Chapter 4

Introduction to Salam-Weinberg-Glashow model

4.1 Neutral weak currents

We are now sufficiently safe with the charged currents (and old version is quite good) but hypothesis about weak isotriplet of W leads to neutral currents in quark sector:

$$j_\mu^{neutr.,ud} = \frac{1}{2}(\bar{u}O_\mu u - \bar{d}_C O_\mu d_C) =$$

$$= \frac{1}{2}(\bar{u}O_\mu u - \bar{d}O_\mu d(cos\theta_C)^2 - \bar{s}O_\mu s(sin\theta_C)^2 -$$

$$\bar{d}O_\mu s cos\theta_C sin\theta_C - \bar{s}O_\mu d cos\theta_C sin\theta_C).$$

and, correspondingly, in lepton sector:

$$j_\mu^{neutr.,lept.} = \frac{1}{2}(\bar{\nu}_e O_\mu \nu_e - \bar{e}O_\mu e) +$$

$$+\frac{1}{2}(\bar{\nu}_\mu O_\mu \nu_\mu - \bar{\mu}O_\mu \mu).$$

(We do not write here explicitly weak neutral operator O_μ as it can diverge finally from the usual charged one $\gamma_\mu(1+\gamma_5)$.) Up to the moment when neutral currents were discouvered experimentally presence of these currents in theory was neither very intriguing nor very disturbing.

But when in 1973 one of the most important events in physics of weak interaction of the 2nd half of the XX century happened – neutral currents were discovered in the interactions of neutrino beams of the CERN machine with the matter, it was become immediately clear the contradiction to solve: although neutral currents interacted with neutral weak boson (to be established yet in those years) with more or less the same coupling as charged currents did with the charged W bosons, there were no neutral strange weak currents which were not much suppressed by Cabibbo angle. Even more: neutral currents written above opened channel of decay of neutral K mesons into $\mu^-\mu^+$ pair with approximately the same coupling as that of the main decay mode of the charged K^- meson (into lepton pair $\mu^-\bar{\nu}_\mu$). Experimentally it is suppressed by 7 orders of magnitude!!!

$$\Gamma(K^0_s \to \mu^-\mu^+)/\Gamma(K^0_s \to all) < 3.2 \times 10^{-7}$$

Once more have we obtained serious troubles with the model of weak interaction!?

In what way, clear and understandable, it is possible to save it? It turns out to be sufficient to remind of the J/ψ particle and its interpretation as a state with the 'hidden' charm ($\bar{c}c$). **New quark with charm would save situation!**

4.1.1 GIM model

Indeed now the number of quarks is 4 but in the weak isodoublet only 3 of them are in action. And if one (Glashow, Iliopoulos, Maiani) assumes that the 4th quark also forms a weak isodoublet, only with the combination of d and s quarks orthogonal to $d_C = dcos\theta_C + ssin\theta_C$, namely, $s_C = scos\theta_C - dsin\theta_C$? Then apart from charged currents

$$j_\mu = \bar{c}\gamma_\mu(1+\gamma_5)s_C$$

neutral currents should exist of the form:

$$j_\mu^{neutr.,cs} = \frac{1}{2}(\bar{c}O_\mu c - \bar{s}_C O_\mu s_C) =$$

$$\frac{1}{2}(\bar{u}O_\mu u - \bar{s}O_\mu s(cos\theta_C)^2 - \bar{d}O_\mu d(sin\theta_C)^2 +$$

$$+\bar{d}O_\mu s cos\theta_C sin\theta_C + \bar{s}O_\mu d cos\theta_C sin\theta_C).$$

The total neutral current yields:

$$j_\mu^{neutr.,ud} = \frac{1}{2}(\bar{u}O_\mu u +$$

$$+\bar{c}O_\mu c - \bar{d}O_\mu d - \bar{s}O_\mu s).$$

There are no strangeness-changing neutral currents at all!

This is so called GIM mechanism proposed in 1970 by Glashow, Iliopoulos, Maiani in order to suppress theoretically decays of neutral kaons already suppressed experimentally. (For this mechanism Nobel price was given!)

