqrt{2}}\tilde{G}_F(n,0,0,0)$$

$$\left(i\gamma^\mu\partial_\mu - m - \frac{\mu}{2}\sigma^{\mu\nu}G_{\mu\nu}\right)\Psi(x) = 0$$

$$G^{\mu\nu} = \gamma\rho^{(1)}n(0,\boldsymbol{\beta})$$

$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$E = \sqrt{\mathbf{p}^2 + m^2\left(1 - s\frac{\alpha p}{m}\right)^2}$$

In the limit of low density :

$$E^{s=\pm1} \approx E_0 - \alpha s\frac{pm}{E_0} + \alpha m$$

are not equal

$$E^{s=\pm1} \approx E_0 - \alpha s\frac{pm}{E_0}$$

However, the differences of energies of the two neutrino helicity states equals

$$\Delta E = E^{s=-1} - E^{s=+1} \approx 2\alpha m \Big|_{\alpha \ll \frac{p}{m},\ p \gg m}$$

Modified Dirac-Pauli equation in matter and magnetic field

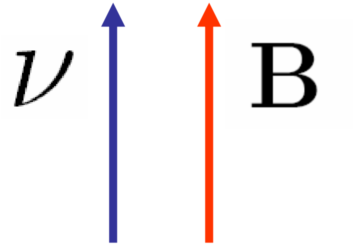
$$\boxed{\left\{ i\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}(G_{\mu\nu} + F_{\mu\nu}) \right\} \Psi(x) = 0}$$

A.Studenikin,
A.Ternov, '04

simultaneously accounts for interactions with **external electromagnetic fields** and also for **weak interaction with background matter** .

If a constant magnetic field is present in the background and a **neutrino is moving** parallel (or anti-parallel) to the field vector $\mathbf{B}$, then the neutrino energy spectrum can be obtained by

$$\boxed{\alpha \to \alpha' = \alpha + \frac{\mu B_{\|}}{p}},$$



$$B_{\|} = (\mathbf{B}\mathbf{p})/p$$

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}$$

and

$$\boxed{E = \sqrt{\mathbf{p}^2 + m^2\left(1 - s\frac{\alpha p + \mu B_{\|}}{m}\right)^2}} \, .$$

The energy gap between the two neutrino **helicity** states in magnetized matter

$$\boxed{\Delta_{eff} = E^{s=-1} - E^{s=+1} = \frac{\tilde{G}_F}{\sqrt{2}} n + 2\frac{\mu B_{\|}}{\gamma}}$$

Neutrino oscillation in magnetized matter

A.Studenikin, A.Ternov, 2004

The **Dirac-Pauli** energy spectrum of a neutrino in magnetized matter can be used for derivation of the neutrino oscillation $\nu_L \leftrightarrow \nu_R$ probability .

In the case of relativistic neutrino energies and constant magnetic field $\mathbf{B} = \mathbf{B}_{\|} + \mathbf{B}_{\perp}$

oscillation probability (adiabatic approx.)

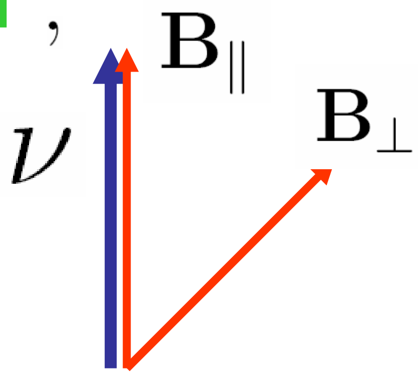
$$\boxed{P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}} \quad ,$$

$$\boxed{sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, L_{eff} = \frac{2\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}} \quad , \quad \boxed{E_{eff} = 2\mu B_{\perp}} \quad ,$$

$$\boxed{\Delta_{eff} = E^{s=-1} - E^{s=+1} = \frac{\tilde{G}_F}{\sqrt{2}} n + 2\frac{\mu B_{\|}}{\gamma}}$$

