Chapter 5

Colour and gluons

5.1 Colour and its appearence in particle physics

Hypothesis of colour has been the beginning of creation of the modern theory of strong interaction that is quantum chromodynamics. We discuss in the beginning several experimental facts which have forced physicists to accept an idea of existence of gluons - quanta of colour field.

1) **Problem of statistics for states** uuu, ddd, sss **with** $J^P = \frac{3}{2}^+$

As it is known fermion behaviour follows Fermi-Dirac statistics and because of that a total wave function of a system describing half-integer spin should be antisymmetric. But in quark model quarks forming resonances $\Delta^{++} = (uuu)$, $\Delta^- = (ddd)$ and the particle $\Omega^- = (sss)$ should be in symmetric S states either in spin or isospin spaces which is prohibited by Pauli principle. One can obviously renounce from Fermi-Dirac statistics for quarks, introduce some kind of 'parastatistics' and so on. (All this is very similar to some kind of 'parapsychology' but physicists are mostly very rational people.) So it is reasonable to try to maintain fundamental views and principles and save situation by just inventing new 'colour' space external to space-time and to unitary space (which include isotopic one). As one should antisymmetrize qqq and we have the simplest absolutely antisymmetric tensor of the 3rd rank ϵ_{abc} which (as we already know) transforms as singlet representation of the group $SU(3)$ the reduction $\epsilon_{abc}q^a q^b q^c$ would be a scalar of $SU(3)$ in new quantum number called 'colour' (here $a, b, c = 1, 2, 3$ are colour indices and have no relation to previous unitary indices in mass formulae, currents and so on!) Thus the fermion statistics is saved and there is no new quantum number (like strangeness or isospin) for ordinary baryons in accord with the experimental data. But quarks become coloured and number of them is tripled. Let it be as we do not observe them on experiment.

2) **Problem of the mean life of π^0 meson**

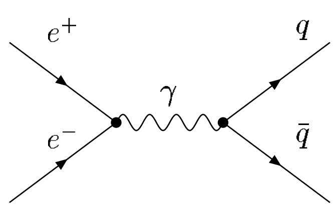
We have already mentioned that a simple model of π^0 meson decay based on Feynman diagram with nucleon loop gives very good agreement with experiment though it looks strange. Nucleon mass squared enters the denominator in the integral over the loop. Because of that taking now quark model (transfer from $m_N = 0.940$ GeV to $m_u = 0.300 GeV$ of the constituent quark) we would have an extra factor ~ 10! In other words quark model result would give strong divergence with the experimental data. How is it possible to save situation? Triplicate number of quark diagrams by introducing 'colour'!!! Really as $3^2 = 9$ the situation is saved.

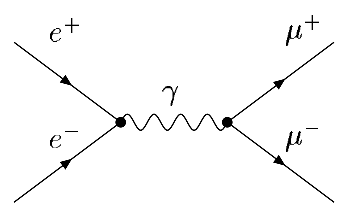
3) **Problem with the hadronic production cross-section in e^+e^- annihilation**

Let us consider the ratio of the hadronic production cross-section of e^+e^- annihilation to the well-known cross section of the muon production in e^+e^- annihilation:

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

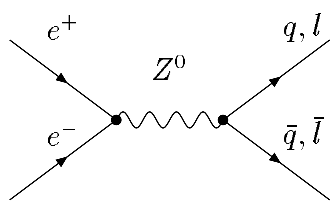
It is seen from the Feynman diagrams in the lowest order in α





that the corresponding processes upon neglecting 'hadronization' of quarks are described by similar diagrams. The difference lies in different charges of electrons(positrons) and quarks. In a simple quark model with the pointlike quarks the ratio R is given just by the sum of quark charges squared that is for the energy interval of the electron-positron rings up to 2-3 GeV it should be $R = (\frac{2}{3})^2 + (-\frac{1}{3})^2 + (-\frac{1}{3})^2) = \frac{2}{3}$. However experiment gives in this interval the value around 2.0. As one can see, many things could be hidden into the not so understandable 'hadronization' process (we observe finally not quarks but hadrons!). But the simplest way to do has been again triplication of the number of quarks, then one obtains the needed value: $3 \times \frac{2}{3} = 2$.