4.1.2 Construction of the Salam-Weinberg model

Now we should understand what is the form of the operator O_μ. But this problem is already connected with the problem of a unification of weak and electromagnetic interactions into the electroweak interaction. Indeed the form of the currents in both interactions are remarkably similar to each other. Maybe it would be possible to attach to the neutral weak current the electromagnetic one? It turns out to be possible, and this is the main achievement of the Salam-Weinberg model.

But we cannot add electromagnetic current promptly as it does not contain weak isospin. Instead we are free to introduce one more weak- interacting neutral boson Y_μ ascribing to it properties of weak isosinglet. We shall consider only sector of u and d quarks and put for a moment even $\theta_C = 0$ to simplify discussion.

$$L = g\frac{1}{2}(\bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L)W_{3\mu} +$$

$$+g'(a\bar{u}_L\gamma_\mu u_L + b\bar{u}_R\gamma_\mu u_R + c\bar{d}_L\gamma_\mu d_L + q\bar{d}_R\gamma_\mu d_R)Y_\mu =$$

$$e[\frac{2}{3}(\bar{u}_L\gamma_\mu u_L + \bar{u}_R\gamma_\mu u_R) - \frac{1}{3}(\bar{d}_L\gamma_\mu d_L + \bar{d}_R\gamma_\mu d_R)]A_\mu +$$

$$+\kappa J_\mu^{neutr.,ud} Z_\mu^0.$$

Having two vector boson fields $W_{3\mu}, Y_\mu$ we should transfer to two other boson fields A_μ, Z^0_μ (one of them, namely, A_μ we reserve for electromagnetic field) and take into account that in fact we do know the right form of the electromagnetic current. It would be reasonable to choose orthogonal transformation from one pair of fields to another. Let it be

$$W_{3\mu} = \frac{gZ^0_\mu + g'A_\mu}{\sqrt{g^2+g'^2}}, Y_\mu = \frac{-g'Z^0_\mu + gA_\mu}{\sqrt{g^2+g'^2}}.$$

Substituting these relations into the formula for currents we obtain in the left-hand side (LHS) of the expression for the electromagnetic current the following formula

$$\frac{gg'}{\sqrt{g^2+g'^2}}[(\frac{1}{2}+a)\bar{u}_L\gamma_\mu u_L + b\bar{u}_R\gamma_\mu u_R +$$

$$+(-\frac{1}{2}c\bar{d}_L\gamma_\mu d_L + q\bar{d}_R\gamma_\mu d_R)A_\mu] = eJ^{em}A_\mu,$$

wherefrom

$$a = \frac{1}{6}, \quad b = \frac{2}{3}, \quad c = \frac{1}{6}$$

$$q = -\frac{1}{3}, \quad e = \frac{gg'}{\sqrt{g^2+g'^2}}.$$

Then for the neutral current we obtain

$$\frac{(g^2+g'^2)}{\sqrt{g^2+g'^2}}\frac{1}{2}(\bar{u}_L\gamma_\mu u_L -$$

$$-\bar{d}_L\gamma_\mu d_L)W_{3\mu} - \frac{g'^2}{\sqrt{g^2+g'^2}}J^{em} =$$

$$\frac{\sqrt{g^2+g'^2}}{g}g\frac{1}{2}(\bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L)W_{3\mu} -$$

$$-\frac{\sqrt{g^2+g'^2}}{g}g\frac{g'^2}{g^2+g'^2}J^{em}$$

Let us now introduce notations

$$sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}, \quad cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}.$$

Now neutral vector fields are related by formula

$$W_{3\mu} = cos\theta_W Z^0_\mu + sin\theta_W A_\mu, \quad Y_\mu = -sin\theta_W Z^0_\mu + cos\theta_W A_\mu.$$

Finally weak neutral current in the sector of u and d quarks reads

$$\frac{g}{cos\theta_W}[\frac{1}{2}(\bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L) - sin^2\theta_W J^{em}].$$