A.Lobanov, A.Studenikin,

Phys.Lett.B **515** (2001) 94



Neutrino propagation in matter

I.Pivovarov, A.Studenikin, ’05

Equation for neutrino **Green function** in matter

$$\boxed{\left\{i\gamma_{\mu}\partial^{\mu}-\frac{1}{2}\gamma_{\mu}\left(1+\gamma_{5}\right)f^{\mu}-m\right\}G\left(x\right)=-\delta(x)}\ ,\quad f^{\mu}=\frac{G_F}{\sqrt{2}}\Big((1+4\sin^2\theta_W)j^{\mu}-\lambda^{\mu}\Big),$$

matter current and polarization

in the **momentum representation**

$$\boxed{\left(\hat{p}-m-\hat{f}P_L\right)G(p)=-1}\ ,\qquad P_L=\frac{1+\gamma_5}{2},\quad P_R=\frac{1-\gamma_5}{2}\ .$$

Neutrino Green function in matter

$$\boxed{G_{matt}(p)=\frac{-\left(p^2-m^2\right)\left(\hat{p}+m\right)+\hat{f}\left(\hat{p}-m\right)P_L\left(\hat{p}+m\right)-f^2\hat{p}\,P_L+2\left(fp\right)P_R\left(\hat{p}+m\right)}{\left(p^2-m^2\right)^2-2\left(fp\right)\left(p^2-m^2\right)+f^2p^2}}$$

*S*pin *L*ight

of *N*eutrino in matter

Quantum theory of $\boxed{SL\nu}$

- A.Studenikin, A.Ternov, *Phys. Lett.**B*** **608** (2005) 107;
- A.Grigoriev, A.Studenikin, A.Ternov, *Phys. Lett.**B*** **622** (2005) 199,
  hep-ph/0502231, hep-ph/0507200;
- A.Grigoriev, A.Studenikin, A.Ternov, *Grav. & Cosm.* **11** (2005) 132;

A.Grigoriev, A.Studenikin, A.Ternov, hep-ph/0502210, hep-ph/0511311,
hep-ph/0511330

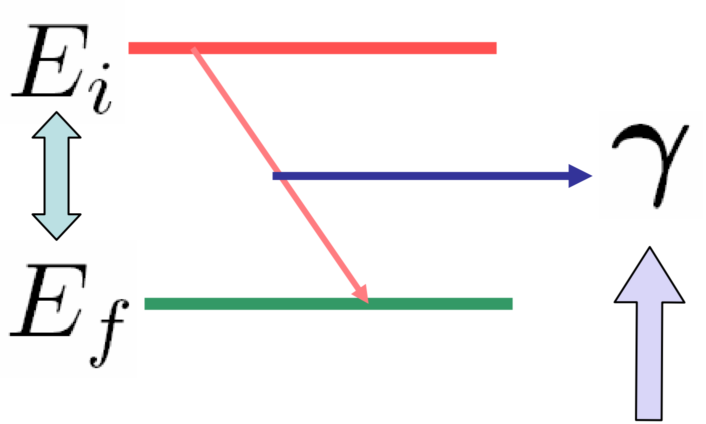
A.Studenikin, A.Ternov, hep-ph/0410296, hep-ph/0410297

Quantum theory of spin light of neutrino (I)

Quantum treatment of *spin light of neutrino* in matter
showns that this process originates from the **two subdivided phenomena:**

$SL\nu$

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite neutrino helicity states,