With the 'discouvery' of the charmed quark we should recalculate the value of R for energies higher then thresholds of pair productions of charm particles that is for energies $\geq$ 3 GeV, $R = 3 \times [(\frac{2}{3})^2 + (-\frac{1}{3})^2 + (-\frac{1}{3}^2) + \frac{2}{3})^2)] = \frac{10}{3}$. Production of the pair of ($\bar{b}b$) quarks should increase the value R by 1/3 which proves to hardly note experimentally. Experiment gives above 4 GeV and p to e^+e^- energies around 35-40 GeV the value $\simeq$4. At higher energies influence on R of the Z boson contribution (see diagram) is already seen



Thus introduction of three colours can help to escape several important and even fundamental contradictions in particle physics.

But dynamic theory appears only there arrives quant of the field (gluon) transferring colour from one quark to another and this quant in some way acts on experimental detectors. Otherwise everything could be finished at the level of more or less good classification as it succeeded with isospin and hyper charge with no corresponding quanta)

There is assumption that dynamical theories are closely related to local gauge invariance of Lagrangian describing fields and its interactions with respect to well defined gauge groups.

5.1.1 Gluon as a gauge field

Similar to cases considered above with photon and ρ meson let us write a Lagrangian for free fields of 3-couloured quarks q^a where quark $q^a, a = 1, 2, 3$

is a 3-spinor of the group $SU(3)_C$ in colour space;

$$L_0 = \bar{q}_a(x)\partial_\mu\gamma_\mu q^a(x) - m_q\bar{q}_a(x)q^a(x).$$

This Lagrangian is invariant under global gauge transformation

$$q'^a(x) = e^{i(\alpha^k\lambda^k)^a_b}q^b(x)$$

, where $\lambda^k k = 1, ...8$, are known to us Gell-Mann matrices but now in colour space. Let us require invariance of the Lagrangian under similar but local gauge transformation when α^k are functions of x:

$$q'^a(x) = e^{i(\alpha^k(x)\lambda^k)^a_b}q^b(x),$$

or

$$q'(x) = U(x)q(x), \quad U(x) = e^{i\alpha^k(x)\lambda^k}.$$

But exactly as in previous case L_0 is not invariant under this local gauge transformation

$$L'_0 = \bar{q}_a(x)\partial_\mu\gamma_\mu q^a(x) + \bar{q}_a(x)(U(x)\gamma_\mu\partial_\mu U(x))^a_b(x)q^b(x)+$$

$$+m_q\bar{q}_a(x)q^a(x).$$

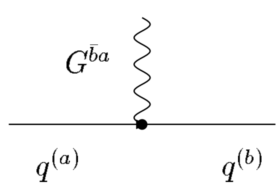
In order to cancel terms breaking gauge invariance let us introduce massless vector fields $G^k_\mu, k = 1, ...8$, with the gauge transformation

$$\lambda^k G'^k = U\lambda^k G^k U^\dagger - \frac{1}{g_s}\frac{\partial U}{\partial x_\mu}U^\dagger.$$

Let us define interaction of these fields with quarks by Lagrangian

$$L = g_s\bar{q}_a(x)(G^k_\mu\lambda_k)^a_b\gamma_\mu q^b(x),$$

to which corresponds Feynman graphs



$$G = \frac{1}{\sqrt{2}} \begin{pmatrix} G_3 + 1/\sqrt{3}G_8 & G_1 + iG_2 & G_4 + iG_5 \\ G_1 - iG_2 & -G_3 + 1/\sqrt{3}G_8 & G_6 + iG_7 \\ G_4 - iG_5 & G_6 - iG_7 & -2/\sqrt{3}G_8 \end{pmatrix} =$$

$$= \begin{pmatrix} \mathcal{D}_1 & G^{\bar{2}1} & G^{\bar{3}1} \\ G^{\bar{1}2} & \mathcal{D}_2 & G^{\bar{2}3} \\ G^{\bar{1}3} & G^{\bar{2}3} & \mathcal{D}_3 \end{pmatrix}, \qquad (5.1)$$

where

$$\mathcal{D}_1 = \frac{1}{2}(G^{\bar{1}1} - G^{\bar{2}2}) + \frac{1}{6}(G^{\bar{1}1} + G^{\bar{2}2} - 2G^{\bar{3}3});$$

$$\mathcal{D}_2 = -\frac{1}{2}(G^{\bar{1}1} - G^{\bar{2}2}) + \frac{1}{6}(G^{\bar{1}1} + G^{\bar{2}2} - 2G^{\bar{3}3});$$