Now we repeat these reasonings for the sector of c and s quarks and restore Cabibbo angle arriving at the neutral weak currents in the model with 4 flavors:

$$J_W^{neutr.,GWS} = \frac{g}{cos\theta_W}[\frac{1}{2}(\bar{c}_L\gamma_\mu c_L + \bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L - \bar{s}_L\gamma_\mu s_L) -$$

$$-sin^2\theta_W J^{em}]. \tag{4.1}$$

Remember now that the charged current enters Lagrangian as

$$L = \frac{g_W}{2\sqrt{2}}(\bar{c}\gamma_\mu(1+\gamma_5)s_C W^+_\mu + \bar{u}\gamma_\mu(1+\gamma_5)d_C W^+_\mu +$$

$$+\bar{s}_C\gamma_\mu(1+\gamma_5)cW^-_\mu + \bar{d}_C\gamma_\mu(1+\gamma_5)uW^-_\mu)$$

and in the 2nd order of perturbation theory in ud-sector one would have

$$L^{(2)} = \frac{1}{8}\frac{g_W^2}{(M_W^2+q^2)}\bar{u}\gamma_\mu(1+\gamma_5)d_C\bar{d}_C\gamma_\mu(1+\gamma_5)u + H.C.,$$

what should be compared to

$$L^{eff} = \frac{G_F}{\sqrt{2}}\bar{u}\gamma_\mu(1+\gamma_5)d_C\bar{d}_C\gamma_\mu(1+\gamma_5)u + H.C.$$

Upon neglecting square of momentum transfer q^2 in comparing to the W-boson mass one has

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} =$$

$$= \frac{e^2}{8M_W^2 sin^2\theta_W},$$

wherefrom

$$M_W^2 \leq \frac{\sqrt{2}e^2}{8G_F} = \frac{\sqrt{2}4\pi\alpha}{8G_F} \sim 1200GeV^2$$

that is

$$M_W \geq 35GeV!!!$$

(Nothing similar happened earlier!)

Measurements of the neutral weak currents give the value of Weinberg angle as $sin^2\theta_W = 0,2311 \pm 0,0003$. But in this case the prediction becomes absolutely definite: $M_W = 73GeV$. As is known the vector intermediate boson W was discouvered at the mass $80,22 \pm 0,26$ГэВ which agree with the prediction as one must increase it by ∼10% due to large radiative corrections.

4.1.3 Six quark model and CKM matrix

But nowadays we have 6 and not 4 quark flavors. So we have to assume that there is a mix not of two flavors (d and s) but of all 3 ones (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

This very hypothesis has been proposed by Kobayashi and Maskawa in 1973. The problem is to mix flavors in such a way as to guarantee disappearence of neutral flavor-changing currents. Diagonal character of neutral current is achieved by choosing of the orthogonal matrix V_{CKM} of the flavor transformations for the quarks of the charge -1/3.

Even more it occurs that it is now possible to introduce a phase in order to describe violation of CP-invariance (with number of flavours less then 3 one can surely introduce an extra phase but it could be hidden into the irrelevant phase factor of one of the quark wave functions). Usually Cabibbo-Kobayashi-Maskawa matrix is chosen as

$$V_{CKM} =$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \quad (4.2)$$

Here $c_{ij} = cos\theta_{ij}, s_{ij} = sin\theta_{ij}, (i,j = 1,2,3)$, while θ_{ij} - generalized Cabibbo angles. At $\theta_{23} = 0, \theta_{13} = 0$ one returns to the usual Cabibbo angle $\theta_C = \theta_{12}$. Let us write matrix V_{CKM} with the help of Eqs. (1,5,7) as

$$V_{CKM} = R_1(\theta_{23})D^*(e^{i\delta_{13}/2})R_2(\theta_{13})D(e^{i\delta_{13}/2})R_3(\theta_{12}) =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta_{23} & sin\theta_{23} \\ 0 & -sin\theta_{23} & cos\theta_{23} \end{pmatrix} \begin{pmatrix} e^{-i\delta_{13}/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_{13}/2} \end{pmatrix} \times \tag{4.3}$$