$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$s = \pm 1$$

the radiation of the photon in the process of the neutrino transition from the **”exited” helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;
A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

hep-ph/0507200, hep-ph/0502210,

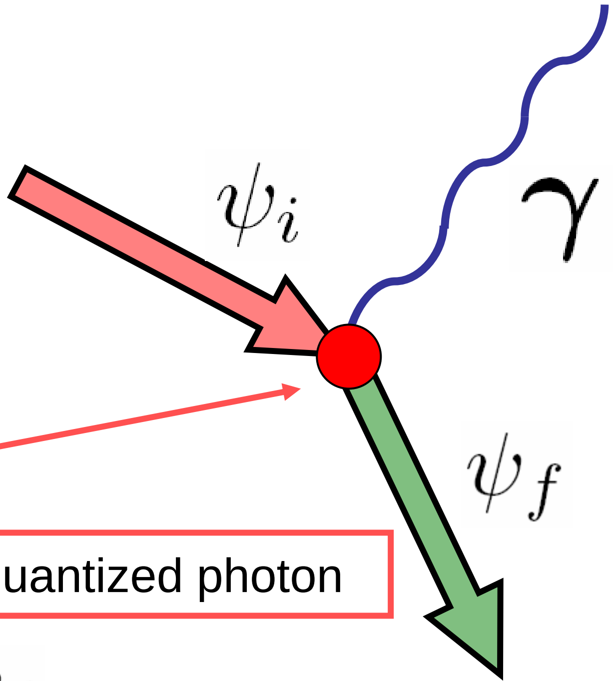
hep-ph/0502231

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

Quantum theory of spin light of neutrino $SL\nu$

Within the **quantum approach**, the corresponding Feynman diagram is the one-photon emission diagram with the initial and final neutrino states described by the **"broad lines"** that account for the neutrino interaction with matter.



Neutrino magnetic moment interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$\boxed{S_{fi} = -\mu\sqrt{4\pi}\int d^4x\,\bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)} \; ,$$

$$\boxed{\hat{\mathbf{\Gamma}} = i\omega\{[\mathbf{\Sigma}\times\boldsymbol{\varkappa}] + i\gamma^5\mathbf{\Sigma}\}} \; ,$$

$k^\mu = (\omega, \mathbf{k}), \boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

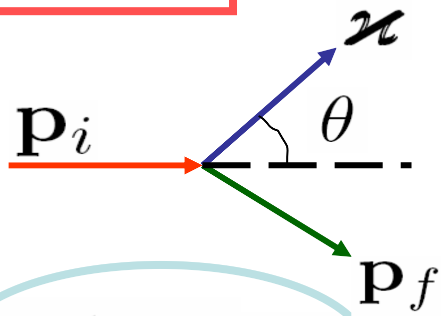
Spin light of neutrino photon's energy

$SL\nu$ transition amplitude after integration :

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}}\, 2\pi\delta(E_f - E_i + \omega) \int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\mathbf{\Gamma}}\mathbf{e}^*)e^{i\mathbf{kr}}\psi_i(\mathbf{r})$$

Energy-momentum conservation

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$



For **electron neutrino** moving in matter composed of **electrons**

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$$

$$\omega = \frac{2\alpha m p_i\left[(E_i - \alpha m) - (p_i + \alpha m)\cos\theta\right]}{(E_i - \alpha m - p_i\cos\theta)^2 - (\alpha m)^2}$$

photon energy

- In the radiation process: $s_i = -1$ ⟶ $s_f = +1$ **neutrino self-polarization**

- For not very high densities of matter , $\tilde{G}_F n/m \ll 1$, in the linear approximation over α

$$\omega = \frac{\beta}{1 - \beta\cos\theta}\omega_0 \quad , \quad \omega_0 = \frac{\tilde{G}_F}{\sqrt{2}}n\beta$$

neutrino speed in vacuum

Spin light transition rate (III)

$\boxed{SL\nu}$ transition rate for different neutrino momentum p
and **matter density parameter** $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$