$$-\frac{2}{6}(G^{\bar{1}1} + G^{\bar{2}2} - 2G^{\bar{3}3}).$$

Final expression for the Lagrangian invariant under local gauge transformations of the non-abelian group $SU(3)_C$ in colour space is:

$$L_{SU(3)_C} = \bar{q}_a(x)\partial_\mu\gamma_\mu q^a(x) + m_q\bar{q}_a(x)q^a(x) +$$

$$g_s\bar{q}_a(x)(G^k_\mu\lambda_k)^a_b\gamma_\mu q^b(x) - \frac{1}{4}\vec{F}^{k\mu\nu}F^k_{\mu\nu},$$

where $F^{k\mu\nu}, k = 1, 2, ...8$, is the tensor of the free gluon field transforming under local gauge transformation as

$$\lambda_k F^{k\prime}_{\mu\nu} = U^\dagger(x)\lambda_l(x)F^l_{\mu\nu}U(x).$$

It is covariant under gauge transformations $U^\dagger\vec{F}'_{\mu\nu}U = \vec{F}_{\mu\nu}$. Let us write in some detail tensor of the free gluon field $\vec{\lambda}\vec{F}_{\mu\nu} = \lambda_k F^k_{\mu\nu} \equiv \tilde{F}_{\mu\nu}, \quad ([\lambda_i, \lambda_j] = 2i\epsilon^{ijk}\lambda_k, \quad i, j, k = 1, 2, ...8)$:

$$\vec{F}^k_{\mu\nu} = (\partial_\nu G^k_\mu - \partial_\mu G^k_\nu) - 2g_s i f^{kij} G^i_\mu G^j_\nu$$

or

$$\tilde{F}_{\mu\nu} = (\partial_\nu\tilde{G}_\mu - \partial_\mu\tilde{G}_\nu) - g_s[\tilde{G}_\mu, \tilde{G}_\nu]$$

and prove that this expression in a covariant way transforms under gauge transformation of the field G:

$$U^\dagger(\partial_\nu \tilde{G}'_\mu - \partial_\mu \tilde{G}'_\nu)U =$$

$$= (\partial_\nu \tilde{G}_\mu - \partial_\mu \tilde{G}_\nu) + [U^\dagger \partial_\nu U, \tilde{G}_\mu] - [U^\dagger \partial_\mu U, \tilde{G}_\nu],$$

$$U^\dagger [\tilde{G}'_\mu, \tilde{G}'_\nu] U = [\tilde{G}_\mu, \tilde{G}_\nu] + \frac{1}{g_s}[U^\dagger \partial_\nu U, \tilde{G}_\mu] - \frac{1}{g_s}[U^\dagger \partial_\mu U, \tilde{G}_\nu].$$

Finally

$$U^\dagger \vec{F}'_{\mu\nu} U = U^\dagger(\partial_\nu \tilde{G}'_\mu - \partial_\mu \tilde{G}'_\nu - g_s[\tilde{G}'_\mu, \tilde{G}'_\nu])U =$$

$$= \partial_\nu \tilde{G}_\mu - \partial_\mu \tilde{G}_\nu + g_s[\tilde{G}_\mu, \tilde{G}_\nu] = \vec{F}_{\mu\nu}.$$

The particular property of non-Abelian vector field as we have already seen on the example of the ρ field is the fact that this field is autointeracting that is in the Lagrangian in the free term $(-1/4)|\vec{F}^{\mu\nu}|^2$ there are not only terms bilinear in the field G as it is in the case of the (Abelian) electromagnetic field but also 3-and 4- linear terms in gluon field G of the form $G_\nu G_\mu \partial_\nu G_\mu$ and $G_\nu^2 G_\mu^2$ to which the following Feynman diagrams correspond:

This circumstance turns to be decisive for construction of the non-Abelian theory of strong interaction - quantum chromodynamics.