$$\begin{pmatrix} cos\theta_{13} & 0 & sin\theta_{13} \\ 0 & 1 & 0 \\ -sin\theta_{13} & 0 & cos\theta_{13} \end{pmatrix} \begin{pmatrix} e^{i\delta_{13}/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}/2} \end{pmatrix} \times$$

$$\begin{pmatrix} cos\theta_{12} & sin\theta_{12} & 0 \\ -sin\theta_{12} & cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Matrix elements are obtained from experiments with more and more precision. Dynamics of experimental progress could be seen from these two matrices divided by 10 years in time:

$$V_{CKM}^{(1990)} =$$

$$= \begin{pmatrix} 0.9747 \ to \ 0.9759 & 0.218 \ to \ 0.224 & 0.001 \ to \ 0.007 \\ 0.218 \ to \ 0.224 & 0.9734 \ to \ 0.9752 & 0.030 \ to \ 0.058 \\ 0.03 \ to \ 0.019 & 0.029 \ to \ 0.058 & 0.9983 \ to \ 0.9996 \end{pmatrix}. \tag{4.4}$$

$$V_{CKM}^{(2000)} =$$

$$= \begin{pmatrix} 0.9742 \ to \ 0.9757 & 0.219 \ to \ 0.226 & 0.002 \ to \ 0.005 \\ 0.219 \ to \ 0.225 & 0.9734 \ to \ 0.9749 & 0.037 \ to \ 0.043 \\ 0.04 \ to \ 0.014 & 0.035 \ to \ 0.043 & 0.9990 \ to \ 0.9993 \end{pmatrix}. \tag{4.5}$$

The charged weak current could be written as

$$J_W^- = (\bar{u}, \quad \bar{c}, \quad \bar{t})\gamma_\mu(1+\gamma_5)V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Neutral current would be the following in the standard 6-quark model of Salam-Weinberg:

$$\frac{g}{\sqrt{cos\theta_W}} J_W^{\text{нейтр.6}} =$$

$$\frac{g}{\sqrt{cos\theta_W}}[\frac{1}{2}(\bar{t}_L\gamma_\mu t_L + \bar{c}_L\gamma_\mu c_L + \bar{u}_L\gamma_\mu u_L -$$

$$\bar{d}_L\gamma_\mu d_L - \bar{s}_L\gamma_\mu s_L - \bar{b}_L\gamma_\mu b_L) - sin^2\theta_W J^{em}].$$

4.2 Vector bosons W and Y as gauge fields

Bosons W and Y could be introduced as gauge fields to assure renormalization of the theory of electroweak interactions. We are acquainted with the method of construction of Lagrangians invariant under local gauge transformations on examples of electromagnetic field and isotriplet of the massless ρ-meson fields.

We have introduced also the notion of weak isospin, so now we require local gauge invariance of the Lagrangian of the left-handed and right-handed quark (and lepton) fields under transformations in the weak isotopic space with the group $SU(2)_L \times SU(1)$.

But as we consider left- and right- components of quarks (and leptons) apart we put for a moment all the quark (and lepton) masses equal to zero.

For our purpose it is sufficient to write an expression for one left-handed isodoublet and corresponding right-handed weak isosinglets u_R, d_R:

$$L_0 = \bar{q}_L(x)\partial_\mu\gamma_\mu q_L(x) + \bar{u}_R(x)\partial_\mu\gamma_\mu u_R(x) + \bar{d}_R(x)\partial_\mu\gamma_\mu d_R(x)$$

This Lagrangian is invariant under a global gauge transformation

$$q'_L(x) = e^{i\vec{\alpha}\vec{\tau}} q_L(x),$$

$$u'_{R,L}(x) = e^{i\beta_{R,L}} u_{R,L}(x),$$

$$d'_{R,L}(x) = e^{i\beta'_{R,L}} d_{R,L}(x),$$

where matrices $\vec{\tau}$ act in weak isotopic space and $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$,$\beta_{R,L}$, $\beta'_{R,L}$ are arbitrary real phases.