“relativistic” case

$\boxed{p \gg m}$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^2m, & \text{for } \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^2m^2p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$\boxed{p \ll m}$

neutrino momentum

mass

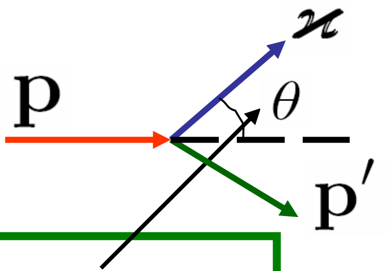
$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^3, & \text{for } \alpha \ll 1, \\ \frac{512}{5}\mu^2\alpha^6p^3, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

neutrino magnetic moment

Spin light radiation power

$SL\nu$

radiation power angular distribution :



$$I = \mu^2 \int_0^\pi \omega^4 \left[(\tilde{\beta}\tilde{\beta}' + 1)(1 - y\cos\theta) - (\tilde{\beta} + \tilde{\beta}')(\cos\theta - y)\right] \frac{\sin\theta}{1 + \tilde{\beta}' y} d\theta$$

$$\tilde{\beta} = \frac{p + \alpha m}{E - \alpha m}, \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}, \quad y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m}, \quad \omega = \frac{2\alpha m p\left[(E - \alpha m) - (p + \alpha m)\cos\theta\right]}{(E - \alpha m - p\cos\theta)^2 - (\alpha m)^2}$$

“relativistic” case

$\boxed{p \gg m}$

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{4}{3}\mu^2\alpha^2 m^2 p^2, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$\boxed{p \ll m}$

$$I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll 1, \\ \frac{1024}{3}\mu^2\alpha^8 p^4, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

Spin light photon average energy

$$\langle\omega\rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

See also:
A.Lobanov,
Phys.Lett.B 619
(2005) 136

"relativistic" case

$$\boxed{p \gg m}$$

$$\langle\omega\rangle \simeq \begin{cases} 2\alpha\frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

"non-relativistic" case

$$\boxed{p \ll m}$$

$$\langle\omega\rangle \simeq \begin{cases} 2p\alpha, & \text{for } \alpha \ll 1, \\ \frac{10}{3}p\alpha^2, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

$\alpha \ll \frac{m}{p}$

$$\omega = 2.37 \times 10^{-7} \left(\frac{n}{10^{30} cm^{-3}}\right) \left(\frac{E}{m_\nu}\right)^2 eV.$$