The base of it is the asymptotic freedom which can be understood from the behaviour of the effective strong coupling constant of quarks and gluons $\alpha_s = g_s^2/4\pi$ for which

$$\alpha_s(Q^2) \sim \frac{\alpha_s \mu^2}{1 + (11N_C - 2n_f)\ln(Q^2/\Lambda^2)},$$

where Q^2 is momentum transfer squared, μ^2 is a renormalization point, Λ is a QCD scale parameter, N_C being number of colors and n_f number of flavours. With Q^2 going to infinity coupling constant α_s tends to zero! Just this property is called asymptotic freedom.(Instead in QED (quantum electrodynamics) with no colors it grows and even have a pole.) But one should also have in mind that in the difference from QED where we have two observable quantities electron mass and its charge (or those of μ- and τ-leptons) in QCD we have none. Indeed we could not measure directly either quark mass or its coupling to gluon.

Here we shall not discuss problems of the QCD and shall give only some examples of application of the notion of colour to observable processes.

5.1.2 Simple examples with coloured quarks

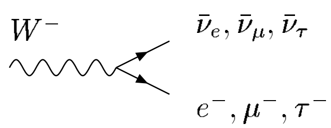
By introducing colour we have obtained possibility to predict ratios of many modes of decays and to prove once more validity of the hypothesis on existence of colour.

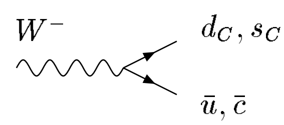
Let us consider decay modes of lepton τ discouvered practically after J/ψ which has the mass ~1800 MeV (exp.(1777,1 +0,4-0,5) MeV). Taking quark model and assuming pointlike quarks (that is fundamental at the level of leptons) we obtain that τ^- lepton decays emitting τ neutrino ν_τ either to lepton (two channels, $e^-\bar{\nu}_e$ or $\mu^-\bar{\nu}_\mu$) or to quarks (charm quark is too heavy, strange quark contribution is suppressed by Cabibbo angle and we are left with u and d quarks).

From our reasoning it follows that in absence of color we have two lepton modes and only one quark mode and partial hadron width B_h should be equal to 1/3 of the total width while with colour quarks we have two lepton modes and three quark modes leading to $B_h \sim 3/5$.

Or in other words definite lepton mode would be 33 % in absence of colour and 20 % with the colour. Experiment gives $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau = (17,37 \pm 0,09)\%$ and $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau = (17,81 \pm 0,07)\%$, supporting hypothesis of 3 colours.

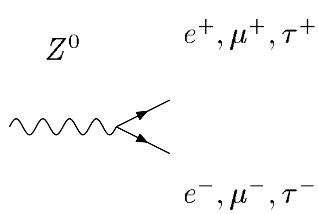
The W decays already in three lepton pairs and two quark ones

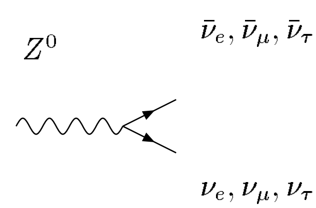


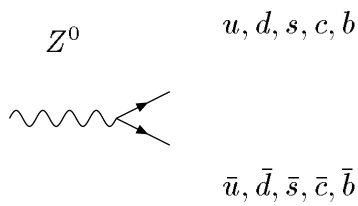


This means that in absence of colour hadron branching ratio B_h would be 40% while with three colours around 66%. Experiment gives $B_h \sim (67,8 \pm 1,5)\%$ once more supporting hypothesis of 3-coloured quarks.

Z boson decays already along 6 lepton and 5 quark modes,







Thus it is possible to predict at once that in absence of colour hadron channel should be $5/11 \simeq 45\%$ of the total width of Z while with three colours number of partial lepton and colour quark channels increase up to $6+3\times5=21$, and hadron channel would be $15/21 \simeq 71\%$. Experimentally it is $(69.89 \pm 0.07\%)$.

Chapter 6

Conclusion

In these chosen chapters on group theory and its application to the particle physics there have been considered problems of classification of the particles along irreducible representations of the unitary groups, have been studied in some detail quark model. In detail mass formulae for elementary particles have been analyzed. Examples of calculations of the magnetic moments and axial-vector weak constants have been exposed in unitary symmetry and quark model. Formulae for electromagnetic and weak currents are given for both models and problem of neutral currents is given in some detail. Electroweak current of the Glashow-Salam-Weinberg model has been constructed. The notion of colour has been introduced and simple examples with it are given. Introduction of vector bosons as gauge fields are explained.

Author has tried to write lectures in such a way as to give possibility to eventual reader to evaluate by him- or herself many properties of the elementary particles.