Let us require now invariance of this Lagrangian under similar but local gauge transformations when $\vec{\alpha}$ and $\beta_{R,L}, \beta'_{R,L}$ are functions of x. As it has been previously L_0 is not invariant under such local gauge transformations:

$$L'_0 = L_0 + i\bar{q}_L(x)\gamma_\mu \frac{\partial \vec{\tau}\vec{\alpha}(x)}{\partial x_\mu} q_L(x) +$$

$$+ i\bar{u}_R\gamma_\mu \frac{\partial \beta_R(x)}{\partial x_\mu} u_R + i\bar{d}_R\gamma_\mu \frac{\partial \beta'_R(x)}{\partial x_\mu} d_R$$
$$+ i\bar{u}_L\gamma_\mu \frac{\partial \beta_L(x)}{\partial x_\mu} u_L + i\bar{d}_L\gamma_\mu \frac{\partial \beta'_L(x)}{\partial x_\mu} d_L.$$

In order to cancel terms violating local gauge invariance let us introduce weak isotriplet of vector fields $\vec{W}_\mu$ and also weak isosinglet Y_μ with the gauge transformations

$$\vec{\tau}\vec{W}'_\mu = U^\dagger \vec{\tau}\vec{W}_\mu U - \frac{1}{g_W}\frac{\partial U}{\partial x_\mu} U^\dagger$$

где $U = e^{i\vec{\alpha}(x)\vec{\tau}}$;

$$Y'_\mu = Y_\mu - \frac{1}{g_Y}\frac{\partial(\beta_R + \beta'_R + \beta_L + \beta'_L)}{\partial x_\mu}$$

Interactions of these fields with quarks could be defined by the Lagrangian constructed above

$$L = \frac{1}{\sqrt{2}} g(\bar{u}_L\gamma_\mu d_L W^+_\mu + \bar{d}_L\gamma_\mu u_L W^-_\mu) + g\frac{1}{2}(\bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L)W_{3\mu} +$$

$$g'(a\bar{u}_L\gamma_\mu u_L + b\bar{u}_R\gamma_\mu u_R + c\bar{d}_L\gamma_\mu d_L + q\bar{d}_R\gamma_\mu d_R)Y_\mu =$$

Thus the requirement of invariance of the Lagrangian under local gauge transformations along the group $SU(2)_L \times SU(1)$ yields appearence of four massless vector fields $\vec{W}, Y$.

Earlier it has already been demonstrated in what way neutral fields $W_{3\mu}$, Y_μ by an orthogonal transformation can be transformed into the fields Z_μ, A_μ. After that one needs some mechanism (called mechanism of spontaneous breaking of gauge symmetry) in order to give masses to $W^\pm, Z$ and to leave the field A_μ massless. Usually it is achieved by so called Higgs mechanisn. Finally repeating discussion for all other flavours we come to the already obtained formulae for the charged and neutral weak currents but already in the gauge-invariant symmetry with spontaneous breaking of gauge symmetry.

4.3 About Higgs mechanism

Because of short of time we could not show Higgs mechanism in detail as an accepted way of introducing of massive vector intermediate bosons $W^{\pm}, Z^0$ into the theory of Glashow-Salam-Weinberg. We give only short introduction into the subject.

Let us introduce first scalar fields ϕ with the Lagrangian

$$L_\phi = T - V = \partial_\mu \phi \partial_\mu \phi + \mu^2 \phi^2 + \lambda \phi^4. \tag{4.6}$$

Let us look at V just as at ordinary function of a parameter ϕ and search for the minimum of the potential $V(\phi)$.

$$\frac{dV(\phi)}{d\phi} = -2\mu^2 \phi - 4\lambda \phi^3 = 0.$$

We have 3 solutions:

$$\phi^1_{min} = 0,$$

$$\phi^{2,3}_{min} = \pm\sqrt{-\frac{\mu^2}{\lambda}}.$$

That is, with $\mu^2 \langle 0$ we would have two minima (or vacuum states) not at the zero point! How to understand this fact? Let us find some telegraph mast and cut all the cords which help to maintain it in vertical state. For a while it happens nothing. But suddenly we would see that it is falling. And maybe directly to us. What should be our last thought? That this is indeed a spontaneous breaking of symmetry!