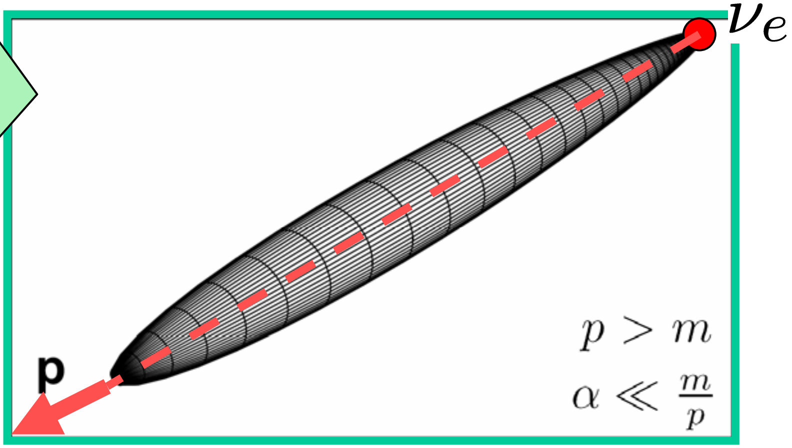
energy range of $\boldsymbol{SL\nu}$

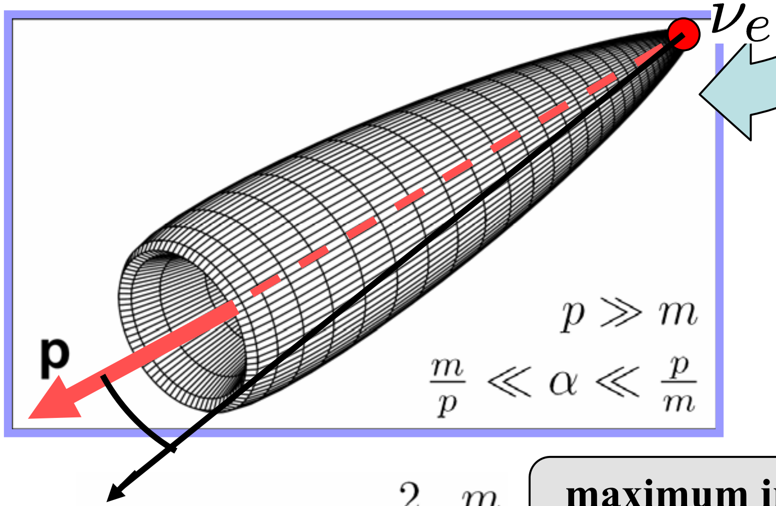
span up to **gamma-rays**

Spatial distribution of radiation power

From the **angular distribution** of $\boxed{SL\nu}$

$$I = \mu^2 \int_0^\pi \omega^4 \left[(\tilde\beta\tilde\beta' + 1)(1 - y\cos\theta) - (\tilde\beta + \tilde\beta')(\cos\theta - y)\right] \frac{\sin\theta}{1 + \tilde\beta' y} d\theta$$





for $p/m = 5$ **and** $\alpha = 0.01$

$\boxed{n \approx 10^{35} cm^{-3}}$

neutrino momentum

mass

matter density

$$\cos\theta_{max} \simeq 1 - \frac{2}{3}\alpha\frac{m}{p}$$

maximum in radiation power distribution

for $p/m = 10^3$ **and** $\alpha = 100$

$\boxed{n \approx 10^{39} cm^{-3}}$

increase of matter density

projector-like distribution → **cap-like distribution**

Propagation of spin light photon in plasma

Only **photons** **with energy that exceeds** **plasmon frequency** can propagate in electron plasma.

$$\omega_{pl} = \sqrt{\frac{4\pi e^2}{m_e} n}$$

$e^2 = \alpha_{QED}$ **fine-structure constant**

The case of relativistic **neutrino** $p \gg m$
and rather dense **plasma** $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$:

$$\omega_{max} \simeq p \quad \Longleftrightarrow \quad \omega(\theta_{max}) \simeq \frac{3}{4}p$$

maximal value of photon's energy

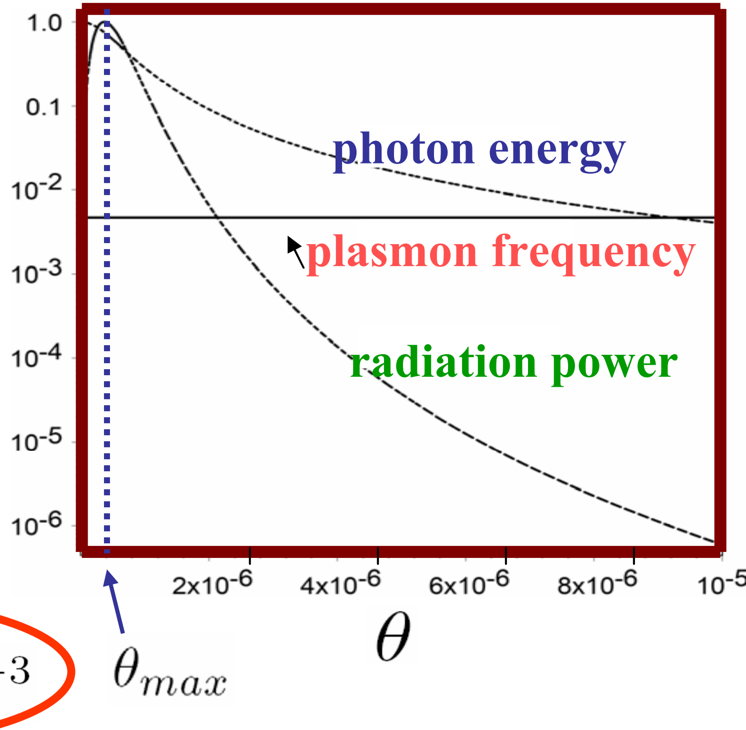
photon's energy in direction of radiation power maximum

$$\cos\theta_{\omega_{max}} = 1 \quad \Longleftrightarrow \quad \cos\theta_{max} \simeq 1 - \frac{2}{3}\alpha\frac{m}{p}$$

$$p \gg p_{min} = 3.5 \times 10^4 \left(\frac{n}{10^{30} cm^{-3}}\right)^{1/2} eV.$$