This example shows to us not only a sort of vanity of our existence but also the way to follow in searching for non-zero masses of the weak vector bosons $W^{\pm}, Z$.

By introducing some scalar field ϕ with nonzero vacuum expectation value (v.e.v.) $< \phi >= v$ it is possible to construct in a gauge-invariant way the interaction of this scalar field with the vector bosons $W^{\pm}, W^3, Y$ having at that moment zero masses. This interaction is bilinear in scalar field and bilinear in fields $W^{\pm}, W^3, Y$ that is it contains terms of the kind $\phi^2 |W^{\pm}_\mu|^2$. At this moment the whole Lagrangian is locally gauge invariant which assures its renormalization. Changing scalar field ϕ to scalar field χ with the v.e.v. equal to zero $\chi = \phi - < \phi >, \quad < \chi >= 0$ we break spontaneously local gauge invariance of the whole Lagrangian but instead we obtain terms of the kind $v^2|W_\mu^\pm|^2$ which are immediately associated with the mass terms of the vector bosons $W_\mu^\pm$. A similar discussion is valid for Z boson.

Now we shall show this "miracle" step by step.

Firs let us introduce weak isodoublet of complex scalar fields

$$L_\phi = \partial_\mu \phi^* \partial_\mu \phi + \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2. \tag{4.7}$$

where ϕ is a doublet, $\phi^T = (\phi^+ \quad \phi^0)$ with non-zero vacuum value, $\langle \phi \rangle = v \neq 0$.

It is invariant under global gauge transformations

$$\phi' = U\phi = e^{i\vec{\alpha}\vec{\tau}}\phi.$$

Now let us as usual require invariance of the Lagrangian under the local gauge transformations. The Lagrangian Eq.(4.7) however is invariant only in the part without derivatives. So let us study "kinetic" part of it. With $\vec{\alpha}(x)$ dependent on x we have

$$\partial_\mu \phi' = U \partial_\mu \phi + \partial_\mu U \phi,$$

it becomes

$$\partial_\mu \phi^{*\prime} \partial_\mu \phi' = (\phi^* \partial_\mu U \dagger + \partial_\mu \phi^* U^\dagger)(\phi \partial_\mu U + \partial_\mu \phi \cdot U) =$$

$$= \partial_\mu \phi^* (U^\dagger U) \partial_\mu \phi + \phi^* (\partial_\mu (U^\dagger \partial_\mu U)\phi +$$
$$\phi^* (\partial_\mu U^\dagger) U \partial_\mu \phi + \phi^* U^\dagger (\partial_\mu U)\phi.$$

Let us introduce as already known remedy massless isotriplet of vector mesons $\vec{W}$ with the gauge transformation already proposed

$$\vec{\tau}\vec{W}_\mu' = U\vec{\tau}\vec{W}U^\dagger - \frac{1}{g_W}(\partial_\mu U)U^\dagger,$$

but with two interaction Lagrangians transforming under local gauge transformations as:

$$\phi^{*\prime} \tau \vec{W}_\mu^{\dagger\prime} \tau \vec{W}_\mu' \phi' = \phi^* \tau \vec{W}_\mu^\dagger \tau \vec{W}_\mu \phi +$$

$$\phi^* U^\dagger U \tau \vec{W}_\mu^\dagger U^\dagger (\partial_\mu U)\phi + \phi^* U^\dagger \partial_\mu \tau \vec{W}_\mu U^\dagger U \phi +$$

$$\phi^* \partial_\mu U^\dagger (\partial_\mu U) \phi,$$

and

$$\phi^{*\prime} \tau \vec{W}'_\mu \partial_\mu \phi' = \phi^* \tau \vec{W}_\mu \partial_\mu \phi + \phi^* \tau \vec{W}_\mu U^\dagger (\partial_\mu U) \phi.$$

The sum of all the terms results in invariance of the new Lagrangian with two introduced interaction terms under the local gauge transformations.