$p_{min} \sim 1\ MeV$ for $n \sim 10^{33}\ cm^{-3}$

Angular distributions
of **photon energy**
and **radiation power**



Polarization properties of $SL\nu$ photons (I)

Radiation power of **linearly polarized** photons:

$$\boxed{I^{(1),(2)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} \left(\frac{1}{2} S \mp \Delta S \right) \sin\theta d\theta}$$

$$\mathbf{e}_1 = \frac{[\boldsymbol{\varkappa} \times \mathbf{j}]}{\sqrt{1-(\boldsymbol{\varkappa}\mathbf{j})^2}} \, , \mathbf{j} = \frac{\boldsymbol{\beta}}{\beta}$$

$$\mathbf{e}_2 = \frac{\boldsymbol{\varkappa}(\boldsymbol{\varkappa}\mathbf{j}) - \mathbf{j}}{\sqrt{1-(\boldsymbol{\varkappa}\mathbf{j})^2}} \, , \ \mathbf{j} \perp \mathbf{e_1}$$

$$, \quad \mathbf{e}_2 = \{\cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta\}$$

$$\mathbf{e}_1 = \{\sin\phi, -\cos\phi\}$$

where

$$\boxed{S = \frac{(\beta\beta'+1)(1-y\cos\theta) - (\beta+\beta')(\cos\theta - y)\sin\theta}{1+\beta' y}} \quad \text{and} \quad \boxed{\Delta S = \frac{m^2 p \sin^2\theta}{2\,(E'-\alpha m)\,(E-\alpha m)\,p'}} \ .$$

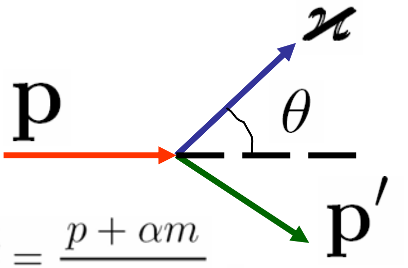
In the limit of **low matter density** $\alpha \ll 1$:

$$\boxed{I^{(1),(2)} \simeq \frac{64}{3}\left(1 \mp \frac{1}{2}\right)\mu^2 p^4 \alpha^4} \longleftarrow SL\nu \text{ is linearly polarized.}$$

In **dense matter** spin light of neutrino is not polarized :

$$\boxed{I^{(1)} \simeq I^{(2)} \simeq \frac{1}{2}(I^{(1)} + I^{(2)})} \ .$$

Polarization properties of $SL\nu$ photons (II)



Radiation power of **circularly polarized** photons:

$$\boxed{I^{(l)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} S_l \sin\theta d\theta} ,$$

$$\tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m} , \quad \tilde{\beta} = \frac{p + \alpha m}{E - \alpha m} ,$$

$$y = \frac{\omega - p\cos\theta}{p'} , \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m} ,$$

$$\omega = \frac{2(E - \alpha m)(K\beta - 1)}{K^2 - 1}$$

where

$$\boxed{S_l = \frac{1}{2}(1 + l\beta')(1 + l\beta)(1 - l\cos\theta)(1 + ly)}$$

$l = \pm 1$ correspond to the photon **right** and **left circular polarizations** .

In the limit of **low matter density** $\alpha \ll 1$:

$$E_0 = \sqrt{p^2 + m^2}$$

$$\boxed{I^{(l)} \simeq \frac{64}{3}\mu^2 \alpha^4 p^4 \left(1 - l\frac{p}{2E_0}\right)} , \quad I^{(+1)} > I^{(-1)} ,$$

however $I^{(+1)} \sim I^{(-1)}$.