We can do it in a shorter way by stating that

$$(\partial_\mu + ig\tau \vec{W}'_\mu)\phi' =$$

$$(U\partial_\mu + (\partial_\mu U) + igU\tau \vec{W}_\mu U^\dagger U + (\partial_\mu U)U^\dagger)\phi =$$

$$U(\partial_\mu + ig\tau \vec{W}_\mu)\phi.$$

Then it is obvious that the Lagrangian

$$(\partial_\mu - ig\tau \vec{W}^\dagger_\mu)\phi^* (\partial_\mu + ig\tau \vec{W}_\mu)\phi$$

is invariant under the local gauge transformations.

In the same way but with less difficulties we can obtain the Lagrangian invariant under the local gauge transformation of the kind

$$\phi' = e^{i\beta}\phi$$

that is under Abelian transformations of the type use for photon previously:

$$(\partial_\mu - ig' Y_\mu)\phi^* (\partial_\mu + ig' Y \mu).\phi$$

But our aim is to obtain masses of the vector bosons. It is in fact already achieved with terms of the kind $\phi^2 |W_\mu^\pm|^2$ and $\phi^2 Y_\mu^2$. Now we should also assure that our efforts are not in vain. That is searching for weak boson masses we should maintain zero for that of the photon. It is sufficient to propose the Lagrangian

$$(\partial_\mu - ig\tau \vec{W}^\dagger_\mu + ig' Y_\mu)\phi^* (\partial_\mu + ig\tau \vec{W}_\mu - ig' Y_\mu)\phi,$$

where $g^2 |W_\mu^\pm|^2 \phi^* \phi$ terms would yield with $\phi = \chi + v$ masses of $W^\pm$ bosons $M_W = v \cdot g$, while term $|gW_\mu^3 - g' Y_\mu|^2 \phi^* \phi$ would yield mass of the Z^0 boson

$$M_Z = v \cdot \sqrt{g^2 + g'^2} = v \cdot g \frac{\sqrt{g^2 + g'^2}}{g} = \frac{M_W}{cos\theta_W}.$$

There is a net prediction that the ratio M_W/M_Z is equal to $cos\theta_W$. Experimentally this ratio is (omitting errors)$\sim$ 80/91 which gives the value of Weinberg angle as $sin^2\theta_W \sim 0.23$ in agreement with experiments on neutrino scattering on protons. Due to the construction there is no term $|g'W_\mu^3 + gY_\mu|^2\phi^*\phi$ that is photon does not acquire the mass!

Talking of weak bosons and scalar Higgs mesons we omit one important point that is in the previous Lagrangians dealing with fermions we should put all the fermion masses equal to zero! Why?

It is because we use different left-hand-helicity and right-hand-helicity gauge transformations under which the mass terms are not invariant as

$$m_q\bar{q}q = m_q\bar{q}_L q_R + m_q\bar{q}_R q_L,$$

$$q_L = \frac{1}{2}(1+\gamma_5)q, \quad q_R = \frac{1}{2}(1-\gamma_5)q.$$

What is the remedy for fermion masses? Again we could use Higgs bosons. In fact, interaction Lagrangian of quarks (similar for leptons) with the same scalar field ϕ, $\phi^T = (\pi^+ \quad \phi^0)$ with non-zero vacuum value, $\langle\phi\rangle = v \neq 0$, can be written as

$$L_{dm} = \lambda_d \bar{q}_L \phi d_R + HC = \lambda_d(\bar{u}_L\phi^+ d + \bar{d}_L\phi^0 d_R) =$$

$$\lambda_d(\bar{u}_L\chi^+ d + \bar{d}_L\chi^0 d_R + m_d\bar{d}_L d_R)$$

with $m_d = \lambda_d v$.

(In a similar way lepton masses are introduced:

$$\lambda_l(\bar{\nu}_L^l\chi^+ l + \bar{l}_L\chi^0 l_R + m_l\bar{l}_L l_R)$$

with $m_l = \lambda_l v$.)

So we could obtain now within Higgs mechanism all the masses of weak bosons and of all fermions either quarks or leptons.

By this note we finish our introduction into the Salam-Weinberg model in quark sector and begin a discussion on colour.