In **dense matter** $\left(\alpha \gg \frac{m}{p} \text{ for } p \gg m, \text{ and } \alpha \gg 1 \text{ for } p \ll m\right)$:

$$\boxed{\begin{array}{ccc} I^{(+1)} & \simeq & I \\ I^{(-1)} & \simeq & 0 \end{array}}$$

In a dense matter $SL\nu$ is right-circular polarized.

Summary of $SL\nu$ features

- Dirac ν with **nonzero mass** and **magnetic moment** emits **spin light** when moving in dense matter.

- $\boxed{SL\nu}$ in matter is due to neutrino energy dependence on **matter density**

  (neutrinos of the same momentum but opposite helicities have different energies) .

- In the particular case of ν_e moving in matter composed of electrons, the **matter density** parameter α_{ν_e} is positive →

  the negative-helicity ν_- (the left-handed relativistic ν_L) is converted to the positive-helicity ν_+ (the right-handed relativistic ν_R) , giving rise to **neutrino-spin polarization effect** .

- The **matter density** parameter α can, in general, be negative; therefore the types on initial and final neutrino states, conversion between which effectively produces $\boxed{SL\nu}$, are determined by the **matter composition** and the **type of neutrino.**

(II) Summary of $SL\nu$ features

- In a wide range of **matter density** $\boxed{SL\nu}$ radiation is beamed along the neutrino momentum,
however the actual shape of the radiation spatial distribution may vary form **projector-like** to **cap-like**, depending on the neutrino momentum-to-mass ratio and the **matter density** parameter α .

- In a wide range of **matter density** parameters α the $\boxed{SL\nu}$ radiation is characterized by **total circular polarization**.

- The emitted **photon energy** essentially depends on the neutrino energy and **matter density** ;
the **photon energy** increases from $\omega \sim 2\,p$ to $\omega \sim \alpha m$ with the **density** ;
in the most interesting for astrophysical and cosmological applications case

(when $p \gg m$ and $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$)

● the average **energy** of the emitted photon is $\omega \sim \frac{1}{3} p$;

● in the case of very high **matter density** $\omega \sim \frac{1}{2} E$.

one half of neutrino energy

Experimental identification of $SL\nu$ from astrophysical and cosmological sources

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.**B 622** (2005) 199, **hep-ph/0507200**

- **Fireball model of GRBs**

B.Zhang, P.Meszaros, Int.J.Mod.Phys. **A19** (2004) 2385;
T.Piran, Rev.Mod.Phys. **76** (2004) 1143.

- **Gamma-rays** can be expected to be produced during **collapses** or **coalescence** processes of neutron stars, owing to $\boxed{SL\nu}$ in dense matter.

- Another favorable situation for effective $\boxed{SL\nu}$ production can be realized during

a **neutron star** being **"eaten up"** by the **black hole** at the center of our Galaxy .

For estimation, consider a neutron star with mass $M_{NS} \sim 3M_{\odot}$, $M_{\odot} = 2 \cdot 10^{33} g$

$\Rightarrow$ $n \sim 8 \cdot 10^{38}\ cm^{-3}$, matter density parameter $\alpha \sim 23$,

if $m_{\nu} \sim 0.1\ eV$.

Then for **relativistic neutrinos** $(p \gg m)$

the $\boxed{SL\nu}$ **photon energy** $\langle\omega\rangle \sim \frac{1}{3}p$ $\Leftarrow$ **totally polarized** **gamma-rays.**

Conclusion

ν **exhibits unexpected properties**

W. Pauli, 1930 : *"… I have done a terrible thing –*
I have introduced a particle
that can't be observed …"

- neutron **now we know that it is** **neutrino** *E.Fermi, 1933*
- neutral **now we know that** $\boxed{q_\nu \neq 0}$ **in** **matter** **and** **external fields**

  and probably

- massless **now we know that** $\boxed{\Delta m^2_{12} \neq 0}$

  particle $\boxed{\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta}$

- ν **contrary to our introductory claim is**
  very important player **(astrophysics, cosmology etc. . .)**